

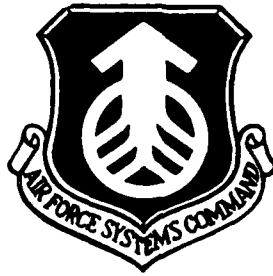
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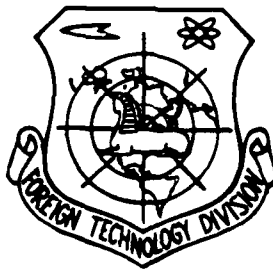


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METRICS OF CLOSED WORLD OF FRIEDMANN, AGITATED BY ELECTRIC CHARGE
(TOWARDS A THEORY ELECTROMAGNETIC "FRIEDMANNS")

by

M.A. Markov, V.P. Frolov



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By: M.A. Markov, V.P. Frolov

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cossec	csc	esch	esch	arc esch	esch ⁻¹

Russian English

rot curi
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THEORETICAL AND MATHEMATICAL PHYSICS.

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METRICS OF CLOSED WORLD OF FRIEDMANN, AGITATED BY ELECTRIC CHARGE
(TOWARDS A THEORY ELECTROMAGNETIC "FRIEDMANN").

M. A. Markov, V. P. Frolov.

The generalization is considered of the well-known Tolman problem to the case of electrically charged dust-like matter of the central-symmetrical system. The first integrals of the correspondent system of the Einstein - Maxwell equations are found. The problem is specified in such a way that with the full charge of the system going to zero, the metrics of the closed Friedman world arises. Such a system is considered at the initial moment, that of maximal enlargement. With any non-vanishing but no matter how small value of the electric charge the metrics is unclosed. The metrics of the almost-Friedmanian part of the world allows the continuation through the narrow manhole (at the small charge) as the Nordström-Reissner metrics with the parameters $\sqrt{\kappa} m_0 = e_0$. The expression for the electric potential in the manhole $\varphi_A = c^2/\sqrt{\kappa}$ does not depend upon the value of the electric charge. The radius of the manhole ($r_A = e_0\sqrt{\kappa}/c^2$) increases with the increase of the charge. The state of the manhole as given by the classical description appears as essentially unstable from the quantum physics viewpoint. The production of various pairs in the enormous electric fields of the manhole gives rise to the polarisation of the latter up to effective charge $Z < 137e$ irrespective of the initial (no matter how great) charge of the system.

1. Generalization of the solution of Tolman to the case of the electrically charged/loaded dustlike material.

Solution of Einstein's equations for case of centrally symmetric gravitational field in associated coordinate system for dustlike material (pressure $p=0$) was found with R. Tolman [1].

For series of problems is of interest generalization of solution of Tolman in case of electrically charged/loaded dustlike material. As is known, closed space/world of Friedman is described by

particular solutions of problem of Tolman. Known also that for the charged/loaded material the metrics of world cannot be closed even in such a case, when the density of material exceeds critical density.

Question arises, how metrics of closed peace/world of Friedmann under the effect of, let us allow, of weak disturbance/perturbation of it by the presence of electric charge.

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Response/answer to the presented question must give the joint decision of the system of equations of Einstein - Maxwell

$$G_i^k = R_i^k - \frac{1}{2} \delta_i^k R = \frac{8\pi\kappa}{c^4} (T_i^k + E_i^k), \quad (1)$$

$$F^{ik}{}_{;k} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g} F^{ik}) = -\frac{4\pi}{c} j^i, \quad (2)$$

$$\frac{\partial F_{ik}}{\partial x^l} + \frac{\partial F_{kl}}{\partial x^i} + \frac{\partial F_{li}}{\partial x^k} = 0. \quad (3)$$

The tensor of energy in right side of (1) let us select in the form

$$T_i^k + E_i^k = \begin{pmatrix} e + \Lambda/8\pi & 0 & 0 & 0 \\ 0 & \Lambda/8\pi & 0 & 0 \\ 0 & 0 & -\Lambda/8\pi & 0 \\ 0 & 0 & 0 & -\Lambda/8\pi \end{pmatrix}, \quad (4)$$

where

$$\Lambda = e^2 / r^4 = -F_{0i} F^{0i} \quad (5)$$

it appears as a result of solving Maxwell's equation under the condition of the spherical symmetry of system. Here

$$T_i^k = \delta_0^k \delta_i^0 e \quad (6)$$

- tensor of material in the associated system of coordinates ($x^i = q$).

In expanded form equation (1) takes form

$$-\frac{8\pi\kappa}{c^4}(T_1^1 + E_1^1) = \frac{1}{2}e^{-\lambda}\left(\frac{\mu'^2}{2} + \mu'v'\right) - e^{-\nu}\left(\ddot{\mu} - \frac{1}{2}\dot{\mu}\dot{v} + \frac{3}{4}\dot{\mu}^2\right) - e^{-\nu} = -\frac{\kappa}{c^4}\Lambda \equiv -\tilde{\Lambda}, \quad (I)$$

$$-\frac{8\pi\kappa}{c^4}(T_2^2 + E_2^2) = \frac{1}{4}e^{-\lambda}(2v'' + v'^2 + 2\mu'' + \mu'^2 - \mu'\lambda' - v'\lambda' + \mu'v') + \frac{1}{4}e^{-\nu}(\dot{\lambda}\dot{v} + \dot{\mu}\dot{v} - \dot{\lambda}\dot{\mu} - 2\ddot{\lambda} - \dot{\lambda}^2 - 2\ddot{\mu} - \dot{\mu}^2) = \frac{\kappa}{c^4}\Lambda \equiv \tilde{\Lambda}, \quad (II)$$

$$\frac{8\pi\kappa}{c^4}(T_0^0 + E_0^0) = -e^{-\lambda}\left(\mu'' + \frac{3}{4}\mu'^2 - \frac{\mu'\lambda'}{2}\right) + \frac{1}{2}e^{-\nu}\left(\dot{\lambda}\dot{\mu} + \frac{\dot{\mu}^2}{2}\right) + e^{-\nu} = \frac{8\pi\kappa}{c^4}\varepsilon + \frac{\kappa}{c^4}\Lambda \equiv \tilde{\varepsilon} + \tilde{\Lambda}, \quad (III)$$

$$\frac{8\pi\kappa}{c^4}(T_0^1 + E_0^1) = \frac{1}{2}e^{-\lambda}(2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - v'\dot{\mu}) = 0. \quad (IV)$$

Here metric is selected in the form

$$ds^2 = e^{\nu}dx^{\nu 2} - e^{\lambda}dx^{12} - e^{\mu}d\sigma^2, \quad (7)$$

where $d\sigma^2 = dx^{\nu 2} + \sin^2 x^2 dx^{\nu 2}$. Differentiation on x^{ν} is designated by point, by prime - differentiation with respect to q .

Using laws of conservation, easy to obtain [1]

$$\tilde{\varepsilon} = -\tilde{\varepsilon}\left(\frac{\dot{\lambda}}{2} + \dot{\mu}\right), \quad (V)$$

$$2\frac{e'}{e}\tilde{\Lambda} = \frac{1}{2}v'\tilde{\varepsilon}. \quad (VI)$$

In our case associated system is not synchronous ($v \neq 0$).

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From (V) integration on x^{ν} is obtained relationship/ratio

$$\tilde{\varepsilon} = 2\frac{\kappa}{c^4}\frac{C(q)}{r^2}e^{-\lambda,2}. \quad (8)$$

where $r^2 \equiv e^\mu$. Equation (VI) gives

$$v' = \frac{2ee'}{r^2 C(q)} e^{\lambda/2}, \quad (9)$$

equation (IV) can be rewritten in the form

$$2(\ln r')' - \lambda - v'/r' = 0. \quad (10)$$

Integrating equation (10) on x^0 , we obtain

$$\ln(r')^2 = \lambda + \int \frac{v'/r'}{r'} dx^0 + \ln(1+f), \quad (11)$$

where $f = f(q)$, $1+f \geq 0$.

Designating through ϕ expression

$$\phi = \int \frac{v'/r'}{r'} dx^0 = \frac{2ee'}{C(q)} \int \frac{e^{\lambda/2} r'}{r'^2} dx^0, \quad (12)$$

let us rewrite relationship/ratio (11) in the form

$$e^\lambda = \frac{r'^2}{1+f} e^{-\phi}. \quad (13)$$

Expression for ϕ can be obtained as follows: using (13), let us rewrite relationship/ratio (12) in the form of integral equation for ϕ :

$$\phi = \frac{2ee'}{C(q)\sqrt{1+f}} \int \frac{e^{-\phi/2} r'}{r'^2} dx^0, \quad (14)$$

whence for ϕ is obtained the differential equation

$$\phi = \delta(q) e^{-\phi/2} / r^2, \quad (15)$$

where

$$\delta(q) \equiv 2ee' / C(q)\sqrt{1+f}, \quad (16)$$

and hence

$$2e^{q/2} = -\delta/r + 2\psi(q). \quad (17)$$

Substituting (17) into (13), we obtain

$$e^\lambda = \frac{r^n}{(\sqrt{1+f}\psi - ee'/C(q)r)^2} \quad (18)$$

or, designating

$$\sqrt{1+f}\psi \equiv \sqrt{1+f}, \quad (19)$$

$$\delta(q) \equiv \frac{2ee'}{\sqrt{1+f}C(q)}, \quad (20)$$

finally we have

$$e^\lambda = \frac{r^n}{1+f} \frac{1}{(1 - \delta/2r)^2}. \quad (21)$$

Equation (1) can be rewritten as follows:

$$e^{-\lambda}(r^n + r'rv') - e^{-\nu}(2\bar{r}r + \bar{r}^2 - rrv') - 1 = -\frac{\kappa e^2}{c^4 r^2}. \quad (22)$$

It is easy to check that

$$e^{-\lambda}(r^n + r'rv') = (1+f)(1 - \delta^2/4r^2),$$

$$e^{-\nu}(2\bar{r}r + \bar{r}^2 - rrv') = \frac{1}{\bar{r}}(e^{-\nu}\bar{r}^2 r).$$

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Integrating (22) on x^0 , we obtain

$$e^{-\nu\bar{r}^2} = f + \frac{2m(q)}{r} - \frac{1}{r^2} \left(\frac{\kappa}{c^4} e^2 - \frac{\delta^2(1+f)}{4} \right). \quad (23)$$

where $m(q)$ - integration constant on x^0 .

Let us rewrite equation (III) in the form

$$-e^{-\lambda}(2r''r + r'^2 - r'r\lambda') + e^{-\nu}(\lambda\dot{r}r + r^2) + 1 = (\tilde{\Lambda} + \varepsilon)r^2. \quad (24)$$

Being convinced, that

$$e^{-\lambda}(2r''r + r'^2 - r'r\lambda') \equiv \frac{(e^{-\lambda}r'^2r)'}{r'},$$

$$e^{-\nu}(\lambda\dot{r}r + r^2) = \frac{(e^{-\nu}r^2r)'}{r'};$$

substituting in obtained expressions (21) and (23) and designating

$$m_1(q) \equiv \frac{c^2}{\kappa} \left[m(q) + \frac{\delta(1+f)}{2} \right], \quad (25)$$

we obtain the relationship/ratio

$$m_1'(q) = \frac{1}{c^2} C(q) \sqrt{1+f}. \quad (26)$$

Equation (II) does not give new relationships/ratios - it is the corollary of others of those used by us equations.

Into first integrals of equations (I) found by us and (III) enter three unknown functions

$$f(q), \quad m(q), \quad e(q). \quad (27)$$

Problem is defined concretely by these functions - they must be assigned initial conditions. Let us select surface $x^0=0$ as three-dimensional-like hypersurface Σ for the assignments of initial conditions.

With condition (IV) $G_0^4 = 0$ together relationship/ratio

$$e^{\lambda(0,q)} = \frac{r'(0,q)}{1+f(q)} \frac{1}{(1-\delta(q)/2r(0,q))^2}. \quad (28)$$

Equation (III) on surface Σ can be registered in the form

$$(e^{-\nu} r^2 r')' - (e^{-\nu} r^2 r)'' + r' = \frac{2\kappa}{c^4} C(q) e^{-\lambda/2} r' + \frac{\kappa e^2}{c^4 r^2} r'. \quad (29)$$

Let us select as q canonical coordinate - distance from the center at zero time ($\exp \lambda(0,q) = 1$ then relationship/ratio (28) becomes determination of $f(q)$):

$$\sqrt{1+f} = r'(0,q) + \frac{ee'}{C(q)r(0,q)}. \quad (30)$$

Subsequently we specialize our problem mainly in that case, when with tendency of electric charge of system in question toward zero locked peace/world of Friedman.

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2. Friedmann's world, deformed by the presence of electric charge.

Internal solution.

Subsequently we will attempt to assign unknown functions $f(q)$, $m(q)$ and $e(q)$ thus in order with $e(q) \rightarrow 0$ to obtain metric of locked world of Friedmann. Since in the locked peace/world complete electric charge is equal to zero, then priori is clear and that the metric of the peace/world in question even with a small electric charge must be not completely closed and that the Friedmann metric deformed by charge must have Nordstrom-Reissner continuation beyond substance. Our problem - of finding at least particular examples, for which by continuous form it is possible to describe entire space of this peace/world. Such by form, we is expected, what internal solution, close one of Friedman for the locked peace/world, the neck/throat must pass into the known external solution of Nordstrom - Reissner. Therefore for the internal solution we will attempt to formulate initial conditions to $x^0=0$ - to moment/torque of the maximum expansion of system, closest to the Friedmann. Specifically, let

- 1) with $x^0=0$ entire space belong to R-region [2],
- 2) the initial velocities of all particles are equal to zero,
- 3) energy density at the initial moment does not depend on q :

$$T_0^0 + E_0^0 = \epsilon_0 = \text{const.}$$

Below we will show that the problem with such conditions in the case of the electrically charged/loaded dust has a solution, i.e., there

is such function $C(q)$ or $M(q)$, which is consistent with the data by conditions.

For given initial conditions equation (29) is rewritten in the form

$$1 - \frac{(r^2 r)'}{r'} = \frac{\kappa}{c^4} 8\pi e_0 r^2. \quad (31)$$

Let us designate

$$\frac{8\pi\kappa e_0}{c^4} \equiv \frac{3}{4a_0^2} \quad (32)$$

and will integrate equation (31), then

$$1 - \frac{r^2}{4a_0^2} = r^2 \quad (33)$$

or, since $r(q=0)=0$,

$$r = 2a_0 \sin q / 2a_0, \quad (34)$$

expression (29) now can be rewritten in the form

$$2 \frac{\kappa}{c^4} C(q) + \frac{\kappa}{c^4} \frac{e^2}{r^2} = 3 \sin^2 \frac{q}{2a_0}. \quad (35)$$

Let us assign charge distribution further. Let all specks of system have one and the same ratio of charge to the mass, β . Let us designate

$$\frac{1}{c^2} \int_0^q C(q) dq = M(q), \quad (36)$$

then new condition it will be registered in the form

$$e(q) = \beta M(q). \quad (37)$$

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Now equality (35) takes form of equation for determination $M(q)$

$$2\bar{M}'(q) + \beta^2 \frac{\bar{M}^2(q)}{r^2} = 3 \sin^2 \frac{q}{2a_0}, \quad (38)$$

where

$$\bar{M} = \frac{\kappa}{c^2} M; \quad \beta = \frac{\beta}{\gamma \kappa}. \quad (39)$$

It is possible to check by substitution which satisfies the following expression equation (38):

$$\bar{M} = \frac{4a_0}{\beta^2} \sin \chi (b \operatorname{ctg} b\chi \sin \chi - \cos \chi),$$

where

$$\chi = \frac{q}{2a_0}, \quad b = \sqrt{1 - \frac{3}{4} \beta^2}.$$

It is easy to check that \bar{M} with $\beta \rightarrow 0$ passes in

$$\bar{M}_0(q) = \frac{3}{2} a_0 \left(\chi - \frac{\sin 2\chi}{2} \right), \quad (40)$$

i.e. into expression for "internal" mass [3] in uncharged Friedmann peace/world '.

FOOTNOTE 1. The total mass, which considers gravitational mass defect in the closed world, is equal to zero [1]. ENDFOOTNOTE.

Further, using (9), it is possible to obtain

$$e^{\chi(0,q)} = \left(\frac{\sin b\chi}{b \sin \chi} \right)^4. \quad (41)$$

With these observations is finished examination of internal solution at zero time - at moment of greatest expansion of material system. In the following points/items the solution in the void (in the regions, where $\bar{r}=0$) is analyzed and the problem of the join of the external and internal solutions.

External Nordstrom-Reissner solution.

As is known, geometry of space out of spherically symmetric distributed mass m_0 by electric charge e_0 is described by metrics of Nordstrom - Reissner

$$ds^2 = \Phi(r) dt^2 - dr^2 / \Phi(r) - r^2 d\sigma^2, \quad (42)$$

where

$$\Phi(r) = 1 - \frac{2\kappa m_0}{c^2 r} + \frac{\kappa e_0^2}{c^4 r^2}. \quad (43)$$

In problem in question should be distinguished three cases:

- 1) $\sqrt{\kappa} m_0 > e_0,$
 - 2) $\sqrt{\kappa} m_0 = e_0,$
 - 3) $\sqrt{\kappa} m_0 < e_0.$
- (44)

In first case metric is characterized by two pseudo-special features of type of Schwarzschild's pseudo-special feature: $\Phi(r_1) = \Phi(r_2) = 0$. With $r_2 < r < r_1$, coordinate r has time-like character. In the coordinates of the type of the coordinates of Kruskal it is possible to describe complete space-time for this case of [4]. The test particle, placed in $r=r_1$, with $x^0 = 0$, for time $T = \pi \kappa m_0 / c^3$ reaches $r=r_2$, it stops and is returned to r_1 .

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At initial moment (moment/torque of temporary/time symmetry) geometry of space takes form of "mole burrow" ("bridges of Einstein - Rosen"). Its neck/throat pulsates with the period $2T$ and never is closed (in contrast to the case of Schwarzschild). Complete closure neck/throat I impede the electrical lines of force, which go through neck/throat² into Euclidean infinity.

FOOTNOTE ². With $e \rightarrow 0$, $\Phi(r) = 1 - (2\kappa m_0) / (c^2 r)$ - the Schwarzschild solution can be interpreted as the external solution for semiclosed peace/world [3, 5]. In work [5] metrics of Kruskal obtains its physical interpretation. ENDFOOTNOTE.

Second case differs from first in terms of fact that T-region is absent. At point

$$r = \frac{\kappa m_0}{c^2} = \frac{e_0 \sqrt{\kappa}}{c^2}$$

we have zero of second order:

$$\Phi(r) = (1 - r_h/r)^2, \text{ where } r_h = \kappa m_0 / c^2 = \sqrt{\kappa} e_0 / c^2. \quad (45)$$

As it follows from further analysis, in this case the geometry at the moment of temporary/time symmetry can be two forms:

α^0 -type of "mole burrow",

β^0 -geometry with the monotonic change r , in particular, realized in the model of Papapetrou (static model of the charged/loaded bullet with $\bar{\beta} = e\sqrt{\kappa}M = 1$). If the discussion deals with the semiclosed charged world, then the external solution, which satisfies the condition of Euclidean nature at infinity, there will be type (α^0).

Third case ($e_0 > \sqrt{\kappa}m_0$) passes into case β , decrease e_0 . Special features here are absent, entire space of R-type. In this case semiclosed peaces/worlds (with the condition of Euclidean nature at infinity) are not realized. Limit $\bar{\beta} = 1$ in this case gives the everywhere static system (model of Papapetrou).

Us interests problem - of lacing external Nordstrom-Reissner solution with internal solution, which describes almost locked

peace/world, i.e., peace/world, whose metric with $e_0 \rightarrow 0$ would pass into metric of locked peace/world of Friedman. With $e_0 \neq 0$ our problem - to maximally continue internal Friedmann solution (to decrease dimensions of neck/throat), to how much this permits the presence of electric field. From this point of view it is expedient to consider the deformation of Friedmann metric by weak electric charge $\bar{\beta} \ll 1$.

Of all cases examined case of 2 ($\alpha^0=1$) satisfies our conditions only. All remaining cases do not lead to the locked peace/world with $e_0 \rightarrow 0$.

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Join of the internal and external solutions.

For convenience in use of boundary conditions for join we convert formula (7) to form, to close one (42), precisely, to

$$ds^2 = \beta dt^2 - \alpha dr^2 - r^2 d\sigma^2, \quad (46)$$

where as x^1 (or q -coordinate) was selected coordinate, whose square stands by coefficient near $d\sigma^2$. The conversion

$$dr = \dot{r} dx^0 + r' dx^1, \quad dx^1 = \frac{dr - \dot{r} dx^0}{r'}$$

converts the first two members of formula (7) to the form

$$e^\nu dx^{02} - e^\lambda dx^{12} = \left(\sqrt{\nu e^\nu - e^\lambda r'^2} dx + \frac{r e^\lambda}{r' \sqrt{\nu e^\nu - e^\lambda r'^2 / r'^2}} dr \right)^2 - \frac{dr^2}{r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu}} \quad (47)$$

Bracketed expression can be converted with the help of the integrating factor $\bar{\mu}(t, r)$ to the form

$$\left(e^{\nu} - e^{\lambda} \frac{\dot{r}^2}{r'^2} \right)^{1/2} dx^0 + \frac{\dot{r}}{r'^2} e^{\lambda} \left(e^{\nu} - e^{\lambda} \frac{\dot{r}^2}{r'^2} \right)^{-1/2} dr = \frac{1}{\bar{\mu}(t, r)} dt,$$

contained in the right side total differential. For future reference substantially obtained expression for α :

$$\alpha = \frac{1}{r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu}}. \quad (48)$$

From conditions for join of internal and external solutions on interface Σ :

$$a^{\text{in}} = a^{\text{out}}|_{\Sigma}, \quad r^{\text{in}} = r^{\text{out}}|_{\Sigma}, \quad (49)$$

we obtain

$$r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu}|_{q=q_0} = 1 - \frac{2\kappa m_0}{c^2 r} + \frac{\kappa e_0^2}{c^4 r^2}. \quad (50)$$

Using (21), (22) and (25), we find

$$\begin{aligned} m_1(q_0) &= m_0, \\ e(q_0) &= e_0. \end{aligned} \quad (51)$$

Thus far we did not obtain response/answer to basic question, precisely, - with what q_0 we were forced to join internal and external of solution, if we wish to continue Friedmann world to its maximally possible closure/isolation with minimal size of neck/throat at moment of temporary/time symmetry. In the third case the join of semiclosed peace/world with the space flat/plane at infinity is

impossible. Consequently, on boundary of $\sqrt{\kappa m_1}(q_0) \geq e(q_0)$, and unknown q_0 is found from the equation

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$$\sqrt{\kappa m_1}(q_0) = e(q_0). \quad (52)$$

Condition (52) can be registered in the form³⁾

$$r_0'^2 = (1 - r_h / r_0)^2, \quad (53)$$

where $r_h = \sqrt{\kappa e_0} / c^2 = \kappa m_0 / c^2$, and r_0 and r_0' - values r and r' on the boundary of substance.

FOOTNOTE³⁾. Equation (31) in this case is rewritten in the form

$[(1 - r'^2)r]' = \frac{\kappa e_0^2}{c^4 r^2} r'$, whence $r'^2 = 1 - \frac{2\kappa m_0}{c^2 r} + \frac{\kappa e_0^2}{c^4 r^2}$. In view of continuity r and r' on the boundary substance-vacuum in this region r it is correct (52).

ENDFOOTNOTE.

Condition (53) for semiclosed peace/world ($r' < 0$) in the case of space flat/plane at infinity leads to relationship/ratio $r_h < r_0$, which corresponds to the presence of "mole", i.e., to case of 2 (α°). Thus, for the semiclosed peace/world with the flat/plane at infinity space condition (53) can be rewritten in the form

$$r_0' = \frac{r_h}{r_0} - 1. \quad (54)$$

We investigate model of weakly charged/loaded peace/world $\bar{\beta} \ll 1$ (or $\pi \bar{\beta} \ll 1$) in more detail. In this case

$$\begin{aligned} \bar{M} &= \frac{3}{2} a_0 (\chi_0 - \sin \chi_0 \cos \chi_0) + O(\bar{\beta}^2), \\ r_h &= \frac{3}{2} \bar{\beta} a_0 (\chi_0 - \sin \chi_0 \cos \chi_0) + O(\bar{\beta}^3), \\ r_0 &= 2a_0 \sin \chi_0, \quad r_0' = \cos \chi_0, \end{aligned} \quad (55)$$

where $0 < \chi_0 < \pi$, $\chi_0 = q_0 / 2a_0$.

Condition (54) for $\pi/2 < \chi_0 < \pi$ and small $\bar{\beta}$ will be registered in the form

$$1 + \left(1 + \frac{3}{4} \bar{\beta}\right) \cos \chi_0 = \frac{3}{4} \bar{\beta} \frac{\chi_0}{\sin \chi_0}. \quad (56)$$

With $\bar{\beta}=0$, $\chi_0 = \pi$, i.e. χ_0 it reaches its maximum value - peace/world becomes the completely locked Friedmann peace/world.

With weak charge $\bar{\beta} (\bar{\beta} \ll 1)$ unknown boundary of internal (Friedmann) solution must be somewhere near π , i.e. $\chi_0 = \pi - \delta$, where δ is small. Actually, as it follows from the graph (Fig. 1), equation (56) has one solution χ_0 . With $\bar{\beta} \ll 1$ χ_0 it is close to π with $\bar{\beta} \rightarrow 1$, χ_0 it approaches $\pi/2$.

With increase in charge of peace/world e , increases its external (Schwarzschild) mass and respectively radius of neck/throat

$$r_h = \sqrt{\chi} e_0 / c^2. \quad (57)$$

It is essential to stress that in this case the potential of electric field in the neck/throat

$$\varphi_h = e_0 / r_h \quad (58)$$

with an increase in charge e , does not vary, but it remains equal to the constant value

$$\varphi_h = c^2 / \sqrt{\chi}. \quad (59)$$

Value φ_h plays the role of maximum potential in the theory; it is comprised from the world constants and, what is interesting, it does not contain electric charge.

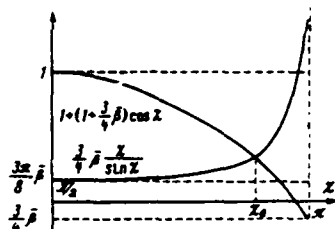


Fig. 1.

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NECK/THROAT.

Condition $\sqrt{\kappa} m_0 = e_0$ ensures statical character of neck/throat. External observer always sees the charged/loaded continued semiclosed peace/world in the form of the thickening charged/loaded sphere ⁴.

FOOTNOTE ⁴. Let us recall: in the case of $\sqrt{\kappa} m_0 > e_0$ of neck/throat oscillates between r_1 and r_2 . With $e_0 \rightarrow \sqrt{\kappa} m_0$ $r_1 \rightarrow r_2$. In the case of $\sqrt{\kappa} m_0 = e_0$ with $e_0 \rightarrow 0$ external (Schwarzschild) mass vanishes. World becomes completely closed, i.e., in the case of $\sqrt{\kappa} m_0 = e_0$ entire mass of electrical origin. Under these conditions any initial value of the internal mass of nonelectromagnetic origin is damped completely by gravitational mass defect. ENDFOOTNOTE.

Dynamics of part of almost locked evenly charged/loaded peace/world and in this case remains unsteady. After the

moment/torque of the maximum expansion the charge cloud, described by the internal solution, is compressed. But the collapse of system is stopped by electric forces on the maximally small radius, determined by the sizes/dimensions of neck/throat, i.e., by the complete electric charge of system.

One should stress that in the neck/throat there is no matter. The nonstaticity of the material cloud is not reflected in the statical character of neck/throat. In the neck/throat the beam of electrical lines of force is compressed in a maximum permissible manner ($\varphi_h = c^2/\gamma_k$). From the neck/throat the beam of lines of force diverges both in outside, in the direction of Euclidean infinity, and inside almost Friedmann space/world. Thus, neck imitates the source of electric field (charges), although no material charge carriers in the neck it is localized.

More detailed examination shows that field in external space and field between substance and neck have different signs (Fig. 2):

$$\begin{aligned} F_{tr} &= e/r^2 \text{ (1) (во внешнем пространстве область 1),} & (60) \\ F_{tr} &= -e/r^2 \text{ (2) (между веществом и горловиной, область 0).} \end{aligned}$$

Key: (1). (in the external space region 1). (2). (between substance and neck, region 0).

Actually, connection between $F_{x^i q}$ and F_{tr} is given by the conversion

$$F_{tr} = \frac{D(t, r)}{D(x^0, q)} F_{x^i q}. \quad (61)$$

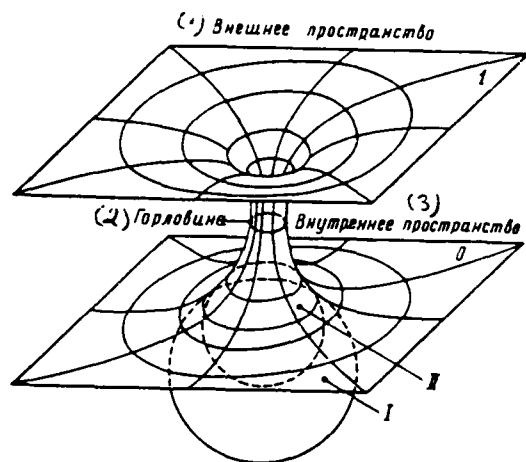


Fig. 2.

Key: (1). External space. (2). Neck/throat. (3). Internal space.

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It is possible to show farther⁵, that

$$\text{sign} \frac{D(t, r)}{D(x^0, q)} = \text{sign } r', \tag{62}$$

i.e. $\text{sign } D(t, r) / D(x^0, q)$ coincides with $\text{sign } r'$, hence follows (60).

FOOTNOTE ⁵.

$$g_{x^0 x^0} = \left(\frac{\partial t}{\partial x^0}\right)^2 g_{tt} + \left(\frac{\partial r}{\partial x^0}\right)^2 g_{rr}, \quad g_{qq} = \left(\frac{\partial t}{\partial q}\right)^2 g_{tt} + \left(\frac{\partial r}{\partial q}\right)^2 g_{rr}.$$

but $g_{x^0 x^0} > 0$, $g_{qq} < 0$, $g_{tt} > 0$, $g_{rr} < 0$. Consequently $\left(\frac{\partial t}{\partial x^0}\right)^2 g_{tt} > \left(\frac{\partial r}{\partial x^0}\right)^2 (-g_{rr})$ and $\left(\frac{\partial r}{\partial q}\right)^2 (-g_{rr}) > \left(\frac{\partial t}{\partial q}\right)^2 g_{tt}$, whence $\left|\frac{\partial t}{\partial x^0} \frac{\partial r}{\partial q}\right| > \left|\frac{\partial r}{\partial x^0} \frac{\partial t}{\partial q}\right|$ and, consequently, the sign $\frac{D(t, r)}{D(x^0, q)} = \frac{\partial t}{\partial x^0} \frac{\partial r}{\partial q} - \frac{\partial t}{\partial q} \frac{\partial r}{\partial x^0}$ is determined by the first term, since always $\frac{\partial t}{\partial x^0} > 0$ - time always grows ("arrow of time"). ENDFOOTNOTE.

In the very neck test electric charge must rest. In regions 1 and 0 it is easy to realize static reference system, using the respectively charged/loaded weightless specks. This system coincides with Reissner-Nordstrom system. As is known, the complete description of Nordstrom-Reissner metrics - (i.e. including in the region between its two pseudo-special features) is given by coordinates of the type of Kruskal (nonstatic reference system).

In our case region (r_1, r_2) subtends to one value $r_1 = r_2 = r_h$ - the neck/throat. Static reference system does not cover/coat only this section directly near the neck.

Polarization of neck (need for the quantum description of neck).

On the basis of relationship/ratio (57) ($r_h = \sqrt{\kappa e_0} / c^2$) we come to conclusion that radius of neck increases proportional to complete electric charge. The description of neck from the point of view of classical theory so appears. But from the point of view of quantum physics this state of neck cannot be stable. Actually, if at some initial moment/torque neck with the properties described above arose, then in its ultrapowerful electric field unavoidably begins to occur the violent process of generating any kind of the electrically

charged/loaded pairs, pairs of proton-antiprotons, any kind of meson pairs, finally, pairs electron-positron. Opposite charges will attempt to decrease the effective charge of neck, and the charges of another component of pairs will be flowed into Euclidean infinity. In this process the charge of neck is gradually decreased, together with it a radius of neck is reduced, and the internal metrics of system becomes ever more than and that more closed.

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We will be delayed several during the discussion of this effect, it can be, not so many in order to give the comprehensive quantitative description of pair production in this field, but are faster for that, so as to somewhat more fix/record attention on this, from our point of view, very curious situation - need for application in the ultra-macrocosm of quantum theory, precisely, for describing such processes, which, it would seem, they play the significant role only in the microcosm. Although quantitative estimations thus far are far from that desired, they by themselves are not deprived of a certain interest.

Pair production of electrically charged particles in strong uniform electric field was examined by Nikishov [6].³ FOOTNOTE .
The authors express appreciation to A. I. Nikishov for the communication/report of its results. ENDFOOTNOTE.

If there is uniform electrostatic field by intensity/strength E , for filling space of cube by sizes/dimensions L^3 , then probability of formation in it of pair (let us say, electrons) with data by impulse/momentum/pulse (p) and spin (r) for always it is given by expression

$$W_{pr} = \exp(-\pi\lambda), \quad \lambda = \frac{c^2(p_1^2 + p_2^2 + m_0^2c^2)}{eE\hbar c} \quad (E = (0, 0, E)), \quad (63)$$

where m_0 - mass particle to vapor, p - value of particle momentum of originated pair after disconnection of field. In such a problem p should belong to discrete spectrum, i.e., $Lp_n^{(i)} = 2\pi\hbar n$.

Formula (63) can be rewritten in the form

$$W_{n_1, n_2, n, r} = \exp\left(-\frac{\pi m_0^2 c^4}{eE\hbar c}\right) \exp\left[-\frac{\pi c^2}{eE\hbar c} \left(\frac{2\pi\hbar}{L}\right)^2 n_1^2\right] \times \\ \times \exp\left[-\frac{\pi c^2}{eE\hbar c} \left(\frac{2\pi\hbar}{L}\right)^2 n_2^2\right]. \quad (64)$$

Here the state of the originated particle is characterized by numbers (n_1, n_2, n, r) . Summing $W_{n_1, n_2, n, r}$ for all quantum numbers, and further substituting the sum for n by integral, we obtain

$$W = 4N \exp\left(-\frac{\pi m_0^2 c^4}{eE\hbar c}\right) eE\hbar c \left(\frac{L}{2\pi\hbar c}\right)^2 \Phi^2(\xi_0), \quad (65)$$

where

$$\xi_0 = \sqrt{\frac{\pi}{eE\hbar c}} \frac{2\pi\hbar c}{L} N, \quad \Phi(\xi_0) = \frac{2}{\sqrt{\pi}} \int_0^{\xi_0} e^{-z^2} dz, \\ N = n_{\max}; \quad p_{\max} = \frac{2\pi\hbar N}{L} = c \left(m_0 + \frac{e\Phi_{\max}}{c^2} \right). \quad (66)$$

With large maximum impulse/momentum/pulse (p_{\max}), $\xi_0 \gg 1$ and $\Phi(\xi_0) \sim 1$. On the basis (65), (66) probability of pair production per unit of volume

$$w = \frac{4}{(2\pi\hbar)^3} \frac{p_{\max}}{c} eE\hbar \exp\left(-\frac{\pi m_0^2 c^4}{eE\hbar c}\right) \Phi^2(\xi_0). \quad (67)$$

Further, for evaluations let us illegally formula (67) to heterogeneous permanent field $E=Ze/r^2$ or $\phi=Ze/r$.

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During the calculation of total number of pairs (N_p) in entire space we will assume it p_{\max} into (67) that depending on r :

$$p_{\max} = Ze^2 / cr, \quad (68)$$

where Ze - the net charge of material system. For total number of pairs, originated in this field for always, we obtain

$$\begin{aligned} N_p &= \int_V w d^3v = \frac{16\pi Z^2 e^4 \hbar}{(2\pi\hbar)^3 c^2} \int_{a_0}^{\infty} \frac{1}{r} \exp\left(-\frac{\pi m_0^2 c^4 r^2}{Ze^2 \hbar c}\right) dr = \\ &= \frac{16\pi Z^2 e^4 \hbar}{(2\pi\hbar)^3 c^2} \int_{A_0}^{\infty} \frac{e^{-\xi^2}}{\xi} d\xi, \end{aligned} \quad (69)$$

where $A_0 = \sqrt{\pi m_0^2 c^4 / Ze^2 \hbar c a_0}$, $a_0 = \sqrt{\pi Ze / c^2}$ - minimum radius. Since

$$\int_{A_0}^{\infty} \frac{e^{-\xi^2}}{\xi} d\xi = \frac{1}{2} \int_{A_0^2}^{\infty} \frac{e^{-x}}{x} dx = -\frac{1}{2} \text{Ei}(-A_0^2),$$

that

$$N_p = -\frac{1}{\alpha^2} (Z\alpha)^2 \text{Ei}(-A_0^2). \quad (70)$$

p_c :

With small A_0^2 , i.e., with

$$Z \ll \frac{1}{\pi\alpha} \left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2 \sim 10^{45}, \quad (71)$$

$$\text{Ei}(-A_0^2) \sim c + \ln A_0^2, \quad (72)$$

c - Euler's constant and, consequently,

$$N_p \sim \frac{1}{\pi^2} (Z\alpha)^2 \left[\ln \left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2 - c - \ln \pi Z\alpha \right],$$

or, keeping in mind condition (71),

$$N_p \sim \frac{1}{\pi^2} (Z\alpha)^2 \ln \left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2. \quad (73)$$

Condition $(Z - N_p) = \max = Z_f$, gives the value of charge Z_f , which remains the unextinguished effect of pair production:

$$Z_f = \max \left\{ Z \left[1 - Z \frac{\alpha^2}{\pi^2} \ln \left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2 \right] \right\} = \frac{\pi^2}{4\alpha^2 \ln \left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2}, \quad (74)$$

since $\left(\frac{e/\sqrt{\kappa}}{m_0} \right)^2 \sim \frac{1}{\alpha}$, then $Z_f \sim 137$. In other words, the effect of pair production in this high electric field reduces the effective charge of neck to finite value $Z_f \sim 137$ independent of the value of the initial charge Z .

FOOTNOTE. This result is sufficiently natural, since with $Z > 137$, as is known, the process of real pair production [7,8] begins.

ENDFOOTNOTE.

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Independence of value of final charge from as conveniently high value of bare charge follows also from known formula of Landau [9], which connects value of bare charge e_1 , localized in small region,

with value of physically effective charge e , to which effect of polarization of vacuum reduces initial charge e_1 ,

$$e^2 = \frac{e_1^2}{1 + \frac{e_1^2}{3\pi} \ln \left(\frac{\Lambda}{m_0} \right)^2}. \quad (75)$$

With high value of charge e_1 , or more precisely with

$$\frac{e_1^2}{3\pi} \ln \left(\frac{\Lambda}{m_0} \right)^2 \gg 1, \quad e^2 \sim \frac{3\pi}{\ln(\Lambda/m_0)^2}. \quad (76)$$

It is interesting to note that in roughly estimated formula (74) the same characteristic logarithm appears, that in Landau's formula, and argument in logarithm in formula (73) gives for introduced to Landau expression

$$\Lambda = e/\sqrt{\kappa} \sim 10^{28} \text{ eV}. \quad (77)$$

This the very thing value Λ , which is considered in Landau's article in connection with possible role of gravitation in theory of elementary particles. The form of the considered object even from the point of view of Schwarzschild observer is very complex. Matter in the fact that at the initial moment of the existence of this system with the large electric charge its overall sizes, proportional to charge, can be very large: $r_h^i = Z_i e \sqrt{\kappa} / c^2$.

Pair formation reduces initial charge (Z_i) to $Z_f \sim 137$, consequently⁸,

$$r_h' \leq \frac{137 e \sqrt{\kappa}}{c^2} \approx 10^{-30} \text{ cm.}$$

FOOTNOTE *. $r_h' < 10^{-30} \text{ cm}$, since in the previous estimations was not considered the possibility of the generation of any kind of particles. ENDFOOTNOTE.

But in this region Z ($Z_f \sim 137$) filling of shells (around the source of field) with radii $\sim \hbar / mc$, where m - masses of the particles of the given birth to pairs, begins. If we take into account that the hadron particles (for example, protons) have their inherent sizes/dimensions, then around the system in question appears the peculiar atmosphere, which increases the orders by twenty its external sizes/dimensions. It is by chance or it is nonrandom, the object, characteristic according to its extrinsic properties for physics of microcosm, appears from the object of cosmological, and similar its internal content continues to remain.

In article [10] for objects with described properties is introduced special term of "Friedmons".

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