

AD-A168 013

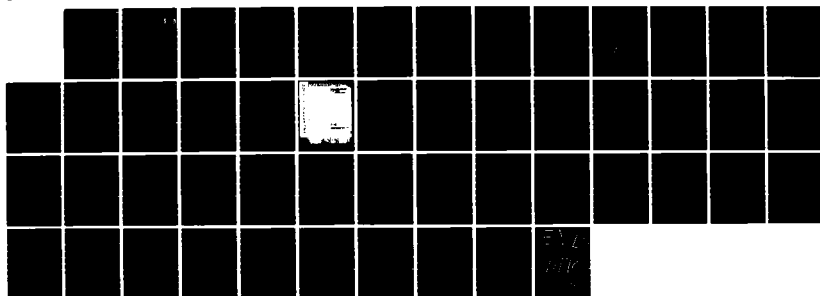
FREE SURFACE TURBULENCE(U) MICHIGAN TECHNOLOGICAL UNIV
HOUGHTON DEPT OF CHEMISTRY AND CHEMICAL ENGINEERING
D W HUBBARD ET AL. 28 FEB 86 N00014-85-K-0236

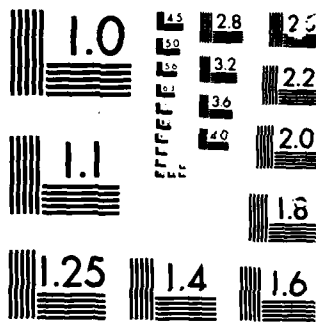
1/1

UNCLASSIFIED

F/G 20/4

NL





MICROCOPY

CHART

Michigan Technological University
Department of Chemistry and Chemical
Engineering,
Houghton, MI 49931

Date: 1 May 1985

DTIC
ELECTE
S MAY 23 1986 D
A

To: Dr. C. M. Lee

From: D. W. Hubbard and G. Trevino

Subject: ONR Project Number NR 4322777/1-4-85 (432)

Free Surface Turbulence

ONR Reference 432:CML:edg
LN/432/85/223/07

Contract Number N00014-85-K-0236

Annual report for the first year of the project--
1 March 1985 to 28 February 1986

When turbulent jets or wakes interact with a free surface, the turbulent eddies are apparently damped in the vertical direction and extended in the horizontal direction. Turbulent velocities are being measured for this zone of interaction to enable verifying turbulence modelling techniques. A two-dimensional jet is formed by pumping water through a rectangular slit into a channel filled with still water. Hot-film anemometry techniques are used to measure velocity fluctuations in the jet and in the immediate neighborhood outside the jet. Some effects of jet flow rate and jet submergence have been studied. .

The raw data are sampled time records of the anemometer output voltages obtained at different positions in the jet. These time records together with the sensor calibration information are analyzed using digital signal processing techniques. The mean velocities, turbulence intensities, and velocity correlation functions are currently being calculated,

The results are being used to develop a two-point closure scheme and a mathematical model for the jet flow turbulence. The model contains small non-homogeneities and anisotropy because of the proximity to the free surface. By analogy, the results are being used to predict the interaction of a wake with a free surface.

Background and Objectives

The wake of a ship or torpedo gives evidence of its passing. For detection and tracking purposes, it is of interest to know as much as possible about the characteristics of wakes. It is difficult to study wakes formed by moving objects, because the detectors themselves

This document has been approved
for public release and sale; its
distribution is unlimited.

86 5 12 163

AD-A168 013

DTIC FILE COPY

must be moving. A towing channel or basin is required for this, and moving the detector can introduce extraneous vibrations which can make data analysis difficult. The flow in a wake can be simulated by the flow in a jet. This is one unique feature of the current research being supported by ONR.

The similarities between turbulent flow in jets and wakes have been described by Schlichting(1968). The mean velocity profiles have similar shapes, but the jet profile is the negative of the velocity defect of the wake profile. The width of a jet or wake is proportional to the distance downstream from the source, and the centerline velocity is proportional to the square root of the distance downstream. The similarity becomes clear if the velocity defect profile for the wake is compared to the velocity profile for the jet as shown by Abramovich(1963). Since the characteristics of jets and wakes are similar, studying jet turbulence should give insight into wake turbulence.

In the current research, the interaction of a two dimensional plane jet with a free surface is being studied by measuring the distribution of fluctuating velocities in the flow from a submerged jet located near the surface. This is being done by making time records of the output of hot-film anemometers placed at various positions in the jet. From these time records and sensor calibration data, the mean velocity distribution, jet width, turbulence intensity, and turbulent velocity correlations are being calculated. A turbulence model based on a two-point closure scheme is being developed, and the results of these modelling calculations is being compared with the experimental data and with data which has been reported by others.

The objectives of the current project are as follows.

- (1) Determine the effect of jet flow rate, depth of submergence, angle of impingement, and jet dimensions on their turbulence characteristics in the zone where a two-dimensional turbulent jet interacts with a free surface. These turbulence characteristics include--
 - steady-state mean flow velocity distribution
 - distribution of turbulent intensity
 - spatial distribution of turbulent velocity correlations
 - time decay characteristics of velocity fluctuations and velocity correlations
- (2) Develop a two-point closure model for turbulence near a free surface. This model includes some interesting effects.
 - Small nonhomogeneities occur because of the

presence of the free surface

- Anisotropy is present, because there is a preferred direction perpendicular to the free surface.

- (3) Determine the time decay properties of turbulent velocity correlations as a consequence of the closure scheme.

The literature pertinent to this project was reviewed in the original proposal. Most of the papers dealing with experimental measurements describe studies made using air jets where there was no anisotropy caused by the presence of an air-water interface. Many of the mathematical modelling papers in the literature deal with one-point closure models--the k-epsilon model, for example. These cannot incorporate fully the turbulent energy transfer from low wave number eddies to high wave number eddies. The contemporary view of steady-state turbulence is that flow structures (coherent structures) or eddies decay in the flow and eventually disappear. In order to sustain a steady-state turbulent flow, new structures must be formed, and this requires energy. The energy is thought to come from the mean flow, but the mechanism for the transfer from the mean flow to the turbulent structures is not fully understood. WHATEVER THE ENERGY TRANSFER MECHANISM IS, ITS EFFECTS SHOULD BE REFLECTED IN ANY CLOSURE MODEL DEVELOPED IN ORDER FOR THE MODEL TO BE UNIVERSALLY APPLICABLE TO ALL TURBULENT FLOWS.

Two-point turbulence closure modelling techniques for isotropic turbulence have been discussed by Trevino (1982a, 1983) and by Domaradski and Mellor (1984). The Navier-Stokes equations governing momentum transport in fluids are written in terms of the two-point velocity correlation tensors.

$$\frac{\partial}{\partial t} c_{ij}(r, t) = \frac{\partial}{\partial r_k} S_{ikj}(r, t) - \frac{\partial}{\partial r_k} S_{jki}(-r, t) + \text{Pressure terms} \\ + \text{Viscous terms}$$

This equation is being solved by assuming self-similar structure in the turbulence, that is, by assuming that the turbulence and the mean flow scale in the same way (Trevino, 1986a), and by including the "free-surface" effects through the classical "method of images".



Distribution For ☒ GR&I ☐ DTIC TAB ☐ Unannounced ☐ Justification	By <i>John D. G. H.</i>	Distribution/ Availability Codes	Avail and/or Special
Dist		4-1	

Accomplishments During The First Year

Work on the jet flow research project began on 1 March 1985. The first four months were devoted to equipment design, construction, and installation. This was a time consuming process, since an entirely new project was being started.

A special device for producing a plane jet was designed and fabricated. There are entrance sections and a narrow slit as shown in Figure 1. The parallel flat plate section forming the slit is separate so that slit sections having different slit widths can be attached to the entrance sections enabling the effect of slit width to be investigated. This jet flow unit is installed in a water channel 0.45 m wide, 0.45 m deep, and 7.5 m long. A schematic flow diagram of the system is shown in Figure 2. A special framework is installed over the channel to support the jet flow unit. This framework keeps the unit rigidly in position and incorporates a means for raising, lowering, and tilting the jet flow unit. Water is supplied to the jet flow unit from a pump supported above the channel on a separate framework. The piping incorporates a flexible tubing section to isolate the jet flow unit from pump vibrations. The pump suction line is located downstream from the jet to minimize reverse flow effects.

The fluctuating velocities in the jet are measured using hot-film sensors connected to a TSI, Inc. Model IFA-100-4 anemometer system which was purchased partly with funds from ONR and partly with funds from Michigan Tech. This four-channel anemometer system enables obtaining time histories of the fluctuating velocities simultaneously at four different points in the flow. This enables determining six points on the spatial correlation function curve. This gives a definitive test of the degree to which the experimental data deviate from the corresponding curve which can be calculated for isotropic turbulence and enables making a good estimate of the inherent anisotropy which occurs because of the presence of the free surface. The hot-film probes are mounted on a framework separate from the other two to eliminate any vibration which might be transmitted from the pump or elsewhere. The probe can be positioned anywhere in the jet by sliding the mounting device horizontally along a set of rails. At any horizontal position, the probe support can be moved to any vertical position and clamped.

One problem which occurs when operating hot-film probes in water is fouling. This cannot be avoided, so the probes must be calibrated frequently. It is essential to have timely and accurate calibrations in order to be able to interpret the anemometer output voltage data. It would be very time consuming to calibrate each probe frequently

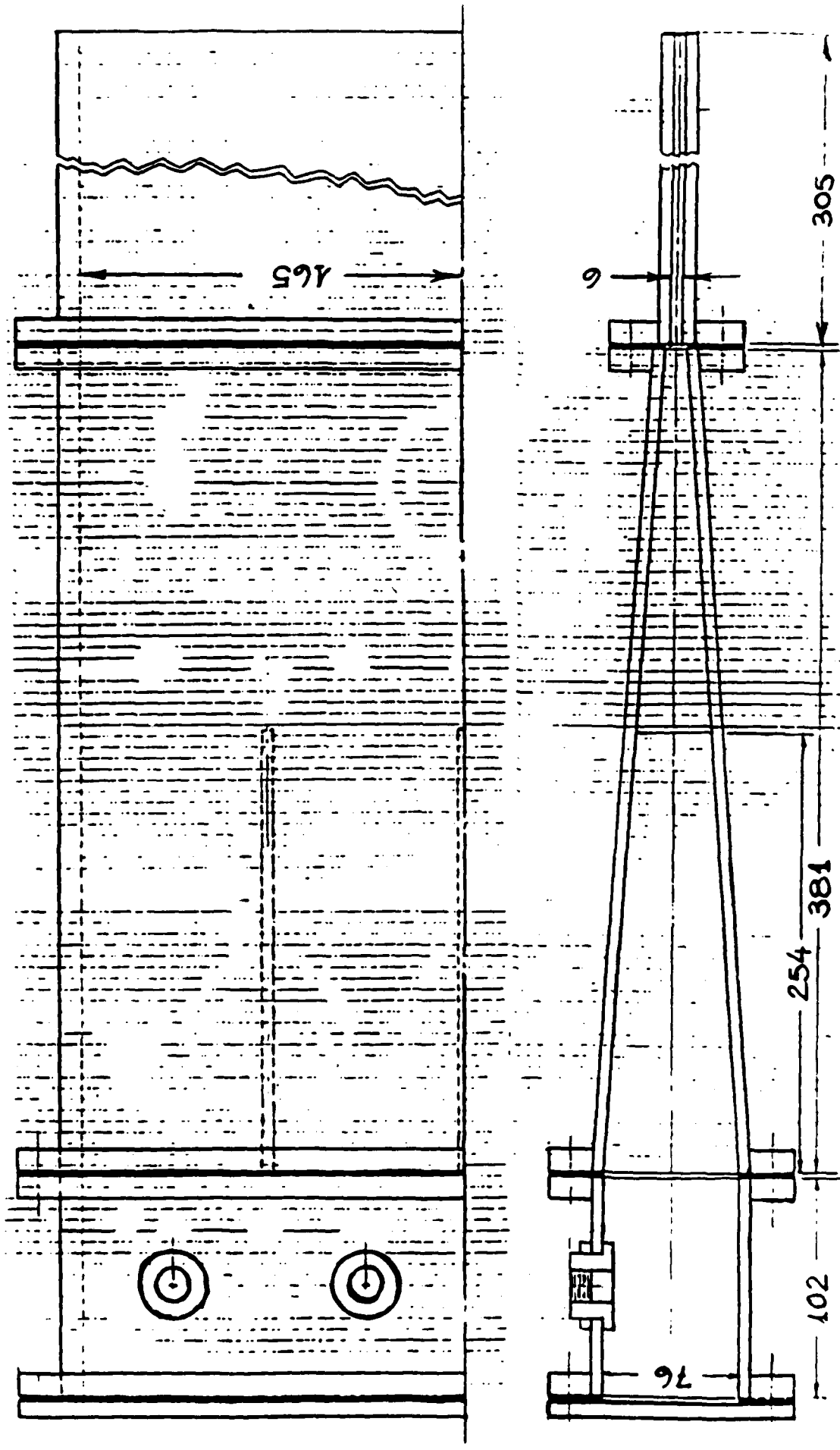


Figure 1. Jet Flow Unit

Dimensions in mm.

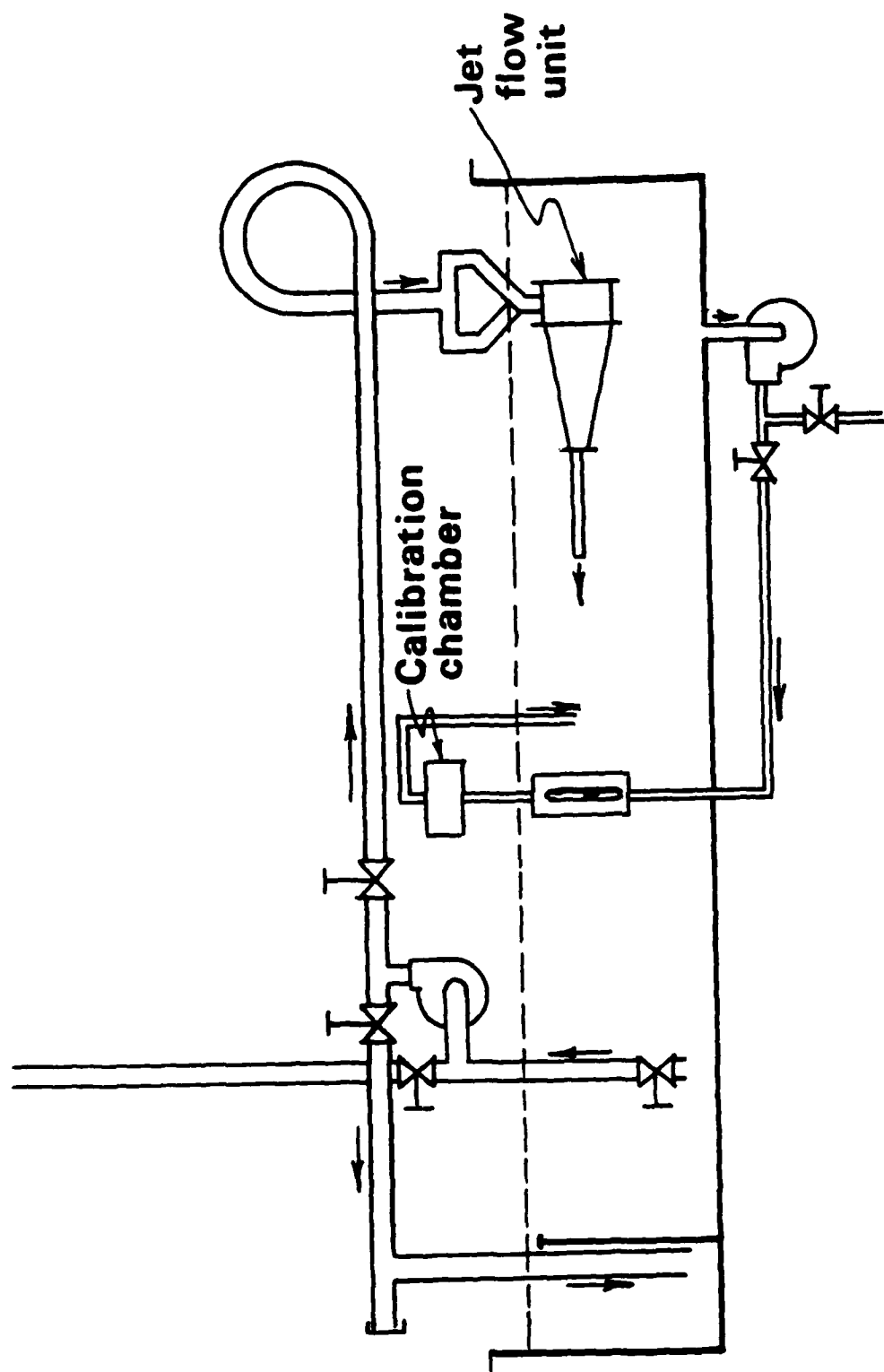


Figure 2. Flow System

during the course of an experiment. Because of this, quite a bit of time during the summer of 1985 was spent to develop a new type of laminar jet calibration system and a new calibration procedure. This device uses water from the same source as that which is flowing through the slit which produces the turbulent jet. The construction of the calibration device is shown schematically in Figure 3. It is positioned as shown in the flow diagram(Figure 2). The method of using this calibration device to incorporate probe fouling while making turbulent velocity measurements is discussed in a paper by Hubbard, Trevino, and Hine(1985) which was presented at the 22nd Annual Meeting of the Society of Engineering Science. This paper also includes a discussion of all the operating procedures which have been developed in connection with the jet flow research project. A copy of this paper is attached as an appendix. The development of the probe calibration scheme and the method for handling probe fouling is one of the unique accomplishments for the project.

Besides completing the design, construction, and installation of the equipment and the development of operating and calibration procedures, computer software necessary for carrying out the data analysis has been written. The anemometer output data and the calibration data are transferred to the mainframe computer into separate files. The voltage-time data are converted to velocity-time data by applying the appropriate calibration function. The new files thus formed contain the instantaneous velocity values. These data can then be analyzed using the time-averaging codes or the velocity correlation codes.

Each set of voltage-time data consists of data obtained at the same time at four different locations downstream from the source of the jet. Using these data, time-averaged velocities have been calculated to show the validity of the original idea of using the turbulent jet as a model system to represent a turbulent wake and to obtain some idea about the persistence of the jet with distance from the source. The results of some of these calculations appear in Figure 4 where the mean velocity profiles for the jet are compared with data obtained at the Naval Research Laboratory by Swaan and Keramidas(1984). These data show that the two systems are indeed similar, and the original idea of using jets to model wakes is verified.

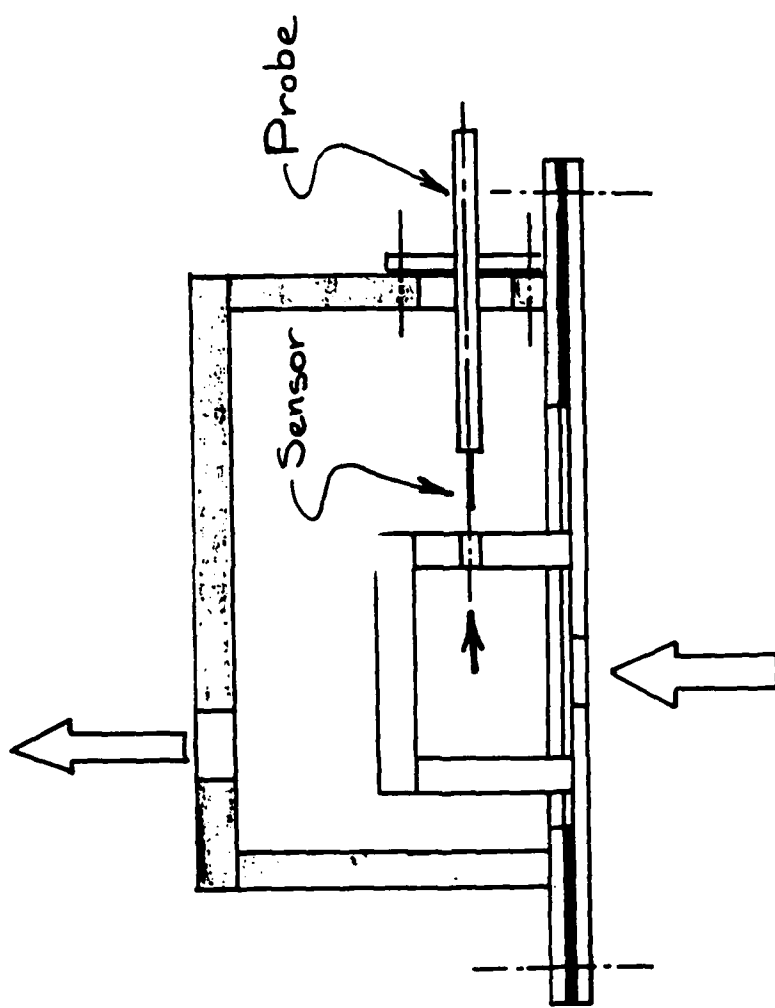


Figure 3. Calibration Device

Jet • Wake ▲

U_{∞} (m/s) 0.63 0.59
 $\bar{U}_{max}$ (m/s) 0.35 0.44
 X (m) 0.76 0.62
 y at free surface 5.5 cm 5.0 cm

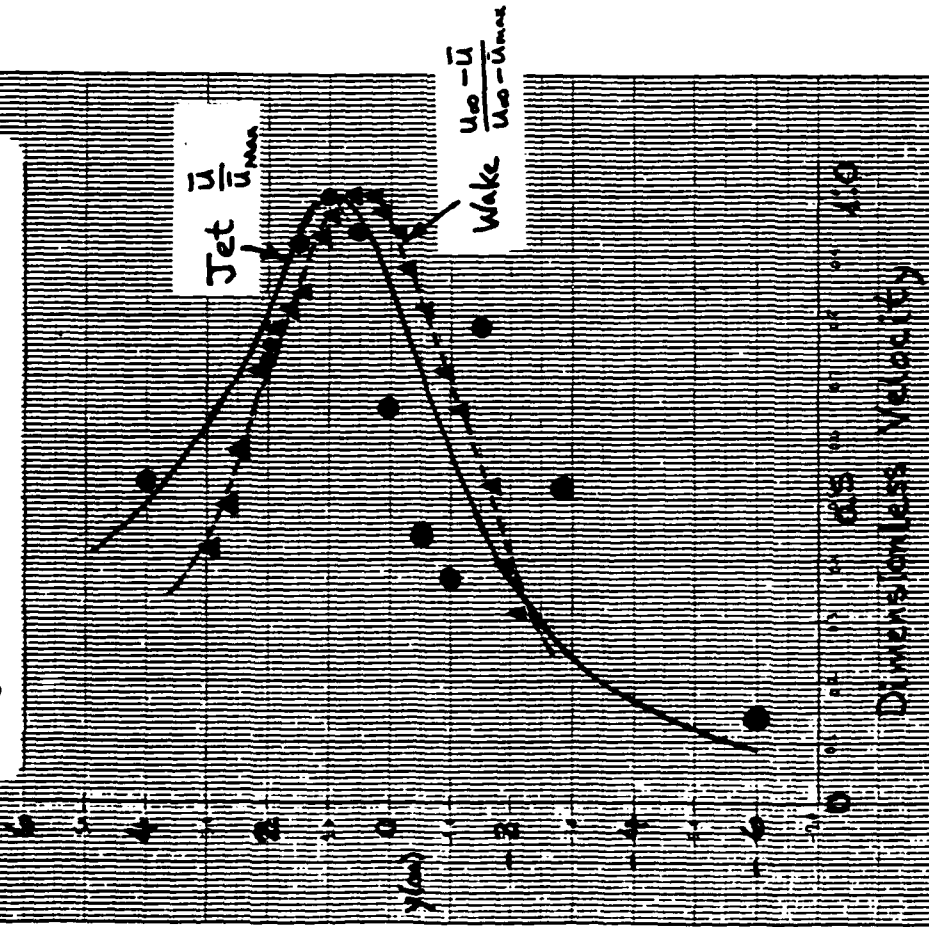
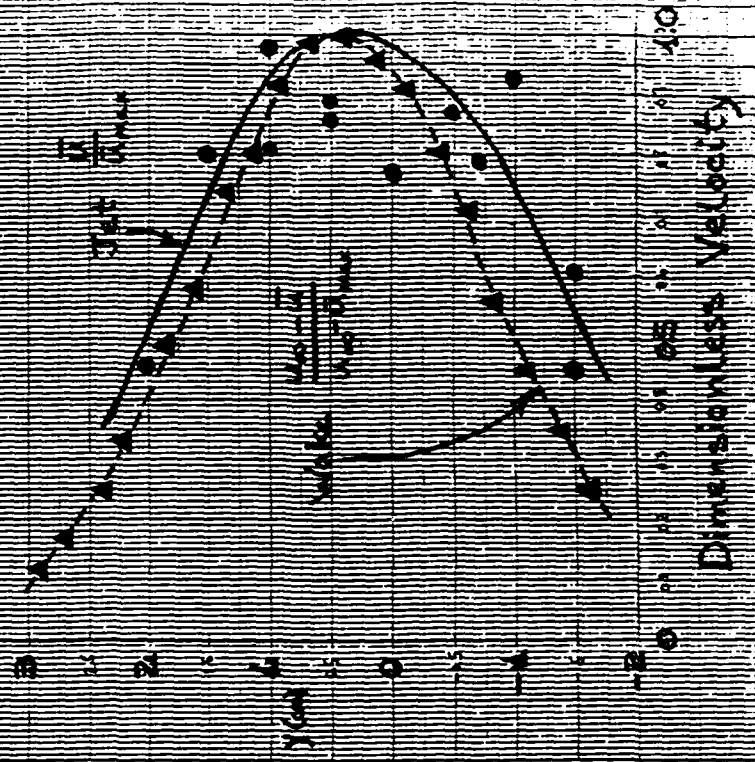


Figure 4. Comparison of Mean Velocity Profiles

Jet • Wake ▲
 U_{∞} (m/s) 0.63 0.59
 $\bar{U}_{max}$ (m/s) 0.42 0.39
 X (m) 0.16 0.23
 y at free surface 5.5 cm 5.0 cm



Correlation calculations are presently being made in preparation for the detailed turbulence closure modelling. To make the correlation calculations, it is necessary first to calculate the time-averaged (or mean) value for the data in each file and subtract this value from all the velocity values to obtain the turbulent velocity fluctuations. The correlation function values or double velocity correlations are calculated at six different values of the separation, x .

$$C_{ij}(x) = \langle u_i'(x) u_j'(x+x) \rangle$$

where u' is the velocity fluctuation and the $\langle \rangle$ brackets indicate an ensemble average which is approximated here by the time-average. The skewness or triple velocity correlations are also calculated for 24 different combinations of the separations available from the data.

$$S_{ijk}(x) = \langle u_i'(x) u_j'(x) u_k'(x+x) \rangle$$

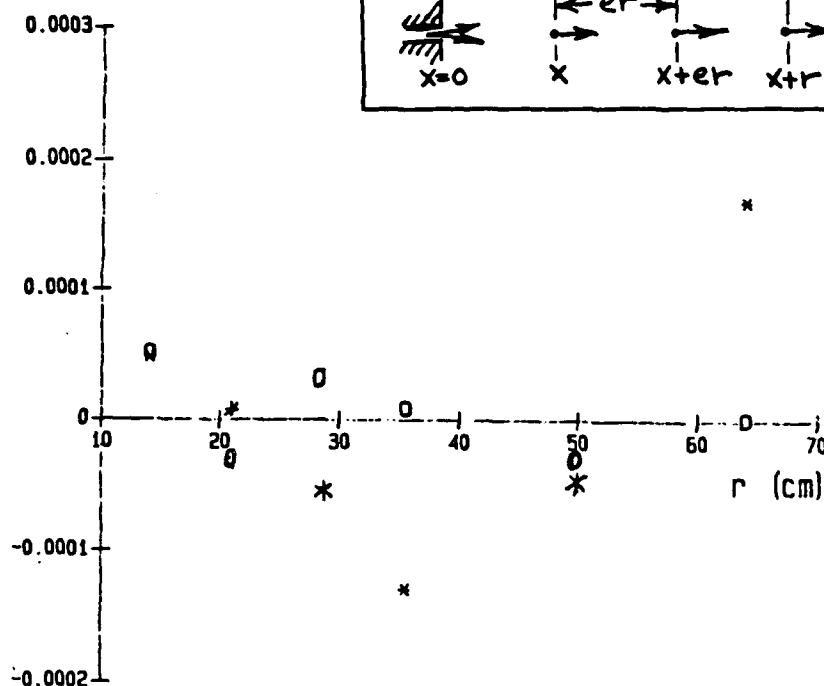
As part of the work on the contract, George Trevino has developed the general theoretical framework for the turbulence closure scheme based on self-similar velocity correlations. A paper entitled "Analysis of Turbulent Decay Using Affine Transformations" has been prepared and is currently being reviewed for publication in Physics of Fluids. Another paper entitled "The Skewness Function of Turbulence" has been completed and submitted for publication in Boundary Layer Meteorology. Copies of these manuscripts are attached in the appendix.

The idea of self-similar structure means that the turbulence characteristics retain similarities throughout the turbulent field. This does not mean that these characteristics must be identical throughout the field. It only means that something about the turbulence characteristics is preserved throughout the field. For example, the ratio of the integral scale to the Taylor microscale might be preserved as the turbulence decays--as the distance downstream from the source of the jet increases. This ratio of scales is related to the triple velocity correlation or skewness of the probability density function which characterizes the jet turbulence.

In Figure 5 are shown some triple correlation data which illustrate this idea of self-similar structure. These data show that the triple correlation, S_{ijk} , depends on the separation distances but not on the distance downstream from the source of the jet, because all the functions, $S(x, e, r)$ have the same shape without regard to the position in the jet. Since just four sample records are available, only a few points on the triple velocity correlation function can be determined, but the data available are consistent with the idea of self-similar decay.

$S(x, e, r)$

$(M/S)^3$



$$Q = 0.0026 \text{ m/s}$$

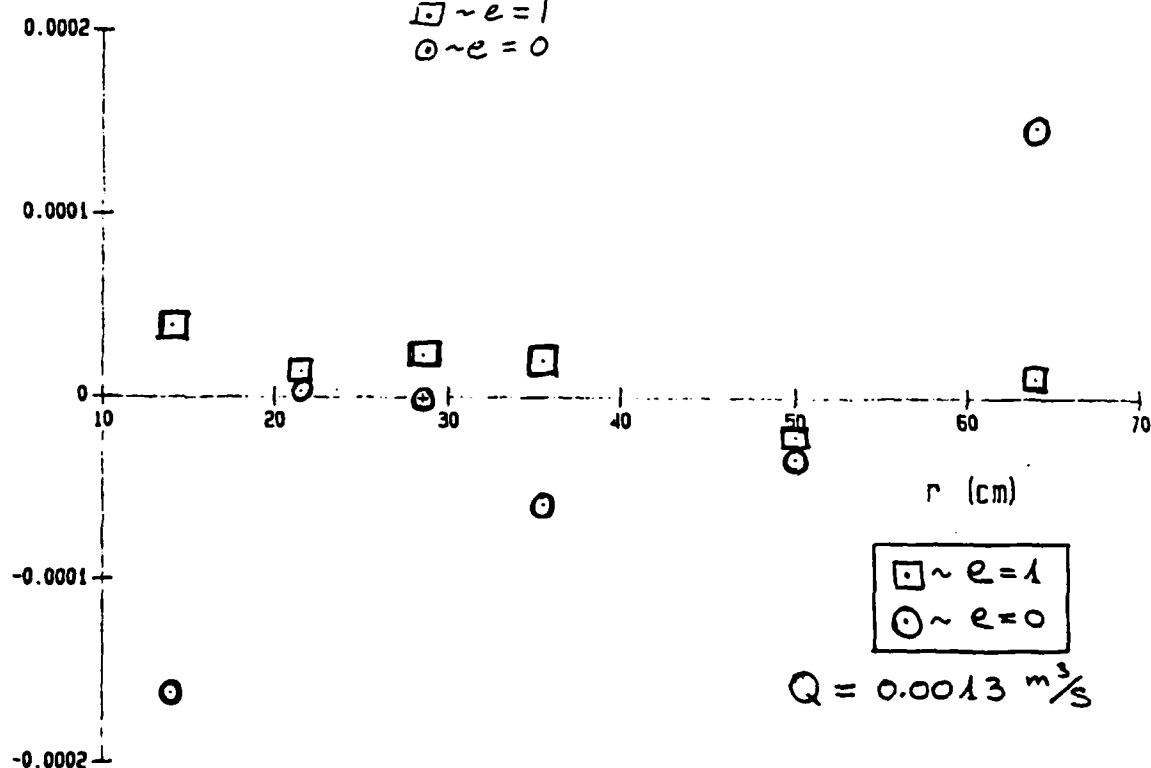
$$\circ \sim e = 1$$

$$* \sim e = 0$$

Figure 5(a). Triple Velocity Correlation Functions

$S(x, e, r)$

$(M/S)^3$



$$Q = 0.0013 \text{ m/s}$$

S(X, e, r)

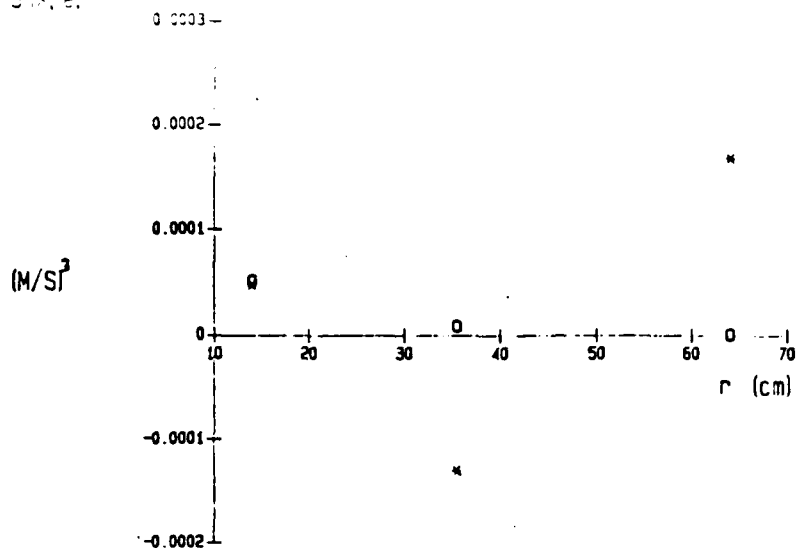
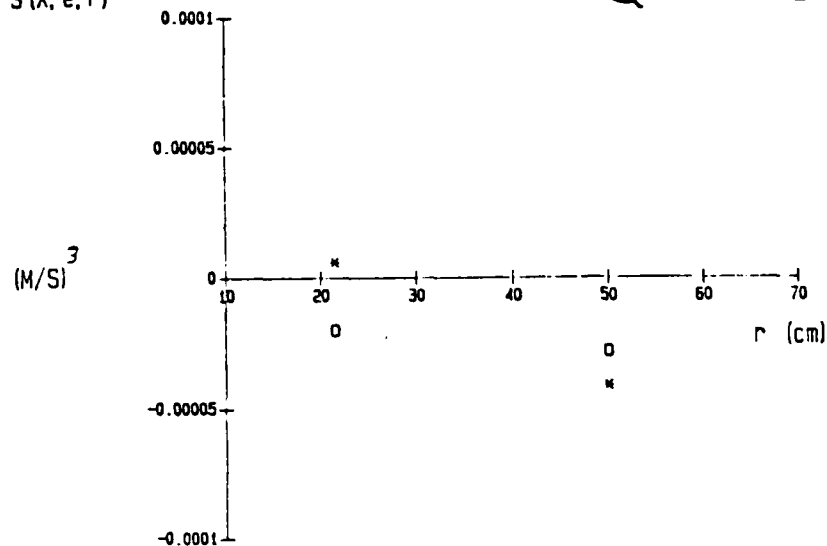
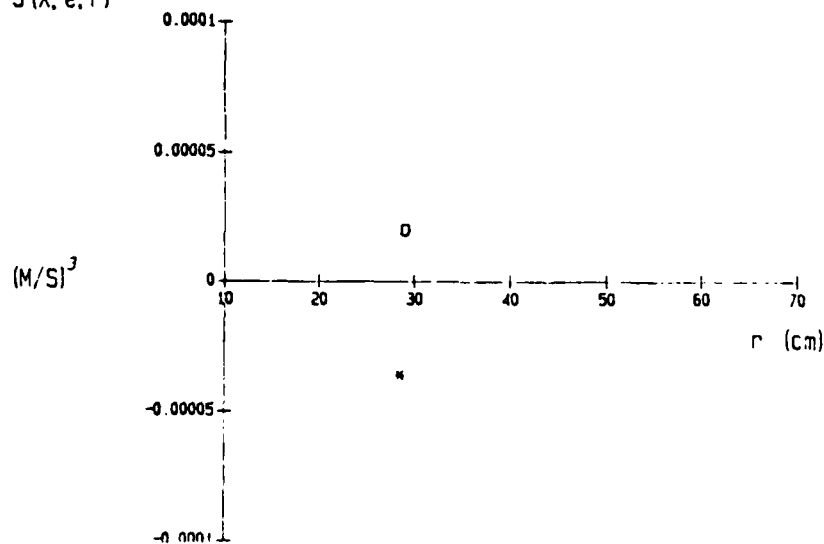


Figure 5(b). Triple Velocity Correlations -- details
 $Q = 0.0026 \text{ m}^3/\text{s}$

S(X, e, r)



S(X, e, r)



The complete results obtained during the first year for the jet flow project will be presented in the thesis currently being written by Eric J. Hine who was supported by funds from the contract while pursuing his graduate work. A paper based on this work is being prepared and will be presented at the annual meeting of the American Institute of Chemical Engineers in November 1986. A copy of the abstract of this paper and a copy of the program for this session are attached in the appendix. Eric Hine has accepted employment at the Knolls Atomic Power Laboratory and will be working on nuclear ship propulsion systems. He will begin in this position as soon as he finishes his thesis and his security clearance is completed.

Plans for Continuing the Project

During the next year, it is proposed to continue turbulent velocity measurements to provide more data for comparison with turbulence models. These data will be obtained for different values of jet submergence and for different values of the angle of impingement on the free surface, so that these effects can be studied. It is also proposed to add pressure measurements to the experimental program. The closure and turbulence modelling calculations will be continued, and the model will be improved by incorporating pressure-velocity correlations into the model.

Cheng(1985) and Trevino(1986b) point out the importance of pressure-velocity correlations in modelling the spatial evolution of the transfer of turbulent energy to lower and higher wave numbers. For isotropic turbulence, the pressure-velocity correlation is zero, and many turbulence closure models are based on setting it equal to zero. For non-isotropic turbulence, the pressure-velocity correlations are not zero. It is proposed to compute the pressure-velocity correlations and use the results to introduce more realism into the closure model being developed as part of the jet flow project.

In the "spatial decay" (spatial evolution) of a steady-state, self-similar, nonhomogeneous turbulence such as may be encountered in a submerged turbulent jet, the convective mechanism is spatially invariant; that is, independent of spatial location downstream within the jet. This is true even though the turbulence itself is fully nonhomogeneous. This point is discussed by Trevino in some forthcoming papers(see Trevino,1986a,b). This spatial independence therefore requires that in the "high" Reynolds number situation, it is the pressure-velocity correlation which provides the all-important spatially varying energy transfer mechanism. Spatial invariance of the convective tensor, $S_{ij}(\dots) = \langle u_i(x-r/2)u_j(x-r/2)u_l(x+r/2) \rangle$, can exist only if there is a continuous cascade of energy toward the lower

frequencies of the spectrum as well as toward the higher frequencies. This phenomenon has already been suggested theoretically by Tsuge(1984) and shown experimentally by Okude(1981). The role of $P_i(x, r) = \langle p(x-r/2) u_i(x+r/2) \rangle$ in the decay of $C_{ij} = \langle u_i(x-r/2) u_j(x+r/2) \rangle$ accordingly ought not be neglected(as it is typically neglected in the analysis of nonhomogeneous decay), and its effects must be explicitly measured and specifically included in the equation of jet-decay. Because of the anisotropy present in the turbulence, the pressure-velocity term is not zero. The related steady-state dynamical equation of turbulence for high Reynolds number is--

$$\bar{U} \cdot \nabla \bar{U} + \bar{u} \cdot \nabla \bar{U} + \bar{U} \cdot \nabla \bar{u} + \bar{u} \cdot \nabla \bar{u} = -\frac{1}{\rho} \nabla P$$

where the total flow is $\bar{U} = \bar{U} + \bar{u}$ and the total pressure is $P = \bar{P} + p$. In the process of stochastic-averaging, this equation produces the "spatial-decay" equation--

$$\langle \bar{u} \cdot \nabla \bar{U} \bar{u}' \rangle + \bar{U} \cdot \nabla \langle \bar{u} \bar{u}' \rangle + \langle \bar{u} \cdot \nabla \bar{u} \bar{u}' \rangle = -\frac{1}{\rho} \nabla \langle p \bar{u}' \rangle$$

where $\bar{u} \sim u(x-r/2)$ and $\bar{u}' \sim u(x+r/2)$, etc. For the self-similar turbulence being considered, the term $\langle \bar{u} \cdot \nabla \bar{u} \bar{u}' \rangle$ is independent of x , but the term $(-1/\rho) \nabla \langle p \bar{u}' \rangle$ is not. This latter term is in fact what introduces into the right-hand side of the equation the x -dependence necessary to sustain the x -dependence of the terms $\langle \bar{u} \cdot \nabla \bar{U} \bar{u}' \rangle$ and $\bar{U} \cdot \nabla \langle \bar{u} \bar{u}' \rangle$. THIS PRESSURE TERM IS THE TERM FOR WHICH MEASUREMENT AND MODELLING ARE CRUCIAL TO THE ANALYSIS.

Until recently, pressure sensors sensitive enough to measure turbulent pressure fluctuations were not available, so the effect of the pressure terms could not be included in two-point turbulence closure models for nonisotropic turbulence. Jones (1981) and Spencer(1970) report the development of a bleed-type pressure sensor based on a hot-film anemometer system. A hot-film velocity probe connected to a constant temperature anemometer circuit is enclosed in a tube open to the pressure to be measured at the downstream end. The upstream end is connected to a constant pressure source of fluid, and there is a constant flow of fluid from the constant pressure supply, past the velocity sensor, and into the flow where the pressure is being measured. The velocity of the fluid flowing through the tube is related to the pressure difference, so the downstream pressure can be related directly to the anemometer output. It is well-known that hot-film velocity probes are very sensitive to changes in the cooling velocity in the neighborhood of the sensor. This means that pressure measurements based on the velocity determined using a hot-film sensor are equally as sensitive. Pressure sensors such as described above are available commercially from TSI, Inc. For measurements in water, the bleed fluid used is alcohol which is miscible with the main water flow.

Expected Results and Applications

The research program in progress is yielding a body of results which will indicate the effects of jet flow rate, jet submergence, and jet orientation on the turbulence characteristics and decay in the zone of interaction between a plane jet and a free surface. When the jet is submerged well below the surface, it is expected that the time-averaged velocities, turbulence intensities, Reynolds stresses, and other turbulence characteristics will be the same as similar data reported by others for air and water jets. The results for jets at shallow submergence are new data which can be compared with theoretical results for anisotropic closure schemes. The comparison will establish the range of validity of the anisotropic decay laws generated by the theory.

Besides the one-point, steady-state turbulence characteristics, the experimental results will include information about the decay of the turbulence parameters and information about the spatial dependence of the two-point velocity correlation functions. The two-point closure model to be developed will include pressure-velocity correlations as well as spatial decay effects. Because of the presence of the free surface, small nonhomogeneities and anisotropy will also be included. The two-point model is more effective for modelling the details of the turbulence far from the source of a jet or wake.

Personnel

The project is being carried out by D. W. Hubbard (experimental work) and G. Trevino (closure modelling) with the help of graduate research assistants who are candidates for advanced degrees in chemical engineering or mechanical engineering. The research is the basis for preparing theses which will fulfill part of the requirements for these degrees. The students receive academic credit for the work and spend approximately one-half time engaged in the research. The students currently working on the project are Eric J. Hine and Lisa Heydenburg who are candidates for graduate degrees in chemical engineering.

Bibliography

Abramovich, G. N. 1963. The Theory of Turbulent Jets, MIT Press, Cambridge, Massachusetts, pp. 110,140.

Cheng, S-I. 1985. "Fluid Turbulence--Deterministic or Statistical". Proceedings of the IUTAM Conference on

Shockwave and Turbulent Boundary Layer Interaction (Paris, September 1985). Springer-Verlag, Berlin, To be published.

Domaradski, J. A. and Mellor, G. L. 1984. "A Turbulence Closure Hypothesis for the Triple-Velocity Correlation Functions in Homogeneous Isotropic Turbulence". J. Fluid Mech., 140, 45.

Hubbard, D. W., Trevino, G., and Hine, E. J. 1985. "Experimental Techniques for Measuring Turbulent Velocity Correlations in a Jet Flow Near an Air-Water Interface". Paper 6AM6-3. 22nd Annual Meeting, Soc. Eng. Sci., Penn State University.

Jones, B. G. 1981. "A Bleed-Type Transducer for In-Stream Fluctuating Static Pressure Sensing". TSI Quarterly, 7, 5

Okuda, M. 1981. "Rearrangement of the Karman Vortex Street". Trans Japan Soc Aero Space Sci, 24, 64

Schlichting, H. 1968. Boundary Layer Theory, 6th Edition. McGraw-Hill Book Company, New York, p. 681.

Spencer, B. W. 1970. "Measurement of Fluctuating Pressure". Tech. Bull. 34, TSI, Inc., St. Paul, Minnesota.

Trevino, G. 1982a. "An Invariant Theory Approach to the Problem of Closure". Boundary Layer Meteorology, 22, 643.

Trevino, G. 1982b. "An Introduction to the Theory of Non-Homogeneous Turbulence". Texas Journal of Science, 34, 35

Trevino, G. 1983. "Closure and Isotropic Turbulence". Ann. New York Acad. Sci., 404, 83

Trevino, G. 1986a. "Inertial-Transfer in Nonhomogeneous Turbulence". In preparation.

Trevino, G. 1986b. "Comments on 'A Note on the Height Dependence...' by O. Chiba". Boundary Layer Meteorology, In press.

Tsuge, S. 1984. "Separability into Coherent and Chaotic Time Dependences of Turbulent Fluctuations". Physics of Fluids, 27, 6.

Appendices

- A. Paper Presented at the 22nd Annual Meeting of the Society for Engineering Science
- B. Manuscript submitted to Boundary Layer Meteorology
- C. Manuscript submitted to Physics of Fluids
- D. Abstract of the paper to be presented at the 1986 annual meeting of the American Institute of Chemical Engineers

PREPRINTS-22

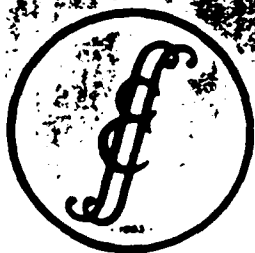
22nd Annual Technical Meeting
Society of Engineering Science
October 7-9, 1985
The Pennsylvania State University
University Park, Pennsylvania

85-22-35056

EXPERIMENTAL TECHNIQUES FOR MEASURING TURBULENCE
VELOCITY CORRELATIONS IN A JET FLOW NEAR AN
AIR-WATER INTERFACE

D. W. Hubbard
G. Trevino
E. J. Hines

Michigan Technological University
Houghton, Michigan 49931



Published by the Society of Engineering Sciences, Inc. The full responsibility for the accuracy, veracity and originality of the materials published in these *Preprints-22* is solely that of the author(s).

EXPERIMENTAL TECHNIQUES FOR MEASURING TURBULENT VELOCITY CORRELATIONS IN A JET FLOW NEAR AN AIR-WATER INTERFACE

D. W. Hubbard, G. Trevino, and E. J. Hine

Michigan Technological University
Houghton, Michigan 49931 USA

ABSTRACT

When a turbulent water jet interacts with an air-water interface, the turbulent eddies are apparently damped in the vertical direction and extended in the horizontal direction. This phenomenon is being studied in a two-dimensional jet formed by pumping water through a rectangular slit into a channel filled with water. Hot-film anemometry techniques are being used to measure velocity fluctuations in the jet and in the immediate neighborhood outside the jet. Since hot-film probe fouling cannot be avoided when making measurements in water, a sensor calibration method which includes the effect of probe fouling has been devised.

INTRODUCTION

When turbulent jets or wakes interact with an air-water interface, the turbulent eddies are apparently damped in the vertical direction and extended in the horizontal direction. Experimental methods useful for studying turbulent velocity correlations in two-dimensional jets have been devised. A unique feature of this work is the development of a sensor calibration method which reduces the labor required for calibrating hot-film probes used in a multi-channel array in water.

JET FLOW SYSTEM

A two-dimensional jet is formed using the flow system shown in Figure 1. Water is pumped upward from a water channel 0.45 by 0.45 by 7.5 meters containing still water using a stainless steel centrifugal pump driven by a 2 HP motor. Water flows from the pump discharge through polyvinyl chloride (PVC) piping, through an inlet manifold, and into a converging flow chamber which is submerged in the channel. The water flows through this converging section part

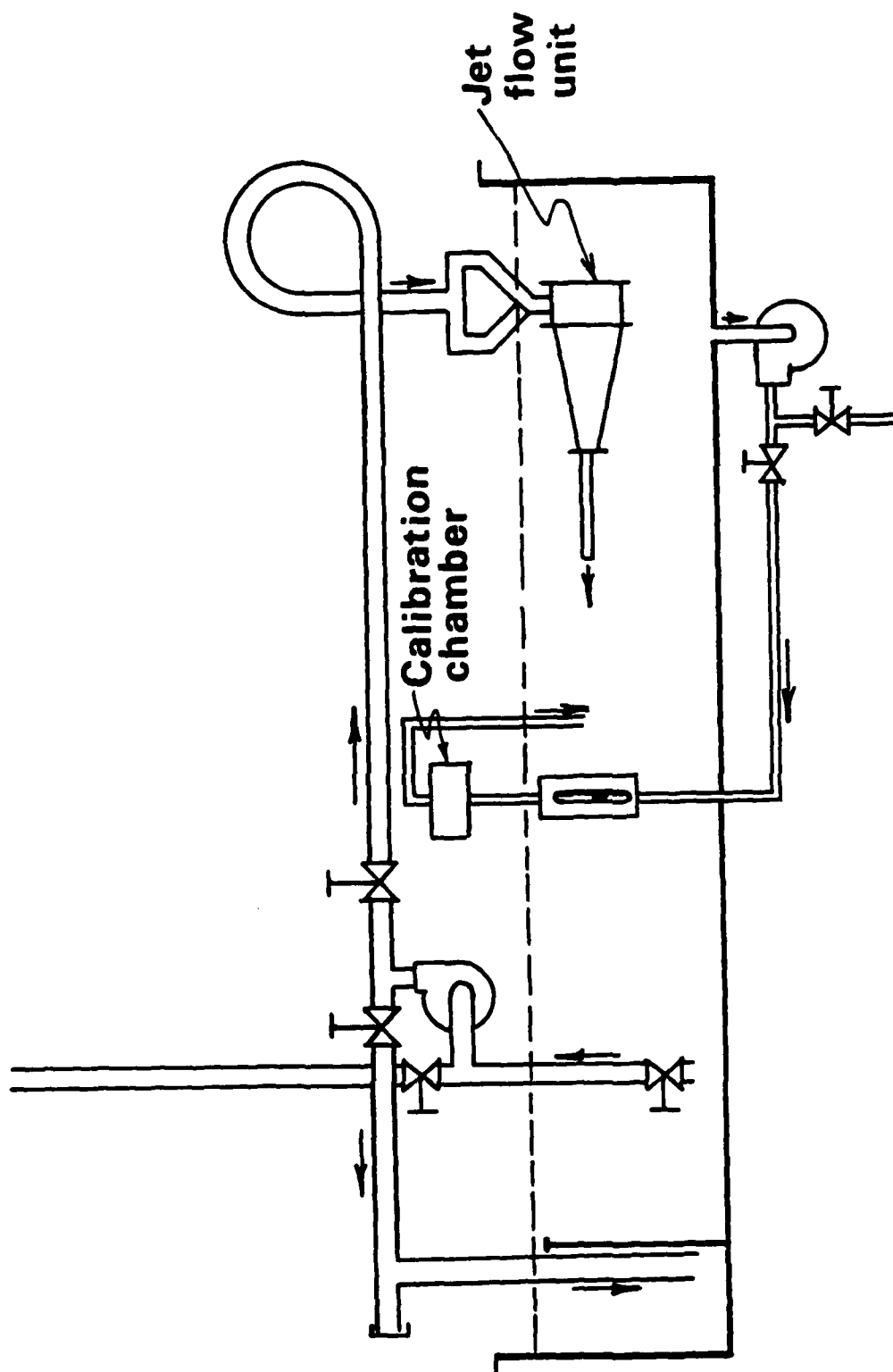


Figure 1. Flow System

of which is packed with shredded stainless steel and through a rectangular slit formed by two flat plates 356 mm wide by 305 mm long spaced 6mm apart and back into the channel filled with still water. The flow rate through the slit can be adjusted by adjusting the valves which control the fraction of the flow passing through the by-pass line. Since the main circulating pump operates with a suction lift, appropriate valves and piping are provided for priming the pump.

DATA ACQUISITION AND ANALYSIS METHODS

Turbulent velocity fluctuations are measured using hot-film probes connected to a TSI, Inc. Model IFA-100 four-channel anemometer circuit. By positioning four probes at unequal spacing in the jet--for example, at 0.1, 0.3, 0.7, and 1.5 meters downstream from the slit--data can be acquired which will enable calculating the spatial correlations of the turbulent velocity fluctuations at six different separation distances--for example, at separations of 0.2, 0.4, 0.6, 0.8, 1.2, and 1.4 meters. The analog data are available continuously on each of the four channels of the anemometer unit. The data acquisition microcomputer used to record the data--Heathkit Model H-8--samples the output of the four channel analog-digital converter channel by channel at 2 ms intervals, so there is at least a 2 ms lag between the data from different channels and an 8 ms sampling interval for the data on any one channel. This means that in order to have simultaneous values of the velocity fluctuations to use for calculating the values of the correlation functions, some sort of interpolation scheme must be used for each channel. At any particular time chosen for a correlation function calculation, there will be one actual data value and three interpolated data values. The calculation can be done at 2 ms intervals using a real data value from a different channel in turn for each calculation. Using these data, the wave number spectrum can be determined at six different wave numbers using a Fast Fourier Transform method. If more than six wave number values are desired, the correlation function must be fitted to some equation and interpolated values at intermediate separation distances must be calculated. An alternate method for obtaining a spectrum using more wave numbers would be to change the probe spacing and collect more data. A spectrum calculated from such data will contain values calculated from velocity fluctuation data not all measured at the same time.

PROBE FOULING AND CALIBRATION

Commercially available hot-film probes specifically designed for use in water are being used for the measurements. These probes are 0.152 mm in diameter by 2.0 mm long and are coated with quartz

to minimize corrosion. Each probe is heated to a constant temperature when it is immersed in the flowing water. The temperature is set by controlling the probe resistance using an automatic balancing Wheatstone bridge circuit. The sensor loses heat to the surrounding fluid, and the rate of heat loss depends on the heat transfer coefficient for heat transfer from the sensor to the fluid flowing past it. This heat transfer coefficient is mainly influenced by the fluid velocity perpendicular to the sensor.

Since the rate of heat loss is not a linear function of fluid velocity, and since there are a number of complicating factors--the sensor supports influence the heat transfer, for example--which affect the relationship between sensor heat loss rate and fluid velocity, each sensor must be calibrated. The anemometer circuit output is the voltage required to maintain the required current through the sensor in order to maintain the sensor temperature (or sensor resistance) constant. Sensor calibration is accomplished by immersing the probe in water flowing at a known velocity and noting the anemometer circuit output. The sensor calibration system being used is simple to operate and can be operated simultaneously with the jet flow unit using the same water that flows through the slit which forms the turbulent jet. The calibration unit forms a laminar jet in a small enclosed chamber. Water from the water channel is pumped through a calibrated variable area flow meter and into the inlet of this chamber. The water passes from the inlet section through an orifice 8.7 mm in diameter into another section of the chamber. The water which passes through the orifice forms the laminar jet, and the velocity of this jet can be varied by varying the flow rate. The probe to be calibrated is positioned in this laminar jet, and the relationship between anemometer output and jet velocity is determined. The water from the calibration device passes out the exit port and is returned to the water channel.

One annoying aspect of making turbulence measurements in water is that the sensors become fouled quite rapidly. This cannot be avoided, so some method must be devised for including probe fouling in the sensor calibration scheme. Since probe fouling always occurs when hot-film anemometers are operated in water, it is important that probe calibration be carried out continuously during the course of the turbulence measurements. To reduce the labor of calibrating several probes several times during the course of the experiments carried out on any particular day, is desirable to be able to calibrate the probes without moving or remounting them. The objective of the work was to find out whether or not the probes foul in a predictable manner and whether or not all probes foul in the same way when operated in the same water.

The results of the calibration experiments show that probe fouling can be expressed by the bridge output decreasing linearly with time. In many cases, the linear decrease is interrupted when the water is not moving relative to the probe. This is illustrated in Figure 2. At any particular velocity, each segment of the

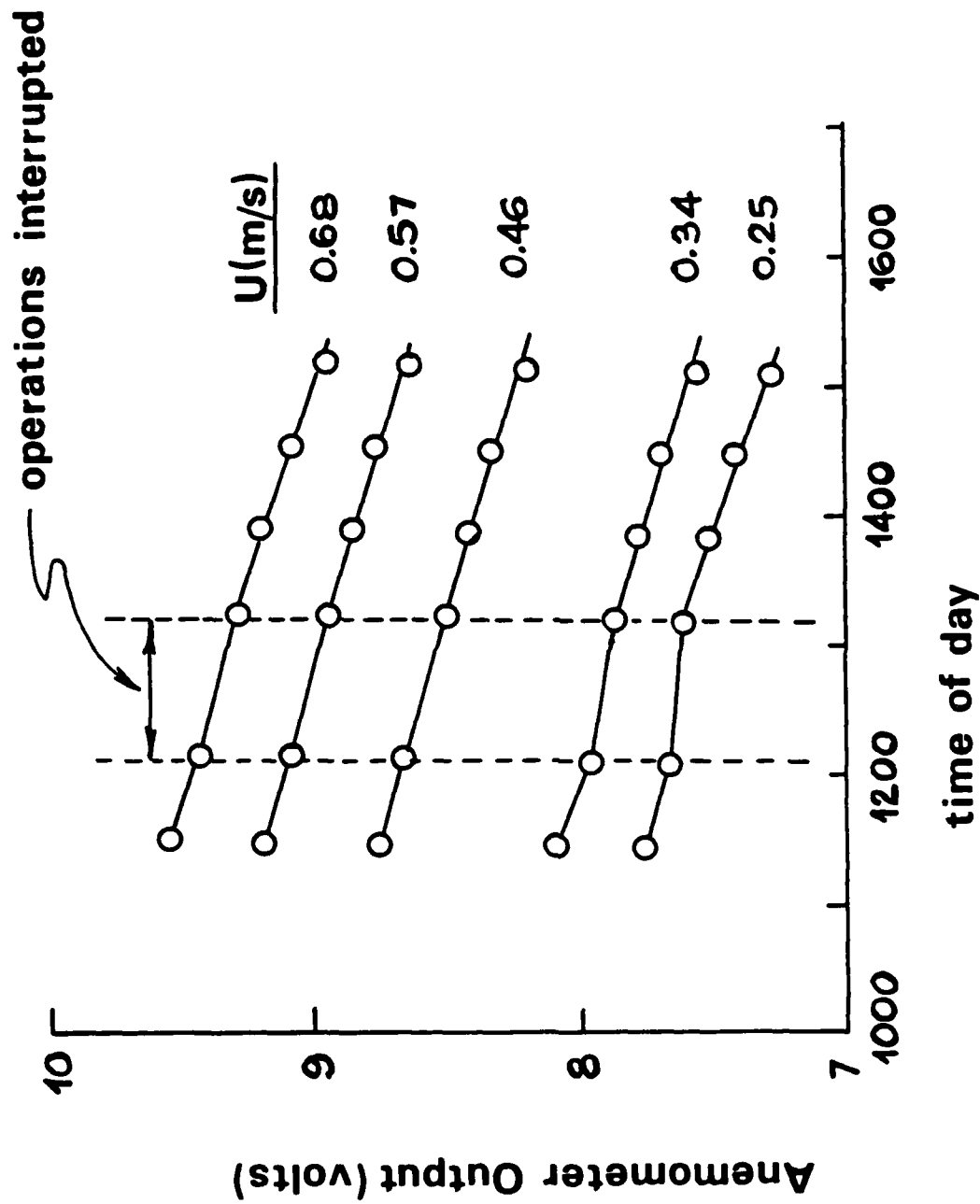


Figure 2. Probe Calibration and Fouling

bridge output voltage versus time function can be represented by a linear function stored in the mainframe computer as part of the data analysis code. Such calibration curves are prepared during the course of turbulence measurements with the calibration extending over the whole period during which turbulent velocity measurements are being made in the two-dimensional jet. The clock time at which any particular measurement is made in the jet is recorded, and the appropriate calibration curve for that particular measurement is developed by reading the anemometer output voltage at different velocity values at the clock time noted for the measurement. These data are then fitted to a polynomial calibration equation--

$$U = A_0 + A_1E + A_2E^2 + A_3E^3 + A_4E^4$$

This equation is then used for interpreting the anemometer output data obtained with the probe immersed in the turbulent jet.

The calibration data obtained for several different probes coincide closely enough so that it is possible to monitor the fouling of probes used for turbulence measurements in the plane jet by using a separate probe located in the calibration chamber. An example of the calibration data for two different probes is shown in Figure 3. At the start of the calibration period, there is an interval for which the data for the two probes do not coincide very closely. After this "seasoning" period, the calibration data do coincide.

The method to be used for incorporating fouling into the probe calibrations while avoiding the need to move the probes during a series of turbulence measurements is as follows.

1. Mount all probes in the water channel and let them operate in the measuring mode for the seasoning period.
2. Install each probe in turn in the calibration chamber and obtain calibration data for each.
3. Mount the probes to be used in the channel in the required positions. One probe to be used for monitoring the extent of fouling on all probes is left mounted in the calibration device. During the course of turbulence measurements, calibration data are obtained frequently using this monitoring probe.
4. Plot the calibration data as $E(t)$ with U as a parameter, fit the calibration data with linear functions, and use these functions for interpreting the turbulent velocity fluctuation data obtained from the two-dimensional jet.

In order to use this method to include probe fouling in the calibration functions used for interpreting turbulence data, there must be at least one actual calibration point for each probe. This point should be obtained after the period of "seasoning" in the same water as is being used for turbulence experiments.

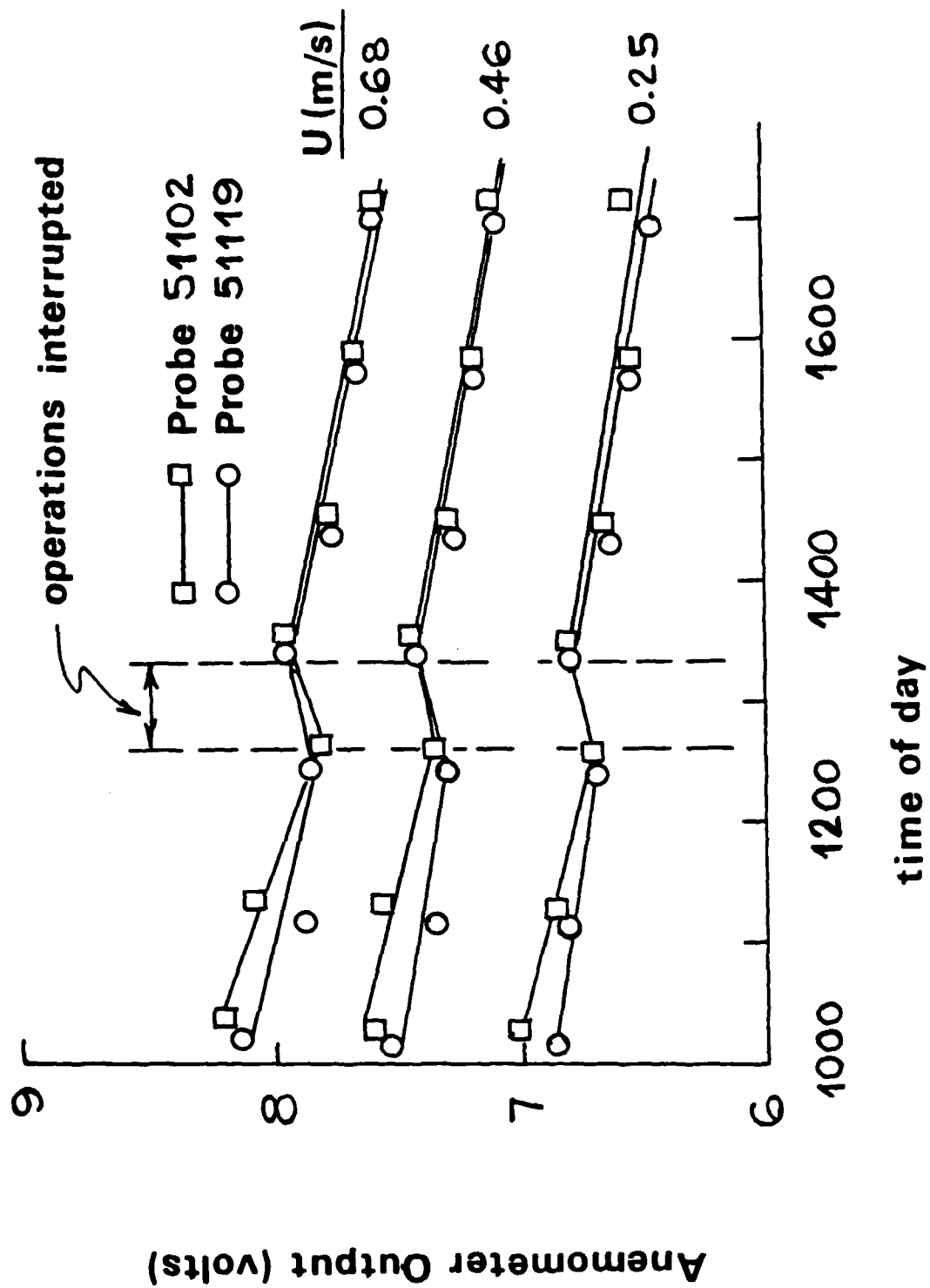


Figure 3. Comparison of Two Probes

Acknowledgement: This work is being supported by the Office of Naval Research - Contract Number N00014-85-K-0236.

Michigan Technological University



Houghton, Michigan 49931

College of Engineering
Department of Mechanical Engineering and
Engineering Mechanics 906/487-2551, 2561

4 December, 1985

R. E. Munn
IIASA
A-2361
Laxenburg, AUSTRIA

Dear Sir:

Enclosed are two (2) copies of the manuscript, "The skewness function of turbulence", which I am submitting for review and publication as a RESEARCH NOTE in Boundary-Layer Meteorology. If you have any questions or requests of me, please feel free to contact me at my university address.

Thank you.

Sincerely,

George Trevino
George Trevino
Associate Professor

GT/cal

Enclosure

The Skewness Function of Turbulence

George Treviño

Mechanical Engineering-Engineering Mechanics Dept.

Michigan Technological University

Houghton, MI 49931

Abstract

An invariant algebraic form of the skewness function for a particular type of nonhomogeneous turbulence is proposed. This form is identical for both the intended type of turbulence and classical isotropic turbulence, and as such provides a unifying link between turbulence whose statistics are spatially varying and turbulence whose statistics are spatially constant.

1. Introduction

In a recent publication (Treviño, 1985), it was suggested that traditional schemes for algebraically modelling the skewness function of turbulence typically lead to unfortunate analytical difficulties, especially when the turbulence is nonhomogeneous; in particular it was hinted that classical methods do not permit the absolutely invariant structure of nonhomogeneous turbulence to be explicitly emphasized (see also Treviño, 1982). It was therein also suggested that there nonetheless exists an invariant algebraic representation for the skewness function of such turbulence, but it can be formulated only for a peculiar class of nonhomogeneous behavior. That even the most general nonhomogeneous turbulence itself exhibits a characteristic invariant structure somewhere in its hierarchy of stochastic moments can also be fundamentally established (cf. Treviño, 1984), and how one uses this knowledge is the subject of a forthcoming communication (Treviño, to appear). The purpose of this note is to propose a representation for the skewness function, $S_{ijl}(q,r) = \langle u_i(\underline{x})u_j(\underline{x} + q)u_l(\underline{x} + r) \rangle^+$, which ^{is} identical for both isotropic turbulence and a unique type of nonhomogeneous turbulence. The skewness function only is to be addressed here, but in principle any relevant three-point tensor can be similarly represented. The advantage provided by the resulting algebraic form of $S_{ijl}(q,r)$ is that it furnishes a unifying link between turbulence whose statistics are independent of spatial location (viz. $\underline{x}$) and turbulence whose statistics are not. The information available in such a formulation is valuable in experimental studies of turbulence, where the time-decay of an isotropic nonstationary turbulence is duly analyzed in terms of the spatial-decay of a nonhomogeneous

⁺The t-dependence of u is here suppressed for convenience.

stationary turbulence (Comte-Bellot and Corrsin, 1966); to be sure, it requires that in the advected frame of reference, the convective-mechanism for the intended type of turbulence is characteristically independent of time (recall that $\underline{x} = \underline{U} t$), and that a closure for same is ultimately obtained by simply enforcing the attendant invariance of $S_{ijl}(\dots)$.

2. The form of $S_{ijl}(\dots)$

The skewness function is a tensor function of the two separation vectors, $\underline{q}$ and $\underline{r}$; for isotropic turbulence it is written in general as (cf. Batchelor, 1967)

$$S_{ijl}(\underline{q}, \underline{r}) = \sum_{n=1}^N A_n(\underline{q} \cdot \underline{q}, \underline{q} \cdot \underline{r}, \underline{r} \cdot \underline{r}) F_{ijl}^{(n)}(\underline{q}, \underline{r}), \quad (1)$$

where $\underline{q} \cdot \underline{q}$, $\underline{q} \cdot \underline{r}$, $\underline{r} \cdot \underline{r}$ are the absolute scalar invariants of the full orthogonal group. The well-known convective-tensor of classical turbulence results from Equ. (1) by setting either $\underline{q} = \underline{0}$, or $\underline{q} = \underline{r}$, to accordingly obtain

$$S_{ijl}(\underline{0}, \underline{r}) = A(r^2) r_i r_j r_l + B(r^2) \{r_i \delta_{jl} + r_j \delta_{il}\} + C(r^2) r_l \delta_{ij},$$

etc.. In this note the tensor $S_{ijl}(\underline{q}, \underline{r})$ is represented rather in terms of the invariants, $e_1 = \underline{q} \cdot \underline{r} / \underline{q} \cdot \underline{q}$ and $e_2 = \underline{q} \cdot \underline{r} / \underline{r} \cdot \underline{r}$, which are unique to the three points specified by $\underline{x}$, $\underline{x} + \underline{q}$, and $\underline{x} + \underline{r}$, and are also more general than the standard metric invariants listed earlier. e_1 and e_2 have the advantage that they remain invariant even though the turbulence itself may be undergoing some scale-change in its statistical structure, i.e. some nonhomogeneous characteristic which can be qualitatively described by changes in coordinate-axes scale only; a nonhomogeneity of the type, $C_{ij}(\underline{x} + \Delta \underline{x}, \underline{r}) = \eta C_{ij}(\underline{x}, \xi \underline{r})$, where $C_{ij}(\underline{x}, \underline{r}) = \langle u_i(\underline{x}) u_j(\underline{x} + \underline{r}) \rangle$ and (η, ξ) are simple scale-factors, is a typical example. In this type of nonhomogeneity, (η, ξ) are dependent only on $\underline{x}$, $\Delta \underline{x}$ (and possibly t), but are strictly independent of $\underline{r}$. In particular, it fol-

lows that e_1 and e_2 are such that $\xi q \cdot \xi r / \xi q \cdot \xi q \equiv e_1$, etc., so that these invariants do not lose their underlying isotropic flavor even under the effects of coordinate-axes scale-changes. Note that the classical isotropic invariants, $q \cdot q, q \cdot r, r \cdot r$, are not preserved under scale-change transformations, but that e_1 and e_2 are invariant under any combination of translations, rotations, reflections, and the prescribed scaling effects. This latter mode of isotropy is more correctly denoted an affine isotropy, in contradistinction to the metric isotropy of classical turbulence theories.

The invariant form of $S_{ijl}(q, r)$ is herein to be derived through successive differentiations of the related first-order tensor, $T_i(q, r)$; although not germane to the analysis, this latter tensor could conceivably correspond in turbulence theory to the triple-correlation, $\langle p(\underline{x})p(\underline{x} + \underline{q})u_i(\underline{x} + \underline{r}) \rangle$, where $p(\dots)$ is the fluid pressure; accordingly, the required associated invariants (Batechelor, loc. cit.) of this tensor are

$$T^{(1)}(q, r, a) = \underline{T} \cdot \underline{a} / \underline{a} \cdot \underline{a} \equiv \underline{T} \cdot \underline{a}$$

and

$$T^{(2)}(q, r, a) = \underline{T} \cdot \underline{a} / \underline{T} \cdot \underline{T} \equiv T^{(1)}(q, r, a) / |\underline{T}|^2 ;$$

here $\underline{a}$ is an arbitrary unit vector. Note that both of these forms are absolutely invariant under scale-change transformations of the type already considered, as well as under classical distance-preserving transformations; also note that $T^{(2)}(\dots)$ is not independent of $T^{(1)}(\dots)$. For the special case where $q = e, r$ both $T^{(1)}(\dots)$ and $T^{(2)}(\dots)$ are expressible in terms of the set of related independent invariants, $\{e, r \cdot a / r^2, r \cdot a\}$, and the resulting forms of $T_i(e, r)$ are

$$T_i^{(1)}(e, \underline{r}) = \{A(e) + r^{-2}B(e)\}r_i \quad \text{and} \quad T_j^{(2)}(e, \underline{r}) = \{r^2A(e) + B(e)\}^{-1}r_j ;$$

A and B are arbitrary functions of e only; because $T_i^{(1)}(\dots)$ and $T_j^{(2)}(\dots)$ are not independent, only one of them, say $T_i^{(1)}(\dots) \sim T_i(\dots)$, need be fully known (and subsequently used). The appropriate form of $S_{ijl}(q, \underline{r}) \sim$

$\hat{S}_{ijl}(e, \underline{r})$ is now found by determining $\frac{\partial T_i(e, \underline{r})}{\partial r_j \partial r_l}$, and since $\frac{\partial r}{\partial r_j} = r_j/r$ and $\frac{\partial e}{\partial r_j} = -er_j/r^2$ it can eventually be established that

$$\hat{S}_{ijl}(e, \underline{r}) = A(e, r)r_i r_j r_l + B(e, r)r_i \delta_{jl} + C(e, r)r_j \delta_{il} + D(e, r)r_l \delta_{ij}$$

is the intended form of the skewness function of turbulence. The conservation of mass condition provides two partial differential equations between the four (4) functions, $A(\dots)$, $B(\dots)$, $C(\dots)$, $D(\dots)$, so that only two of these are independently arbitrary. This same result was originally reported by Proudman and Reid (1954), but the functions here are not the same as those intended by them since these apply to a certain type of nonhomogeneous turbulence as well. The functions A, B, ..., can be determined from experimental measurements by choosing $\underline{r} = (r, 0, 0)$ (and e any suitable set of numbers) to obtain the non-zero correlations.

$$\hat{S}_{111}(e, \underline{r}) = Ar^3 + (B + C + D)r, \quad \hat{S}_{122}(e, \underline{r}) = \hat{S}_{133}(e, \underline{r}) = B r,$$

$$\hat{S}_{212}(e, \underline{r}) = \hat{S}_{313}(e, \underline{r}) = C r, \quad \hat{S}_{221}(e, \underline{r}) = \hat{S}_{331}(e, \underline{r}) = D r.$$

3. Concluding remarks

In dynamic analyses the applicable forms of $\hat{S}_{ijl}(e, \underline{r})$ are $\hat{S}_{ijl}(0, \underline{r})$ and $\hat{S}_{ijl}(1, \underline{r})$; these correspond to $\langle u_i(\underline{x})u_j(\underline{x})u_l(\underline{x} + \underline{r}) \rangle$ and $\langle u_i(\underline{x})u_j(\underline{x} + \underline{r})u_l(\underline{x} + \underline{r}) \rangle$ respectively, and the number of independently arbi-

trary functions needed to specify each of these is accordingly one (1). These forms are characteristically independent of $\underline{x}$, even though the turbulence itself may be fully nonhomogeneous. This more general invariance, again, results from the use of related non-metric invariants to algebraically represent $\hat{S}_{ijl}(\underline{q}, \underline{r})$, and is true only for nonhomogeneous turbulence which undergoes coordinate-axes scale-changes only. The particularly derived algebraic form of $\hat{S}_{ijl}(\dots)$ is valid for the collinear vector configurations only (i.e. $\underline{q} = e \underline{r}$), but the method readily generalizes to non-collinear vectors by introducing the collinear ratios, $e_x = q_x/r_x$, $e_y = q_y/r_y$, $e_z = q_z/r_z$ (Treviño, 1985); collinear ratios, note, are invariant even for scale-changes which differ from one coordinate-axis to another, viz. $\xi \rightarrow (\xi_x, \xi_y, \xi_z)$.

The physical meaning of the above result is that the role of inertia forces in the depicted type of nonhomogeneous turbulence (call it self-similar or self-preserving nonhomogeneous turbulence), as herein algebraically expressed, is identical to that for isotropic turbulence, viz. to transfer energy from one part of wave-number space to another without changing the total amount associated with any particular directional component. In other words, the related convective terms do indeed affect the distribution of energy but they do not affect the convolution and/or modulation that due to the nonhomogeneous nature of the turbulence exists between the energy associated with a given wave-number component, say $d Z(\underline{\kappa})$, and the energy associated with a different wave-number component, say $d Z(\underline{\kappa}')$; pressure terms and viscous terms are the only terms that in the concomitant hydrodynamic turbulence can influence this component interaction. Unlike homogeneous turbulence, the double correlation between different wave-number components of nonhomogeneous turbulence is such that

$$\langle dZ(\underline{\kappa}) dZ^*(\underline{\kappa}') \rangle \neq 0 \text{ when } \underline{\kappa}' \neq \underline{\kappa} ,$$

i.e. $dZ(\underline{\kappa})$ and $dZ^*(\underline{\kappa}')$ are non-orthogonal (Treviño, 1981, 1982), but the triple-correlation, $\langle dZ(\underline{\kappa}) dZ^*(\underline{\kappa}') dZ(\underline{\kappa}'') \rangle$, is such that $dZ(\underline{\kappa})$, $dZ^*(\underline{\kappa}')$, $dZ(\underline{\kappa}'')$ are orthogonal for $\underline{\kappa} \neq \underline{\kappa}' = \underline{\kappa}''$; * ~ complex conjugate.

In closing it remains exclusively to demonstrate skewness invariance for the suggested turbulence through purely ad hoc geometrical arguments on the spatial evolution of the probability density function (pdf) itself. For the attendant case of two (2) scaling effects, the most general self-preservation in the pdf requires that

$$p(u(\underline{x} + \Delta \underline{x})) \sim \beta p(\alpha u(\underline{x})) ,$$

where α , β are strictly $\underline{x}$ -dependent scale-factors and u is the longitudinal component of $\underline{u}$, i.e. $\underline{u} = (u,v,w)$. In terms of coordinate-axes transformations the relevant algebraic scheme is

$$u(\underline{x} + \Delta \underline{x}) \sim u' = \alpha u$$

$$p(u(\underline{x} + \Delta \underline{x})) \sim p' = \beta p ,$$

where $\alpha \beta \equiv 1$, i.e. the transformation is area-preserving, this constraint demanded by the pdf property that $\int p(\xi) d\xi \equiv 1$ always. Skewness, recall, is the first measure of dis-symmetry in the pdf (the second measure is super skewness, etc.), and simple coordinate-axes scale-changes of the intended type cannot alter this characteristic (e.g. an odd function remains odd under homogeneous coordinate-axes "stretching" or "shrinking"). Skewness therefore is preserved in the considered self-similar evolution. Corresponding arguments apply to the case where $p \sim p(v)$, $p \sim p(w)$, $p \sim p(u,v,w) = p(\underline{u})$,

etc., and skewness invariance is easily verified in the special case where $p\{u(x)\}$ is symmetric about the mean-value for all x (for which the skewness is always zero).

Acknowledgements

The author gratefully acknowledges the Office of Naval Research (USA) through whose support (Contract No. N00014-85-K-0236) this research was completed. The author also acknowledges the collaboration of Professor D.W. Hubbard of Michigan Technological University.

References

- Batchelor, G.K.: 1967, Theory of Homogeneous Turbulence, Cambridge University Press, 197 pp.
- Comte-Bellot, G., and Corrsin, S.: 1966, 'The Use of a Contraction to Improve the Isotropy of Grid-Generated Turbulence', J. Fluid Mech. 25, 657-682.
- Proudman, I., and Reid, W.H.: 1954, 'On the Decay of a Normally Distributed Homogeneous Turbulent Velocity Field', Phil. Trans. Roy. Soc. London 247A, 163-189.
- Treviño, G.: 1981, 'Effects of Inhomogeneities in Atmospheric Turbulence on the Dynamic Response of an Aircraft', J. Aircraft 18, 844-848.
- Trevino, G.: 1982, 'An Introduction to the Theory of Nonhomogeneous Turbulence', Tex. J. Science 34, 35-56.
- Treviño, G.: 1984, 'Topological Invariance in Stochastic Analysis', in Proceedings of the Fourth International Conference on Mathematical Modelling (X.J.R. Avula, et. al., eds.) Pergamon Press, 1006 pp.
- Treviño, G.: 1985, 'Comments on a Note on the Height Dependence of the Skewness and Kurtosis of the Vertical Turbulent Velocity in the Neutral Surface Layer', Bound.-Layer Met., in press.

Treviño, G.: to appear, 'The Invariant Theory of Nonhomogeneous Turbulence'.

Decay of Self-Similar Turbulence

George Treviño

Mechanical Engineering-Engineering Mechanics Dept.

Michigan Technological University

Houghton, MI 49931

Abstract

The invariance concepts of affine geometry are invoked to establish time-independence for the convective mechanism of self-similar isotropic turbulence. The decay-law for the integral scale during the initial period follows immediately.

Submitted to Physics of Fluids

1. Introduction

Affine geometry and affine transformations have recently begun to find their way into the scheme of "mechanics-type" problems in such fields as flutter analysis¹⁻⁵, shaky structures⁶⁻⁹, and turbulence¹⁰⁻¹³; their use in qualitatively describing elastic deformations in solid mechanics has been known for some time¹⁴. The purpose of this note is to adapt recent research results¹⁵⁻¹⁹, regarding invariant algebraic representations of nonstationary stochastic processes, to the determination of the characteristic time-invariant structure of self-similar isotropic turbulence; a closure for same is accordingly formulated, and the decay of the integral scale during the initial period follows immediately. Self-similar turbulence, because it can be geometrically described by uniform scale-changes along the abscissa and ordinate axes respectively, can be characterized as turbulence which is invariant under the affine group^{20,21}.

The notion of a self-similar, or self-preserving, turbulence was introduced by Kármán and Howarth in 1938²². For the most part the self-similar device is principally a mathematical one, even though the assumption itself is quite valid in the "dissipation range" of the turbulence and in the "large-scale" structure of the turbulence; in the dissipation range the self-similar structure is $f(r,t) \approx 1 - r^2/2\lambda^2$ (during all phases of the decay²³), while in the large-scale structure self-similarity is reflected in the time-independence of $f(\dots)$ ²⁴.

The essential idea of such a flow is that during the decay, the correlation function, $f(r,t)$, evolves in time, but its geometric shape at any time instant is similar to its geometric shape at any other time instant; i.e., the integral scale

$$\Lambda(t) = \int_0^{\infty} f(r,t) dr ,$$

and the dissipation scale,

$$\lambda(t) = \left\{ - \frac{2f}{r^2} \right\}_{r=0}^{-1/2} ,$$

both increase with time, but the r -dependence of $f(\dots)$ is defined by the same function for all time; self-similar turbulence, therefore, is more idealized than is isotropic turbulence. Nonetheless, its practical value is that it provides theoretical results that can be applied in some "asymptotic fashion" to the analysis of the "limiting behavior" of real turbulent flows. Kármán and Howarth went on to suggest that such a decay could be adequately described by an infinite family of self-preserving correlation functions, one of which is of the form,

$$f(r,t) = \exp\left\{ - \frac{r^2}{8\nu(t-t_0)} \right\} .$$

2. The analysis

The inertial-transfer tensor of turbulence theory is defined here as

$$S_{ijl}(\underline{r},t) = \lim_{\tau \rightarrow 0} \{ \langle u_i(\underline{x},t) u_j(\underline{x},t + \alpha\tau) u_l(\underline{x} + \underline{r},t + \tau) \rangle \} ;$$

this particular definition is deliberately formulated in terms of the two time-lags, $\tau_1 = \alpha\tau$ and $\tau_2 = \tau$, in order to illustrate the inherent three-(space-time) point character of $S_{ijl}(\underline{r},t)$. For self-similar turbulence this quantity is independent of t , and therefore does not decay with time; this result derives from the fact that the "affine nature"¹⁶ of self-similarity requires that three-point statistical moments, when algebraically written in terms of the ratio invariant between the two time-lags, viz. α , necessarily be independent of the choice of reference origin (i.e.

the time, t , at which the moments are measured). In terms of the geometry of decay this invariance is a consequence of the fact that any self-similar time-evolution in the probability density function (pdf) of the turbulence automatically preserves skewness; i.e., any time-decay such that

$$p\{u(\underline{x}, t + \Delta t)\} \approx \beta p\{\gamma u(\underline{x}, t)\},$$

leaves the amount of "dis-symmetry" in $p\{\dots\}$ unaltered¹⁹. Note that self-similarity requires that $C_{ij}(\underline{r}, t) = \langle u_i(\underline{x}, t) u_j(\underline{x} + \underline{r}, t) \rangle$ evolve in time in a self-preserving mode, or in more practical terms that $F(\underline{r}, t) = u^2(t) f(\underline{r}, t)$ undergoes changes in its coordinate-axes scales only, and no changes in its $\underline{r}$ -functional form. In particular, self-similar decay requires that $F(\underline{r}, t + \Delta t) \approx \eta F(\xi \underline{r}, t)$, the scale-factors η and ξ being independent of $\underline{r} = |\underline{r}|$, and dependent only upon t (and Δt); typically, $\eta \sim$ velocity scale factor while $\xi \sim$ length scale factor.

It then follows¹⁵⁻¹⁹ that for the considered turbulence $S_{ijl}(\underline{r}, t) \rightarrow \hat{S}_{ijl}(\underline{r}, \alpha)$, and more importantly that ultimately $\{u^3(t)k(\underline{r}, t)\}$ is also independent of t and dependent only on $\underline{r}$ and α ; here $u = \langle u_i u_i \rangle^{1/2} \sim u(t)$, and $k(\underline{r}, t)$ is the single arbitrary function required to completely specify in the classical theory the $\underline{r}$ -dependence of $S_{ijl}(\dots)$. To be sure, for self-similar turbulence the tensor $S_{ijl}(\underline{r}, t)$ can be written instead as²⁵

$$\hat{S}_{ijl}(\underline{r}, \alpha) = \sum_{n=1}^N g_n(\underline{r}, \alpha) A_{ijl}^{(n)}(\underline{r}),$$

where N is here an unspecified finite number and the $g_n(\dots)$ are invariant functions on the affine $(\underline{r}, t)$ -space; the key result in the above is not the value of N nor the unique form of the $g_n(\dots)$, but rather it is the explicit time-independence of the algebraic representation. The equation for the

decay of the integral scale is now found by first integrating the Kármán-Howarth equation,

$$\frac{\partial(u^2 f)}{\partial t} = u^3 \{ r^{-4} \frac{\partial}{\partial r}(r^4 k) \} + 2\nu u^2 r^{-4} \frac{\partial}{\partial r}(r^4 \frac{\partial f}{\partial r}) ,$$

from $r = 0$ to $r = \infty$, and accordingly

$$\frac{d(u^2 \Lambda)}{dt} = I(\alpha) - \left(\frac{2}{t - t_0} \right) u^2 \Lambda , \quad (1)$$

where

$$I(\alpha) = \int_0^\infty u^3 r^{-4} \frac{\partial}{\partial r}(r^4 k) dr \equiv \text{a parameter-dependent constant} ,$$

and for a self-preserving correlation function of the type already specified

$$\int_0^\infty r^{-4} \frac{\partial}{\partial r}(r^4 \frac{\partial f}{\partial r}) dr = - \frac{\Lambda(t)}{\nu(t - t_0)} ;$$

because of the mode in which the convective-tensor is constructed through the Navier-Stokes equation, the turbulent decay of interest ^{is recovered} Λ for $\alpha = 0$ only. The power-law decay of kinetic energy, $u^2 \propto (t - t_0)^{-n}$, subsequently provides

$$\Lambda(t) = (t - t_0)^{1+n} \hat{I}/3 , \quad \hat{I} \equiv I(0) , \quad (2)$$

as the equation describing the general integral scale decay of self-similar turbulence; for the specific case where $n = 1$ and $u^{-2} \propto (t - t_0)$, the so-called "linear decay law", the corresponding initial decay of the integral scale is

$$\Lambda(t) = \frac{\hat{I}}{3} (t - t_0)^2 . \quad (3)$$

3. Concluding remarks

Decay laws of the form specified by Equ. (2) have not been found experimentally²⁶. One reason for this is that even without the self-similarity

characteristic, isotropic turbulence is itself difficult to generate in practice, since virtually all mechanisms for creating turbulence in the laboratory usually confer a preference of at least one particular spatial direction in the flow. Another is that it is common practice in experimental analyses of turbulence to study the time-decay of an isotropic nonstationary turbulence by examining the spatial decay of a nonhomogeneous (necessarily) stationary (i.e. steady-state) turbulence, and accordingly imposing the "frozen turbulence" assumption to transform time-derivatives into spatial derivatives (and vice-versa) as²⁶

$$\left\{ \frac{\partial}{\partial t} \right\} \text{following the mean motion} \approx \bar{U} \frac{\partial}{\partial x}, \quad \bar{U} \sim \text{mean-flow velocity.}$$

This scheme, although appealing and quite practical, has the defect that in a nonhomogeneous turbulence pressure terms play an important role in the decay, even for the total-energy case; indeed, if the spatial evolution of the covariance of such a flow is in fact of a self-similar nature, then theoretical results spatially analogous to the foregoing¹² establish that convective terms are not at all the proponents of the "spatial decay" of the turbulence—pressure terms are the only carriers of nonhomogeneous energy-transfer while convective-transfer is necessarily spatially constant, i.e. in a truly nonhomogeneous self-similar flow pressure terms cannot be neglected. In the study of total-energy decay of homogeneous isotropic turbulence pressure terms are not only negligible, they are indentially zero²³. Decay laws of the above form have also not otherwise been reported in the literature (see, for example, P.G. Saffman²⁷); in fact, in order to obtain with the herein proposed analysis decay results comparable to Saffman's, $\underline{n}$ in the derived power-law decay for kinetic energy would have to be $\underline{n} = -3/5$.

Saffman's results are obtained by imposing the analytical consequences of the invariance of a particular integral of the velocity-covariance tensor, $C_{ij}(\underline{r}, t)$ and not by imposing, as done here, the consequences of the newly found time-invariance of the inertial-transfer tensor.

In closing it remains to iterate that the above results can only be established through affine invariants modelling, i.e. by modelling using parameters that remain invariant even though the turbulence itself is undergoing temporal changes in its coordinate-axes scales—it cannot be established through metric invariants modelling without introducing some germane postulate about the metric structure of $S_{ijl}(\dots)$. In other words, by modelling the related self-similar structure of $S_{ijl}(\dots)$ through non-metric invariants, invariants which are unique and peculiar to this type of evolution, it is then possible to analytically establish a rather fundamental characteristic about the convective-transfer in self-similar turbulent decay.

Acknowledgement

This activity was kindly supported by the Office of Naval Research, Contract No. N00014-85-K-0236. The author also acknowledges the collaboration of Professor D.W. Hubbard of Michigan Technological University.

- ¹G.A. Oyibo, AIAA Journal 21, 283 (1983).
- ²G.A. Oyibo, AIAA Journal 21, 767 (1983).
- ³P.A.A. Laura, AIAA Journal 22, 574 (1984).
- ⁴G.A. Oyibo, AIAA Journal 22, 575 (1984).
- ⁵G.A. Oyibo and E.J. Brunelle, AIAA Journal 23, 296 (1985).
- ⁶W. Wunderlich, Acta Mechanica 42, 171 (1982).
- ⁷B. Wegner, Acta Mechanica 53, 163 (1984).
- ⁸G. Treviño, Acta Mechanica 55, 273 (1985).
- ⁹B. Wegner, Acta Mechanica 55, 275 (1985).
- ¹⁰N. Pan Thien and R.A. Antonia, ZAMM 62, 129 (1982).
- ¹¹G. Treviño, J. Aircraft 22, 827 (1983).
- ¹²G. Treviño, Bound.-Lay. Meteor., in press.
- ¹³O. Chiba, Bound.-Lay. Meteor., in press.
- ¹⁴I.S. Sokolnikov, Mathematical Theory of Elasticity (McGraw-Hill, New York, 1956). p. 6.
- ¹⁵G. Treviño, J. Sound Vib. 90, 590 (1983).
- ¹⁶G. Treviño, in Proceedings of the Fourth International Conference on Mathematical Modelling, edited by X.J.R. Avula, et. al. (Pergamon Press, New York, 1984), p. 278.
- ¹⁷G. Treviño, J. Sound Vib. 94, 154 (1984).
- ¹⁸G. Treviño, J. Sound Vib. 99, 576 (1985).
- ¹⁹G. Treviño, J. Sound Vib. 102, 599 (1985).
- ²⁰G.B. Gurevich, Foundations of the Theory of Algebraic Invariants (P. Noordhoff, The Netherlands, 1964).

- ²¹D. Gans, Transformations and Geometries (Appleton-Century-Crofts, New York, 1969).
- ²²T. von Kármán and L. Howarth, Proc. Roy. Soc. A 164, 192 (1938).
- ²³G.K. Batchelor, Theory of Homogeneous Turbulence (Cambridge, 1967).
- ²⁴F.N. Frenkiel, P.S. Klebanoff, and T.T. Huang, Phys. Fluids 22, 1606 (1979).
- ²⁵D. Hilbert, Math. Ann. 36, 473 (1890).
- ²⁶G. Comte-Bellot and S. Corrsin, J. Fluid Mech. 25, 657 (1966).
- ²⁷P.G. Saffman, Phys. Fluids 10, 1349 (1967).

Speaker: Davis W. Hubbard
Employer: Michigan Technological University
Phone: (906) 487-2140
AIChE Member: Yes X No

Address: Dept. of Chemistry and Chemical Engr.
Michigan Technological University
Houghton, MI 49931

Co-Author: Eric J. Hine
Employer: Michigan Technological University
Phone: (906) 487-2047
AIChE Member: Yes X No

Address: Dept. of Chemistry and Chemical Engr.
Michigan Technological University
Houghton, MI 49931

Co-Author: Lisa R. Heydenburg
Employer: Michigan Technological University
Phone: (906) 487-2047
AIChE Member: Yes No X

Address: Dept. of Chemistry and Chemical Engr.
Michigan Technological University
Houghton, MI 49931

Co-Author: George Trevino
Employer: Michigan Technological University
Phone: (906) 487-2865
AIChE Member: Yes No X

Address: Dept. of Mech. Eng. and Eng. Mech.
Michigan Technological University
Houghton, MI 49931

Abstract for meeting program book (Maximum 60 words, including title):

Title: TURBULENCE IN WATER JETS: A TWO-POINT CLOSURE

"Submergence and flow effects on turbulence in a two-dimensional water jet
were studied using hot-film anemometry. The velocity fluctuations measured were
used to formulate a two-point closure scheme for the stochastically averaged
Navier-Stokes equations assuming self-similar decay for the probability density
function. Measurements of the triple velocity correlations support this."

TO BE COMPLETED BY THE SESSION CHAIRMAN. IT WILL THEN BE RETURNED BY THE SESSION CHAIRMAN TO THE SUBMITTING AUTHOR, THE MEETING PROGRAM CHAIRMAN, AND THE AIChE MEETINGS DEPARTMENT.

Meeting: Session #: Paper #: a b c d e f (circle one)

Sponsoring Group/Division: Area:

Date of Session: Time:

SESSION CHAIRMAN'S REVIEW

Date:

- Accepted for meeting above.
 AIChE Meetings Department notified.
 Submitting author notified.
 Meeting Program Chairman notified.

Signed:

FINAL PROGRAM

AICHE 1986 ANNUAL MEETING
NOVEMBER 2-7, 1986
MIAMI BEACH, FLORIDA

SYMPOSIUM

TURBULENT FLOW AND
TURBULENT TRANSPORT PROCESSES

2 Sessions

Sponsored by Group 1J: Fluid Mechanics

Session Co-Chairs

Professor Robert S. Brodkey Professor James C. Hill
Chemical Engineering Dept. Chemical Engineering Dept.
The Ohio State University Iowa State University
Columbus, Ohio 43210 Ames, Iowa 50011
(614) 422-2609 (515) 294-4959

Session 1 - Turbulence

AM - Introduction R.S. Brodkey

WHAT DOES DETERMINISTIC CHAOS AND TURBULENCE HAVE TO DO WITH EACH OTHER?

AM J. M. OTTINO (SPEAKER), CALTECH, PASADENA, CA Paper No. a

"In this talk we will review, at an introductory level, the essential elements of the theory of chaos in dynamical systems (both dissipative and Hamiltonian) giving precise definitions of what is meant by chaos and discuss examples in such a way as to highlight the state of current development."

THE ROLE OF WALL STREAKS IN TURBULENT TRANSPORT

AM B.R. CIRCELLI, MOUNTAIN VIEW, CA; P. MOLIN AND J. KIM, AMES RES. CENTER, NASA MUFFETT FIELD, CA AND J.B. McLAUGHLIN (SPEAKER), CLAREMONT UNIV., POTSDOM, NY Paper No. b

"We have performed computer experiments with turbulent channel flow using a high resolution (128³) pseudospectral computer program. Two-dimensional wavenumber spectra (downstream and spanwise coordinates) were used to identify spatial coherence in the viscous wall region, and a wave suppression technique was used to determine the dynamical significance and origin of the coherence

DOWNSTREAM EXCITATION OF AN AXISYMMETRIC JET

AM B. GHORASHI (SPEAKER) AND G. RAMAN, CLEVELAND STATE UNIV., CLEVELAND, OH; E.J. RICE AND J.H. MILES, LEWIS RES. CENTER, NASA, CLEVELAND, OH

"Acoustic excitation and parameters influencing jet mixing were experimentally investigated. The basic premise was that an external perturbation triggers the evolution and growth of coherent structures in the free shear layer. The results showed a marked increase in turbulent intensity in the Strouhal number range of 0.4 to 0.8. The effect of acoustic excitation was more pronounced as the sound levels were increased."

VORTICITY FIELD PROPERTIES IN THE INTERMITTENT REGION OF A PLANE JET

AM J.F. FOSS (SPEAKER), S.K. ALI, R.C. HAN AND C.M. LAWS, MICHIGAN STATE UNIV., E. LANSING, MI

"Transverse vorticity ω_z measurements in a large single streamwise layer are used to characterize the intermittent region of the development. With $Re_0(0) > 5500$, the flow contains a wide range of scales. The relationship of the ω_z properties to the coherent and dissipative regions is investigated."

TURBULENCE IN WATER JETS: A TWO-POINT CLOSURE

AM E.J. HINE, D.W. HUBBARD (SPEAKER), L.R. HEYDENBURG AND G. TREVINO, MICHIGAN TECHNOLOGICAL UNIV., HOUGHTON, MI

"Submergence and flow effects on turbulence in a two-dimensional jet were studied using hot-film anemometry. The velocity fluctuations measured were used to formulate a two-point closure scheme for the locally averaged Navier-Stokes equations assuming self-similar decay and probability density function. Measurements of the triple velocity correlations support this."

LAMINARIZATION OF TURBULENT FLOW IN A TUBE OWING TO A STEP INCREASE IN TEMPERATURE

AM L.R. COLLINS (SPEAKER) AND S.W. CHURCHILL, UNIV. OF PENNSYLVANIA, PHILADELPHIA, PA

"An essentially abrupt increase in temperature from 300 K to 400 K, owing to thermally stabilized combustion, produces a corresponding decrease in Reynolds number from 4000 to 800 in a 9.52 mm tube. The subsequent decay of turbulence and process of laminarization in this transitional axisymmetric stream have been investigated using a modified

FINAL PROGRAM

AICHE 1986 ANNUAL MEETING
NOVEMBER 2-7, 1986
MIAMI BEACH, FLORIDA

SYMPOSIUM

TURBULENT FLOW AND TURBULENT TRANSPORT PROCESSES

2 Sessions

Sponsored by Group 1j: Fluid Mechanics

Sessions Co-Chairs

Professor James C. Hill
Chemical Engineering Dept.
Iowa State University
Ames, Iowa 50011
(515) 294-4959

Professor Robert S. Brodwin
Chemical Engineering Dept.
The Ohio State University
Columbus, Ohio 43210
(614) 422-2609

Session 2 - Mixing

MECHANISMS OF TURBULENT MIXING FROM NUMERICAL SIMULATIONS P.M.

C.H. GIBSON (SPEAKER), UNIVERSITY OF CALIFORNIA, SAN DIEGO, LA JOLLA, CA 92037

"University similarity hypotheses proposed by Gibson (1966) that turbulent mixing should depend critically on the local rate-of-strain and scalar fields of arbitrary Prandtl number is supported by recent numerical simulations. Mechanisms of strain-mixing occur where the scalar field is very large and very small, and where the vorticity is very large."

MECHANISM OF TURBULENT MASS TRANSFER TO A WALL P.M.

E. VASSILIADOU, KONINKLIJKE SHELL LAB., AMSTERDAM AND T.J. HANSEN, UNIV. OF ILLINOIS, URBANA, IL

"In previous work from this laboratory it was shown that the energy dissipation in oscillations containing a small fraction of the energy budget is concentrated between a turbulent fluid and a solid boundary. This idea is further developed in this presentation."

APPLICATION OF PARTICLE ARRIVAL RATE STATISTICS TO TURBULENT SHEAR FLOW A.M.

Paper No. 198

H.S. BERMAN (SPEAKER), ARIZONA STATE UNIV., TEMPE, AZ

"The probability of finding a particle in one probe volume and not in a second volume downstream of the first can be measured in a turbulent shear flow. It is shown how approximations to the Lagrangian velocity correlation functions are obtained from a series of such particle counting experiments."

MIXING AND ISOTROPY IN HOMOGENEOUS TURBULENCE A.M.

Paper No. 199

J. LEE (SPEAKER), Flight Dynamics Lab., WRIGHT PATTERSON AFB, OH

"We examine a sequence of lower order truncations of the 3D Navier-Stokes equations in Fourier space. The main part of this talk is on the conservative (inviscid) dynamics to demonstrate mixing (ergodicity) on the energy-helicity surface, isotropy of the covariance tensor, orbital instability, Kolmogorov entropy, etc. Some results of dissipative (viscous) dynamics are also presented."

IMAGE PROCESSING OF TRACER PARTICLES IN AGITATED TANKS A.M.

Paper No. 200

G. PATTERSON (SPEAKER), TEXAS A&M UNIV., COLLEGE STATION, TX

"Simultaneous multi-point velocity measurements are necessary to understand the three dimensional nature of time-dependent turbulent flows. A technique based upon image processing of neutrally buoyant tracer particles recorded by stereoscopic cine photography is reviewed which is capable of providing such information. The major result of the technique is an accumulation of three dimensional particle paths."

STUDY OF TWO PHASE FLOW BY LASER IMAGE PROCESSING A.M.

Paper No. 201

E.L. HANZEVACK, C.B. BOHRS, JR. AND C.H. JU (SPEAKER), UNIV. OF SOUTH CAROLINA, COLUMBIA, SC

"Digital image processing is used to study turbulent two-phase flow. A pulsed laser freezes the motion, which is viewed by a CCD scanner. The intensity distribution of light affected by droplets is measured by an array of sensors and stored. Computer software reconstructs droplet patterns from these spatially distributed grey levels."

MIXING WITH DIFFUSION AND CHEMICAL REACTION IN SIMPLE SHEAR FLOW AS APPLIED WITHIN THE VELOCITY MICROSCALE

P.M.
C.S. LEE (SPEAKER), S.H. CHEN AND J.J. OU, UNIVERSITY OF ROCHESTER, ROCHESTER, NY
Paper No. 10

"Simple shear flow is frequently presumed for the fluid dynamics within the velocity microscale to model the performance of a chemical reactor involving turbulent mixing. The present work is aimed evaluating the accuracy of such an approach in terms of the two dimensionless groups relevant to such transport processes in reactive systems. The results suggest that in the region where $Pe \geq 10^6$ or $Sc \geq 1$ the model is quite satisfactory in representing the spatial distribution of the chemical species."

TURBULENT MIXING PARAMETERS FROM REACTIVE MIXING MEASUREMENTS

P.M.
U.V. SHENOY (SPEAKER) AND H.L. LOCH, CARNEGIE-MELLON UNIV., PITTSBURGH, PA
Paper No. 11

"The scalar probability density function can be computed by differentiating twice experimental data on mean reactant concentrations of very rapid, irreversible reactions. The intensity of segregation can be calculated (without knowledge of the PDF) by integrating the same experimental data. Relevant equations are derived and applied to available experimental measurements."

EVALUATION OF CLOSURE MODELS FOR PARALLEL-CONSECUTIVE REACTIONS

P.M.
J.M. TABBELL (SPEAKER), PENN STATE UNIV., UNIV. PARK, PA AND R.V. MEHTA, PESU, IMPERIAL COLLEGE, LONDON, ENGLAND
Paper No. 12

"Turbulent mixing and chemical reaction experiments using the azo-coupling of α -naphthol with diazotized sulfanilic acid have been conducted in a highly segregated plug flow turbulent reactor. Experimental data from these experiments is used to evaluate 7 closure models for complex reactions. All closure models work well for single reactions, but only 2 are successful for complex reactions."

A SPECTROPHOTOMETRIC, IN-SITU, CORRELATION MEASUREMENT TECHNIQUE FOR STUDYING SELECTIVITY IN REACTIVE TURBULENT MIXING

P.M.
Y.C. LIN (SPEAKER), R.J. ADLER AND R.V. EDWARDS, CASE WESTERN RESERVE UNIV., CLEVELAND, OH
Paper No. 13

"An in-situ, spectrophotometric method applicable to turbulent flows has been developed to measure conversion and selectivity of Bourne's series-parallel azo dyestuff reactions. The method employs multiple light beams, fiber optics, light filters, and time-shared detection. New transient data reported for a batch reactor will be useful in testing mixing-reaction models."

EFFECTS OF HEAT RELEASE IN CHEMICALLY-REACTING, TURBULENT MIXING

P.M.
P.M. MCMURTRY (SPEAKER), FLOW IND., KENT, WA; J.J. RILEY, FLOW IND., SEATTLE, WA AND R.R. METCALFE, FLOW IND., KENT, WA

"The effects of chemical heat release on the fluid dynamics of a turbulent mixing layer are studied by Direct Numerical Simulation of a three-dimensional, time dependent simulations have been performed to study three-dimensional structures which play an important role in mixing."

NUMERICAL SIMULATION OF MASS TRANSFER AT A SHEARED GAS-LIQUID INTERFACE

P.M.
D.D. BACK AND M.J. MCCREADY (SPEAKER), UNIV. OF NOTRE DAME, INDIANAPOLIS, IN

"Transfer of a slightly soluble gas across a sheared gas-liquid interface is determined by numerical solutions to the time varying advection-diffusion equation. The flow field is calculated from an Orr-Sommerfeld equation with a variable stress surface boundary condition. Good agreement between calculations and gas absorption data is found."

MAXIMAL DISORDER IN THE REPRESENTATION OF CONVECTIVE TRANSPORT IN TURBULENCE

P.M.
E.E. O'BRIEN (SPEAKER) AND A. DECKERT, STATE UNIV. OF NEW YORK, ALBANY, NY AND V. ESWARAN, CORNELL UNIV., ITHACA, NY

"A critical study of the applicability of a maximal 'advection-diffusion' to two-point pdf representations of turbulent convection of reactants. The study compares correlation function evolution, and other statistical measures, obtained using the hypothesis with results obtained by direct numerical simulation."

A THEORETICAL ANALYSIS OF GAS ABSORPTION ACCOMPANIED BY AN EXOTHERMIC BIOMOLECULAR REACTION IN A TURBULENT LIQUID FILM

P.M.
C.A. PETTY AND E.A. GRULKE (SPEAKER), MICHIGAN STATE UNIV., EAST LANSING, MI

"Recent experiments show that the absorption rate of O_2 into an aqueous film containing NaOH is less than anticipated by classical film theory. This is explained using the hypothesis that the turbulent reacting systems has a different constitutive behavior than film theory counterpart."

END

DTIC

6-86