

AD-A172 253

NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND RELATED
PROBLEMS OF PADE APP. (U) COLUMBIA UNIV NEW YORK DEPT
OF MATHEMATICS D V CHUDNOVSKY ET AL. 30 SEP 85

1/1

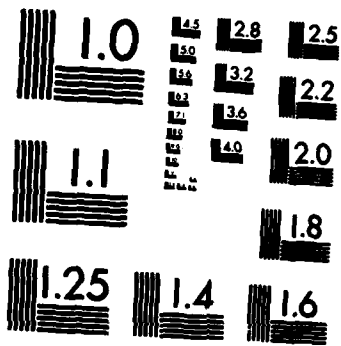
UNCLASSIFIED

AFOSR-TR-84-0654 AFOSR-81-0190

F/G 12/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

AFOSR-TR. 86-0654

Grant No.

AFOSR-81-0190

DTIC
ELECTE
SEP 15 1986
S B D

Final Scientific Report

Nonlinear Partial Differential Equations and
Related Problems of Padé Approximations

by D.V. Chudnovsky and G.V. Chudnovsky

proved for public release
Distribution Unlimited

AD-A172 253

DTIC FILE COPY

the authors'
The focus of ~~our~~ work is the understanding and possibly explicit mathematical solution of the physically realistic classical and quantum models of field theories in dimensions one to four. ~~We~~ ^{They} aim at the description of hidden symmetries and analytic structures of the solutions of various physical systems in dimensions one-four described by systems of nonlinear p.d.e. in cases when a system is suspected of being completely integrable, or at the investigation of obstructions to complete integrability.

We continued and expanded our earlier studies of the structure of infinite-dimensional Lie algebras generated by Bäcklund transformations and the geometric interpretation of the structure of S-matrices, applied to the extension of the Bethe Ansatz determining the solutions of completely integrable quantum systems to three- and four-dimensional systems. Very little is currently known in these cases, and there arise new multidimensional generalizations of factorization equations of S-matrices that have to be solved. A totally new area for mathematical studies has been opened by recent progress in the physics of —

86 9 15 090

superstrings, where we hope to apply our methods of multi-particle and multi-string interactions, and where the use of methods from algebraic topology and theta functions (some of which were introduced by us in lower-dimensional cases earlier) seems very promising.

Our work includes the theoretical and computer study of Padé approximations and their applications, particularly to differential equations, differential algebra and number theory. Methods of Padé approximations proved to be very powerful in the study of completely integrable systems (particularly in the description of Bäcklund transformations). These methods allowed us to achieve a major progress in the study of the analytic and arithmetic properties of solutions of differential equations. Extending our previous work (e.g. the solution to Kolchin's problem and the proof of the "almost perfectness" of Padé approximations to algebraic functions), we have solved in many cases the Grothendieck conjecture. According to it, the monodromy structure of a linear differential equation (more precisely, a Lie algebra of a Galois group of an equation) can be determined from mod p properties of this equation for (a finite set of) primes p . This led to a development and implementation of new algorithms that determine the algebraicity of solutions of differential equations, and reduction of Abelian integrals. Methods of Padé approximations in combination with formal groups made it possible for us to solve effectively Tate's conjecture in many cases. The work on formal groups and Padé approximations leads to many number-theoretic applications, and to some more practical applications to new classes of computer algorithms for primality testing.

Our work greatly benefits from the use of various computer

algebra systems. We are working on the design, optimization and implementation of specialized computer algebra programs, particularly for very fast power series manipulations, Padé and continued fraction approximations, and fast numeric and symbolic solutions of differential equations.

During the period of the Grant (since 1984) we had prepared, with the support of Air Force, and published 13 research papers, 2 papers were prepared for publications. We were authors or editors of 4 books or proceedings of seminars.

In 1984 we were among organizers and speakers of a conference "Computer Algebra as a Tool for Research in Mathematics and Physics", held in New York (D.V. was a co-chairman of the Conference). There were more than 500 participants at the Conference. A new conference on "Computers and Mathematics" will be held in July - August 1986 in Stanford.

Department of Mathematics
Columbia University
New York, N.Y. 10027

1000	
1001	
1002	
1003	
1004	
1005	
1006	
1007	
1008	
1009	
1010	
1011	
1012	
1013	
1014	
1015	
1016	
1017	
1018	
1019	
1020	
1021	
1022	
1023	
1024	
1025	
1026	
1027	
1028	
1029	
1030	
1031	
1032	
1033	
1034	
1035	
1036	
1037	
1038	
1039	
1040	
1041	
1042	
1043	
1044	
1045	
1046	
1047	
1048	
1049	
1050	
1051	
1052	
1053	
1054	
1055	
1056	
1057	
1058	
1059	
1060	
1061	
1062	
1063	
1064	
1065	
1066	
1067	
1068	
1069	
1070	
1071	
1072	
1073	
1074	
1075	
1076	
1077	
1078	
1079	
1080	
1081	
1082	
1083	
1084	
1085	
1086	
1087	
1088	
1089	
1090	
1091	
1092	
1093	
1094	
1095	
1096	
1097	
1098	
1099	
1100	
1101	
1102	
1103	
1104	
1105	
1106	
1107	
1108	
1109	
1110	
1111	
1112	
1113	
1114	
1115	
1116	
1117	
1118	
1119	
1120	
1121	
1122	
1123	
1124	
1125	
1126	
1127	
1128	
1129	
1130	
1131	
1132	
1133	
1134	
1135	
1136	
1137	
1138	
1139	
1140	
1141	
1142	
1143	
1144	
1145	
1146	
1147	
1148	
1149	
1150	
1151	
1152	
1153	
1154	
1155	
1156	
1157	
1158	
1159	
1160	
1161	
1162	
1163	
1164	
1165	
1166	
1167	
1168	
1169	
1170	
1171	
1172	
1173	
1174	
1175	
1176	
1177	
1178	
1179	
1180	
1181	
1182	
1183	
1184	
1185	
1186	
1187	
1188	
1189	
1190	
1191	
1192	
1193	
1194	
1195	
1196	
1197	
1198	
1199	
1200	
1201	
1202	
1203	
1204	
1205	
1206	
1207	
1208	
1209	
1210	
1211	
1212	
1213	
1214	
1215	
1216	
1217	
1218	
1219	
1220	
1221	
1222	
1223	
1224	
1225	
1226	
1227	
1228	
1229	
1230	
1231	
1232	
1233	
1234	
1235	
1236	
1237	
1238	
1239	
1240	
1241	
1242	
1243	
1244	
1245	
1246	
1247	
1248	
1249	
1250	
1251	
1252	
1253	
1254	
1255	
1256	
1257	
1258	
1259	
1260	
1261	
1262	
1263	
1264	
1265	
1266	
1267	
1268	
1269	
1270	
1271	
1272	
1273	
1274	
1275	
1276	
1277	
1278	
1279	
1280	
1281	
1282	
1283	
1284	
1285	
1286	
1287	
1288	
1289	
1290	
1291	
1292	
1293	
1294	
1295	
1296	
1297	
1298	
1299	
1300	
1301	
1302	
1303	
1304	
1305	
1306	
1307	
1308	
1309	
1310	
1311	
1312	
1313	
1314	
1315	
1316	
1317	
1318	
1319	
1320	
1321	
1322	
1323	
1324	
1325	
1326	
1327	
1328	
1329	
1330	
1331	
1332	
1333	
1334	
1335	
1336	
1337	
1338	
1339	
1340	
1341	
1342	
1343	
1344	
1345	
1346	
1347	
1348	
1349	
1350	
1351	
1352	
1353	
1354	
1355	
1356	
1357	
1358	
1359	
1360	
1361	
1362	
1363	
1364	
1365	
1366	
1367	
1368	
1369	
1370	
1371	
1372	
1373	
1374	
1375	
1376	
1377	
1378	
1379	
1380	
1381	
1382	
1383	
1384	
1385	
1386	
1387	
1388	
1389	
1390	
1391	
1392	
1393	
1394	
1395	
1396	
1397	
1398	
1399	
1400	
1401	
1402	
1403	
1404	
1405	
1406	
1407	
1408	
1409	
1410	
1411	
1412	
1413	
1414	
1415	
1416	
1417	
1418	
1419	
1420	
1421	
1422	
1423	
1424	
1425	
1426	
1427	
1428	
1429	
1430	
1431	
1432	
1433	
1434	
1435	
1436	
1437	
1438	
1439	
1440	
1441	
1442	
1443	
1444	
1445	
1446	
1447	
1448	
1449	
1450	
1451	
1452	
1453	
1454	
1455	
1456	
1457	
1458	
1459	
1460	
1461	
1462	
1463	
1464	
1465	
1466	
1467	
1468	
1469	
1470	
1471	
1472	
1473	
1474	
1475	
1476	
1477	
1478	
1479	
1480	
1481	
1482	
1483	
1484	
1485	
1486	
1487	
1488	
1489	
1490	
1491	
1492	
1493	
1494	
1495	
1496	
1497	
1498	
1499	
1500	
1501	
1502	
1503	
1504	
1505	
1506	
1507	
1508	
1509	
1510	
1511	
1512	
1513	
1514	
1515	
1516	
1517	
1518	
1519	
1520	
1521	
1522	
1523	
1524	
1525	
1526	
1527	
1528	
1529	
1530	
1531	
1532	
1533	
1534	
1535	
1536	
1537	
1538	
1539	
1540	
1541	
1542	
1543	
1544	
1545	
1546	
1547	
1548	
1549	
1550	
1551	
1552	
1553	
1554	
1555	
1556	
1557	
1558	
1559	
1560	
1561	
1562	
1563	
1564	
1565	
1566	
1567	
1568	
1569	
1570	
1571	
1572	
1573	
1574	
1575	
1576	
1577	
1578	
1579	
1580	
1581	
1582	
1583	
1584	
1585	
1586	
1587	
1588	
1589	
1590	
1591	
1592	
1593	
1594	
1595	
1596	
1597	
1598	
1599	
1600	
1601	
1602	
1603	
1604	
1605	
1606	
1607	
1608	
1609	
1610	
1611	
1612	
1613	
1614	
1615	
1616	
1617	
1618	
1619	
1620	
1621	
1622	
1623	
1624	
1625	
1626	
1627	
1628	
1629	
1630	
1631	
1632	
1633	
1634	
1635	
1636	
1637	
1638	
1639	
1640	
1641	
1642	
1643	
1644	
1645	
1646	
1647	
1648	
1649	
1650	
1651	
1652	
1653	
1654	
1655	
1656	
1657	
1658	
1659	
1660	
1661	
1662	
1663	
1664	
1665	
1666	
1667	
1668	
1669	
1670	
1671	
1672	
1673	
1674	
1675	
1676	
1677	
1678	
1679	
1680	
1681	
1682	
1683	
1684	
1685	
1686	
1687	
1688	
1689	
1690	
1691	
1692	
1693	
1694	
1695	
1696	
1697	
1698	
1699	
1700	
1701	
1702	
1703	
1704	
1705	
1706	

Recent publications of D.V. Chudnovsky and G.V. Chudnovsky for the Project "Nonlinear Partial Differential Equations and Related Problems of Padé Approximations", supported by the U.S. Air Force.

1. D.V. Chudnovsky, Topological and algebraic structures of linear problems associated with completely integrable systems, in Proceedings of XI International Conference in Group Theoretical Methods in Physics, Istanbul, 1982.

Lecture Notes Physics, v. 180, 1983, 65-90.
2. D.V. Chudnovsky, Bäcklund transformations and geometric and complex-analytic background for construction of completely integrable lattice systems,

In Problems of Mathematics and Physics, Festschrift in honor of F. Gursey, Gordon and Breach, 1984, 46pp (in press).
3. G.V. Chudnovsky, Bäcklund transformations and deformations of linear differential equations with applications to diophantine approximations, in Problems of Mathematical Physics.

Festschrift in honor of F. Gursey, Gordon and Breach, 1984, 30 pp. (in press).
4. G.V. Chudnovsky, Number theoretical applications of polynomials with rational coefficients defined by extremality conditions, in Arithmetic and Geometry.

Progress in Mathematics, v. 35, Birkhauser, Boston, 1983, pp. 67-107.
5. D.V. Chudnovsky, G.V. Chudnovsky, Algebraic reductions of scalar three-dimensional systems,

Phys. Lett. A, 93A(1983), 371-374.
6. D.V. Chudnovsky, G.V. Chudnovsky, Laws of composition of Bäcklund transformations and the universal form of completely integrable systems in dimensions two and three.
7. G.V. Chudnovsky, On the method of Thue-Siegel.

Ann. of Math. v. 117 (1983), 325-382.
8. D.V. Chudnovsky, G.V. Chudnovsky, Rational approximations to solutions of linear differential equations.

Proc. Nat'l. Acad. Sci. U.S.A., v. 80 (1983), 5158-5162.

9. D.V. Chudnovsky, G.V. Chudnovsky, M. Tabor, Painlevé property and multicomponent isospectral deformation equations.
Phys. Lett. A., 97A, 1983, 268-274.
10. G.V. Chudnovsky, The Thue-Siegel-Roth Theorem for values of algebraic functions.
Proc. Japan Academy, v.59, 1983, 281-284.
11. G.V. Chudnovsky, Rational approximations to linear forms of exponential and binomials.
Proc. Nat'l. Acad. Sci. U.S.A., v.80, 1983, 3183-3141.
12. D.V. Chudnovsky, G.V. Chudnovsky, Padé and rational approximations to system of functions and their arithmetic applications.
Lecture Notes in Mathematics, Springer, N.Y., v.1052, 1984, 37-84.
13. D.V. Chudnovsky, G.V. Chudnovsky, Padé approximations to solutions of linear differential equations and applications to diophantine analysis.
Lecture Notes in Mathematics, Springer, N.Y., v.1052, 1984, 85-167.
14. D.V. Chudnovsky, G.V. Chudnovsky, Some remarks on theta functions and S-matrices.
In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 100pp.
15. D.V. Chudnovsky, G.V. Chudnovsky, Note on Eisenstein's system of differential equations.
In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 17pp.
16. D.V. Chudnovsky, G.V. Chudnovsky, Recurrences, Padé Approximations and their applications.
In Classical and Quantum Models and Arithmetic Problems, M. Dekker, v.92, 1984, 25pp.
17. G.V. Chudnovsky, On some applications of diophantine approximations.
Proc. Nat'l. Acad. Sci. U.S.A., v.81, 1984, 1926-1930.
18. D.V. Chudnovsky, G.V. Chudnovsky, The Wronskian Formalism for Linear Differential Equations and Padé Approximations.
Advances in Mathematics, v.52, 1984, 111-138.

19. G.V. Chudnovsky, On applications of diophantine approximations.
Proc. Nat'l. Acad. Sci. U.S.A., v. 81 (1984), 7261-7265.
20. D.V. Chudnovsky, G.V. Chudnovsky, The Grothendieck conjecture and Padé approximations.
Proc. Japan Acad., v. 61, Ser. A (1985), 87-91.
21. D.V. Chudnovsky, G.V. Chudnovsky, Padé approximations and diophantine geometry.
Proc. Nat'l. Acad. Sci. U.S.A., v. 82 (1985), 2212-2216.
22. D.V. Chudnovsky, G.V. Chudnovsky, Applications of Padé Approximations to Diophantine Inequalities in Values of G-Functions.
Lecture Notes in Mathematics, Springer, N.Y., v. 1135 (1985), 44pp.
23. D.V. Chudnovsky, G.V. Chudnovsky, Applications of Padé Approximations to the Grothendieck Conjecture on Linear Differential Equations.
ibid, 54pp.
24. D.V. Chudnovsky, G.V. Chudnovsky, Sequences of numbers generated by addition in formal groups and new primality and factorization tests, IBM Research Report, RC 11262, 7/12/85.
25. D.V. Chudnovsky, G.V. Chudnovsky, A random walk in higher arithmetic, Advances in Appl. Math., 1986 (to appear).
26. D.V. Chudnovsky, G.V. Chudnovsky, Elliptic Modular Functions and Elliptic Genera, Topolgy (submitted).

BOOKS

1. Number Theory, Proceedings of New York Seminar, 1981-1982, Lecture Notes in Mathematics, v. 1052, Springer-Verlag, NY, 1984, 309 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky, H. Cohn and M. Nathanson).
2. Classical and Quantum Models and Arithmetic Problems, Lecture Notes in Pure and Applied Mathematics, M. Dekker, NY, 1984, 461 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky).
3. Contributions to the Theory of Transcendental Numbers, by G.V. Chudnovsky, mathematical Surveys and Monographs, v. 19, American Mathematical Society, Providence, Rhode Island, 1984, XI, 450 pp.
4. Number Theory, Proceedings of New York Seminar, 1983-1984, Lecture Notes in Mathematics, v. 1135, Springer-Verlag, NY, 1985, 283 pp. (edited by D.V. Chudnovsky, G.V. Chudnovsky, H. Cohn and M. Nathanson).

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR. 86-0654	
6a. NAME OF PERFORMING ORGANIZATION Columbia University	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research	
6c. ADDRESS (City, State and ZIP Code) Department of Mathematics Columbia University New York, N.Y. 10027		7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-81-0190	
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332		10. SOURCE OF FUNDING NOS.	
		PROGRAM ELEMENT NO.	PROJECT NO.
		61102F	2304
		TASK NO.	WORK UNIT NO.
		A4	
11. TITLE (Include Security Classification) NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND RELATED PROBLEMS OF PADE APPROXIMATIONS			
12. PERSONAL AUTHOR(S) Dr.'s D. V. Chudnovsky and G. V. Chudnovsky			
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM 6-30-84 TO 9-30-85	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT 3
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB. GR.	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
During this period the two investigators published 13 research papers and prepared two (2) for publication and authored or edited four (4) books on proceedings of seminars. Titles include "Ellipric Modular Functions and Ellipric Geneia", "A Random Walk in Higher Arithmeric," "Pade Approvimations and Diophantine Geometry", "The Grothendieck Conjecture and Pade Appromations."			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL CAPTAIN JOHN P. THOMAS, Jr.	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5026	22c. OFFICE SYMBOL NM	

END

DTIC

10-86