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A BREAKDOWN SURFACE MODEL FOR THERMAL BACKSCATTERING
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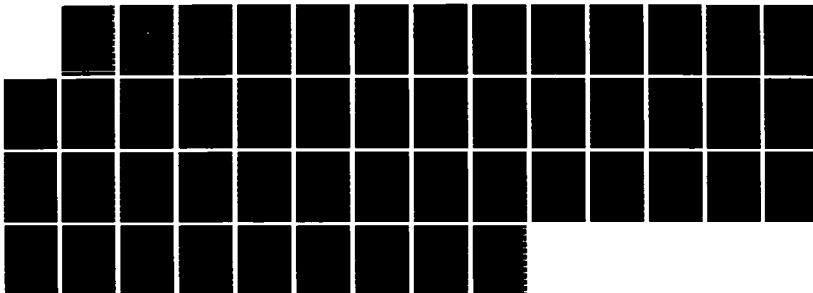
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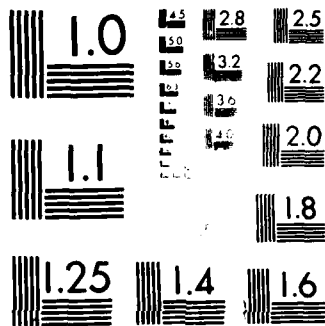
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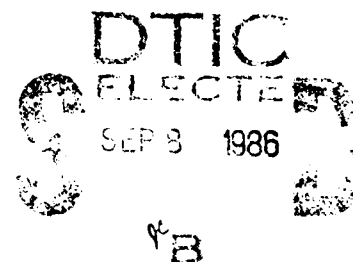
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NAVAL POSTGRADUATE SCHOOL

Monterey, California



CONTRACTOR REPORT

A BREAKDOWN SURFACE MODEL FOR THERMAL
BACKSCATTERING FROM THE EXHAUST PLUME OF A
SPACE-BASED HF LASER

Joseph Falcovitz

June 1986

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The work reported herein was performed for the Naval Postgraduate School by Dr. Joseph Falcovitz under Contract N62271-84-M-3345. The work presented in this report is in support of "Rarefied Gas Dynamics of Laser Exhaust Plume" sponsored by the Strategic Defense Initiative Office/Directed Energy Office. This is the final report for that contract. The work provides information concerning backscattering to spacecraft from a multispecies laser exhaust plume. The project at the Naval Postgraduate School is under the cognizance of Distinguished Professor A. E. Fuhs who is principal investigator.

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ABSTRACT

The purpose of this report is to present a breakdown surface model for evaluating thermal backscattering flux from the supersonic exhaust plume of a space-based HF laser. The plume is of ring symmetry. It consists of a gaseous mixture of H, HF, H₂, DF and He. Fluxes of these species are considered separately. The model is carefully analyzed and is shown to overestimate the flux. Actual flux levels of the heavy corrosive molecules (HF, DF) have been found to be exceedingly low. It is concluded that the contribution of thermal backscattering to contaminating flux of HF and DF can be neglected. This work is an extension and modification of the recent thesis work done by S. E. McCarty at the Naval Postgraduate School.

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The ideas leading to this work crystalize through numerous discussions with LCDR Scott E. McCarty and Distinguished Professor Allen E. Fuhs. This Contractor report constitutes in fact an extension and generalization of LCDR McCarty's MSAE Thesis. Their help and cooperation are gratefully acknowledged.

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1. INTRODUCTION

This report is a presentation of one part of a study on the contaminating backflow from the exhaust plume of a large space-based HF laser (Figure 1). The exhaust plume is an underexpanded supersonic ring-jet, designed to stay clear of the spacecraft by maintaining a Prandtl-Meyer turning angle at the nozzle lips of well below 90° . However, it is well known from experience with rocket plumes in space[1,2] that cavity regions (where continuum gasdynamic theory predicts vacuum) are filled with a free-molecular flow. This back flow is largely due to viscous effects, which give rise to a "spill-over" of the boundary layer around the nozzle lip[3]. Assuming the boundary layer can be eliminated (e.g., an expanding step design of the nozzle lip), there are two more mechanisms which lead to backflow: thermal backscattering and scattering by ambient molecules traveling at orbital speeds. Since these effects are a small perturbation to the exhaust flow field, they can be considered independently (the total backflow will be a superposition of contributions due to these two effects). As a first phase of our broader study, we consider solely the contribution of thermal backscattering to backflow from a ring-plume of an HF laser, via a simple model of molecular effusion from a breakdown surface, fashioned after ideas suggested by Noller[4]. Our results indicate that the backflow of the heavier contaminants (HF, DF) due to thermal backscattering is negligible.

Naturally, our study pertains to presumably typical operating conditions of the HF laser. These operating conditions were largely determined from a report on some HF laser tests conducted at TRW in 1971[5] (in particular, Table 5, Test III, of this report). The typical parameters at the nozzle exit

are:

Composition (mole fractions):	$[H] = .091, [HF] = .091,$ $[H_2] = .104, [DF] = .135,$ $[He] = .579$
Specific Heats Ratio:	$\gamma = 1.54$ (assuming ideal gas)
Mach Number	$M_1 = 4.0$
Average Molecular Weight	$W_A = 7.27$ [kg/kg mole] (1.1)
Stagnation Temperature	$T_0 = 2300$ [K]
Stagnation Density	$\rho_0 = 0.0075$ [kg/m ³]
Molecular Diameter, assuming it is uniform for all species (hard- sphere collisions)	$D = 2.5 \times 10^{-10}$ [m]

The exit Mach number can be chosen higher than $M_1 = 4$, but not considerably lower than this value, since $M_1 = 4$ results in a modest clearance angle of 41° between the limiting (vacuum) characteristic of the lip-centered rarefaction fan and the spacecraft. We assume isentropic flow throughout the diffuser^[5], so that upon specifying the composition and flow variables at the diffuser inlet, along with M_1 at the diffuser exit, the exit flow is fully determined. One exception to this definition, however, is the stagnation temperature, which was estimated as $T_0 = 1400$ [K] at the diffuser inlet^[5]. We set $T_0 = 2300$ [K], which corresponds to complete hydrogen recombination, even though the flow in the diffuser is of a nearly frozen composition due to the low rate of hydrogen recombination^[5]. The reason for this choice is that given the uncertainty in determining T_0 , which results from an uncertainty in the degree of hydrogen recombination, it is the most pessimistic choice, resulting in higher thermally backscattered flux.

The model that we propose for evaluating the backscattered flux arriving

at the spacecraft (Figures 1, 2) is based on the effusive breakdown surface concept suggested by Noller[4]. The gradual transition from continuum to collisionless flow, which invariably takes place at the outer fringes of exhaust plumes having a near-vacuum background environment, is replaced by an abrupt change. We assume that the flow regime in each stream tube changes from continuum (with local thermodynamic equilibrium) to collisionless, upon crossing some breakdown surface.

An important simplification is introduced in the case of a large-radius spacecraft (about 2.5[m]), by observing that the temperature along the breakdown surface decreases so sharply with the distance from the nozzle lip, that the segment contributing significantly to thermal backscattering is only about 0.01 to 0.1[m] long. Consequently, the lip-centered rarefaction ring-fan may well be approximated by the standard (planar) Prandtl-Meyer flow field.

The structure of this report is as follows. The breakdown surface and the molecular effusion flux from it are obtained in closed-form expressions in Section 2. Section 3 is devoted to the spatial integration scheme, which is the evaluation of the flux arriving at a certain point on the spacecraft. Results of flux distribution along the spacecraft for the presumed laser operating range are presented and discussed in Section 4, followed by a critical examination of the breakdown surface model. Conclusions are given in Section 5, and Section 6 is a list of references. The code RINGBD, which computes the flux by numerical integration over the breakdown (effusing) surface, is given in Appendix A.

2. BREAKDOWN SURFACE AND EFFUSION FLUX

Our model for the thermally backscattered flux arriving at the surface of the spacecraft is essentially a modification of Noller's concept of a breakdown effusive surface^[4]. We substitute his definition of a breakdown surface by a similar one introduced by Bird^[6, Section 8.3]. We obtain the one-sided effusion flux from the breakdown surface by integrating over velocity space as suggested by Noller^[4], except for the fact that we compute flux rather than density and we also consider the flux of species having molecular weight different from the average. In the following, each one of these steps is described in some detail, beginning with the breakdown surface.

As mentioned in the introduction, the lip-centered rarefaction fan is approximated by a planar Prandtl-Meyer flow field (Figure 2). The standard expressions for this flow field have Mach number (M) as the independent parameter, thus M varies between $M = M_1$ at the exit and $M \rightarrow \infty$ at the limiting (vacuum) characteristic. (Index 1 always refers to exit conditions, i.e., to parameters evaluated at $M = M_1$).

$$\psi(M) = -\zeta(M) + \zeta_1 + \mu_1 + \frac{\pi}{2}$$

$$\zeta(M) = \Gamma^{1/2} \text{ARCTAN}[\Gamma^{-1/2}(M^2-1)^{1/2}] \quad \Gamma = \left(\frac{\gamma+1}{\gamma-1}\right)$$

$$\mu(M) = \text{ARCSIN}(M^{-1}) \quad (2.1)$$

$$\theta(M) = \psi(M) - \mu(M)$$

where ψ is the angle of characteristic lines, and θ is the angle of the velocity vector (or streamline).

Adopting Bird's definition of a breakdown parameter, which was first introduced in conjunction with a spherical source flow[6 , Section 8.3] and later was shown to be meaningful also in a Prandtl-Meyer flow[7], we define the breakdown surface as having a constant value of B , where B is given by:

$$B = \frac{U}{v} \frac{1}{\rho} \left| \frac{d\rho}{dS} \right| \quad (2.2)$$

Here ρ , U , v , S are local flow density, speed, collision frequency, and coordinate along streamlines (thus restricting this definition of B to stationary flows). From the geometrical relationships in a Prandtl-Meyer fan (Figure 2) and from (2.1) we get:

$$\begin{aligned} \frac{d\rho}{dS} &= - (1/R) \left(\frac{d\rho}{d\psi} \right) \sin \mu = - \frac{2}{\gamma+1} [M^{-1}(M^2 - 1)^{1/2}] (\rho/R) \\ \rho(M) &= \rho_0 \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{1}{\gamma-1}} \end{aligned} \quad (2.3)$$

Using the expression for collision frequency[6]:

$$v_0 = 4(\pi/\gamma)^{1/2} (N_0 D^2 C_0) \quad (2.4)$$

where N_0, C_0, D are stagnation number density, stagnation sound speed, molecular diameter, and using $U = MC$ in conjunction with (2.2) and (2.3), we get:

$$R_B(M) = (BN_0 D^2)^{-1} \frac{(\gamma/\pi)^{1/2}}{2(\gamma+1)} (M^2-1)^{1/2} \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{\gamma-1}} \quad (2.5)$$

This expression is almost identical to that of Bird[7], the main difference being in assuming a constant collision diameter (hard spheres), which we

believe to be commensurate with the overall crudeness of the model. The breakdown surface as defined by equations 2.5 and 2.1, starts at point $[R_B(M_1), \psi(M_1)]$ on the exit characteristic $M = M_1$, which we refer to as the initial point (see Figure 4). However, a breakdown in continuum flow also takes place on the segment of the exit characteristic between the corner and the initial point, since the value of the breakdown parameter there (Equation 2.2) is clearly larger than the value of B used in defining the breakdown surface (Equation 2.5). Hence, the breakdown surface defined by (2.5) should be supplemented by that segment. We refer to the combined surface as the augmented breakdown surface. The segment on the exit characteristic is referred to as the supplementary breakdown surface.

The one-sided directed effusion flux is defined as the number flux of molecules per unit area of an area element normal to the flux direction, per unit solid angle about the flux direction. It is obtained as a function of local Mach number and the angle κ between the flux direction and the local velocity vector, by repeating Noller's velocity integration scheme [4, EQ. (6)], with an added factor of molecular speed in order to obtain flux (rather than density as in Noller's work). The resulting expression for species i is readily obtained by using standard definite integrals:

$$\begin{aligned}
 F_1(M) = & h_1 N_0 C_0 (W_A/W_1)^{1/2} \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \\
 & \left[(2\gamma\pi^3)^{-1/2} \left(1 + (1/2)\gamma\tilde{M}^2 \cos^2 \kappa\right) \text{EXP} \left(-\gamma\tilde{M}^2/2\right) + \right. \\
 & (2\pi)^{-1} \tilde{M} \left(3/2 + (1/2)\gamma\tilde{M}^2 \cos^2 \kappa\right) \cos \kappa \text{EXP} \left(- (1/2)\gamma\tilde{M}^2 \sin^2 \kappa\right) \\
 & \left. \text{ERFC} \left[- (\gamma/2)^{1/2} \tilde{M} \cos \kappa\right] \right]
 \end{aligned} \tag{2.6}$$

$$\text{ERFC}(v) = 2 \pi^{-1/2} \int_v^{\infty} \text{EXP}(-x^2) dx \quad (\text{complementary error function})$$

$$\tilde{M} = (W_1/W_A)^{1/2} M$$

h_i - Mole fraction of species i .

(2.6 Continued)

W_i - Molecular weight of species i .

The dependence of $F_i(M)$ on the flux angle κ is so sensitive that for some Mach number around $M \approx 10$, the backflow (in the typical operating range) is virtually negligible. In the following section we describe how the flux $F_i(M)$ is integrated over the augmented breakdown surface, yielding the backscattered flux arriving at the surface of the spacecraft.

3. FLUX INTEGRATION

The effusion flux $F_1(M)$ given by (2.6) above, is defined in such a way that the number of molecules effusing from an area element ΔA_B of the breakdown surface and arriving at an area element ΔA_S on the spacecraft (per second), is given by:

$$F_1(M) (\Delta A_B \cos \alpha_B) (\Delta A_S \cos \alpha_S) L_{BS}^{-2} \text{ [molecules per second]} \quad (3.1)$$

where α_B , α_S are the angles between the line-of-sight $\vec{L}_{BS}$ (Figure 3) and the normals to the breakdown surface and the spacecraft surface respectively. L_{BS} is the distance between the elements ΔA_B and ΔA_S . Dividing equation (3.1) by ΔA_S and integrating over the breakdown surface, the flux per unit area of the spacecraft is given by:

$$Q_1 = \int F_1(M) \cos \alpha_B \cos \alpha_S L_{BS}^{-2} dA_B \text{ [Molecules per second per m}^2\text{]} \quad (3.2)$$

The integration scheme for Q_1 over the breakdown surface is expressed in terms of the set of polar coordinates R, ψ, ϕ (Figure 3). For a point (R, ψ, ϕ) on the breakdown surface, using Cartesian coordinates (X, Y, Z) and the angle ω between X-axis and the line-of-sight $\vec{L}_{BS}$ we obtain the following geometrical relationships:

$$X = R \cos \psi ; \quad Y = (A_0 + R \sin \psi) \cos \phi ; \quad Z = (A_0 + R \sin \psi) \sin \phi$$

$$\cos \phi_{\text{MAX}}(M) = \left(\frac{A_0}{A_0 + R(M) \sin \psi} \right) \quad (3.3)$$

$$\vec{U} = U(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$

$$\vec{L}_{BS} / |\vec{L}_{BS}| = (\cos \omega, \sin \omega \cos \beta, \sin \omega \sin \beta)$$

$$\tan \beta = \frac{Z}{(Y - A_0)}$$

The cosines $\cos \kappa$, $\cos \alpha_B$, $\cos \alpha_S$ in (3.2) are expressed as scalar products of $[\vec{L}_{BS}/|\vec{L}_{BS}|]$ and unit vectors along the local velocity vector $\vec{U}$, along the local normal to the breakdown surface and along the local normal to the spacecraft surface, correspondingly.

The integration is performed numerically in two phases, the first being the integration along the supplementary breakdown surface (Figure 4). For this first phase, the straight line segment which constitutes the supplementary breakdown surface is divided into several intervals of length ΔR (typically 10 intervals). Each interval generates a half-strip by rotating it from $\phi = 0$ through $\phi = \phi_{\max}(M_1)$. This strip is in turn subdivided into several sub-intervals of $\Delta \phi$ each (typically 10 intervals). The total flux arriving at X_S is obtained by summing contributions from each sub-interval (two-dimensional integration). When the integration along the supplementary breakdown surface is concluded, it is continued into the breakdown surface, where ΔR intervals are replaced by breakdown surface intervals that correspond to a fixed Mach number increment ΔM (typically $\Delta M = 0.1$). The integration proceeds along the breakdown surface (2.5) until the contribution of the last ΔM strip is negligibly small. The computation time is modest (about 1 second CPU per X_S point, on IBM 3033 mainframe computer). The computations were carried out by a code RINGBD written specifically for this purpose. Further details of the scheme and programming can be obtained by reading this code which is given in Appendix A.

4. RESULTS AND DISCUSSION

4.1 Presentation of Results

The molecular flux backscattered to the spacecraft from the surface of continuum flow breakdown in the lip-centered rarefaction fan, has been computed for all five species H, HF, H₂, DF, He. The results are depicted in Figures 5 to 9 respectively. For each species two more cases were computed in addition to the nominal case (1.1), where the stagnation density ρ_0 was replaced by $\rho_0/10$ and by ρ_0*10 (see Figures 5 to 9). This has been done in order to demonstrate the effect of variations in exit flow conditions on the flux. The particular choice of ρ_0 was motivated by the fact that the effects of changing ρ_0 are not obvious. The effects of changing the exit Mach number M_1 or the stagnation temperature T_0 are rather obvious (a higher flux would result from either a decrease in M_1 or an increase in T_0). It turns out that for points lying not too near the nozzle lip ($X_s > 0.1$ m), the lower density flow generates a higher backscattered flux!

In addition to varying ρ_0 , we also varied the breakdown parameter B, obtaining a surprising result. The computation was performed for a particular species (HF), and the results obtained upon replacing $B=0.05$ (nominal value) by $B/2$ and by $B*2$ are brought in Figure 10.

It turns out that the $B/2$ case has the higher flux. This is somewhat surprising, since a lower value of B in a centered rarefaction fan (equation 2.5) means that the breakdown of continuum flow takes place in a region further out from the corner. In a source flow (e.g., a spherical source), that implies lower density and temperature, which would give rise to lower thermally backscattered flux.

An explanation to these seemingly counterintuitive results, along with some deeper insight into the breakdown surface model as it is applied to a centered rarefaction flow, can be obtained by taking a close look at the flow field and the breakdown surface in the vicinity of the corner. We take up this matter in the following sections.

We conclude the presentation of results, by comparing the flux (in the nominal case) of the five species with each other (Figure 11). This figure underlines the fact that the flux of light species (H, H₂, He) is many orders of magnitude (typically 10¹⁵) times that of heavy species (HF, DF). Indeed, these results demonstrate a well known effect: When an expanding gaseous mixture of light and heavy molecules experiences a breakdown of continuum flow, a separation of species takes place (see e.g., the work of Cattolica et. al. [8]).

4.2 The Breakdown Surface and Streamlines

Consider the parametric description $R_B(M)$ for the breakdown surface (Equation 2.5). Normalizing R relative to the exit mean free path λ_1 , we get:

$$\begin{aligned}
 R_B(M) &= R_B(M_1) \left[(M^2 - 1) / (M_1^2 - 1) \right]^{1/2} \left[\left(1 + \frac{\gamma-1}{2} M^2 \right) / \left(1 + \frac{\gamma-1}{2} M_1^2 \right) \right]^{\frac{1}{\gamma-1}} \\
 R_B(M_1) / \lambda_1 &= \left[(\gamma\pi/2)^{1/2} / (\gamma+1)B \right] (M_1^2 - 1)^{1/2} \\
 \lambda_1 &= \left(2^{1/2} \pi D^2 N_0 \right)^{-1} \left[1 + \frac{\gamma-1}{2} M_1^2 \right]^{\frac{1}{\gamma-1}}
 \end{aligned}
 \tag{4.1}$$

The normalized surface $R_B(M)/\lambda_1$ is thus independent of stagnation density, depending only on γ , M_1 , B .

Let us now derive a parametric equation $R_S(M)$ for a streamline that enters the fan at point $R_S(M_1)$ on the exit characteristic. The following geometrical relationship is readily obtained by considering two characteristic lines ψ and $\psi + \Delta\psi$ and a streamline inclined at the Mach angle μ to them:

$$\frac{dR_S(\psi)}{d\psi} = - R_S(\psi) (\tan\mu)^{-1} \quad (4.2)$$

Using the standard Prandtl-Meyer functions (2.1), we get the following differential equation for $R_S(M)$:

$$\frac{1}{R_S(M)} \frac{dR_S(M)}{dM} = \left(\frac{\gamma+1}{2}\right) M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \quad (4.3)$$

This equation is readily integrated, giving:

$$R_S(M) = R_S(M_1) \left[\left(1 + \frac{\gamma-1}{2} M^2\right) / \left(1 + \frac{\gamma-1}{2} M_1^2\right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (4.4)$$

As pointed out by Bird^[7], there is a particular streamline $R_{Sa}(M)$ which asymptotically approaches the breakdown surface for large M , since the ratio $R_S(M)/R_B(M)$ tends to a constant (not zero) when $M \rightarrow \infty$. (Strictly speaking, this holds only for hard-sphere molecules, i.e., only when $\omega = 0.5$ in^[7]). The limit is:

$$\lim_{M \rightarrow \infty} \frac{R_S(M)}{R_B(M)} = \frac{R_S(M_1)}{R_B(M_1)} \left[\frac{\frac{M_1^2}{2} - 1}{\frac{M_1^2}{2} + \frac{2}{\gamma-1}} \right]^{1/2} \quad (4.5)$$

For the limiting streamline $R_{Sa}(M)$, the ratio $R_{Sa}(M)/R_B(M)$ should tend to 1. This determines the point $R_{Sa}(M_1)$ at which the limiting streamline enters the fan, as well as the entire line $R_{Sa}(M)$:

$$R_{Sa}(M_1) = R_B(M_1) \left[\frac{\frac{M_1^2}{2} + \frac{2}{\gamma-1}}{\frac{M_1^2}{2} - 1} \right]^{1/2} \quad (4.6)$$

Clearly, $R_{sa}(M)$ is larger than $R_B(M)$ for any $M > M_1$, so that no streamline beyond $R_{sa}(M)$ can cross the breakdown surface. This pattern is shown in Figure 4, where $R_{sa}(M)$ is denoted "streamline 2", and "streamline 1" is the streamline $R_s(M_1) = R_B(M_1)$.

All this leads to the following observation regarding the continuum breakdown of the flow in a centered rarefaction fan^[7]. Referring to Figure 4, the fluid entering the fan through the supplementary breakdown surface (i.e., through the exit characteristic between the corner and the initial point of streamline 1), experiences breakdown immediately upon crossing this surface. Every streamline between streamline 1 and streamline 2 crosses the breakdown surface at some Mach number $M > M_1$, and at that point the continuum flow breaks down. All fluid entering the fan beyond streamline 2 will never pass through the breakdown surface, and hence will maintain a continuum flow regime all the way to infinity. Of course, that is only true for planar centered rarefaction fans. When the exhaust flow emerges from a nozzle of finite width, and especially when the exhaust jet has a ring symmetry (as in our case), the breakdown surface gradually curves in a balloonlike shape towards the opposite nozzle lip, forming the familiar plume pattern (Figure 1).

4.3 Analysis and Discussion of Results

The foregoing analysis is now used to explain the variation in back-scattered flux due to a change in exhaust flow conditions at the nozzle exit. Specifically, we consider a tenfold decrease in stagnation density (i.e., the case $\rho_0/10$), and hence a tenfold increase in the exit mean free path λ_1 .

The effusion flux from the breakdown surface is proportional to the local density, so one would expect to observe a decrease in flux, rather than

an increase (see Figures 5 to 9, for $X_s > 0.1$ m). Other factors causing increased flux, must then be larger than 10 so that they more than offset the 1/10 factor in density. It turns out that these effects are mainly geometrical, in that a tenfold increase in λ_1 causes the domain of integration on the breakdown surface to increase more than tenfold. In the (X,Y) plane there is a tenfold "blowup" of the breakdown surface, due to the self-similar structure of the Prandtl-Meyer flow field. As a result of this "blowup" in (X,Y), the angular integration range $\phi_{\max}$ also increases, albeit not linearly (Equation 3.3). Another geometrical effect is an increase in the flux incidence cosine factor $\cos \alpha_s$ (see Equation 3.2), which for points X_s sufficiently far from the nozzle lip, increases roughly tenfold (while the other cosine factor $\cos \alpha_B$ is almost constant). All this provides a qualitative explanation for the observed increase in flux at far points ($X_s > 0.1$ m).

As for the near range ($X_s < 0.1$ m), another effect becomes increasingly significant as X_s approaches the nozzle lip. The turning angle κ , by which backscattered molecules have to be deflected relative to the flow velocity vector in order to reach point X_s on the spacecraft (Figure 2), increases with the size of the breakdown surface (fixed M and X_s). Since the local effusion flux (Equation 2.6) decreases rather sharply as κ is increased, the net result is a tendency to get a reduced backscattered flux at near points such as $X_s = 0.01$ m (Figures 5 to 9).

We now turn to the effect of changing the value of the breakdown parameter B. From equation 4.1 it is clear that multiplying B by some factor

will have the same "blowup" effect as dividing λ_1 by the same factor. A tenfold decrease in B is thus geometrically equivalent to a tenfold decrease in ρ_0 . However, since the local effusion flux at the breakdown surface is proportional to ρ_0 while it is independent of B , the $B/10$ case will have ten times as much backscattered flux as the $\rho_0/10$ case. In order to illustrate the sensitivity of the flux estimates to an uncertainty in the appropriate value of B , we computed the cases $B/2$ and $B*2$ for one species (HF), and they are presented in Figure 10. The variation in flux relative to the nominal case ($B = 0.05$), is by a factor no larger than about 5. Results for other species were found to exhibit comparable variations.

Does this observation about the dependence of the breakdown surface on B agree with the breakdown surface appropriate to the far field of the exhaust plume? In stationary source flow into vacuum, and when $M \gg 1$, the breakdown parameter varies with radius as $B \sim R^{\delta-1}$ ($\delta = 1$ for cylindrical source, $\delta = 2$ for spherical source). In a ringjet, the stream tubes of the exhaust plume generally diverge at a rate higher than that of stream tubes in a cylindrical source flow, so the effective value of δ in a ringjet is $\delta > 1$. Hence, in this case the far field breakdown surface moves downstream along each stream tube as the value of B increases. This is indeed geometrically compatible with the fact that near the corner of the lip-centered rarefaction fan $B \sim R^{-1}$, as shown schematically in Figure 12. The dependence of the breakdown surface on B near the corner and in the far field, thus assures that complete breakdown surfaces corresponding to different values of B , do not intersect (Figure 12).

In the foregoing discussion it was pointed out that variations in flux caused by changes in parameters such as ρ_0 and B , were directly related

to the self-similar structure of the Prandtl-Meyer flow field. It has been further shown that these variations are well-understood within the framework of the breakdown surface model and that they are not excessively large. Are we to conclude that the thermally backscattered flux estimates of the present model are also physically plausible and reliable? In the following section we take up this matter, arriving at some interesting conclusions about this model and its range of validity.

4.4 Critical Examination of the Model

Consider the centered rarefaction flow field of a compressible fluid negotiating an expansive corner at supersonic speed (Prandtl-Meyer flow). The streamlines of this flow field have an orderly "layered" structure, with each streamline curving around the corner, starting at its point of entrance into the fan (see Figure 4).

The present model is based on the stipulation that there is a point of continuum flow breakdown on each streamline, provided this streamline is not beyond a certain limiting streamline. Consider a sample molecule effusing from this breakdown point toward the spacecraft. It advances at constant speed along a straight line trajectory, traversing all inner streamlines. Since the flow velocity vector points away from the spacecraft, and since the flow is highly supersonic so that the velocity of most individual molecules does not differ much from the flow velocity (i.e., it is a "cold" flow), any collision of the sample molecule with a mainflow molecule will most probably divert the sample molecule away from the spacecraft. What is the probability that a sample molecule would traverse this cross flow collisionlessly? This probability is simply $\exp(-n)$, where n is the expected number of collisions along the straight-line trajectory from the point of breakdown to the spacecraft. In the typical operating conditions assumed here, we estimated n to be roughly about 10. Since this no-collision probability factor is ignored in the formulation of the present model, the backscattered flux may be exaggerated by a factor of $\exp(10)$ or about 10^4 . We conclude that in all likelihood, the prediction of the breakdown surface model for thermally backscattered flux from a centered rarefaction flow, is substantially overestimated.

Can anything be done to improve the present model? One may be inclined to suggest at this point that the obvious remedy is to incorporate the no-collision probability factor into the model. Rather, we prefer to retain the breakdown surface model in its present form as a simple means of obtaining an overestimate to the thermally backscattered flux from a centered rarefaction flow. An improved model can be constructed by considering thermal backscattering from the entire flow field (tempered by the probability of no-collision), without resorting to the physically untenable notion of an abrupt transition from continuum flow to free molecular flow.

5. CONCLUSIONS

Some surprising similarity laws of the breakdown surface model were observed. It has been shown that they were a direct result of the self similar structure of the Prandtl-Meyer flow field to which the model was applied. Specifically, it was found (and shown plausible) that reduced values of either the exhaust stagnation density ρ_0 , or the breakdown parameter B, caused higher backscattered flux.

The breakdown surface model for thermally backscattered flux from a centered rarefaction fan, has been shown to overestimate the flux arriving at the spacecraft. It is suggested that an improved model be constructed by considering thermal backscattering from the entire flow field, along with the probability factor for a side-scattered molecule traversing the main flow collisionlessly.

The molecular flux of corrosive species (HF, DF) arriving at the spacecraft (Figures 6 and 8) is no larger than about 10^7 ($\text{sec}^{-1} \text{ m}^{-2}$), which is negligible since it corresponds to about 10^{-5} molecular monolayers per year. This conclusion is reliable since even this flux level is an overestimate.

The maximum thermally backscattered flux of light species (H, H₂, He) is in the range of 10^{20} to 10^{22} ($\text{sec}^{-1} \text{ m}^{-2}$) (see Figures 5, 7, 9). Thus, we conclude that while thermal backscattering would contribute significantly to the flux of light molecules arriving at the spacecraft, it is utterly negligible as far as heavy molecules are concerned.

5. REFERENCES

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APPENDIX A. The Computer Code RINGBD

We present a printout of the code RINGBD along with the results of the nominal case (printout of actual run). This is preceded by a brief description of the subroutines and a summary of major variables with their code and report notations.

A.1 Description of Subroutines

- | | |
|--------------|--|
| MAIN PROGRAM | - Computes flux integration by summation of segment contributions (centered). Printing of results. |
| INIDAT | - Definition of all data (no input file). Preparatory evaluation of parameters. Printing of data. |
| FLUX | - Evaluates flux emitted from a single point on breakdown surface (mean segment values) to a point on spacecraft (XS). |
| BREAKR | - Computes point on breakdown surface for given Mach number. |
| BREAKM | - Computes point on breakdown surface for mean Mach number of a segment. |

A.2 Code Versus Report Notation

XC(I)	- [AB]	- Mole fraction of species AB. (I=1,2,3,4,5 corresponds to H, HF, H ₂ , DF, He).
WC(I)	- W _i	- Molecular weight of species i.
WAV	- W _A	- Average molecular weight
TO	- T _O	- Stagnation temperature
RHOO	- ρ _O	- Stagnation density
G	- γ	- Specific heat ratio
EM1	- M ₁	- Exit Mach number
LAMDAL	- λ ₁	- Exit mean free path
AO	- A _O	- Spacecraft radius
R	- R	- Distance from corner (X=0, Y ² +Z ² = A _O ²).
DIST	- L _{BS}	- Distance between emitting point on breakdown surface and receiving point (XS) on spacecraft.
XS	- X _S	- Point on spacecraft (X=X _S , Y=A _O , Z=0).
PSI	- ψ	- Characteristic angle
AMU	- μ	- Mach angle
TETA	- θ	- Velocity vector angle
PHI	- φ	- Rotation angle for flux integration
W	- ω	- Angle between x-axis and line-of-sight $\vec{L}_{BS}$
BETA	- β	- Angle between Y-axis and projection of $\vec{L}_{BS}$ on (Y,Z) plane.
DMO	- ΔM	- Mach number increment for flux integration
EM	- M	- Mach number
PBIRD	- B	- Breakdown parameter

A.3 Code Listing (Run of Nominal Case):

```

$JOB          RINGBD,NOXREF                                RIN00010
1  IMPLICIT REAL*8(A-H,O-Z,$)                             RIN00020
2  COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, RIN00030
   1          G16,G17,G18,G19,G20                         RIN00040
3  COMMON /PAR/CO,ENO,EM1,D,TLIM,ETALIM,CLIM,ELO,QO,TO,     RIN00050
   1          PBIRD,RBIRD,DMO,DEG,OMEGA,XSV(51)           RIN00060
4  COMMON /NPAR/NETA,NC,NT,NEMO,NPHI,NXS,NRO,NSPEC          RIN00070
5  COMMON /GEOM/APF,PAI,PAI2,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1, RIN00080
   1          CPSI1,PSIF,SPSIF,CPSIF,AK,SK,CK,A0,RF,XF,YF,ZF, RIN00090
   2          PHI,SPHI,CPHI,RMIN,RMAX,XS,DIST,            RIN00100
   3          AMU1,ZETA1,XN,YN,ZN,PSIM,SPSIM,CPSIM,R0      RIN00110
6  COMMON /EPSIL/EPSQ,EPSETA,EPST,EPSC,EPSEM              RIN00120
7  COMMON /EXTREM/TEXT,ETAEXT,CEXT,REXT,PSIEXT,EMEXT,BEXT,QEXT RIN00130
8  COMMON /SPEC/WAV,XC(5),WC(5),WCR(5),XNAME(5),QC(5),FLUXC(5) RIN00140
9  DIMENSION DSUM(5)                                       RIN00150
10 PRINT 101                                              RIN00160
11 101 FORMAT('1'/1X,'RINGBD - FLUX INTEGRATION FROM BREAKDOWN', RIN00170
   1          1X,'SURFACE'//)                             RIN00180
C                                                         RIN00190
12 CALL INIDAT                                           RIN00200
C                                                         RIN00210
13 PRINT 110,XNAME                                         RIN00220
14 110 FORMAT(///1X,' NX',' NEM',' XS ',' PHIMAX',' QMAX ', RIN00230
   1          5(4X,A6,1X,'/ LOG',1X))                    RIN00240
15 DO 200 NX=1,NXS                                         RIN00250
16 EM=EM1                                                  RIN00260
17 CALL BREAKR(EM,RF)                                     RIN00270
18 IF(NRO.GT.0) RF=R0                                     RIN00280
19 XF=RF*CPSI1                                             RIN00290
20 YF=RF*SPSI1+A0                                         RIN00300
21 XS=XSV(NX)                                             RIN00310
22 QMAX=0.                                                 RIN00320
23 DO 45 N=1,NSPEC                                         RIN00330
24 FLUXC(N)=0.                                             RIN00340
25 45 CONTINUE                                             RIN00350
26 DO 1 NEM=1,NEMO                                         RIN00360
27 RN=RF                                                   RIN00370
28 XN=XF                                                   RIN00380
29 YN=YF                                                   RIN00390
30 IF(NEM.GT.NRO) GO TO 41                                RIN00400
31 RF=R0+DFLOAT(NEM)*(RMIN-R0)/DFLOAT(NRO)               RIN00410
32 XF=RF*CPSI1                                             RIN00420
33 YF=RF*SPSI1+A0                                         RIN00430
34 RMEAN=(RN+RF)/2.D0                                     RIN00440
35 EMMEAN=EM1                                              RIN00450
36 PSIM=PSI1                                              RIN00460
37 SPSIM=SPSI1                                            RIN00470
38 CPSIM=CPSI1                                            RIN00480
39 GO TO 42                                                RIN00490
40 41 CONTINUE                                             RIN00500
41 EM=EM+DMO                                              RIN00510
42 CALL BREAKR(EM,RF)                                     RIN00520
43 EMMEAN=EM-DMO/2.D0                                     RIN00530
44 CALL BREAKM(EMMEAN,RMEAN)                             RIN00540
45 42 CONTINUE                                             RIN00550
46 ALONG=DSQRT((XF-XN)**2+(YF-YN)**2)                     RIN00560
47 SALFA=(YF-YN)/ALONG                                    RIN00570
48 CALFA=(XF-XN)/ALONG                                    RIN00580
49 PHIMAX=DARCOS(A0/(A0+RMEAN*SPSIM))                     RIN00590
50 DPHI=PHIMAX/NPHI                                       RIN00600

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51		DO 44 N=1, NSPEC	RIN00610
52		DSUM(N)=0.	RIN00620
53	44	CONTINUE	RIN00630
54		DO 2 NP=1, NPHI	RIN00640
55		PHI=(DFLOAT(NP)-0.5D0)*DPHI	RIN00650
56		CALL FLUX(EMMEAN, RMEAN)	RIN00660
57		CROSS1=SW*CBETA	RIN00670
58		CROSS2=(SALFA)*(-CW)+(-CALFA*CPHI)*(-SW*CBETA)+	RIN00680
		1 (-CALFA*SPHI)*(-SW*SBETA)	RIN00690
59		GOREM=CROSS1*CROSS2*DPHI*(A0+RMEAN*SPSIM)*ALONG/DIST**2	RIN00700
60		DO 24 N=1, NSPEC	RIN00710
61		DSUM(N)=DSUM(N)+QC(N)*GOREM	RIN00720
62	24	CONTINUE	RIN00730
63		IF(QMAX.GE.QEXT) GO TO 25	RIN00740
64		QMAX=QEXT	RIN00750
65	25	CONTINUE	RIN00760
	C	PRINT 22, NEM, NP, EMMEAN, RMEAN, PHIMAX*DEG, DARCOS(CW)*DEG,	RIN00770
	C	1 BETA*DEG, PHI*DEG, CROSS1, CROSS2, ALONG, DIST, GOREM,	RIN00780
	C	2 SPSIM, A0, (QC(N), FLUXC(N), N=1, NSPEC)	RIN00790
	C22	FORMAT(/1X, 'NEM, NP, EMMEAN, RMEAN, PHIMAX=', 2I3, 3D13.4/	RIN00800
	C	1 1X, 'W, BETA, PHI=', 3D15.5/	RIN00810
	C	2 1X, 'CROSS1, CROSS2, ALONG, DIST, GOREM=', 5D15.5/	RIN00820
	C	3 1X, 'SPSIM, A0=', 2D15.5/	RIN00830
	C	4 1X, 'QC, FLUXC=', 5(1X, D10.3, 1X, D10.3))	RIN00840
66	2	CONTINUE	RIN00850
67		DO 26 N=1, NSPEC	RIN00860
68		FLUXC(N)=FLUXC(N)+DSUM(N)	RIN00870
69	26	CONTINUE	RIN00880
70		IF(NEM.LE.NR0+2) GO TO 1	RIN00890
71		DO 27 N=1, NSPEC	RIN00900
72		IF((DSUM(N)/FLUXC(N)).GT.EPSEM) GO TO 28	RIN00910
73	27	CONTINUE	RIN00920
74		GO TO 10	RIN00930
75	28	CONTINUE	RIN00940
76	1	CONTINUE	RIN00950
77	10	CONTINUE	RIN00960
78		DO 31 N=1, NSPEC	RIN00970
79		FLUXC(N)=2.D0*XC(N)*FLUXC(N)	RIN00980
80	31	CONTINUE	RIN00990
81		PRINT 11, NX, NEM, XS, PHIMAX*DEG, QMAX,	RIN01000
		1 (FLUXC(N), DABS(DLOG10(FLUXC(N))), N=1, NSPEC)	RIN01010
82	11	FORMAT(/1X, 2I4, F9.4, F7.2, D11.3, 5(1X, D10.3, '/', F5.2))	RIN01020
83	200	CONTINUE	RIN01030
84		PRINT 102	RIN01040
85	102	FORMAT(/1X, 'END RINGBD RUN', ///)	RIN01050
86		STOP	RIN01060
87		END	RIN01070
88		SUBROUTINE INIDAT	RIN01080
89		IMPLICIT REAL*8(A-H, O-Z, \$)	RIN01090
90		REAL*8 LAMDA0, LAMDA1	RIN01100
91		COMMON /GAMA/G, G1, G2, G3, G4, G5, G6, G7, G8, G9, G10, G11, G12, G13, G14, G15,	RIN01110
		1 G16, G17, G18, G19, G20	RIN01120
92		COMMON /PAR/CO, ENO, EM1, D, TLIM, ETALIM, CLIM, EL0, Q0, T0,	RIN01130
		1 PBIRD, RBIRD, DMO, DEG, OMEGA, XSV(51)	RIN01140
93		COMMON /NETA/NC, NT, NEMO, NPHI, NXS, NR0, NSPEC	RIN01150
94		COMMON /GEOM/APF, PAI, PAI2, SW, CW, BETA, SBETA, CBETA, PSI1, SPSI1,	RIN01160
		1 CPSI1, PSIF, SPSIF, CPSIF, AK, SK, CK, A0, RF, XF, YF, ZF,	RIN01170
		2 PHI, SPHI, CPHI, RMIN, RMAX, XS, DIST,	RIN01180
		3 AMU1, ZETA1, XN, YN, ZN, PSIM, SPSIM, CPSIM, R0	RIN01190

95	COMMON /EPSIL/EPSQ,EPSETA,EPST,EPSC,EPSEM	RIN01200
96	COMMON /EXTREM/TEXT,ETAEXT,CEXT,REXT,PSIEXT,EMEXT,BEXT,QEXT	RIN01210
97	COMMON /SPEC/WAV,XC(5),WC(5),WCR(5),XNAME(5),QC(5),FLUXC(5)	RIN01220
98	DATA XC/.091D0,.091D0,.104D0,.135D0,.579D0/	RIN01230
99	DATA WC/1.00D0,20.0D0,2.00D0,21.0D0,4.00D0/	RIN01240
100	DATA XNAME/' H ',' HF ',' H2 ',' DF ',' HE '/	RIN01250
101	PAI=4.*DATAN(.1D 1)	RIN01260
102	AR=8.3143D3	RIN01270
103	AV=6.022D 26	RIN01280
	C OMEGA=0.5 IS FOR HARD SPHERE COLLISIONS,	RIN01290
	C AN AVERAGE RECOMMENDED VALUE IS ABOUT OMEGA=0.75	RIN01300
104	OMEGA=0.5D0	RIN01310
105	NSPEC=5	RIN01320
106	WAV=0.	RIN01330
107	DO 51 N=1,NSPEC	RIN01340
108	WAV=WAV+XC(N)*WC(N)	RIN01350
109	51 CONTINUE	RIN01360
110	DO 52 N=1,NSPEC	RIN01370
111	WCR(N)=DSQRT(WC(N)/WAV)	RIN01380
112	52 CONTINUE	RIN01390
113	A0=2.5D0	RIN01400
114	EM1=4.0D0	RIN01410
115	RH00=0.0075D0	RIN01420
116	T0=2.300D3	RIN01430
117	D=2.50D-10	RIN01440
118	G=1.54D0	RIN01450
119	EN0=RH00*AV/WAV	RIN01460
120	C0=DSQRT(G*AR*T0/WAV)	RIN01470
121	PBIRD=0.05D0*2.D0	RIN01480
	C R0 IS THE RADIUS FOR BEGINNING THE INTEGRATION ALONG THE M=M1	RIN01490
	C CHARACTERISTIC(THE AUGMENTED BREAKDOWN SURFACE).	RIN01500
	C NR0 IS THE NUMBER OF INTEGRATION INTERVALS ON THIS SEGMENT.	RIN01510
	C FOR NO INTEGRATION ALONG M=M1 CHARACTERISTIC, SET NR0=0.	RIN01520
122	R0=0.	RIN01530
123	NR0=10	RIN01540
124	DM0=0.1D0	RIN01550
125	NEM0=20.D0/DM0+NR0	RIN01560
	C TO GET FLUX DUE TO AUGMENTED BREAKDOWN SURFACE SOLELY, ACTIVATE:	RIN01570
	C	RIN01580
126	NPHI=10	RIN01590
127	NXS=13	RIN01600
128	XSI=1.D-2	RIN01610
129	XSF=1.D1	RIN01620
130	XSV(1)=XSI	RIN01630
131	IF(NXS.EQ.1) GO TO 111	RIN01640
132	DXL=(DLOG(XSF)-DLOG(XSI))/(NXS-1.D0)	RIN01650
133	XLI=DLOG(XSI)	RIN01660
134	DO 11 NX=2,NXS	RIN01670
135	XSV(NX)=DEXP(XLI+(NX-1.D0)*DXL)	RIN01680
136	11 CONTINUE	RIN01690
137	111 CONTINUE	RIN01700
138	EPSEM=1.D-5	RIN01710
139	DEG=180.D0/PAI	RIN01720
140	PAI2=PAI/2.D0	RIN01730
141	GAMMA=G	RIN01740
142	G1=(G-1.D0)/2.D0	RIN01750
143	G2=(G+1.D0)/(2.D0*(G-1.D0))	RIN01760
144	G3=G/2.D0	RIN01770
145	G4=(G+1.D0)/(G-1.D0)	RIN01780
146	G5=DSQRT((G+1.D0)/(G-1.D0))	RIN01790

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147      G6=1.D0/(G-1.D0)                                RIN01800
148      G7=2.D0/(G+1.D0)                                RIN01810
149      G8=(5.D-1*(G+1.D0)**2/(G-1.D0))*((1.D0/(G+1.D0)) * RIN01820
1      ((G+1.D0)/(G-1.D0))*((G-1.D0)/(G+1.D0))          RIN01830
150      G9=(G+3.D0)/(2.D0*(G-1.D0))                    RIN01840
151      G10=(7.D0-3.D0*G)/(2.D0*(G-1.D0))              RIN01850
152      G11=DSQRT(G/PAI)/(2.D0*(G+1.D0))               RIN01860
153      G12=DSQRT(G/2.D0)                                RIN01870
154      G13=1.D0/DSQRT(2.D0*G*PAI**3)                  RIN01880
155      LAMDA0=1.D0/(DSQRT(2.D0)*PAI*D**2*EN0)          RIN01890
156      LAMDA1=LAMDA0*(1.D0+G1*EM1**2)**(G6-OMEGA+0.5D0) RIN01900
157      RBIRD=G11/(D**2*EN0*PBIRD)                     RIN01910
158      ZETA1=G5*DATAN(DSQRT(EM1**2-1.D0)/G5)          RIN01920
159      AMU1=DARSIN(1.D0/EM1)                           RIN01930
160      PSI1=PAI2+AMU1                                   RIN01940
161      SPSI1=DSIN(PSI1)                                 RIN01950
162      CPSI1=DCOS(PSI1)                                 RIN01960
163      PSIF=PAI2+AMU1+ZETA1-G5*PAI2                   RIN01970
164      SPSIF=DSIN(PSIF)                                 RIN01980
165      CPSIF=DCOS(PSIF)                                 RIN01990
166      CALL BREAKR(EM1,RMIN)                            RIN02000
167      RSMIN=RMIN*DSQRT((2.D0/(G-1.D0)+EM1**2)/(EM1**2-1.D0)) RIN02010
C
168      PRINT 201,NSPEC,XNAME                            RIN02020
169      201  FORMAT(/1X,'SPECIES DATA   NSPEC=',I3/    RIN02030
1      1X,'SPECIES NAMES   ',11(2X,A6,2X))              RIN02040
170      PRINT 202,XC                                     RIN02050
171      202  FORMAT( 1X,'MOLE FRACTION XC=',11(F8.4,2X)) RIN02060
172      PRINT 203,WC                                     RIN02070
173      203  FORMAT( 1X,'MOL. WEIGHT   WC=',11(F8.4,2X)) RIN02080
174      PRINT 21,AR,AV,WAV,G,RH00,T0,EN0,C0,D           RIN02090
175      21  FORMAT(/1X,'THERMODYNAMIC DATA'/          RIN02100
1      1X,'AR,AV,WAV,GAMMA=',2X,2D14.5,2F9.3/          RIN02110
2      1X,'RH00,T0,EN0,C0,D=',D12.4,F8.0,D13.5,2D12.4) RIN02120
176      PRINT 22,EM1,PSI1*DEG,PSIF*DEG,PBIRD,          RIN02130
1      A0,RMIN,RSMIN,R0,                                RIN02140
2      LAMDA0,LAMDA1,RMIN/LAMDA1,RSMIN/LAMDA1           RIN02150
177      22  FORMAT(/1X,'FLOW AND GEOMETRY DATA'/      RIN02160
1      1X,'EM1,PSI1,PSIF,PBIRD=',4F9.3/                RIN02170
2      1X,'A0,RMIN,RSMIN,R0=',F9.3,3D14.5/             RIN02180
3      1X,'LAMDA0,LAMDA1,RMIN/LAMDA1,RSMIN/LAMDA1=',4D13.4) RIN02190
178      PRINT 23,DM0,NPHI,NR0,EPSEM                    RIN02200
179      23  FORMAT(/1X,'INTEGRATION DATA'/            RIN02210
1      1X,'DM0,NPHI,NR0,EPSEM=',F9.3,2I5,4X,D12.3)     RIN02220
180      RETURN                                           RIN02230
181      END                                              RIN02240
182      SUBROUTINE FLUX(EM,R)                            RIN02250
183      IMPLICIT REAL*8(A-H,O-Z,$)                      RIN02260
184      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, RIN02270
1      G16,G17,G18,G19,G20                               RIN02280
185      COMMON /PAR/C0,EN0,EM1,D,TLIM,ETALIM,CLIM,EL0,Q0,T0, RIN02290
1      PBIRD,RBIRD,DM0,DEG,OMEGA,XSV(51)                RIN02300
186      COMMON /NPAR/NETA,NC,NT,NEM0,NPHI,NXS,NR0,NSPEC RIN02310
187      COMMON /GEOM/APF,PAI,PAI2,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1, RIN02320
1      CPSI1,PSIF,SPSIF,CPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,  RIN02330
2      PHI,SPHI,CPHI,RMIN,RMAX,XS,DIST,                 RIN02340
3      AMU1,ZETA1,XN,YN,ZN,PSIM,SPSIM,CPSIM,R0          RIN02350
188      COMMON /EXTREM/TEXT,ETAEXT,CEXT,REXT,PSIEXT,EMEXT,BEXT,QEXT RIN02360
189      COMMON /SPEC/WAV,XC(5),WC(5),WCR(5),XNAME(5),QC(5),FLUXC(5) RIN02370
RIN02380

```

190	SPHI=DSIN(PHI)	RIN02390
191	CPHI=DCOS(PHI)	RIN02400
192	XMEAN=R*CPSIM	RIN02410
193	YMEAN=(A0+R*SPSIM)*CPHI	RIN02420
194	ZMEAN=(A0+R*SPSIM)*SPHI	RIN02430
195	TBETA=ZMEAN/(YMEAN-A0)	RIN02440
196	BETA=PAI2-DATAN(1.D0/TBETA)	RIN02450
197	SBETA=DSIN(BETA)	RIN02460
198	CBETA=DCOS(BETA)	RIN02470
199	DIST=DSQRT((XS-XMEAN)**2+(YMEAN-A0)**2+ZMEAN**2)	RIN02480
200	CW=-(XS-XMEAN)/DIST	RIN02490
201	SW=DSQRT(1.D0-CW**2)	RIN02500
202	GM=(1.D0+G1*EM**2)**(-G2)	RIN02510
203	AMU=DARSIN(1.D0/EM)	RIN02520
204	TETA=PSIM-AMU	RIN02530
205	STETA=DSIN(TETA)	RIN02540
206	CTETA=DCOS(TETA)	RIN02550
207	CKAPA=(CTETA)*(-CW)+(STETA*CPHI)*(-SW*CBETA)+	RIN02560
	1 (STETA*SPHI)*(-SW*SBETA)	RIN02570
208	SKAPA=DSQRT(1.D0-CKAPA**2)	RIN02580
209	QEXT=0.	RIN02590
210	DO 1 N=1,NSPEC	RIN02600
211	EMT=EM*CKAPA*G12*WCR(N)	RIN02610
212	IF(DABS(EMT).GT.13.D0) EMT=EMT*(13.D0/DABS(EMT))	RIN02620
213	POW=G3*EM**2*WCR(N)**2	RIN02630
214	POWT=POW*SKAPA**2	RIN02640
215	IF(POW.GT.1.0D2)POW=1.0D2	RIN02650
216	IF(POWT.GT.1.0D2)POWT=1.0D2	RIN02660
217	EXP1=DEXP(-POW)	RIN02670
218	EXP2=DEXP(-POWT)	RIN02680
219	ERFC1=DERFC(-EMT)	RIN02690
220	IF(ERFC1.LT.1.D-43) ERFC1=1.D-43	RIN02700
	C IF(XS.LT.5.D0) GO TO 234	RIN02710
	C IF(EXP1.GT.1.D-20.AND.ERFC1.GT.1.D-20) GO TO 234	RIN02720
	C PRINT 235,N,POW,POWT,EMT,EXP1,EXP2,ERFC1	RIN02730
	C235 FORMAT(I4,8D11.3)	RIN02740
	C234 CONTINUE	RIN02750
221	EVER1=ENO*(C0/WCR(N))*G13*GM*(1.D0+EMT**2)*EXP1	RIN02760
222	EVER2=(ENO*(C0/WCR(N))*0.5D0/PAI)*(EM*WCR(N))*CKAPA*GM*	RIN02770
	1 (1.5D0+EMT**2)*EXP2*ERFC1	RIN02780
223	QC(N)=EVER1+EVER2	RIN02790
224	IF(QEXT.GE.QC(N)) GO TO 1	RIN02800
225	QEXT=QC(N)	RIN02810
226	1 CONTINUE	RIN02820
227	RETURN	RIN02830
228	END	RIN02840
229	SUBROUTINE BREAKR(EM,R)	RIN02850
230	IMPLICIT REAL*8(A-H,O-Z,\$)	RIN02860
231	COMMON /GEOM/APF,PAI,PAI2,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,	RIN02870
	1 CPSI1,PSIF,SPSIF,CPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,	RIN02880
	2 PHI,SPHI,CPHI,RMIN,RMAX,XS,DIST,	RIN02890
	3 AMU1,ZETA1,XN,YN,ZN,PSIM,SPSIM,CPSIM,R0	RIN02900
232	COMMON /PAR/C0,ENO,EM1,D,TLIM,ETALIM,CLIM,EL0,Q0,T0,	RIN02910
	1 PBIRD,RBIRD,DM0,DEG,OMEGA,XSV(51)	RIN02920
233	COMMON /GAMA/G,C1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	RIN02930
	1 G16,G17,G18,G19,G20	RIN02940
234	R=RBIRD*DSQRT(EM**2-1.D0)*(1.D0+G1*EM**2)**(G6-OMEGA+0.5D0)	RIN02950
235	ZETA=G5*DATAN(DSQRT(EM**2-1.D0)/G5)	RIN02960
236	PSI=PAI2+AMU1+ZETA1-ZETA	RIN02970

```

237      XF=R*DCOS(PSI)
238      YF=R*DSIN(PSI)+A0
239      ZF=0.
240      1  CONTINUE
241      RETURN
      C
242      ENTRY BREAKM(EM,R)
      C
243      R=RRIRD*DSQRT(EM**2-1.D0)*(1.D0+G1*EM**2)**(G6-OMEGA+0.5D0)
244      ZETA=G5*DATAN(DSQRT(EM**2-1.D0)/G5)
245      PSIM=PAI2+AMU1+ZETA1-ZETA
246      SPSIM=DSIN(PSIM)
247      CPSIM=DCOS(PSIM)
248      RETURN
249      END

```

RIN02980
RIN02990
RIN03000
RIN03010
RIN03020
RIN03030
RIN03040
RIN03050
RIN03060
RIN03070
RIN03080
RIN03090
RIN03100
RIN03110
RIN03120

\$ENTRY

RIN03130

<INGBD - FLUX INTEGRATION FROM BREAKDOWN SURFACE

SPECIES DATA NSPEC= 5
SPECIES NAMES
MOLE FRACTION XC= 0.0910 0.0910 0.1040 0.1350 0.5790
MOL. WEIGHT MC= 1.0000 20.0000 2.0000 21.0000 4.0000

THERMODYNAMIC DATA
AR,AV,NAV,GAMMA= 0.83143D 04 0.60220D 27 7.270 1.540
RH00,T0,END,C0,D= 0.7500D-02 2300. 0.62125D 24 0.2013D 04 0.2500D-09

FLOW AND GEOMETRY DATA
EM1,PSI1,PSIF,PBIRD= 4.000 104.478 41.044 0.100
A0,RMIN,RSMIN,R0= 2.500 0.30574D-02 0.34812D-02 0.00000D 00
LAMD00,LAMD01,RMIN/LAMD01,RSMIN/LAMD01= 0.5797D-05 0.1281D-03 0.2372D 02 0.2718D 02

INTEGRATION DATA
DM0,NPHI,NR0,EPSEM= 0.100 10 10 0.100D-04

NX	NEM	XS	PHIMAX	QMAX	H / LOG	HF / LOG	H2 / LOG	DF / LOG	HE / LOG
1	47	0.0100	11.19	0.120D 25	0.824D 22/21.92	0.269D 07/ 6.43	0.777D 21/20.89	0.491D 06/ 5.84	0.653D 20/19.81
2	50	0.0178	12.15	0.121D 25	0.474D 22/21.68	0.162D 07/ 6.21	0.424D 21/20.63	0.418D 06/ 5.62	0.349D 20/19.54
3	54	0.0316	13.50	0.122D 25	0.257D 22/21.38	0.776D 06/ 5.89	0.196D 21/20.29	0.202D 06/ 5.30	0.154D 20/19.19
4	59	0.0562	15.27	0.123D 25	0.106D 22/21.02	0.295D 06/ 5.47	0.780D 20/19.89	0.770D 05/ 4.89	0.568D 19/18.75
5	64	0.1000	17.16	0.124D 25	0.421D 21/20.62	0.892D 05/ 4.95	0.268D 20/19.43	0.233D 05/ 4.37	0.175D 19/18.24
6	70	0.1778	19.55	0.124D 25	0.149D 21/20.17	0.219D 05/ 4.34	0.791D 19/18.90	0.575D 04/ 3.76	0.451D 18/17.65
7	77	0.3162	22.52	0.124D 25	0.471D 20/19.67	0.464D 04/ 3.67	0.203D 19/18.31	0.122D 04/ 3.09	0.100D 18/17.00
8	84	0.5623	25.65	0.124D 25	0.132D 20/19.12	0.901D 03/ 2.95	0.461D 18/17.66	0.237D 03/ 2.37	0.201D 17/16.30
9	92	1.0000	29.39	0.124D 25	0.335D 19/18.52	0.167D 03/ 2.22	0.959D 17/16.98	0.440D 02/ 1.64	0.381D 16/15.58
10	100	1.7783	33.26	0.124D 25	0.771D 18/17.89	0.305D 02/ 1.48	0.187D 17/16.27	0.801D 01/ 0.90	0.699D 15/14.84
11	108	3.1623	37.22	0.124D 25	0.165D 18/17.22	0.548D 01/ 0.74	0.353D 16/15.55	0.144D 01/ 0.16	0.126D 15/14.10
12	116	5.6234	41.19	0.124D 25	0.332D 17/16.52	0.981D 00/ 0.01	0.648D 15/14.81	0.258D 00/ 0.59	0.227D 14/13.36
13	124	10.0000	45.14	0.124D 25	0.641D 16/15.81	0.175D 00/ 0.76	0.117D 15/14.07	0.461D-01/ 1.34	0.405D 13/12.61

END RIHGBD RUN

STATEMENTS EXECUTED= 1281233

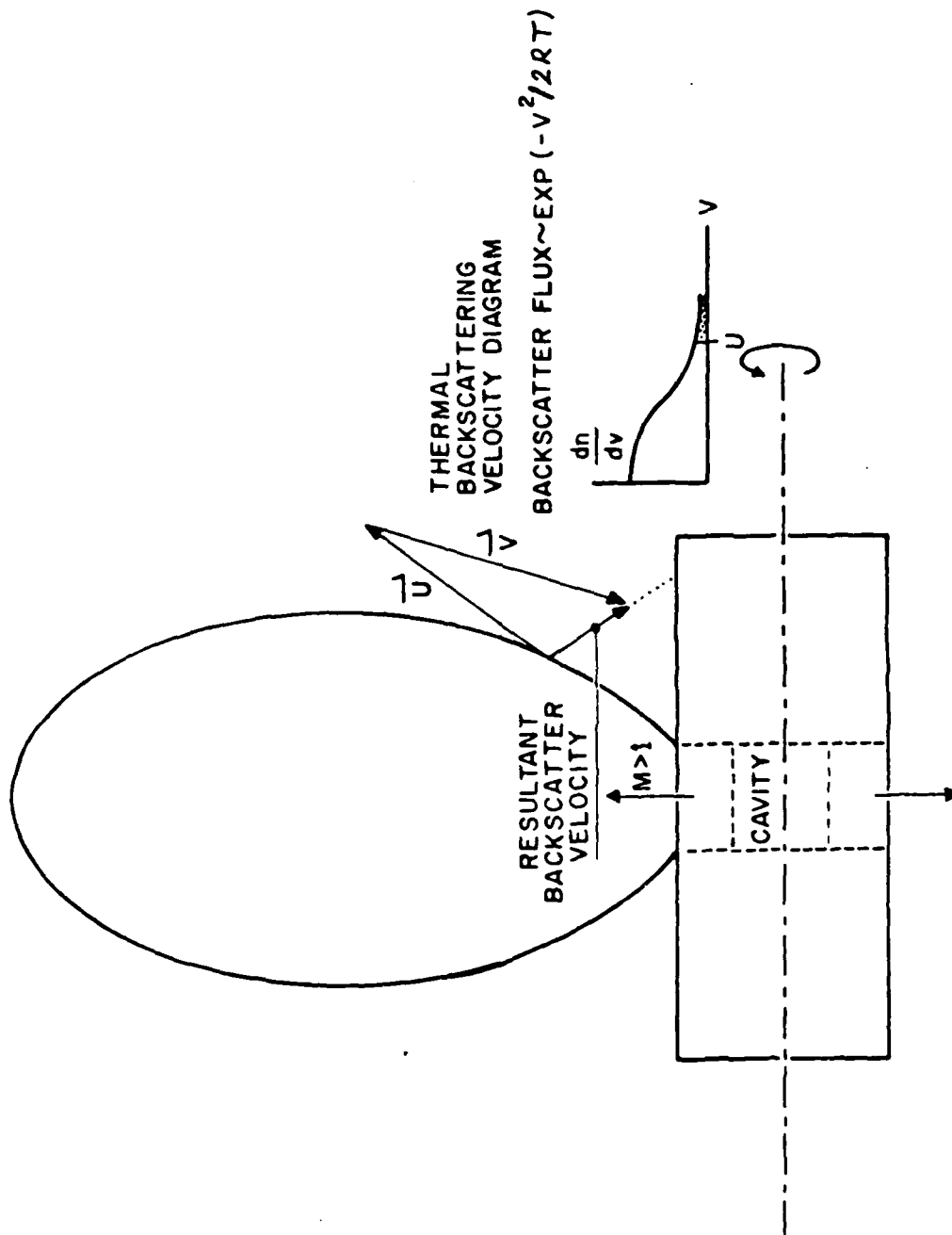


Figure 1. Thermal Backscattering from Laser Exhaust Plume

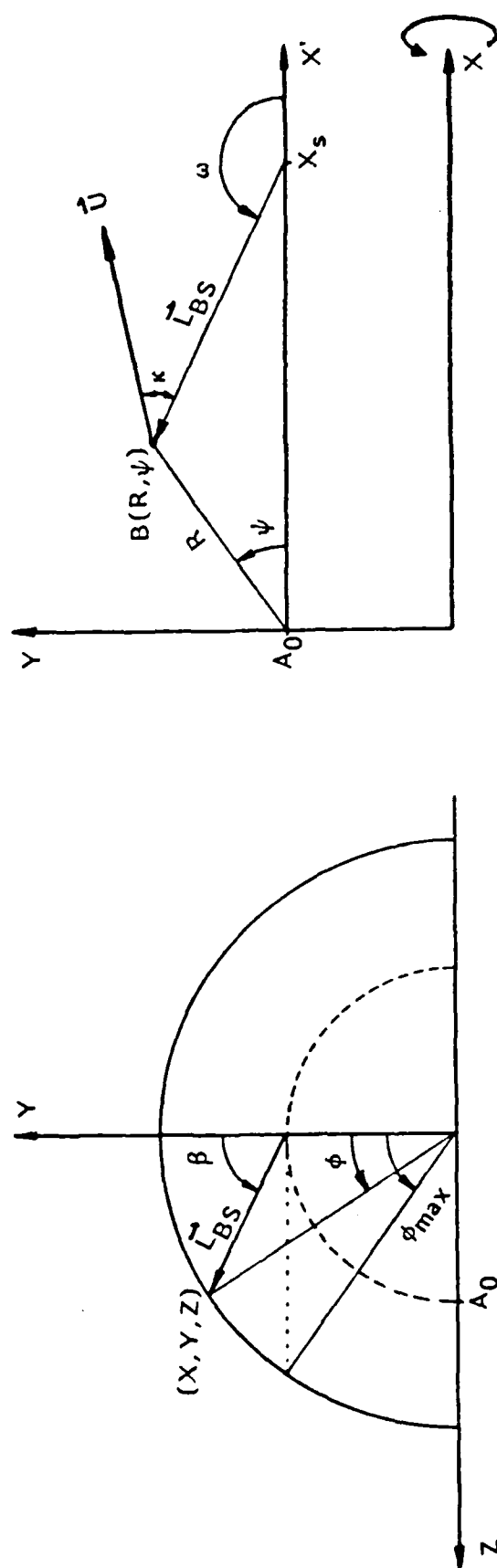


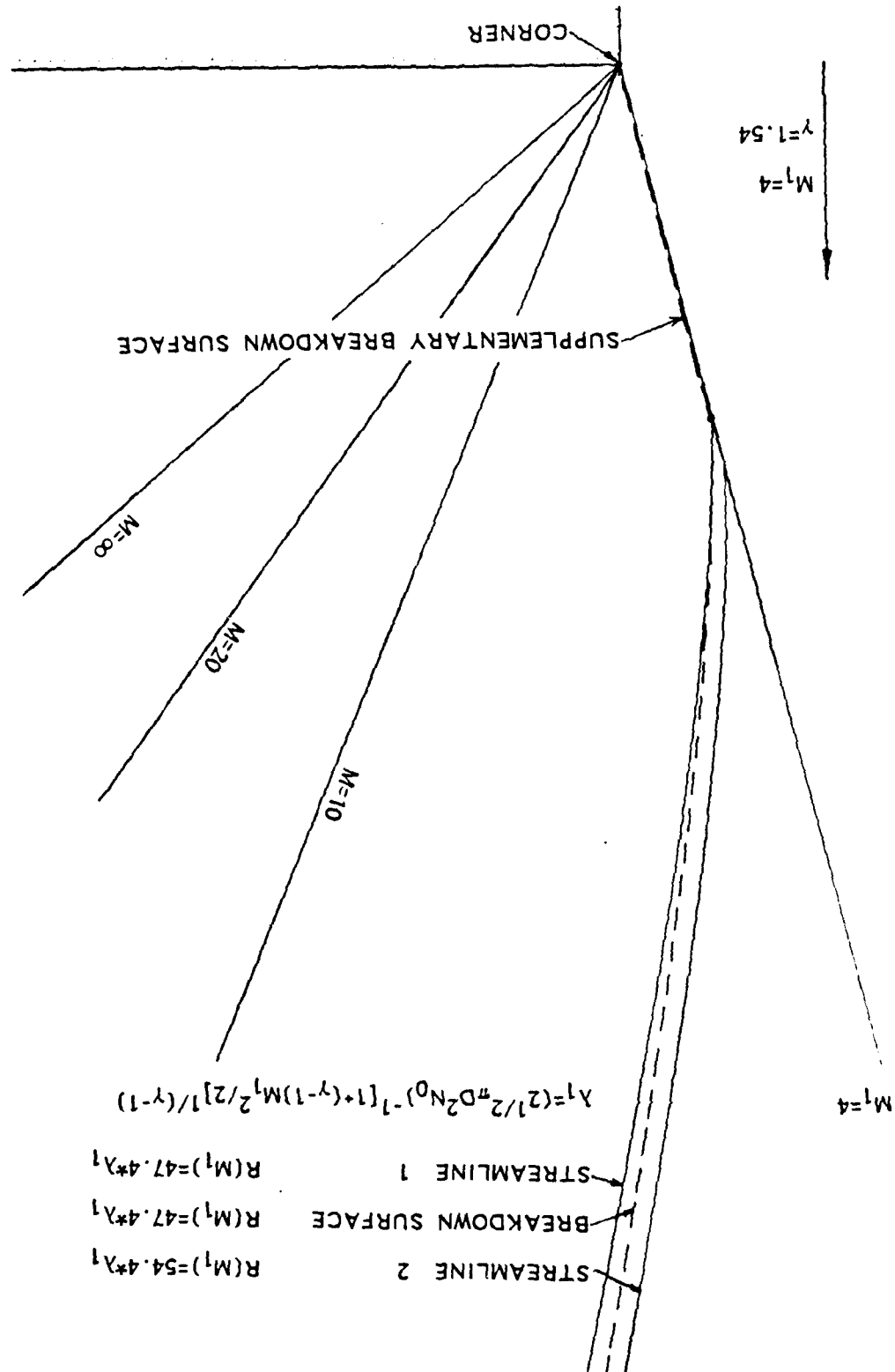
Figure 3. Flux Integration Scheme

B - Point on Breakdown Surface

(X, Y, Z) - Point on Breakdown Surface (Same point as $B(R, \psi)$)

$\vec{L}_{BS}$ - Line of Sight

Figure 4. Prandtl-Meyer Flow Field Near the Corner, Including Actual



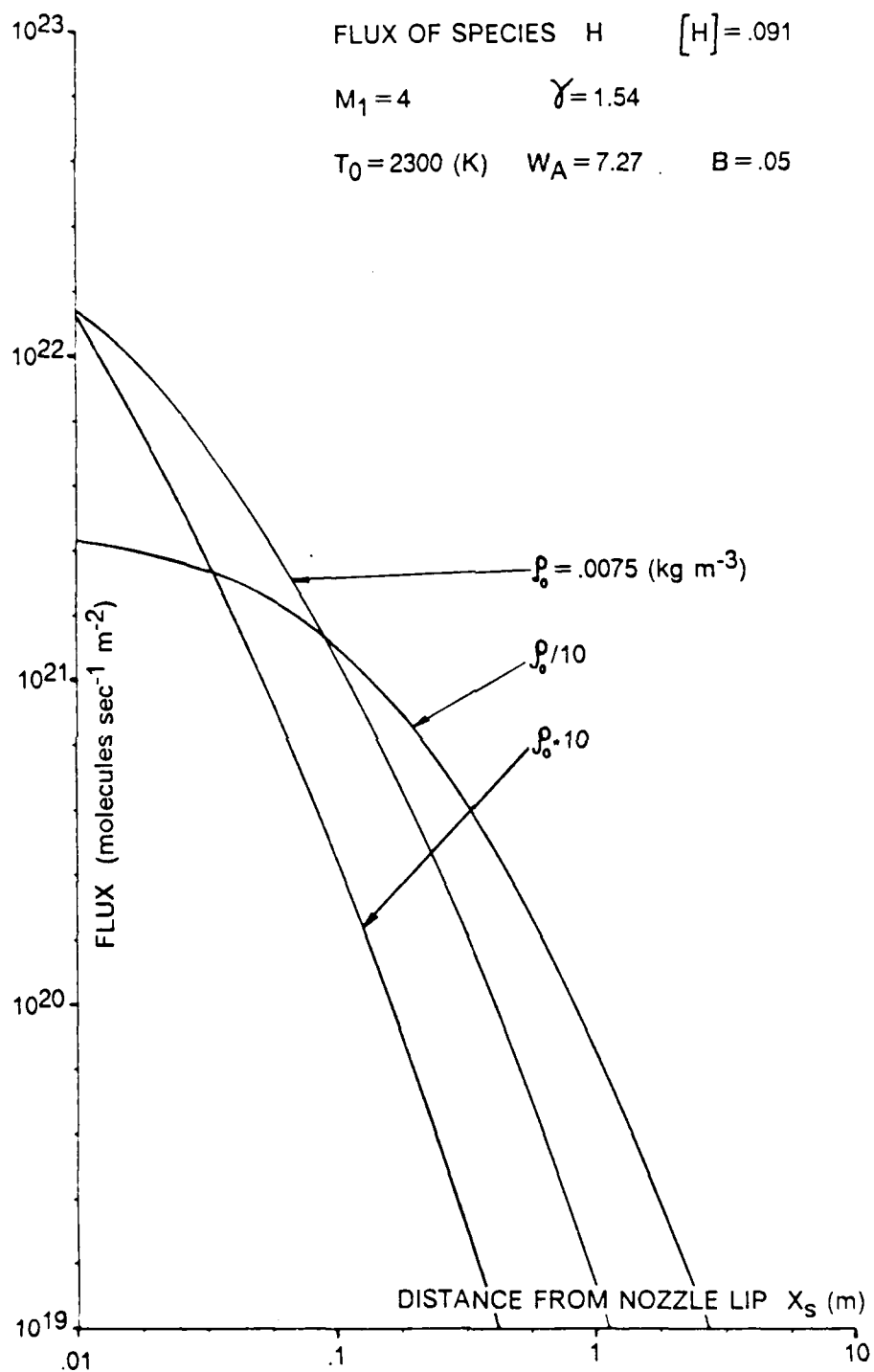


Figure 5. Flux of Species H at Various Stagnation Densities

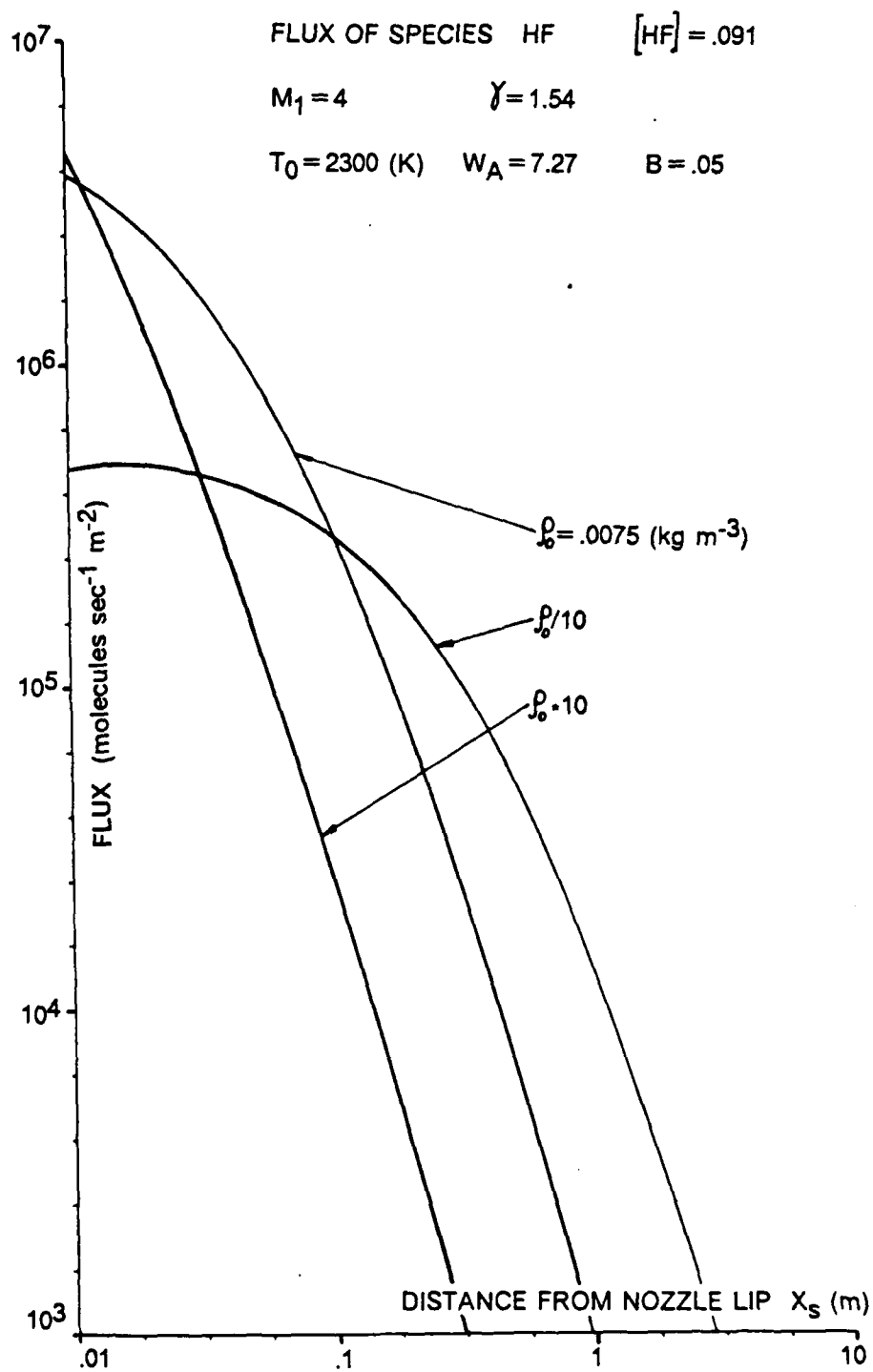


Figure 6. Flux of Species HF at Various Stagnation Densities

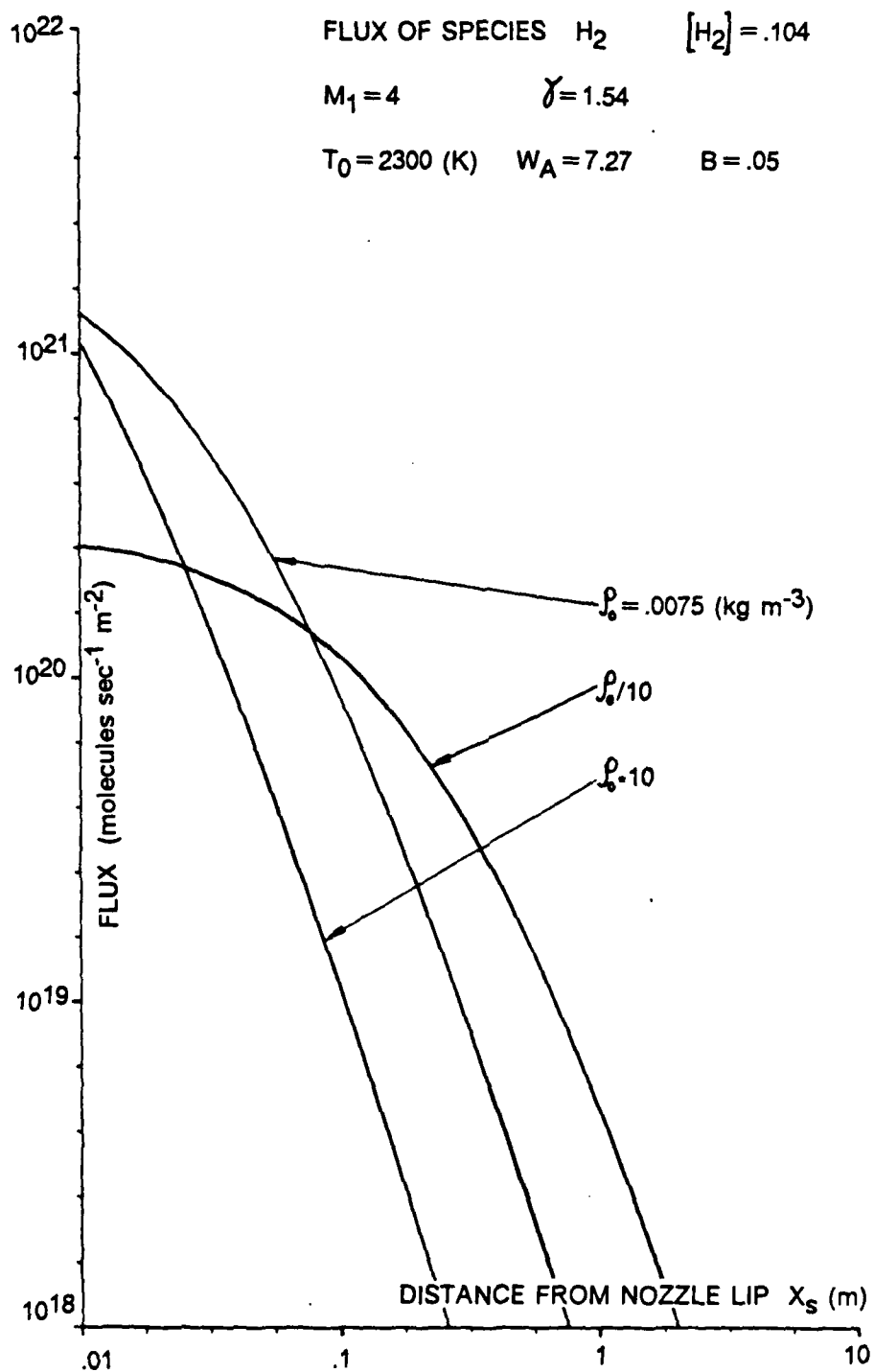


Figure 7. Flux of Species H_2 at Various Stagnation Densities

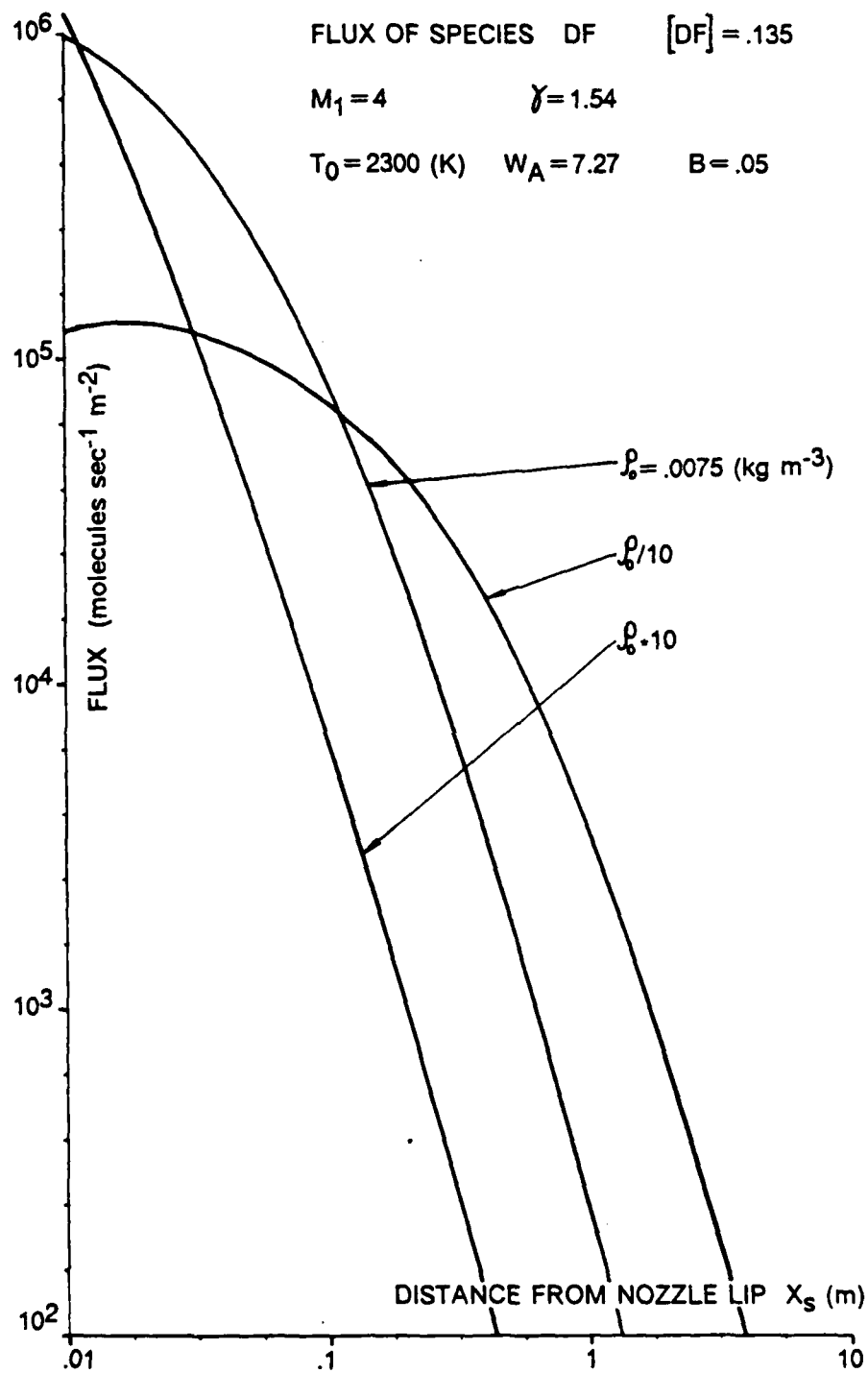


Figure 8. Flux of Species DF at Various Stagnation Densities

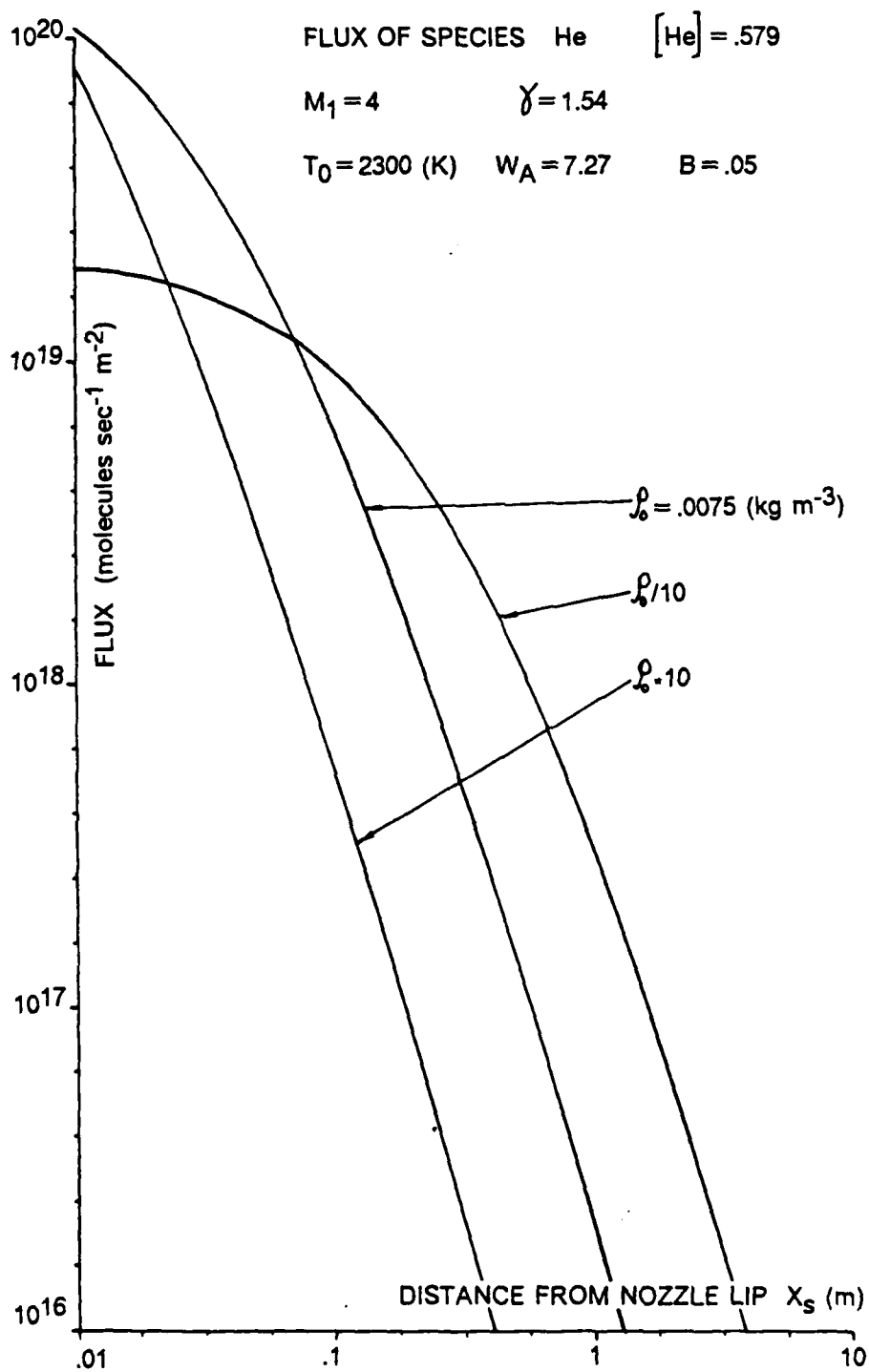


Figure 9. Flux of Species He at Various Stagnation Densities

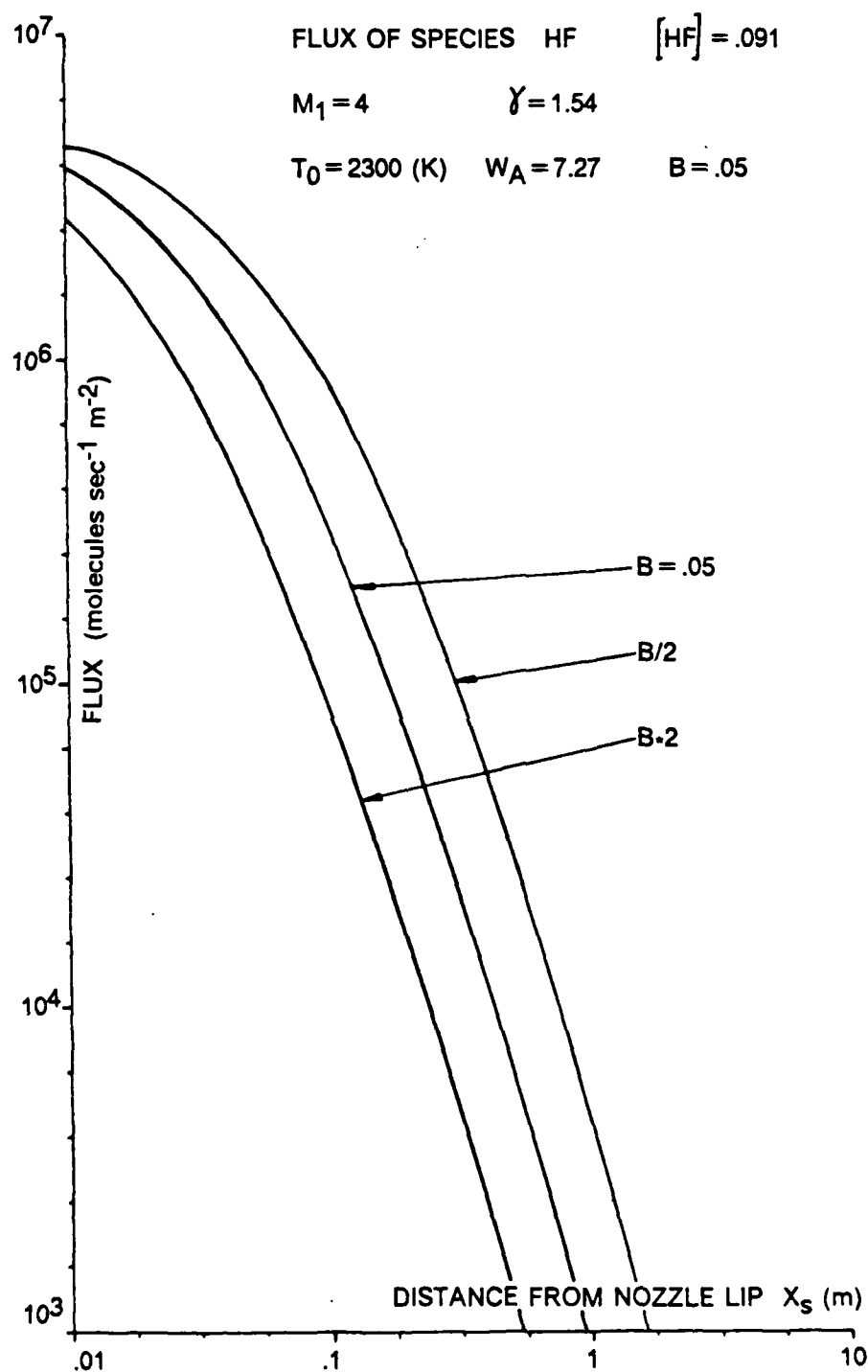


Figure 10. Flux of Species HF at Various Values of Breakdown Parameter

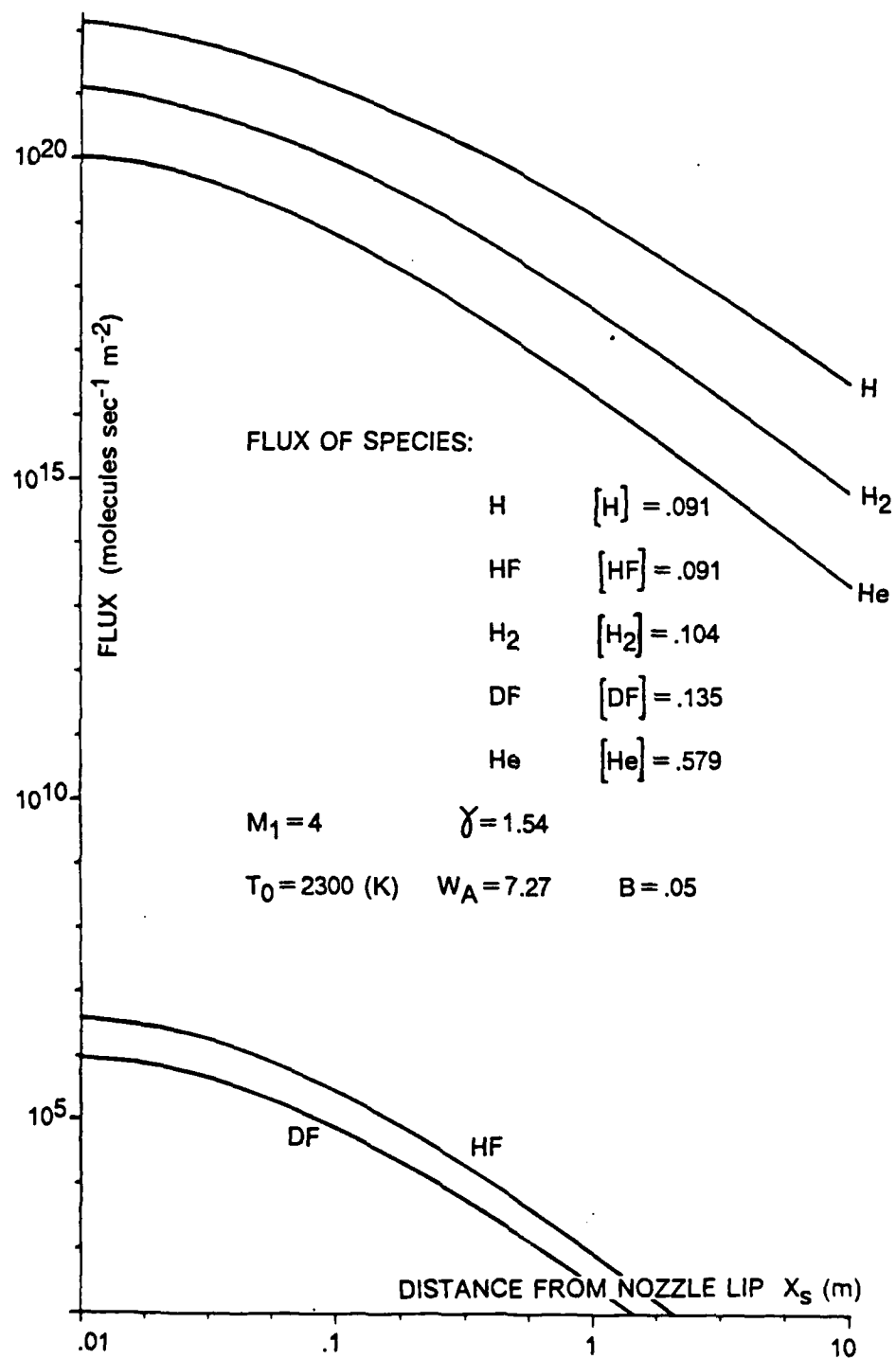


Figure 11. Flux of All Species at Typical Operating Conditions

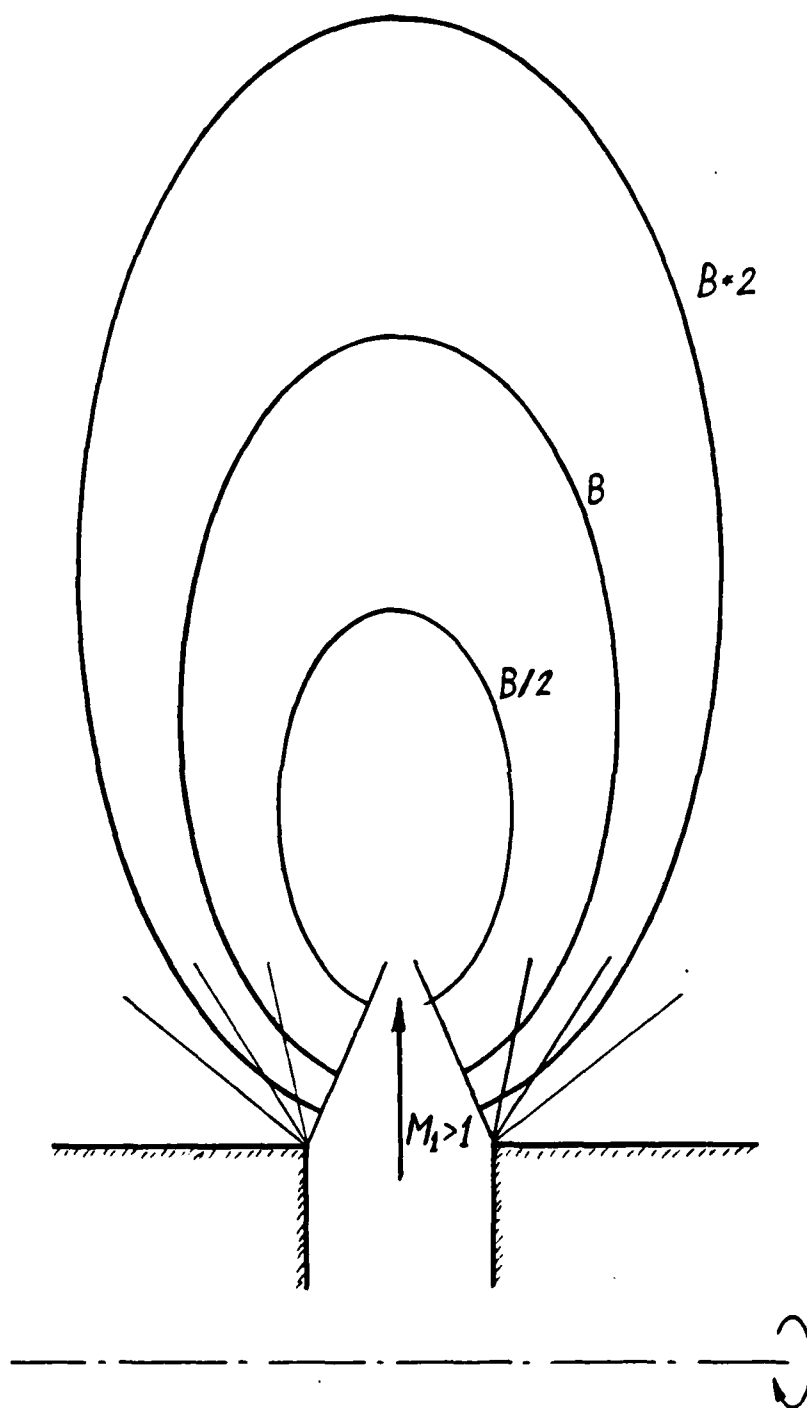


Figure 12. Schematic Display of Complete Breakdown Surface in a Ringjet Exhaust Plume

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