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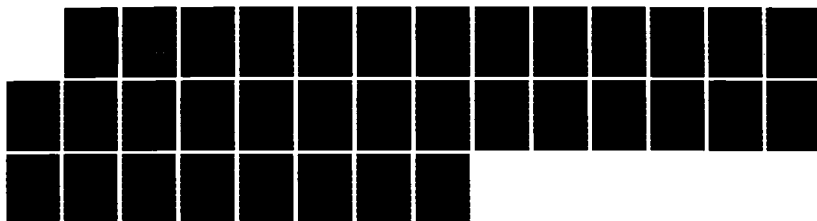
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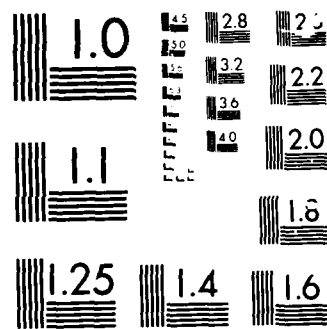
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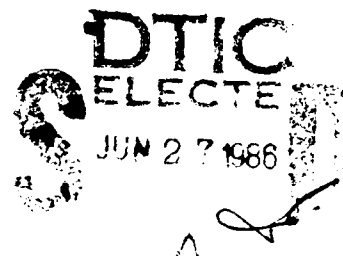
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TECHNICAL REPORT ARAED-TR-86010

**AN ANALYTICAL TECHNIQUE FOR MODELING GYROSCOPES
IN GUIDED PROJECTILE SIMULATIONS**

MICHAEL J. AMORUSO



JUNE 1986



US ARMY
ARMAMENT
MUNITIONS &
CHEMICAL COMMAND
ARMAMENT R&D CENTER

U. S. ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER

ARMAMENT ENGINEERING DIRECTORATE

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INTRODUCTION

A substantial need exists within the Armament Research and Development Center (ARDC) for a readily modifiable and computationally efficient flight simulation for smart munitions. This simulation must be easily adapted to varying designs of projectiles undergoing rapid evolution. Furthermore, the simulations must be highly efficient to prevent ARDC's computer facilities from being overwhelmed by the workload associated with the development of a guided projectile or smart munition.

To meet this need, ARDC is formulating methods and developing modular software for simulating autopilot components for incorporation into its standard flight simulation, TRAJ. An innovative technique was developed in which exact analytical solutions for various autopilot functions are applied in piecewise fashion within the more complex but lower frequency simulation of the projectile airframe, which is solved numerically (ref 1). The extension of this approach to treat two-axis gimballed gyroscopes with nutation damping is documented in reference 2. The module presented here reproduces gyro nutation as well as precession without the extremely small integration time steps previously required for a successful numerical integration.

DISCUSSION

Gyroscopes are often used in guided projectiles to provide inertial sensing for the autopilot or a stabilized platform for a seeker. A gyro exhibits both precession and nutation when torqued. Since nutation is not ordinarily desirable, and a well designed gyroscope should have sufficient nutational damping to make the nutation negligible, simulations of guided projectiles usually neglect nutation and treat gyroscopic motion as a simple precession rate which is proportional to the applied torque. This angular rate is then numerically integrated to produce a gyroscopic angle.

Attempts to treat the gyroscope more realistically have met with practical difficulties. The equations for a gimballed gyro require a very fine integration time step when solved by numerical integration techniques. Time steps can be smaller than a 0.00001 second. Clearly, it is usually not practical to use such a small time step in a large and complex flight simulation.

The size of the time step required to perform a numerical integration is driven downward by two constraints: (1) the driving term must not vary significantly during the time step, and (2) the time step must be small enough to produce a stable integration. Reflection will convince the reader that the second requirement is more stringent in the case of a gyro in a flight simulation. The time constants associated with the airframe, which determine the rate of change of the driving term, are an inherently slow process compared to the dynamics of the gyro. Therefore, stable numerical integration is a stronger driver to fine integration than the variation of the driving term. The proposed piecewise analytical approach can relax the time step size by orders of magnitude by eliminating the second constraint. Such a solution is exact no matter how large the time

step as long as the driving term variation is negligible during that interval. In practice, an improvement of orders of magnitude in execution speed can be achieved. If it is desirable to study the nutation of the gyro, the time step of the closed form solution can be made sufficiently small compared to the nutation period to achieve sufficient resolution. When the nutation does not need to be resolved from the precession, the size of the time step can be determined by the requirements imposed by the rest of the simulation.

ANALYTIC SOLUTION TO THE EQUATIONS OF MOTION

The equations of motion of the gimballed gyro are (ref 3)

$$J_1 \dot{\omega}_1 + f\omega_1 + H\omega_2 = T_1 \quad (1a)$$

$$J_2 \dot{\omega}_2 + f\omega_2 - H\omega_1 = T_2 \quad (1b)$$

where

J_1 and J_2 are the pitch and yaw moments of inertia, respectively;

f is the frictional restraint due to gimbal bearing friction;

T_1 and T_2 are the applied pitch and yaw torques, respectively;

ω_1 and ω_2 are components of the gyroscopic angular velocities; and

H is the rotor, angular momentum, $2\pi SJ$, where S is the rotor spin rate and J is the axial moment of inertia.

For convenience we substitute

$$\begin{aligned} \omega_1 &\equiv \dot{x}_1 = x_3 = \omega_y \\ \omega_2 &\equiv \dot{x}_2 = x_4 = \omega_z \end{aligned} \quad (2)$$

The equations of motion then become

$$J_1 \dot{x}_3 + fx_3 + Hx_4 = T_1 \quad (3)$$

$$J_2 \dot{x}_4 + fx_4 - Hx_3 = T_2 \quad (4)$$

These two differential equations can be decoupled to facilitate obtaining a solution by increasing the order of the equations. Assuming T_1 to be constant, solving equation 3 to obtain x_4 , differentiating v_{1t} and solving for x_4 , and then substituting these results in equation 4 yields

$$\ddot{x}_3 + f \left[\frac{1}{J_1} + \frac{1}{J_2} \right] \dot{x}_3 + \left[\frac{f^2 + H^2}{J_1 J_2} \right] x_3 = \frac{f T_1 - H T_2}{J_1 J_2} \quad (5)$$

The elimination of x_4 has resulted in an equation of motion in the form of a damped harmonic oscillator with a constant applied force, viz.

$$m\ddot{x} + b\dot{x} + kx = F \quad (6)$$

The solution to equation 6 has the well-known form

$$x(t) = \frac{F}{m} + e^{-\alpha t} \left[k_1 \sin \beta t + k_2 \cos \beta t \right] \quad (7)$$

where k_1 and k_2 are to be determined by the initial conditions and

$$\begin{aligned} \alpha &= b/2m \\ \beta &= \sqrt{\omega_o^2 - \alpha^2} \\ \omega_o &= \sqrt{k/m} \end{aligned} \quad (8)$$

By comparing equations 5 through 7, an expression can be written for x_3

$$\omega_y = x_3(t) = k_5 + e^{-\alpha t} \left[k_1 \sin \beta t + k_2 \cos \beta t \right] \quad (9)$$

Similarly, the following expression can be obtained for x_4

$$\omega_z = x_4(t) = k_6 + e^{-\alpha t} \left[k_3 \sin \beta t + k_4 \cos \beta t \right] \quad (10)$$

where

$$\begin{aligned}
 k_5 &= \frac{fT_1 - HT_2}{f^2 + H^2} \\
 k_6 &= \frac{HT_1 + fT_2}{f^2 + H^2} \\
 \alpha &= \frac{f}{2} \left[\frac{1}{J_1} + \frac{1}{J_2} \right] \\
 \beta &= \sqrt{\frac{f^2 + H^2}{J_1 J_2} - \alpha^2}
 \end{aligned} \tag{11}$$

and k_1 through k_4 are constants of integration to be determined by the initial conditions. Note that only two out of the four constants of integration are independent since the equations are second order only because of the artifice used to decouple the original equations of motion. If $t = 0$ is set in equations 9 and 10, k_2 and k_4 will be determined in terms of $x_3(0)$ and $x_4(0)$. The quantities k_1 and k_3 are linearly dependent on k_2 and k_4 . To see this explicitly, substitute the solutions (equations 9 and 10) into the original equation of motion (equation 3) to obtain the expression

$$\begin{aligned}
 &\left[\alpha J_1 k_1 + \beta J_1 k_2 - f k_1 - H k_3 \right] \sin \beta t \\
 &+ \left[\alpha J_1 k_2 - \beta J_1 k_1 - f k_2 - H k_4 \right] \cos \beta t = 0
 \end{aligned} \tag{12}$$

Because of the linear independence of the trigonometric functions, the quantities in brackets must each vanish identically. This condition yields

$$\begin{aligned}
 k_1 &= \frac{1}{\beta J_1} \left[(\alpha J_1 - f) k_2 - H k_4 \right] \\
 k_3 &= \frac{1}{H} \left[(\alpha J_1 - f) k_1 + \beta J_1 k_2 \right] \\
 &= -\frac{1}{\beta J_1} \left[(\alpha J_1 - f) k_4 - H \frac{J_1}{J_2} k_2 \right]
 \end{aligned} \tag{13}$$

Therefore, only k_2 and k_4 are specified independently and can now be evaluated from the initial conditions by setting $t = 0$ in equations 9 and 10.

$$\begin{aligned}
 k_2 &= x_3(0) - k_5 \\
 k_4 &= x_4(0) - k_6
 \end{aligned} \tag{14}$$

This completes the solution to equations 3 and 4; however, this form is not convenient. The solution expressed in terms of the gyroscopic Euler angles θ and ψ rather than the angular velocity components ω and ω_z is preferred. The gyroscope roll angle ϕ may be zero with no loss of generality since a two axis gimbaled gyroscope only has two degrees-of-freedom. Furthermore, if the gyroscope motion is integrated stepwise in time, discarding the old coordinate system at the end of each integration step and replacing the coordinates with a new set to coincide with the new position of the gyroscope, small angle substitution $\dot{\theta} = \omega_y$, $\dot{\psi} = \omega_z$ can be made. These equations can then be integrated for θ and ψ using standard integral tables to obtain the final results given below.

$$\begin{aligned}\theta(t) &= \theta(0) + k_5 t + B_1 \\ &\quad - \frac{e^{-\alpha t}}{\alpha^2 + \beta^2} \left[(k_1 \alpha - k_2 \beta) \sin \beta t + (k_1 \beta + k_2 \alpha) \cos \beta t \right] \\ &= \theta(0) + k_5 t + B_1 \\ &\quad - e^{-\alpha t} \left[\frac{k_1^2 + k_2^2}{\alpha^2 + \beta^2} \right]^{1/2} \sin (\beta t + \phi_1)\end{aligned}\tag{15}$$

$$\begin{aligned}\psi(t) &= \psi(0) + k_6 t + B_2 \\ &\quad - \frac{e^{-\alpha t}}{\alpha^2 + \beta^2} \left[(k_3 \alpha - k_4 \beta) \sin \beta t + (k_3 \beta + k_4 \alpha) \cos \beta t \right] \\ &= \psi(0) + k_6 t + B_2 - e^{-\alpha t} \left[\frac{k_3^2 + k_4^2}{\alpha^2 + \beta^2} \right]^{1/2} \sin (\beta t + \phi_2)\end{aligned}\tag{16}$$

where

$$\begin{aligned}\phi_1 &= \tan^{-1} \left[\frac{k_1 \beta + k_2 \alpha}{k_1 \alpha - k_2 \beta} \right] \\ \phi_2 &= \tan^{-1} \left[\frac{k_3 \beta + k_4 \alpha}{k_3 \alpha - k_4 \beta} \right]\end{aligned}\tag{17}$$

$$\begin{aligned}\dot{\theta}(t) &= k_5 + e^{-\alpha t} \left[k_1 \sin \beta t + k_2 \cos \beta t \right] \\ \dot{\psi}(t) &= k_6 + e^{-\alpha t} \left[k_3 \sin \beta t + k_4 \cos \beta t \right]\end{aligned}\tag{18}$$

where

$$B_1 = \frac{k_1 \beta + k_2 \alpha}{\alpha^2 + \beta^2}$$

$$B_2 = \frac{k_3 \beta + k_4 \alpha}{\alpha^2 + \beta^2} \quad (19)$$

A brief discussion of the physical significance of the terms in the solutions for $\theta(t)$ and $\psi(t)$ follows: The term linear in t with coefficient k_5 or k_6 is the precession that ordinarily dominates gyroscopic kinematics. The terms involving β are the nutation terms with frequency $f = \beta/2T$, and $[k_1^2 + k_2^2]$ and $[k_3^2 + k_4^2]$ are the amplitudes for the pitch and yaw components. For consistency with the small angle approximation used to obtain equations 15 and 16, these nutation amplitudes must be small.

The gyro pitch and yaw torques are cross-coupled in the expressions k_5 and k_6 whenever $f > 0$. In the subroutine GYRO (appendix), noise sources and error terms are included with the typical treatment given by gyroscope manufacturers. They are for illustration purposes only. Care must be taken with each gyroscope manufacturer to determine which of these terms might be pertinent and to verify that they are not merely a phenomenological way of including in simpler gyro models the effects that flow naturally from this more complete treatment. For example, a typical term is torquer cross coupling. Such an effect can arise in part because of the nonorthogonality of the pitch and yaw components of the magnetic fields of the torquer, especially when the gimbal angles are not zero. However, torque is also cross coupled in the expressions k_5 and k_6 whenever $f > 0$. Such effects should not be accounted for twice.

The computer realization of the algorithm is as follows: At the beginning of each integration time step, a coordinate system is fixed that momentarily coincides with the gyroscope axes at the beginning of the interval. The gyroscopic motion is integrated in this artificially fixed inertial coordinate system with the angles initially zero. Therefore, if the integration time step is not too large, the small angle approximation is maintained. At the end of the time step, the gyro gimbal angles, gyro Euler angles, and Euler rotation matrices can be reconstructed by using the gyroscopic rotation matrix at the beginning of the integration time step and the projectile body rotation matrix at the end of the integration time step.

CONCLUSIONS

This piecewise analytic solution to a two-axis gimbaled gyroscope results in considerable savings in computer run time.

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SYMBOLS

Algebraic Symbols

f	Frictional restraint due to gimbal bearing friction, (slug-ft ² /s)
H	Gyro rotor angular momentum, $2 \pi sJ$, (slug-ft ² /s)
J	Gyro axial moment of inertia (slug-ft ²)
J_1, J_2	Gyro pitch and yaw moments of inertia, respectively (slug-ft ²)
s	Gyro rotor spin rate (Hertz)
T_1, T_2	Applied gyro pitch and yaw torques, respectively (slug-ft ² /s ²)
ω_1, ω_2	Pitch and yaw components of the gyroscopic angular velocities, respectively (rad/s)

FORTRAN Symbols

In this simulation, the "inertial small angle frame" is defined as a set of nonrotating inertial coordinates that momentarily coincides with the gyro axes at the beginning of each integration time step. This coordinate frame is abbreviated as ISAF.

F	Frictional restraint due to gyro pitch/yaw gimbal-bearing friction (slug-ft ² /s)
GERROR	Matrix of gyro error sources (various units)
GIMBAL	Gimbal limit angle (radians)
GIMBY	Pitch gyro gimbal angle (radians)
GIMBZ	Yaw gyro gimbal angle (radians)
GJO	Gyro spin-axis moment of inertia (slug-ft ²)
GJ1, GJ2	Gyro pitch and yaw axes moments of inertia (slug-ft ²)
GP	Components of a unit vector coinciding with the gyro spin axis and resolved in projectile body axes; used internally only.
GSPIN	Gyro spin rate (Hertz). GJO and GSPIN are used only to calculate H .
H	Gyro rotor angular momentum (slug-ft ² /s)

IGLE	Gimbal limits exceeded indicator:	1 = limits exceeded 0 = not exceeded
ISAE	Small angle exceeded indicator:	1 = limits exceeded 0 = not exceeded
PSI	Gyro yaw angle in ISAF (radians)	
PSID	Time derivative of PSI (rad/s)	
PSIDCM	Commanded gyro yaw slew rate (rad/s)	
PSIDMX	Maximum commanded gyro yaw rate (rad/s)	
ROTC	Value of the matrix ROTEG at the beginning of each time step	
ROTEG	Gyro Euler angle rotation matrix (earth to gyro)	
TDELTA	Integration time step (seconds)	
THETA	Gyro pitch angle in the ISAF (radians)	
THETAD	Time derivative of THETA (rad/s)	
THTDCM	Commanded gyro pitch slew rate (rad/s)	
THTDMX	Maximum commanded gyro pitch rate (rad/s)	

APPENDIX
PROGRAM LISTINGS

COMPUTER PROGRAMS

The formalism described in this report was implemented in FORTRAN 77 and comprises three parts: (1) a main program MAINP that is a surrogate for a six degree-of-freedom simulation in which the gyro model is to be embedded, (2) an initialization subroutine INITG, and (3) the gyro model subroutine GYRO. Subroutine GYRO is suitable for use as it stands. INITG is meant to serve as a guide to creating the proper initialization in the user's own simulation. MAINP is meant to be completely replaced by the user's six degree-of-freedom simulation and is only used here as a driver for testing or demonstrating subroutine GYRO.

A sample run was made with 100-hertz gyro spin rate and 5 slug-ft²/s frictional restraint for the two input data records. In addition to the tabular output, plots (figs A-1 and A-2) show the rapid nutation and the general slow drift which is the precession.

The program GYRO is self-documented with comment statements and can be used as it stands but the user should examine the comments so that interfacing requirements and the definitions of all the variables and constants are understood. COMMON/SIXDOF/ is meant to be replaced by whatever COMMON BLOCK is required in the user's own six degree-of-freedom simulation to communicate the values of these parameters needed by subroutine GYRO. Note that the resulting rotation matrix is regularly renormalized to restore its unitary property at the end of subroutine GYRO.

PROGRAM GYMAIN

```
C      MAIN PROGRAM TO DRIVE GYRO SUBROUTINE.
C
C      FOR TEST PURPOSES ONLY.
C
      DIMENSION ROTX(3,3),ROTY(3,3),ROTZ(3,3),ROTNEW(3,3)
      COMMON/GYROD/ GJ1,GJ2,F,GIMBAL,THTDMX,PSIDMX,GERROR(6,2),
+ THETAD,PSID,ROTEG(3,3),THETA,PSI, THTDCM,PSIDCM,H,
+ GIMBY,GIMBZ, GJ0,GSPIN, IGLE,ISAE
      COMMON/SIXDOF/ROTEB(3,3),VD,WD,G,PI
C
C      INITIAL DATA
      OPEN (UNIT=7,FILE='INPUT')
      REWIND 7
      OPEN (UNIT=6,FILE='OUTPUT')
      REWIND 6
      VD =0.0
      WD =0.0
      GJ0=3.0
      GJ1=2.25
      GJ2=2.00
      GIMBAL=.4363323
      THTDMX=.1745329
      PSIDMX=.1745329
      G=32.1725
      KK=0
```

```

        PI=3.1415927
        DO 10 J=1,2
        DO 10 K=1,6
        GERROR(K,J)=0.0
10 CONTINUE
        WRITE(6,1007)
C
        READ(7,1004)GSPIN
        READ(7,1004)F
        DO 1 I=1,3
        DO 1 J=1,3
        EL=0.
        IF(I.EQ.J) EL=1.
        ROTX(I,J)=EL
        ROTY(I,J)=EL
        ROTZ(I,J)=EL
        ROTEB(I,J)=EL
        ROTNEW(I,J)=EL
1 ROTEG(I,J)=EL
        CALL INITG(TSTEP)
        WRITE(6,1005) GSPIN,F
        EL=.1745329*TSTEP
        ROTY(3,1)=2.3*EL
        ROTY(1,3)=-2.3*EL
        ROTZ(1,2)=1.8*EL
        ROTZ(2,1)=-1.8*EL
        T=0.

```

```

GIMBY=0.

GIMBZ=0.

CALL GYRO(T)

C   CALL GYRO WITH ZERO ARGUMENT AT UNCAGING TO INITIALIZE.

WRITE(6,1001)

THETAD=ROTY(3,1)/TSTEP

PSID  =ROTZ(1,2)/TSTEP

THETA =.0

PSI   =.0

DO 3 KK=0,800

THTDCM =-.19

C   IF (KK.GE.40) THTDCM=0.0

PSIDCM=.013

CALL GYRO(TSTEP)

IF (IGLE.EQ.1) STOP

IF (ISAE.EQ.1) STOP

C   IF (KK.GE.60) PSIDCM=-.151

T=T+TSTEP

WRITE(6,1000)KK,T,THETA,PSI,THTDCM,PSIDCM

DO 2 I=1,3

DO 2 N=1,3

ROTNEW(I,N)=0.

DO 2 J=1,3

DO 2 K=1,3

DO 2 L=1,3

2 ROTNEW(I,N)=ROTNEW(I,N)+ROTZ(I,J)*ROTY(J,K)*ROTX(K,L)*ROTEB(L,N)

DO 4 I=1,3

```

```

DO 4 K=1,3

4 ROTEB(I,K)=ROTNEW(I,K)

3 CONTINUE

STOP

1000 FORMAT(1X,I6,3X,F10.6,2F14.6,3X,2F10.4)

1001 FORMAT(16X,4HTIME,9X,5HTHETA,11X,3HPSI,7X,6HTHTDCM,4X,6HPSIDCM)

1004 FORMAT(F20.10)

1005 FORMAT(11H GSPIN IS      ,F10.2,7H HERTZ. ,/,5H F IS,F10.5,
          $ 16H SLUG*FT**2/SEC. ,//)

1006 FORMAT(8H TIME IS,F15.8,5H SEC. ,/)

1007 FORMAT(1H1)

END

SUBROUTINE INITG(TSTEP)

C
C   GYRO INITIALIZATION ROUTINE
C

COMMON/GYROD/ GJ1,GJ2,F,GIMBAL,THTDMX,PSIDMX,GERROR(6,2),
+ THETAD,PSID,ROTEG(3,3),THETA,PSI, THTDCM,PSIDCM,H,
+ GIMBY,GIMBZ, GJ0,GSPIN, IGLE,ISAE

COMMON/SIXDOF/ROTEB(3,3),VD,WD,G,PI

C

H      = 2.*PI*GSPIN*GJ0

ALPHA  = (F/2.)*(1./GJ1 + 1./GJ2)

BETA   = SQRT( (F*F+H*H)/(GJ1*GJ2) - ALPHA**2 )

C   NUTATION FREQUENCY IS USED ONLY INFORMATIVELY AND IN THE
C   SELECTION OF A TIME STEP TO PERMIT RESOLUTION OF THE
C   NUTATION. GNUTAT IS IN HERTZ (CPS).

```

```

C
GNUTAT = BETA/(2.*PI)
WRITE(6,1000) GNUTAT
C
SELECTING OUTPUT RATE TO RESOLVE NUTATION.
TSTEP = GNUTAT*20.
TSTEP = 1./FLOAT(IFIX(TSTEP))
WRITE(6,1001) TSTEP
RETURN

1000 FORMAT(30H GYRO NUTATION FREQUENCY IS ,F10.5 ,7H HERTZ. )
1001 FORMAT(14H TIME STEP IS ,F10.5 ,5H SEC. )

END

SUBROUTINE GYRO(TDELTA)
C
C THIS ROUTINE MOVES THE GYRO GIMBAL PICKOFF ANGLES GIMBY & GIMBZ,
C AND THE GYRO ROTATION MATRIX ROTEG ONE TIME STEP TDELTA
C FORWARD.
C
C
DIMENSION ROT(3,3),ROTC(3,3),GP(3),X(13)
COMMON/GYROD/ GJ1,GJ2,F,GIMBAL,THTDMX,PSIDMX,GERROR(6,2),
+ THETAD,PSID,ROTEG(3,3),THETA,PSI, THTDCM,PSIDCM,H,
+ GIMBY,GIMBZ, GJ0,GSPIN, IGLE,ISAE
C
COMMON/SIXDOF/ROTEB(3,3),VD,WD,G,PI
C
C INITIALIZE ELSEWHERE ONCE AND FOR ALL: GJ1,GJ2,F,GIMBAL,THTDMX,
C PSIDMX,GERROR(6,2).
C

```

C INITIALIZE ELSEWHERE AT UNCAGING: THETAD,PSID TO PROJECTILE BODY
 C RATES; ROTEG TO ROTEB+OFFSET ERROR; THETA,PSI TO ZERO+

 C OFFSETS.
 C
 C INPUT ARGUMENTS: TDELTA,THTDCM,PSIDCM,H,VD,WD.
 C
 C OUTPUT ARGUMENTS: GIMBY,GIMBZ,ROTEG,IGLE,ISAE.
 C
 C GSPIN AND GJ0 ARE ONLY NEEDED TO CALCULATE H. TO MODEL GYRO SPIN
 C DOWN, ASSIGN ELSEWHERE AS INPUT TO THIS ROUTINE A VARYING
 C VALUE OF GSPIN. H IS THEN CALCULATED IN THIS ROUTINE.
 C
 C IN THIS PROGRAM, THE 'INERTIAL SMALL ANGLE FRAME' IS DEFINED
 C AS A SET OF NONROTATING INERTIAL COORDINATES MOMENTARILY
 C COINCIDING WITH THE GYRO AXES AT T=0. IT WILL BE ABBREVIATED
 C AS 'ISAF'.
 C
 C TDELTA INTEGRATION TIME STEP (SEC).
 C THETA GYRO PITCH ANGLE IN THE ISAF (RAD) -INFINITESIMAL ANGLE.
 C PSI GYRO YAW ANGLE IN ISAF (RAD) -INFINITESIMAL ANGLE.
 C THETAD TIME DERIVATIVE OF THETA (RAD/SEC).
 C PSID TIME DERIVATIVE OF PSI (RAD/SEC).
 C GJ1,GJ2 GYRO PITCH AND YAW AXIS MOMENTS OF INERTIA (SLUG*FT**2).
 C F FRICTIONAL RESTRAINT DUE TO PITCH/YAW GIMBAL-BEARING
 C FRICTION. (SLUG*FT**2/SEC).
 C H GYRO ROTOR ANGULAR MOMENTUM (SLUG*FT**2/SEC)

C PSIDCM COMMANDED GYRO YAW SLEW RATE (RAD/SEC).
 C THTDCM COMMANDED GYRO PITCH SLEW RATE (RAD/SEC).
 C GIMBAL GIMBAL LIMIT ANGLE (RAD).
 C THTDMX MAXIMUM COMMANDED GYRO PITCH RATE (IMPLIED TORQUER
 C LIMIT). (RAD/SEC)
 C PSIDMX MAXIMUM COMMANDED GYRO YAW RATE. (RAD/SEC).
 C GERROR MATRIX OF GYRO ERROR SOURCES (VARIOUS UNITS). SEE BELOW.
 C ROTEG GYRO EULER ANGLE ROTATION MATRIX (EARTH TO GYRO).
 C ROTC ROTEG AT BEGINNING OF TIME STEP.
 C GP COMPONENTS OF A UNIT VECTOR COINCIDING WITH THE GYRO
 C SPIN AXIS AND RESOLVED IN PROJECTILE BODY AXES.
 C (USED INTERNALLY FOR CALCULATIONS IN THIS ROUTINE ONLY.)
 C GIMBY PITCH GYRO GIMBAL ANGLE (ANGLE GYRO MAKES WITH RESPECT
 C TO PROJECTILE BODY). (RAD)
 C GIMBZ YAW GYRO GIMBAL ANGLE (RAD).
 C IGLE GIMBAL LIMITS EXCEEDED INDICATOR (1 = LIMITS EXCEEDED,
 C 0 = NOT EXCEEDED).
 C ISAE SMALL ANGLE EXCEEDED INDICATOR (1 = LIMITS EXCEEDED,
 C 0 = NOT EXCEEDED).
 C CURRENT VALUE OF .01 ASSURES AT LEAST 4 DIGITS.
 C
 C GJ0 GYRO SPIN-AXIS MOMENT OF INERTIAL (SLUG*FT**2).
 C GSPIN GYRO SPIN RATE (CYCLES/SEC).
 C GJ0 AND GSPIN USED ONLY TO CALCULATE H.
 C
 C VARIABLES FROM THE SIXDOF PROGRAM (SEE COMMON/SIXDOF/..)
 C ROTEB PROJECTILE EULER ROTATION MATRIX (EARTH TO BODY).

```

C      VD,WD      DERIVATIVES OF V AND U, PROJECTILE VELOCITY COMPONENTS
C                  IN BODY AXES (FT/SEC) .
C      G          ACCELERATION OF GRAVITY (FT/SEC**2) .
C
C      INITIALIZE INDICATORS
C      IGLE = 0
C      ISAE = 0
C      IF ARGUMENT < OR = 0, THEN JUST INITIALIZE ROTC.
C      IF (TDELTA.LE.0.) GO TO 10
C      TT1 = PSIDCM
C      TT2 = THTDCM
C
C      SLEW RATE LIMIT (IMPLIED TORQUER LIMIT) .
C      IF (ABS(TT1).GT.THTDMX) TT1=SIGN(THTDMX,TT1)
C      IF (ABS(TT2).GT.PSIDMX) TT2=SIGN(PSIDMX,TT2)
C
C
C      GYRO ERROR SOURCES - STATIC DRIFT (RAD/SEC) .
C      TT1  = TT1 + GERROR(1,1)
C      TT2  = TT2 + GERROR(1,2)
C
C      IN-PLANE AND CROSS-PLANE AUTO ERECTION (1/SEC) .
C      TT1  = TT1 + GERROR(2,1)*(-GIMBY) + GERROR(3,1)*(-GIMBZ)
C      TT2  = TT2 + GERROR(2,2)*(-GIMBZ) - GERROR(3,2)*(-GIMBY)
C
C      G-SENSITIVE DRIFT (RAD/SEC) .
C      IF (ABS(WD).GT.G) TT1 = TT1 + (WD/G - SIGN(1.,WD))*GERROR(4,1)
C      IF (ABS(VD).GT.G) TT2 = TT2 + (VD/G - SIGN(1.,VD))*GERROR(4,2)
C
C      TORQUER CROSS-COUPLING. DO NOT INCLUDE CONTRIBUTIONS ALREADY
C      ACCOUNTED FOR BY THE EXPRESSIONS X(8) AND X(9) (DIMENTIONLESS) .
C      TT1 = TT1 + PSIDCM*GERROR(5,1)

```



```

      TT2 = TT2 - THTDCM*GERROR(5,2)

C      COULOMB FRICTION (RAD/SEC) .

      TT1 = TT1 - SIGN(1.,PSID)*GERROR(6,1)
      TT2 = TT2 - SIGN(1.,THTD)*GERROR(6,2)

C      APPLIED TORQUES.

      H = 2.*PI*GSPIN*GJ0

      T1 = H*TT1
      T2 = -H*TT2

      ALPHA = (F/2.)*(1./GJ1 + 1./GJ2)
      BETA = SQRT( (F*F+H*H)/(GJ1*GJ2) - ALPHA*ALPHA )
      X(1) = F*F+H*H
      X(2) = BETA*GJ1
      X(3) = ALPHA*GJ1-F
      X(4) = ALPHA**2 + BETA**2
      X(5) = SIN(BETA*TDELTA)
      X(6) = COS(BETA*TDELTA)
      X(7) = EXP(-ALPHA*TDELTA)
      X(8) = (F*T1-H*T2)/X(1)
      X(9) = (H*T1+F*T2)/X(1)
      GK2 = THETAD-X(8)
      GK4 = PSID-X(9)
      GK1 = (X(3)*GK2-H*GK4)/X(2)
      GK3 = (X(3)*GK1+X(2)*GK2)/H
      X(10) = (GK1*BETA+GK2*ALPHA)/X(4)
      X(11) = (GK1*ALPHA-GK2*BETA)/X(4)
      X(12) = (GK3*BETA+GK4*ALPHA)/X(4)
      X(13) = (GK3*ALPHA-GK4*BETA)/X(4)

```

```

      THETA = THETA+X(8)*TDELTA+X(10)-X(7)*(X(11)*X(5)+X(10)*X(6))
      PSI   = PSI+X(9)*TDELTA+X(12)-X(7)*(X(13)*X(5)+X(12)*X(6))
      THETAD = X(8)+X(7)*(GK1*X(5)+GK2*X(6))
      PSID   = X(9)+X(7)*(GK3*X(5)+GK4*X(6))

C
C   TEST IF SMALL ANGLE APPROXIMATION IS VIOLATED.
C   THIS TEST (.01) ASSURES AT LEAST 4 DIGITS OF ACCURACY.
      AMNUTY = (GK1*GK1 + GK2*GK2)/X(4)
      AMNUTZ = (GK3*GK3 + GK4*GK4)/X(4)
      AMNUTY = SQRT(AMNUTY)
      AMNUTZ = SQRT(AMNUTZ)
      IF (AMNUTY.GT. .01 .OR. AMNUTZ.GT. .01) ISAE=1
      IF (ABS(THETA).GT. .01 .OR. ABS(PSI).GT. .01) ISAE=1

C
      ROT(1,1) = 1.
      ROT(1,2) = PSI
      ROT(1,3) = -THETA
      ROT(2,1) = -PSI
      ROT(2,2) = 1.
      ROT(2,3) = 0.
      ROT(3,1) = THETA
      ROT(3,2) = 0.
      ROT(3,3) = 1.

C   ROT IS THE INFINITESIMAL EULER ANGLE ROTATION GOING FROM 'ISAF'
C   AXES TO CURRENT GYRO AXES AT END OF INTEGRATION.
      DO 2 J=1,3
      DO 2 K=1,3

```

```

    ROTEG (J,K)=0.

    DO 2 L=1,3

    ROTEG (J,K)=ROTEG (J,K)+ROT (J,L)*ROTC (L,K)

2  CONTINUE

    DO 3 J=1,3

    GP (J)=0.

    DO 3 K=1,3

3  GP (J)=ROTEB (J,K)*ROTEG (1,K)+GP (J)

    TEMP=SQRT (GP (1)*GP (1)+GP (2)*GP (2))

    GIMBY=ATAN2 (+GP (3),TEMP)

    GIMBZ=ATAN2 (-GP (2),GP (1))

C

C  FOLLOWING IS APPROPRIATE TEST IF TOTAL GIMBAL ANGLE IS THE

C  LIMITING FACTOR.

C

    IF (GP (1).GT.COS (GIMBAL)) GOTO 10

C

C  IN SOME DESIGNS, THE INDIVIDUAL GIMBAL ANGLES MAY BE

C  THE LIMITING FACTOR AND SHOULD BE TESTED SEPARATELY.

C

C  IF (GIMBY.GT.COS (GIMBAL) .AND. GIMBZ.GT.COS (GIMBAL)) GOTO 10

C

    WRITE (6,1000)

    IGLE=1

    RETURN

C  RENORMALIZING MATRIX PERIODICALLY TO RESTORE UNITARITY.

10 IF (NCT.LT.0.OR.NCT.GT.20) NCT=0

```

```

      GIMBY=0.0
      GIMBZ=0.0
      NCT=NCT+1
      IF (NCT.LT.20) GO TO 13
      REN = 0.0
      DO 11 I=1,3
      DO 11 K=1,3
11  REN = REN + ROTEG(I,K)*ROTEG(I,K)
      REN = SQRT(REN)/3.
      DO 12 I=1,3
      DO 12 J=1,3
12  ROTEG(I,J)=ROTEG(I,J)/REN
13  DO 14 J=1,3
      DO 14 K=1,3
14  ROTC(J,K)=ROTEG(J,K)
C
C   THE FOLLOWING WILL CALCULATE THE GYRO ROTATION MATRIX
C   THAT ROTATES VECTORS FROM THE PROJECTILE BODY AXES TO
C   THE GYRO ACES. IF NEEDED, REMOVE 'C' AND INSERT 'ROTBG'
C   INTO COMMON BLOCK 'GYRO'.
C
C   DO 15 I=1,3
C   DO 15 J=1,3
C   ROTBG(I,J)=0.0
C   DO 15 K=1,3
C 15  ROTBG(I,J)=ROTBG(I,J,+ROTEB(I,K)*ROTEG(J,K)
      RETURN

```

1000 FORMAT(//,37H *** GYRO GIMBAL LIMITS EXCEEDED. ***,//)

END

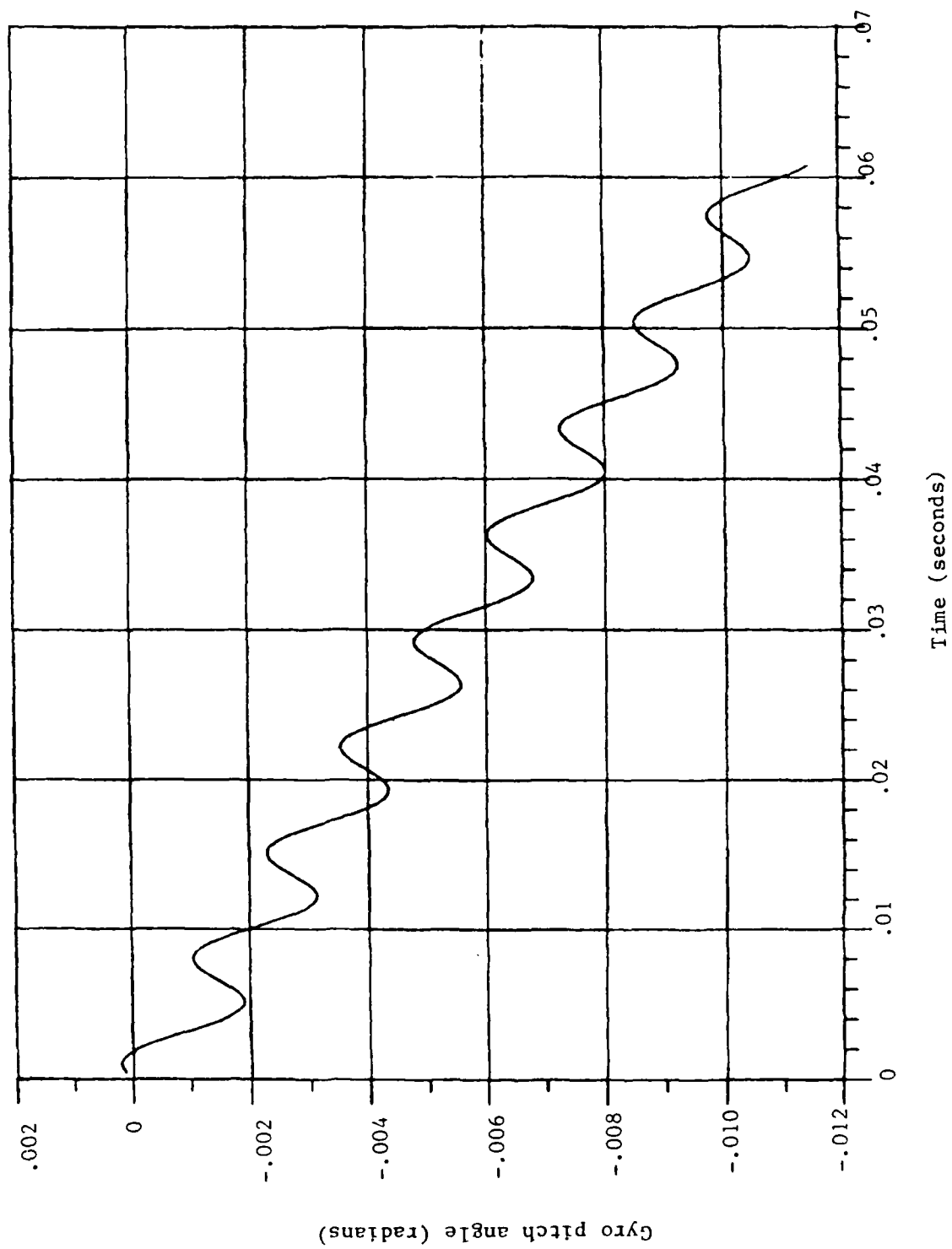


Figure A-1. Gyro pitch gimbal angle

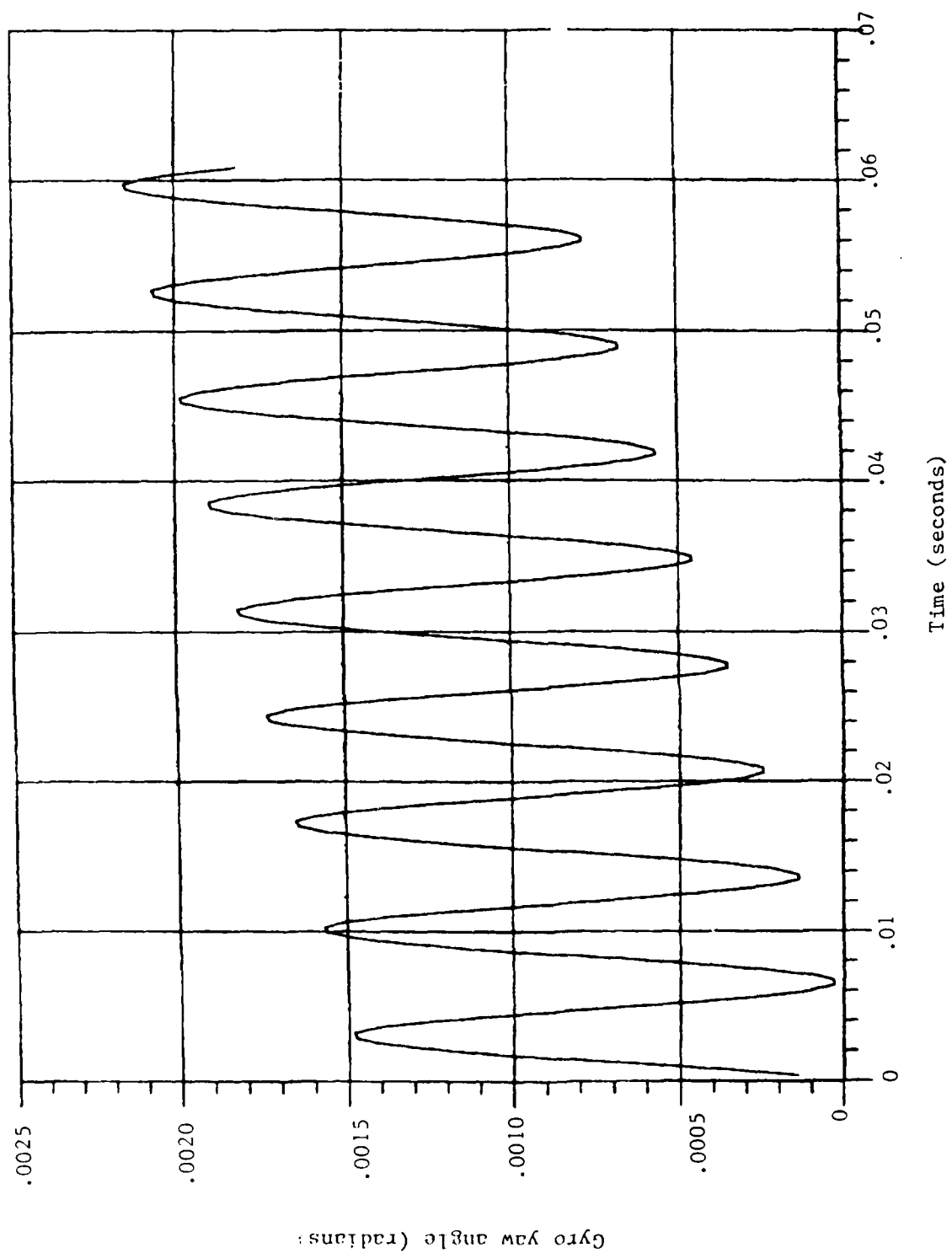


Figure A-2. Gyro yaw gimbal angle

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