AD-A168 936 RADIOGRAPHIC DETERMINATION OF MASS OF INERTIAL TENSORS 1/1
OF ANATOMICAL SEGMENTS(U) ANCO ENGINEERS INC CULVER
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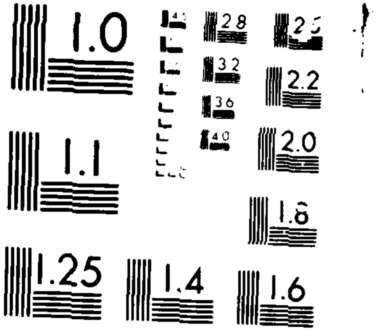

MICROCOP

ANCO

TO: L. Lustik, Dr. Weiss, Naval Biodynamics Laboratory

FROM: Paul Ibanez, ANCO Engineers, Inc. Culver City, Calif

N00014-85-C-0594

DATE: 21 April 1986

RE: "Radiographic Determination of Mass of Inertial Tensors of Anatomical Segments", Progress Reports 3-5, 0001AC, 0001AD, 0001AE - ANCO project 1663.15

1.0 INTRODUCTION

ANCO is investigating the use of radiographic techniques to determine properties of anatomical segments - mass, center of gravity, and the inertial tensor. As shown in our proposal, three orthogonal projections (x-ray photographs) through the segment will determine these properties if sufficiently high energy x-rays are used so that attenuation is independent of atomic number and just dependent on mass density. This occurs above a few hundred keV energy.

The intent of this study is to investigate the practicality and potential accuracy of the method. Progress to date is described herein and covers all but a small final fraction of the work to be conducted under this project.

2.0 X-RAY SOURCE

The decision must be made as to the source of the x-rays, either an x-ray tube can be used or an x-ray (gamma ray) source. Most commercial x-ray tubes operate below several hundred keV. Special industrial x-ray tubes can reach several MeV. All tubes provide a distributed energy source containing energies above and below their specified energy (the bulk being below).

Gamma ray sources can provide single energy x-rays at high energies. Examples include Cesium 137 at .66 MeV (33 year half life, .39 r/hr/Curie at 1 meter dosage) and cobalt 60 (5.3 year half life, 1.35 r/hr/Curie).

Our studies indicate that ~~a~~ radioactive source is preferable to an x-ray tube for the following reasons:

- The source can have higher energy than the more commonly available tubes.
- The source energy is monochromatic (single energy) while the tube has much radiation at lower energies where absorption is atomic number dependent. This is true even for high energy tubes.

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- Sources are less expensive and smaller than tubes making the exposure set up up less expensive and simpler. Also the smaller sources can be placed closer to the subject without blurring due to source size and are easier to shield.

- The source is more easily shielded than is a machine because of the small size of the source.

3.0 EXPOSURE TIME AND DOSE

Calculations and actual testing indicate that for Agfa D-7 film (with 5 mg Pb screens and X10 intensifier) and the source placed 1 meter from the segment and screen, the following exposure time and doses are required (the source strength below are those currently available to us and could, of course, be varied in future work).

Source	Strength	Exposure Time (min.)	Dose (roentgens)
Cs137	.146 Ci	343	2.2
Co60	.80 Ci	26	0.6

The exposure time can be reduced by placing the source closer to the object or increasing source strength (dose will not be significantly changed). Thus, for example, use of a one curie source at 1/2 meter would result in an exposure time of 12.5 minutes for Cs137 and 5.2 minutes for Co60. Both this distance and source strength are practical. As several exposures (typically 3) must be made.

We are looking at total exposure times of 15-40 minutes, it appears feasible to hold an anesthetized animal still for this length of time. A dose of 2-7 roentgens should not cause the animal harm on a one or two time basis. It is possible that alternate films and intensifiers may further reduce these times and dosages. Larger sources could be used to reduce exposure time.

4.0 EXPOSURE CONFIGURATION

As anticipated, multiple exposures are desired. The issues to be considered are:

- Can non-orthogonal views be used?
- Can more than 3 views be useful?
- Can exposure be made simultaneously using multiple sources?
- How can the anatomical segment be "spotted" to establish a coordinate reference?

The first three points above have a common solution. Non-orthogonal views are useful so as to better isolate a given segment. For example, the head is connected to the body by the neck. If the mass properties are desired above a given reference (say a specified vertebrae) and if one could severe the head at the point and "float" it in space, one could easily take 3 orthogonal views, each encompassing the entire head. Since we wish to do non-destructive testing, this can not be done. Orthogonal views of an unsevered head will result in some non-coverage and/error, as illustrated in Figure 1.

Non-orthogonal views, as shown in Figure 1, reduce this error. The error could be reduced to zero by having the views in the same horizontal plane. However, they would no longer be independent and this would lead to an inability to estimate certain mass properties. (Remember that, as shown in our proposal, two orthogonal views in a horizontal plane could determine mass, center of gravity, and all but the off-diagonal terms of the inertia tensor. Remember also that if many views, each differing by a small angle, were taken in a horizontal plane, we would have a "CAT" scan and could determine the density distribution and hence all properties. We are however trying to determine all properties with a few scans.) Hence, some angle, however small, must exist between all scans.

Multiple scans produce redundancy and help to make up for almost dependent scans and are consequently potentially useful.

Alternately, notes by L. Lustik suggest that if symmetry of the head is assumed (bilateral symmetry) then only 2 views may suffice to determine the inertial tensor. We are currently evaluating this approach.

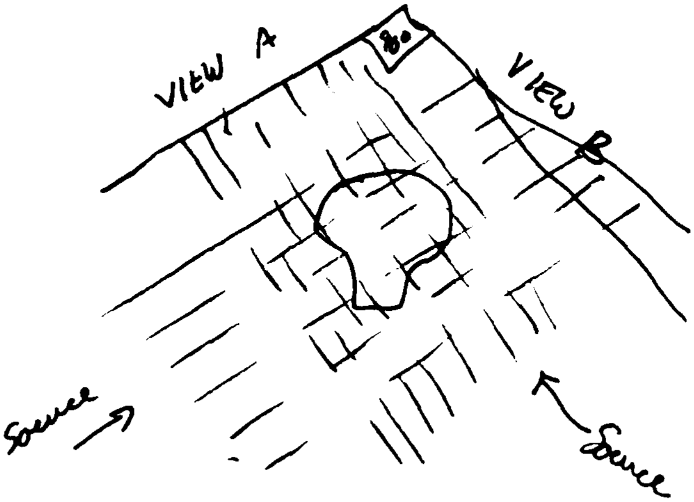
In addition to these consideration, we must concern ourselves with beam spreading (parallax, fan beam) as we plan to use a source close to the segment. A source closer than about 2 meters will have a significant non-parallel beam. Non-orthogonal views, multiple views (more than 3) and beam speed all introduce transformations of the data that must be accounted for. Fortunately there is a simple technique for doing so.

Assume that any number of views are taken in any number of orientations. The views are discretely converted to give a vector R of measurements. For example, if 5 views were taken and a 20x20 cm grid used for each, then there would be 5x20x20 or 2000 measurements, and R would be a vector of length 2000. Assume also that in a convenient orthogonal coordinate system, the space containing the segment is discretely converted into cells of assumed uniform density P.

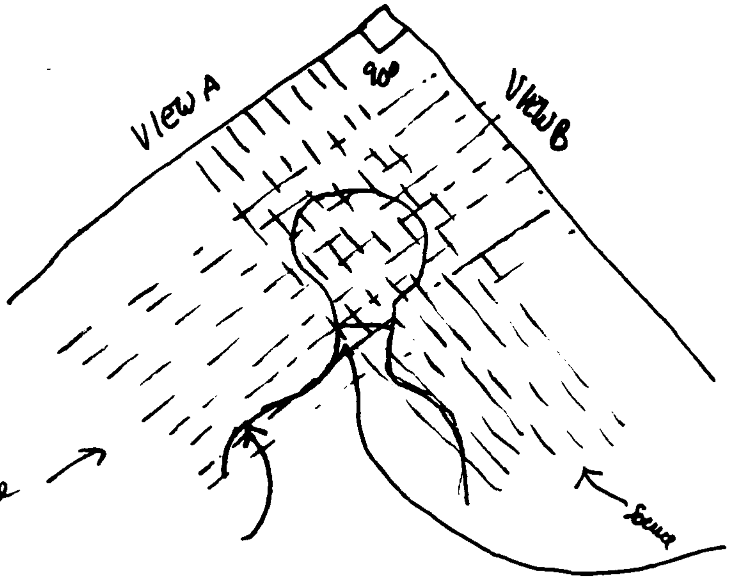
Thus, if the space is 20x20x20 cm, we can define a vector P of size 8000. Depending on the geometry of the views and the spreading ray path, each element of R is linearly dependent on some (in fact a very few) of the elements of P.

$$R = TP$$

FIGURE. 1



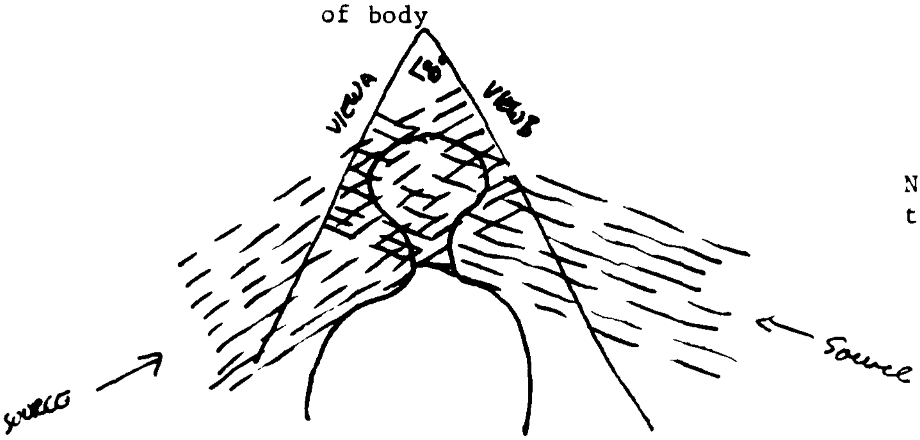
Orthogonal views of a severed head have complete coverage.



Orthogonal views of an attached head can introduce errors.

Non overlap at neck

Possible effect of body



Non orthogonal views reduce this error

(for simplicity these examples are just two dimensional)

Where T is a large (2000 x 8000) matrix with most terms equal to zero. The P can not be solved for as there are too few equations (2000) for the unknowns (8000) and, further, all the equations may not represent linearly independent data. We do not, however, wish to know P but rather certain linear combinations of P (i.e. mass, center of gravity, inertia tensor - see equations 1-4 of our proposal). For example the elements of the inertia tensor can be placed in a six component vector J and equation 3 and 4 of our proposal written as,

$$J = UP$$

Where, for example U is a 6 x 8000 matrix. For the moment, assume that the matrix T can be inverted and call this inversion TI.

Then

$$P = TIR$$

And

$$J = UP$$

Or

$$J = UTIR$$

Where UTI is a 6 x 2000 matrix. We would in fact have an over constrained system (more equations than unknowns). Note that UTI, if it existed, is only a property of the geometry of the sources and views and is independent of the properties of the segment under study.

This form of problem occurs frequently in various processing and estimation tasks. In fact a matrix TI can be defined which has many of the properties of the (non-existent) inverse of T. It is called the Pseudo Inverse (also the Penrose inverse, or singular value decomposition inverse). Without further elaboration here, note that J = UTIR should provide a good estimate of the inertial properties. We know that in an orthogonal parallel scan, the answer is exact (as provided in our proposal) and feel that non-orthogonal scans with spreading beams should also yield a good estimate (as long as the angles are not too far from 90 degrees) and that multiple views (beyond 3) can help. Thus, the pseudo inverse allows solution in one formalism, accounting for:

- Non-orthogonal views
- Spreading beams
- Redundant views (in the limit - a CAT Scan)

The evaluation of UTI, while a lengthy computation, needs only be done once and will not change once a view geometry is chosen.

These are two other issues mentioned above - simultaneous exposure and spotting for coordinate reference. In principal all views could be exposed at once, thus reducing the time that animal must be held still (anesthetized). There is no mathematical reason why the data could not be analyzed even if one view received direct exposure from more than one source. However, the effect of scattering would be greater in such a situation. This is because with one source, most of the scattered radiation would fall upon one of the views. This effect will be quantified but will probably suggest against simultaneous exposure. Note that if used, simultaneous exposures reduce exposure time but not dosage.

The spotting of a coordinate reference is required. Our idea is to place a small dense object (ball bearings) at known points on the segment (e.g. vertebrae, top of head, temples, etc.) and then locate these in each of the view exposures. The data reduction computer program would then relate these to the coordinate system in which the mass properties are defined. The weight of the objects would have little effect on the calculated mass properties.

5.0 ACCURACY

A primary goal is to determine the potential accuracy of this technique. Note that researchers using a CAT scanner to determine inertial properties have, at great cost, produced results with a 5% accuracy. It is unlikely that the accuracy of our few scan techniques would be better.

5.1 Discretization - A coarser grid will give greater error than a finer grid. We are tentatively working with a 1 cm grid. In section 3.5 we present theoretical calculations of a homogeneous sphere with a 1 cm grid. The error in mass calculation is about .8%, and the error in moment of inertia is 7%. This suggests that a 1/2 cm grid may be preferred.

5.2 Attenuation Sensitivity - Even at several hundred keV and higher, there is some variation in attenuation that depends on atomic number (not just density). In section 8.5 we present theoretical calculations of a 6.35 cm radius sphere in which the outer 1 cm is assumed to be 5% less attenuating than the inner part. This represents an upper bound on the attenuation sensitivities expected. The discretization error discussed above is present as well. In this case mass was still predicted to within 1.3% and the moment of inertia error is predicted to 6% (i.e. 1.% different from the case in which only discretization error is present). Medical x-ray technology always has sought tissue differentiation capability, quite understandably. This has played a major role in selection of the operating voltage of the x-ray sources. In the commonly used 70 to 100 kev region, the mass absorption coefficients are quite sensitive to the atomic number of the materials in the path of the beam. One gram of calcium per square centimeter of beam cross section absorbs nearly twice the number of x-rays that are absorbed by one gram of carbon per square centimeter. As a result, bones are very prominent in medical x-rays. The contrast between bones and soft tissue also is a function of their different densities, of course, but the different chemical composition also is a significant factor.

At energies above approximately 300 keV, and up to approximately 2 MeV, the variation in these mass absorption coefficients is very much reduced. Radiographs produced in this energy range should exhibit densities quite independent of minor variations in composition. At the energies emitted by decaying Cs137 or Co60, 662 keV or approximately 1.25 MeV, respectively, the variation in mass absorption coefficient between carbon and calcium is no more than about one percent.

The only exception is hydrogen, which tends to have a coefficient about twice that of the other elements. Then 8% hydrogen tissue has a radiographic density approximately 8% higher than it would be if the tissue had some other element substituted for the hydrogen. Fortunately, it seems unlikely that similar parts of different bodies will vary in hydrogen content by more than perhaps one percent. The same probably can be said about all but the gravest of injuries or dehydration effects in the same body. The effect of typical hydrogen content can be covered by the calibration process. Then the error associated with hydrogen content variations should not exceed about one percent.

The overall composition effect on error at Cs or Co energies also should be of the class of one percent, perhaps two at the very most, in concert with the numerical case mentioned above.

5.3 Scattering - X-ray or gamma ray scattering declines more rapidly with increasing energy than does absorption, but scattering remains significant throughout the practical range of energies. For cesium or cobalt sources, scattering totals approximately 1.5 times absorption. Some of the rays will be scattered in a direction that traverses the film, and statistically will contribute to the exposure. This does have the potential to limit the accuracy of the entire process, if not properly taken into account in the experiment plan or the data reduction.

The angular distribution of the scattered gamma rays is energy dependent, becoming more and more forward, relatively, at higher and higher energies. The 662 kev gammas from cesium scatter with the distribution shown below:

Scattering Angle, Degrees		Percent of Scatters	Cum. Percent
0	10	2.8	2.8
10	20	7.6	10.4
20	30	10.4	20.8
30	40	11.8	32.6
40	50	10.2	42.8
50	60	8.4	51.2
60	70	7.5	58.7
70	80	6.4	65.1
80	90	5.7	70.8
0	90	70.8	

Seventy percent are scattered forward, versus thirty percent backward. The energy of the scattered radiation is less than that of the original photon, declining from the undegraded value for near zero degree scatter to 288 kev at 90 degrees.

The significance of the scattering is not hard to understand. Envision an equidimensional body resting on a film cassette, and irradiated by the collimated bundle of gamma rays. Scattering events occur throughout the body. Events in the upper central region can be scattered at angles up to 30 or 40 degrees and, if they escape further scatter, arrive at a point in the film directly beneath the outer regions of the body. One third or more of this scattered radiation will fall in the image zone on the film if no action is taken to alleviate the problem. In addition, the lower energy gamma has a higher probability of interacting with the film, both because of the higher cross section at lower energy and because of the longer path through the emulsion at the slant angle.

Examination of the results of scattering in the bottom region of the film leads to the conclusion that about two-thirds of the scattered rays will fall in the image zone of the film. In this case, at least, the small angle scatters may not fall outside of the rather large resolution element that is acceptable for the current work. Overall, it appears that the scattered radiation reaching the film may be of the same order as the radiation absorbed by the body.

There are several reasons that the scattering phenomenon does not destroy medical x-rays. First and foremost, lower energy photons have a substantially lower ratio of scattering to absorption cross section. Then, when scattering does occur, the cross section is increased so sharply that absorption is virtually assured before the scattered photon can reach the film. Finally, because of the relative ease with which the low energy photons can be absorbed, filters are provided to selectively absorb those photons approaching the film at an angle indicative of scattering.

Thin absorber bars are of little value for collimating the beam from a cesium or cobalt source, and the lower absolute cross section and less favorable cross section ratio at the higher energies already has been observed. The only viable approach to high energy absorption control is to withdraw the film from its position immediately behind the film. In the extreme, if the film were withdrawn a great distance, that fraction of the original collimated beam which penetrated the object would be essentially free of scattered photons by the time it reached the film. The scattering would have to be within some very small angle to remain within the image at the distant film, and very few of the scattering events involve very small angles.

Consider a 20 centimeter object withdrawn 40 centimeters, about 16 inches away from the film. Only those photons scattered at angles less than about 14 degrees can fall in the image plane. This includes only about seven percent of those gammas scattered from the bottom of the body. The figure drops to less than three percent for scattering events in the upper part of the body. The mean is of the order of 5 percent, and can be accounted for empirically in the calibration process. The residual errors probably can be held to one or two percent.

In principle the separation could be increased to many feet, totally eliminating the scattered component of exposure, for all practical purposes. There are two potential objections. First, the density achieved on the film would decline, if the level of object exposure to radiation were held constant. The "thinner" film might not be readable with the same precision. This seems resolvable. The image would be enlarged as well as less dense; the same number of unreacted gammas reach the film after penetrating the object. It should be possible to alter the instrument to read a larger section of film, corresponding to the larger size of the image. There might be some loss of exposure resolution due to the nonlinearity of the film in the low exposure region if the separation were pushed to excess.

The second concern involves growth in the defocussing of the image due to source size. The angle subtended by the source as seen from the object is projected forward to the film. The greater the objects separation from the film, the larger the image blur zone will become. However, this also musts be considered relative to the enlarged image size. The blur does not grow relative to the image, and so poses no problem in a properly redesigned experiment. The same can be said about photon statistical errors.

There seems to be little true objection to separating the film from the object, within reasonable limits. It is clear that separation of the object from the film alleviates the scattering problem. Separations of 30 to 60 centimeters may be desirable, depending on the object size. The resulting errors will be acceptable.

5.4 Calibration and Dynamic Range - All films have certain sensitivity and dynamic range. The simulations are a non-linear function of exposure and have a threshold and saturation. Thus, too small an exposure will produce a zero signal regardless of variation of the density. Too large an exposure will produce a "100%" signal regardless of the variations of density. A film and exposure time must be chosen to produce a result in the quantitative region of sensitivity and the resulting radiograph must be capable of being quantitatively read.

Initial simulations suggest that the attenuation of the beam will vary between 9% and 90% (leaving 100% to 10% of the beam). Thus a dynamic range of about 10 is required. The maximum attenuation is expected to equal about 1" of steel (the approximate equivalent path density of a ray through the head). To provide calibration of the film, a 1" steel and 1" aluminum step wedge have been constructed with 1/8" steps. As the aluminum is about 1/3 as dense as the steel, the steps provide a 24 fold dynamic range calibration in the region of interest. These wedges will be used in every view of exposure. The data reduction photodensitometer will read the wedge radiographs for each view to provide a direct density calibration. It is felt that the calibration and dynamic range error can be kept below 10%. Variation of film sensitivity across a single piece of film is felt to be negligible.

5.5 Reading Error - A photodensitometer has been constructed and appears to give repeatable results to within about 5%. Its accuracy will be further evaluated based on the planned experiments, as discussed below.

5.6 Slicing Error - As discussed earlier, the radiography of any segment that is still connected to the rest of the body will cause some error due to non-overlap of the various views at the interface between the segment and the rest of the body. It is felt that by use of shallow angle views (but not zero angle), this effect can be kept below 5%. This will be quantified.

5.7 Photon Statistical Errors - Photon statistics are satisfactory for the present purposes. The film used requires the order of 0.1 R for reasonable exposure. This indicates energy absorption of the order of 10 ergs per gram of emulsion. The emulsion thickness is of the order of 0.001 grams per square centimeter, indicating that the exposure requirement is equal to about 0.01 ergs per square centimeter. Now noting the half Mev class of the event energy, and remembering that there are approximately 600,000 Mev per erg, it is clear that the order of 10,000 events are required per square centimeter to obtain adequate exposure.

These nuclear events are random in character, and the uncertainty in number of events occurring is equal to the square root of the number, or the order of 100 events for a one square centimeter resolution element. This indicates that the one square centimeter element will have a statistical error of about one percent. If the element were reduced in size, the error would increase relative to the measurement; clearly it would become significant if the resolution requirement were comparable to those of medical x-rays. However, for the present purposes, the error is quite acceptable.

Note that even if the resolution element were significantly reduced in size, say to 0.1 square centimeter, the error would increase only to the order of three percent. If desired, this could be compensated by an order of magnitude increase in exposure. The film still would be readable, and the one roentgen exposure to the test subject probably would be acceptable. Statistical errors are not a problem.

5.8 Distributed Source - The maximum dimensions of the source, either tube target or isotopic, can be kept to less than one centimeter. The source to target distance is unlikely to be less than about one meter. These values imply a probable maximum angle subtended by the source at the target of 0.01 milliradians. Rays penetrating a 20 centimeter target in contact with a film will be spread over the film in proportion to this angle; and edge in the object structure will be imaged as a band 0.2 centimeters in width for the assumptions above. This is acceptable, consistent with the resolution requirements. In fact, the separation between the film and the farthest portion of the object could be increased several fold without causing unacceptable blurring for the present purposes.

6.0 PHOTODENSITOMETER

A photodensitometer has been constructed. It allows sweeping a photocell over a radiograph. The position of the photocell is measured using two orthogonal Celesco lanyard transducers (.005" accuracy). The transducers have + 5 volt output for + 10" travel. The photocell reads light intensity from a reflected light source next to the photocell. The area of illumination and sensing is approximately a 1 cm diameter circle. The photocell output is 0 to 5 volts with adjustable sensitivity.

The three signals generated (two coordinates and one intensity) are continuously read by an IBM-PC based digital data acquisition program, ANFILM. Each 1/10 second, the data is sampled and averaged. The intensity is then assigned to the geometric grid point given by the two measured coordinates. The value is also displayed on a CRT screen grid. Thus the photocell can be swept by hand over the film at random without regard to grid lines to "fill in" the picture. Moving slowly in significant areas results in multiple measurements and averaging of the same grid point, hence improving accuracy. Once the grid is filled in, it is written in a standard data set for future processing (called an "R" data set).

ANFILM also allows positioning of the photocell on the radiographic wedge steps for calibration. Thus, the photocell readings are directly and immediately converted to actual mass densities, cancelling out many potential errors.

ANFILM also allows positioning on the "spot" points on the segment so as to establish their exact relation to the measurement coordinate system.

7.0 SIMULATION PROGRAMS

Two other programs, ANRAY and ANSEG have been implemented. ANRAY has the purpose of solving the equations of Section 4.1.2 of our proposal. Thus, given three "R" data sets, ANRAY calculates the corresponding mass and inertial properties. (Initially for Phase I, we are assuming 3 orthogonal views and parallel beams.)

The second program ANSEG, allows the user to arbitrarily define the mass density of a segment in a 3 dimensional space grid. It then calculates the mass and inertial properties and the 3 resulting "R" data sets. This program will be used in the simulations to evaluate grid size error, absorption variation (with atomic number) error, and dynamic range requirements. Thus, the various "R" sets from ANSEG will be analyzed by ANRAY to see how well the mass and inertial properties can be reconstructed after various error sources are introduced.

8.0 EXPERIMENTS

8.1 Radiography - Test films were prepared typical medical x-ray energies and at 662 kev, using a cesium 137 source. A medium speed, high resolution film was used, in a cassette with intensifying and lead screens.

8.2 Test Objects - The test objects were of several classes, intended to simulate natural animal materials, to provide simple shapes, and to support density versus mass thickness calibration.

The calibration aids were manufactured from steel and aluminum. Step wedges were machined with care, and were configured with eight steps, each 1/8 inch thicker than the last. The maximum thickness was one inch. The wedges are 1/2 inch wide and approximately 4 inches long. The maximum mass thickness of the steel and aluminum wedges are approximately 20.0 and 6.86 grams per square centimeter, respectively. The maximum steel thickness was chosen to match the mass thickness of a human head, approximately.

Spherical and cylindrical bodies were tested to allow simplified data reduction and software validity testing. Acrylic and rubber balls were used, as was a "duck pin" bowling ball. The duck pin ball is a 5 inch diameter sphere of an apparently uniform composition, with a density of 1.39 grams per cubic centimeter. It has a maximum mass thickness of 17.65 grams per square centimeter.

The cylindrical specimen was comprised of a lucite rod 1.75 inches in diameter by 6 inches long, with a density of 1.18 grams per cubic centimeter, inside of an aluminum tube measuring 2.125 inch o.d. by 4.125 inches long by 1.844 inches i.d.. Quarter inch diameter holes were drilled into the ends of the lucite rod to depth of 1 and 3 inches at the two ends.

Two pseudo-natural specimens were prepared, the first comprised of gelatin with chicken bones and some small metallic objects for composition, in a rectangular polyethylene container. The second, used later in the program, was comof sucrose, roughly simulating body soft tissues in atomic mass distribution, and a simulated bone. The "bone" was a plaster of paris loaded polyethylene vial with a 2 centimeter i.d. and an inside length of 5 centimeters. The bone composition included approximately 38 percent water, resulting in a density of 1.77 grams per cubic centimeter. The mineral part of the bone is believed to have an atomic number distribution that simulates the mineral composition of true bone with adequate accuracy. The bone was placed in the center of an approximately 4 inch square by 2.5 inch high polyethylene container, which then was filled with sucrose to a density of 0.96 grams per square centimeter.

8.3 X-Ray Preparation - The initial series of radiographs was prepared with a medical x-ray unit, operating at 70 kev. The objectives of the first test series included establishment the correct range of exposure and confirmation of the expected magnitude of the "Z" effect, the effect of atomic number on x-ray absorption at typical medical x-ray energies. About a dozen pictures were made, using the loaded gelatin and cylindrical objects described earlier. The wedges were included in all shots, and the integrated current was adjusted until qualitatively satisfactory appearing pictures were obtained for both samples. Exposures were duly noted.

8.4 Gamma Radiograph Preparation - A 143 millicurie cesium 137 source was used in several test series. This source produces a tissue dose rate of approximately 0.5 roentgens per hour at a distance of one foot.

As with the x-ray series, the test objects were placed directly against the cassette in the early test series. The test objects were withdrawn approximately six inches from the cassette in later experiments to confirm the validity of this technique for suppression of exposure by scattered radiation. In the final test, the wedges were left on the cassette and the sample was pulled back about six inches from the film, in an attempt to reduce confusion of the calibration densities by the radiation scattered from the sample.

8.5 Preliminary Results - Three numerical simulations and one actual radiograph use are reported here. In all cases, the object of concern is the 5 inch diameter plastic sphere referred to above. The cases were:

A - Exact theoretical model
B - Numerical evaluation of sphere using 1 cm grid size
C - Same as A but reducing outer 1 cm shell attenuation by 5%.
D - Reduced data from actual radiograph of plastic sphere

The resulting mass, c.g., and inertia tensor values are shown in Table 8.1., Case B (compared to Case A) shows that 1 cm grid discretization causes a 1 % mass error and a 7% inertia (Izz) error. The c.g. and inertia cross product (Ixy) error is zero because a symmetric case was taken. Additional simulations will be made to evaluate a non-symmetrical case.

Case C shows that a 5% attenuation variation in a 1 cm shell has less of an effect than the grid discretization.

Case D shows, based on the radiograph experiment, a total mass error of 13% and an inertia term (Izz) error of 8%. The maximum c.g. error is about 6% of the sphere diameter. The cross product term (IXY) is equal to about 2% of the diagonal (Izz) term, whereas, it should be zero. In a typical human head*, the cross product terms are 3% of the diagonal terms. Hence, this cross product error (of 2%) is significant.

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* "Measurement of Mass Distribution Parameters of Anatomical Segments", by E.B. Beeker, NAMRL - 1193, October 1973. I have taken his case 3356. TABLE 8.1

TABLE 8.1

MASS AND INERTIAL PROPERTIES OF 6.35 CM RADIUS 'ERE

A - Exact Theoretical Model

M = Total Mass = 1.49 kg

c.g. X = Y = 0.0 cm

Izz = 24.0 kg-cm^2

Ixy = 0.0 kg-cm^2

B - Numerical Evaluation with 1 cm Grid

M = Total Mass = 1.48 kg

c.g. = X = Y = 0.0 cm

Izz = 25.7 kg-cm^2

Ixy = 0.0 kg-cm^2

C - Same as B with 1 cm shell reduced 5%

M = Total Mass = 1.47 kg

c.g. = X = Y = 0.0 cm

Izz = 25.5 kg-cm^2

Ixy = 0.0 kg-cm^2

D - Actual Radiograph Reduction

M = Total Mass = 1.68 kg

c.g. X = .37 cm, Y = -.80 cm

Izz = 25.9 kg-cm^2

Ixy = 0.6 kg-cm^2

END

DTIC

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