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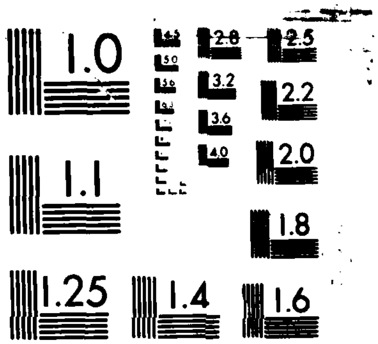

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The Optimal Construction of Synthetic Discriminant Functions (SDF's)

Robert R. Kallman

NORTH TEXAS STATE UNIVERSITY
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APRIL 1986

FINAL REPORT FOR PERIOD MAY-JULY 1984

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AIR FORCE ARMAMENT LABORATORY
Air Force Systems Command*United States Air Force*Eglin Air Force Base, Florida

NOTICE

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Franklin M. Gay

FRANKLIN M. GAY
Technical Director, Advanced Seeker
Division

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Synthetic discriminant functions (SDF's) for optical matched filters have potential use for pattern recognition. Several digital computer programs were written to compute SDF's, subject to certain constraints, and to compare the resulting SDF's to each other. For example, one of the programs chooses the four best from 36 tank images and manufactures an SDF from them. A guiding principle throughout this work was that objective numerical criteria be applied to determine what is a good or optimal SDF. There are many interesting and important questions which remain to be investigated in this area of research. They are discussed in this report.

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PREFACE

This document describes the results of an effort at the Electro-Optical Terminal Guidance Branch, Guided Weapons Division, U.S. Air Force Armament Laboratory, Eglin AFB, Florida.

The work reported herein was performed under Contract 749620-82-0C-0035 during the period 15 May 1984 to 31 July 1984 by the author, Dr. Robert R. Kallman, while visiting the Armament Laboratory as a Southeastern Center for Electrical Engineering Education (SCEEE) Postdoctoral Fellow.

The author would like to thank the Air Force Systems Command, the Air Force Office of Scientific Research, and SCEEE for providing him with the opportunity to spend a very worthwhile and interesting 10 weeks at the Armament Laboratory. The author would like to acknowledge in particular the Electro-Optical Terminal Guidance Branch and the Image Processing Laboratory for their hospitality and excellent working conditions. Special thanks go to Captain James Riggins and Steve Butler for introducing this present research topic to the author and patiently explaining its many aspects to him.

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SECTION I

INTRODUCTION

The work reported in this document is the result of a mathematical approach to an engineering problem. The work is described in mathematical language. The purpose of this introduction is to describe the engineering problem being attacked, thus explaining the motivation for the mathematical work, and to define enough terms so that the reader has the proper mindset to appreciate what is reported herein.

A large body of work has been dedicated to shape recognizing or object identification. One particular approach to this problem is optical correlation. This is usually attempted with a discriminant function, implemented in a holographic optical filter, which contains sufficient information about the object so that an image of the object can be processed by the filter. If the object sought is in the filter input plane, a large correlation peak should occur in the filter output plane. An excellent summary of the how this can be accomplished with holography is found in the survey article by Casasent and Caulfield, Reference 1.

Practical problems which appear include how to design the discriminant function to contain sufficient information, how to implement the discriminant function to make a tangible filter, and how to enhance filter efficiency. The discriminant function design is in many respects a mathematical problem and is addressed in this report.

In the approach relevant to this report, the discriminant function is constructed by judiciously adding information from selected aspects of the object of interest, thus creating the SDF, Reference 2. This report addresses how to best choose the information used to create the SDF.

The input data is imagery. The format for the imagery is usually a 512 pixel by 512 pixel scene. In order to use this information, it is convenient to break out the 512 by 512 = 262,144 pixels into a string of integers on a magnetic tape. Mathematically, therefore, one can think of a scene as a vector with 262,144 components. Ideas such as vector inner products then follow naturally.

As filters are designed, criteria must be established by which the quality of the filters can be measured. One of the unique aspects of the work reported herein is the fact that the filter performance is quantified.

Testing each filter can be done optically, but for this effort it was much more efficient to simulate all the optics on a digital computer. This was done in the E-O Terminal Guidance Branch Image Processing Laboratory, using its VAX 11-750.

A glossary is included in the report.

SECTION II

OPTICAL FILTERING AND SDF BASICS

Imagine a two-dimensional infrared image $f(x_1,x_2)$ of a scene which contains an object of interest. Consider the following operations on f. Map $f(x_1,x_2)$ to its Fourier transform $F(f)(k_1,k_2)$, multiply by the Fourier transform of a suitable function $F(h^{\#})(k_1,k_2)$, take the inverse Fourier transform of the product to obtain the convolution $(f\ h^{\#})(x_1,x_2)$, and measure the magnitude $|f\ h^{\#}|^2(x_1,x_2)$. Here, $h^{\#}(y_1,y_2) = h(-y_1,-y_2)$, so $(f\ h^{\#})(x_1,x_2)$ may be viewed as the inner product of f and the translate of h by $(-x_1,-x_2)$. All of these operations can be carried out almost instantaneously, for Fourier transforms and their inverses can be carried out by lenses, and the important multiplication and filtering step can be carried out by passing the light wave $F(f)$ through a suitable hologram transparency, Reference 1, incorporating information about $F(h^{\#})$. If h, the synthetic discriminant function or SDF, is suitably constructed and scaled, the objects of interest should be centered at the points (x_1,x_2) such that

$$|f\ h^{\#}|^2(x_1,x_2) = 1. \tag{1}$$

In reality f will probably be a 512 by 512 pixel image, and h will be 32 by 32 pixels in size. One can think of the filter as operating by instantaneously placing translates of h all over f, taking the corresponding inner products, and indicting those places where the magnitude of the inner product is large. These should be the places where objects of interest are located. This, in a very rough form, is the optical matched filtering process via an SDF.

The correct construction of h is obviously of the utmost importance if this scheme is to be successful. An early attempt along these lines was made by simply taking a number of transparencies of the object of interest, at a variety of aspects and angles, and overlaying them, Reference 3. Hence, in these early attempts one started with m images $f_1, \ldots, f_m$ and let $h = f_1 + \ldots + f_m$. This is a good idea if $f_1, \ldots, f_m$ are mutually orthogonal, but in general they are not.

In the past few years a generalization of this original approach was suggested by Caulfield and Maloney, Reference 4, and by Hester, Casasent, et al., References 5 - 10, who proposed to take a number of pictures $f_1, \ldots, f_m$ of the object interest and to choose h to be a suitable linear combination of $f_1, \ldots, f_m$. So in theory

$$h = a_1 f_1 + \ldots + a_m f_m \tag{2}$$

for some suitable choice of constants $a_1, \ldots, a_m$. Thinking of the f_i $(1 \leq i \leq m)$ as vectors in some high dimensional (e.g., for images that are 512 pixels by 512 pixels, the dimension of the space is 262,144) Euclidean space, h is expressed as a linear combination of the vectors $f_1, \ldots, f_m$. To determine the a_i $(1 \leq i \leq m)$, make the ad hoc assumption that $\langle h, f_j \rangle = 1$ $(1 \leq j \leq m)$ and use the bilinearity of the inner project to obtain

$$1 = \langle h, f_j \rangle = a_1 \langle f_1, f_j \rangle + \ldots + a_m \langle f_m, f_j \rangle, \tag{3}$$

where $\langle .,. \rangle$ denotes the inner project between vectors. If the m x m matrix $(\langle f_i, f_j \rangle)$ is nonsingular, as it most probably is for quite different images $f_1, \ldots, f_m$, then there is a unique choice for the a_i $(1 \leq i \leq m)$, and they can be determined easily by solving a system of m equations in m unknowns.

The notion has persisted that *one should not use all of the original* images $f_1, \ldots, f_m$ to manufacture the SDF h, but instead should use a subset p (p less than m) of them, usually selected by some sort of orthogonalization procedure. Reasons given for not using all of the images include problems with correlating on clutter. Given that this rather dubious notion has some merit, the question remains to carefully formulate how to choose the p best from all m images.

SECTION III

OBJECTIVES

The objective of the author during the SCEEE fellowship was to design from scratch a variety of programs to generate the best possible SDF's from a training set of images and to compare the SDF's to each other. The training set (a set of 36 images to be used to construct the SDF's) consisted of 512 by 512 pixel 8-bit infrared tank images, but pixel values outside of rows 200 to 400 inclusive were all zero. These images were dirty, in the sense that they did not consist of tanks in a zero background, as would be desired, but were images with a very bright background included. The images had been previously edge-enhanced and biased. Two typical members of the tank imagery with the background removed are shown in Figure 1. An aspect of one type of SDF (CSDF1 - c.f. the next section) made from this training set is shown in Figure 2. The training set images were furnished on a computer magnetic tape, each image consisting of a string of 512^2 = 262,144 integers (each integer with a value between 0 and 255), representing intensity levels at each pixel of the image. The guiding principle throughout the effort was that notions such as good or best be determined by concrete numerical criteria. In general, the programs try to drive a least squares error down to 0. The computing was done on a VAX 750/VMS 3.5 in the E-O Terminal Guidance Branch Image Processing Laboratory.

Figure 1. Two Tanks in the Training Set

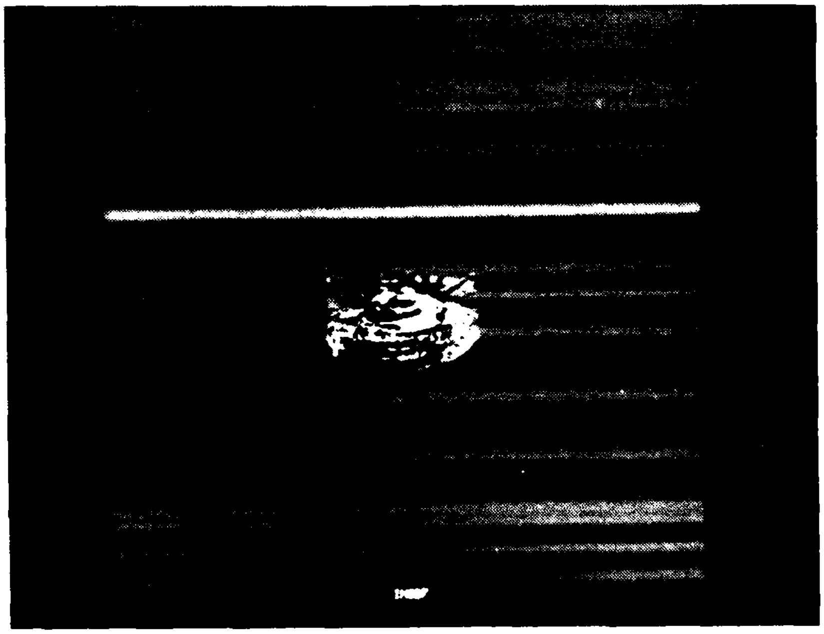

Figure 2. An SDF

SECTION IV

THE PROGRAMS AND NUMERICAL RESULTS

Let $f_1, \ldots, f_{36}$ be the tank images, thought of as vectors in a high dimensional ($512^2 = 262{,}144$) Euclidean space. The following concepts and programs, with minor modifications, apply to any number of images, not just 36, and of any size, not just 512 by 512. In all of the calculations it fortuitously turns out that the only thing one really needs to know about the f_i's is the symmetric 36 by 36 matrix ($<f_i, f_j>$), which should be calculated and stored first. As a measure of error made by a potential SDF h, the least squares error was chosen:

$$LSE(h) = |<h,f_1> - 1|^2 + |<h,f_2> - 1|^2 + \ldots + |<h,f_{36}> - 1|^2. \quad (4)$$

This procedure is extremely plausible, for such a measure of error has proven useful over the past 200 years in astronomy and statistics.

The following is a list of some of the SDF's calculated, giving their method of calculation and the least squares error for each. They are named and numbered in the order in which they were calculated.

SDF1: This is the theoretically perfect SDF and is a linear combination of all 36 tank images. So SDF1 = $a_1f_1 + a_2f_2 + \ldots + a_{36}f_{36}$, for some choice of constants $a_1, \ldots, a_{36}$. The a_j's must satisfy the 36 equations

$$a_1<f_1,f_i> + a_2<f_2,f_i> + \ldots + a_{36}<f_{36},f_i> = 1, \quad (5)$$

for i between 1 and 36. They are easily calculated by Gaussian elimination. The resulting LSE(SDF1) = 0.0.

SDF2: This SDF is a linear combination of 6 tank images. It was calculated by exhaustively checking all (36 choose 6) = 1,947,792 subsets of the 36 tank images, and for each fixed subset of 6, calculating that linear combination which makes the LSE as small as possible. It is easy to check that LSE(h) is a convex function of h, so a local minimum for LSE(h) is a global minimum for LSE(h). If the f_i are independent vectors and h is restricted to be a linear combination of them, it is easy to check that LSE(h) becomes uniformly unbounded as $||h||$ increases, so a global minimum for LSE(h) exists. Suppose $g_1, \ldots g_6$ is a subcollection of 6 out of the 36 tank images. We would like to find numbers $a_1, \ldots, a_6$ so that $h = a_1 g_1 + \ldots + a_6 g_6$ minimizes LSE(h) over all possible choices of the a_i. The above reasoning indicates that a minimum exists and is assumed when the partial derivative of LSE(h) with respect to each a_i is 0. Doing this for each a_i gives us 6 linear equations which must be satisfied. They are

$$m_{i1} a_1 + \ldots + m_{i6} a_6 = b_i, \tag{6}$$

where

$$m_{ij} = \langle g_i, f_1\rangle\langle f_1, g_j\rangle + \langle g_i, f_2\rangle\langle f_2, g_j\rangle + \ldots + \langle g_i, f_{36}\rangle\langle f_{36}, g_j\rangle \tag{7}$$

and

$$b_i = \langle g_i, f_1\rangle + \langle g_i, f_2\rangle + \ldots + \langle g_i, f_{36}\rangle. \tag{8}$$

Given the a_i's and b_i's as above, a little algebra shows that

$$\mathrm{LSE}(h) = 36 - a_1 b_1 - \ldots - a_6 b_6. \tag{9}$$

So the program to compute SDF2 proceeds as follows: select 6 out of the 36 tank images $g_1, \ldots, g_6$, find $a_1, \ldots, a_6$ by solving one set of 6 equations in 6 unknowns, and compute $36 - a_1 b_1 - \ldots - a_6 b_6$. Select that subset of 6 which makes this last number as small as possible, and manufacture SDF2 from them by computing $a_1 g_1 + \ldots + a_6 g_6$. The images selected by SDF2 were 4, 12, 16, 24, 29, and 32. The resulting LSE(SDF2) = 0.061511.

SDF3: This SDF is a linear combination of 6 tank images. It was calculated in a step-by-step orthogonalization procedure from SDF1. Let g_1 be that tank image so that the orthogonal projection of SDF1 onto the line spanned by g_1 is largest. Take the orthogonal projection of SDF1 and of $f_1, \ldots, f_{36}$ onto the orthocomplement of g_1 to obtain SDF1' and $f_1', \ldots, f_{36}'$ (only 35 of which are now nonzero), and repeat the process 5 more times. Keep track of the index chosen each time to get the 6 desired original images $g_1, \ldots, g_6$. The calculations are easy, for g_1 is that image f_i so that angle between SDF1 and f_i is as small as possible. That is, g_1 is that f_i so that

$$|\langle SDF1, f_i \rangle| / ||SDF1|| \cdot ||f_i|| \tag{10}$$

is as large as possible. Once g_1 is chosen, the iteration is simple for

$$\langle f_i', f_j' \rangle = \langle f_i, f_j \rangle - \langle f_i, g_1 \rangle \langle f_j, g_1 \rangle / ||g_1||^2, \tag{11}$$

and

$$\langle SDF1, f_i' \rangle = \langle SDF1, f_i \rangle - \langle SDF1, g_1 \rangle \langle f_i, g_1 \rangle / ||g_1||^2. \tag{12}$$

So repetition is easy. The images selected, in the order in which they were chosen, are 17, 12, 4, 29, 16, and 24. SDF3 is then that linear combination of these images which minimizes the LSE. The resulting LSE(SDF3) = 0.085728.

SDF3A: This SDF is a linear combination of 6 tank images. The choice of the images is done in the same manner as was done for SDF3, so the images are the same. However, SDF3A was chosen to be the theoretically perfect SDF on these 6 images. It is simple to check that SDF3A is the orthogonal projection of SDF1 onto the subspace spanned by the 6 selected images. LSE(SDF3A) = 0.271316.

SDF4: This SDF is a linear combination of 6 tank images. To find the images, an exhaustive search of all (36 choose 6) = 1,947,792 subsets of 6 tank images is made. The subset of 6 chosen is that subset such that the orthogonal projection of the theoretically perfect SDF1 onto their span is as large as possible. As in SDF3A, this projection is simple to calculate, for it coincides with the theoretically perfect SDF made from the 6 working images. SDF4 is then that linear combination of the images chosen which minimizes the LSE. The images chosen were 4, 12, 16, 17, 24, and 29, the same images chosen by SDF3. This is a fluke (c.f. CSDF3 and CSDF4, to be discussed later). LSE(SDF4) = 0.085728.

SDF4A: SDF4A stands in the same relation to SDF4 as SDF3A does to SDF3. LSE(SDF4A) = 0.271316.

SDF5: This SDF was calculated in the same manner as was SDF2, except that SDF5 is a linear combination of 5 images. The images chosen were 4, 12, 17, 18, and 29. Notice that the best 5 images are not a subset of the best 6 images. LSE(SDF5) = 0.080739.

SDF6: This SDF was calculated in the same manner as were SDF2 and SDF5, except that SDF6 is a linear combination of 4 images. The images chosen were 4, 12, 17, and 29. LSE(SDF6) = 0.103963.

SDF7: This SDF is a linear combination of 6 tank images. They are chosen in a step-by-step orthogonalization procedure. Roughly speaking, the first image chosen is that one which contains as much information as possible about all of the other images. The numerical measure for this information is taken to be the sum of the squares of the cosines of the angles between all of the images. So the first image chosen, g_1, is that image f_i, so that

$$\langle f_i, f_1\rangle^2/||f_i||^2 \cdot ||f_1||^2 + \dots + \langle f_i, f_{36}\rangle^2/||f_i||^2 \cdot ||f_{36}||^2 \tag{13}$$

is as large as possible. All vectors are now projected onto the orthocomplement of g_1, as in the calculation of SDF3, and the process is repeated 5 more times. The images selected, in the order in which they were chosen, are 16, 28, 6, 22, 11, and 36. LSE(SDF7) = 0.258666.

SDF7A: SDF7A stands in the same relation to SDF7 as SDF3A does to SDF3. LSE(SDF7A) = 0.753411.

SDF8: This SDF stands in the same relation to SDF7 as SDF4 does to SDF3. This was not run, for the required computation time was estimated to exceed more than 50 hours of CPU time.

SDF8A: SDF8A stands in the same relation to SDF8 as SDF3A does to SDF3. It was not run for the same reason that SDF8 was not run.

The results of the above SDF fabrications are tabulated in Table 1.

Suppose one chose to make an SDF from the images 1, 2, 5, 16, 19, and 36. The very best SDF that can be made with these images has LSE = 0.183932. If one took these same 6 images and took the orthogonal projection of SDF1 onto their span, and used this orthogonal projection as an SDF, the resulting LSE = 0.247391.

Some concern was expressed that the SDF's might be correlating on the clutter in the background and not on the tanks themselves. For this reason, and because working with 512 by 512 images consumed inordinate amounts of CPU time, the tank images were now extracted from the background and placed into 256 by 256 arrays, using DeAnza image processing equipment. All previous calculations were performed on this new data. The results are summarized in Table 2. In general, CSDF- was manufactured in exactly the same manner as SDF-, except that the clean images were used instead of the dirty images. Figure 2 is a picture of the larger pixels in a biased version of CSDF1 - biased to make all of its entries nonnegative.

TABLE 1. SUMMARY OF THE SDF CALCULATIONS

	Technique	Images chosen	LSE
SDF1	Linear combinations of all 36 images.	All 36	0.0
SDF2	Calculate 36 choose 6.	4,12,16,24,29,32	0.061511
SDF3	Calculate 36 chose 6, but choose each of the 6 according to maximum value of $\lvert\langle SDF1, f_i\rangle\rvert / \lVert SDF1\rVert \cdot \lVert f_i\rVert$.	17,12,4,29,16,24	0.085728
SDF3A	Same as SDF3, but SDF3A is chosen to be the theoretically perfect SDF on the 6 chosen images.	17,12,4,29,16,24	0.271316
SDF4	Calculate 36 chose 6, choose to maximize the orthogonal projection of SDF1 onto their span.	4,12,16,17,24,29	0.085728
SDF4A		4,12,16,17,24,29	0.271316
SDF5	Calculate 36 choose 5.	4,12,17,18,29	0.080739
SDF6	Calculate 36 choose 4.	4,12,17,29	0.103963
SDF7		16,28,6,22,11,36	0.258666
SDF7A		16,28,6,22,11,36	0.753411
SDF8		Not Run	
SDF8A		Not Run	

TABLE 2. SUMMARY OF THE CSDF CALCULATIONS

	Images chosen	LSE
CSDF1	This SDF is a linear combination of all 36 clean tank images.	0.0
CSDF2	14, 16, 24, 29, 32, and 33	0.870515
CSDF3	17, 16, 33, 14, 32, and 12	1.289280
CSDF3A	17, 16, 33, 14, 32, and 12	2.731533
CSDF4	12, 14, 16, 17, 31, and 33 (Note that the images used to manufacture CSDF4 do not coincide with the images used to manufacture CSDF3.)	1.414930
CSDF4A	12, 14, 16, 17, 31, and 33	2.160517
CSDF5	14, 17, 24, 29, and 32	1.046511
CSDF6	12, 17, 29, and 31	1.235655
CSDF7	31, 28, 17, 24, 1, and 10	1.420816
CSDF7A	31, 28, 17, 24, 1, and 10 (The lesson to be learned from this computation is that given a collection of images, one should do the best job one can in manufacturing an SDF from them.)	6.821821
CSDF8	not computed	
CSDF8A	not computed	

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

The numbers in Tables 1 and 2 speak for themselves. Assume that minimizing least squares errors is a reasonable approach to quantifying the goodness of an SDF, and suppose that it is desired to make an SDF out of some small subset of all the images, no matter how dubious this concept may seem. The computations summarized in Tables 1 and 2 strongly suggest that a least squares choice and manufacture of an SDF on 5 images always does better than any orthogonalization procedure on 6 images (and usually much better), and that a least squares choice and manufacture of an SDF on 4 images usually does better than most orthogonalization procedures on 6 images (and sometimes much better). They also strongly suggest that the worst orthogonalization procedure is the one which tries to find 6 images which contain the most information about the other images and then take one's SDF to be the theoretically perfect SDF manufactured from these 6 images. It is doubtful that the time-consuming computation of SDF8, SDF8A, CSDF8, or CSDF8A would change these empirical conclusions.

While the ideas described to pick the best p out of m images to manufacture an SDF work fairly well if p and m are not too large, they will not work in a practical sense if $m = 100$ and $p = 10$ say, for then to calculate the analogue of SDF2 would involve finding solutions to (100 choose 10), around 1.731×10^{13}, sets of 10 equations in 10 unknowns, a very large task indeed. There might be fairly short computational procedures for finding results close to the theoretically best. Perhaps the random selection and testing of 10 scenes at a time, combined with some sort of gradient technique, would work fairly well.

One can generalize the LSE estimator by replacing each summand by $w_i \cdot |\langle h, f_i\rangle - v_i\rangle|^2$. Think of w_i as a weight. Usually v_i will take on the value 0 or 1, but it can be any value one desires. This should have some uses and should be tested in an appropriate setting.

The numerical experiments described in Section IV should be tried on training sets of images which have been edge-enhanced and then energy-normalized. Recall that edge-enhancement normally involves starting with an image f, mapping f to its Fourier transform F(f), deleting a suitable disk containing the origin from F(f), and mapping the result back to image space by an inverse Fourier transform. Most practitioners in image processing insist on this. The simple example usually given is that if this is not done, then any (uniformly filled in) circle would correlate quite well with a (uniformly filled in) square of roughly the same size, even though they are quite different objects. This process also has a considerable technical advantage, for if an image f has already been edge-enhanced as above, then F(f) vanishes at (0,0) - i.e., the integral of f over R^2 vanishes. Notice that if an SDF h is made up of a linear combination of such images, it too has the same property. Furthermore, any biasing done to h then does not change its correlations with any zero mean image. To see this, recall that a biasing of h involves replacing h by h + h', where h' is a vector all of whose components have the same constant value, say c. But if g is any zero mean image, then $\langle(h + h'), g\rangle = \langle h, g\rangle + \langle h', g\rangle = \langle h, g\rangle$, for $\langle h', g\rangle = 0$ since it is equal to c times the sum of the pixel values in g. Notice that grave errors may be made by blithely biasing SDF's without taking into consideration the mean of g, for then $\langle h',g \rangle$ may be quite large. There is a subtle but potentially very serious problem in this circle of ideas. Since the SDF will be made from edge-enhanced images, 32 by 32 pixels in size, it will be looking for edge-enhanced images, 32 by 32 pixels in size. But an edge-enhanced 512 by 512 image does not have in general each 32 by 32 subscene edge-enhanced, in the sense that the sum of the pixel values in this subscene equals 0. For suppose f is an image such that $F(f) = 0$ in a disk about the origin in Fourier transform space. Let B be a square box in image space and let I_B be its characteristic function; i.e., I_B is 1 at points in the box and is 0 at points outside the box. If fI_B were edge-enhanced, then $F(fI_B)(0,0) = 0$. But $F(fI_B)(0,0)$ is the integral of f over B, which certainly may be nonzero even if the integral of f over all of R^2 is 0. For a concrete but somewhat artifical example of this, suppose F(f) equals $F(I_B)$ outside of the deleted disk. Then $F(fI_B)(0,0) = (F(f)*F(I_B))(0,0) = \langle F(f),F(f) \rangle$, a positive number. This computation does show that the more the original scene is edge-enhanced, the smaller is $F(fI_B)(0,0)$, but how much edge-enhancement is enough to avoid serious errors in the correlation process? These errors might be especially pronounced if one is using a biased SDF.

The dirty tank images had been edge-enhanced as above and then biased, so that all their entries were nonnegative, and then discretized into 256 equal parts - hence the bit streams which appeared on the data tape. One way to come close to recapturing the original edge-enhanced images would be to take the dirty images, compute the average pixel value over all pixels between rows

200 to 400 inclusive, and subtract this average pixel value from the same pixels. But then the DeAnza image processing equipment could not have been used to outline the images and toss out the clutter in the background. A somewhat inexact but homefully fairly reasonable way out of this conundrum is to take each of the clean images, compute the average nonzero pixel value, and subtract this average from each of the nonzero pixels, leaving those pixels with 0 values unaltered.

Recall that energy-normalization is equivalent to replacing each f_i by $f_i/||f_i||$. Many practitioners in image processing insist on this. The usual reason given is to reduce the climatic effects in which the images are located. There are at least two reasons why this practice should be done with a certain amount of caution. Since the SDF will be made from energy-normalized images, 32 by 32 pixels in size, it will be looking for energy-normalized images, 32 by 32 pixels in size. But an energy-normalized 512 by 512 image certainly does not, in general, have each 32 by 32 subscene proportionally energy-normalized. If the object sought is the brightest object in the 512 by 512 scene, then energy-normalizing the entire image will leave the object subscene more than proportionally energy-normalized, and no harm will result if one is searching for a correlation peak. But if there is a much brighter object, say a fire, in the upper left hand corner of the image, and the object sought is in the lower right hand corner, then energy-normalizing the entire scene perhaps will make the image of the object sought so faint as to be useless. Furthermore, the Schwarz inequality implies that the SDF will have the largest inner product with unit vectors that do not look like the objects sought, but instead look like the SDF itself. This difference might be quite pronounced. For the SDF process to work, without further processing, one must make an act of faith that there are no real objects which look more like the SDF than the sought for images themselves.

The training set images should first be edge-enhanced, and then energy-normalized. Note that simple examples show that edge-enhancing and energy-normalizing are not commutative operations. For example, suppose an image in its right half is uniformly bright with fuzzy edges and contains a faint object with sharp edges in its left half. First energy-normalizing and then edge-enhancing would destroy the left hand object and leave an empty scene, while first edge-enhancing and then energy-normalizing would leave a sharp image of the object on the left and nothing on the right.

The author knows of no scientifically unimpeachable reason why the theoretically perfect SDF should not be used, instead of one made from a small number of pictures, no matter how they are chosen. At first glance it seems implausible that one can do a better job by throwing away information. Perhaps repeating the experiments of Section IV on edge-enhanced and then energy-normalized training sets will shed light on this important issue.

Even if SDF's made from a small number of images have lower correlation with clutter, there still might be several ways to enhance the theoretically perfect SDF. Notice, for instance, that the number of theoretically perfect SDF's is enormous. If h is a theoretically perfect SDF (i.e., if $\langle h, f_i \rangle = 1$, $1 \leq i \leq m$), then one can add to h any vector which is in the orthocomplement of the f_i's and still obtain another theoretically perfect SDF. Furthermore, they all arise in this manner. So if the f_i's are d by d images, then the set of theoretically perfect SDF's is a hyperplane in R^d of dimension $d^2 - m$. This gives one hope that superior SDF's exist.

GLOSSARY OF TERMINOLOGY

Edge-Enhance - A technique used to reduce the low spatial frequencies in an image, so that only high spatial frequencies (small objects and edges of large objects) remain in the image. This is discussed at some length in Section V.

Energy-Normalize - A technique used to adjust the total energy in an image to some fixed value, usually 1. This is discussed at some length in Section V.

Vector Space - In this paper only concrete vector spaces over the reals are needed, i.e., only R^n, the set of ordered n-tuples of real numbers. A typical vector is of the form $a = (a_1, \ldots, a_n)$ where each a_j is a real number. Here n may be much larger than 2 or 3.

Convex Function - A real valued function f on R^n is said to be convex if $f(ta + (1 - t)b) \leq tf(a) + (1 - t)f(b)$ for all vectors $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ in R^n and all real numbers $0 \leq t \leq 1$. A convex function has the property that a local minimum is a global minimum.

Inner Product - If $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are two vectors, then their inner product, $\langle a,b \rangle$, is defined to be $\langle a,b \rangle = a_1 b_1 + \ldots + a_n b_n$.

Length of a Vector - If $a = (a_1 \ldots, a_n)$ is a vector, its length, $||a||$, is defined to be $||a|| = \langle a,a\rangle^{1/2}$.

Angle Between Two Vectors - If $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are two nonzero vectors, the angle between them is defined to be the unique angle θ between 0 and π which satisfies $\langle a,b\rangle = ||a||\cdot||b|| \cos(\theta)$. The Schwarz inequality guarantees that θ exists.

Orthogonal Vectors - In view of the previous definition, it is natural to say that two vectors $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are orthogonal if the angle between them is $\pi/2$, i.e., if $\langle a,b\rangle = 0$.

Orthocomplement - If S is any nonempty collection of vectors, then the orthocomplement of S, denoted $S^\perp$, is the set of all vectors which are orthogonal to every vector in S. $S^\perp$ is always a linear subspace of R^n.

Orthogonal Projection - If S is any linear subspace of R^n, then the orthogonal projection onto S is the operator P_S which carries any vector a to $P_S(a)$, that unique element in S which is closest to a. $P_S(a)$ always exists and P_S is a linear operator.

Synthetic Discriminant Function (SDF) - See Section II for a detailed description of this object.

REFERENCES

1. D. Casasent and H. J. Caulfield, Basic concepts, pp. 1 - 16, Optical Data Processing Applications, edited by D. Casasent, Topics in Applied Physics, volume 23, Springer-Verlag, New York, 1978.

2. C. F. Hester and D. Casasent, Multivariant technique for multiclass pattern recognition, Applied Optics, # 11, volume 19, 1980, pp. 1758 - 1761.

3. K. G. Lieb, M. R. Wohlers, H. K. Hinz, and G. Moyer, Analysis of an optical matched filter guidance system for tactical homing applications, Grumman Research Department Memorandum RM-674J, December 1978.

4. H. J. Caulfield and W. T. Maloney, Improved discrimination in optical character recognition, Applied Optics, # 11, volume 8, 1969, pp. 2354 - 2356.

5. D. Casasent, Unified synthetic discriminant function computational formulation, Optical Society of America, # 10, volume 23, 1984, pp. 1620 - 1627.

6. D. Casasent, B. V. K. V. Kumar, and V. Sharma, Synthetic discriminant functions for 3-D object recognition, SPIE, volume 360, 1982, 7 pp.

7. D. Casasent and V. Sharma, Performance of synthetic discriminant functions for infrared ship classification, SPIE, volume 422, pp. 193 - 196.

8. C. F. Hester and D. Cassasent, Intra-class IR pattern recognition using SDF's, SPIE, volume 292, 1981, pp. 25 - 33.

9. C. F. Hester and D. Cassasent, Interclass discrimination using SDF's, SPIE, volume 302, 1981, pp. 108 - 116.

10. J. R. Leger and S. H. Lee, Hybrid optical processor for pattern recognition and classification using a generalized set of pattern functions, Applied Optics, # 2, volume 21, 1982, pp. 274 - 287.

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