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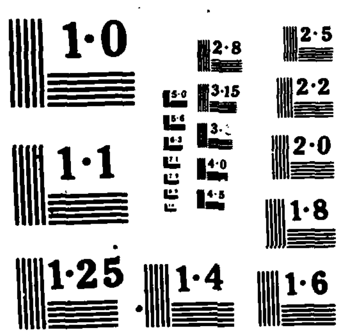

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ASYMPTOTIC ANALYSIS OF THE ROOTS OF A CERTAIN TRANSCENDENTAL EQUATION

Raymond Sedney
Nathan Gerber

April 1986

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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The spatial eigenvalues that occurred first in the Stewartson-Wedemeyer theory and then in later theories consist of a denumerable basic set which are $O(1)$ for $Re \to \infty$. In this report the existence of an additional single, isolated eigenvalue that is $O(Re^{1/2})$ is demonstrated. The former were used in several reports on liquid-filled projectiles; the latter was undetected in previous work. Asymptotic analysis for $Re \to \infty$ is used to give accurate

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20. ABSTRACT (Continued)

estimates for the new eigenvalue. Numerically it is shown that for $10 < Re < 1000$ and $0.1 \leq \tau$ 1.0 the formulas from asymptotic analysis must be used rather than those in the original CDC program. The limiting case of nutational frequency approaching spin frequency is also discussed. The effect of the new eigenvalue on previous results, for $Re > 1000$, is negligible but for $Re < 1000$ the effect is significant.

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I. INTRODUCTION

The study of the 3-D perturbed motion of a rotating fluid which fills a spinning and coning cylinder shows that the solution depends on the cylinder aspect ratio $A = c/a$ and the Reynolds number $Re = a^2\dot{\phi}/\nu$, where c = half height, a = radius, $\dot{\phi}$ = spin of the cylinder and ν = kinematic viscosity of the fluid. The forced oscillation problem was solved for the pressure in Reference 1 and for the moment exerted by the fluid on the cylinder in Reference 2 assuming linear, viscous perturbations of the steady state, i.e., solid body rotation. A modal analysis, matched asymptotic expansions for the flow near the endwalls and an expansion in outer flow spatial eigenfunctions were employed.

The ranges of interest of the parameters are $0.5 < A < 5$ and $1 < Re < 10^6$, determined by the values for which experimental data, of various sorts, are available. The theory of References 1 and 2 is asymptotic for $Re \to \infty$ and requires the solution of an eigenvalue (e.v.) problem. The complex e.v. are denoted by λ and the eigenfunctions are $\sin \lambda x$, $\cos \lambda x$. The "basic set," λ_k, satisfies $\lambda_k \to k\pi/2A$ for $Re \to \infty$, where $k = 1,3,5...$ Only this set was considered in References 1-4.

The existence of another eigenvalue, λ_s, where $\lambda_s = O(Re^{1/2})$ and therefore is not in the basic set, is shown here; in this limit the coning frequency, τ, is fixed and $\tau \neq \pm 1,3$. The limits of all the e.v.'s as coning frequency approaches spin frequency, $\tau \to 1$, with Re fixed, are also considered.

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1. *Gerber, N., Sedney, R., and Bartos, J. M., "Pressure Moment on a Liquid-Filled Projectile: Solid Body Rotation," U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02422, October 1982. (AD A120567)*

2. *Gerber, N., and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02470, February 1983. (AD A125332)*

3. *Wedemeyer, E.H., "Viscous Corrections to Stewartson's Stability Criterion," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, Report No. 1325, June 1966. (AD 489687) (See Also AGARD Conference Proceedings, No. 10, Mulhouse, France, pp. 103-120, September 1966)*

4. *Murphy, C.H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-MR-03194, August 1982. (AD A118676) (See Also Journal of Guidance, Control and Dynamics, Vol. 6, July-August 1983, pp. 280-286)*

The eigenvalue λ_s exists for all Re. It is definitely an outlier with respect to the basic set. The $|\mathrm{Im}\,(\lambda_s)|$ is always larger than that for any λ_k and its inclusion has a definite effect on the solution. As an example, pressure coefficient at the intersection of the endwall and sidewall is 41% less than that obtained without λ_s at $\tau = 0.5$ for Re = 10 and A = 3.148.

The determination of λ_s and some implications of its inclusion in the theory of Reference 1 are discussed in this report. The simple asymptotic methods used to determine λ_s show their efficacy not only in analysis but also in the successful computation of the pressure; straightforward calculation would have required more than the 14 significant figures available in the CDC 7600.

II. PRECIS OF REFERENCE 1

Certain parts of the theory of Reference 1 are needed here; the major points are:

a) The motion of the cylinder (projectile) is given by $K_0 \exp\,(if\phi t)$ where t is time, $f = (1 - i\delta)\tau$, τ is nutational frequency/ϕ and δ is the yaw growth rate. K_0 is the magnitude of the yaw, assumed small, and is the parameter used to linearize the Navier-Stokes equations in the perturbation analysis. Lengths, time, velocity and perturbation pressure are made non-dimensional by a, ϕ^{-1}, $a\phi$ and $K_0\,\rho a^2\phi^2$, respectively, where ρ is the liquid density.

b) The flow variables are assumed to vary as $\exp\,[i(f\phi t-\theta)]$ where θ is the azimuthal angle coordinate; the variations with x and r, the longitudinal and radial coordinates, respectively, are governed by linear partial differential equations and the no-slip conditions on the sidewall and endwalls.

c) These equations are solved using a modal analysis. The x-variation is determined by a second-order, non-self-adjoint system for spatial eigenvalues, λ, and eigenfunctions $\sin \lambda x$ and $\cos \lambda x$.

d) The solution satisfies the boundary conditions on the sidewall but not on the endwalls. In the neighborhood of the latter, it must be taken as the outer solution and then matched to an inner (boundary layer type) solution. The matching is asymptotic for Re $\to \infty$.

e) First term matching provides a solution to $O\,(\mathrm{Re}^{-1})$ and corrected endwall boundary conditions that determine the λ.

f) The λ are the roots of

$$F\,(z,\varepsilon) \equiv \cos z + \varepsilon z \sin z = 0 \tag{2.1}$$

where $z = A\lambda$, $\varepsilon = \delta c/A$ and δc is the complex displacement thickness given by

$$\delta c = -(1+\tau)/2\alpha(1-\tau) + (3-\tau)/2\beta(1-\tau)$$

$$\alpha = [\mathrm{Re}\ (3-\tau)/2]^{1/2}\ (1-i) \tag{2.2}$$

$$\beta = [\mathrm{Re}(1+\tau)/2]^{1/2}\ (1+i),$$ where $\delta = 0$, $f = \tau$, for convenience.

The range of τ is limited to $-1 < \tau < 3$ in order that inertial waves exist in the rotating fluid; this is the range in the inertial coordinate system employed here compared to the frequency range (-2,2) in a rotating coordinate system.

Since (2.1) is solved by iteration, first guesses for the roots are required; for Re >> 1

$$z \simeq (k\pi/2)(1-\varepsilon)^{-1},\ k \text{ odd}, \tag{2.3}$$

was used. This approximation can be derived in several ways; see, e.g. (3.2).

By definition τ is real. In principle there is no restriction on τ, except $\tau \neq \pm 1, 3$. In practice $0 < \tau < 0.2$ for most projectiles and $-0.5 < \tau < 0.5$ for most experimental simulations of a spinning projectile. Here for definiteness, the range of τ will be taken to be $-1 < \tau < 1$. From (2.2) it follows that

$$-\pi/2 < \arg \varepsilon < -\pi/4 \tag{2.4}$$

for $-1 < \tau < 1$.

The question of how small Re can be and still obtain reasonably accurate solutions, using References 1 and 2, will not be discussed because it is not relevant to solving (2.1).

III. ASYMPTOTIC ANALYSIS OF THE ROOTS

A. Limit $\varepsilon \to 0$

The roots of (2.1) depend on the single parameter ε which is proportional to $[(1-\tau)\mathrm{Re}^{1/2}]^{-1}$. The limit of primary interest is $\varepsilon \to 0$, which could correspond to $\mathrm{Re} \to \infty$, τ fixed. The limit $\varepsilon \to \infty$, corresponding to $\tau \to 1$ and Re fixed, will also be considered; the limit $\tau \to -1$ (or even $\tau \to 3$) can be treated by the same techniques but will not be considered here.

If z is a root of (2.1), $-z$ is also. Since the eigenfunctions are $\sin \lambda x$ and $\cos \lambda x$, and because of the symmetry properties of the solution[1], only the roots of one sign are required. The results obtained below show that only roots with

$$-\pi/2 < \arg z < 0 \tag{3.1}$$

are appropriate. First let $z = O(1)$ for $\varepsilon \to 0$ so that $\cos z$ is the dominant term in (2.1). Thus $F \to \cos z$ and $z \to \pm k\pi/2$, $k = 1,3,\ldots$, but only the positive sign need be considered. This gives the zeroth approximation to the basic set of roots, z_k, defined by $z_k = O(1)$ or $z_k = k\pi/2$ for $\varepsilon \to 0$. The basic set was used in References 1-4. The zeroth approximation can be improved by a regular perturbation. Assuming a power series in ε yields

$$z_k = (k\pi/2)(1 + \varepsilon + \ldots) \tag{3.2}$$

so that $-\pi/2 < \arg z_k < 0$.

It is not possible to have $\varepsilon z \sin z$ the dominant term in (2.1). The possibility that neither term dominates requires $\varepsilon z = O(1)$. Since $z = O(1/\varepsilon)$, the asymptotic forms for $\sin z$ and $\cos z$ are required. Using

$$\begin{aligned} \sin z &= (-i/2)(e^{iz} - e^{-iz}) & \cos z &= (1/2)(e^{iz} + e^{-iz}) \\ &= (-i/2)e^{iz}(1 - e^{-2iz}) & &= (1/2)e^{iz}(1 + e^{-2iz}) \end{aligned} \tag{3.3}$$

the asymptotic forms are, for $-\pi < \arg z < 0$, $|z| \to \infty$,

$$\sin z \sim (-i/2)e^{iz} \qquad \cos z \sim (1/2)e^{iz}. \tag{3.4}$$

Substituting (3.4) into (2.1) yields z_0, the zeroth approximation to the asymptotic solution of (2.1):

$$z_0 = -i/\varepsilon. \tag{3.5}$$

(For $0 < \arg z < \pi$, $z_0 = +i/\varepsilon$ is obtained, which need not be considered.) Note that because of (2.4), $-\pi/2 < \arg z_0 < 0$. Therefore $z_0 = O(Re^{1/2})$ for $Re \to \infty$ and is distinct from any member of the basic set. Only $z = O(1)$ and $z = O(1/\varepsilon)$ roots are possible; all other orders give contradictory or inconsistent results.

The eigenvalue, which is $O(Re^{1/2})$ for $Re \to \infty$, is denoted by $z_s \approx z_0$. With respect to the basic set z_s is a singular perturbation. Depending on the iterative method used to solve (2.1) z_s can be determined for small Re (say $Re \approx 10$) and $\tau > .5$ using certain O(1) first guesses. However, using z_0 as the first guess and Newton's method to solve (2.1) will yield z_s for all τ and Re unless Re is so large that $|e^{iz_0}|$ exceeds the capacity of the computer so that the Newton method fails. But for such Re, z_0 is a very close approximation to z_s. Furthermore the approximation $z_s \approx z_0$ can be easily improved so that the iterative solution of (2.1) for z_s is, in fact, no longer necessary for $|\varepsilon|$ small enough.

Let the method of undetermined coefficients assume a series expansion

$$z_s = z_0(1 + c_1\gamma + \dots) \tag{3.6}$$

in a small parameter γ. With (3.3), (2.1) can be written as

$$g(z) \equiv 1 + e^{-2iz} - i\varepsilon z(1 - e^{-2iz}) = 0. \tag{3.7}$$

Since $g(z_0) = 2e^{-2/\varepsilon}$, the small parameter $\underline{\gamma \equiv e^{-2/\varepsilon}}$ is suggested. If (3.6) contains only powers of γ, substitution into (3.7) leads to a contradiction so that the c_i, $i > 1$, cannot be determined. The reason for this is that $\ln \gamma$ terms are required in the expansion. This can be shown if iteration is used to solve (3.7), after rewriting it as

$$\zeta = (1 + \gamma^{\zeta})/(1 - \gamma^{\zeta})$$

$$\zeta \equiv z/z_0.$$

The iteration is defined by

$$\zeta_0 = 1$$

$$\zeta_{n+1} = (1 + \gamma^{\zeta_n})/(1 - \gamma^{\zeta_n}) \tag{3.8}$$

and the result for n = 1 is

$$z_s/z_0 \simeq \zeta_2 = 1 + 2\gamma + 2\gamma^2 + 4\gamma^2 \ln \gamma + O(\gamma^3 \ln \gamma). \tag{3.9}$$

The method of undetermined coefficients works with these functions of γ. Equation (3.9) gives the first four terms in an asymptotic series which appears to diverge for $|\gamma| > 10^{-3}$. By a theorem in Reference 5 the iteration (3.8) converges to a unique root if $|\gamma| < 10^{-2}$, approximately.

Since z_s and the basic set, z_k, are unique and since only roots of O(1) and O(1/ε) are permissible, all roots of (2.1) have been determined for $\varepsilon \to 0$.

B. Limit $\varepsilon \to \infty$

Next the limit $\varepsilon \to \infty$ is considered, i.e., $\tau \to 1$, Re $\to 0$ or both. Because the present work is based on the Re $\to \infty$ theory of Reference 1, it would be inappropriate to allow Re $\to 0$. From (2.2), if $\tau = 1 - s$ where $s > 0$ and $s \ll 1$, then

$$\varepsilon = (-i/A\,Re^{1/2}s)[1 + (3is/4) + O(s^2)]. \tag{3.10}$$

In this section all limits are taken with respect to $\varepsilon \to \infty$. Again there are only two kinds of roots of (2.1).

(i) $z = O(1)$ for which $\varepsilon z \sin z$ is the dominant term and the zeroth approximation to the roots is

$$z_N = N\pi,\ N = 1, 2, \ldots \tag{3.11}$$

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5. *Isaacson, E., and Keller, H. B., Analysis of Numerical Methods, John Wiley & Sons, New York, 1966, p. 86.*

the root $z = 0$ being discarded since it is not $O(1)$. The next approximation gives

$$z_N = N\pi - (1/N\pi\varepsilon). \tag{3.12}$$

This expression can be obtained by substituting $z = N\pi + \Phi$ into (2.1), expanding $\sin \Phi$ and $\cos \Phi$ in power series, and retaining only terms through first order in Φ.

(ii) $z = O(\varepsilon^{-1/2})$ for which neither term in (2.1) is dominant. There is a single root, z_s'; its zeroth approximation is

$$z_{so}' = -i/\varepsilon^{1/2} \tag{3.13}$$

with $-\pi/2 < \arg \varepsilon^{1/2} < 0$. Although it has not been proved it seems clear that $z_s' = z_s$. The next approximation gives

$$z_s' \simeq (-i/\varepsilon^{1/2}) - (i/3\varepsilon^{3/2}). \tag{3.14}$$

The approximations (3.11) and (3.13) or (3.12) and (3.14) can be used as first guesses in an iterative scheme to get the roots of (2.1); for the largest ε tried, $|\varepsilon| = 1.08$, they approximated the roots to only one significant figure. Another way to obtain a reliable first guess for moderate ε is to rewrite (2.1) in terms of $\cot z$ (take $z = N\pi + \phi$) and use the first two terms in the expansion of $\cot \phi$. The result is

$$z = [-N\pi\varepsilon + \{(N\pi\varepsilon)^2 - 4(\varepsilon - 1/3)\}^{1/2}]/[2(\varepsilon - 1/3)] + N\pi. \tag{3.15}$$

Some representative results are shown in Figures 1 and 2. The roots of (2.1) are plotted in the (z_R, z_I) plane for Re = 10 and 100, $\tau = 0.1$ and $A = 3.148$ in Figure 1. All the z_k, $k = 1, 3, \ldots$ lie close to the real axis; the z_s for the two values of Re are noted. As Re increases, the z_k do not change much but $z_s = O(\mathrm{Re}^{1/2})$. For Re = 10, $\varepsilon = 0.0836 - 0.1346\,i$ and $\gamma = (-0.348 + 1.23i)10^{-3}$ and for Re = 100, $\varepsilon = 0.0264 - 0.0462\,i$ and $\gamma = (-5.65 - 4.35i)10^{-10}$. In Figure 2, the roots are plotted for Re = 10 and $A = 3.148$ for $\tau = 0.1$ and 0.9. For the latter either (3.5) or (3.13) can be used as first guess in Newton's method to obtain z_s although (3.13) is a better guess, as expected. Also for this case Real $(z_s) <$ Real (z_k) for all odd k.

IV. SOME EFFECTS OF INCLUDING z_s IN THE THEORY OF REFERENCE 1

The existence of z_s has a number of consequences in the theory of Reference 1, some of which are mentioned here. The effect on the non-dimensional endwall pressure, C_p at $r = 1$, of including z_s is shown in Table 1 for $\tau = .5$, $A = 3.148$, and four values of Re. Ten terms were used in the series to calculate C_p, i.e., ten values of z_k or nine plus z_s. For Re $< 10^3$ inclusion of z_s makes a significant difference. Since the theory of Reference 1 is asymptotic for Re $\to \infty$, there is a separate question of how accurate that theory is for Re $< 10^3$; this question is not discussed in detail here.

Table 1. C_p at $r = 1$

$\tau = .5$ $\quad$ $A = 3.148$

Re	10	10^2	10^3	10^4
With z_s	.0819	.836	.668	.470
Without z_s	.1398	.793	.662	.470

Figure 3 shows C_p vs r for Re = 100, $A = 3.148$ and $\tau = .1$, including z_s and not. The effects are negligible for $0 < r < .45$; for $r = 1$ the C_p that includes z_s is 16% larger than that if z_s is not included. The results for C_p from the spatial e.v. method (not discussed here) are also shown; it can be shown that these are accurate to the number of significant figures plotted and therefore show the error in the results of Reference 1, even including z_s.

In Reference 1, the satisfaction of the sidewall boundary conditions, at $r = 1$, requires the expansion of x, $-A < x < A$, in terms of the eigenfunctions $\sin z_k x/A$ and $\sin z_s x/A$. Two methods of determining the coefficients were given: a least squares fit of the partial sum to x and the eigenfunction expansion obtained from the boundary value problem satisfied by the e.v. and eigenfunctions. This is not a self-adjoint problem so the eigenfunctions are not orthogonal but they are biorthogonal with respect to the solutions of the adjoint problem. This fact enables b_k and b_s, the coefficients in the expansion

$$x = \Sigma\, b_k \sin z_k\, x/A + b_s \sin z_s\, x/A \qquad (4.1)$$

to be determined analytically, using

$$b_k = b\ (z_k)$$

$$b_s = b(z_s) \tag{4.2}$$

$$b(z) = (2A/z^2)\ (1 + \varepsilon^2 z^2)\ \sin z/(1 + \varepsilon^2 z^2 - \varepsilon).$$

The calculation of b_k is straightforward because z_k is $O(1)$ but the evaluation of b_s is tedious, time consuming and could require more than the 14 significant figures available in the CDC 7600 for Re > 100. The reason for this is that the numerator of b_s goes to zero exponentially, which can be seen from (3.5), (3.9) and the fact that $1 + \varepsilon^2 z_s^2 \sim -4\gamma = -4e^{-2/\varepsilon}$. If b_s is evaluated from (4.2), without using asymptotic formulas, a large number of significant figures is required in z_s; this is illustrated in Table 2 for the quantity called $\hat{u}_s(1) = -i\ [2\tau\ (1-\tau)/(1+\tau)]\ b_s$ in Reference 1.

Table 2. Effect of Significant Figures in z_s

Re = 100			A = 3.148		τ = .1
NSIG(z_s)	5	7	9	11	13
Real($\hat{u}_s(1)$)	1.55×10^{-3}	1.63×10^{-5}	-2.85×10^{-6}	-2.12×10^{-6}	-2.132×10^{-6}

NSIG (z_s) is the number of significant figures in z_s used in (4.2); similar behavior is found for Imag ($\hat{u}_s$ (1)). In contrast, if (3.5) and only the first two terms of (3.9) are used to derive the asymptotic form of b_s,

$$b_s \sim i\ 4A\ \varepsilon\ e^{-1/\varepsilon},$$

then Real ($\hat{u}_s$ (1)) = -2.131×10^{-6} is obtained from this simple calculation using four significant figures for ε (i.e., z_s); this differs by 1 out of 2000 from the results given in Table 2 for NSIG (z_s) = 13. The power of asymptotic methods is shown by this example. In addition, for Re > 100, NSIG > 14 would be necessary, at which point it would be impractical to use the CDC 7600. The way to proceed is to make use of asymptotic methods.

A check on the relative contribution of z_s can be made by computing the sum and the z_s terms in (4.1). For x = A these should add to A. With the sum denoted by Σ_k, the following results are obtained for Re = 100, A = 3.148, τ = .1:

Real (Σ_k)	= 2.9815	Im (Σ_k)	= .26793
Real $(b_s \sin z_s)$	= .1665	Im $(b_s \sin z_s)$	= -.26799
	3.1480		-6×10^{-5}

The exact results are 3.148 and 0; 49 terms were used in Σ_k. Clearly it is essential to include the z_s terms.

Since $b_s \sin z_s = 0(\epsilon)$, the contribution of the z_s term to the series expansion and, ultimately, the pressure, approaches zero as Re $^{-1/2}$. Since the calculation in References 1 and 2 were made for Re > 1,000, inclusion of the z_s term would have a negligible effect on those calculations.

ACKNOWLEDGMENT

The authors wish to thank Miss Joan M. Bartos for programming and performing the calculations presented here.

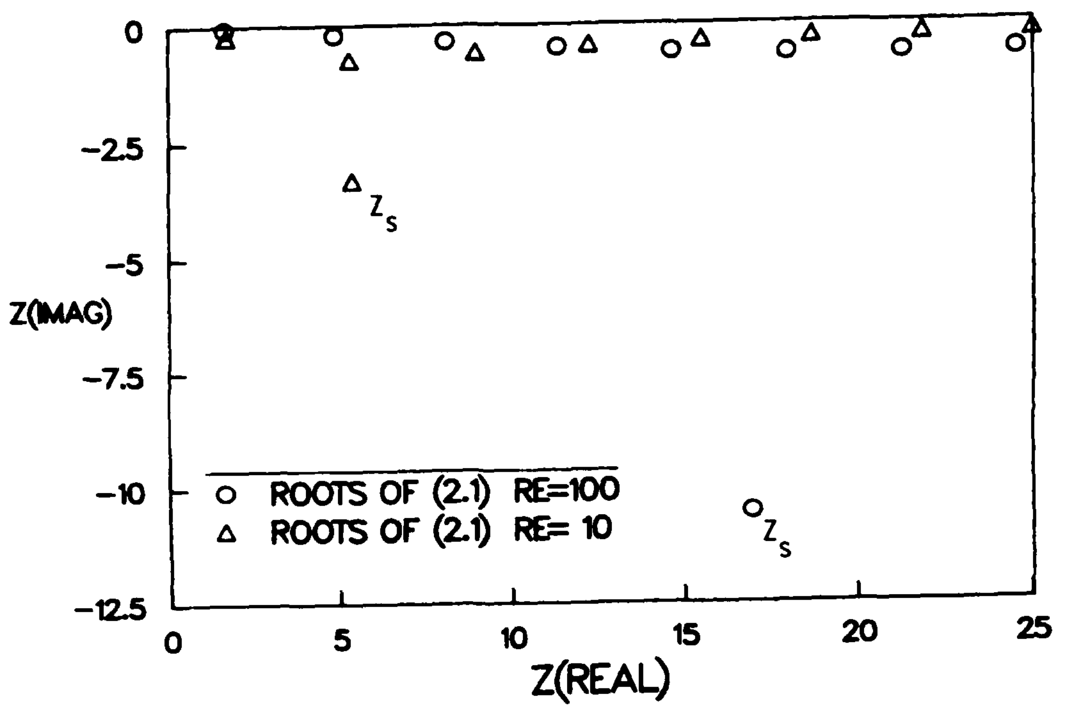

Figure 1. The Roots of (2.1) for Two Values of Re with $\tau = 0.1$ and $A = 3.148$; the $O(Re^{1/2})$ Roots are Designated by z_s

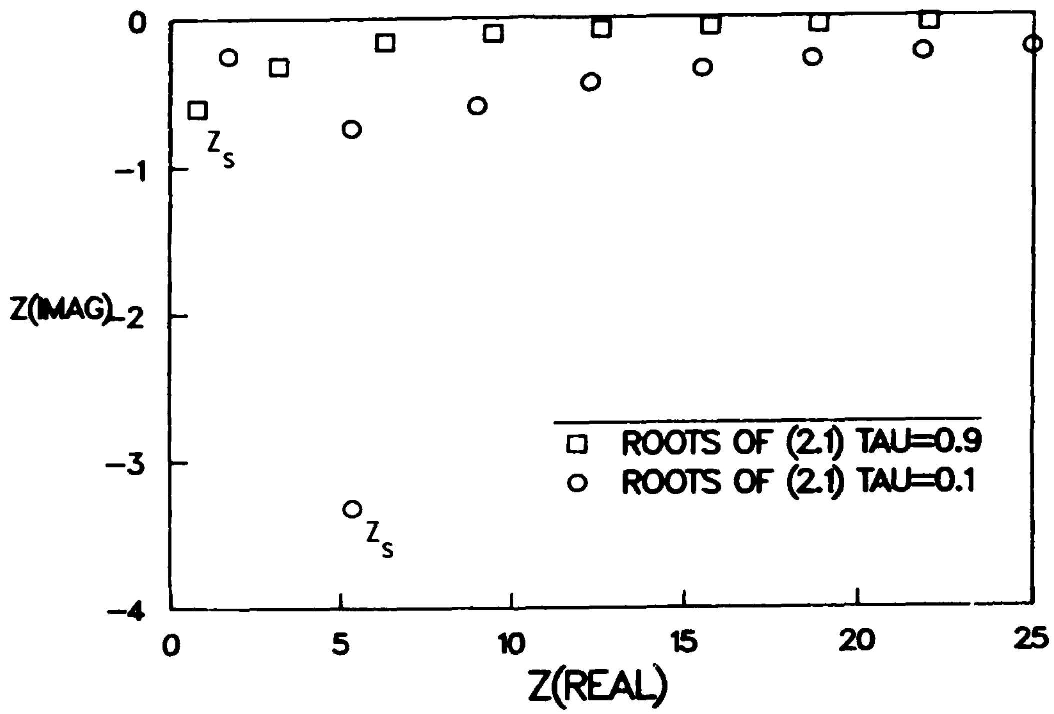

Figure 2. The Roots of (2.1) for Two Values of τ with Re = 10 and A = 3.148; the $O(Re^{1/2})$ Roots are Designated by z_s

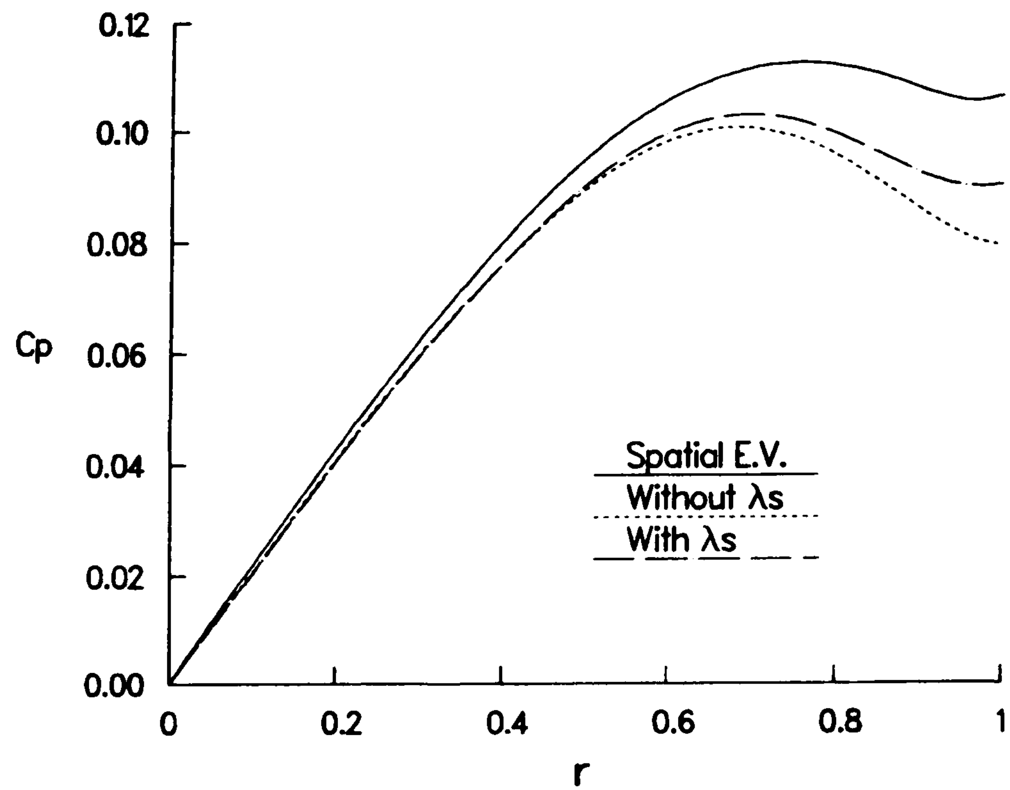

Figure 3. C_p vs r for Re = 100, A = 3.148 and $\tau = 0.1$ Including and not Including z_s; Results for the Spatial Eigenvalue Method are also Shown

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LIST OF SYMBOLS

a	radius of cylinder [cm]
A	c/a = aspect ratio of cylinder
b_k, b_s	coefficients in biorthogonal eigenfunction expansion of $f(x) = x$ (See (4.1) and (4.2))
c	half-height of cylinder [cm]
C_p	pressure coefficient
e.v.	abbreviation for eigenvalue
f	$(1-i\ \delta)\ \tau$
F	$\cos z + \varepsilon z\ \sin z$
k	odd integer
K_o	amplitude of projectile angular motion [non-dimensional]
r	radial coordinate /a
Re	Reynolds number = $a^2\phi/\nu$
s	$1 - \tau$
t	time [sec]
$\hat{u}_s(1)$	$-i\ [2\tau(1-\tau)/(1+\tau)]b_s$
x	longitudinal coordinate/a
z	$A\lambda$
z_k	root in basic set of roots of (2.1)
z_N	root in basic set of roots of (2.1) for $\varepsilon \to \infty$

LIST OF SYMBOLS (Continued)

z_s	$O(\varepsilon)$ root of (2.1)
z_s'	$O(\varepsilon^{-1/2})$ root of (2.1)
z'_{so}	$-i/\varepsilon^{1/2}$, (see (3.13))
z_o	$-i/\varepsilon$, (see (3.5))
γ	$e^{-2/\varepsilon}$
δ	$(1/\tau) \times$ yaw growth per radian of nutation
δc	complex displacement thickness
ε	$\delta c/A$
θ	azimuthal coordinate
λ	spatial eigenvalue
ν	kinematic viscosity of liquid [cm^2/s]
ρ	density of liquid [g/cm^3]
τ	nutational frequency of cylinder/$\dot{\phi}$
$\dot{\phi}$	spin rate of cylinder [radians/s]

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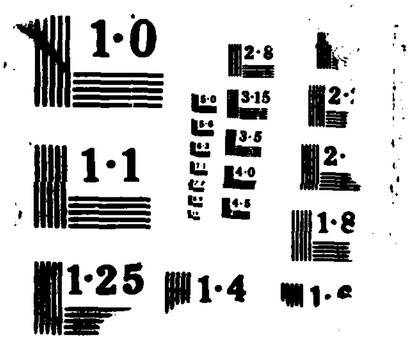
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SUPPLEMENTARY

INFORMATION

Errata for BRL-TR-2727

1. The enclosed sheet contains corrected versions of Figures 1 and 2 for pages 17 and 18 of the following BRL technical report: R. Sedney and N. Gerber, "Asymptotic Analysis of the Roots of a Certain Transcendental Equation," BRL-TR-2727, April 1986, US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD.

2. P.10, 6th line from bottom -- the value of ε should be $\varepsilon = 0.0264 - 0.0426\, i$.

NATHAN GERBER

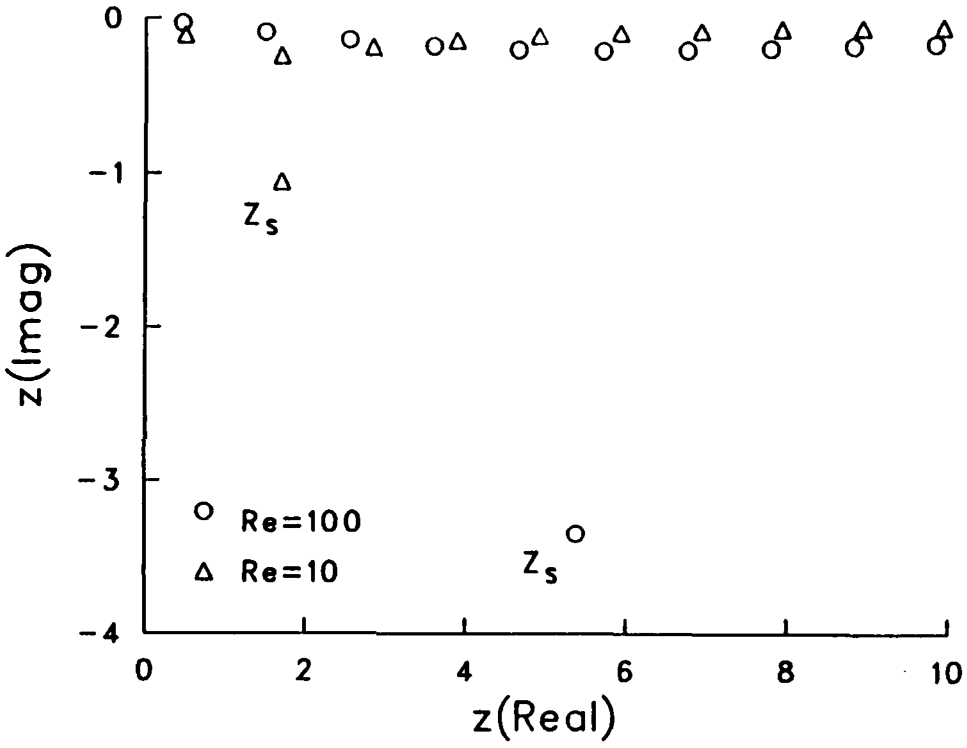

Figure 1. The Roots of (2.1) for Two Values of Re with $\tau = 0.1$ and $A = 3.148$; the $O(Re^{1/2})$ Roots are Designated by z_s

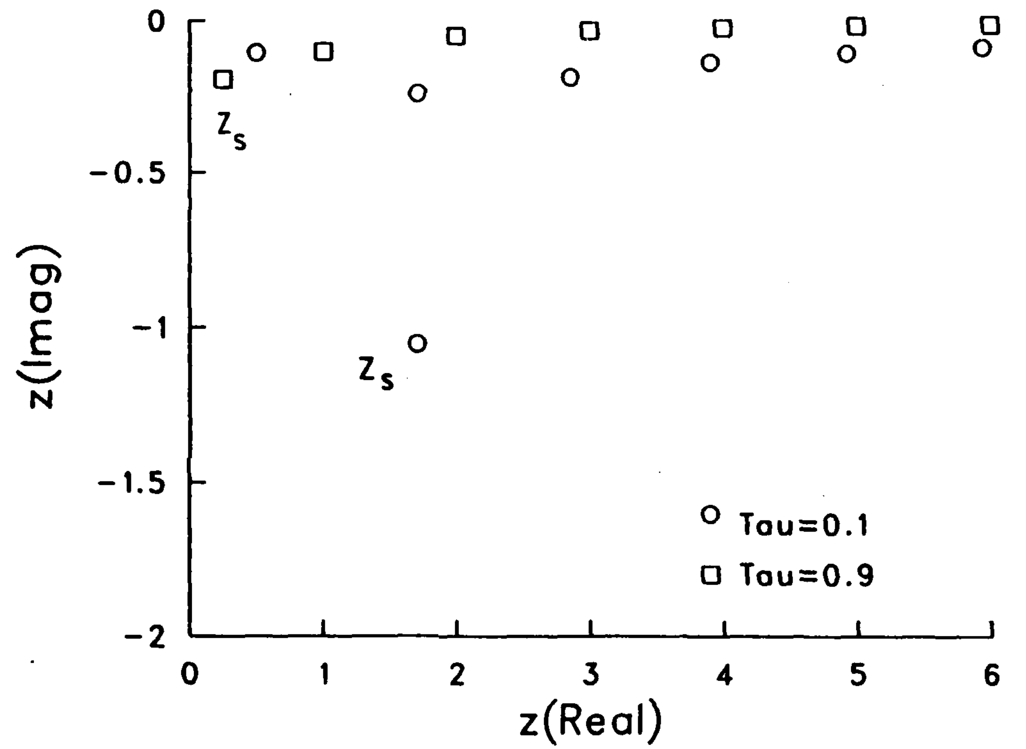

Figure 2. The Roots of (2.1) for Two Values of τ with Re = 10 and $A = 3.148$; the $O(Re^{1/2})$ Roots are Designated by z_s

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