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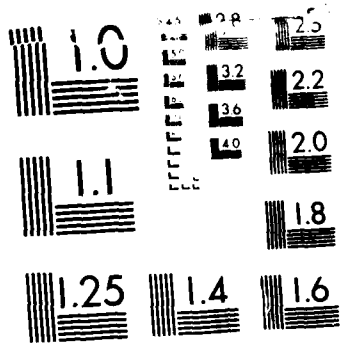
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Role of the Boundary Conditions in the Problem of the Linear Stability of the Sedov Point Blast Solution

D. L. BOOK

Laboratory for Computational Physics

May 16, 1986

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19 ABSTRACT (Continue on reverse if necessary and identify by block number) The linear stability of those Sedov blast wave similarity solutions for which the flow is homologous behind the shock, of which the best-known example is the Primakoff point blast model, along with that of their two-dimensional counterparts, has previously been demonstrated analytically by Bernstein and Book (1980). Their conclusion that the eigenmodes are stable applies to all perturbations in the three-dimensional case, and to all flutelike ($k_z = 0$) modes in the two-dimensional case. Gaffet (1984) has argued that the treatment did not allow for a jump in the entropy of the perturbations at the shock front, and so must be incorrect. The analysis of Bernstein and Book, however, can easily be extended to include anisotropic perturbations. The additional terms, which have the same time dependence as the basic state, and therefore cannot give rise to instability, in any case drop out of the treatment. The resulting eigenvalue problem is identical with that solved by Bernstein and Book, whose conclusion that the Primakoff-Sedov blast waves are stable is therefore restored.					
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ROLE OF THE BOUNDARY CONDITIONS IN THE PROBLEM OF
THE LINEAR STABILITY OF THE SEDOV POINT BLAST SOLUTION

1. Introduction

A highly localized release of energy in an unbounded pressureless ideal gas medium gives rise to an expanding spherical blast wave. Because of the absence of a characteristic length scale in the problem, the resulting flow is self-similar. It consists of a shock followed by a rarefaction wave. If the density $\bar{\rho}$ of the background medium satisfies a power-law distribution ($\bar{\rho} \sim R^{-\alpha}$, where R is the distance from the blast center), then the solution of the problem can be reduced to quadratures. This analysis was carried out by Sedov (1946, 1959) and independently by Taylor (1950) and Von Neumann (1947). The analogous implosion problem was treated by Guderley (1942).

Numerous workers have investigated the stability of the Sedov point blast solutions. In its full generality the problem can be stated as follows: Given a particular Sedov solution, specified by the values of α and the adiabatic index γ , for what wave numbers l, m does the deviation in the shock amplitude associated with an infinitesimal perturbation about this solution grow faster than the shock radius S as $S \rightarrow \infty$? If no such wave numbers exist, the unperturbed point blast solution is stable. At present, a solution of this general problem is lacking. As we shall note, very few results of any sort have been established beyond dispute.

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It is not trivial to demonstrate the stability even of planar shocks, a problem treated originally by D'yakov (1954) and later by Erpenbeck (1962). They found that for a wide class of differing fluids (characterized by different equations of state), planar shock waves propagating in a uniform medium are stable. The physical mechanism is easy to describe. A small ripple in the shock front gives rise to divergence (respectively, convergence) in the curved portions ahead of (behind) the main front. These regions become weaker (stronger) than the unperturbed shock, hence propagate slower (faster) than average, thus reducing the amplitude of the ripple. Evidently the longer the perturbation wavelength, the weaker the stabilizing effect of this mechanism, so by analogy we would expect $\lambda = 0$ perturbations around the Sedov solutions to be the most unstable.

When variation of the density of the background medium is taken into account, however, this mechanism becomes obscure. For $\beta > 3$, the unperturbed Sedov shocks accelerate with increasing radius. It is conceivable that the ripples ahead of the shock can run away, while those behind fall further behind, leading to instability. Lerche and Vasyliunas (1976) and Isenberg (1977), who treated the case $\gamma = 1$, have claimed that for some values of γ , Sedov shocks are unstable for all values of β . Their conclusions are remarkable in predicting instability at short wavelength ($\lambda \gg 1$). Newman (1979) has disputed the results, on the ground that Lerche and Vasyliunas (1976) and Isenberg (1977) improperly treated the boundary conditions at the shock front.

Bernstein and Book (1980) studied the stability of a restricted class of the Sedov solutions, those in which the velocity behind the shock is proportional to the distance R from the origin (homologous or uniform expansion). This type of blast wave was apparently first considered by Primakoff (see Courant and Friedrichs 1948). He found that for $\bar{v} = \text{const.}$, the point blast similarity solution can be written explicitly in closed form when $\gamma = 7$. Keller (1956) generalized the results to nonuniform densities and found for each value of $\gamma > 1$

the value of Ω for which explicit solutions with homologous velocity distributions exist.

Bernstein and Book (1980) linearized the fluid equations about the Primakoff blast solution and applied the first-order Rankine-Hugoniot equations at the shock location. The resulting eigenvalue problem reduces to a quartic equation for the stability index Γ , all of whose roots have negative real part. They therefore concluded that this class of Sedov solutions is stable, contradicting the conclusion of Isenberg (1977).

Subsequently Gaffet (1984) presented a refutation of the work of Bernstein and Book (1980). His criticism of the latter focused on the neglect of the entropy perturbations arising at the shock front. This, he claimed, resulted from an incorrect statement of the perturbed boundary conditions.

To clarify the matter, we rederive the results of Bernstein and Book (1980), taking into account both the corrected boundary conditions and the entropic perturbations. We find that the solution of the eigenvalue problem is the same as that found previously, and the conclusion that the modes are stable remains unchanged.

The plan of the paper is as follows. We begin by summarizing the work followed by D'yakov (1954) in treating the planar shock problem, because his characterization of the acoustic and entropic modes is instructive in the present case. We then go through the revised treatment of the Primakoff blast wave stability problem, using essentially the notation of Bernstein and Book (1980). The paper concludes with a summary of the results and briefly explores their implications.

2. Stability of Planar Shock Waves

D'yakov (1954) analyzed the stability of a planar shock propagating in the positive x direction. He chose as his basic equations the adiabatic law expressed in terms of the entropy s , Euler's equation, and an equation for the pressure p . The latter, taken together with the adiabatic law, is equivalent to the continuity equation. The location of the shock front is perturbed by an amount

$$\zeta = \zeta_0 \exp[i(kx - \omega t)]. \quad (1)$$

All other perturbed quantities then are taken to vary as $\exp[i(kx + \ell y)]$. In the frame of the unperturbed shock, the linearized equations assume the form

$$(\ell v - \omega) \delta s = 0; \quad (2)$$

$$(\ell v - \omega) \delta v_x + kV\delta p = 0; \quad (3)$$

$$(\ell v - \omega) \delta v_y + \ell V\delta p = 0; \quad (4)$$

$$(\ell v - \omega) \delta p + \rho c^2(k\delta v_x + \ell\delta v_y) = 0, \quad (5)$$

where $V = \rho^{-1}$ is the specific volume, $c = (\gamma p/\rho)^{1/2}$ is the speed of sound, and first-order fluid quantities are distinguished with a δ . From Eq. (2) it follows that two types of solutions are possible (indicated respectively by (1) and (2)).

For type (1) $\ell v - \omega = 0$, while $\delta s^{(1)} \neq 0$. It follows from Eqs. (3) and (4) that $\delta p^{(1)} = 0$, and from Eq. (5) that

$$k\delta v_x^{(1)} + \ell_1\delta v_y^{(1)} = 0 \quad (6)$$

(the condition for incompressibility), and the perturbed specific volume $\delta V^{(1)} = (\partial V / \partial s)_p \delta s^{(1)}$ is nonzero. This is the "entropic" mode, which advects entropy and vorticity perturbations away from the perturbed shock.

The second type of solution of Eq. (2) satisfies $\ell_2 v - \omega \neq 0$, while $\delta s^{(2)} = 0$. Multiplying Eq. (3) by k , Eq. (4) by ℓ_2 , adding and using Eq. (5) to eliminate $\delta v_x^{(2)}$ and $\delta v_y^{(2)}$, we obtain the acoustic dispersion relation:

$$(\omega - \ell_2 v)^2 = (k^2 + \ell_2^2) c^2. \quad (7)$$

The quantities $\delta v_x^{(2)}$, $\delta v_y^{(2)}$, and $\delta p^{(2)}$ are then related by any two of Eqs. (3)-(5), for example

$$(\ell_2 v - \omega) \delta v_x^{(2)} + k V \delta p^{(2)} = 0, \quad (8)$$

$$(\ell_2 v - \omega) \delta v_y^{(2)} + \ell_2 V \delta p^{(2)} = 0. \quad (9)$$

By virtue of the vanishing of the perturbed entropy $\delta s^{(2)}$, we have

$$\delta p^{(2)} = -(c^2 / V^2) \delta V^{(2)}. \quad (10)$$

The general solution of the perturbed equation consists of a superposition of the entropic and acoustic modes.

By expanding the Rankine-Hugoniot relations through terms in first order, four boundary conditions can be obtained. These are the condition that the tangential component be continuous across the shock,

$$\delta v_x^{(1)} + \delta v_x^{(2)} = ik \zeta (\bar{v} - v), \quad (11)$$

two conditions obtained from the jump in the momentum normal to the shock and from the continuity equation, which D'yakov (1954) rewrites in the form

$$\delta v_y^{(1)} + \delta v_y^{(2)} = \frac{v - \bar{v}}{2} \left(\frac{\delta p^{(2)}}{p - \bar{p}} + \frac{\delta v^{(1)} + \delta v^{(2)}}{v - \bar{v}} \right) \quad (12)$$

and

$$\frac{2i\Omega\zeta}{\bar{v}} = \frac{\delta p^{(2)}}{p - \bar{p}} + \frac{\delta v^{(1)} + \delta v^{(2)}}{\bar{v} - v}, \quad (13)$$

and an equation derived from the energy jump condition. Since for given \bar{v} , \bar{v} , and \bar{p} , p is a known function of V , D'yakov (1956) takes this last condition in the form

$$\delta p^{(2)} = (\partial p / \partial V)_H (\delta v^{(1)} + \delta v^{(2)}), \quad (14)$$

where the subscript H stands for "Hugoniot." We have consistently used an overbar to indicate quantities ahead of the shock.

Equations (6) and (8)-(14) constitute a set of eight algebraic relations in the eight coefficients $\delta v^{(1)}$, $\delta v^{(2)}$, $\delta v_x^{(1)}$, $\delta v_y^{(1)}$, $\delta v_x^{(2)}$, $\delta v_y^{(2)}$, $\delta p^{(2)}$, and ζ . Equation (7) is to be understood as a definition of λ_2 in terms of k and Ω . D'yakov (1954) goes on to analyze the dispersion relation

$$\frac{2\omega v}{\bar{v}} \left(k^2 + \frac{\omega^2}{v^2} \right) = \left(\frac{\omega^2}{v\bar{v}} + k^2 \right) (\omega - \lambda_2 v) \left(1 + \frac{v^2}{v^2} \left(\frac{\partial v}{\partial p} \right)_H \right) \quad (15)$$

which follows, emphasizing the role of the Hugoniot curve in determining the stability of the perturbations. The interested reader is referred to that paper for the details.

3. Stability of the Primakoff Point Blast

For the sake of brevity we will not go through the whole derivation and solution of fluid equations linearized around the Primakoff-Sedov solutions. In this section we will refer to Bernstein and Book (1980) and to equations contained in it by using the abbreviation "BB." Except as noted, the notation here is that same as in BB.

The correct forms for the Lagrangian perturbed density ρ_1 and pressure p_1 , expressed in terms of the infinitesimal displacement $\underline{\xi}(\underline{R}, t)$ of an element of fluid whose unperturbed trajectory is $\underline{R}(\underline{r}, t)$, are

$$\rho_1 = \rho(-\underline{\nabla}_{\underline{R}} \cdot \underline{\xi} + \kappa) \quad (16)$$

and

$$p_1 = p(-\gamma \underline{\nabla}_{\underline{R}} \cdot \underline{\xi} + \lambda), \quad (17)$$

where κ and λ are time-independent quantities which were taken equal to zero in BB. Substituting (16) and (17) in the perturbed equation of motion then yields

$$\begin{aligned} \omega_0^{-2} r^{\nu(\gamma-1)} + 2 \ddot{\underline{\xi}} + \underline{r} \nabla \cdot \underline{\xi} - \frac{\gamma}{\nu} r^{2-\nu} \nabla(r^\nu \nabla \cdot \underline{\xi}) - (\nabla \underline{\xi}) \cdot \underline{r} \\ = f[\kappa \underline{r} - \frac{1}{\nu} r^{2-\nu} \nabla(r^\nu \lambda)], \end{aligned} \quad (18)$$

where $\nu = 2$ for the cylindrical case and $\nu = 3$ for the spherical case. The solution of Eq. (18) may be written as the sum of a particular and a homogeneous solution:

$$\underline{\xi} = \underline{X}^{(p)}(\underline{r}) f + \underline{X}^{(h)}(\underline{r}) r^3 = (\underline{X}^{(p)} r + \underline{X}^{(h)} r^3) H(\theta, \phi), \quad (19)$$

where the harmonic dependence of the solution is contained in $H(\phi) = \exp(im \phi)$ ($v=2$) or $H(\theta, \phi) = Y_{2m}(\theta, \phi)$ ($v=3$). Note that the definition of β here differs slightly from that in BB.

Since

$$(r^\beta)'' = \beta r^{\beta-1} r'' + \beta(\beta-1) r^{\beta-2} \dot{r}^2, \quad (20)$$

we have by virtue of BB(2.23) and (2.24)

$$\omega^{-2} r^{\nu(\gamma-1)} + 2 (r^\beta)'' = -u r^\beta, \quad (21)$$

where

$$u = - \frac{\beta [2(\beta-1) - \nu(\gamma-1)]}{\nu(\gamma-1)}. \quad (22)$$

Writing

$$X_r^{(h)} = ar^\alpha H, \quad (23)$$

$$r \nabla \cdot \underline{X}^{(h)} = br^\alpha H, \quad (24)$$

we get Eq. BB(3.14)

$$(\mu + \alpha) a + \left[\frac{\gamma}{\nu} (\alpha + \nu - 1) - 1 \right] b = 0 \quad (25)$$

from the r -component of (13), and an equation identical with BB(3.15) by taking the divergence of (13). Multiplying the first of these by $\alpha + \nu - 1$ and subtracting the second, we obtain the equivalent but simpler relation

$$\Lambda \left(\frac{\gamma}{\nu} b + a \right) + (\mu - 1) [(\alpha + \nu - 1) a - b] = 0, \quad (26)$$

where $\Lambda = \mu^2$ for $\nu = 2$ and $\Lambda = 2(2+\mu)$ for $\nu = 3$.

The "particular" solutions of Eq. (18), which evidently correspond to D'yakov's entropic mode, depend on time as $f(t)$. Assuming

$$\kappa = K r^{\delta-1} H, \quad (27)$$

$$\lambda + L r^{\delta-1} H, \quad (28)$$

we can write

$$\xi_r^{(P)} = A r^{\delta} f H, \quad (29)$$

$$r \nabla \cdot \underline{\xi}^{(P)} = B r^{\delta} f H, \quad (30)$$

where K , L , A and B are constants. Then substitution in the r -component and divergence of (18) yields two relations satisfied by the particular solution,

$$(\delta + 1) A - B + K = \frac{1}{\nu} (\delta + \nu - 1) (L - \gamma B) \quad (31)$$

and

$$(\delta + \nu - 1) [(\delta + 1)A - B + K] = \frac{1}{\nu} [(\delta + \nu - 1)^2 - \Lambda] (L - \alpha B) + \Lambda A. \quad (32)$$

Multiplying (31) by $\delta + \nu - 1$ and subtracting, we are left with

$$L = \nu A + \gamma B \quad (33)$$

if $\Lambda = 0$; substitution in (32) then yields

$$K = (\nu - 2)A + B. \quad (34)$$

Taking into account the additional terms arising from κ and λ in Eqs. (16) and (17), we can rewrite the first-order Rankine-Hugoniot relations [BB(3.34 - (3.36))] as

$$-r\sigma + (\nu-2) (\zeta - \xi_r) + \kappa s f \frac{\gamma+1}{2} = q\zeta; \quad (35)$$

$$t\dot{\xi}_r + \frac{2(\zeta - \xi_r)}{\nu(\gamma-1) + 2} = \frac{2tV_{ir}}{\gamma+1}; \quad (36)$$

$$-\gamma r\sigma + \nu (\zeta - \xi_r) + \lambda s f \frac{\gamma+1}{2} = \frac{4\bar{p} s \dot{s} V_{ir}}{(\gamma+1)\rho} + q\zeta, \quad (37)$$

where $\sigma = 7 \cdot \xi$, with all expressions evaluated at $r = s f \frac{\gamma-1}{2}$, i.e., on the shock front. If we write

$$\zeta = z s^\alpha r^\alpha \frac{\gamma-1}{2} + \beta H, \quad (38)$$

$$tV_{ir} = c s^\alpha r^\alpha \frac{\gamma-1}{2} + \beta H, \quad (39)$$

and substitute $\xi = \xi^{(p)} + \xi^{(h)}$ in Eqs. (35) - (37), we find

$$\begin{aligned} & [(\nu-2)a + b + (q-\nu+2)z] s^\alpha r^\alpha \frac{\gamma-1}{2} + \beta \\ & [K - (\nu-2)\lambda - \beta] s^\beta r^\beta \frac{\gamma-1}{2} + \beta = 0; \end{aligned} \quad (40)$$

$$(\beta-1)a + z + \frac{\nu(\gamma-1)+2}{\gamma+1} c = 0; \quad (41)$$

$$\left\{ \nu a + \gamma b + (q - \nu)z - \frac{2[\nu(\gamma-1)+2]}{\gamma+1} c \right\} s^\alpha r^\alpha \frac{\gamma-1}{2} + \beta =$$

$$(L - \nu A - \gamma B) s^\delta r^\delta \frac{\gamma-1}{2} + 1. \quad (42)$$

But Eqs. (33) and (34) imply that the right hand sides of (40) and (42) vanish. Thus all the coefficients associated with the particular solution (i.e., the entropic mode) cancel out. We are left with three equations which, are identical with those found in BB. These yield the boundary condition

$$2\beta a + (\gamma - 1) b = 0. \quad (43)$$

Equations (25), (26), and (43) constitute three simultaneous expressions for a/b . Equating these, we can solve for α , β , and μ as before, obtaining the same quartic and hence the same values for the stability index

$$\Gamma = \frac{1}{2} [(\gamma - 1) \alpha - \gamma - 1] + \beta. \quad (44)$$

The arguments presented here must be modified slightly to treat the case $\Lambda = 0$, but the result is the same as that found by taking the limit $\Lambda \rightarrow 0$ in BB(4.8) and (4.13), which are analytic in Λ .

4. Discussion

We have seen that there are indeed terms in the corrected analysis which correspond to D'yakov's (1954) entropic mode, but they drop out of the problem and do not alter our earlier result. Gaffet (1984) was correct in pointing out the term corresponding to $\lambda \neq 0$ in Eq. (17) for the pressure, but he failed to note the analogous term in Eq. (16) for the density. The parallel with D'yakov's (1954) analysis of the planar-shock (section 2) is not complete; D'yakov found a nonvanishing $\delta V^{(1)}$ (whence $\delta \rho^{(1)} \neq 0$), but $\delta p^{(1)} = 0$. We note that in our analysis the entropic mode has the same time dependence $\sim f(t)$ as the unperturbed shock. Hence the density contrast κ satisfies $d\kappa/dt = 0$, whereas in the planar case [by virtue of the incompressibility condition, Eq. (6)] it is the perturbed density of the entropic mode which satisfies $\frac{d}{dt} \rho^{(1)} = i(\Omega - \lambda_2 v) \rho^{(1)} = 0$.

Bernstein and Book (1980) made no use of the tangential velocity condition analogous to Eq. (11). In our notation this can be shown to take the form

$$\underline{n} \times \underline{\xi} + \underline{v} \times \nabla \zeta = 0, \quad (45)$$

where \underline{n} is the normal to the unperturbed shock. Gaffet (1984) also noted that the perturbation ξ found by Bernstein and Book (1980) does not vanish at the shock front, as it should. We readily see that Eq. (45), together with

$$\underline{\xi} = 0 \quad (46)$$

at the shock front, just suffice to establish the connection between a , b and A , B , i.e., to determine the coefficients of the entropic part of the solution and the exponent δ , which turns out to satisfy

$$\delta = \alpha + 2\beta/(\gamma-1). \quad (47)$$

The analysis presented by Bernstein and Book (1980) and here is restricted to the discrete modes, another point which Gaffet (1984) criticized. Although it is conceivable that the continuum modes (which are associated with the initial conditions) could give rise to instability, we believe this is unlikely. Certainly most conventional problems of the stability of ideal fluids in finite geometry can be treated in terms of the normal modes only.

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