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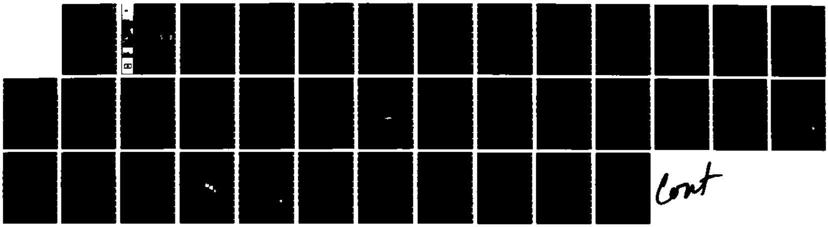

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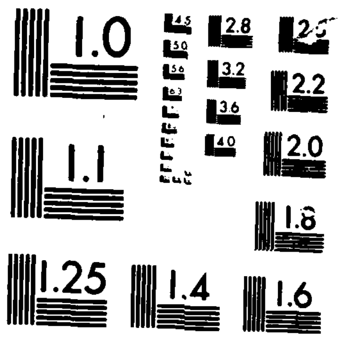


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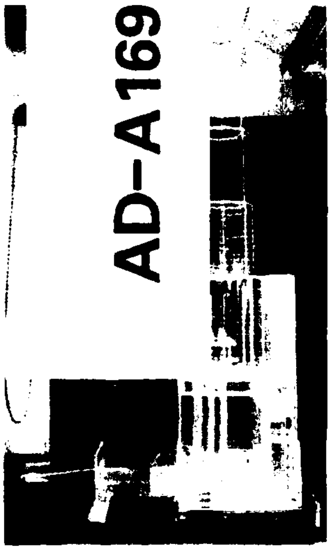

HYDRAULICS LABORATORY

(62)

MISCELLANEOUS PAPER HL-86-2

# PRESSURE CALCULATION FOR TWO-DIMENSIONAL FLOW INSIDE HYDRAULIC STRUCTURES

by

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US ARMY ENGINEER WATERWAYS EXPERIMENT STATION

April 1986

Final Report

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method has been developed for computing two-dimensional pressure distributions inside hydraulic structures. Velocities are first obtained by finite difference solution of the Navier-Stokes equations in stream-function/vorticity form. Pressure is then calculated by numerical integration of the momentum equation. The method has been incorporated for arbitrary geometry in the [illegible] computer code, which uses boundary-fitted grids generated by the WES[illegible] code.

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20. ABSTRACT (Continued).

Computed results compare well with piezometric data from physical-model tests for the Taylorsville outlet works, indicating that the VORTEX code may be useful in identifying and eliminating flow conditions that promote cavitation.

## PREFACE

This investigation was conducted as part of the Flood Control Hydraulics Research and Development Program, sponsored by the Office, Chief of Engineers (OCE), US Army, and administered by the Hydraulics Laboratory of the US Army Engineer Waterways Experiment Station (WES).

The study was accomplished under the direction of Messrs. H. B. Simmons and F. A. Herrmann, Jr., former and present Chiefs of the Hydraulics Laboratory; J. L. Grace, Jr., Chief of the Hydraulic Structures Division; and J. P. Holland, Chief of the Reservoir Water Quality Branch (Physical). Mr. M. B. Boyd, Chief of the Hydraulic Analysis Division, was the laboratory Program Manager, and Mr. Tom Munsey of OCE was Technical Monitor. The study was conducted and this report was prepared by Dr. Robert S. Bernard. This report was edited by Mrs. Beth F. Vavra, Publications and Graphic Arts Division.

Director of WES was COL Allen F. Grum, USA. Technical Director was Dr. Robert W. Whalin.

CONTENTS

## CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to metric (SI) units as follows:

| Multiply | By | To Obtain |
|---|---|---|
| cubic feet per second | 0.02831685 | cubic metres per second |
| feet | 0.3048 | metres |
| feet of water | 2989.0 | pascals |

# PRESSURE CALCULATION FOR TWO-DIMENSIONAL FLOW INSIDE HYDRAULIC STRUCTURES

## PART I: INTRODUCTION

### Background

1. When the static pressure inside a hydraulic structure falls below some critical value, voids containing water vapor form therein. The voids then collapse with great force, pitting and eroding the internal surfaces of the structure. The formation/collapse of these voids is called cavitation, and the prevention thereof is a prime concern in structural design.

2. The prediction of cavitation is generally accomplished by empirical means, using formulas that require knowledge of the local static pressure (Robertson 1965). The latter is obtainable from simple empirical equations in conduits; but without site-specific physical modeling, such equations have not been available for structures with complex internal geometry. Thus, for hydraulic structures in general, one must solve the governing equations (the Navier-Stokes equations) in order to predict the pressure. Given a means of computing pressure as a function of internal geometry, pool elevation, and flow rate, one can then screen ideas for new configurations and structural modifications with regard to cavitation.

### Purpose and Scope

3. Discussed briefly in Appendix A is a method that has been developed for calculating velocities and pressures inside a two-dimensional (2-D) structure of arbitrary shape. The procedure requires the numerical solution of the Navier-Stokes equations (in stream-function/vorticity form), in order to determine the velocity distribution. The pressure is then computed from the known velocity and vorticity field by numerical integration of the momentum equation. This procedure is implemented in the VORTEX computer code, which is a modified version of the WESSEL finite difference code (Thompson and Bernard 1985). Like WESSEL, the VORTEX code makes hydrodynamic computations using boundary-fitted finite difference grids generated by WESCOR code (Thompson 1983). A brief discussion of the theory for VORTEX is given in Appendix A.

4. The VORTEX code has been used to calculate pressure distributions for three distinct flow conditions in the outlet works for Taylorsville Lake, Salt River, Kentucky. Results of these calculations (given in PART II) exhibit favorable agreement with piezometric data from physical model tests conducted by Dortch (1975). Indications are that the VORTEX code may be of considerable value in helping to identify and rectify flow conditions that promote cavitation.

## PART II: CALCULATIONS FOR TAYLORSVILLE OUTLET WORKS

### Prototype Description

5. Dortch (1975) reports an exhaustive physical model study of the outlet works for Taylorsville Lake, Salt River, Kentucky. He describes the prototype as follows:

> The plan for the project consists of a rock-filled dam, an open cut uncontrolled spillway in the right abutment, and a controlled outlet works through the right abutment. The top of the dam will be at el 622.0* with the spillway crest at el 592.0.
>
> Reservoir releases will be regulated by a gated intake tower, consisting of two flood-control intakes at the base of the structure (el 474.0) and two wet wells with five 6- by 6-ft** water-quality intakes in each wet well at elevations ranging from 503.0 to 534.0. Both flood-control and water-quality flows pass through two separate 5.5- by 14.75-ft rectangular gate passages. The two gate passages transition into a single 11.5- by 14.75-ft oblong conduit. The last 20 ft of the oblong conduit contains a transition to a flat bottom conduit before discharging into an outlet transition and stilling basin. A profile depicting the general plan and original design of the outlet works is shown in Plate 1 [Figure 1].
>
> During selective withdrawal operation, the emergency gates will be closed and flow will be discharged through the multilevel intakes into the wet wells and through an opening located in the roof of the gate passages between the emergency and service gates. The service gates will be used to regulate the selective withdrawal releases. The locations of the ten multilevel intakes (five intakes in each wet well) are shown in Plate 2 [Figure 2]. An 18-in.-diameter pipe bypass around each service gate will be provided to regulate the release of low flows with the service gates closed.

### Computer Calculations

6. The VORTEX code has been used to compute the steady-state flow

---

* All elevations (el) cited herein are in feet referred to the National Geodetic Vertical Datum (NGVD).

** A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

through the right half (facing downstream) of the outlet works. The flow region of interest is section A-A in Figure 3. Finite difference grids were generated for (a) flow through the flood-control facility with the emergency and service gates open 100 percent (Figure 4), and (b) flow through the water quality facility with the emergency gate closed (Figure 5). In each case, only the anticipated 2-D portion of the flow field was simulated, and three-dimensional effects in the approach flow and the upper part of the intake structure were neglected. VORTEX does not compute flow rate (discharge) as a function of pool elevation, so both quantities were specified in advance using information given by Dortch (1975). Flow distribution was assumed uniform along designated inflow and outflow boundary segments.

7. For calculating pressure, a reference point was chosen on one of the inlets, at which Bernoulli's equation was used to calculate the reference pressure:

$$p_{ref} = \rho g(y_{pool} - y_{ref}) - \frac{1}{2}\rho v_{ref}^2$$

where

$y_{pool}$ = pool elevation

$y_{ref}$ = reference-point elevation

$v_{ref}$ = reference-point flow velocity

The velocity and pressure at the water surface were assumed to be zero. With the reference pressure known, the differential momentum equation (A13) was integrated along a path lying just inside the perimeter of the flow field to obtain pressure values along the perimeter itself (by extrapolation). The perimeter segment of greatest interest was the circular bend in the wet well (piezometers 31-35), since this is where cavitation was thought to be most likely. The finite difference grids (Figures 4 and 5) were therefore designed to provide the greatest resolution along this bend.

## Discussion of Results

8. Figures 6 and 7 show computed velocity vectors for a flood-control operation with service and emergency gates both open 100 percent. Figure 8 depicts streamlines for the same hydraulic conditions. In this case the wet well develops into a region of recirculation, while the flow elsewhere is nearly uniform (except along walls where the no-slip condition applies).

Comparisons of VORTEX pressure predictions with piezometric data from the physical model (Dortch 1975) are presented in Figures 9-11. The magnitudes and trends of the calculations are substantially the same as the piezometric data, though some of the details are lacking. As uncalibrated predictions, however, the code results are quite acceptable.

9. Figures 12 and 13 show computed velocity vectors and streamlines, respectively, for a water quality operation with the emergency gate closed and the service gate open 25 percent. Eighty percent of the flow enters through the top inlet, and the rest enters from the left (Figures 5 and 12). Regions of recirculation occur downstream of all corners, causing the streamlines to form a nearly straight channel between the inlets and outlet (Figure 13). Pressure predictions are compared with piezometer data in Figures 14-16. The same comments apply to the accuracy of these results as to those for the flood-control flow calculation.

10. A series of flow calculations was performed for operation of the water quality facility alone, with the emergency gate closed and the service gate open 50 percent. The tabulation below gives the flow rates and associated pool elevations, and Figures 17 and 18 show the velocity vectors and streamlines, respectively, at a flow rate of 1,520 cfs. Data in the tabulation were taken from a discharge curve given by Dortch (1975). Figure 19 presents a comparison of predicted and measured pressures for piezometer 33, which registered the lowest measured pressure along the circular bend (piezometers 31-35) in the wet well. The lowest computed pressure occurred about halfway between piezometers 32 and 33; nonetheless, the agreement between calculation and experiment is quite good for piezometer 33. No other piezometric data were reported for this series of tests.

Pool Elevations and Flow Rates with Emergency Gate Closed and Service Gate Open 50 Percent

| Pool El | Flow Rate, cfs |
|---|---|
| 530 | 1,520 |
| 540 | 1,700 |
| 550 | 1,840 |
| 560 | 1,960 |
| 570 | 2,080 |
| 580 | 2,200 |
| 590 | 2,300 |

## PART III: DISCUSSION AND CONCLUSIONS

11. A method has been developed for calculating pressures for two-dimensional flow inside hydraulic structures. This method has been incorporated in the VORTEX computer code, which solves the Navier-Stokes equations in stream-function/vorticity form. The code has been used to calculate pressure distributions for three different hydraulic conditions in the Taylorsville outlet works. Agreement between code predictions and physical-model data was fair to good, even though the real flow was probably turbulent and somewhat three-dimensional. No effort was made to calibrate the code, and the accuracy of the computed results illustrates the confidence with which predictions can be made for the hydraulic conditions simulated.

12. The VORTEX code is relatively easy to use after a few months experience. The main prerequisite is that the user have a basic understanding of fluid mechanics in order to interpret the results. Previous experience in numerical fluid mechanics is helpful but not essential, since guidance can be given for grid generation and for the selection of code parameters. (The latter task concerns numerical stability rather than predictive accuracy.) The VORTEX code can be used by engineers to screen ideas for new designs or structural modifications at a modest expenditure of computer funds. The steady-state calculations reported herein required about 4,000 central processing seconds on the CYBER 760.

13. No effort was made herein to model turbulence or to account for three-dimensional effects. Moreover, the numerical scheme for solving the Navier-Stokes equations is fairly simple, employing two-point upwind differencing for the advective terms. Nevertheless, the VORTEX code generated pressure information that would have been quite useful in trouble-shooting and predicting the onset of cavitation in the Taylorsville outlet works. Since there is nothing unusual about the Taylorsville configuration, it is likely that the code will do about as well for other structures. Further study and application are needed to establish error bounds and code utility for hydraulic structures in general.

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Robertson, J. M. 1965. Hydrodynamics in Theory and Application, Prentice-Hall, Englewood Cliffs, N. J., pp 525-530.

Thompson, J. F. 1983 (Mar). "A Boundary-Fitted Coordinate Code for General Two-Dimensional Regions with Obstacles and Boundary Intrusions," Technical Report E-83-8, US Army Engineer Waterways Experiment Station, Vicksburg, Miss.

Thompson, J. F., and Bernard, R. S. 1985. "WESSEL - Code for Simulation of 2D Time-Dependent Width-Averaged Flows with Arbitrary Boundaries, Obstacles, Intrusions, Entrances, and Exits" (in preparation), US Army Engineer Waterways Experiment Station, Vicksburg, Miss.

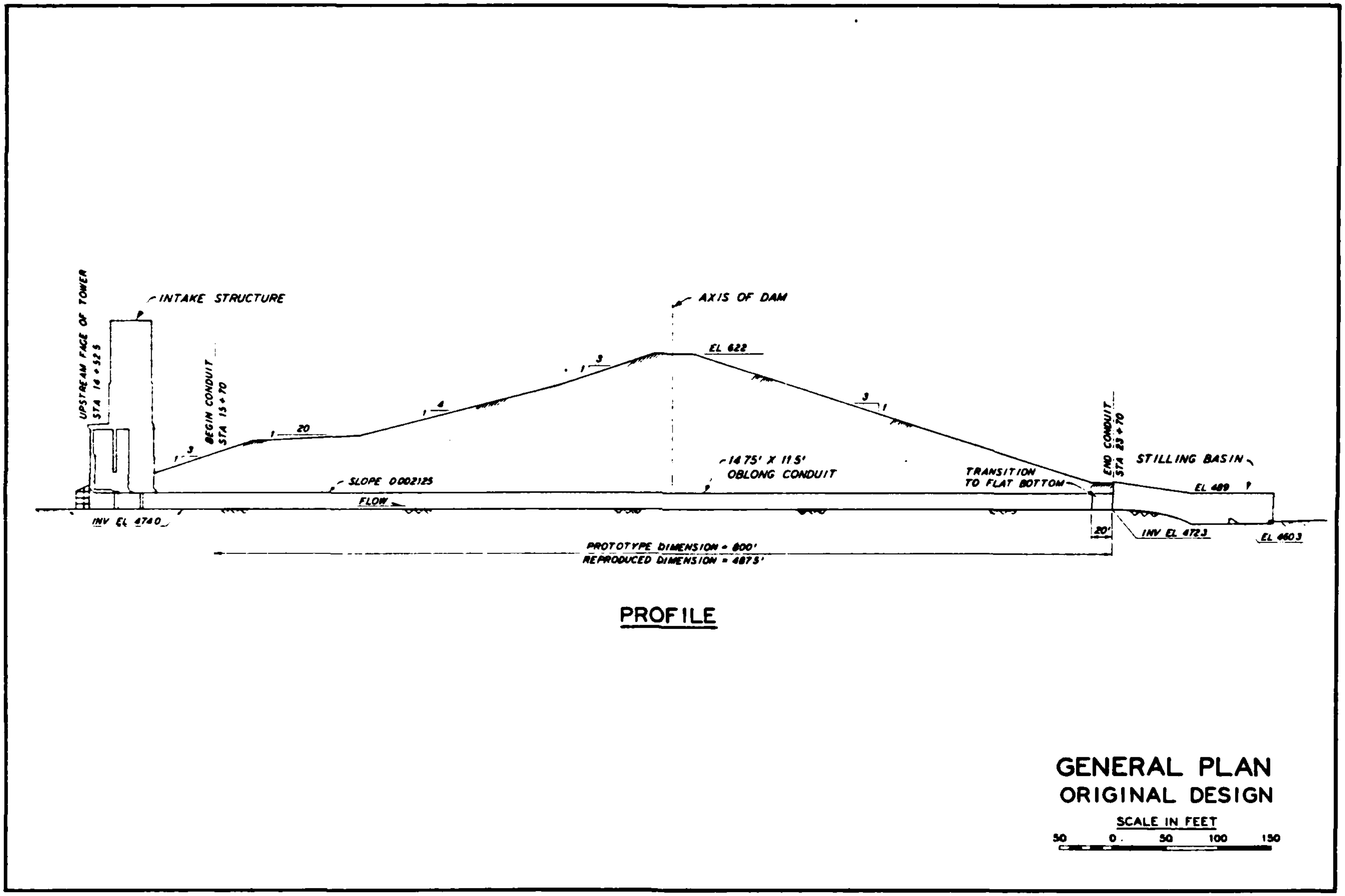



Figure 1. Taylorsville Dam and outlet works (Plate 1, Dortch (1975))

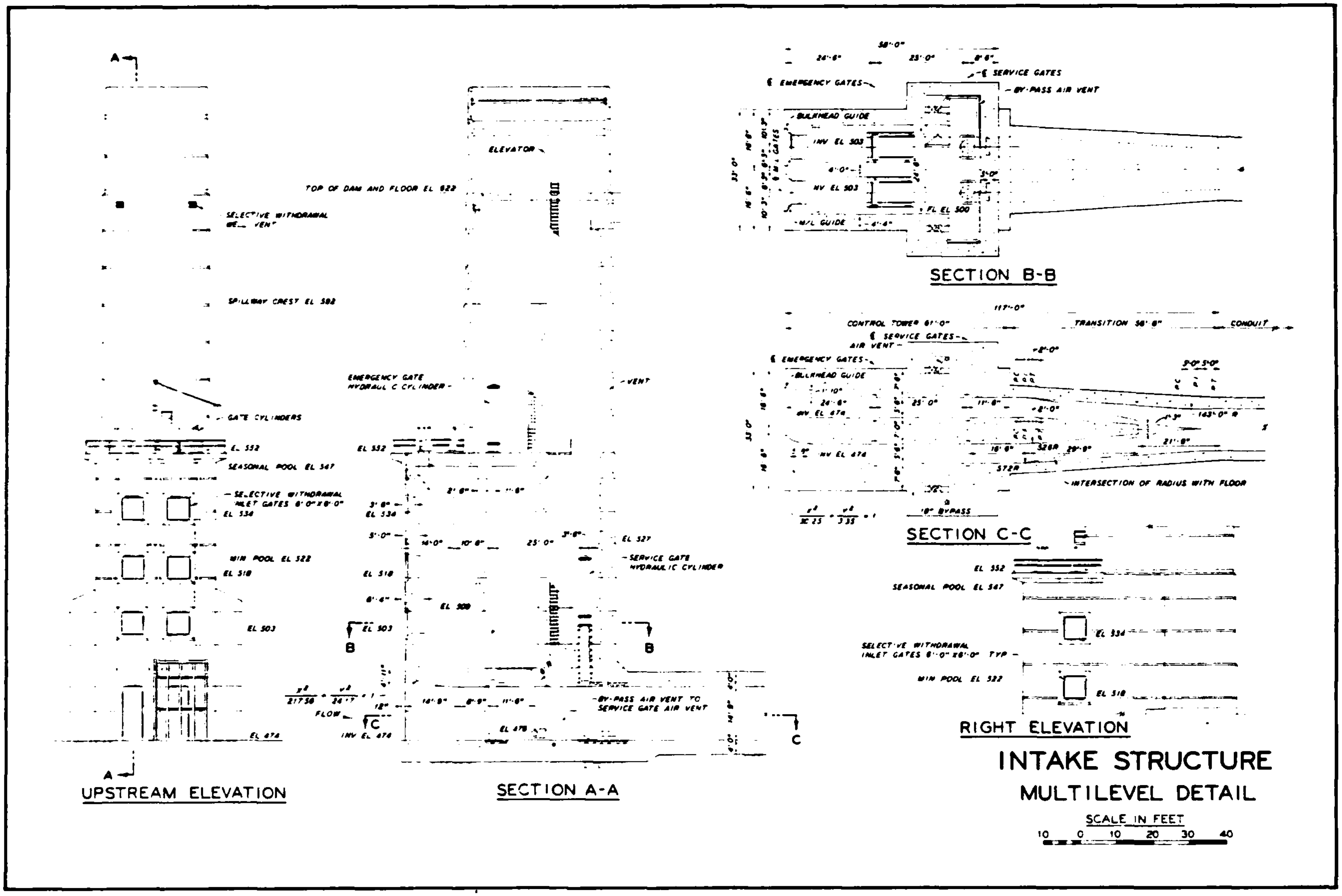
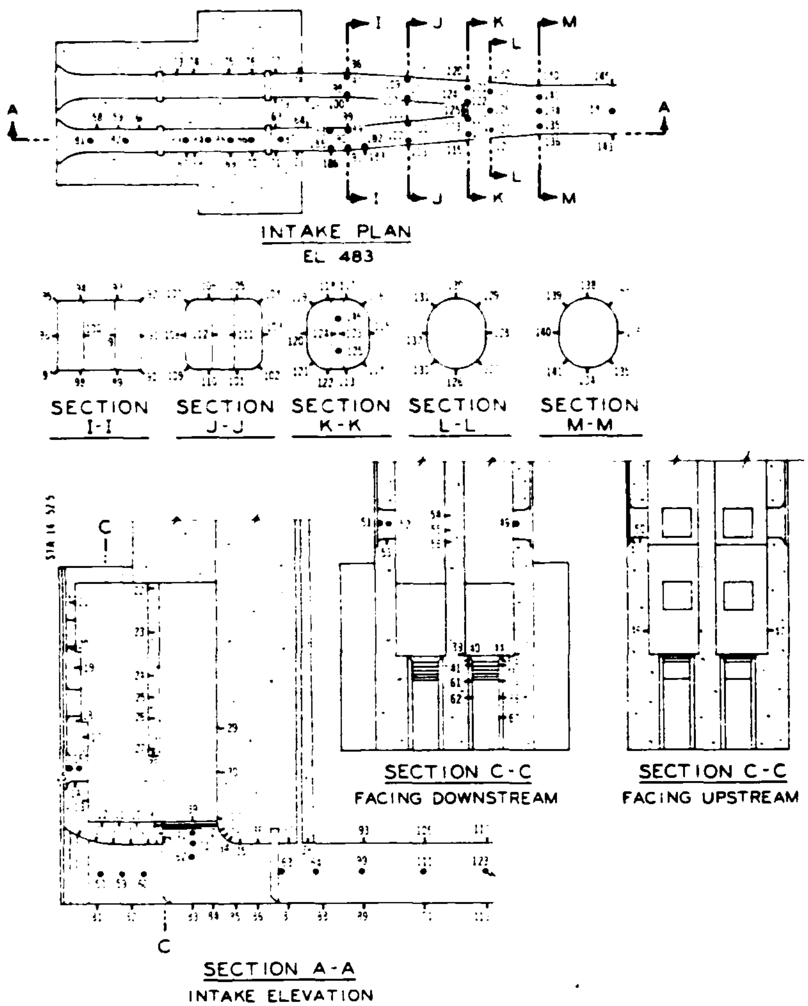



Figure 2. Taylorsville outlet works (Plate 2, Dortch (1975))



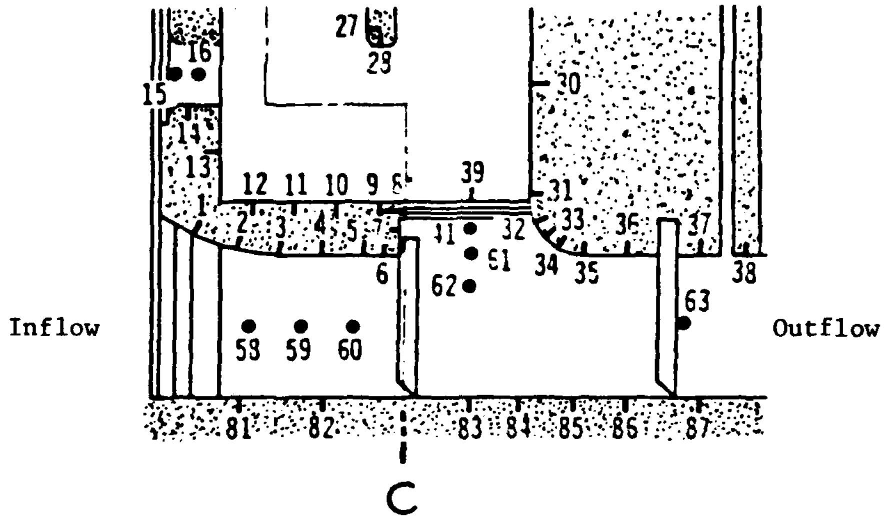

| PIEZ NO | ELEVATION | STA | PIEZ NO | ELEVATION | STA | PIEZ NO | ELEVATION | STA |
|---|---|---|---|---|---|---|---|---|
| 1 | 490.00 | 14 56.5 | 52 | 521.00 | 14 69.5 | 103 | 481.40 | 15 34.4 |
| 2 | 485.00 | 14 60.5 | 53 | 518.00 | 14 66.5 | 104 | 487.20 | 15 34.4 |
| 3 | 488.50 | 14 64.5 | 54 | 524.50 | 14 66.5 | 105 | 488.75 | 15 34.4 |
| 4 | 488.75 | 14 68.5 | 55 | 521.00 | 14 66.5 | 106 | 488.75 | 15 34.4 |
| 5 | 488.75 | 14 72.5 | 56 | 517.50 | 14 66.5 | 107 | [illegible] | 15 34.4 |
| 6 | 488.75 | 14 74.5 | 57 |  | -- | 108 | 481.40 | 15 34.4 |
| 7 | 491.20 | 14 76.2 | 58 | 481.40 | 14 61.5 | 109 | 475.50 | 15 34.4 |
| 8 | 493.20 | 14 75.7 | 59 | 481.40 | 14 66.5 | 110 | 474.00 | 15 34.4 |
| 9 | 493.70 | 14 73.8 | 60 | 481.40 | 14 71.5 | 111 | 481.40 | 15 34.4 |
| 10 | 493.70 | 14 69.8 | 61 | 488.70 | 14 83.1 | 112 | 481.40 | 15 34.4 |
| 11 | 493.70 | 14 65.8 | 62 | 484.90 | 14 83.1 | 113 | 474.00 | 15 48.4 |
| 12 | 493.70 | 14 61.8 | 63 | 481.40 | 15 03.5 | 114 | 476.40 | 15 48.4 |
| 13 | 498.10 | 14 58.8 | 64 | 481.40 | 15 11.5 | 115 | 481.40 | 15 48.4 |
| 14 | 503.00 | 14 55.7 | 65 | 488.70 | 14 83.1 | 116 | 486.30 | 15 48.4 |
| 15 | 506.00 | 14 54.7 | 66 | 484.90 | 14 83.1 | 117 | 488.75 | 15 48.4 |
| 16 | 506.00 | 14 55.7 | 67 | 481.20 | 14 83.1 | 118 | 488.75 | 15 48.4 |
| 17 | 513.10 | 14 58.8 | 68 | 481.40 | 14 85.1 | 119 | 486.30 | 15 48.4 |
| 18 | 517.00 | 14 57.5 | 69 | 481.40 | 14 93.1 | 120 | 481.40 | 15 48.4 |
| 19 | 529.10 | 14 57.5 | 70 | 481.40 | 14 98.1 | 121 | 476.40 | 15 48.4 |
| 20 | 533.00 | 14 56.0 | 71 | 481.40 | 15 03.5 | 122 | 474.00 | 15 48.4 |
| 21 | 544.10 | 14 56.0 | 72 | 481.40 | 15 09.0 | 123 | 481.40 | 15 48.4 |
| 22 | 547.50 | 14 73.0 | 73 | 481.40 | 14 91.0 | 124 | 481.40 | 15 48.4 |
| 23 | 537.50 | 14 73.0 | 74 | 481.40 | 14 85.0 | 125 | 477.70 | 15 48.4 |
| 24 | 527.50 | 14 73.0 | 75 | 481.40 | 14 93.0 | 126 | 474.00 | 15 53.4 |
| 25 | 522.50 | 14 73.0 | 76 | 481.40 | 14 98.0 | 127 | 476.90 | 15 53.4 |
| 26 | 517.50 | 14 73.0 | 77 | 481.40 | 15 03.5 | 128 | 481.40 | 15 53.4 |
| 27 | 510.50 | 14 74.2 | 78 | 481.40 | 15 09.0 | 129 | 485.90 | 15 53.4 |
| 28 | 509.00 | 14 88.5 | 79 | 481.40 | 15 03.5 | 130 | 488.75 | 15 53.4 |
| 29 | 515.50 | 14 88.5 | 80 | 481.40 | 15 11.5 | 131 | 485.90 | 15 53.4 |
| 30 | 505.50 | 14 88.5 | 81 | 474.00 | 14 60.5 | 132 | 481.40 | 15 53.4 |
| 31 | 494.70 | 14 88.5 | 82 | 474.00 | 14 68.5 | 133 | 486.90 | 15 53.4 |
| 32 | 493.20 | 14 90.0 | 83 | 474.00 | 14 83.0 | 134 | 474.00 | 15 65.0 |
| 33 | 491.70 | 14 91.5 | 84 | 474.00 | 14 88.0 | 135 | 476.90 | 15 65.0 |
| 34 | 490.20 | 14 93.0 | 85 | 474.30 | 14 93.0 | 136 | 481.40 | 15 65.0 |
| 35 | 488.75 | 14 94.5 | 86 | 474.00 | 15 98.0 | 137 | 485.90 | 15 65.0 |
| 36 | 488.75 | 14 98.0 | 87 | 474.30 | 15 05.0 | 138 | 488.75 | 15 65.0 |
| 37 | 488.75 | 15 05.0 | 88 | 474.00 | 15 17.5 | 139 | 485.90 | 15 65.0 |
| 38 | 488.75 | 15 08.9 | 89 | 474.00 | 15 20.5 | 140 | 481.40 | 15 65.0 |
| 39 | 494.70 | 14 83.1 | 90 | 474.10 | 15 20.5 | 141 | 476.90 | 15 65.0 |
| 40 | 493.20 | 14 83.1 | 91 | 481.40 | 15 20.5 | 142 | 473.96 | 15 90.52 |
| 41 | 492.70 | 14 83.1 | 92 | 488.60 | 15 20.5 | 143 | 481.33 | 15 90.52 |
| 42 | 494.70 | 14 83.1 | 93 | 488.75 | 15 20.5 | 144 | 488.71 | 15 90.52 |
| 43 | 493.70 | 14 83.1 | 94 | 488.75 | 15 20.5 | 145 | 481.33 | 15 90.52 |
| 44 | 493.20 | 14 83.1 | 95 | 488.60 | 15 20.5 | 146 | 485.10 | 15 48.4 |
| 45 | 492.70 | 14 83.1 | 96 | 481.40 | 15 20.5 | 182 | 474.4 | 15 24.5 |
| 46 | 498.70 | 14 66.5 | 97 | 474.10 | 15 20.5 | 183 | 481.4 | 15 24.5 |
| 47 | 498.70 | 14 66.5 | 98 | 474.30 | 15 20.5 | 184 | 488.1 | 15 24.5 |
| 48 | 518.00 | 14 66.5 | 99 | 481.40 | 15 20.5 | 185 | 474.1 | 15 16.5 |
| 49 | 521.00 | 14 69.5 | 100 | 481.40 | 15 20.5 | 186 | 487.4 | 15 16.5 |
| 50 | 518.00 | 14 66.5 | 101 | 474.00 | 15 34.4 | 187 | 486.75 | 15 16.5 |
| 51 | 521.00 | 14 69.5 | 102 | 475.50 | 15 34.4 |  |  |  |

ALL ELEVATIONS ARE IN FEET REFERRED TO MEAN SEA LEVEL
PIEZOMETER OMITTED

PIEZOMETER LOCATIONS
INTAKE AND TRANSITION
ORIGINAL DESIGN

SCALE IN FEET
10 0 10 20 30 40 50

Figure 3. Piezometer locations in physical model (Plate 8, Dortch (1975))




Numbers indicate piezometer locations

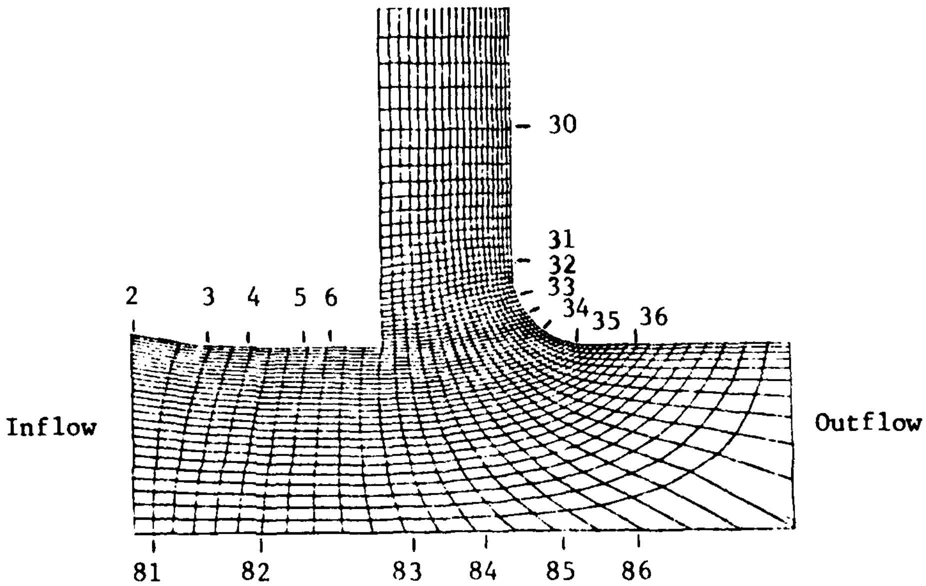
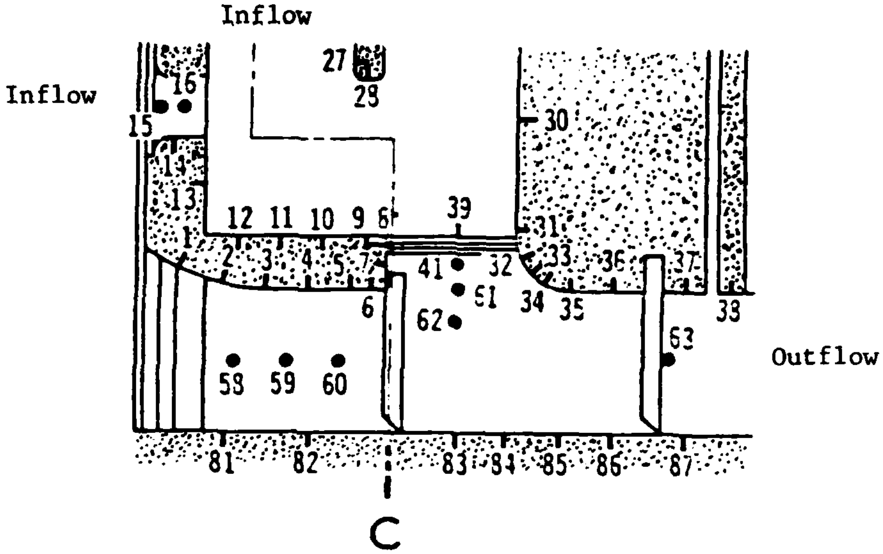



Figure 4. Physical-model section and boundary-fitted finite difference grid for flow simulation through flood-control facility with both gates open 100 percent




Numbers indicate piezometer locations

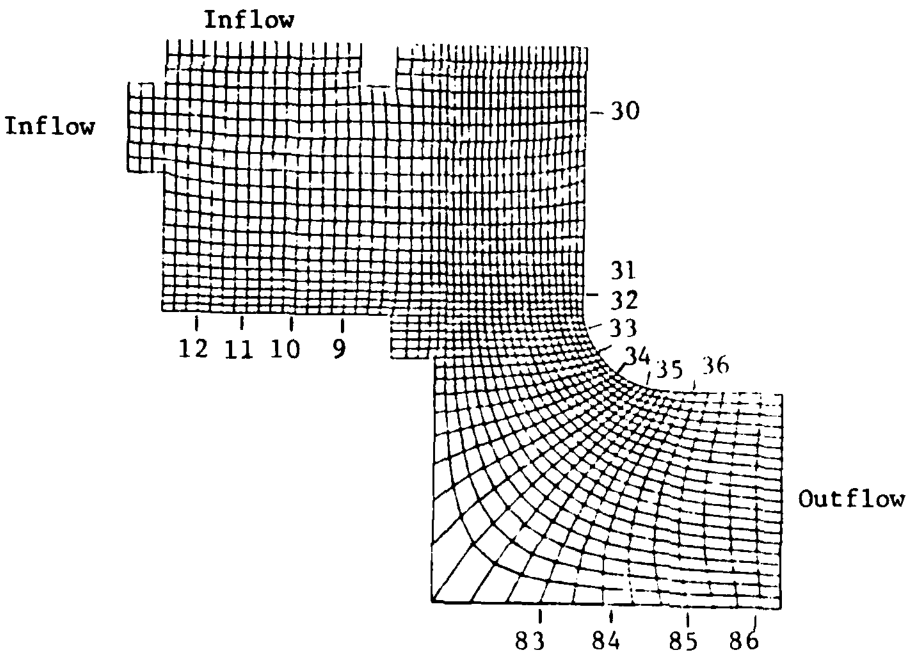



Figure 5. Physical-model section and boundary-fitted finite difference grid for flow simulation through water-quality facility with emergency gate closed

Gate Opening = 100 percent

Pool Elevation = 570.5 ft

Flow Rate = 5,000 cfs

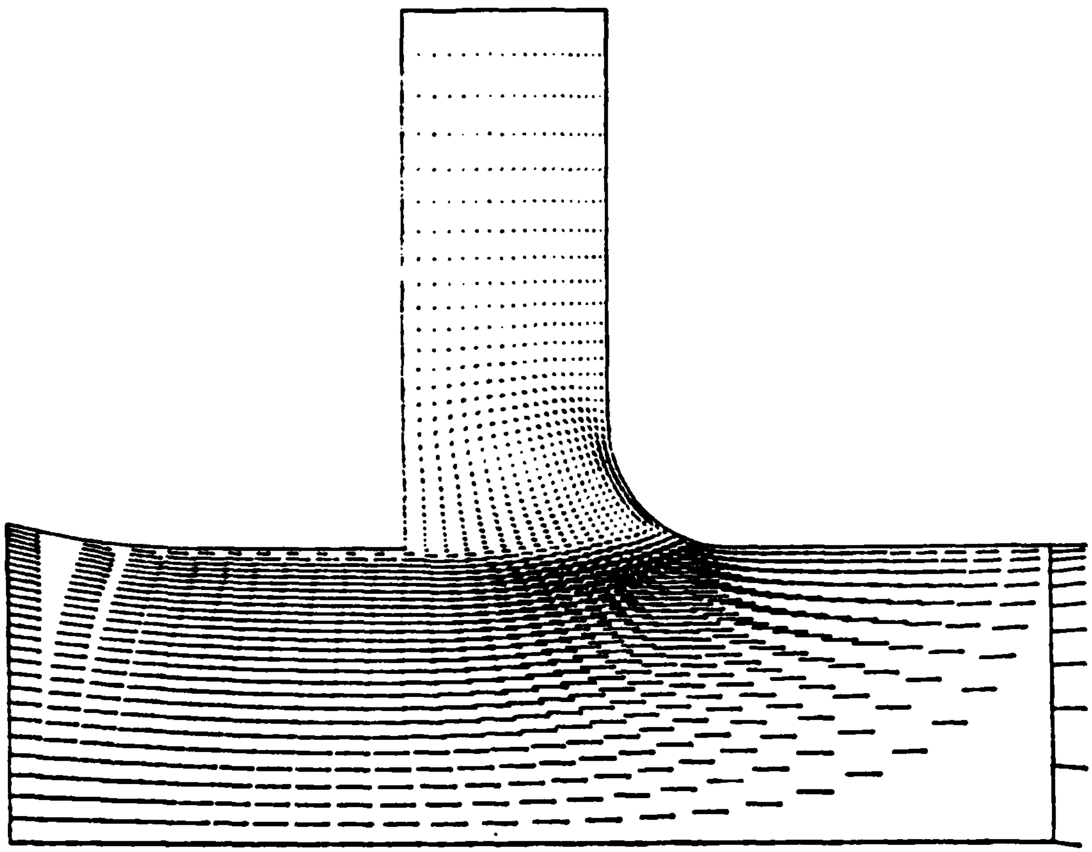


Figure 6. Velocity vectors for flow through flood-control facility with both gates open 100 percent

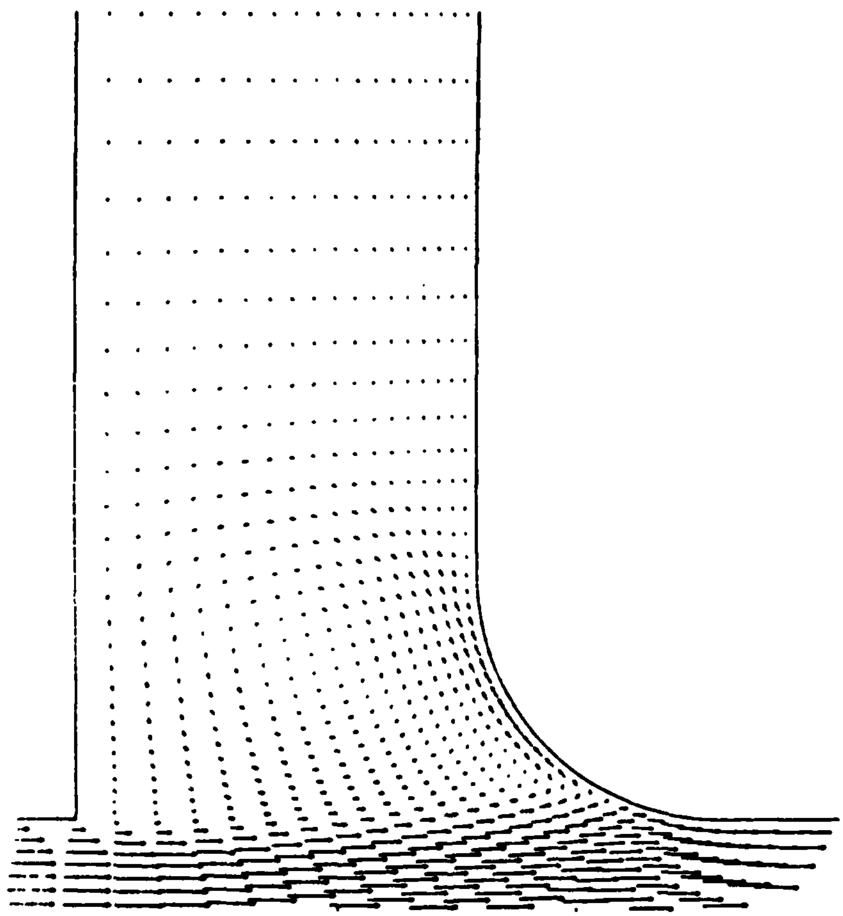


Figure 7. Closeup view of velocity vectors in wet well for flow through flood-control facility with both gates open 100 percent

Gate Opening = 100 percent
Pool Elevation = 570.5 ft
Flow Rate = 5,000 cfs

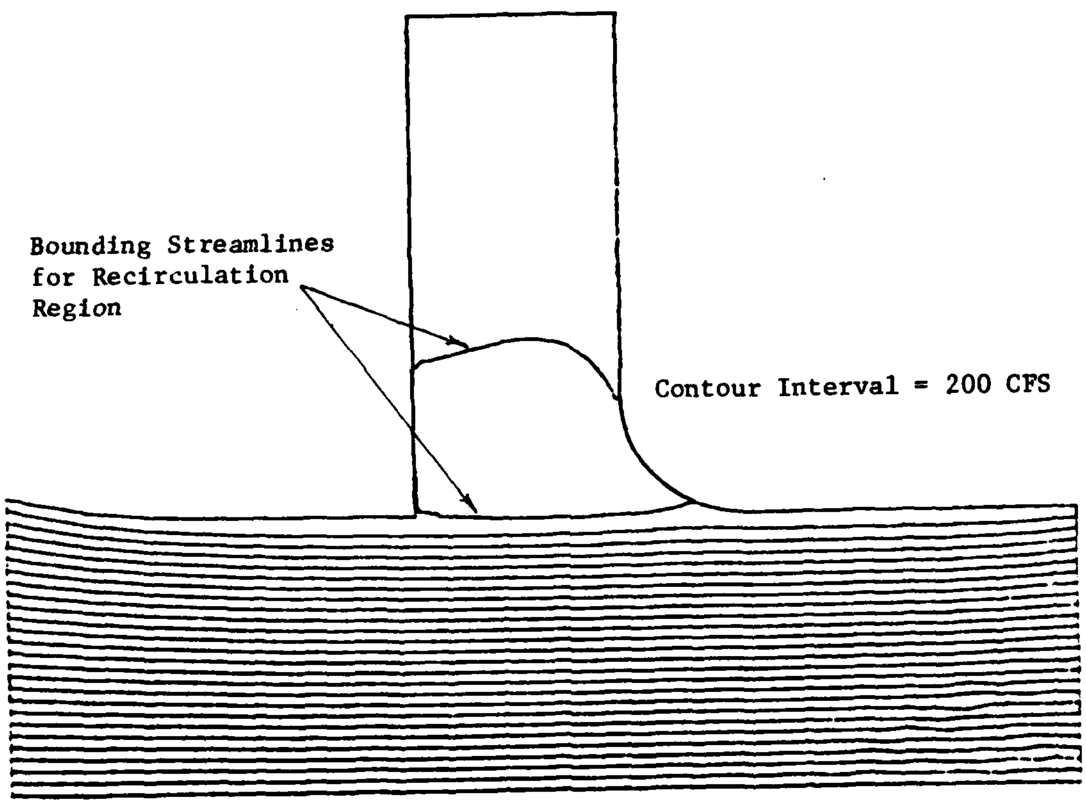



Figure 8. Streamlines for flow through flood-control facility with both gates open 100 percent

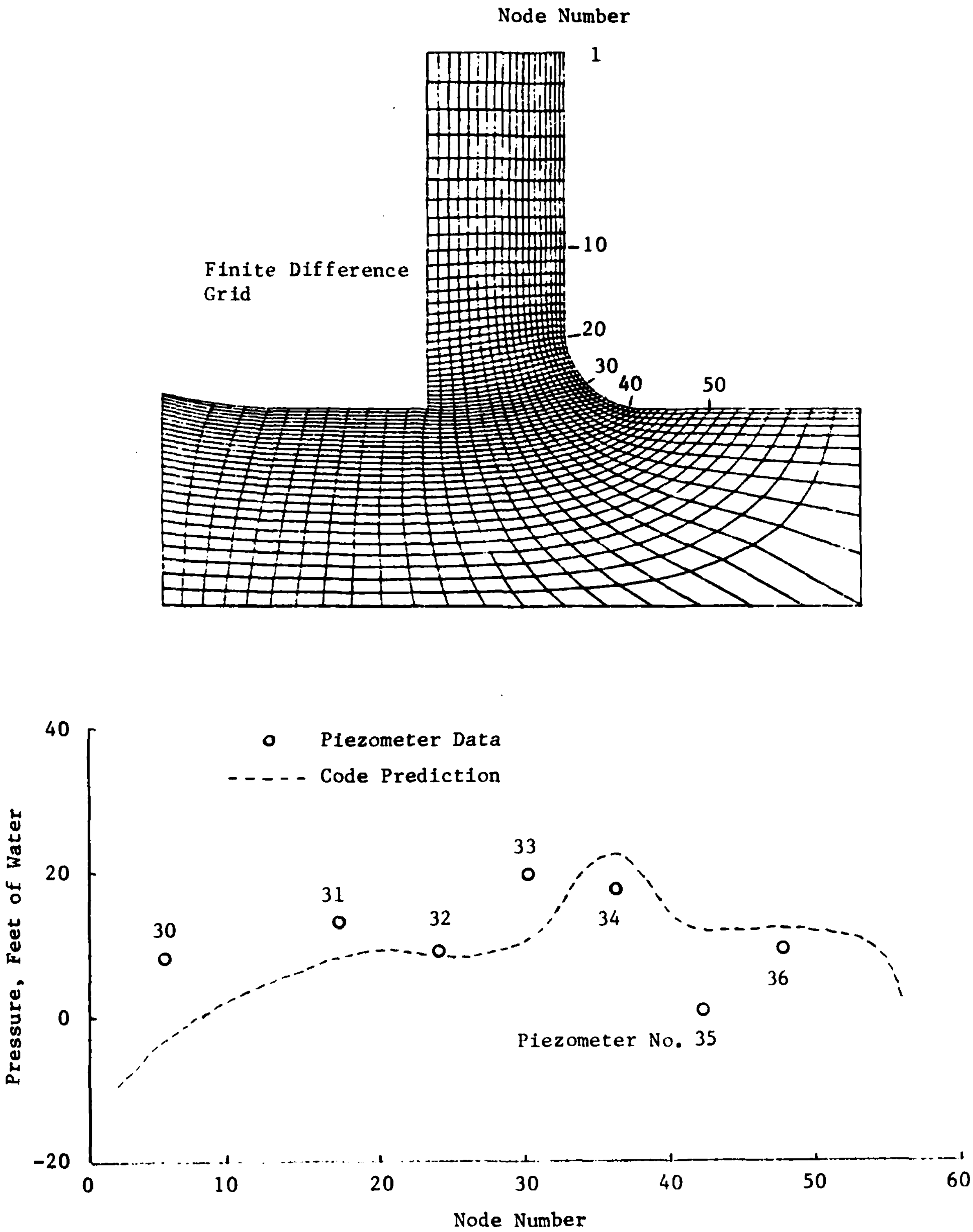
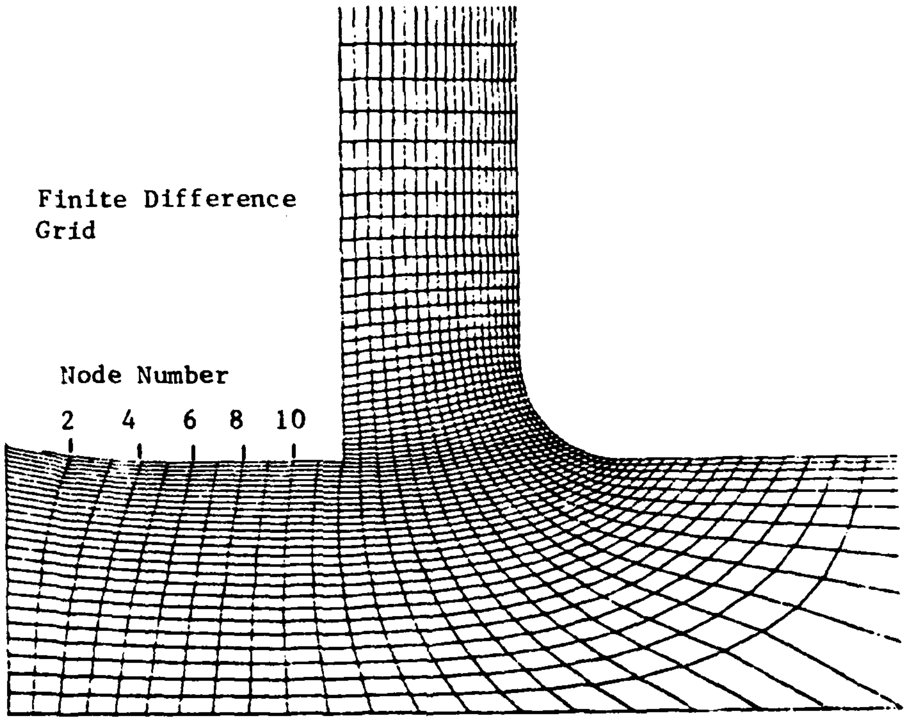



Figure 9. Comparison of computed and measured pressure distributions for flood-control flow, both gates open 100 percent, nodes 1-50



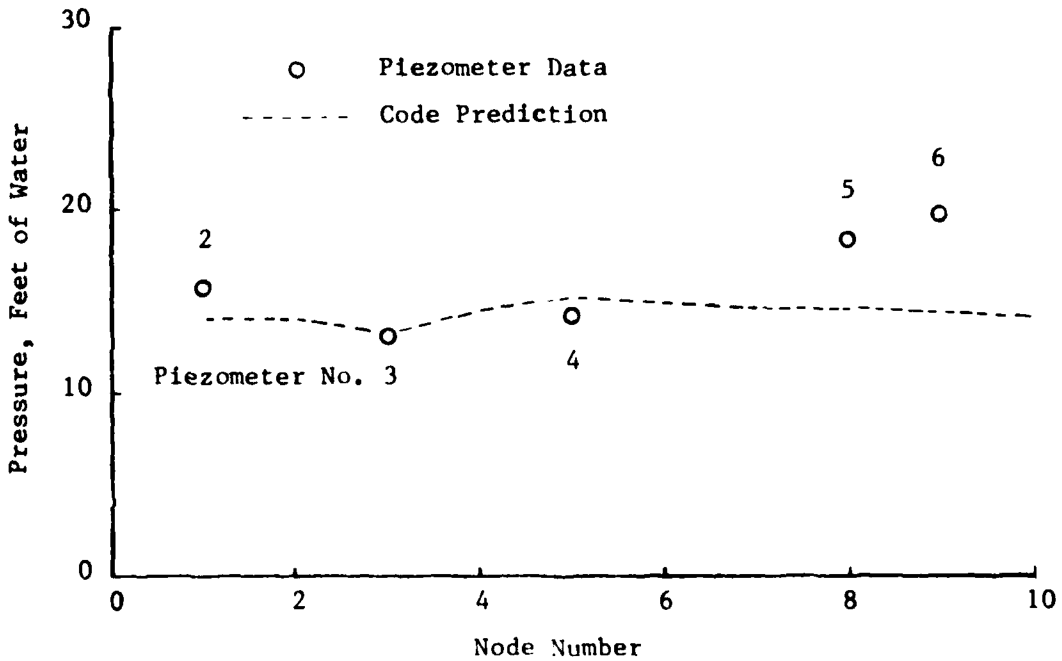
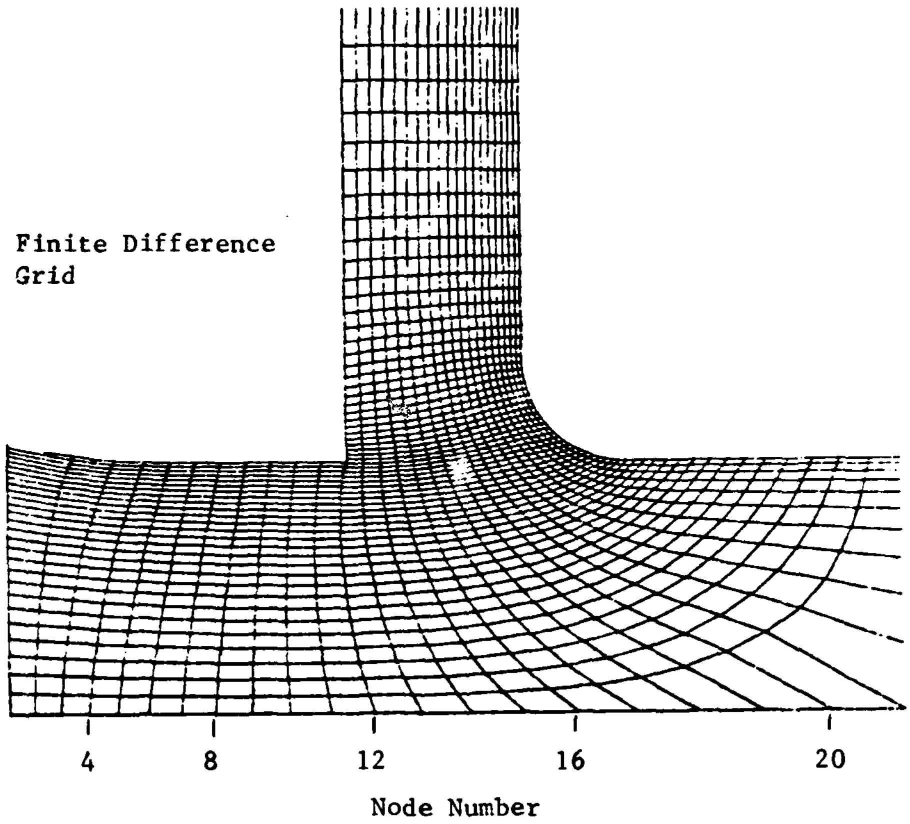




Figure 10. Comparison of computed and measured pressure distributions for flood-control flow, both gates open 100 percent, nodes 2-10



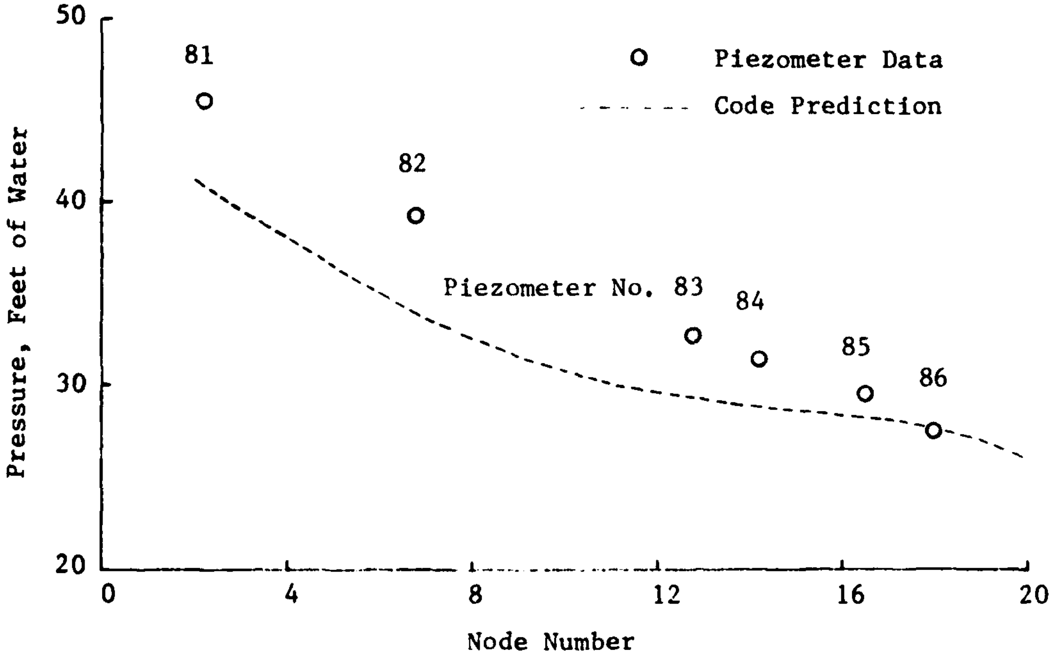




Figure 11. Comparison of computed and measured pressure distributions for flood-control flow, both gates open 100 percent, nodes 4-20

Gate Opening = 25 percent

Pool Elevation = 590 ft above NGVD

Flow Rate = 1,260 cfs

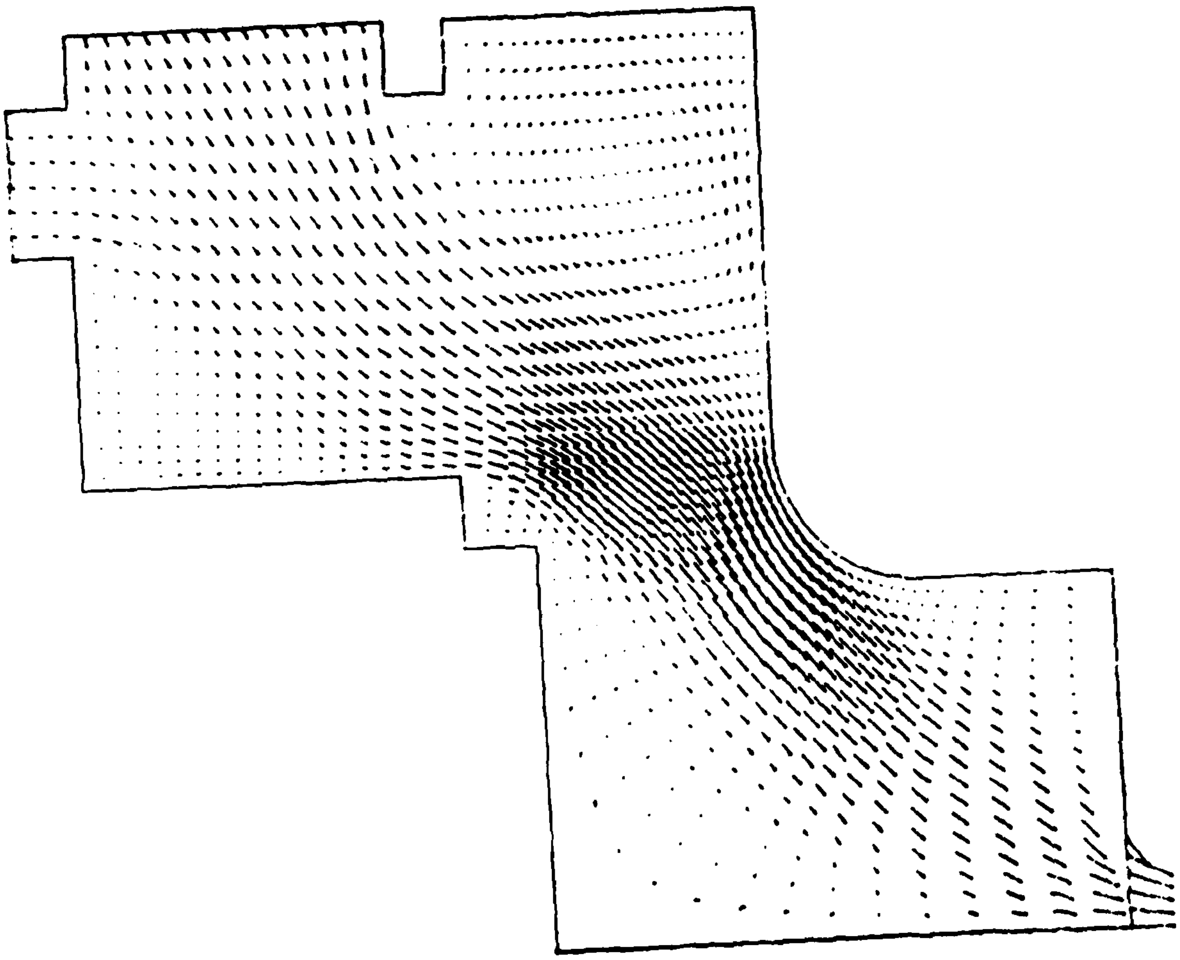


Figure 12. Velocity vectors for flow through water quality facility with service gate open 25 percent

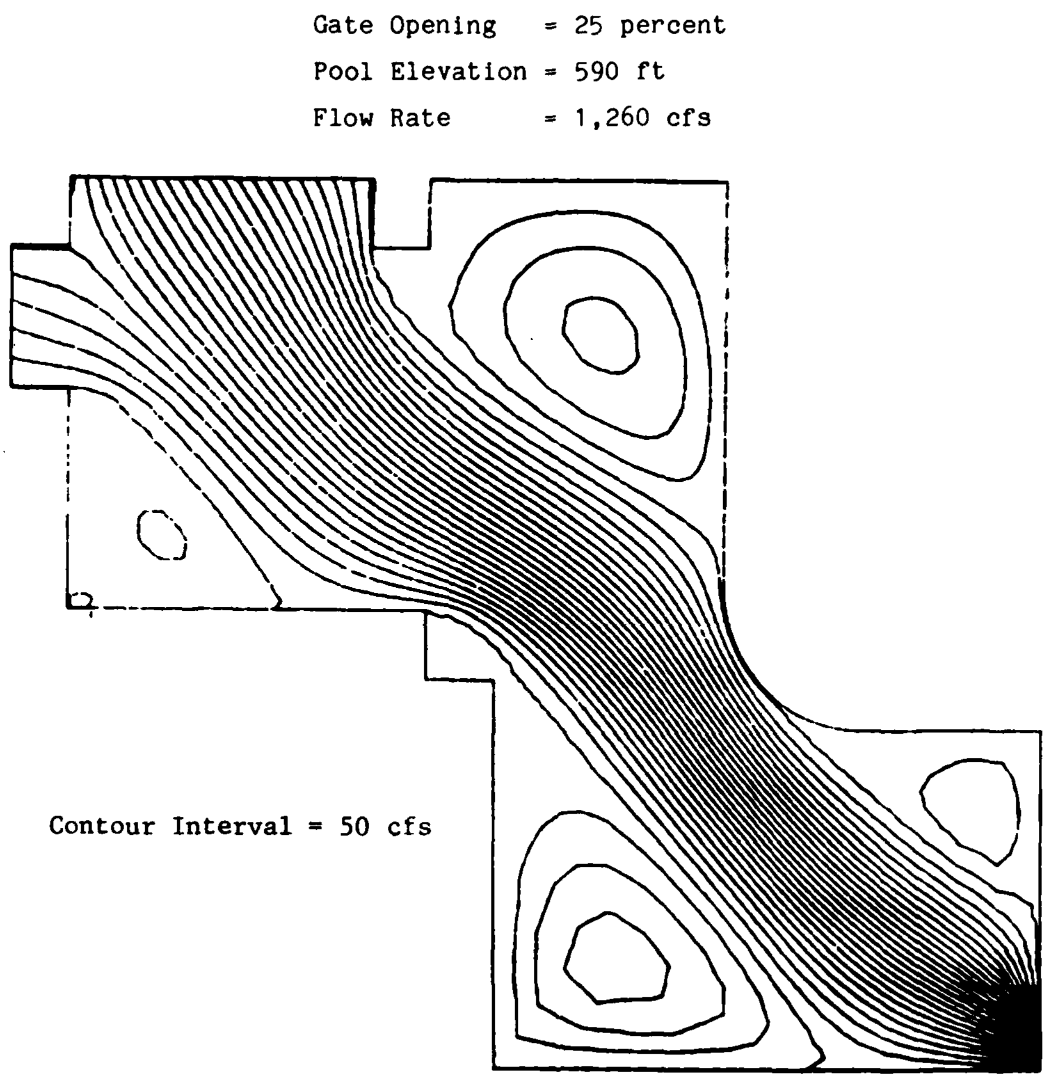



Figure 13. Streamlines for flow through water quality facility with service gate open 25 percent

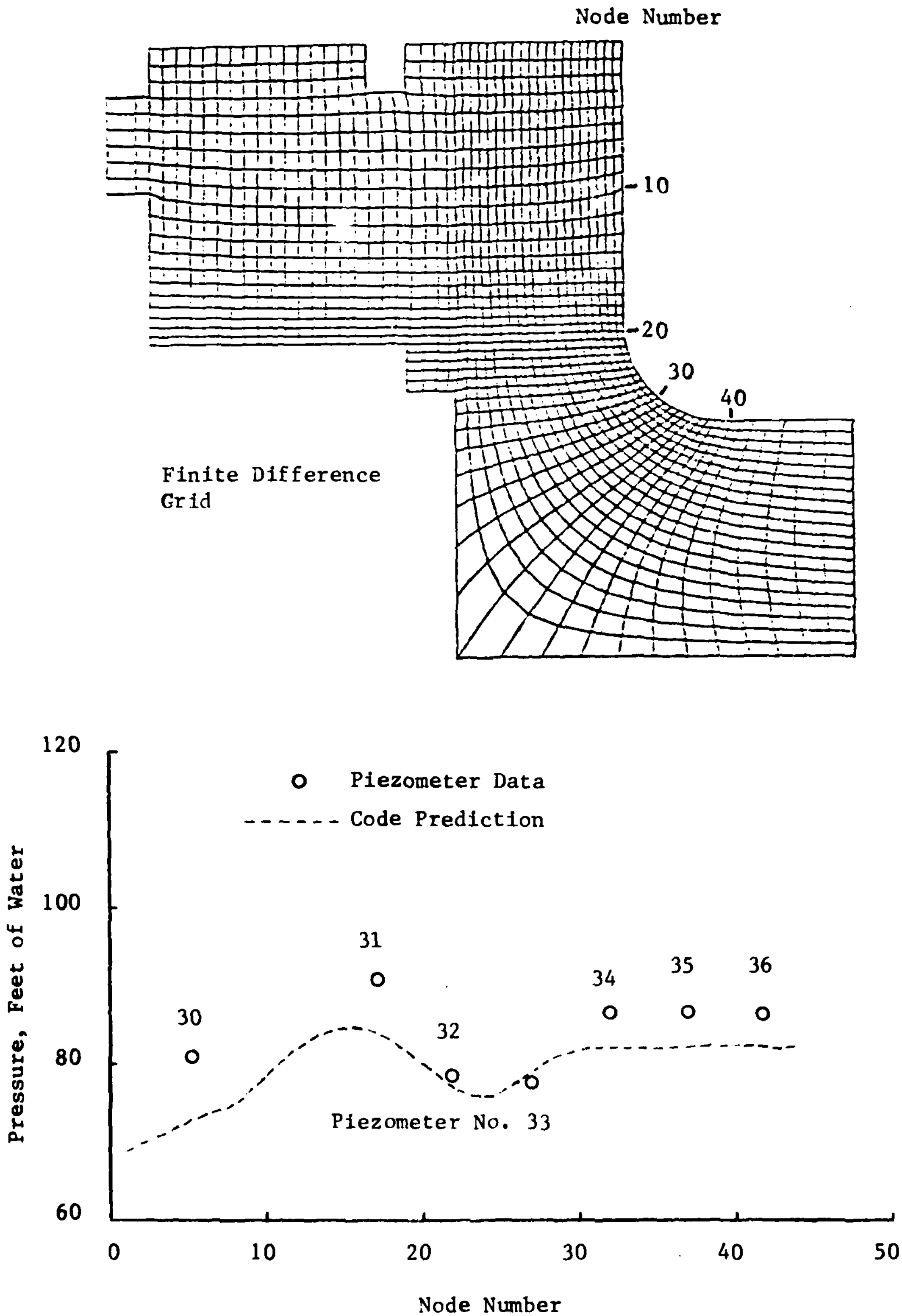
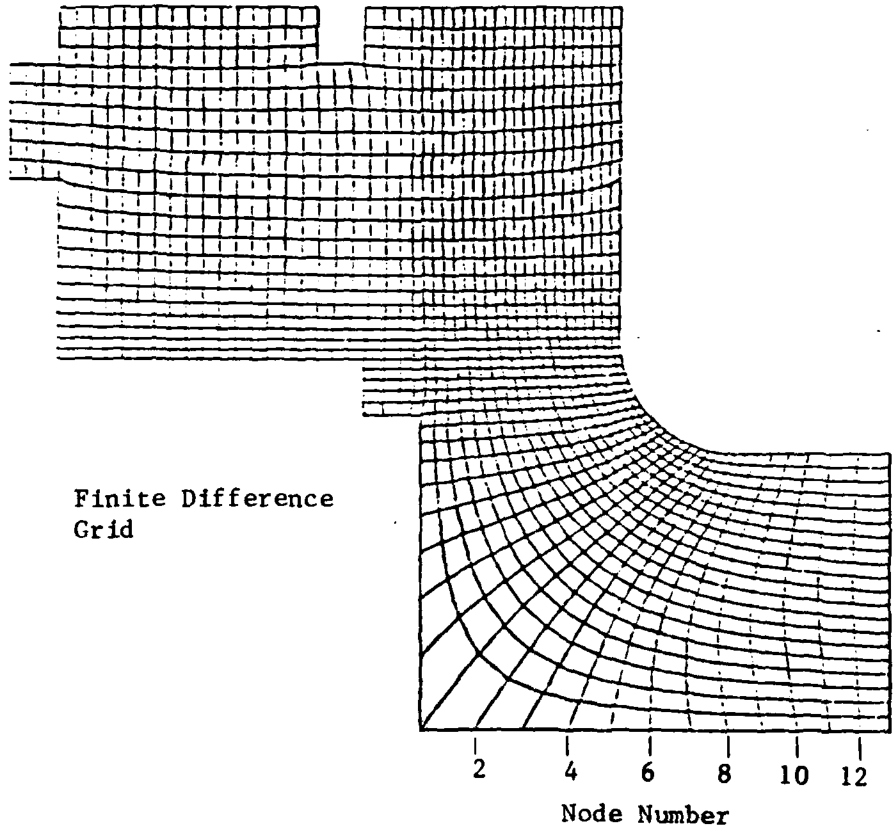



Figure 14. Comparison of computed and measured pressure distributions for water quality flow with service gate open 25 percent, nodes 10-40



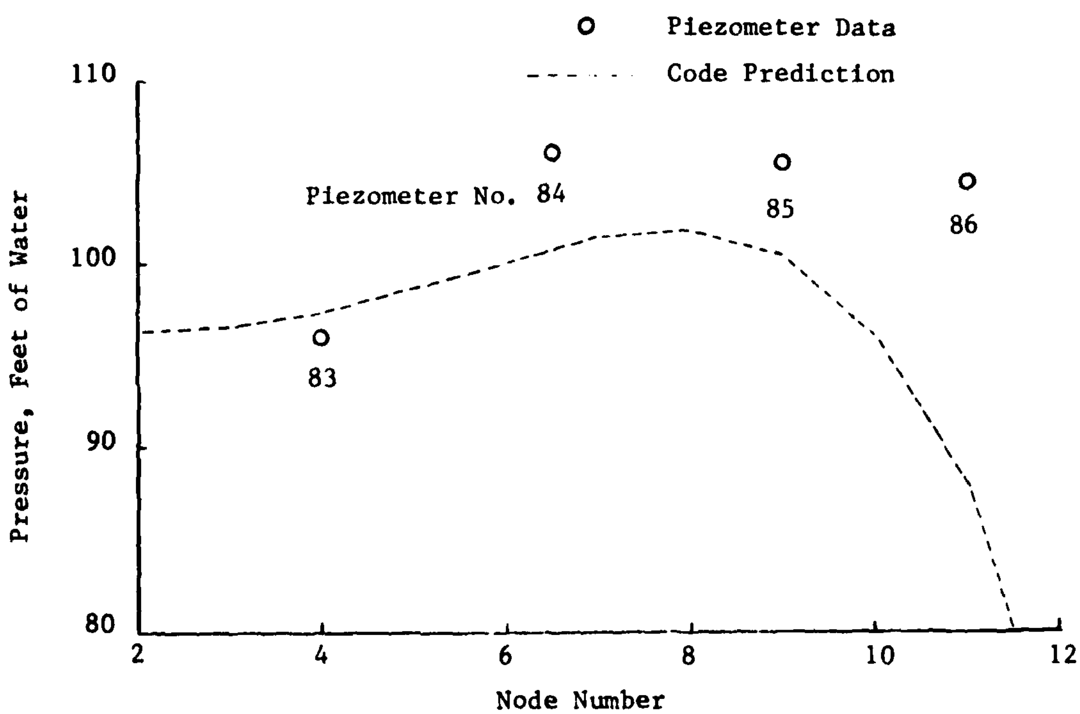




Figure 15. Comparison of computed and measured pressure distributions for water quality flow with service gate open 25 percent, nodes 2-12

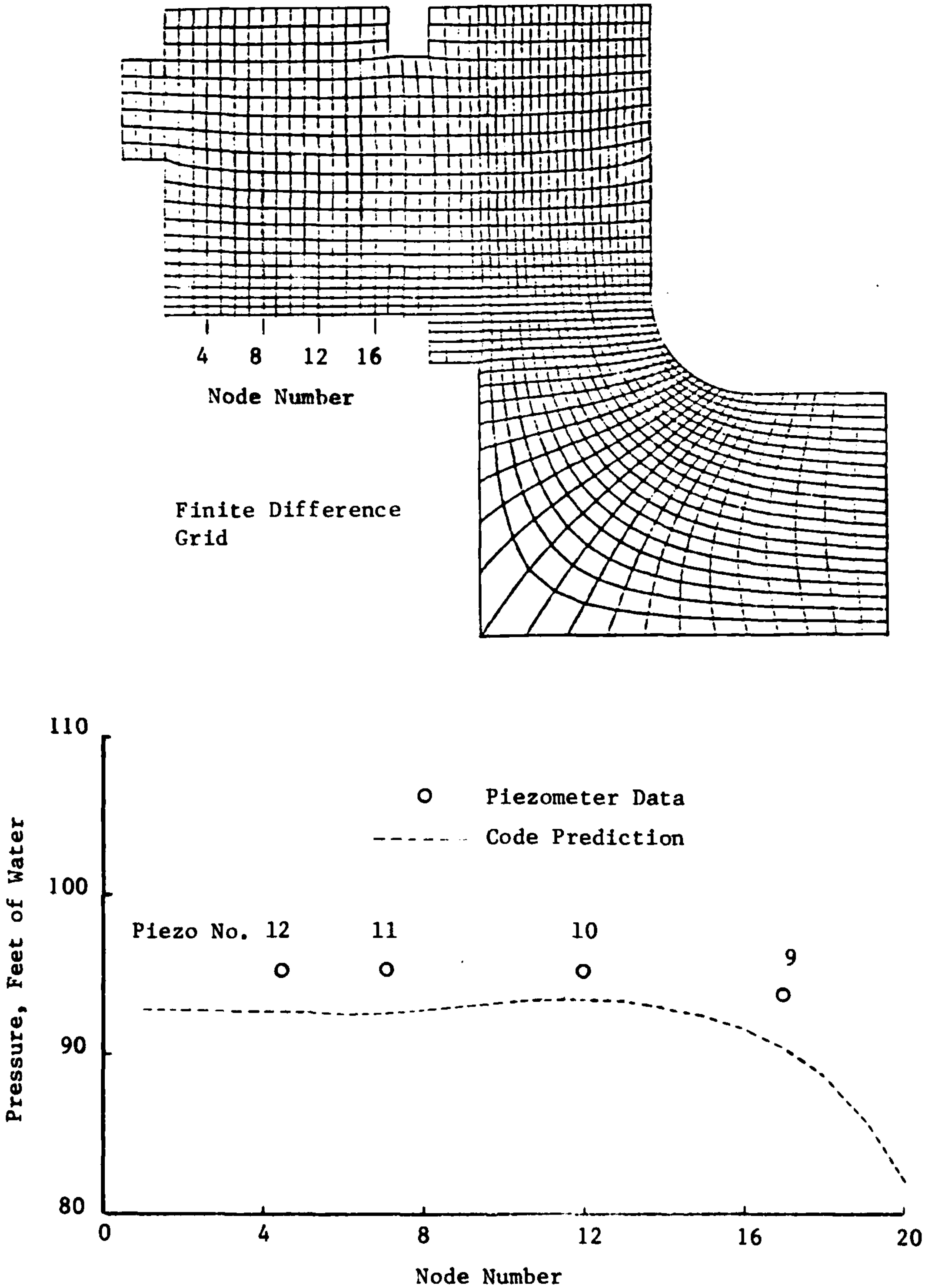



Figure 16. Comparison of computed and measured pressure distributions for water quality flow with service gate open 25 percent, nodes 4-16

Gate Opening = 50 percent

Pool Elevation = 530 ft

Flow Rate = 1,520 cfs

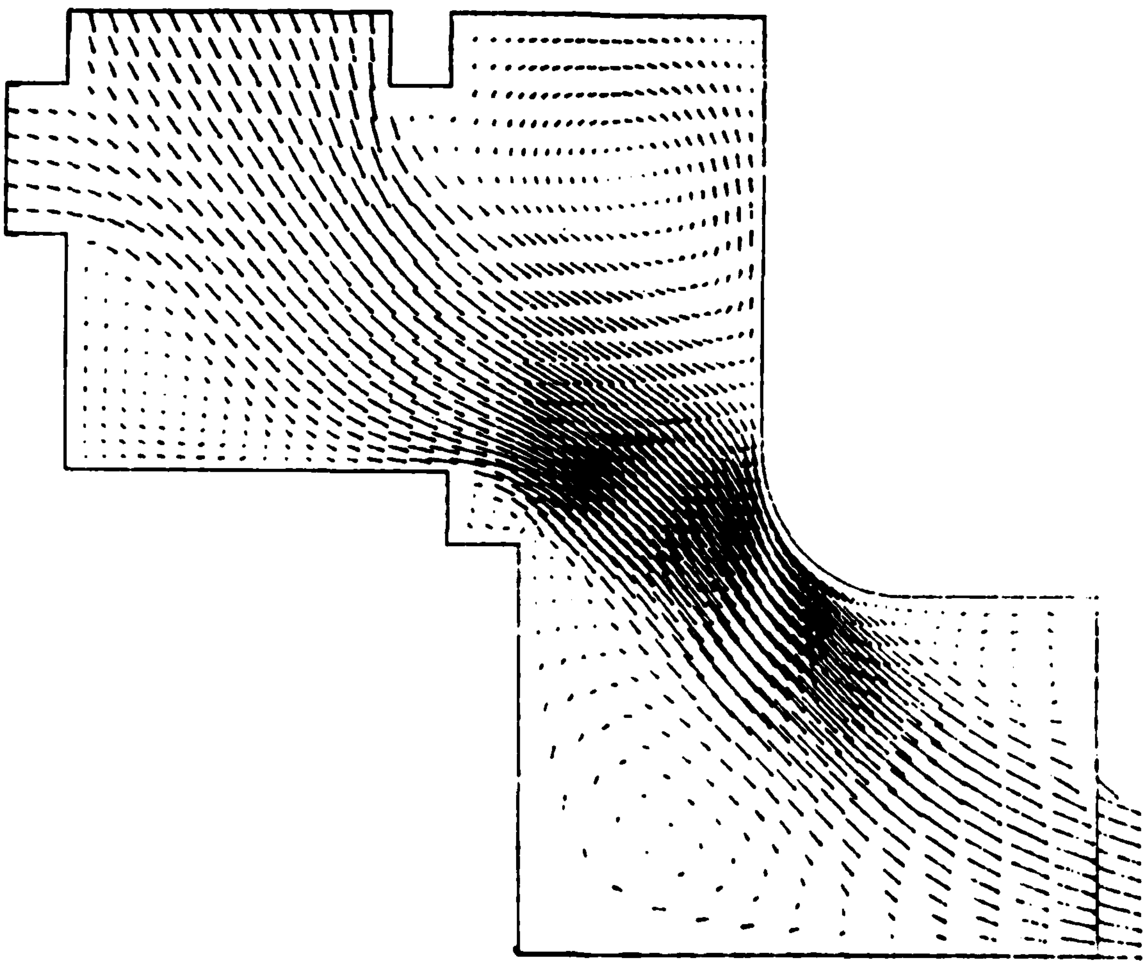


Figure 17. Velocity vectors for flow through water quality facility with service gate open 50 percent

Gate Opening = 50 percent

Pool Elevation = 530 ft

Flow Rate = 1,520 cfs

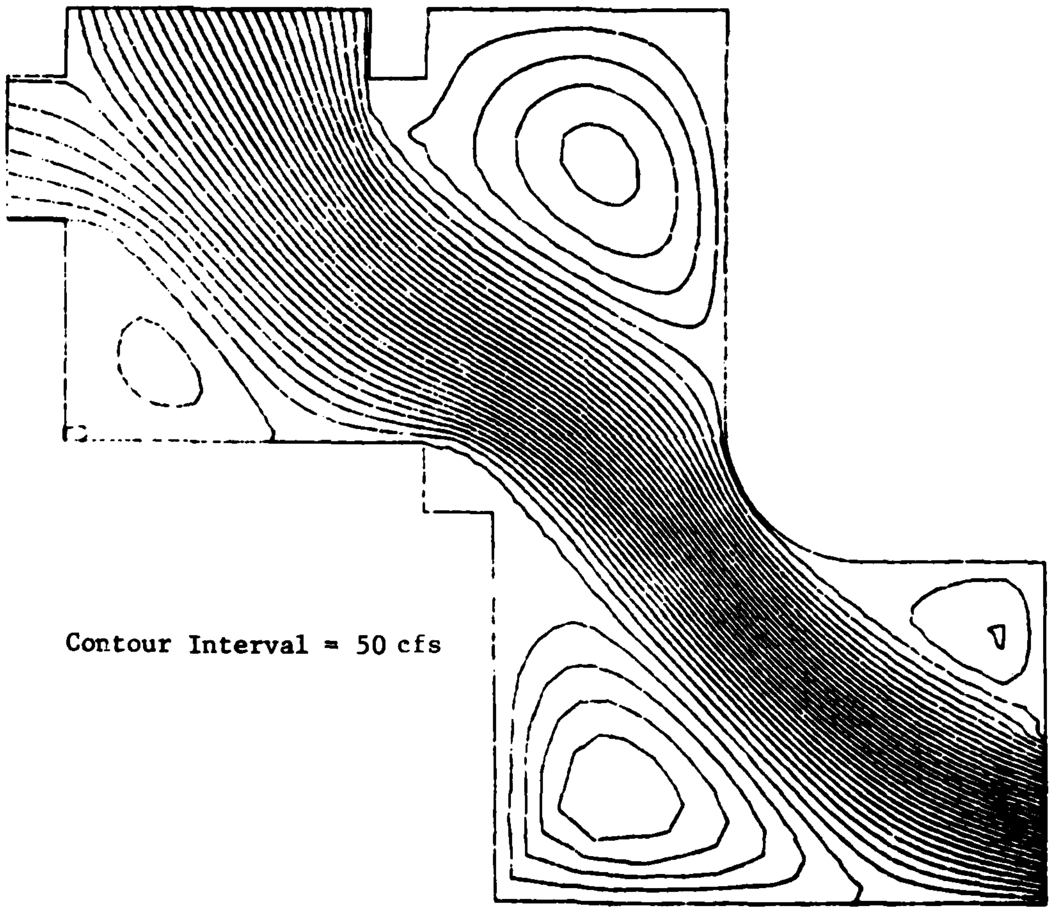
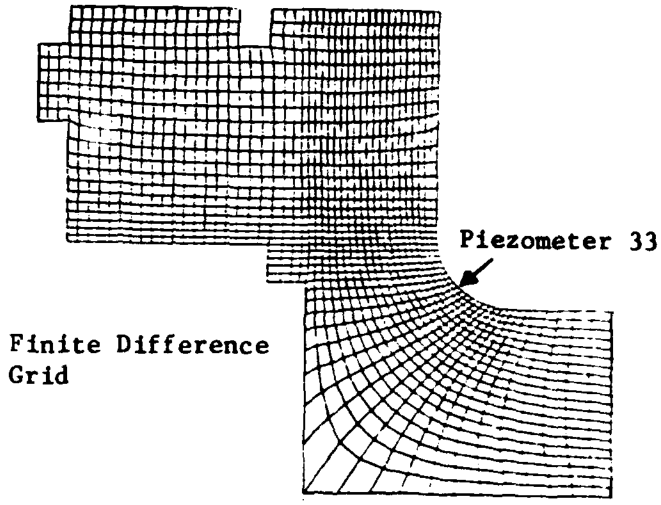



Figure 18. Streamlines for flow through water quality facility with service gate open 50 percent



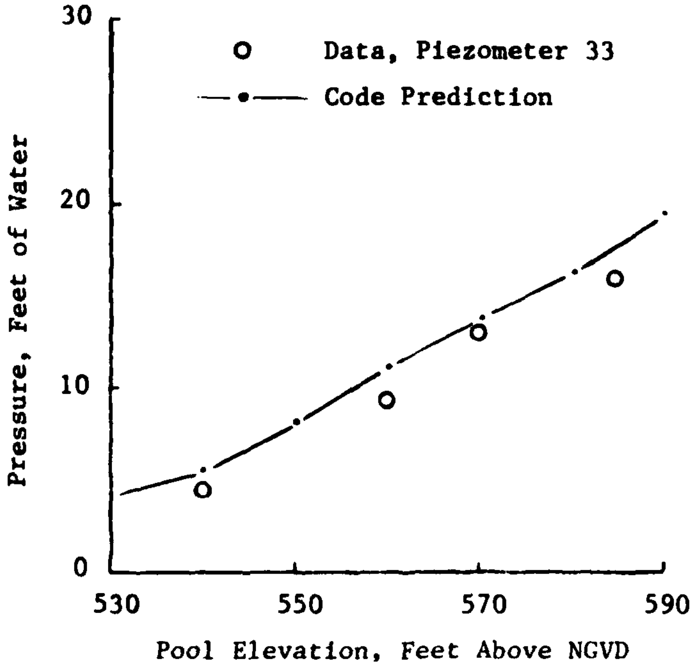




Figure 19. Comparison of computed and measured pressures at piezometer 33 for water quality flow with service gate open 50 percent

## APPENDIX A: THEORY USED IN THE VORTEX CODE

1. For incompressible flow, the governing equations are the Navier-Stokes equations for the conservation of momentum and mass, respectively:

$$\rho\left[\underset{\sim}{u}_t + \nabla \cdot (\underset{\sim}{u}\underset{\sim}{u})\right] = \mu\nabla^2\underset{\sim}{u} - \nabla p + \rho \underset{\sim}{g} \tag{A1}$$

$$\nabla \cdot \underset{\sim}{u} = 0 \tag{A2}$$

In two dimensions, the continuity equation (A2) is readily satisfied when the components of $\underset{\sim}{u}$ are derivatives of the stream function $\Psi$ :

$$u = \Psi_y \tag{A3}$$

$$v = -\Psi_x \tag{A4}$$

The pressure can be eliminated from the governing equations by taking the curl of the momentum equation (A1) and replacing the two-dimensional (2-D) Navier-Stokes equations by the new system

$$\rho\left[\zeta_t + (u\zeta)_x + (v\zeta)_y\right] = \mu\nabla^2\zeta + \left(\rho g_2\right)_x - \left(\rho g_1\right)_y \tag{A5}$$

$$\nabla^2\Psi = -\zeta \tag{A6}$$

Equation A5 replaces Equation A1, subject to the Boussinesq approximation*; and Equation A6 expresses the relation between the vorticity and the derivatives of the velocity components

$$\zeta = v_x - u_y \tag{A7}$$

subject to Equations A3 and A4. The new system of equations represents the Navier-Stokes equations in stream-function/vorticity form, which is often the

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* Variable density is retained in the gravity terms to allow calculations for density-stratified flow. All results presented herein, however, were obtained assuming constant density.

most convenient and manageable form for calculating 2-D internal flow. After solving these equations for $\Psi$ and $\zeta$, the velocity components can be found from Equations A3 and A4; and the pressure can be calculated by integrating the x- and y-components of the momentum equation:

$$\rho\left[u_t + (u^2)_x + (uv)_y\right] = \mu\rho^2 u - p_x + \rho g_1 \tag{A8}$$

$$\rho\left[v_t + (uv)_x + (v^2)_y\right] = \mu\nabla^2 v - p_y + \rho g_2 \tag{A9}$$

Equations A8 and A9 can be put in more tractable form by using the 2-D continuity equation

$$u_x + v_y = 0 \tag{A10}$$

along with Equation A7 to obtain

$$p_x = -\rho(u_t + uu_x + vv_x) + \rho v\zeta - \mu\zeta_y + \rho g_1 \tag{A11}$$

$$p_y = -\rho(v_t + uu_y + vv_y) - \rho u\zeta + \mu\zeta_x + \rho g_2 \tag{A12}$$

Equations A11 and A12 now combine to form a single equation for the pressure increment

$$\frac{dp}{\rho} = \frac{1}{2}\, d(u^2 + v^2) + (g_1 + v\zeta - \nu\zeta_y - u_t)\, dx + (g_2 - u\zeta + \nu\zeta_x - v_t)\, dy \tag{A13}$$

This final form is particularly convenient for integration, because $dp$ is an exact differential. As a result, the pressure change can be calculated along any path defined by incremental displacements $dx$ and $dy$.

2. The stream-function/vorticity equations are solved numerically by the VORTEX computer code, which has been developed from the WESSEL code (Thompson and Bernard 1985*). WESSEL solves Equations A8, A9, and A10, whereas VORTEX solves Equations A5 and A6. As a rule, the latter equations

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* See References at end of main text.

are easier and less expensive to solve when using finite differences. The input/output structure of VORTEX is nearly identical with that of WESSEL, and both codes use boundary-fitted finite difference grids generated by the WESCOR computer code (Thompson 1983). Additional documentation for the VORTEX code will be provided in the near future.

# APPENDIX B: NOTATION

| | |
|---|---|
| $d$ | Incremental operator |
| $g$ | Gravitational acceleration |
| $\underset{\sim}{g}$ | Gravity vector |
| $g_1, g_2$ | x- and y-components of $\underset{\sim}{g}$, respectively |
| $p$ | Pressure |
| $t$ | Time |
| $\underset{\sim}{u}$ | Velocity vector |
| $u, v$ | x- and y-components of $\underset{\sim}{u}$, respectively |
| $x, y$ | Cartesian coordinates |
| $\zeta$ | Vorticity |
| $\mu$ | Dynamic viscosity |
| $\nu$ | Kinematic viscosity |
| $\rho$ | Density |
| $\psi$ | Stream function |
| $\nabla$ | Gradient operator |
| $\nabla^2$ | Laplacian operator |

END

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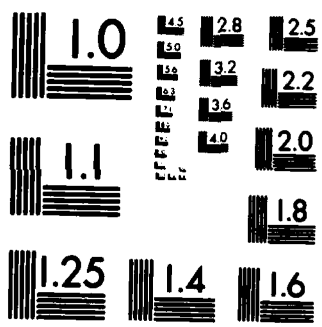


MICROCOPY RESOLUTION TEST CHART
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# SUPPLEMENTARY

# INFORMATION

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VICKSBURG, MISSISSIPPI 39180-0631

REPLY TO
ATTENTION OF
WESHS-R

10 July 1987

AD-A169 039

Errata Sheet

No. 1

PRESSURE CALCULATION FOR TWO-DIMENSIONAL
FLOW INSIDE HYDRAULIC STRUCTURES

Miscellaneous Paper HL-86-2

April 1986

1. Page A2, Equation A8: Change the term $\mu\rho^2 u$ to $\mu\nabla^2 u$ .

2. Page A2, Equation A13: Change the term $v\zeta_y$ to $\nu\zeta_y$ .

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