

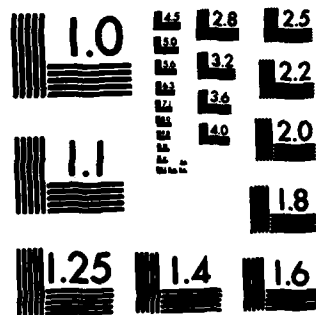
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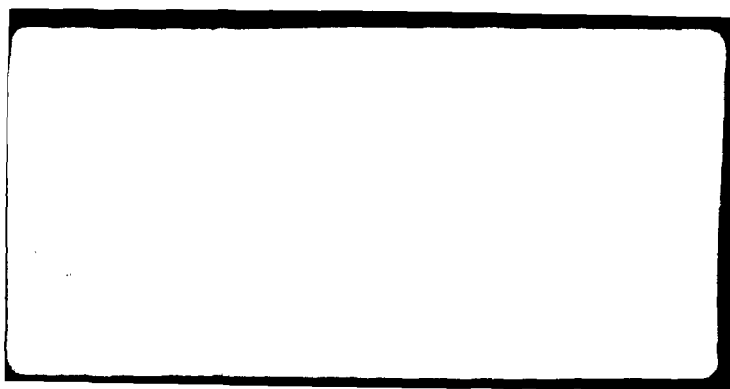
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NUMERICAL SOLUTION OF LAMINAR AND
TURBULENT BOUNDARY LAYER EQUATIONS
INCLUDING TRANSITION, AND
EXPERIMENTAL STUDY OF A FLAT PLATE
WITH A BLUNT FIN AT INCIDENCE

THESIS

Alice J. Chen
Captain, USAF

AFIT/GAE/ENY/86M-1

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BOUNDARY LAYER EQUATIONS INCLUDING TRANSITION, AND
EXPERIMENTAL STUDY OF A FLAT PLATE WITH A
BLUNT FIN AT INCIDENCE
THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Aeronautical Engineering

Alice J. Chen, B.S.
Captain, USAF

March 1986



Mission For	
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List of Symbols

a	Speed of sound
A	Rake orientation angle
A	Van Driest damping parameter
⁺ A	Nondimensional Van Driest damping parameter
AOS	Chapman damping parameter
⁺ AOS	Nondimensional Chapman parameter
A*/ β	Calibration constant for the electrical analog
C	Electrical capacitance
C'	Distributed electrical capacitance per unit length
c_f	Skin friction coefficient
cp	Specific heat capacity at constant pressure
d	Derivative symbol
d	Diameter of fin leading edge
EPS	Maximum error allowed for computer code
H	Total enthalpy
h	Static enthalpy
h	Height of fin
h	Number of stages of arithmetically increasing components in the analog circuit
HC	Heat transfer coefficient
HREF	Reference heat transfer coefficient
H.O.T.	Higher order terms
I	Number of spatial node points in X or ξ direction
i	Node point in X or ξ direction

i	Electrical current
J	Number of spatial node points in η or η direction
j	Node point in η or η direction
k	Conductivity
k_T	Turbulent conductivity
K	von Karman's constant
L	Flat plate length
l	Separation distance ahead of the fin leading edge
l	Turbulent length scale, mixing length
l _o	Outer region mixing length
L _m	Mixing length parameter
L_{m_C}	Chapman's mixing length parameter
$L_{m_{VD}}$	Van Driest's mixing length parameter
M	Mach number
M _∞	Freestream Mach number
n	Number of stages in an identical component analog circuit
P	Static pressure
P _o	Stagnation pressure
Pr	Prandtl number
Pr_T	Turbulent Prandtl number
Q	Cumulative heating
q	Heat rate
q_w	Heat rate at the wall

R	Gas constant for air
R	Electrical resistance
RREF	Thin film resistance at temperature TREF
r	Recovery factor
r	Radial distance in the H-3 tunnel
r'	Distributed electrical resistance per unit length
Re	Reynolds Number per unit length
Rex	Local Reynolds Number, based on x distance downstream of the leading edge of the plate
s	Laplace variable
St	Stanton Number
T	Static temperature
Ta	Ambient temperature
T _{AW}	Adiabatic wall temperature
TF	Reference temperature for theoretical Stanton number calculation
T _∞	Freestream temperature
To	Stagnation temperature
TREF	Reference temperature, usually ambient
t	Time
U	Dorodnitsyn's transformation of streamwise flow velocity
u	Streamwise flow velocity
U ⁺	Nondimensional turbulent velocity scale
U _τ	Friction velocity

V	Dorodnitsyn's transformation of normal flow velocity
V	Electrical voltage
VOUT	Output voltage from analog circuit
VREF	Initial thin film voltage, taken at T_a
v	Normal flow velocity
ΔV	Film output voltage
X	Dorodnitsyn's transformation of streamwise spatial coordinate
x	Streamwise spatial coordinate
x	Streamwise direction in wind tunnel
x _{T,I}	Initial location of transition zone
x _{T,F}	Final location of transition zone
Y	Dorodnitsyn's transformation of normal spatial coordinate
Y	Normal spatial coordinate
⁺ Y	Nondimensional turbulent scale
α	Flat plate incidence angle, pitch up is positive
α	Thermal diffusivity
α_R	Film temperature coefficient of resistance
α^*	Clauser model factor to account for low Reynolds Number
β	Chapman's length scale
r	Transition parameter
γ	Specific heat ratio

Δ	Delta difference of some quantity
δ	Boundary layer thickness
δ^*	Kinematic displacement thickness
η	Transformed Y in the computational plane
θ	Normalized streamwise coordinate in transition zone
θ	Shock angle
θ_s	Numerical stability constant
μ	Dynamic viscosity
μ_T	Turbulent viscosity
ξ	Transformed X in the computational plane
ρ	Density
σ	Intermittency parameter
ξ	Extent of transition
θ	Delta wing incidence angle
∂	Partial derivative symbol
Ω	Delta wing sweep angle

Superscripts

n	Time level
s	Iteration level
#	Dimensional quantity
*	Laplace transformed variable
'	Mass averaged variable-fluctuation
~	Mass averaged variable-mean
-	Time averaged variable-fluctuation
-	Time averaged variable-mean

Subscripts

e	Edge condition
i,j	Spatial node point
∞	Freestream condition
o	Stagnation property

Abstract

This study combines the techniques of computational fluid dynamics and experimentation.

An unsteady laminar boundary layer code was shown to be more efficient when modified to solve the momentum and energy equations using Thomas' algorithm instead of optimized successive over-relaxation (SOR). A steady eddy viscosity turbulence model was then added to the code: two models for the inner region, Van Driest and Chapman, and two models for the outer region, a mixing length and Clauser were used. The computer code was verified against theoretical and experimental data from the literature. The code shows excellent results for laminar flow and each of the models showed very good agreement with turbulent experimental data.

The experimental study was conducted in a Mach 6 blowdown wind tunnel. Oil flow, sublimation, liquid crystals, and thin film gages were used to examine the flow structure and heat transfer on a flat plate. The heat transfer rates from these gages were lower than from the liquid crystals. The viscid-inviscid interaction flow region of flow past a blunt fin mounted on the flat plate was examined using the same flow visualization methods and possible flow interaction models are presented. Striation heating appeared when the fin incidence was 40° and the sweep was 60 or 75° .

The numerical and experimental techniques of this study were then merged. The heat transfer results from the flat plate experiment were compared with results from the computer code and were found to be in good agreement.

I. Introduction

This research effort, as well as this report based on it, consists of three main parts; a computational part, an experimental part, and a comparison between experimental data obtained and results from the computer program. The computational part was completed at the Air Force Institute of Technology (AFIT), Wright-Patterson Air Force Base, Ohio, while the experimental part was completed at the von Karman Institute for Fluid Dynamics (VKI), Rhode-Saint-Genese, Belgium. The experimental research was sponsored by HQ USAF, AFIT, VKI, and the Flight Dynamics Laboratory at Wright-Patterson Air Force Base, Ohio.

The computational effort consisted of modifying an unsteady laminar boundary layer code to a turbulent boundary layer code. The computational effort originally consisted of adding an unsteady turbulence model (38) to the unsteady laminar boundary layer code. However, due to time limitations, only a steady eddy viscosity turbulence model was added. Two models for the turbulent length scale were included, the Van Driest model and the Chapman model. The theory and results of these two models were compared with one another. Two models for the turbulent viscosity in the outer region are also examined; the mixing length model and Clauser's formulation of the mixing length model. A transition (laminar to turbulent) model was also included in the modification of the boundary layer code.

The experimental effort consisted of the study of a sharp nosed flat plate at zero degrees angle of attack. Heat transfer measurements were taken to determine the location of transition and turbulence. A swept blunt fin at incidence was placed on the flat plate and studied using flow visualization. This configuration was chosen due to its similarity to the configuration of the NASA Space Shuttle side fuselage and blunt fin. The Shuttle experiences high heat transfer rates on its side fuselage during re-entry, so the experimental effort originally was envisioned to concentrate on examining heat transfer rates on the flat plate on the expansion side of the blunt fin in laminar flow. Flight data has shown the existence of laminar flow on the Shuttle during many flight regimes including re-entry. Due to time limitations, only flow visualization tests were completed for this configuration. Effects of fin incidence and sweep angle are examined and possible models for the flow interaction region are suggested. Previous studies at VKI have examined this configuration on the expansion side (14), the compression side (15), (53), and the impingement portion (1) of the fin during turbulent flow.

The third part of this study ties the first and the second portions of this study together. Although computational fluid dynamics has recently become more important in determining flow properties about flight

vehicles, it can never exist without experimental data. As each computer code becomes developed, it must be verified for accuracy using experimental data. The third portion of this thesis blends a portion of the separate computational and experimental efforts by verifying the computer code using some of the experimental results from this study.

II. Computational Effort

2.1 Introduction

The computational aspect of this research dealt with the modification of an unsteady compressible laminar boundary layer code to a compressible turbulent boundary layer code. Time limitations caused only a steady turbulence model to be added to the code. A transition to turbulence model was also included. The applicable portions of the laminar boundary layer code are described in a later section.

Three major modifications were made to the original program. The momentum and energy equations were originally solved by the program using optimized successive over-relaxation. This was changed to take advantage of the diagonal dominance inherent in the finite differencing. Thomas' algorithm, which is a tridiagonal solver, was modified to a quad-diagonal solver. The momentum and energy equations were solved using this quad-diagonal solver.

The second major modification was the inclusion of the Cebeci-Smith turbulence model. Prandtl's mixing length theorem was used which included the Van Driest model for the turbulent length scale in the inner region. The Chapman model was also used for the inner region length scale and the differences between the two models are examined. For the outer region, two different models were examined and compared: a mixing length model and the Clauser model.

The third major modification was the addition of a transition to turbulence model. Given the location of the beginning of transition and the end of transition (fully turbulent), the model uses an exponential distribution to match the turbulent viscosity between the two regions.

2.2 Governing Equations

The Navier-Stokes equations for a viscous compressible flow are simplified using normal boundary layer assumptions. That is, gradients perpendicular to the surface are much larger than gradients parallel to the surface. The terms of the order of magnitude of the inverse square of the characteristic Reynolds number are neglected. The resulting boundary layer equations in conservative form are:

continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (2.1)$$

x-momentum

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = - \frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left[\mu \frac{\partial u}{\partial y} \right] \quad (2.2)$$

y-momentum

$$\frac{\partial P}{\partial y} = 0 \quad (2.3)$$

energy

$$\frac{\partial \rho^\# H^\#}{\partial t^\#} + \frac{\partial \rho^\# u^\# H^\#}{\partial x^\#} + \frac{\partial \rho^\# v^\# H^\#}{\partial y^\#} = \frac{\partial P^\#}{\partial t^\#} + \frac{\partial}{\partial y^\#} \left[k^\# \frac{\partial T^\#}{\partial y^\#} \right] + \frac{\partial}{\partial y^\#} \left[\mu^\# u^\# \frac{\partial u^\#}{\partial y^\#} \right] \quad (2.4)$$

These equations have boundary conditions at the edge of the boundary layer and at the wall.

2.2.1 Time and Mass-Weighted Averaging

In order to obtain the governing equations for turbulent flows, it is convenient to replace the instantaneous quantities in the equations by their mean and fluctuating quantities. A combination of conventional time averaging and mass-weighted averaging will be used as outlined in Cebeci and Smith (7:49-55).

The time averaged variables are defined as follows:

$$\begin{aligned} \bar{\rho} &= \bar{\rho} + \rho'' \\ \bar{P} &= \bar{P} + P'' \end{aligned} \quad (2.5)$$

where $\bar{\rho}$ and $\bar{P}$ are the time averages of the bulk density and pressure, respectively, and ρ'' and P'' are the superimposed density and pressure fluctuations, respectively. The time average or "mean" of any quantity, $q(t)$, is defined by

$$\bar{q}(t) = \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} q(t) dt \quad (2.6)$$

where $\bar{\omega}$, as defined by Cebeci and Smith (7:49), is taken to mean

"a time that is long compared to the reciprocal of the predominant frequencies in the spectrum of q ; in wind tunnel experiments, averaging times of a few seconds to a minute are usual."

For a fluctuating quantity $q''(t)$, the mean is zero by definition, i.e.,

$$\overline{q''(t)} = 0 \quad (2.7)$$

therefore,

$$\overline{\rho''} = 0 \quad (2.8)$$

$$\overline{p''} = 0$$

The mass-weighted variables are defined as follows:

$$\begin{aligned} \tilde{u} &= \bar{u} + u' \\ \tilde{v} &= \bar{v} + v' \\ \tilde{H} &= \bar{H} + H' \\ \tilde{T} &= \bar{T} + T' \end{aligned} \quad (2.9)$$

A mass-weighted mean quantity is defined as:

$$\tilde{q} = \frac{\overline{\rho q}}{\bar{\rho}} \quad (2.10)$$

where the bar denotes conventional time averaging and the tilde denotes mass-weighted averaging. A consequence of combining the time and mass-weighted averaging is that

$$\overline{\rho u'} = 0$$

$$\overline{\rho v'} = 0$$

$$\overline{\rho H'} = 0$$

$$\overline{\rho T'} = 0$$

(2.11)

Applying the mass-weighted averaging on the boundary layer equations and then taking the mean, results in a complicated set of equations. An order of magnitude analysis can be used to determine which terms are small enough that they may be neglected. An assumption used in the averaging technique is that fluctuations in the transport properties μ and k are ignored. Since the temperature is allowed to fluctuate, this is an important assumption since μ and k are both functions of the temperature. Details of the averaging are described in Appendix A. The order of magnitude analysis used in this study parallels the analysis in Cebeci and Smith (7:71-78). The resulting set of boundary layer equations for turbulent flow are:

continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (2.12)$$

x-momentum

$$\begin{aligned} \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho u v}{\partial y} + \frac{\partial \rho u u' v'}{\partial y} = - \frac{\partial P}{\partial x} \\ + \frac{\partial}{\partial y} \left[\mu \frac{\partial u}{\partial y} \right] \end{aligned} \quad (2.13)$$

y-momentum

$$\frac{\partial \bar{P}}{\partial Y} = 0 \quad (2.14)$$

energy

$$\begin{aligned} \frac{\partial \bar{\rho} \bar{H}}{\partial t} + \frac{\partial \bar{\rho} \bar{u} \bar{H}}{\partial x} + \frac{\partial \bar{\rho} \bar{v} \bar{H}}{\partial Y} + \frac{\partial \bar{\rho} \bar{v} \bar{H}'}{\partial Y} = \frac{\partial \bar{P}}{\partial t} \\ + \frac{\partial}{\partial Y} \left[\frac{k}{cp} \frac{\partial \bar{H}}{\partial Y} \right] + \frac{\partial}{\partial Y} \left[\left[\mu - \frac{k}{cp} \right] \frac{\partial}{\partial Y} \left(\frac{1}{2} \bar{u} \bar{u} \right) \right] \end{aligned} \quad (2.15)$$

These eqns (2.12), (2.13), (2.14), and (2.15) are similar to the corresponding original eqns, (2.1), (2.2), (2.3) and (2.4), except for an additional term in the x-momentum equation and one in the energy equation. The additional term in the momentum equation, is called the Reynold's stress. These two additional terms from the momentum and energy equations are listed in eqns (2.16) and (2.17) and must be defined before the governing equations can be solved.

$$\text{Reynold's stress term} \quad -\bar{\rho u' v'} \quad (2.16)$$

$$\text{energy transport term} \quad \bar{\rho v' H'} \quad (2.17)$$

Note the last two terms in the energy eqn (2.4) are the same as the last two terms in eqn (2.15), just rewritten in another form.

2.2.2 Closure Equations

Before the continuity, momentum, and energy equations, (2.12), (2.13), (2.14) and (2.15), can be solved, two more equations are needed for complete closure of the set of equations. Boussinesq (3:221) suggested that the apparent turbulent shear stresses might be related to the rate of mean strain through an apparent scalar turbulent or "eddy" viscosity. For the Reynold's stress term, eqn (2.16), the Boussinesq assumption gives

$$\overline{-\rho u'v'} = \mu_T \# \frac{\partial \bar{u}}{\partial Y} \quad (2.18)$$

where $\mu_T \#$ is a turbulent viscosity which will be defined later. The second closure equation (3:225; 7:239) uses a form of the Reynold's analogy which is based on the similarity between the transport of heat and momentum. Combining the Boussinesq assumption and Reynolds analogy:

$$\overline{\rho c_p v' T'} = -k_T \# \frac{\partial \bar{T}}{\partial Y} = -\mu_T \# \frac{c_p \#}{Pr} \frac{\partial \bar{T}}{\partial Y} \quad (2.19)$$

where $k_T \#$ is the turbulent conductivity. Substituting in the closure eqns (2.18) and (2.19), the eqns (2.12), (2.13), (2.14), and (2.15) become:

continuity

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} u}{\partial x} + \frac{\partial \bar{\rho} v}{\partial Y} = 0 \quad (2.20)$$

x-momentum

$$\frac{\partial \bar{\rho} \bar{u}}{\partial t} + \frac{\partial \bar{\rho} \bar{u} \bar{u}}{\partial x} + \frac{\partial \bar{\rho} \bar{u} \bar{v}}{\partial y} = - \frac{\partial \bar{P}}{\partial x} + \frac{\partial}{\partial y} \left[\left[\mu + \mu_T \right] \frac{\partial \bar{u}}{\partial y} \right] \quad (2.21)$$

y-momentum

$$\frac{\partial \bar{P}}{\partial y} = 0 \quad (2.22)$$

energy

$$\begin{aligned} \frac{\partial \bar{\rho} \bar{H}}{\partial t} + \frac{\partial \bar{\rho} \bar{u} \bar{H}}{\partial x} + \frac{\partial \bar{\rho} \bar{v} \bar{H}}{\partial y} &= \frac{\partial \bar{P}}{\partial t} + \frac{\partial}{\partial y} \left[\left[\frac{\mu}{Pr} + \mu_T \right] \frac{\partial \bar{H}}{\partial y} \right] \\ &+ \frac{\partial}{\partial y} \left[\mu (1 - 1/Pr) \frac{\partial (1/2 \bar{u} \bar{u})}{\partial y} \right] \\ &+ \frac{\partial}{\partial y} \left[\mu_T \left[1 - 1/Pr \right] \frac{\partial (1/2 \bar{u} \bar{u})}{\partial y} \right] \end{aligned} \quad (2.23)$$

Note that in many references, since the last term in equation (2.23) is very small, it is usually omitted.

The substitution of the Reynolds stress term into the x-momentum equation is mathematically trivial. However, for completeness, the mathematical steps of substituting the closure equations into the energy equation are displayed in detail in Appendix B.

2.2.3 Nondimensionalization

The turbulent boundary layer equations (2.20), (2.21), (2.22), and (2.23), are nondimensionalized using these factors.

$$\begin{aligned} x &= \frac{x\#}{L\#} & Y &= \frac{Y\#}{L\#} & t &= \frac{t\#U_{\infty}\#}{L\#} & u &= \frac{u\#}{U_{\infty}\#} & v &= \frac{v\#}{U_{\infty}\#} \\ \mu &= \frac{\mu\#}{\mu_{\infty}\#} & \phi &= \frac{\phi\#}{\phi_{\infty}\#} & H &= \frac{H\#}{U_{\infty}\#^2} & P &= \frac{P\#}{\phi_{\infty}\#U_{\infty}\#^2} \\ Pr &= \frac{c_p\#\mu\#}{k\#} & Re &= \frac{u\#L\#\phi\#}{\mu\#} \end{aligned} \quad (2.24)$$

The resulting turbulent boundary layer equations are:
continuity

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi u}{\partial x} + \frac{\partial \phi v}{\partial Y} = 0 \quad (2.25)$$

x-momentum

$$\frac{\partial \phi u}{\partial t} + \frac{\partial \phi u u}{\partial x} + \frac{\partial \phi u v}{\partial Y} = - \frac{\partial P}{\partial x} + \frac{\partial}{\partial Y} \left[\left[\mu + \mu_T \right] \frac{1}{Re} \frac{\partial u}{\partial Y} \right] \quad (2.26)$$

Y-momentum

$$\frac{\partial P}{\partial Y} = 0 \quad (2.27)$$

energy

$$\begin{aligned} \frac{\partial \phi H}{\partial t} + \frac{\partial \phi u H}{\partial x} + \frac{\partial \phi v H}{\partial Y} &= \frac{\partial P}{\partial t} + \frac{\partial}{\partial Y} \left[\frac{1}{Re} \left[\frac{\mu}{Pr} + \mu_T / Pr_T \right] \frac{\partial H}{\partial Y} \right] \\ &+ \frac{\partial}{\partial Y} \left[\frac{1}{Re} \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \frac{\partial}{\partial Y} \left(\frac{1}{2} \overline{uu} \right) \right] \end{aligned} \quad (2.28)$$

2.2.4 Dorodnitsyn Transformation

The Dorodnitsyn transformation (27:2-4) is used to scale these unsteady compressible boundary layer equations into a form similar to the incompressible boundary layer equations. This transformation is

$$\begin{aligned} X &= x & Y &= \int_0^Y \phi \, dy & t &= t & H &= H \\ U &= u & V &= \phi v + \frac{\partial Y}{\partial t} + \frac{u \partial Y}{\partial x} \end{aligned} \quad (2.29)$$

The nondimensional governing equations for turbulent boundary layer flow become

continuity

$$\frac{\partial \bar{U}}{\partial X} + \frac{\partial \bar{V}}{\partial Y} = 0 \quad (2.30)$$

x-momentum

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial X} + \bar{V} \frac{\partial \bar{U}}{\partial Y} = - \frac{1}{\phi} \frac{\partial \bar{P}}{\partial X} + \frac{\partial}{\partial Y} \left[\left[\mu + \mu_T \right] \frac{\phi}{Re} \frac{\partial \bar{U}}{\partial Y} \right] \quad (2.31)$$

Y-momentum

$$\frac{\partial \bar{P}}{\partial Y} = 0 \quad (2.32)$$

energy

$$\begin{aligned} \frac{\partial \bar{H}}{\partial \bar{t}} + \frac{\bar{U} \partial \bar{H}}{\partial \bar{X}} + \frac{\bar{V} \partial \bar{H}}{\partial \bar{Y}} = \frac{1}{\bar{\rho}} \frac{\partial \bar{P}}{\partial \bar{t}} + \frac{\partial}{\partial \bar{Y}} \left[\left[\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T} \right] \frac{\bar{\rho}}{Re} \frac{\partial \bar{H}}{\partial \bar{Y}} \right] \\ + \frac{\partial}{\partial \bar{Y}} \left[\frac{\bar{\rho}}{Re} \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \frac{\partial \bar{U}^2 / 2}{\partial \bar{Y}} \right] \end{aligned} \quad (2.33)$$

Now that the governing equations have been derived, the set of equations can be solved once the turbulent viscosity is known. Several models for the turbulent viscosity will be presented in section 2.4.2.

2.3 Description of Original Program

The original computer program solves the unsteady compressible boundary layer equations for laminar flow. The boundary layer equations are first nondimensionalized and then transformed using the Dorodnitsyn transformation. Then these equations are transformed to the computational plane and finite differenced. The momentum and energy equations are solved using optimized successive over-relaxation, while the method of solving the continuity equation has been modified by Coffey (11:34) and solves for the V velocity using trapezoidal integration.

The wall, edge, and initial conditions for the current work are all specified exactly as in the original program (34:34-36). However, the plate was assumed to be stationary

throughout all the investigations, ie., not pitching. Since all the cases performed for this effort were for a flat plate, the edge conditions were the same as the infinity conditions.

2.3.1 Grid

In the program, the flat plate length is 1.25 feet in the x# direction and approximately two times the fully turbulent boundary layer thickness in the Y# direction. The program originally had only 61 points in the streamwise direction and 30 points in the normal direction. Although the number of grid points were significantly fewer than other schemes, it produced similar results when compared to other computer codes and theory, but at a faster rate and less cost due to its sparse grid. However, more points in the normal direction were added when the turbulence model was introduced to the program. The program currently has a 61 by 59 point grid.

One of the unique features of the original code is a generalized grid. This adds great capability to the code by enabling it to handle several configurations. For the present analysis, the physical grid varied exponentially in the x# direction. The Y# step size varied according to an exponential and power law velocity profile (21). In this study, the grid had a 1/2 power law which is optimum for laminar flow. For fully turbulent flow, a 1/7 power law should be used for the grid. In cases of transition to

turbulence, an optimized grid could be used. This optimized grid could be developed using a grid generation program (22) and then optimized using the adaptive-grid method to minimize truncation error as explained by Coffey (11).

Since the flow is compressible, density variations will essentially cause the physical grid to change. The Dorodnitsyn transformation eliminates the density fluctuations and allows the program to utilize a fixed grid. The Dorodnitsyn fixed grid is shown in Figures 2.1 and 2.3. The physical grid at steady state is shown in Figures 2.2 and 2.4. Both the full grid and an expanded view are shown. Note that the grid is smooth in the computational (Dorodnitsyn grid) plane, but is not in the physical plane at the transition location of $x/L = 0.1$.

2.3.2 Computational Grid

A general transformation can be made to a rectangular computational grid. This grid is in the $\xi - \eta$ plane where ξ is in the streamwise direction and η is in the normal direction. The metrics ξ_X , ξ_Y , η_X , and η_Y that appear in the differential equations are solved using

$$\begin{aligned}\xi_X &= Y_\eta / J \\ \xi_Y &= -X_\eta / J \\ \eta_X &= -Y_\xi / J \\ \eta_Y &= X_\xi / J\end{aligned}\tag{2.34}$$

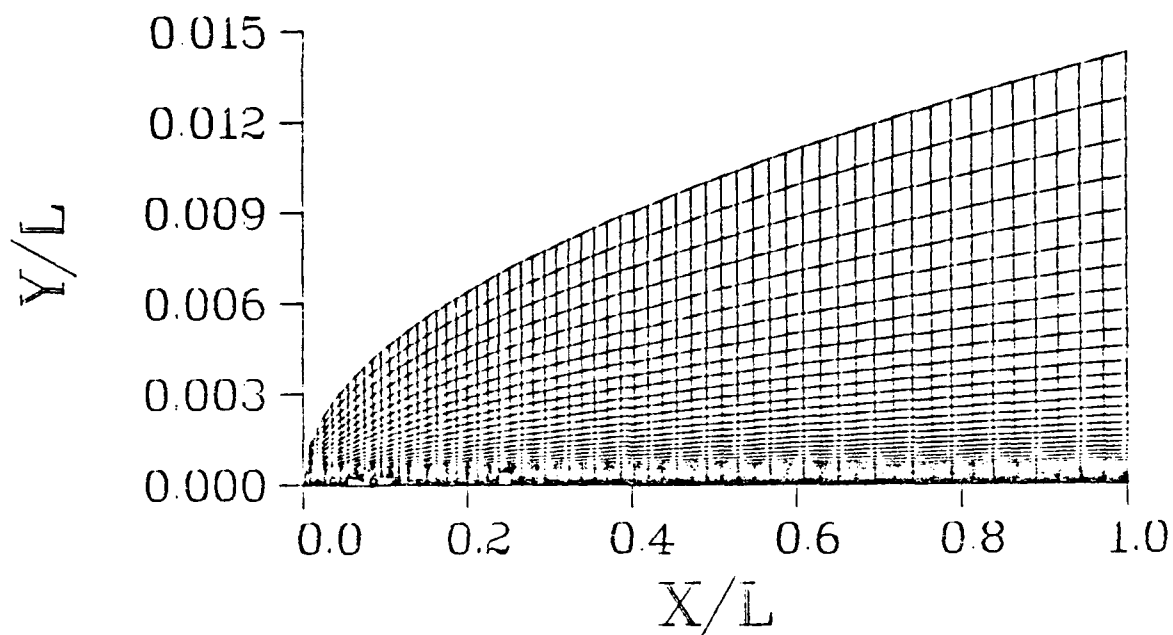


Figure 2.1. Dorodnitsyn Fixed Grid

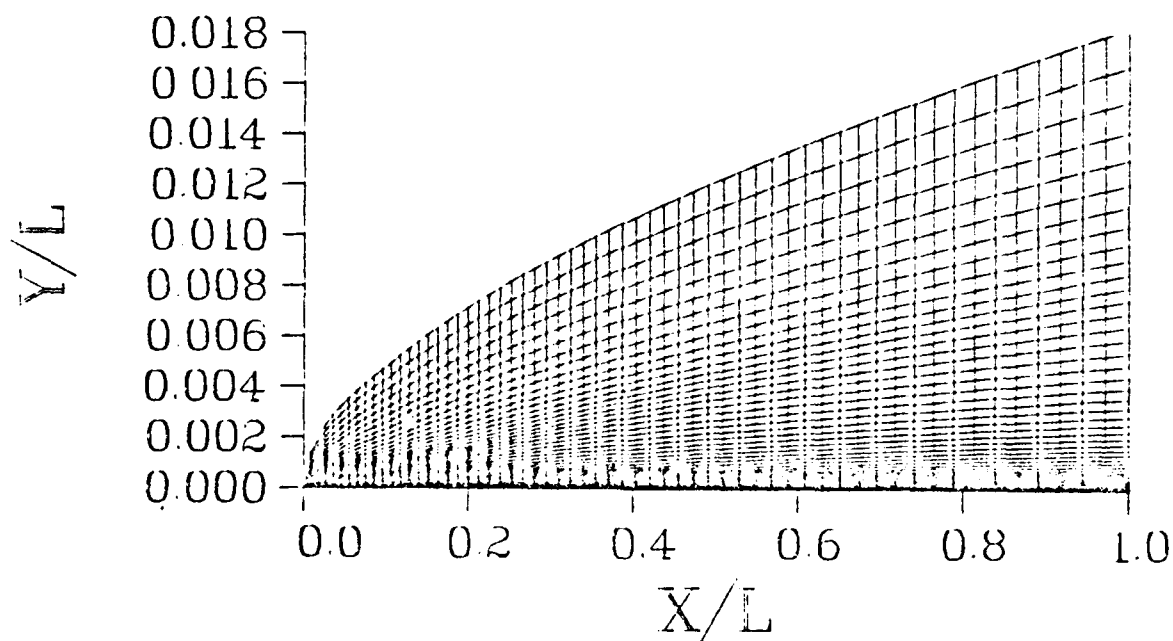


Figure 2.2. Physical Grid

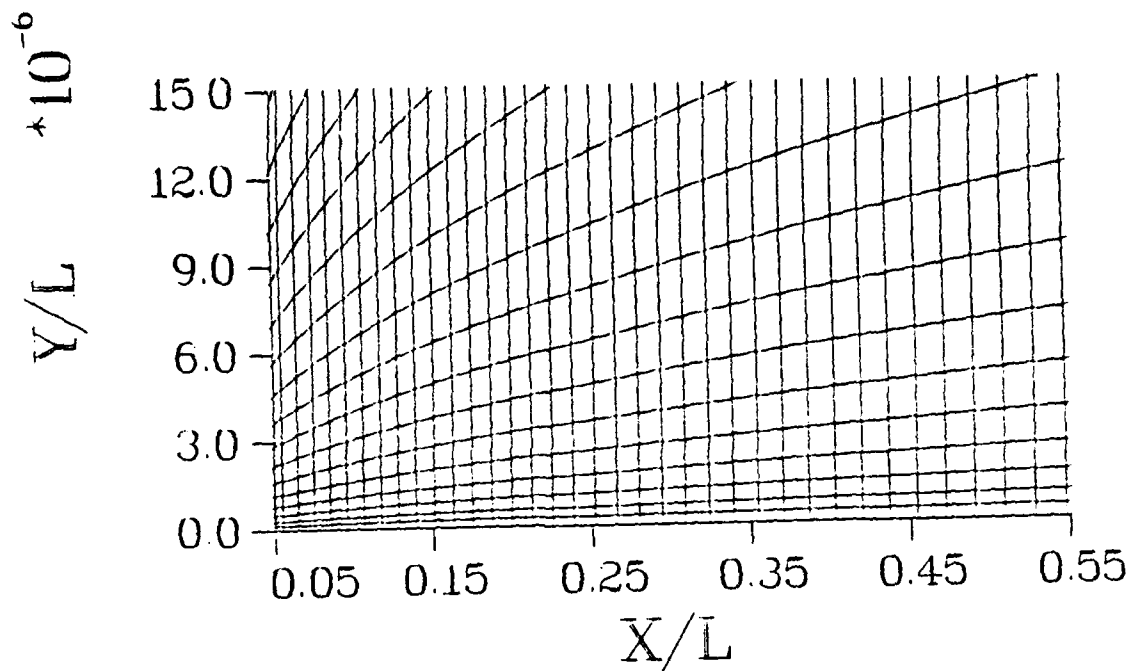


Figure 2.3. Dorodnitsyn Fixed Grid - Expanded View

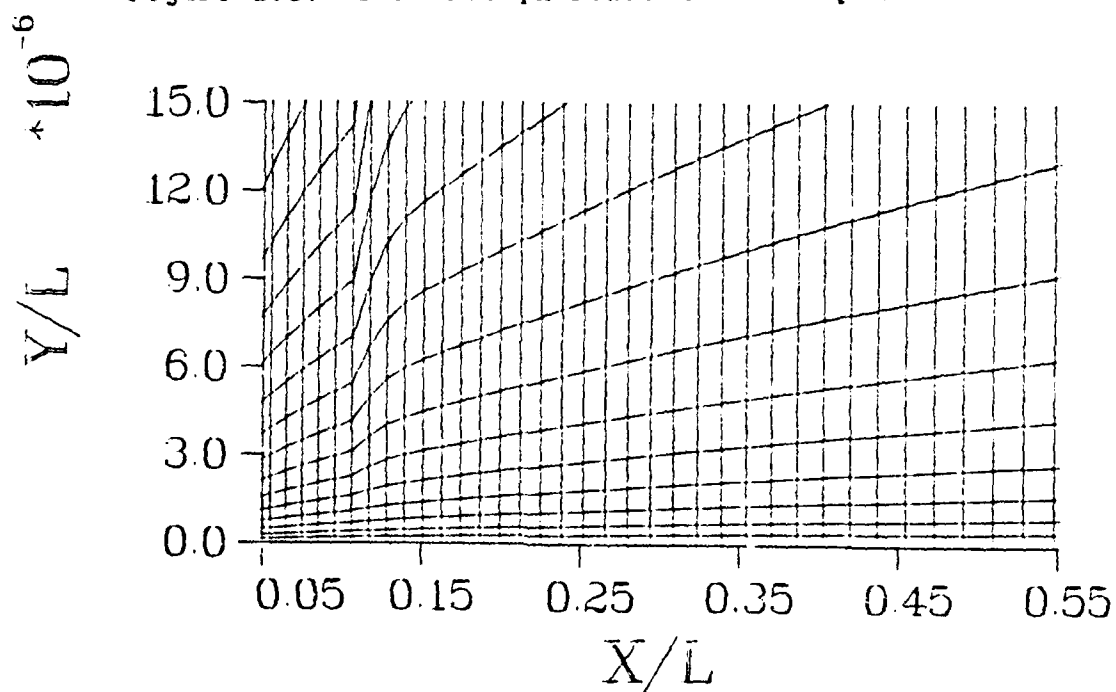


Figure 2.4. Physical Grid - Expanded View

where J is the Jacobian of the transformation,

$$J = X_\xi Y_\eta - Y_\xi X_\eta \quad (2.35)$$

The terms X_ξ , X_η , and Y_η are determined using finite differences. As in the original program, X does not change in the η direction so $X_\eta = 0$. The Y_ξ metric is determined differently than in Lange's thesis. Hodge (21) developed an analytical metric

$$Y_\xi = \frac{0.5 Y}{X_\xi X} \quad (2.36)$$

which increases the code's applicability to also include variable X step sizes. Calculating the Y_ξ metric analytically near the leading edge and then using a numerical solution for the remaining points downstream of the leading edge avoids the large leading edge error as found by Lange (34:27). The numerical differencing is the same as described in Lange's thesis. Hodge (21) further modified the program to accept as an input the point from the leading edge at which the program switches from the analytic to the numerical calculation of the Y_ξ metric.

2.3.3 Numerical Method

The unsteady two-dimensional boundary layer equations are solved numerically by using an implicit finite difference scheme because it is unconditionally stable according to linear theory. The governing equations are first nondimensionalized, then transformed by the Dorodnitsyn transformation. Finally, they are then

transformed from the X - Y plane to the $\xi - \eta$ computational plane.

Once the equations are transformed to the $\xi - \eta$ plane, the first order spatial derivatives in the momentum and energy equations are differenced using three point upwind differencing. This differencing scheme guarantees diagonal dominance and is second order accurate in the computational plane. The time derivative is differenced using a two point backward scheme which implies it is only first order accurate in time. The second derivatives are approximated with second order accurate central differencing. Similar terms are then combined.

2.4 Modifications to Original Program

The modifications to the original laminar program that exist in the turbulent boundary layer code are explained in this section in detail. A copy of the turbulent boundary layer code is listed in Appendix C.

The unsteady two-dimensional boundary layer equations have been time and mass-weighted averaged, non-dimensionalized, and transformed using the Dorodnitsyn transformation. The governing equations are then transformed from the X - Y plane to the $\xi - \eta$ computational plane. The continuity, x-momentum, and energy equations (2.30), (2.31), and (2.33) become

continuity

$$\xi \frac{\bar{U}}{X} + \eta \frac{\bar{U}}{X} + \eta \frac{\bar{V}}{Y} = 0 \quad (2.37)$$

x-momentum

$$\begin{aligned} \bar{U}_t + \bar{U} \left[\bar{U}_x \xi_X + \bar{U}_\eta \eta_X \right] + \bar{V} \bar{U}_\eta \eta_Y \\ = \eta_Y \left[\left[\mu + \mu_T \right] \varphi \left[\bar{U}_\eta \eta_Y \right] \right] \eta \frac{1}{Re} \end{aligned} \quad (2.38)$$

energy

$$\begin{aligned} \bar{H}_t + \bar{U} \left[\bar{H}_x \xi_X + \bar{H}_\eta \eta_X \right] + \bar{V} \bar{H}_\eta \eta_Y = \frac{1}{\varphi} \bar{P}_t \\ + \left[\frac{\varphi}{Re} \left[\frac{\mu}{Pr} + \mu_T / Pr \right] \bar{H}_\eta \eta_Y \right] \eta_Y \\ + \left[\frac{\varphi}{Re} \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr \right] \right] \eta_Y (1/2 \bar{U}^2)_\eta \right] \eta_Y \end{aligned} \quad (2.39)$$

On a stationary flat plate, the pressure gradients in the normal direction are zero from the Y-momentum equation eqn (2.32). The pressure gradient in the streamwise direction has been assumed to be zero since any shock-boundary layer interaction is ignored. Now that the equations have been transformed to the $\xi - \eta$ plane, the derivatives are finite differenced similar to the original program (cf. section 2.3). The equations for the x-momentum and the energy equation (2.38) and (2.39) become,

Momentum

$$\begin{aligned}
 U_{i,j}^n = & \left[U_{i,j}^{n-1} + 2\bar{\epsilon}_X \bar{U} \Delta t U_{i-1,j}^n - .5 \bar{\epsilon}_X \bar{U} \Delta t U_{i-2,j}^n \right. \\
 & - .5 \left[\eta_X \bar{U} \Delta t + \eta_Y \tilde{V} \Delta t \right] U_{i,j-2}^n \\
 & + \left[2\eta_X \bar{U} \Delta t + 2\eta_Y \tilde{V} \Delta t + \eta_Y \frac{\Delta t}{Re} \left[\rho \left[\mu + \mu_T \right] \right]_{j+1/2} \eta_Y \right] U_{i,j-1}^n \\
 & \left. + \left[\eta_Y \frac{\Delta t}{Re} \left[\rho \left[\mu + \mu_T \right] \right]_{j+1/2} \eta_Y \right] U_{i,j+1}^n \right] / UCOEFF
 \end{aligned}$$

(2.40)

Energy

$$\begin{aligned}
 H_{i,j}^n = & \left[H_{i,j}^{n-1} + 2\bar{\epsilon}_X \bar{U} \Delta t H_{i-1,j}^n - .5 \bar{\epsilon}_X \bar{U} \Delta t H_{i-2,j}^n + \frac{1}{\bar{\rho}} \frac{dP}{dx} \right. \\
 & - .5 \left[\eta_X \bar{U} \Delta t + \eta_Y \tilde{V} \Delta t \right] H_{i,j-2}^n \\
 & + \left[2\eta_X \bar{U} \Delta t + 2\eta_Y \tilde{V} \Delta t + \eta_Y \frac{\Delta t}{Re} \left[\rho \left[\mu/Pr + \mu_T/Pr \right] \right]_{j-1/2} \eta_Y \right] H_{i,j-1}^n
 \end{aligned}$$

$$\begin{aligned}
& + q_Y \frac{\Delta t}{Re} \left[\exp \left[\mu / Pr + \mu_T / Pr_T \right] \right]_{j+1/2} q_Y H_{i,j+1}^n \\
& + q_Y \frac{\Delta t}{2Re} \left[\exp \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j+1/2} q_Y U_{i,j+1}^n \\
& - \left[\exp \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j+1/2} q_Y \\
& + \left[\exp \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j-1/2} q_Y U_{i,j}^n \\
& + \left[\exp \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j-1/2} q_Y U_{i,j-1}^n \right] / UCoeff
\end{aligned}
\tag{2.41}$$

where

$$UCOEFF = 1 + \frac{3}{2} \xi \bar{U} \Delta t + \frac{3}{2} \eta \bar{U} \Delta t + \frac{3}{2} \eta \tilde{V} \Delta t$$

$$+ \eta_Y \frac{\Delta t}{Re} \left[\left[\rho \left[\mu / Pr + \mu_T / Pr_T \right] \right]_{j+1/2} \eta_Y \right. \\ \left. + \left[\rho \left[\mu / Pr + \mu_T / Pr_T \right] \right]_{j-1/2} \eta_Y \right] \quad (2.42)$$

Note that i and j are the node points in the $\xi - \eta$ plane, and n is the time level. The momentum and energy equations are nonlinear. To linearize the equations for the first iteration, the coefficients are lagged "in time". The coefficients in this program are based on the solution for U , V , and H at the previous time and same location for the first iteration, and subsequently from each previous iteration.

The linearization is the same as the original program. The calculation of the metrics also remain the same as developed by Hodge and Lange (cf. section 2.3.2). Once the momentum and energy equations are linearized, the equations can be written as:

Momentum

$$\begin{aligned}
 U_{i,j}^{s+1} = & \left[U_{i,j}^{n-1} + \frac{2\Delta t}{X_s} U_{i,j}^s U_{i-1,j}^{s+1} - \frac{\Delta t}{2X_s} U_{i,j}^s U_{i-2,j}^{s+1} \right. \\
 & - .5\Delta t \left[U_{i,j}^s \eta_X + \frac{1}{Y\eta} V_{i,j}^s \right] U_{i,j-2}^s \\
 & + 2\Delta t \left[\frac{1}{\left(\frac{Y_{i,j+1} - Y_{i,j-1}}{Y_{i,j} - Y_{i,j-1}} \right) \text{Re} \left(\frac{Y_{i,j} - Y_{i,j-1}}{Y_{i,j} - Y_{i,j-1}} \right)} \left[\rho \left[\mu + \mu_T \right] \right]_{j-1/2}} \right. \\
 & \left. + U_{i,j}^s \eta_X + \frac{1}{Y\eta} V_{i,j}^s \right] U_{i,j-1}^s \\
 & \left. + 2\Delta t \left[\frac{\left[\rho \left[\mu + \mu_T \right] \right]_{j+1/2}}{\left(\frac{Y_{i,j+1} - Y_{i,j-1}}{Y_{i,j+1} - Y_{i,j}} \right) \text{Re} \left(\frac{Y_{i,j+1} - Y_{i,j}}{Y_{i,j+1} - Y_{i,j}} \right)} U_{i,j+1}^s \right] \right] / \text{UCOEFF}
 \end{aligned} \tag{2.43}$$

Energy

$$H_{i,j}^{s+1} = \left[H_{i,j}^{n-1} + \frac{2\Delta t}{X_s} U_{i,j}^s H_{i-1,j}^{s+1} - \frac{\Delta t}{X_s} U_{i,j}^s H_{i-2,j}^{s+1} \right]$$

$$\begin{aligned}
& - .5 \Delta t \left[U_{1,j}^s \frac{q}{X} + \frac{1}{Y_{\infty}} V_{1,j}^s \right] H_{1,j-2}^s + \frac{1}{\bar{\rho}} dP \\
& + 2\Delta t \left[\frac{1}{\frac{(Y_{1,j+1} - Y_{1,j-1}) Re}{(Y_{1,j+1} - Y_{1,j})}} \left[\phi \left[\mu/Pr + \mu_T/Pr_T \right] \right]_{j-1/2} \right. \\
& \quad \left. + U_{1,j}^s \frac{q}{X} + \frac{1}{Y_{\infty}} V_{1,j}^s \right] H_{1,j-1}^s \\
& + 2\Delta t \left[\frac{1}{\frac{(Y_{1,j+1} - Y_{1,j-1}) Re}{(Y_{1,j+1} - Y_{1,j})}} \left[\phi \left[\mu/Pr + \mu_T/Pr_T \right] \right]_{j+1/2} \right] H_{1,j+1}^s \\
& + 2\Delta t \left[\frac{1}{\frac{(Y_{1,j+1} - Y_{1,j-1}) Re}{(Y_{1,j+1} - Y_{1,j})}} \left[\phi \left[\mu(1-1/Pr) + \mu_T \left[1-1/Pr_T \right] \right] \right]_{j+1/2} \right] \frac{U_{1,j+1}^s}{2}
\end{aligned}$$

$$\begin{aligned}
& - \left[\frac{\left[\phi \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j+1/2}}{(Y_{1,j+1} - Y_{1,j})} \right] \\
& + \left[\frac{\left[\phi \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j-1/2}}{(Y_{1,j} - Y_{1,j-1})} \right] \left[\frac{U_n^2}{2} \right]_{1,j} \\
& + \left[\frac{\left[\phi \left[\mu(1 - 1/Pr) + \mu_T \left[1 - 1/Pr_T \right] \right] \right]_{j-1/2}}{(Y_{1,j} - Y_{1,j-1})} \right] \left[\frac{U_n^2}{2} \right]_{1,j-1} / \text{UCOEFF}
\end{aligned}
\tag{2.44}$$

where

$$\text{UCOEFF} = 1 + \frac{3}{2} \frac{U_n^2}{X_e} \Delta t + \frac{3}{2} \frac{U_n^2}{1,j} \frac{\eta}{X} \Delta t + \frac{3}{2} \frac{V_n^2}{Y_e} \Delta t$$

$$\begin{aligned}
& + \frac{2\Delta t}{(Y_{1,j+1} - Y_{1,j-1}) \text{Re}} \left[\frac{\left[\varphi \left[\mu/Pr + \mu_T/Pr_T \right] \right]_{j+1/2}}{(Y_{1,j+1} - Y_{1,j})} \right. \\
& \quad \left. + \frac{\left[\varphi \left[\mu/Pr + \mu_T/Pr_T \right] \right]_{j-1/2}}{(Y_{1,j} - Y_{1,j-1})} \right] \quad (2.45)
\end{aligned}$$

and

$$\begin{aligned}
X_x &= (X_{1+1,j} - X_{1-1,j})/2 \\
Y_{\eta} &= (Y_{1,j+1} - Y_{1,j-1})/2 \\
\eta &= \frac{-0.5 Y}{X_x X Y_{\eta}} = - \frac{Y_x}{X_x Y_{\eta}} \\
[\varphi(\mu^*)]_{j+1/2} &= ([\varphi(\mu^*)]_{j+1} + [\varphi(\mu^*)]_j)/2 \\
[\varphi(\mu^*)]_{j-1/2} &= ([\varphi(\mu^*)]_j + [\varphi(\mu^*)]_{j-1})/2 \quad (2.46)
\end{aligned}$$

where

$$\mu^* = \mu/Pr + \mu_T/Pr_T$$

or

$$\mu^* = \mu(1 - 1/Pr) + \mu_T(1 - 1/Pr_T)$$

and s is the iteration level. These equations are then solved in the program using an extension of Thomas' algorithm.

2.4.1 Thomas' Algorithm

Thomas' algorithm (3:99) is a technique for rapidly solving a tridiagonal system of linear algebraic equations. It is important to note that in any tridiagonal elimination scheme, as is Thomas' algorithm, truncation errors can grow quickly if diagonal dominance is not maintained. However, the finite differencing methods used in this program guarantee the existence of diagonal dominance. A diagonally dominant system guarantees errors are not propagated (3:343). The scheme of the Thomas' algorithm is as follows. Given a system of equations, M is the matrix of coefficients, U is the vector of independent variables, and E is the vector of dependent variables.

$$M_{1,j-1} U_{j-1} + M_{1,j} U_j + M_{1,j+1} U_{j+1} = E_i \quad (2.47)$$

This system of equations is a tridiagonal system, ie, the matrix M has non-zero values only on the main diagonal and the lower and upper diagonal. The system of equations can be rewritten

$$\begin{bmatrix}
 d_1 & a_1 & 0 & 0 & 0 \\
 b_2 & d_2 & a_2 & 0 & 0 \\
 \cdot & \cdot & \cdot & \cdot & \cdot \\
 0 & b_{JMAX-2} & d_{JMAX-2} & a_{JMAX-2} & 0 \\
 0 & 0 & b_{JMAX-1} & d_{JMAX-1} & a_{JMAX-1} \\
 0 & 0 & 0 & b_{JMAX} & d_{JMAX}
 \end{bmatrix}
 \begin{bmatrix}
 U_1 \\
 U_2 \\
 \vdots \\
 U_{JMAX-2} \\
 U_{JMAX-1} \\
 U_{JMAX}
 \end{bmatrix}
 =
 \begin{bmatrix}
 E_1 \\
 E_2 \\
 \vdots \\
 E_{JMAX-2} \\
 E_{JMAX-1} \\
 E_{JMAX}
 \end{bmatrix}
 \quad (2.48)$$

Thomas' algorithm works by eliminating the "b" diagonal and then solving for the U vector beginning at the Jmax location and stepping backwards. After the "b" diagonal is eliminated, the new values of the "d" and "a" diagonal are computed by

$$d_j = d_j - \frac{b_j a_{j-1}}{d_{j-1}} \quad (2.49)$$

$$a_j = a_j \quad (2.50)$$

$$e_j = e_j - \frac{b_j e_{j-1}}{d_{j-1}} \quad (2.51)$$

The values for U may be obtained now by back substitution,

$$U_{Jmax} = \frac{e_{Jmax}}{d_{Jmax}} \quad (2.52)$$

$$U_j = \frac{e_j - a_j U_{j+1}}{d_j} \quad \text{for } j = Jmax-1, Jmax-2, \dots, 1 \quad (2.53)$$

When examining the governing equations for the momentum and energy equation (2.43) and (2.44), it can be seen that this particular system of equations is not a tridiagonal, but rather a penta-diagonal. An example of this penta-diagonal matrix can be seen in eqn (2.54).

The existence of the fourth or fifth diagonal depends on the sign of the V velocity. If the velocity is positive, the velocity gradients are backward differenced, and an $i, j-2$ term, "f" diagonal in eqn (2.54), will appear. If the velocity is negative the velocity gradients are forward differenced, and an $i, j+2$ term, "c" diagonal in eqn (2.54), will appear.

$$\begin{bmatrix}
 d1 & a1 & c1 & 0 & 0 & 0 \\
 b2 & d2 & a2 & c2 & 0 & 0 \\
 f3 & b3 & d3 & a3 & c3 & 0 \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 0 & fJMAX-2 & bJMAX-2 & dJMAX-2 & aJMAX-2 & cJMAX-2 \\
 0 & 0 & fJMAX-1 & bJMAX-1 & dJMAX-1 & aJMAX-1 \\
 0 & 0 & 0 & fJMAX & bJMAX & dJMAX
 \end{bmatrix} \quad (2.54)$$

This situation is handled in the program differently depending on the sign of the velocity. If the velocity is positive, the $i, j-2$ term is a known value at the previous iteration and included in the e vector and the matrix is solved as a tridiagonal system of equations. If the velocity is negative, the $i, j+2$ term is an unknown and the

matrix is solved as a quad-diagonal. Eqns (2.50) and (2.53) need to be redefined if a quad-diagonal exists. The recursion formulas become:

$$a_j = a_j - \frac{b_j c_{j-1}}{d_{j-1}} \quad (2.55)$$

$$c_j = c_j \quad (2.56)$$

and then for the back substitution,

$$U_{J_{\max}-1} = \frac{e_{J_{\max}-1} - a_{J_{\max}-1} U_{J_{\max}}}{d_{J_{\max}-1}} \quad (2.57)$$

$$U_j = \frac{e_j - a_j U_{j+1} - c_j U_{j+2}}{d_j} \quad (2.58)$$

where $j = J_{\max}-2, J_{\max}-3, \dots, 1$. This modification of the use of Thomas' algorithm should improve the efficiency of the program, relative to the efficiency using the optimized successive over-relaxation. A comparison of execution time was completed and the results are shown in a later section.

2.4.2 Turbulence Model

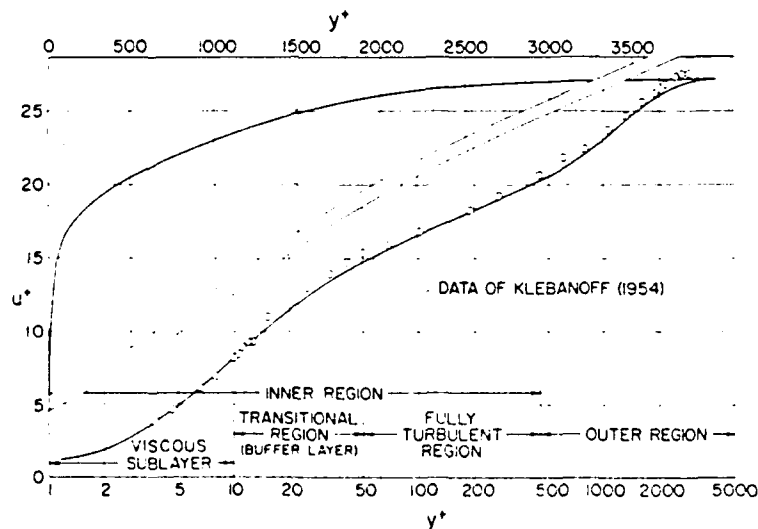
Turbulent boundary layer flow is divided into two regions: an inner region, and an outer region. The inner region is further subdivided into three regions, the viscous sublayer, the transitional region, and the fully turbulent region. The inner region of a turbulent boundary layer is much smaller than the outer region, with a thickness of about 10 to 20 % of the entire boundary layer thickness. An

example of these regions are illustrated in Figure 2.5. To deal with the different regions, some special parameters (7:95) have been defined. They are:

$$U^+ = \left(\frac{\tau_w}{\rho_w} \right)^{1/2} \quad (2.59)$$

$$Y^+ = \frac{y U^+ \rho_w}{\mu_w} \quad (2.60)$$

$$U^+ = U / U^+ \quad (2.61)$$



Semilogarithmic and linear plots of mean velocity distribution across a turbulent boundary layer with zero pressure gradient. The linear plot is included to show a true picture of the thickness of various portions. (7:94)

Figure 2.5. Turbulent Boundary Layer Regions

If $Y^+ < 10.0$, the region is typically considered to be the

viscous sublayer. If $10.0 < Y^+ < 500.0$, the region is typically defined as the rest of the inner region. If $Y^+ > 500.0$, the region is typically the outer region. These limits are very dependent upon the specific flow conditions.

In the viscous sublayer, molecular viscosity is assumed to be dominant over the turbulent viscosity, so that the turbulence viscosity approaches zero in this region. For the rest of the inner region, two models will be considered in this study; the Van Driest and Chapman models. In the outer region, the mixing layer model and Clauser's mixing length model have been included. The mixing length model has the turbulent length scale varying proportionately with the boundary layer thickness. Clauser's model has the turbulent viscosity varying proportionately with the kinematic displacement thickness. Clauser's model is recommended for compressible flow cases (7:120; 49:8), since it has been shown in several references to be more accurate.

Turbulence models center around the definition of the turbulent viscosity. Prandtl's Mixing Length Theorem states that the turbulent viscosity is related to two parameters: a turbulent length scale - called a mixing length, l , and the U velocity gradient in the normal direction.

$$\mu_T = \rho l^2 \left| \frac{\partial U}{\partial Y} \right| \quad (2.62)$$

The two models for the inner region mixing length; the Van Driest model and the Chapman model, will now be examined.

This will be followed by the two models for the outer region, the mixing layer model and Clauser's model.

2.4.2.1 Inner Region Van Driest Model

The Van Driest model for the mixing length for turbulent flow was developed by considering flow over an oscillating plate (Stokes flow). Van Driest modified Prandtl's mixing length equation so that

$$l\# = KY\# [1 - \exp (-Y\#/A\#)] \quad (2.63)$$

where K is von Karman's constant. K is nominally 0.41, but can be changed in the program for each case since K is an input. If the turbulent viscosity, eqn (2.62) is rewritten,

$$\mu\# = \rho\# (KY\#)^2 L_m \left| \frac{\partial U\#}{\partial Y\#} \right| \quad (2.64)$$

where L_m can be called a new mixing length parameter defined by Van Driest (7:213-215) as

$$L_{m\text{VD}} = [1 - \exp (-Y\#/A\#)]^2 \quad (2.65)$$

The $A\#$ defined originally by Van Driest is limited to incompressible turbulent boundary layers with negligible pressure gradient and zero mass transfer. Cebeci and Smith (7:215-217) include compressibility effects by adding a density ratio to the parameter $A\#$ so it becomes,

$$A\# = A^+ (\mu\#/\rho\#) \left(\tau\#/\rho\# \right)^{-1/2} \left(\rho\#/\rho\# \right)^{1/2} \quad (2.66)$$

where $A^+ = 26.0$.

If the definition for Y^+ , eqn (2.60), is solved for $Y\#$ and substituted in eqn (2.65), $L_{m\text{VD}}$ can be rewritten as

$$L_{m_{VD}} = (1 - \exp[-(Y/A)^2])^2 \quad (2.67)$$

Now that the model has been defined, it is of interest to determine what kind of behavior the Van Driest's mixing length parameter contributes to the Reynold's stress or turbulent viscosity near the wall. If the series expansion for $\exp(-Y\#/A\#)$ near the wall, is

$$\exp(-Y/A) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (Y/A)^n = 1 - Y/A + \text{H.O.T.} \quad (2.68)$$

(where H.O.T. means higher order terms) and is substituted into eqn (2.65), it can be seen that

$$L_{m_{VD}} = (Y\#/A\#)^2 + \text{H.O.T.} \quad (2.69)$$

where only the first term is retained since it determines the behavior of the function. Substituting this equation for $L_{m_{VD}}$ back into equation (2.64) one obtains

$$\mu_T = \phi\# K^2 \frac{Y\#^4}{A\#^2} \left| \frac{\partial U\#}{\partial Y\#} \right| \quad (2.70)$$

Therefore, the Van Driest model gives the turbulent viscosity fourth order behavior in $Y\#$.

2.4.2.2 Inner Region Chapman Model

The motivation that Chapman had in deriving a new mixing length model arose from an analysis of oscillating shear flow over an infinite plate; recall Van Driest studied flow over an oscillating plate. Using only the continuity

equation and Taylor series expansions, Chapman, and Reichardt (9:7) earlier, showed that the leading term of the Reynolds stress is proportional to Y^3 . Therefore, the turbulent viscosity should also be proportional to Y^3 .

Prandtl's Mixing Length equation with Van Driest's initial modification, eqn (2.63), is used in the Chapman model. However, the mixing length for turbulent flow needed in eqn (2.64) as given by Chapman (9:8) is

$$L_m = \left[1 - 2 \cos \beta \exp(-\beta) + \exp(-2\beta) \right]^{1/2} \quad (2.71)$$

where $\beta = Y^+ / AOS$, and AOS is usually 111.0 for incompressible flow. [Reference (9) has a typographical error, eqn (2.71) is correct (8)]. For this analysis it was assumed that the same density ratio compressibility factor, as developed by Cebeci and Smith for the Van Driest model, could also be included in AOS to account for compressibility effects.

$$AOS = AOS^+ \left(\frac{\rho}{\rho_w} \right)^{1/2} \quad (2.72)$$

where $AOS^+ = 111.0$ and β becomes

$$\beta = \frac{Y^+}{AOS} \quad (2.73)$$

To determine the type of behavior the Chapman model contributes to the turbulent viscosity, the series representation of the cosine function and the exponential function,

$$\cos(\beta) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (\beta)^{2n} = 1 - .5\beta^2 + \text{H.O.T.} \quad (2.74)$$

$$\exp(-\beta) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \beta^n = 1 - \beta + \text{H.O.T.} \quad (2.75)$$

$$\exp(-2\beta) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (2\beta)^n = 1 - 2\beta + \text{H.O.T.} \quad (2.76)$$

are substituted into equation (2.71) so that L_m can be rewritten in the form

$$L_m = [1 - 2(1 - .5\beta^2 + \dots)(1 - \beta + \dots) + (1 - 2\beta + \dots)]^{1/2} \quad (2.77)$$

$$L_m = [\beta^2 + \text{H.O.T.}]^{1/2} \approx \beta + \text{H.O.T} \quad (2.78)$$

Substituting the leading term of eqn (2.78) into the equation for the turbulent viscosity, eqn (2.64), one obtains

$$\frac{\mu}{T} = \frac{\phi}{\text{AOS}} + K^2 Y^{\#3} \left| \frac{\partial U^{\#}}{\partial Y^{\#}} \right| \quad (2.79)$$

Therefore, it can be seen that Chapman developed his model to achieve third order behavior in $Y^{\#}$ for the turbulent viscosity.

The Van Driest and Chapman mixing length theories will both be used in the program and the differences between their results examined in a later section.

2.4.2.3 Outer Region Mixing Length Model

In the outer region, the flow is very different than the inner region, so a new scaling for the mixing length is

needed. Again, using Prandtl's mixing length theorem, the applicable equation for turbulent viscosity is equation (2.62). During the calculations of the mixing length $l\#$, when the parameter $l\#$ becomes equal to $.089\delta\#$, the equation for $l\#$ is changed to

$$l_{o\#} = 0.089 \delta\# \quad (2.80)$$

This new equation for $l\#$, eqn (2.80) (3:223), insures that the inner and the outer regions will be piecewise continuous in the turbulent length scale and the turbulent viscosity.

2.4.2.4 Outer Region Clauser Model

The Clauser model (3:225) assumes the turbulent viscosity is proportional to the kinematic viscosity, $\delta\#$. The equation for the turbulent viscosity for the Clauser model is:

$$\mu_T\# = \alpha\# \rho\# U_e\# |\delta\#| \quad (2.81)$$

where $\alpha\#$ accounts for low Reynolds number effects. For Reynold's numbers greater than 5000,

$$\alpha\# \approx 0.0168 \quad (2.82)$$

and the kinematic displacement thickness is defined by

$$\delta\# = \int_0^{\infty} \left[1 - \frac{U\#}{U_e\#} \right] dy\# \quad (2.83)$$

For the cases run in this study, the edge velocity is equivalent to the freestream velocity, so $U_e\#$ is used in eqn (2.83). The integral is evaluated using a trapezoidal rule

with U averaged about the $j-1/2$ point prior to the integration. When the Clauser's model is used in the program, the switch between the inner region and the outer region occurs when $\mu_T(\text{outer}) = \mu_T(\text{inner})$ so that the turbulent viscosity is piecewise continuous between the two regions.

2.4.2.5 Intermittency

As the edge of the boundary layer is approached in the outer region, the turbulence becomes intermittent; that is, only a fraction, σ , of the time the flow is turbulent. Without a factor to account for this intermittency, the amount of turbulence calculated to be present in the outer region would be overpredicted. The intermittency factor is defined by an error function (7:110) but is approximated by (50:4)

$$\sigma = [1 + 5.5 (Y^\#/\delta^\#)^6]^{-1} \quad (2.84)$$

The intermittency parameter is introduced into the model by multiplying μ_T by σ .

2.4.3 Transition Model

The transition model bridges the gap between laminar flow and fully turbulent flow. One such model (18:29) considers the transition region, and introduces a transition parameter that varies from 0, the edge of the laminar region, to 1, the beginning of the fully turbulent region. This transition parameter multiplies the turbulent

viscosity. Therefore, only a fraction of the turbulent viscosity is added to the governing equations, depending on the position of the flow in the transition region.

The transition parameter, r , is defined as (18:29)

$$r = 1 - \exp(-.412 \theta^2) \quad (2.85)$$

where θ is the normalized streamwise coordinate in the transition region. θ is defined by

$$\theta = \frac{x - x_{T,I}}{\bar{x}} \quad \text{where} \quad x_{T,I} \leq x \leq x_{T,F} \quad (2.86)$$

with $x_{T,I}$ being the location of the beginning of transition and $x_{T,F}$ is the final location of transition, or the beginning of the fully turbulent region. The x is the location that is being analyzed. The measure of the extent of the transition region is $\bar{x}$ where it is defined by

$$\bar{x} = x|_{r=3/4} - x|_{r=1/4} \quad (2.87)$$

Usually $\bar{x}$ is allowed to vary until agreement with experimental data is reached. The use of this model insures a smooth transition from laminar to fully turbulent flow.

2.4.4 Heat Transfer and Viscosity

The original program references the heat transfer calculations to reference conditions. These references are not useful for turbulent flow data. To examine the turbulent heat transfer data on a flat plate, it is customary to compare the Stanton number with the local Reynold's number, Re_x , as shown in Figure 2.6. The Stanton number is defined as

$$St = \frac{HC}{cp\phi-U_{\infty}} \quad (2.88)$$

with the heat transfer coefficient being defined as

$$HC = \frac{q}{T_{AW} - T_{\infty}} \quad (2.89)$$

The local Reynold's number is defined as

$$Rex = \frac{\rho U_{\infty} x}{\mu} \quad (2.90)$$

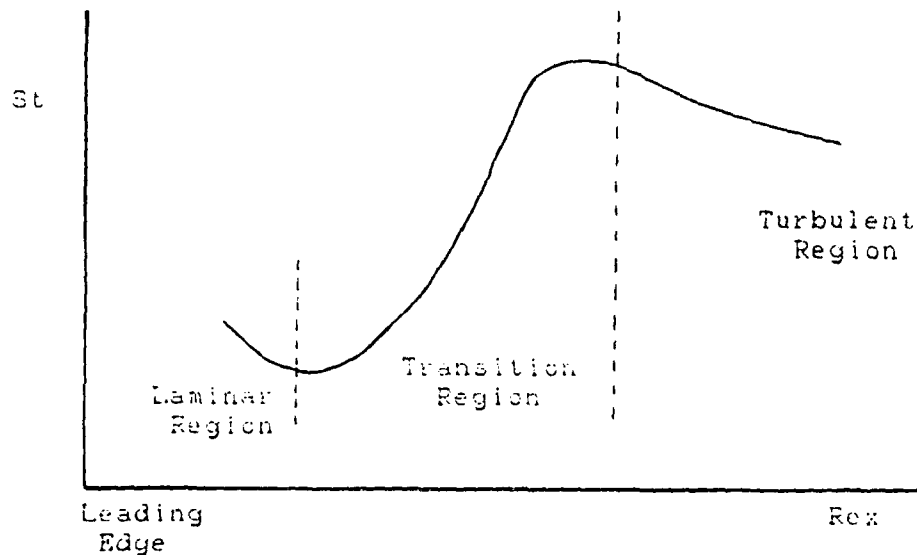


Figure 2.6. Heat Transfer on a Flat Plate

where x is the position on the plate where the Stanton number was obtained. If r is the recovery factor, the adiabatic wall temperature is defined as

$$T_{AW} = T_o [1 + .5 r (\gamma - 1) M_{\infty}^2] \quad (2.91)$$

The recovery factor in this study is defined as

$$r = Pr^{1/2} \quad \text{for laminar flow} \quad (2.92)$$

$$r = Pr^{1/3} \quad \text{for turbulent flow} \quad (2.93)$$

The transition model is used to vary the value of the recovery factor in the transition region. The value of the Prandtl number is 0.72.

In turbulence models, a turbulent Prandtl number is defined as

$$Pr_T = \mu_T c_p / k_T \quad (2.94)$$

and its nominal value is 0.90. The turbulent Prandtl number is a counterpart of the molecular Prandtl number to take into account the turbulent energy transport. However, the turbulent Prandtl number is used as a tool to obtain the turbulent conductivity, k_T . Since the turbulent Prandtl number and the heat capacity for constant pressure are both constants, it is interesting to note that

$$k_T \approx \mu_T \quad (2.95)$$

This says that the diffusion of the turbulent energy is directly related to the turbulent viscosity.

The local skin friction coefficient, c_f , is calculated by the program using (7:28)

$$c_f = \frac{2 \tau_w}{\rho U_\infty^2} \quad (2.96)$$

The theoretical skin friction coefficient for incompressible flow on a smooth flat plate is given by (7:187)

$$c_f = 0.664 (Re_x)^{1/2} \quad (2.97)$$

The viscosity is calculated throughout the program using Keyes viscosity equation, which is

$$\mu = \frac{2.32 \times 10^{-8} T^{1/2}}{1 + \frac{220}{T \times 10^9}} \quad \frac{\text{lb sec}}{\text{ft}^2} \quad (2.98)$$

where T is in degrees Rankine. Keyes viscosity law is more accurate (34:5) in the temperature range encountered in hypersonic wind tunnels than the popular Sutherland's viscosity law. In hypersonic tunnels, the static freestream temperature is generally between 30°R and 200°R, which is below 300°R, the demonstrated range of applicability of Sutherland's law.

2.5 Program Verification

The thesis computer code is verified by comparison of its results to experimental data and another computer code. The experimental data was obtained from several sources, and will be described later in each of the pertinent sections. The other computer code, hereafter called the ITRACT computer code, was developed by the Hypersonic Research Laboratory at Wright-Patterson Air Force Base. The code is a steady state compressible boundary layer code using the Van Driest eddy viscosity turbulence model (49).

2.5.1 Incompressible Laminar

The program inputs used for comparison against incompressible laminar flow over a flat plate (Blausius' solution) are $M_\infty = .05$ and $T_\infty = 530^\circ \text{R}$. The Y_w metric was calculated analytically over the entire plate. In Figure 2.7, the Blasius solution (46:42) for the velocity is plotted against the numerical solution for the program in laminar flow. The computer results fall almost directly on top of the theoretical results. In Figure 2.8, the ratio of the numerical skin friction coefficient to the theoretical skin friction coefficient is plotted against $x^\# / L^\#$. Since the results are very nearly a constant 1.0, the computer results almost exactly match the theoretical results.

At this Mach condition, the flow is incompressible and laminar so the Dorodnitsyn transformation and turbulence model are still unverified. However, the Thomas algorithm has been verified to be correct.

2.5.2 Incompressible Turbulent Isothermal

It is necessary to verify the validity of the turbulence model. This is done by comparing its results with that of experimental data. The data are taken from a NASA memorandum (44:24). The probable errors in the local Stanton numbers is $\pm 3\%$, and the probable errors in the local Reynolds number is $\pm 1\%$ (44:17). The NASA reference calculated the heat transfer coefficient using

$$HC = \frac{q}{T_w - T_\infty} \quad (2.99)$$

The ITRACT computer code calculated HC using

$$HC = \frac{q}{T_o - T_w} \quad (2.100)$$

The thesis computer code calculates HC using

$$HC = \frac{q}{T_{AW} - T_w} \quad (2.101)$$

The two computer codes were first modified so that the heat transfer coefficient was calculated by the eqn (2.99) which was used by the NASA reference. The data from the three different sources could now be compared. The Y_δ metric in the thesis computer program was calculated analytically over the entire plate. Note that the computer code actually calculates compressible flow conditions, but the density variations are on the order of 1% .

Figure 2.9 shows the results of the comparison when the Van Driest model is used for the inner region, and Clauser's model is used for the outer region. The ITRACT computer solution is approximately 20% lower than the experimental data, while the thesis computer solution is approximately 10% lower than the experimental data. This difference could be due to the value of the empirical constants used in the turbulence models, and also truncation errors. However, the thesis code does an adequate job in predicting heat transfer for incompressible turbulent flows.

2.5.3 Incompressible Turbulent Nonisothermal

The thesis code can now be verified using experimental data for incompressible turbulent flow with a nonisothermal wall, or thermal mismatch of approximately 20°F. The data are obtained from a NASA memorandum (45:27). The NASA reference calculates the heat transfer coefficient as shown in eqn (2.99). Again the thesis computer code was modified so that the calculation of the heat transfer coefficient was consistent with the experimental data. The Y_* metric in the thesis computer program was calculated analytically over the entire plate.

When the Van Driest model is used in the inner region and Clauser's model is used in the outer region, Figure 2.10 shows the results of the the comparison. The thesis solution is very close to the experimental data.

2.5.4 Compressible Turbulent Isothermal

The turbulence model and the Dorodnitsyn transformation need to be validated for compressible flow. This will be done by comparing the code to experimental shock tube data of compressible turbulent flow. The data are obtained from a NASA Contractor Report (4:131). The report verifies that the shock tube data agree with perfect gas results to within 3 %.

The experimental data shows a spike in the heating at the beginning of transition. This spike is characteristic of shock tube data. Transition in a shock tube is first

indicated by a series of spikes in the heat transfer data. These spikes are associated with "turbulent bursts" created at the edge of the viscous sublayer (23:6,12). The experimental data also calculates the heat transfer coefficient differently than the thesis computer program. The reference calculates the heat transfer coefficient using

$$HC = \frac{q_{cp}}{H_o - H_w} \quad (2.102)$$

Assuming constant heat capacity, the ITRACT and thesis codes were modified to be consistent with the experimental data.

When the Van Driest model is used for the inner region and Clauser's model is used for the outer region, Figure 2.11 shows the results of the comparison. The ITRACT code for the fully turbulent case is approximately 20% higher than the experimental data. The thesis code is 10% higher than the experimental data, but does model the peak heating at the end of transition very well. Both codes do an excellent job in predicting the laminar portion of the curve.

Figure 2.12 shows a comparison between all the models used in this study. Four cases were examined: Van Driest / Clauser, Van Driest / Mixing Length, Chapman / Clauser, and Chapman / Mixing Length. The inner region models, Van Driest and Chapman, produce results to within 1% of each other, with Chapman giving slightly lower results. This indicates that the argument between the models, the Y⁴

versus Y^3 dependence on the turbulent viscosity is mainly an academic discussion for this case. The outer region models, mixing length and Clauser, also produce similar results, with the mixing length model giving 1.5% higher results than Clauser's model.

2.6 Results of Computational Effort

2.6.1 Decrease in Computer Time

A comparison between the original program with the optimized successive over-relaxation method and the modified program with the Thomas algorithm was completed for laminar flow. Table (2.1) gives the results of the comparison. NT is the number of time steps, DT was the temporal increment in artificial units, and the maximum error allowed was EPS. For these runs, NT = 10, and DT = 1000. As the maximum error allowed increases, the difference between the two algorithms decreases, so there would be no advantage to using the Thomas algorithm versus optimized successive over-relaxation when high accuracy is not important. However, it is evident for the error criteria specified in these cases, the Thomas algorithm is more efficient for solving this particular differencing scheme, especially for grids with a larger number of points.

Table 2.1. Comparison of SOR and Thomas Algorithm

	SOR (CPU Seconds)	Thomas (CPU Seconds)
-5 EPS = 10 61x30 Grid	20	16
-6 EPS = 10 61x30 Grid	32	21
-5 EPS = 10 61x59 Grid	297	35

2.6.2 Laminar

The thesis program is excellent in predicting the flow properties of incompressible laminar flows. The program verification of the U velocity and skin friction coefficient showed the results were nearly identical to the theoretical results.

2.6.3 Turbulence

The thesis program predicts the heat transfer of incompressible and compressible turbulent flows very well. The thesis program results were approximately 10% different than the experimental data. This 10% error could perhaps be due to truncation error. The use of an optimized grid could probably improve the results. However, the program accurately predicted the heat transfer results of the incompressible turbulent flow when a thermal mismatch of 20° was present. Comparisons of the inner region and outer region models showed very little difference between the results, with the Chapman model giving 1% lower results

compared to Van Driest, and the mixing length model giving
1.5% higher results compared to Clauser.

INCOMPRESSIBLE LAMINAR FLOW

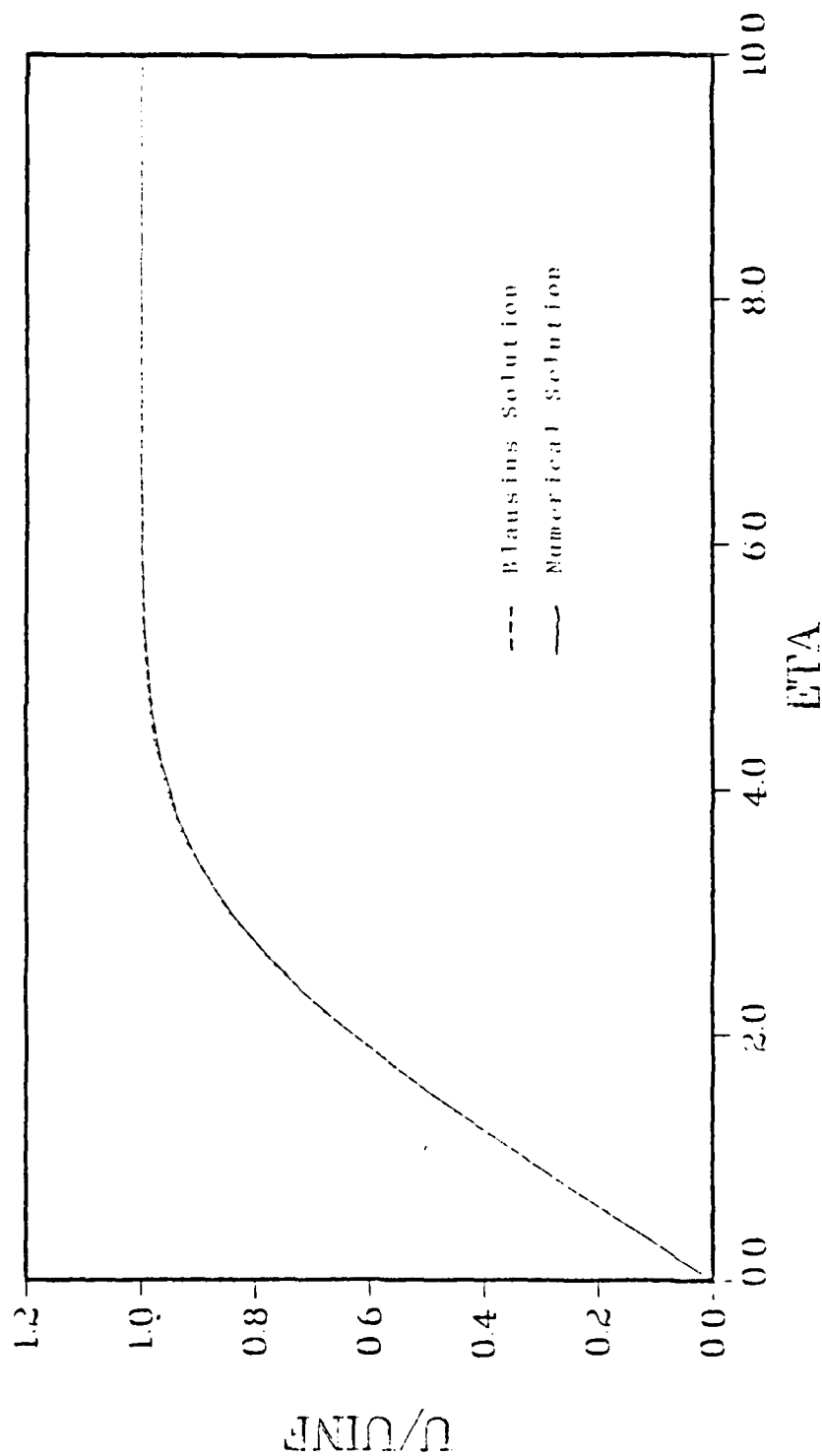


Figure 2.7. Incompressible Flow Verification

SKIN FRICTION COEFFICIENT

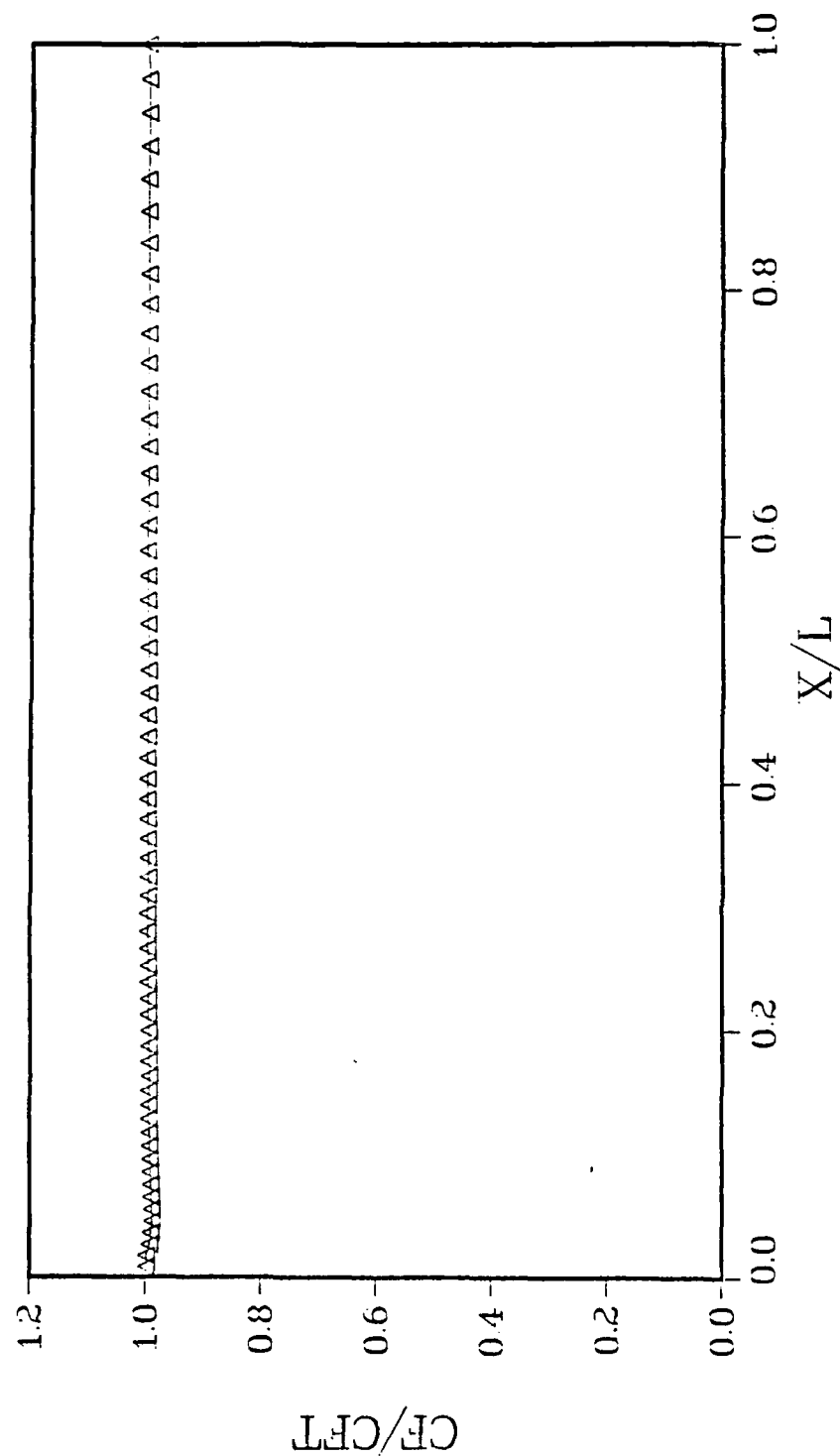


Figure 2.3. Incompressible Skin Friction Verification

FLAT PLATE MACH=11 VAN DRIEST

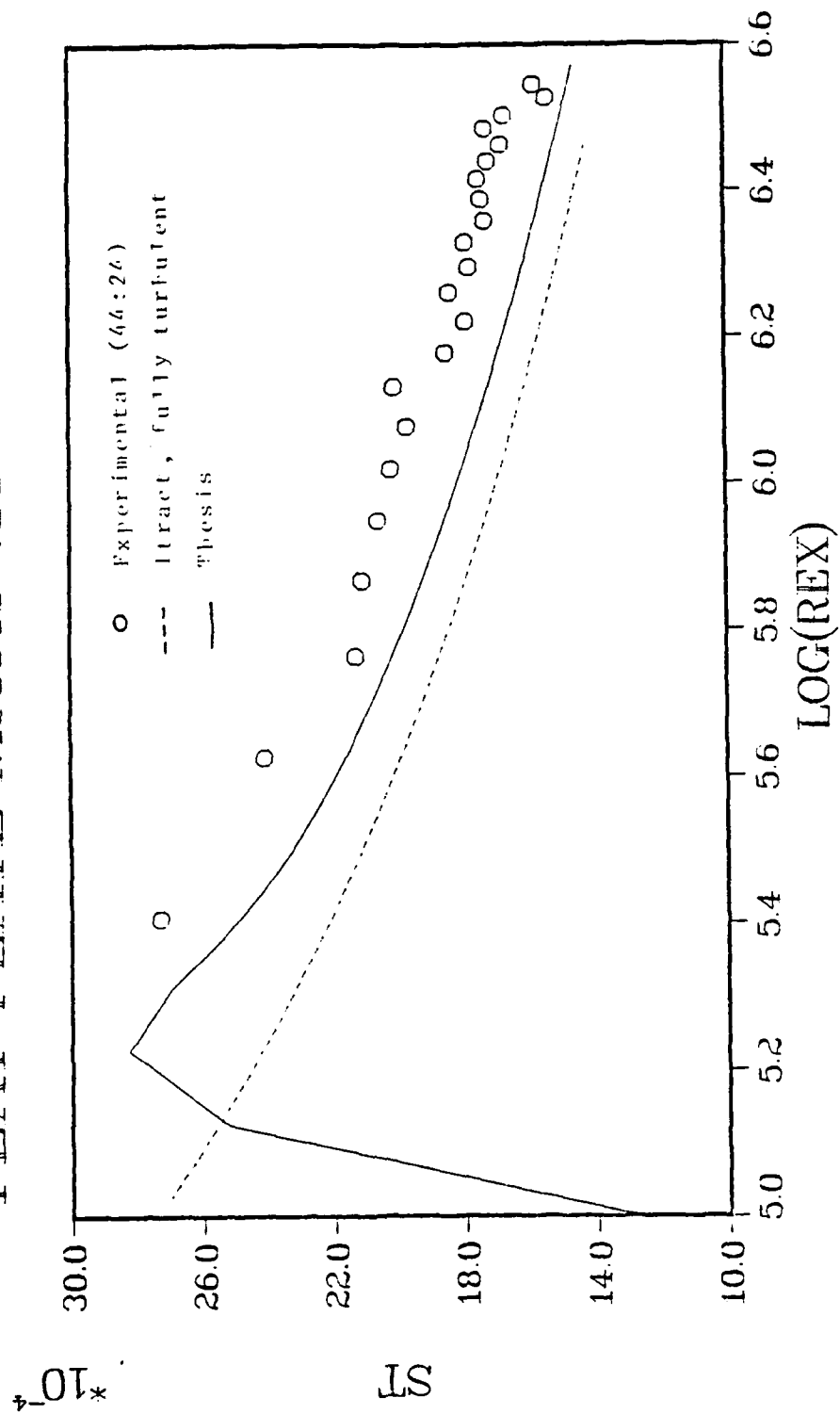


Figure 2.9. Incompressible Turbulent Flow Verification - Van Driest/Clauser Models

WALL THERMAL MISMATCH

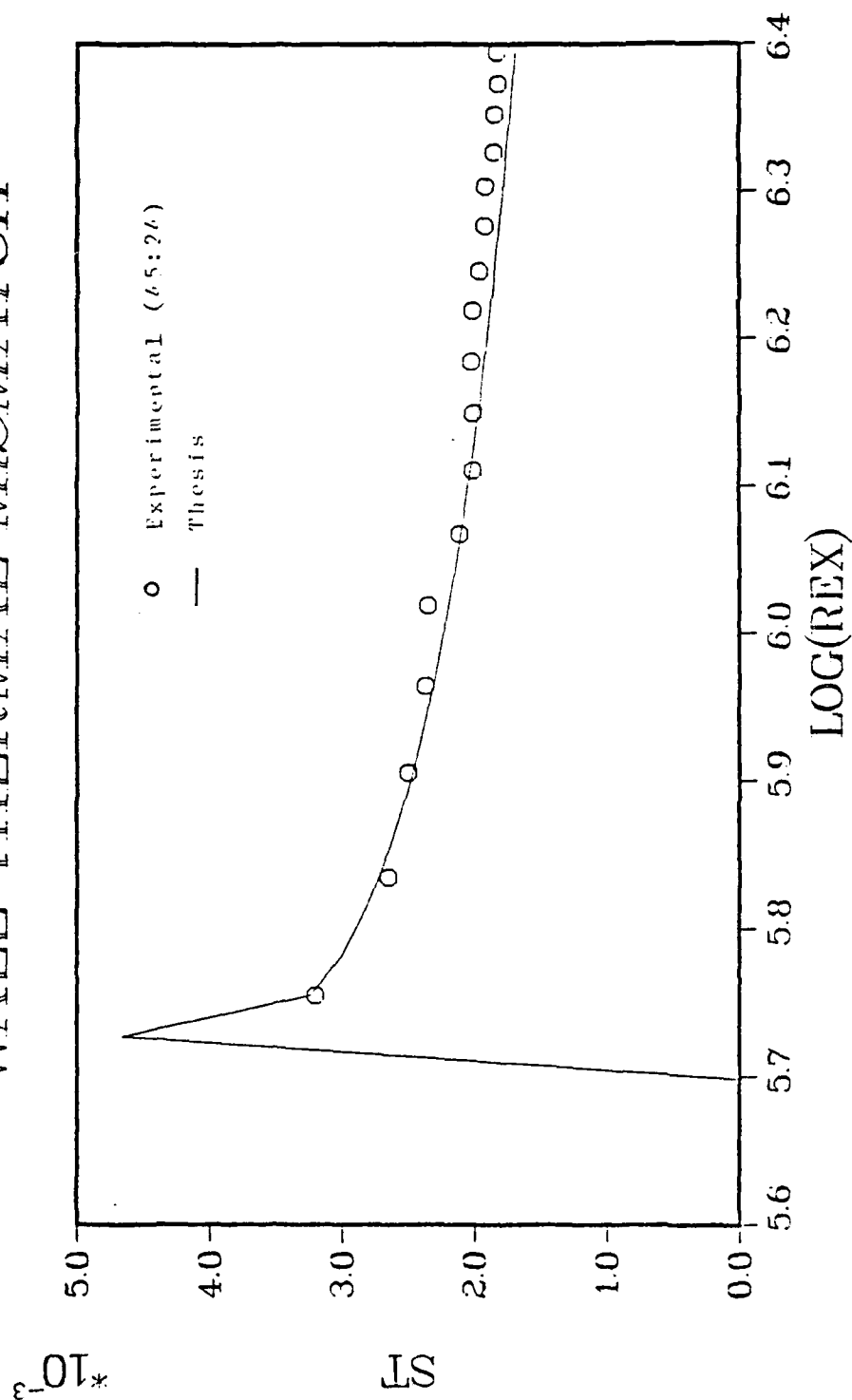


Figure 2.10. Incompressible Turbulent Flow with Thermal Mismatch - Van Driest/Clauser Models

FLAT PLATE MACH=7 COMPARISON

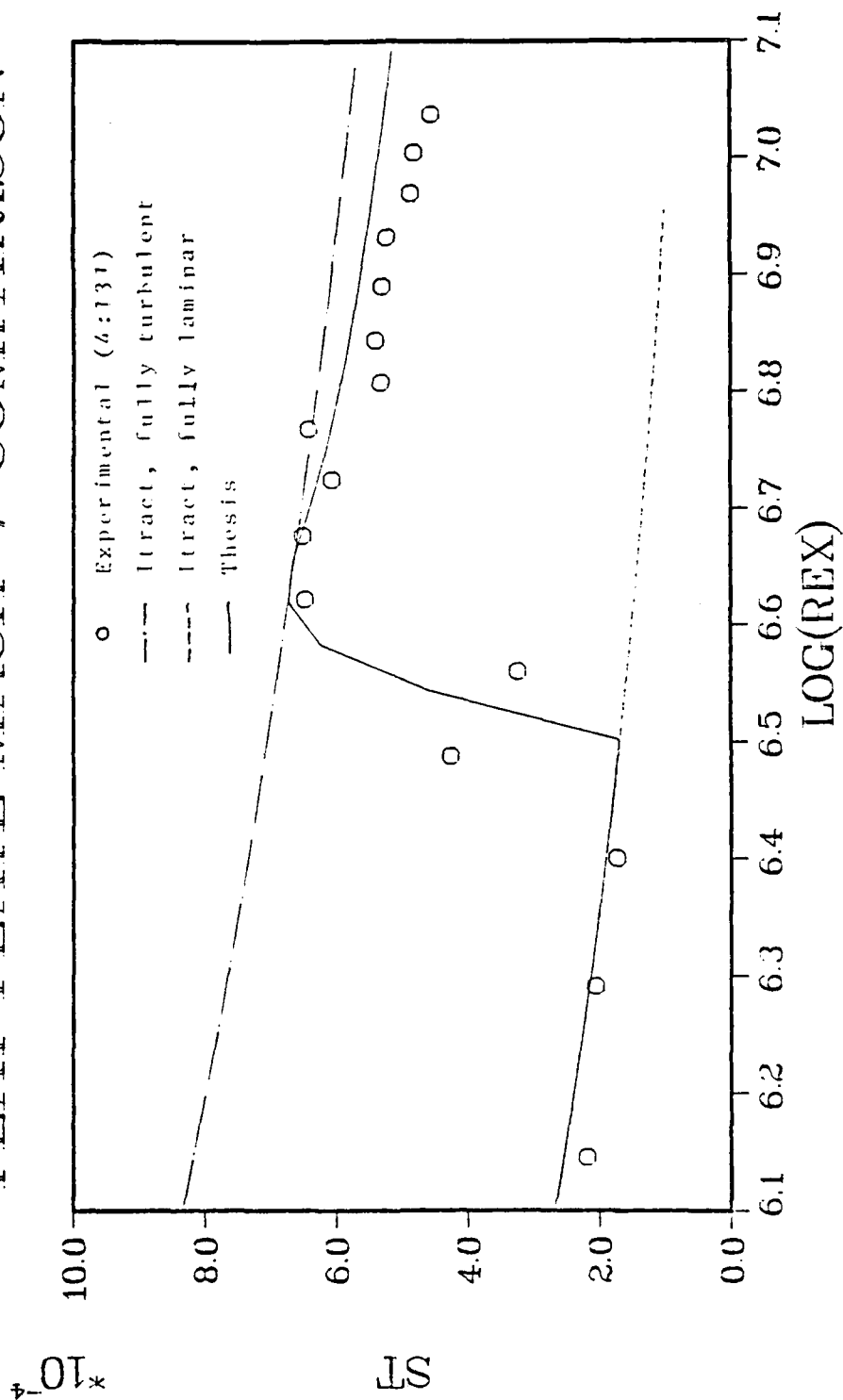
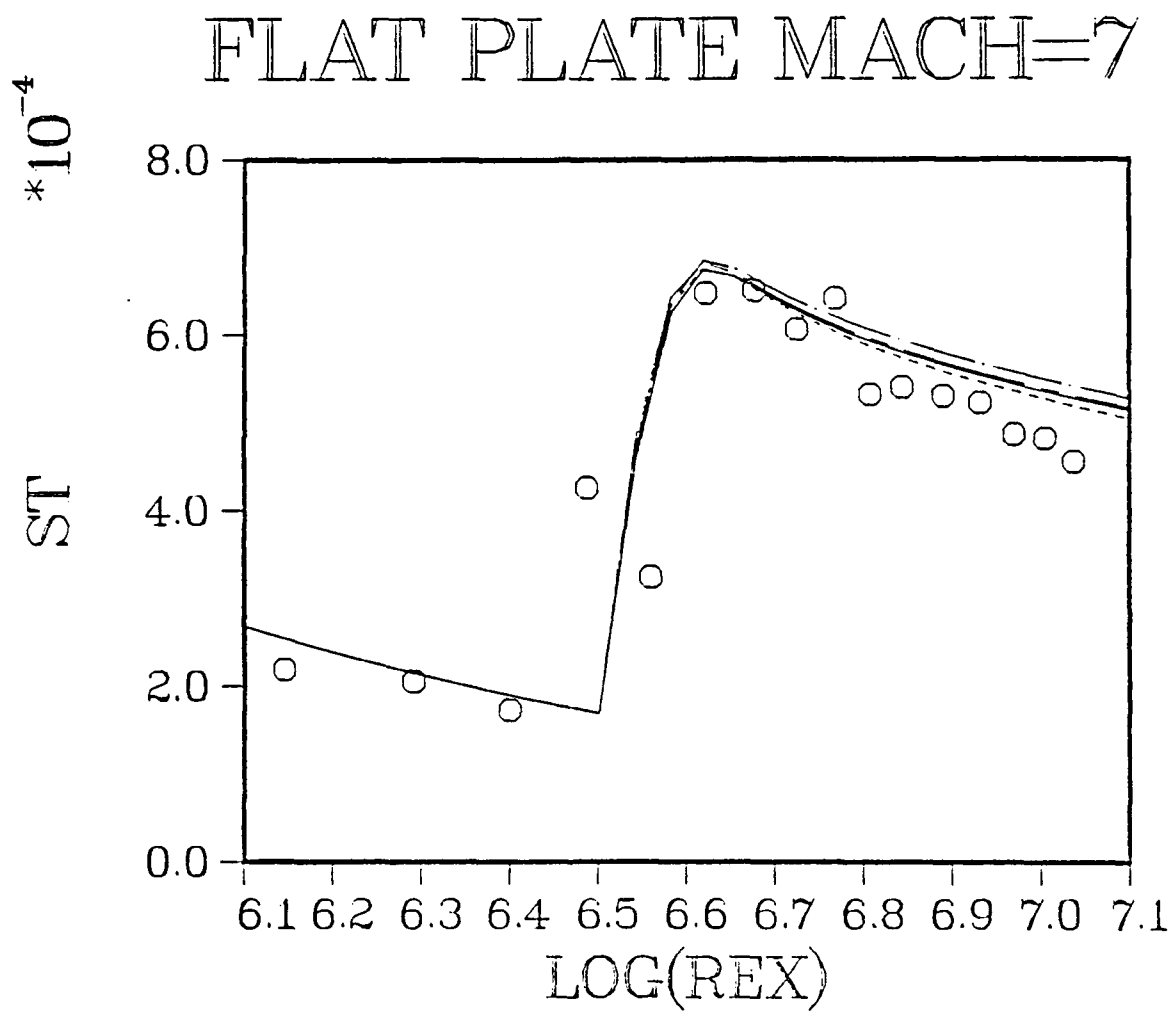


Figure 2.11. Compressible Turbulent Flow Verification - Van Driest/Clauser Models



- Experimental Data (4:131)
- Van Driest/Mixing Length
- Van Driest/Clauser
- - - Chapman/Mixing Length
- · · Chapman/Clauser

Figure 2.12. Compressible Turbulent Comparison - All Models

III. Experimental Effort

3.1 Introduction

The experimental effort consisted of the study of a blunt fin at incidence on a sharp nosed flat plate. The phenomena of interest was the areas of high heating on the expansion side of the blunt fin. The research consisted of three main phases:

Phase I: Calibration of the von Karman Institute H-3 wind tunnel to determine the extent of the uniform velocity jet.

Phase II: Study of the flow on a flat plate to determine the location of transition and turbulence.

Phase III: Study of the interaction flow on a flat plate on the expansion side of a blunt fin in laminar flow.

The flow studies include flow visualization techniques, such as oil flow, sublimation, shadowgraph, and liquid crystals, and then also quantitative thin film and coax gage heat transfer tests. Tests were completed at a Mach number of 6.0 at two Reynolds numbers, $0.844 \times 10^7 / m$ and $1.69 \times 10^7 / m$ which correspond to two reservoir pressures, 10 and 20 bars, respectively. DeGraef (14) had completed liquid crystal tests on this particular interaction flow at Mach 5.26, but only in turbulent flow.

3.2 H-3 Blowdown Wind Tunnel

This research was conducted in the VKI H-3 blowdown intermittent tunnel illustrated in Figure 3.1. The test gas is air and is supplied from a 40 bars compressor. The air is heated by a pebble bed, to avoid condensation of air components (principally nitrogen) occurring during the test (35:137). When condensation occurs, isentropicity is lost, and the test data can no longer be interpreted correctly. Daum and Gyarmathy (13:464) have developed a method to determine the minimum stagnation temperature which may be used to avoid condensation. For the test conditions in this study, their method gives a minimum stagnation temperature of 406°K. All the tests in this study were completed at a stagnation temperature of approximately 473°K. The tunnel uses an air ejector to evacuate the test chamber to ease startup conditions. The high pressure air in the reservoir is expanded in the axisymmetric nozzle giving a uniform free jet of Mach 6. The nozzle exit diameter is approximately 154 mm. The size of the uniform Mach 6 jet will be determined from the tunnel calibration.

There is a model injection system which is driven by a pneumatic mechanism. The test section contains a three degree of freedom (incidence angle, yaw angle, and axial position) traversing mechanism for the model and probe support.

HYPersonic TUNNEL H3

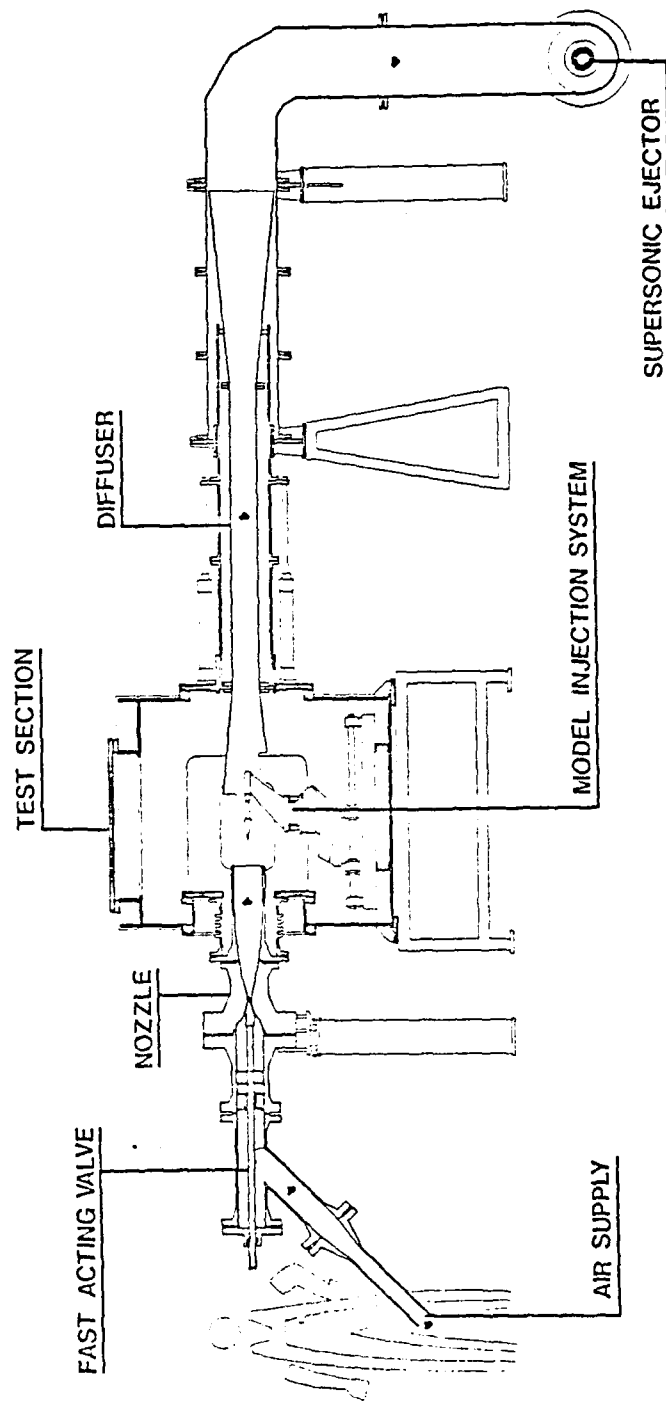


Figure 3.1. VKI H-3 Wind Tunnel

The reservoir chamber pressure can vary from 7 to 35 bars corresponding to a unit Reynold's number from 10^7 to 4×10^7 / meter. Running times up to five minutes are available. However, in order to reduce energy consumption, experiments usually have a short duration.

3.3 Calibration of the H-3 Wind Tunnel

3.3.1 Purpose

To determine the size of the flat plate model to construct, the scope of the Mach 6 uniform free jet must be determined. The first time the tunnel was calibrated was after its installation in 1970 (29). That previous calibration showed the existence of a small centerline disturbance and the slow streamwise growth of the outer shear layer. Details of the calibration procedure used and results obtained in this study are included in Appendix E.

3.3.2 Conclusions of Calibration

The results from this study also shows the presence of a slow growing shear layer and centerline disturbances. The experimental errors in the Mach number are less than .5%. The extent of the uniform flow has been obtained and the centerline Mach number variations have been shown to be within 2.2% of the mean flow in this uniform flow region. The streamwise flow at a radial distance of 39 mm from the centerline shows a slow rise in Mach number as the distances from the nozzle increases, from a mean Mach number of approximately 5.97 to 6.00. The streamwise flow at the

center of the tunnel also shows a rise in Mach number, but the rise is more dramatic. The rise is sudden between $x = 3.5$ mm and 53.5 mm, the mean centerline flow increases from an approximate Mach number of 5.89 to 6.04. Then, the rise is much slower, perhaps even becoming constant after $x = 143.5$ mm at a Mach number of 6.05.

From the calibration, the flow can be considered to be a uniform axisymmetric jet of Mach 6. The uniform flow has been determined to within ± 2 mm to be approximately 117 mm in diameter at the nozzle exit and it remains relatively constant until a distance of 103.5 mm downstream of the nozzle exit. The uniform flow then decreases to approximately 98 ± 2 mm in diameter at 143.5 mm downstream of the nozzle exit. Using the data from Table E.2, the region of uniform flow is sketched in Figure 3.2. The boundary of the uniform flow region beginning at the nozzle exit is interpolated to 143.5 mm behind the nozzle exit. After $x = 143.5$ mm, the region of uniform flow could be extrapolated to $x = 300$ mm. However, the region of uniform flow resulting from the extrapolation appears unrealistic. It is difficult to believe that the shear layer would grow as fast as the extrapolation indicated. This seems to indicate that there is not enough information available to draw a conclusion about the extent of the growth of the shear layer farther than the 143 mm after the nozzle exit. More calibration tests are required if tunnel models exceed

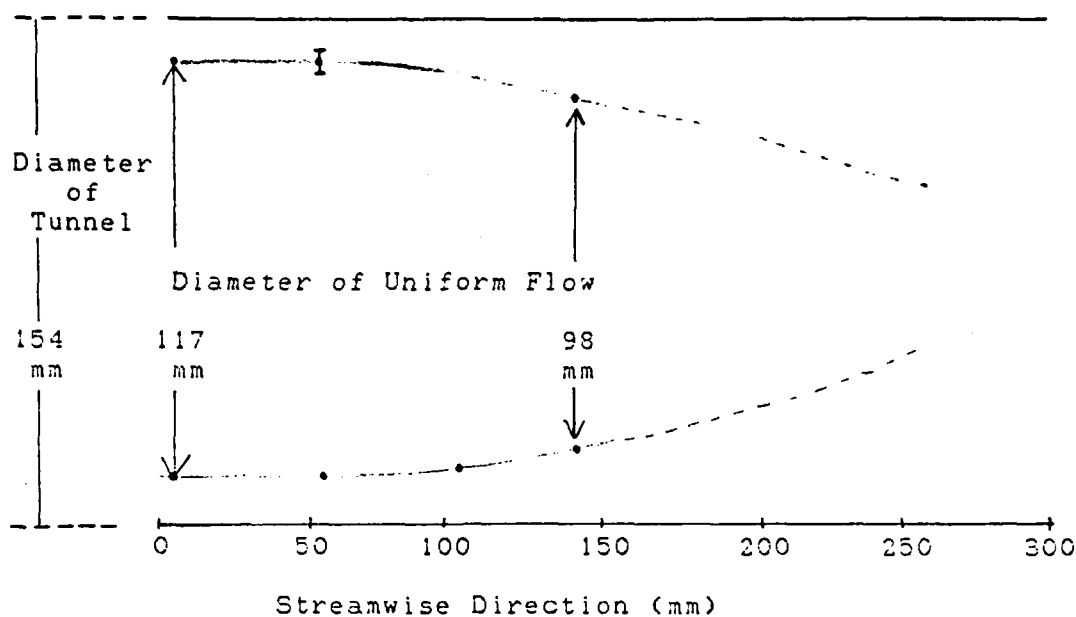


Figure 3.2. Sketched Regions of Uniform Flow

150 mm in length, but this would entail, not only conducting more tests, but designing a new pressure rake to avoid the occurrence of tunnel blockage.

3.4 Theory

3.4.1 Isentropic Flow Calculations

The static properties of the flow are calculated using isentropic flow relations. Since the reservoir total pressure and temperature are known, the static pressure and temperature can be obtained using these equations (35:53)

$$T = T_o (1 + .5(\gamma-1)M^2)^{-1} \quad (3.1)$$

$$P = P_o (1 + .5(\gamma-1)M^2)^{-[\gamma/(\gamma-1)]} \quad (3.2)$$

The static density can be computed using the perfect gas law

$$\rho = P/(RT) \quad (3.3)$$

where $R = 287.0 \text{ Nm/kg}^\circ\text{K}$ and is the gas constant for air.

The local speed of sound can be calculated using

$$a = (\gamma RT)^{1/2} \quad (3.4)$$

and the local velocity is $U = aM$. For viscosity, Keyes law is used, which is in foot-pound-second units

$$\mu_{\text{FPS}} = \frac{2.32 \times 10^{-8} T^{1/2}}{1 + \frac{220.0}{T \times 10}} \quad (3.5)$$

μ has units of lb sec/ft^2 or slug/ft-sec with T in $^\circ\text{R}$.
FPS

The static temperature is converted initially from $^\circ\text{K}$ to $^\circ\text{R}$ using

$$^\circ\text{R} = (9/5) T^\circ\text{K} \quad (3.6)$$

The viscosity is then computed in units of lb sec/ft^2 using eqn (3.5) and converted to metric units of kg/m-sec when multiplied by a conversion factor

$$\mu = \mu_{\text{FPS}} (3.281 \text{ ft/m})(14.59 \text{ kg/slug}) \quad (3.7)$$

The Reynolds number is calculated using

$$Re = Ux/\mu \quad \text{per meter} \quad (3.8)$$

The local Reynolds number, Re_x , is found by multiplying Re by the x location from the leading edge of the plate.

$$Re_x = Ux^2/\mu \quad (3.9)$$

3.4.2 Oblique Shock Theory

From the calibration tests, it is assumed throughout the study that the uniform free jet has a Mach number of 6.0. When the model is at 0° incidence, the flat plate will theoretically see a flow of Mach 6.0. Almost all of the tests completed in this study were with the plate at zero degree incidence. Some preliminary tests were completed at a different Mach number. Since the H-3 tunnel has a fixed geometry nozzle, the only way to change the Mach number of the flow over the plate is to put the plate at a negative angle of attack (leading edge down). Theoretically, when the plate is at a negative angle of attack, a weak oblique shock is formed and the Mach number downstream of the shock will need to be calculated using oblique shock theory. In reality, due to the presence of the boundary layer, an extremely weak oblique shock is formed when the plate is also at zero degree incidence.

Figure 3.3 gives the orientation of the plate and defines the incidence angle of the plate, α , and the shock angle, θ . The Mach number behind the shock can be calculated using the following equation (17:492).

$$M_2^2 = \frac{1 + .5(\gamma-1)M_1^2}{\gamma M_1^2 \sin^2 \theta - .5(\gamma-1)} + \frac{M_1^2 \cos^2 \theta}{1 + .5(\gamma-1)M_1^2 \sin^2 \theta} \quad (3.10)$$

3.4.3 Heat Transfer

The heat transfer distribution on a flat plate, Figure 2.6, distinguishes three regions; laminar, transition, and turbulent. For the case of an isothermal flat plate in the absence of leading edge and viscous interactions, i.e., conditions at the edge of the boundary layer are those of the free stream, Korkegi (31:124) gives approximate expressions for laminar and turbulent Stanton numbers for compressible flows. These equations are obtained from approximate expressions for laminar and turbulent skin friction based on a "reference" temperature T_f .

For laminar flow:

$$St = \frac{0.41}{Rex^{1/2} (T_f/T_\infty)^{.12}} \quad (3.12)$$

$$T_f/T_\infty = 1 + .032 M_\infty^2 + .58 \left[\left(\frac{T_w}{T_\infty} \right) - 1 \right] \quad (3.13)$$

For turbulent flow:

$$St = \frac{0.037}{Rex^{1/5} (T_f/T_\infty)^{.648}} \quad (3.14)$$

$$T_f/T_\infty = 1 + .035 M_\infty^2 + .45 \left[\left(\frac{T_w}{T_\infty} \right) - 1 \right] \quad (3.15)$$

where T_∞ is the freestream static temperature and Rex is defined by eqn (3.9). The experimental data will be plotted with these approximate expressions for comparison purposes.

3.5 Description of Test Models

The dimensions of the flat plate were chosen to be 150 mm in length and 100 mm in width. The rear edges of the

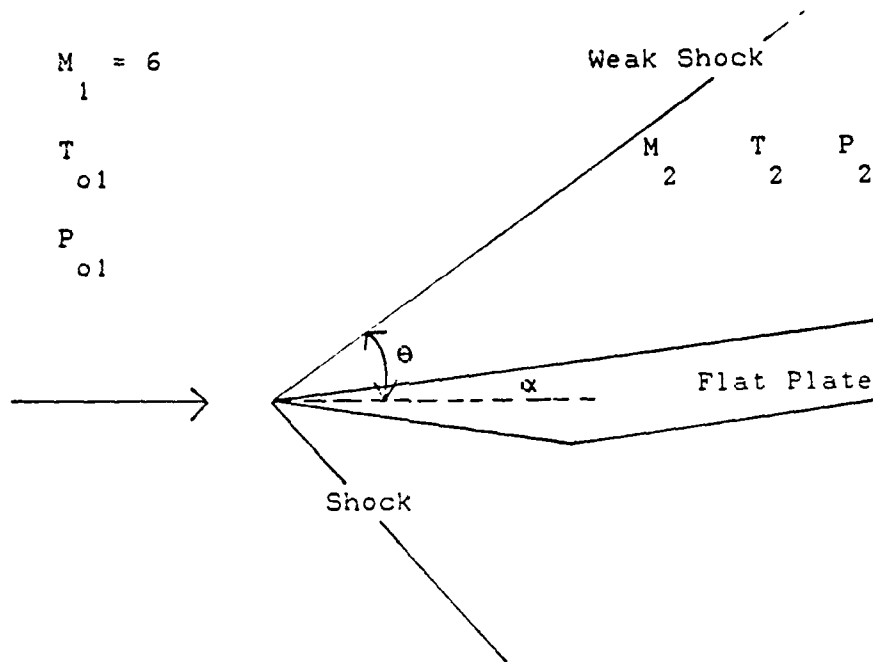


Figure 3.3. Oblique Shock Configuration

The NACA tables (2:22,42,43) have oblique shock data, so if α and M_1 are known, θ can be obtained. Then, the normal component of the Mach number, $M_1 \sin \theta$, can be calculated and the ratios P_2/P_1 , and T_2/T_1 can be obtained from normal shock tables. For this study,

$$\alpha = -5.5^\circ$$

$$\theta = 13.5^\circ$$

$$P_2/P_1 = 2.12$$

$$T_2/T_1 = 1.255$$

(3.11)

Therefore, $M_2 = 5.26$.

plate will therefore be slightly in the shear layer, but no data will be taken near those regions.

The oil flow and sublimation visualization tests for the flat plate were completed on a plexiglas model and repeated with an aluminum model. The dimensions of the model are shown in Figure 3.4. The top surface of the model for the oil and sublimation experiment was sprayed with a dark blue color. The tests done with the plexiglas model were repeated since the leading edge was easily damaged during the tests by particles in the flow. Small nicks in the leading edge caused large disturbances in the oil flow tests. The plexiglas model was chosen initially for the test since it was already machined and had the correct dimensions.

The liquid crystal test was completed on a stainless steel model with a plexiglas insert. The dimensions of the model are shown in Figure 3.5. A single coax gage was installed in the stainless steel portion of the model for calibration purposes. The model was painted black and then the liquid crystals were sprayed on the entire model.

The heat transfer test for a flat plate was completed on a stainless steel model. The dimensions of the model are shown in Figure 3.6. The model had two coax gages installed in it and also had a ceramic insert for the thin film gages.

The aluminum flat plate, with its previously described dimensions shown in Figure 3.4, was used in the three

dimensional flow visualization tests. Five fins of varying sweep angle and height, were constructed out of bronze and sequentially placed on the model for the sublimation and oil flow tests. The dimensions of the fins are shown in Figure 3.7.

It was desired to determine the location of transition on the undisturbed flat plate so that a laminar interaction with the fin could be studied. Experimental data from previous VXI studies completed in the H-3 wind tunnel were examined. Table 3.1 shows the results that were found.

Table 3.1. Location of Transition

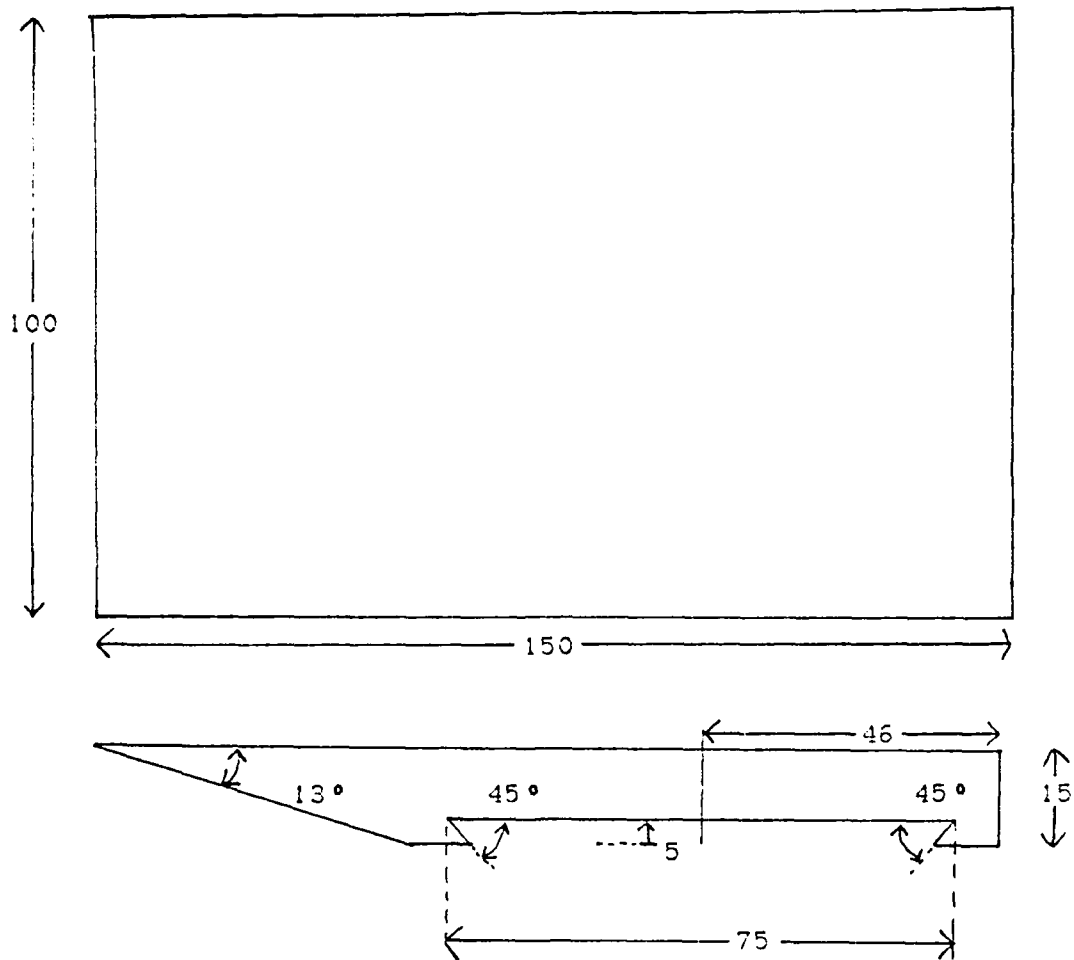
Mach Number	Re (/m)* -7 x10	x transition (mm)	Reference
5.2	3.2	110	(14)
5.2	1.6	120	(14)
6.0	2.3	95	(15)
6.0	1.7	] test conditions of this study	
6.0	0.84		

* based on Keyes viscosity law

Table 3.1 shows that as the Reynolds number is decreased, the transition location moves downstream. Therefore, transition is expected to occur at least 95 mm downstream of the plate leading edge for the test conditions in this study. A more accurate determination of the transition

location was determined in later tests. The results are discussed in section 3.7.4. Therefore, to obtain a laminar interaction, the fins were installed so that the leading edge of the fin was approximately 60 mm downstream of the leading edge and 40 mm from the right side of the plate when looking forward from the rear of the plate.

The heights of the fins were chosen so there would be no interference with the oblique shock originating at the plate leading edge when the plate was at -5.5° angle of attack (Mach 5.26). This height was calculated using the theoretical shock angle and then verified for "no interference" using the shadowgraph technique. However, once the sublimation tests began and the phenomena of interest was detected for the Mach 6 test case, the Mach 5.26 test case was dropped from the remainder of the test plan. Since the fins were already machined at the shortened height and had already been used for the oil flow tests they were retained for the remainder of the test plan. Future tests at Mach 6 should be completed with taller fins to determine if fin height produces any difference in the flow field for varying values of fin sweep and incidence angle.



Dimensions are in millimeters

Figure 3.4. Flow Visualization Flat Plate Model

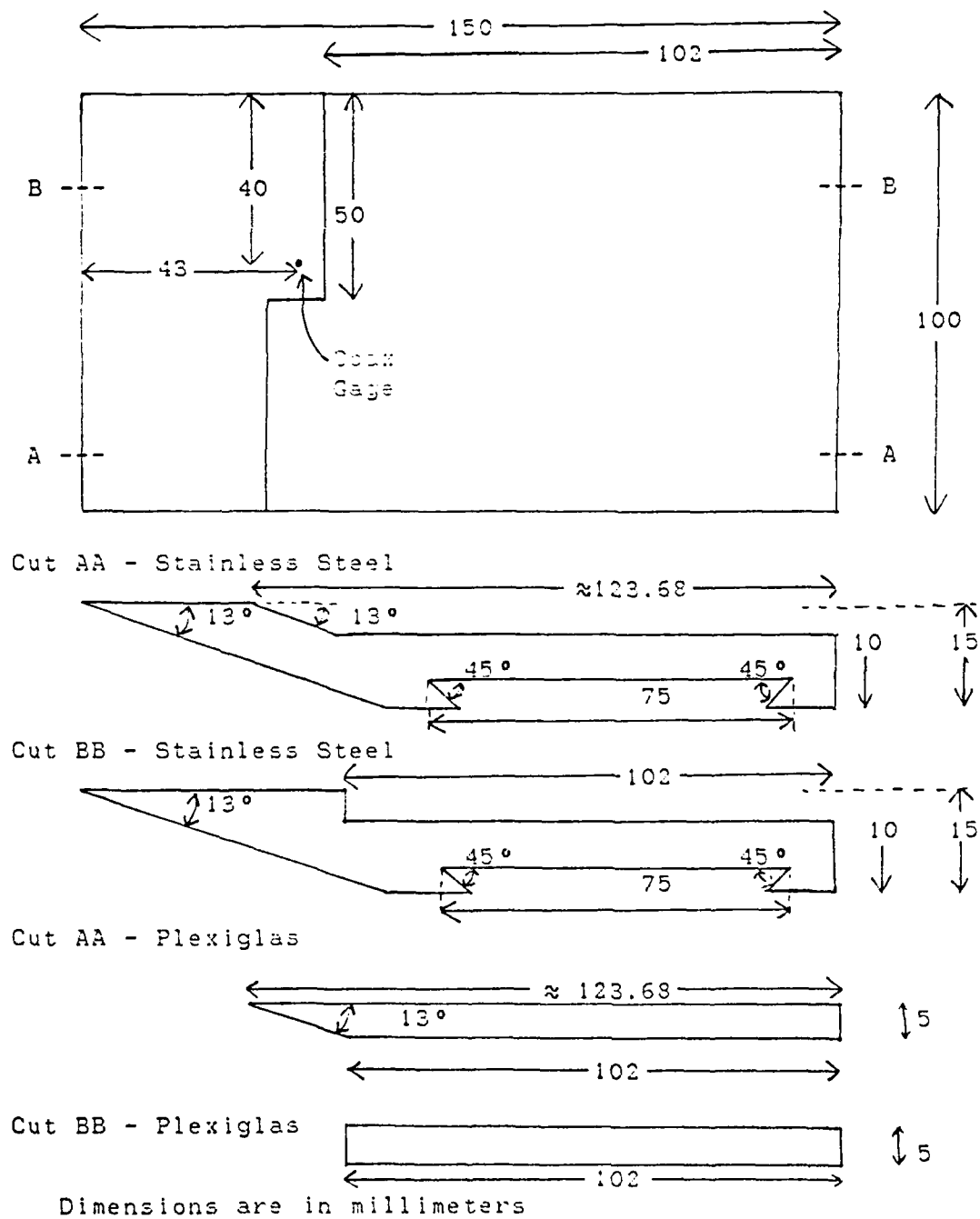


Figure 3.5. Liquid Crystal Flow Visualization Plate Model

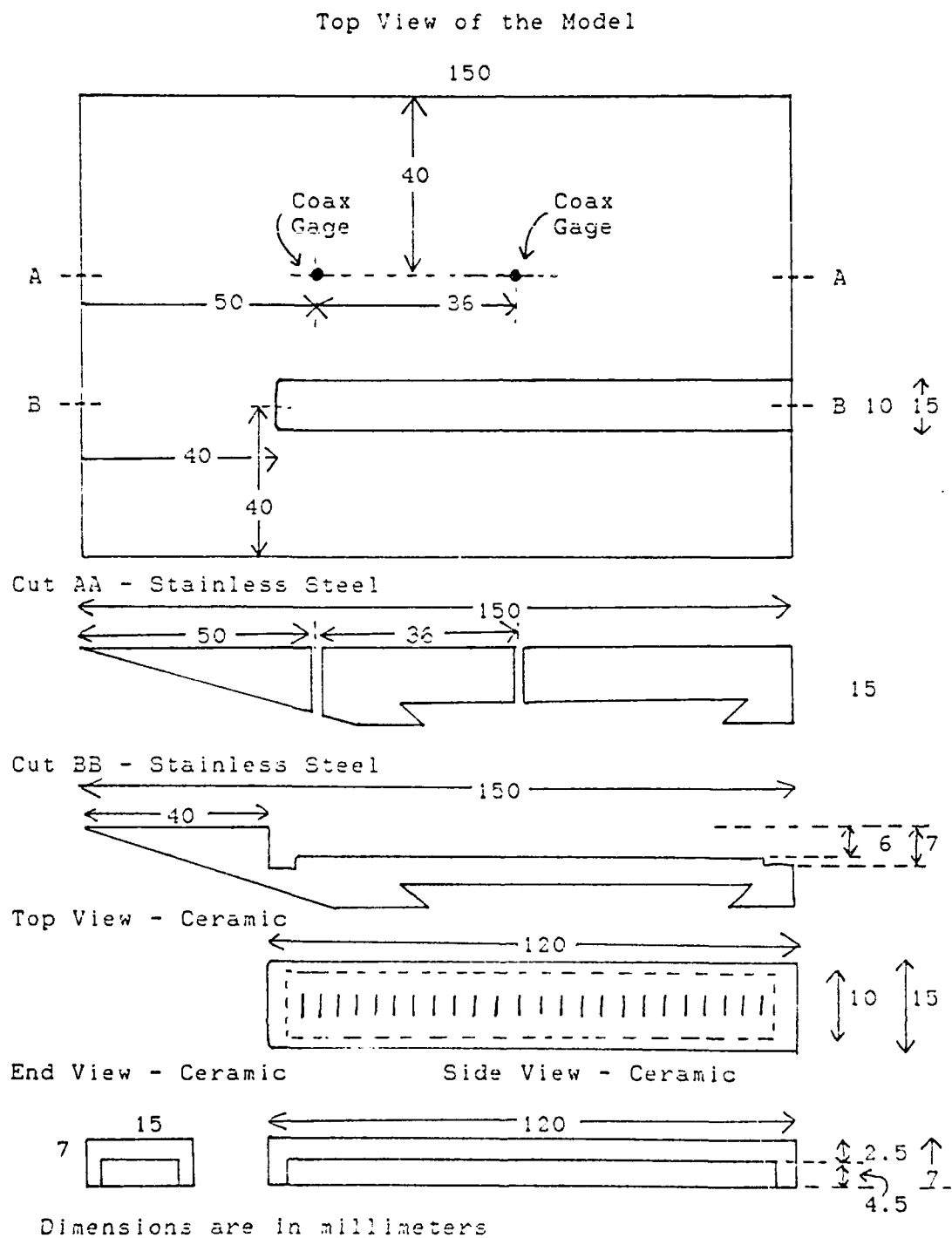


Figure 3.6. Flat Plate Heat Transfer Model

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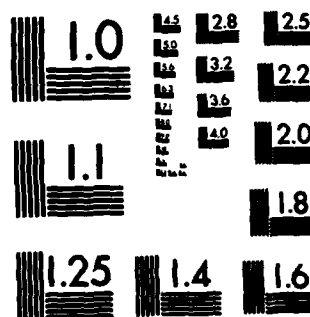
NUMERICAL SOLUTION OF LAMINAR AND TURBULENT BOUNDARY
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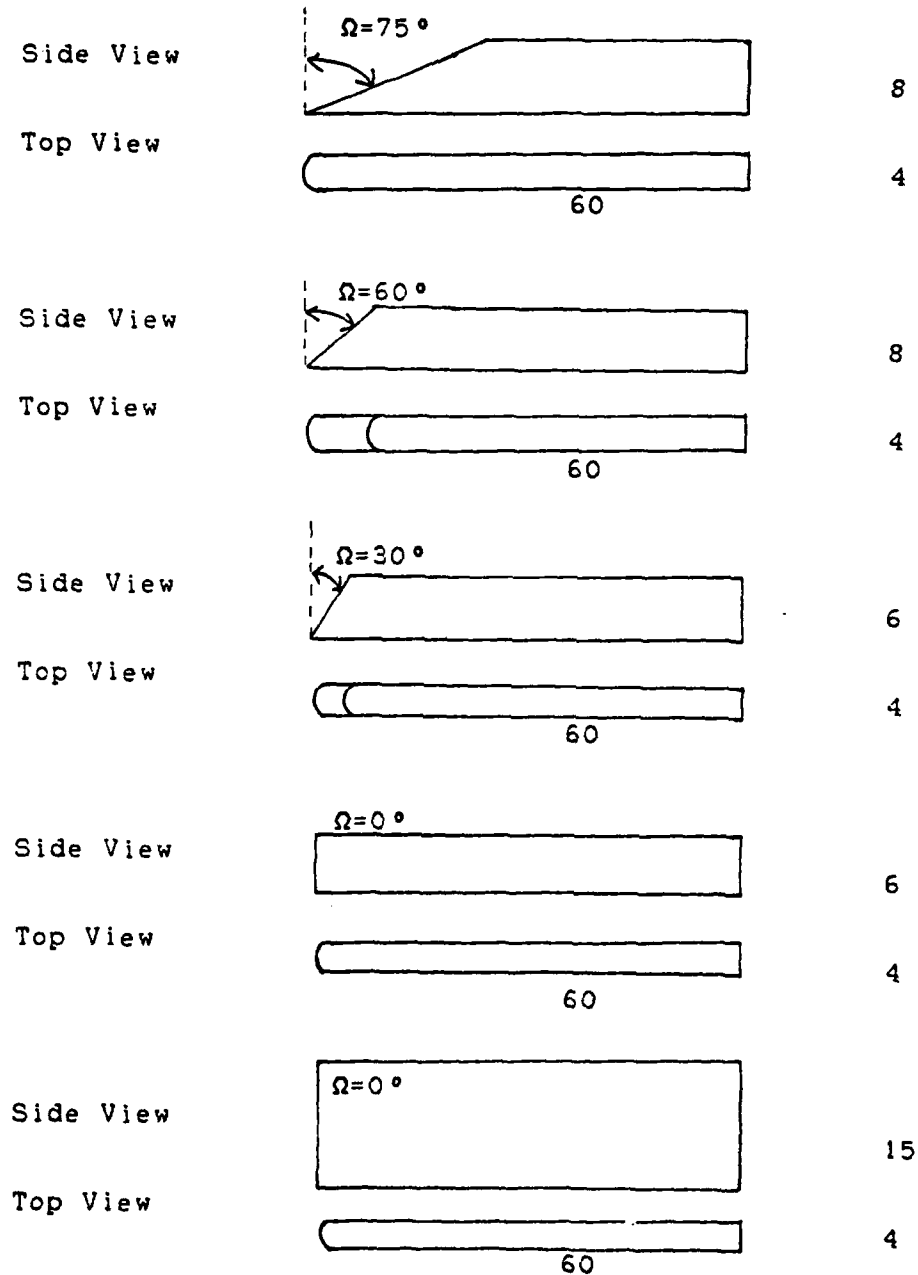
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MICROCOPY RESOLUTION TEST CHART
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Dimensions are in millimeters

Figure 3.7. Blunt Fin Models

3.6 Experimental Techniques

3.6.1 Oil Flow Visualization Technique

The surface oil flow technique is intended to quickly and easily investigate the nature of the flow over the surface of a wind tunnel model. A mixture of gasoline, titanium (IV) oxide (talcum powder), and oleic acid was used for the flat plate tests, but was found to be inadequate for the three dimensional flow visualization. A new mixture of motor oil and titanium oxide was used with excellent results. The surface of the model was lightly painted with the oil mixture. The air flowing over the surface carries the oil with it and leaves a streaky deposit of the powder remaining to mark the direction of flow. The patterns made by the streaks indicate the surface streamlines and the local direction of flow on the surface.

In interpreting oil flow patterns, Maskell's definition of a three dimensional separation line is used (40:161). This says that

"the three dimensional separation line is an envelope of converging limiting surface streamlines."

In many three dimensional situations, oil may accumulate along a line because the surface shear becomes too low to overcome the friction and surface tension of the oil. Care must be taken not to interpret this accumulation line as a separation line. The true separation line, in the Maskell sense, has streamlines converging on it from either side.

Another complication in interpreting oil flow patterns in three dimensional flow interactions is the separation line in the absence of an oil film and the line of oil accumulation will differ due to two reasons (51:29):

1. The effective body shape of the model will be altered by the change of the oil film height, thus altering the inviscid flow properties.
2. The separation of the air flow will be delayed in the presence of an oil film, due to the finite velocity of the air at the oil surface. This will increase the momentum of the air boundary layer leading to flow separation at a place of higher pressure gradient.

This difference is illustrated in Figure 3.8. However, since these influences are small, the oil flow technique is still a very valuable visualization tool.

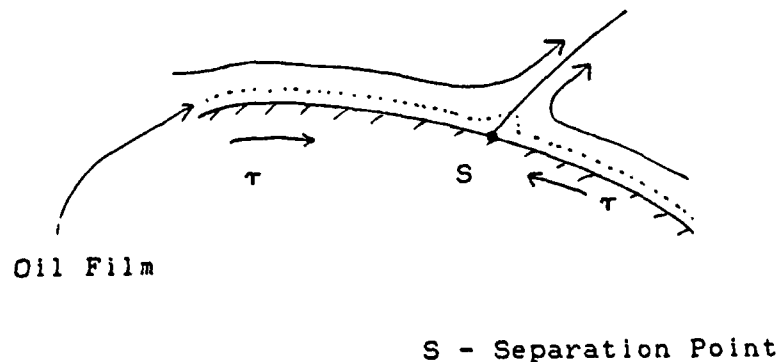


Figure 3.8. Separation in Oil Flow Experiments

The ratio of ingredients, oil : powder, to achieve good test results is a matter of trial and error. For these tests, a thin consistency mixture produced the best results. However, if the mixture was too thin, the oil tended to drip too fast over the plate after the plate is taken out of the injected position, therefore ruining the important patterns. Painting the model with a minimum quantity of the mixture, also seemed to work best.

3.6.2 Sublimation Visualization Technique

The sublimation method is a visualization technique to detect the location of the transition and turbulent regions (51:19-21). The technique consists of coating the model surface with a dark colored paint, and then coating the model with a thin, constant thickness layer of a subliming material, in this case, acenaphthene. The acenaphthene is dissolved in ether and sprayed on the model. The ether evaporates quickly and leaves the white acenaphthene on the model. During exposure to an airstream, sublimation occurs along the surface at varying rates, depending on the diffusion mass transfer of the substance in gaseous form into the air boundary layer. The diffusion mass transfer rates depend on the character of the boundary layer; whether it is laminar, transitional, or turbulent. The mass transfer of an attached turbulent boundary layer is approximately one order of magnitude larger than for an attached laminar boundary layer (51:19).

For zero pressure gradient boundary layers, Reynold's analogy states that the amount of heat transfer (characterized by the Stanton number), can be directly related to the momentum transfer (skin friction coefficient). The heat transfer can also be related to the mass transfer. This relation is explained in detail by Stock (51:22-23). The skin friction coefficient, c_f , distribution in a flat plate boundary layer is shown in Figure 3.9. From Figure 3.9, it can be seen that the regions of highest skin friction coefficient will sublime first - exposing the dark model underneath. Therefore, an indication of the location of transition will be determined using the sublimation technique.

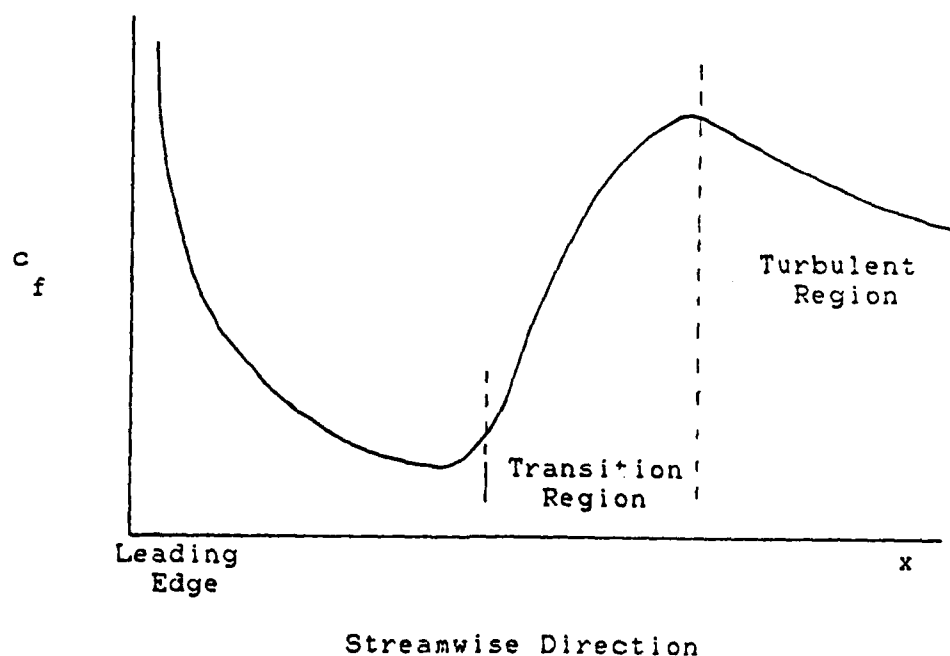


Figure 3.9. Skin Friction versus Streamwise Direction

For three dimensional flows, regions near the fin on the flat plate will also have different skin friction coefficients so the sublimation technique will also be used to examine this flow interaction region.

3.6.3 Shadowgraph Optical Technique

A popular optical system used in wind tunnels is the shadowgraph system. The principal behind the shadowgraph technique is very similar to the schlieren technique, except that there is no knife edge. The shadowgraph system gives an image representative of the variation of density gradients throughout the complete flow field. The VKI H-3 wind tunnel has a "double-pass" shadowgraph/schlieren system, that is, the light rays are passed through the test section twice. When axisymmetric models are being tested, the double-pass system is used rather than a single-pass system because in hypersonic tunnels the test section density is very low. Therefore it is usually difficult to obtain high quality shadowgraph or schlieren pictures with a single-pass system (42:233).

The shadowgraph photographs were used to examine the height of the laminar boundary layer, the shock formed from the leading edge of the flat plate at incidence, and the flow interaction with the fin.

3.6.4 Liquid Crystal Technique

The liquid crystal technique is a qualitative and semi-quantitative technique to visualize the global heating of an

area of interest. An advantage of using liquid crystals over phase change paints is that a coat of liquid crystals can be used for several tests. Phase change materials must be reapplied for every test, which is very inconvenient. Liquid crystals also have a response time on the order of milliseconds (47:15) making it more effective for transient heat transfer tests.

The liquid crystals that will be used in this test are cholesterics. Cholesteric molecules have a helical structure. One of the most interesting properties of the cholesterics is the influence of the temperature on their structure. When a light beam is sent parallel to the helical axis, the reflected wave length depends on the characteristics of the lattice and also on the angle of observation. Therefore, when the helical structure changes with temperature, the reflected wavelength will also be modified, and if the wavelength is in the visible spectrum, the temperature will be associated with a change of colors. The typical colors that will be encountered in this test are red (24.5°C), yellow, green, and blue (28.5°C).

The helical structure of normal liquid crystals are sensitive to shear stress. The shear stresses produced by hypersonic flows produce shifting of the helical structure, thus changing the reflected wavelength. It is impossible to determine in a wind tunnel whether a change of color is due to temperature or shear stress. To overcome this problem,

C encapsulated liquid crystals are used. Small droplets of the cholesteric material are embedded in gelatin bubbles with a diameter of three to five micron. This gelatin bubble shields the liquid crystals from shear. Disadvantage of the encapsulation will be loss in the brilliancy of the colors.

(To determine the temperature changes of a surface due to a flow, the surface material should have a low thermal conductivity. Since it is desired to record the changes seen by the liquid crystals, it is important that the heat does not conduct through the material. If this should occur, then the heat transfer effects seen will be much lower than in reality due to the conduction effects. The tests will therefore be completed on a plexiglas insert rather than directly on the metal model.

C The surface color changes for the flat plate tests will be recorded by color photographs during each test. With an accurate correlation between color and temperature, the temperature time history can be determined. For complicated flows, the color changes could be recorded by a color video camera and the determination of the temperature could be enhanced with the use of a digital image processing (DIP) technique which can distinguish between shades of red, green, or blue from the intensity. For completeness, the DIP is explained in a little more detail in Appendix G.

3.6.4.1 Theory

Once the surface temperatures are determined from the liquid crystals, the heat transfer rate can be determined. To obtain the relation to solve for the heat transfer rate from surface temperature, the one dimensional heat equation must be solved for an initially isothermal semi-infinite slab. The heat equation is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3.16)$$

The solution of the heat equation is shown in great detail by Degrez (15:57-60). If $T_s(0)=0$, and if the heat flux is a constant in time,

$$q = \frac{(\rho c_p k \pi / t)^{1/2}}{2} T_s(t) \quad (3.17)$$

The Stanton number can then be calculated using

$$St = \frac{HC}{c_p \rho U_\infty} \quad (3.18)$$

where HC is the heat transfer coefficient and is defined by

$$HC = \frac{q}{T_{AW} - T_W} \quad (3.19)$$

T_{AW} is the adiabatic wall temperature and is defined by

$$T_{AW} = T_\infty [1 + .5r(\gamma - 1)M_\infty^2] \quad (3.20)$$

where r is the recovery factor, and for laminar flow is

$$r = Pr^{1/2} \quad (3.21)$$

3.6.4.2 Validity of Assumptions

The assumptions used in the theoretical analysis of the liquid crystal technique are

- 1) one dimensional heat transfer
- 2) semi-infinite slab
- 3) constant heat flux
- 4) liquid crystals do not disturb the surface heat flux to the model

Each assumption will be examined to determine if it is valid for the conditions of the test.

The heat transfer into the model must be one dimensional. This will be assumed to be true for the model, except near the edges of the model or at any discontinuity of the material.

The plexiglas on which the liquid crystals are mounted must be thick enough so that the heat transfer into the substrate is similar to that of a semi-infinite solid. Therefore, the base of the plexiglas should remain at a constant temperature. The thickness at which T_x/T_s and q_x/q_s remain smaller than 1% for a given time is approximately (48:8)

$$x = 4 (\alpha t)^{1/2} \quad (3.22)$$

This equation can then be solved for t . Since the thickness of the plexiglas is five mm, the time at which the semi-infinite slab assumption is valid is shown in Table 3.2 for plexiglas, and also stainless steel and aluminum for

comparison. Table 3.2 clearly shows why the plexiglas insert was chosen for the test.

The third assumption used in the analysis was constant heat flux. The validity of this assumption can be examined at the conclusion of the test. When the heat transfer rate is constant, the surface temperature increases as the square root of t . This can be seen from equation (3.17). If $(T_s)^2$ versus t is plotted, the resulting curve should be a straight line. If the line is not straight, the constant heat flux assumption is not valid.

Table 3.2. Semi-infinite Slab Criteria

	Plexiglas	Stainless Steel	Aluminum
$\rho(\text{kg/m}^3)$	1.19×10^3	7.9×10^3	2.7×10^3
$k(\text{Watts/m}^\circ\text{K})$	0.188	15.0	
$cp(\text{J/kg}^\circ\text{K})$	1463	440	
$\alpha(\text{cm}^2/\text{sec})$	0.0011	0.043	0.85
$t(\text{sec})(x=5\text{mm})$	14.5	0.36	0.018

The fourth assumption used in this technique is that the liquid crystals do not disturb the surface heat flux to the model. This will be assumed to be true since the layer of liquid crystals is very thin, on the order of 10^{-6} meter.

3.6.4.3 Calibration of the Liquid Crystals

The calibration of the liquid crystals is critical in that it is the basis of the correlation between the colors on the photograph and temperature on the surface of the model. The calibration is performed on the metal portion of the model. A coax gage is installed in the stainless steel and the entire model is sprayed with the liquid crystal solution. The details of the model preparation are described in Appendix F. Details concerning the coax gage will be considered in section 3.6.6. Since metals are good heat conductors, the temperature found by the coax gage can be assumed to be uniform. This will also be shown during the calibration if the liquid crystals on the metal portion are a uniform color.

To obtain calibration data that will be applicable to the subsequent tests, the same conditions must be present for the calibration as the actual tests. This requires that the position of the model, camera, and light remain constant. The position of the model for filming will always be in the injected "test" position. The camera and light should not be moved after the calibration until all the tests are completed.

After the model is installed in the tunnel, a vacuum pump is installed to produce a vacuum in the test chamber. The purpose of the vacuum is to prevent any convective heating. Since it is impossible to seal the H-3 chamber,

the vacuum pump evacuates the test chamber to approximately 0.4 bar, this taking about two hours. The amount of convective heating should be very small at this low pressure. The model is placed in the injected position where it is heated by two 1000 watt lamps until the model has reached at least the temperature that corresponds to the color of the liquid crystals. The highest temperature visualized by the liquid crystal corresponds to the blue color. The plate required over 60 seconds of heating before the first color appeared. Then the lights are turned off and the model is allowed to cool. When the liquid crystals on the metal portion of the model reaches a uniform color, or steady state temperature, the model is photographed and the voltage from the coax gage is recorded. The model is then allowed to cool some more and this process is repeated until all the colors have been photographed.

3.6.5 Thin Film Heat Transfer Gages

The thin film technique is well suited for measurements in transient conditions due to the fast response time. However, thin film gages have a disadvantage that large errors can occur if used for long duration tests, due to nonisothermal wall effects. The thin film gage is actually a thin-film surface resistance thermometer. The gages consist of thin metallic films applied to the surface of substrates of low heat conductivity. If the thin film gages are properly made, the heat flow into the substrate is one

dimensional and similar to that of heat flow into a semi-infinite solid. The variation of temperature with time is obtained from the thin film gages. The gages become one leg of a wheatstone bridge. Before a test, at ambient conditions, the bridge is balanced. During a test, as the plate is heated the resistances of each gage change and the unbalance of the bridge can then be related to the variation of temperature. The heat flux may then be deduced using a numerical procedure or an electrical analog circuit.

The construction of a thin film gage requires a lot of time, patience, and very precise workmanship. The instructions will not be elaborated upon in this report but the details of the construction of a thin film gage are explained very well by Ligrani, et al (37:3-10). Since ceramic is easier than quartz to machine to precise dimensions, ceramic was chosen as the substrate for the thin film gages in this study. The dimensions of the ceramic must be chosen carefully because the depth of the substrate is dependent on the tunnel running time. Since the semi-infinite slab assumption is used in the data reduction technique, this aspect must be considered.

3.6.5.1 Theory

The one-dimensional heat equation, equation (3.16) is used in the analysis of thin film gage data. Recall that the detailed solution of the one-dimensional heat equation is given in (15:57-60). However, at this point in the

analysis, the heat flux is not yet assumed to be constant. The equation for the heat flux is given by:

$$q_s(t) = \frac{(\rho c p k / \pi)^{1/2}}{2} \left[\frac{2T_s(t)}{t^{1/2}} + \int_0^t \frac{T_s(t) - T_s(\tau)}{(t-\tau)^{3/2}} d\tau \right] \quad (3.23)$$

The evaluation of the heat transfer rate from the observed surface temperature record may be obtained by integrating eqn (3.23) numerically or by the use of electrical analogs.

Arts (5) mentions that the method using the electrical analog circuit to integrate the temperatures is more accurate than the numerical integration methods due to the uncertainty of the exact time of the beginning of the heat flux. If the exact time of the beginning of the heat trace is not known, an approximate time is used for the numerical integration methods. This error is then propagated in the numerical integration. The electrical analog circuit method, is not as sensitive to this problem.

For either method, the surface temperature rise, $T_s(t)$, in equation (3.23) is found from the resistance change of the thin film. For the film,

$$R = \frac{R_{REF} [1 + \alpha (T - T_{REF})]}{R} \quad (3.24)$$

where R is the film resistance at surface temperature T , R_{REF} is the film resistance at temperature T_{REF} , and α is the film temperature coefficient of resistance. By Ohm's law, when the film is supplied with a constant current

$$V = V_{REF} \left[1 + \frac{\alpha}{R} (T - T_{REF}) \right] \quad (3.25)$$

If the reference voltage V_{REF} is measured at the ambient temperature, T_a , before a test then $T_a = T_{REF}$, and

$$T_s = \frac{\Delta V}{\frac{\alpha}{R} V_{REF}} \quad (3.26)$$

where ΔV is the film output voltage, $V - V_{REF}$, and V_{REF} is the initial film voltage.

The numerical computation of heat transfer rate from the temperature measurements can be made on the basis of equation (3.23). This method may be called a direct method in that $q(t)$ is obtained directly. Note that there is a singularity at $t = \tau$. If $\Delta V(t)$ is assumed to be piecewise linear the final equation is

$$q_s(t_n) = \frac{2(\rho c_p k / \pi)}{\frac{\alpha}{R} V_{REF}} \sum_{i=1}^{1/2 n} \frac{\Delta V(t_i) - \Delta V(t_{i-1})}{\frac{1}{2}(t_n - t_{i-1}) + \frac{1}{2}(t_n - t_i)} \quad (3.27)$$

where $\Delta V(t)/(\frac{\alpha}{R} V_{REF})$ is the rise in surface temperature $T_s(t)$ as shown in equation (3.26). To use eqn (3.27), it is generally necessary to eliminate any noise from the data before integration. Details of the derivation of equation (3.27) can be found in Schultz and Jones (48:35-37).

Another method to numerically compute the heat transfer rate from temperature measurements is an indirect method, in that $Q(t)$, the cumulative heating, is obtained first and then differentiated with respect to time to obtain $q(t)$.

$$Q_n(t) = \frac{(\rho c p k / \pi)}{\alpha V_{REF} R} \sum_{i=0}^{1/2 n} \frac{[\Delta V(t_i) + \Delta V(t_{i+1})] \Delta t}{(t_n - t_i)^{1/2} + (t_n - t_{i+1})^{1/2}} \quad (3.28)$$

$$q(t) = \frac{dQ(t)}{dt} \quad (3.29)$$

This differentiation is performed using a 4 point central finite differencing scheme

$$q(t) = \frac{-2Q(i-8) - Q(i-4) + Q(i+4) + 2Q(i+8)}{40 \Delta t} \quad (3.30)$$

In the indirect method, the integration tends to smooth the data and no preliminary noise reduction is necessary. The only approximation used in equations (3.27) and (3.28) was the local linearization of $\Delta V(t)$.

If the electrical analog circuit is used to do the integration in equation (3.23), an examination of the analog circuit is warranted. The electrical analog circuits are explained in detail in Appendix H. From the solution of the one dimensional heat equation, assuming that the heat flux is constant, equation (3.31) gives the heat flux in terms of surface temperature.

$$q_s(t) = \frac{1}{2} (\rho c p k \pi / t)^{1/2} T_s(t) \quad (3.31)$$

If equation (3.26) for T_s is substituted into equation (3.31), then

$$q_s(t) = \frac{1}{2} (\rho c p k \pi / t)^{1/2} \frac{\Delta V}{\alpha V_{REF} R} \quad (3.32)$$

where α_R is determined for each gage before testing using the equation

$$\alpha_R = \frac{1}{RREF} \frac{(R - RREF)}{(T - TREF)} \quad (3.33)$$

If this calibration is done on a different day than the test itself, the ambient temperature may be different. Therefore, the measurement of RREF may be made at a temperature TREF different than the gage temperatures initially before a test, Ta. In order to correct for this difference, the value of α_R in eqn (3.33) must be corrected before data reduction. Correcting for this effect, equation (3.32) becomes

$$q_s = \frac{(\rho c p k \pi / t)}{2} \frac{\Delta V}{\alpha_R VREF} [1 + \alpha_R (Ta - TREF)]^{1/2} \quad (3.34)$$

which is then the same as equation (3.32) when Ta = TREF. Typically, this correction term may account for 1% of q_s if the difference of the temperatures is between 5 to 10°C.

From the electrical analog circuit analysis in Appendix H, where VOUT is the voltage output from the analog circuit,

$$\frac{\Delta x}{\beta} = \frac{\Delta V}{2 VOUT} (\pi / t)^{1/2} \quad (3.35)$$

solving for ΔV and substituting into equation (3.34) the new equation for the heat flux is

$$q_s = \frac{VOUT}{VREF} \frac{(\rho c p k)}{\alpha_R} \frac{\Delta x}{\beta} [1 + \alpha_R (Ta - TREF)]^{1/2} \quad (3.36)$$

The calibration of the thin film gages and determination of all the terms in equation (3.36), V_{REF} , α , $(\rho_{cpk})^{1/2}$, and $A\pi/\beta$, are described in detail in Appendix I.

3.6.5.2 Validity of Assumptions

The assumptions used in the theoretical analysis of the thin film gage technique are

- 1) one dimensional heat transfer
- 2) semi-infinite slab
- 3) thin film gages do not disturb the surface heat flux to the model

and for the electrical analog integration, an additional assumption is

- 4) constant heat flux

Each assumption will be examined individually to determine its validity.

The heat transfer into the model has been assumed to be one dimensional. This will be valid provided the gages are not placed near the edges of the substrate or near the holes in the substrate which may be used for wire connections. A rule of thumb is to place gages away from substrate discontinuities at distances equivalent to or greater than those required for a semi-infinite substrate behavior.

The ceramic on which the gages are mounted must be thick enough so that the heat transfer into the ceramic is similar to that of a semi-infinite solid. It is desired

that the base of the ceramic should remain at a constant temperature. The thickness at which T_x/T_s and q_x/q_s remain smaller than 1% for a given time has been previously given in equation (3.22). For a substrate thickness of 2.5 mm, the test time for a valid test has been determined to be approximately 0.65 seconds.

The third assumption used in this technique is that the thin film gage does not disturb the surface heat flux to the model. This will be assumed to be true since the layer of metal film is very thin, on the order of 10^{-6} to 10^{-7} m (37:23) and it has a high thermal conductivity.

For the electrical analog circuit, the theoretical analysis had an additional assumption: constant heat flux. Again, the validity of the assumption will be considered later by plotting T_s^2 versus t and examining its deviation from a straight line.

3.6.5.3 Data Acquisition - Numerical

Data acquisition for the numerical technique will consist of determining $\Delta V/(\alpha V_{REF})$. α will have been determined previously as explained in Appendix I. However, before a test can even be started, the data channels of the acquisition system must be calibrated. Using the variable resistors in the current generator box, the Wheatstone bridge must be balanced before any calibration or test. The output from the bridge should pass through the amplifier and be recorded by the data acquisition system. These data

units should be close to zero for a balanced bridge. A resistor, R_a , is then added in series to the thin film gage. The output of the data acquisition system can then be amplified to cause the output to be near the full scale of the data acquisition system. The calibration coefficient may then be expressed in terms of ohms/digital units. Once the calibration coefficient is established, the amplifier setting for each data channel must not be changed. The resistance, R_a , is chosen using the resistance versus temperature calibration plots for α for each gage depending on the expected maximum temperature rise R during an actual test.

During a test, the thin film gages are supplied with a constant current by a differential amplifier circuit. The voltage output of the gage may be read from the readout window on the current generator. At ambient conditions before a test, the voltage read from this window is V_{REF} , the initial film voltage. The resistance of the thin film gage, R_{FILM} , should also be recorded by an ohm meter at ambient conditions before test, to determine the initial temperature of the gage.

The voltages are then numerically integrated using equation (3.27) or (3.28). The program used to perform this integration is shown in Appendix L. Once the heat transfer rate is determined, the Stanton number can be calculated using equations (3.18) through (3.21).

3.6.5.4 Data Acquisition - Electrical Analog

The voltage output of the gage is determined exactly as previously described in the data acquisition for the numerical technique. However, the calibration of the data channel as described using a resistor in series with the gages is not necessary since the electrical analog circuit will be used to reduce the data from the thin film gages. The data channels will be calibrated in conjunction with the analog circuit, as described in Appendix I when determining the A/β term.

The output from the thin film gage after amplification is connected to the analog circuit which allows the conversion of a parabolic wave into a step wave. The output from the analog circuit is then amplified again, and the output is called VOUT. Using equation (3.36), the heat transfer trace can be obtained. This trace should be examined, and a time interval, from τ_1 to τ_2 , over which steady flow conditions are present should be chosen. The time interval is important for signal processing. It should be long enough to obtain enough data points for a least squares data fit, but also should not include intervals when flow conditions are not well established, i.e., during model injection. This time interval chosen will correspond to temperatures T_1 and T_2 .

The next step in the signal processing is to reconstruct the surface temperature versus time trace from a

given heat transfer rate versus time trace. This is done using an equation given by Oldfield, Jones, and Schultz (37:34)

$$\Delta T_s(t) = \frac{4}{3} (\rho c p k \pi)^{-1/2} \sum_{n=0}^m \left[q_{n+1} + q_{n-1} - 2q_n \right] (t - t_n)^{3/2} H(t - t_n) \quad (3.37)$$

In equation (3.37), q_n is the measured heat transfer rate at time $t_n = n\Delta t$. $H(t - t_n)$ equals 1 for $t \geq t_n$, and zero for $t < t_n$. If the time t may be expressed as $m\Delta\tau$, where $\Delta\tau$ is the time interval between data samples, then equation (3.37) may be rewritten as:

$$\Delta T_s(m\Delta\tau) = \frac{4}{3} [\Delta\tau / (\pi \rho c p k)]^{1/2} \sum_{n=0}^m \left[q_{n+1} + q_{n-1} - 2q_n \right] (m - n)^{3/2} \quad (3.38)$$

Alternatively, if q_{n+1} , q_{n-1} and q_n are expressed in terms of data units instead of dimensionally in watts/m²,

$$\Delta T_s(m\Delta\tau) = D \sum_{n=0}^m \left[Nq_{n+1} + Nq_{n-1} - 2Nq_n \right] (m - n)^{3/2} \quad (3.39)$$

where

$$D = \frac{4}{3} [\Delta\tau / (\pi \alpha V_{REF})]^{1/2} \frac{A^*}{\beta} \quad (3.40)$$

Equations (3.37) through (3.40) are based on a fit of the heat transfer versus time trace using a series of ramps. The derivation of equation (3.37) is given in (37:44). The

computer program used for the reconstruction of the temperature versus time trace is shown in Appendix M.

After obtaining the heat flux versus time and the temperature versus time plots, the heat transfer versus temperature curve may be constructed. A least squares data fit to a straight line is made between the two temperatures, T_1 and T_2 . This line is then extrapolated back to the value of q_s at the initial test surface temperature which is usually the same as ambient. This value of q_s is then equivalent to that obtained for isothermal test conditions. The program used to find the isothermal heat flux is listed in Appendix N. Once the heat transfer rate is determined, the Stanton number can be calculated using equations (3.18) through (3.21).

3.6.6 Co-axial Thermocouple Gages

The coax heat transfer gage is a coaxial surface thermocouple. The temperature time history is obtained from this gage and used to determine the surface heat flux. The coax gage assembly is illustrated in Figure 3.10. An electrically insulated center conductor, chromel, is enclosed within a cylindrical jacket of constantan. The surface thermocouple junction is formed by the vacuum deposition of a metal about one micron thick. An advantage to the use of the coax gage is that it is thermally matched

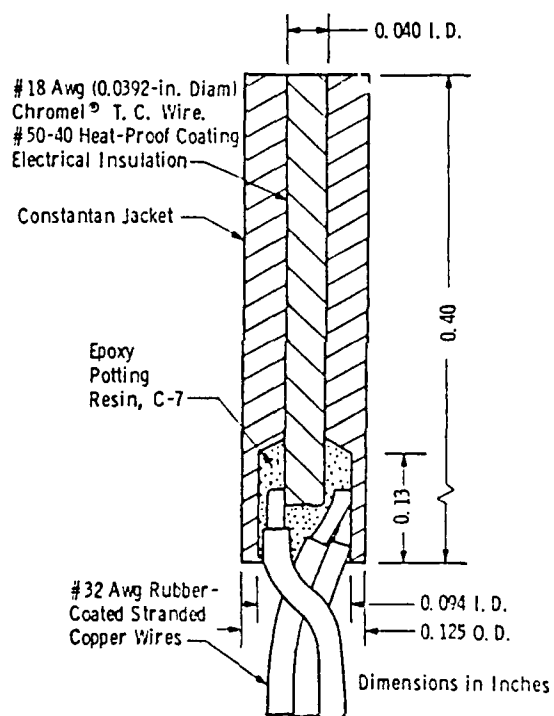


Figure 3.10. Coax Gage Assembly

to the model, therefore allowing it to be used for long duration tests. A disadvantage with the coax gages is that it has a low output.

The choice of material for the center conductor and outer cylindrical jacket is dependent upon the test model material, or vice versa. In this test, the material is stainless steel, so the thermal properties of the gage must be similar to that of stainless steel. Chromel and constantan fulfill this requirement, as shown in Table 3.3 (10:7). The stainless steel used for the model was AISI 304 stainless steel.

Table 3.3. Thermal Properties of Materials

	$\frac{1}{2}$ ($\rho c p k$)	α	cp	k
	$\frac{1}{2}$ $\frac{\text{BTU/sec}}{\text{ft}^2 \text{ } ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{BTU}}{\text{lb } ^\circ\text{F}}$	$\frac{\text{BTU}}{\text{ft hr } ^\circ\text{F}}$
Chromel	0.410	0.19	0.107	11.0
Constantan	0.408	0.23	0.098	13.0
304 Stainless Steel	0.353	0.17	0.105	8.7
17-4 PH Stainless Steel	0.409			
302 Stainless Steel		0.16	0.110	9.4
410 Stainless Steel		0.27	0.110	14.0
Aluminum		3.30	0.214	119.0
Copper		4.30	0.092	219.0

The coax gage is installed through the model wall so that the outer model surface temperature may be measured using wires emerging from the back surface. The gages used in this study have an additional wire that enables the measurement of the temperature of the back side of the gage. This is significant in that these coax gages are not restricted by the semi-infinite slab assumption used by many other heat transfer gages. During long tests, the back side temperature rise can be monitored and accounted for in the data reduction technique.

The popularity of the coax gage has arisen due to several reasons (10:6). The coax gage measures surface temperatures more accurately than other gages because the conduction losses are minimized since the thermocouple materials are actually part of the wall, ie, have the same thermal properties as the wall material. The gage is durable in that it is able to withstand high levels of erosion during testing. The gage may be contoured exactly to the model surface by sanding, so convective or other material flow over the surface is undisturbed. Also the small mass of the thermal junction allows a nominal response time of approximately one microsecond for the vacuum deposited junction or ten microseconds for junctions created by sanding. The coax gage can also be manufactured very small, which allows a very close spacing of thermocouples. The gages are available in probe diameters starting from

0.015 inches (0.381 mm). Also, the voltage versus temperature correlation is a property of the gage materials and is already determined by the manufacturer so no gage calibration is necessary.

3.6.6.1 Theory

The heat equation for one dimensional flow is:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3.41)$$

To account for gage back side heat loss, a finite difference solution of the heat equation was developed (25:3). A simple forward time and central spaced explicit method was applied to the heat equation.

$$\frac{T(N+1,I) - T(N,I)}{\Delta t} = \alpha \frac{T(N,I+1) - 2T(N,I) + T(N,I-1)}{(\Delta y)^2} \quad (3.42)$$

Solving for $T(N+1,I)$ results in the following equation.

$$T(N+1,I) = \Theta_s [T(N,I-1) + T(N,I+1) + (1/\Theta_s - 2) T(N,I)] \quad (3.43)$$

The heat transfer rate is solved using a three point forward differencing scheme.

$$q(N) = -\frac{k}{2\Delta y} [-3T(N,wall) + 4T(N,wall+1) - T(N,wall+2)] \quad (3.44)$$

$$\text{where } \Theta_s = \frac{\alpha \Delta t}{\Delta y^2} \quad (3.45)$$

For this scheme, boundary conditions are imposed to force the temperature at the inner wall surface ($I=1$) to match the coax gage surface temperature while the gage back side temperature ($I=I_{max}$) is forced to match the back side

temperature measurement. An important restriction for the NSWC method is that the stability constant, Θ_s , must be less than 0.5 for the solution procedure to be numerically stable.

3.6.6.2 Validity of Assumptions

For the coax gage, there are only two assumptions used in the analysis and technique,

- 1) one dimensional heat transfer
- 2) coax gage does not disturb the surface heat flux to the model

Each assumption will be considered individually for its validity.

One dimensional heat transfer is assumed to be true since the thermal properties of the gage are similar to those of the stainless steel model. In complicated flows with high heating in localized areas, insulation of the gage may be required so that the peak heating from one point does not conduct laterally to a neighboring gage. For short coax gages, a portion of the heat transfer in the lateral direction can be taken into account by the use of the back face gage. However, for this study, the test duration is very short - approximately 500 milliseconds, so one dimensional heat transfer is assumed to be correct.

It is also assumed that the gage does not disturb the surface heat flux to the model since the gage is installed

in the body of the model. The surface of the gage is installed flush with the model surface.

3.6.6.3 Data Acquisition

The data acquisition involves recording the voltage from the coax gage. However, before the data channel of the acquisition system is used to process the signal from the coax gage, calibration of the amplifier and data channel is required. The calibration is completed by placing a known dc voltage into the amplifier and recording the data units from the data acquisition system to determine a volts/unit calibration constant.

Once the voltage from the coax gage is known, the temperature of the surface and back side can be determined using the calibration tables supplied by the manufacturer. These tables, referenced at 0°C, are included in Appendix Q. Once the surface and back side temperatures are known, the heat transfer rate can be determined using equations (3.43) through (3.45). Appendix R lists the computer program used for this computation. Once the heat transfer rate is known, the Stanton number can also be determined using equations (3.18) through (3.21).

3.7 Results of the Flat Plate Study

Table 3.4 shows the test conditions for all the photographs included in this report for tests on the flat plate. Appendix S shows the test conditions and VKI photograph catalog numbers for every test completed in this study. The shadowgraph photographs showed the undisturbed boundary layer height at approximately 60 mm from the plate leading edge to be less than 1 mm. According to Schlichting (46:140), the boundary layer thickness for incompressible flow, if defined when $U = .99U_{\infty}$, can be calculated using equation (3.46) at $x = .06$ meters

$$\delta = 5.0 \left[\frac{\mu x}{\rho U_{\infty}} \right]^{1/2} \quad (3.46)$$

Table 3.4. Flat Plate Test Conditions

Test	P o (Bars)	α (degrees)	M	Re -7 ($\times 10^7$) (/m)
O-1	20	0.0	6.0	1.69
O-2	20	5.5	5.26	2.23
L-1	20	0.0	6.0	1.69
L-2	10	0.0	6.0	0.844
H-1	20	0.0	6.0	1.69
H-2	10	0.0	6.0	0.844

where O - Oil flow test
L - Liquid crystal test
H - Heat transfer test

Table 3.5 shows the resulting calculated boundary layer thickness using eqn (3.46). The boundary layer thickness

was calculated at the appropriate test conditions using the numerical code explained in the computational effort, and the results are very similar to those from eqn (3.46).

Table 3.5. Boundary Layer Thickness

	δ (mm)
Po = 20 bar, Re = 1.69×10^7 /m	.3
Po = 10 bar, Re = $.844 \times 10^7$ /m	.5

3.7.1 Oil Flow Results

Figure 3.11 shows the results of the oil flow study on the flat plate at Mach = 6, Re = 1.69×10^7 /m and Po = 20 bar. The oil flow photograph shows the entire flat plate model with the direction of the flow being from left to right. At Mach = 6, most of the limiting surface streamlines are straight and parallel. The Mach 6 case was repeated many times because the results were not reproducible. Some photographs showed the flow as totally straight and parallel while some photographs showed some bending of the streamlines at the rear edges of the plate. However, the bent streamlines sometimes appeared on the right rear edge of the plate, and then the next test they appeared on the left rear edge of the plate. In conclusion, the flow near the edges of the plate at Mach 6 seems to be highly unsteady, but for most of the plate, there is parallel flow at Mach = 6.

Figure 3.12 shows the results of the oil flow study at Mach 5.26, $Re = .844 \times 10^7 / m$ and $Po = 20$ bar. In the center of the plate, the streamlines are straight and parallel. However, toward the edges, the streamlines bend toward the edges of the plate. This occurs since the plate is at an incidence and the flow spills over the sides of the plate.

3.7.2 Sublimation Results

The sublimation test did not produce much information. For all the test cases, sublimation only occurred at the leading edge of the flat plate. This indicates that the end of transition for each of the flow cases, occurs farther downstream than 150 mm, the edge of the plate. The results obtained are consistent with the findings of previous studies (15:F10-F12; 14:F20-F21; 31:110).

3.7.3 Liquid Crystal Results

The liquid crystal technique worked well, but some difficulties arose in trying to record the data. The photographs that recorded the color changes were very dark. There was very little contrast between the colors. The use of the digital image processing would probably make the reading of the colors easier. A strobe light was initially used to illuminate the model for photographic purposes. The strobe light proved to be inadequate as it did not uniformly illuminate the model and caused a large glare spot in the center of the model. One 1000 watt lamp was used next, but it did not produce enough light to obtain a good picture.

Finally, two 1000 watt lamps were used to illuminate the model. The lamps were turned on after the model was injected to keep the time that the model is illuminated down to a minimum. The ejector was run for approximately 15 seconds to verify that the two lamps did not heat the model enough to cause color changes during test conditions. The time of a actual test is less than 13 seconds. Although no color changes were detected during this preliminary time, the lamps were probably adding a small unknown amount of heat to the model. Future tests with liquid crystals need to use a different light source; one that has more illumination power, yet produces less heat.

The coax gage was referenced to the ambient temperature during the calibration. The calibration and test photographs were taken on the same roll of color film and developed at the same time so that the correlation between temperature to color would still be valid.

The surface of the liquid crystal is visually marred at the junction of the plexiglas and stainless steel. Apparently the epoxy glue used to fill the crack did not cure properly after application and the glue spreads into the liquid crystal layer. This did not disturb the flow, but just disturbed the visualization of the colors near the material boundaries.

One difficulty arose in the data reduction method used for the liquid crystal technique. This difficulty was in

determining the temperature of the surface of the model at time $t = 0$, just before injection of the model. The coax gage was not connected during the tests, but its voltage output was recorded before the tunnel was started to obtain the surface temperature of the model at ambient conditions. Later, from the thin film gage heat transfer tests, it was found that the model surface is cooled several degrees when the tunnel ejector is started - so the actual surface temperature of the model at the start of the test is quite lower than that at ambient conditions. The temperature of the model at time $t = 0$ could not be ascertained from the liquid crystals for the temperature was too low for the appearance of any color. Therefore, the initial temperature was obtained from the coax gages in the quantitative heat transfer test. Although the zero data from one test (H-1) is used in another test (L-1), this will introduce only a very small error to the liquid crystal results since the startup and initial conditions of the tunnel were the same for both tests.

The results of the liquid crystal test at $Po = 20$ and 10 bars is shown in Figures 3.13 and 3.14. Theoretical results obtained from eqns (3.12) and (3.14) are also plotted with the experimental data. The experimental errors associated with the liquid crystal tests are 8% for the $\log(Rex)$ and 5% for the Stanton number calculations. The results of the liquid crystal tests match very well with the

theoretical results. A check on the validity of the constant heat flux assumption could not be made using liquid crystal data since it was impossible to obtain temperature versus time data at a constant location. This is due to the limited calibration information. The validity of this assumption will be examined in the next section using the thin film gage data since the heating characteristics of the model are the same, just the method of obtaining the data are different.

3.7.4 Heat Transfer Results

Two thin film gage ceramic pieces were constructed as specified in Figure 3.6. Although only one piece was needed, the second was made in case the first piece was broken during testing. The first piece had thin film gages of 4 mm in length. The second piece had thin film gages of 2 mm in length. The 4 mm gage length piece was used in this test. The second piece, because of the shorter gage length, would be better suited for heat transfer measurements in three dimensional flows.

The film coefficient of resistance calibration data for the 4 mm thin film gages are shown in Table 3.6 and plotted in Figures 3.15 through 3.20. In Table 3.7 are the values of the film coefficient of resistance and other values obtained from the film coefficient of resistance data using a least squares algorithm for the 4mm long gages. The least

squares algorithm solves for the coefficients of eqn (3.47) which corresponds to eqn (3.33).

$$\Delta R = A_0 + A_1 \Delta T \quad (3.47)$$

where A_0 is the intercept of the line and should be approximately zero.

$$\Delta R = R - R_{REF}$$

$$\Delta T = T - T_{REF} \quad (3.48)$$

$$A_1 = \frac{\alpha R_{REF}}{R} = \text{slope of the line}$$

and F is a correlation coefficient that indicates how well the data matches the resulting equation of a straight line. Table 3.8 lists the film coefficient of resistance calibration data for the 2 mm thin film gages, and the same data is plotted in Figures 3.21 through 3.26. The second set of data in Table 3.6 and Table 3.8; gages 25, 26, and 27, correspond to the small gages made for the determination of the ceramic thermal product in the CT-2 tunnel. The value of the thermal product for the 4 mm gage ceramic piece was determined in CT-2 and is $1950.0 \text{ J/m}^2 \text{ }^{\circ}\text{K sec}^{1/2}$. For the 2 mm gage ceramic piece, the thermal product is equal to $2100.0 \text{ J/m}^2 \text{ }^{\circ}\text{K sec}^{1/2}$. The value of the film coefficient of resistance and other output from the least squares algorithm are listed in Table 3.9. The computer program with the least squares algorithm is listed in Appendix J.

The equipment setup is shown in Figure 3.27. The large box on the right contains the constant current generator,

100 gain amplifier, wheatstone bridge, electrical analog circuits, and the parabolic voltage generator. The next set of boxes, stacked four high, are an additional 16 adjustable gain amplifiers. The next piece of equipment is the medium speed data acquisition system with a maximum of 16 data channels, and then the computer console was used to start the acquisition program on the PDP computer. The heat transfer flat plate model installed in the tunnel is shown in Figure 2.28. The coax gages are referenced to an ice bath for the heat transfer tests.

Several complications arose during the initial portion of this testing period. There was initially a tremendous amount of noise in the data. This noise was decreased by grounding all the equipment in the data acquisition chain together and grounding the cables from the gages to the tunnel floor. The noise that remains is nearly the same order of magnitude as the data. The noise varies for each gage, but at worst has an unamplified magnitude of 0.2 millivolts, and has a frequency of 50 hertz. The magnitude and frequency of the noise did not change after all the lights and unnecessary equipment were shut off.

The second complication was due to the fact that a negative heat flux occurred on the surface of the plate prior to injection of the model. This negative heat flux occurs when the tunnel ejector is started and the test section is evacuated. This negative heat flux is several

times the magnitude of the data to be recorded. Because of this problem and the relatively large amount of noise present, the integration of the temperature trace using the analog circuit would be too inaccurate so could not be used. The numerical integration method was the only possible way to interpret the data under these conditions. This method is complicated by the reference shift during the test.

Before any good tests could be performed the air compressor for the tunnel broke. It was initially thought that the compressor would not be fixed in a short amount of time so testing was stopped and the medium speed data acquisition system was given to another student. However, the compressor started working after a couple days, so oscilloscopes were used to photograph the temperature trace so some data could be obtained. However, an accurate recording of the reference shift could not be obtained when using the oscilloscope. As a last resort, the high speed data acquisition system was obtained and two tests were performed and the data successfully recorded. Due to time limitations, only the two coax gages and ten thin film gages were recorded during each test. The final equipment setup was similar to that shown in Figure 3.27, except that the high speed data acquisition system was used, and only 12 additional amplifiers were available for the tests.

An example of the temperature trace obtained from a thin film gage is shown in Figure 3.29. The 50 hertz noise

was "smoothed" using an averaging technique. (A least squares algorithm was used initially, but gave incorrect results.) The data was averaged over one period of the noise. An example of the data after smoothing is shown in Figure 3.30. The averaging algorithm is shown in Appendix K. A comparison between raw versus smoothed data shows the smoothed data accurately represents the real data. The raw and smooth data for each channel was plotted and it was verified that the "real" data was kept intact before the numerical integration was performed.

A sample heat transfer rate versus temperature curve is shown in Figure 3.31. The curve shows a rise in heat flux to some maximum and then a decrease in the heat flux corresponding to the fact that the plate is heating up. The isothermal heat flux is obtained from the second portion of the curve and extrapolated back to the temperature of the plate at time = 0. An example of the heat flux versus temperature curve for the two data reduction methods are shown in Figures 3.22 and 3.33. These plots show the rise in heat flux, but the test time was apparently too short to obtain the second portion of the curve. For this reason, the value of the heat flux used to calculate the Stanton number is obtained near the end of the trace. Due to this reasoning, the Stanton number is expected to be lower than expected.

To verify the accuracy of the assumption of constant heat flux, the square of the surface temperature is plotted versus time in Figure 3.34 for one of the gages. If the initial portion of the curve is omitted because of the model injection, the trace is indeed a straight line for the period of the test, approximately .6 seconds.

Unfortunately, most of the data from the coax gages were unusable. An example of a trace is shown in Figure 3.35. Since a very small temperature increase was expected during the test, approximately 1 °C, the amplifiers were set for a gain of 1000, but this was not enough amplification to distinguish the real signal from the noise. A comparison between the thin film gage and the coax gage should be done at test conditions that will produce higher heating rates, ie. higher chamber pressure and lower Mach number.

The computer programs used to convert the data from the VKI PDP computer acquisition file is listed in Appendix K. Appendix L contains the input data files needed to convert the data from digital units to physical units. Appendix M lists the data conversion program, Appendix N is the numerical integration program for the thin film gages. Appendix P lists the program used to calculate the heat flux for an isothermal flat plate. Although not used in this study, Appendix O contains the temperature reconstruction program and Appendix R the coax gage reduction program.

The results of the heat transfer test at $P_o = 20$ and 10 bars are shown in Figures 3.36 and 3.37. These results show the beginning of transition. There is very little difference in the results from the direct method when compared to the results of the indirect method. The experimental errors in the heat transfer test are large, approximately 12%, due to the low amount of heat flux measured. The results of the heat transfer test are quite lower than the results from the theoretical equations (3.12) and (3.14).

Figure 3.36 shows the onset of transition for $Re = 1.69 \times 10^7 / m$ could be at $\log(Rex) = 6.26$ ($x = 107$ mm). However, what is missing from the figure is the fourth data point corresponding to $\log(Rex) = 6.19$ ($x = 91$ mm). The gain on the amplifier was too high for the heat flux trace that was present so no quantitative data could be taken. It is possible that it is just a stray point due to a bad gage, but it could also be an indication of transition.

Figure 3.37 shows the results for $Re = .844 \times 10^7 / m$. Again the fourth data point corresponding to $\log(Rex) = 5.89$ and $x=91$ mm is missing due to a high signal. Because of the lower Stanton numbers after transition, it appears that the position of fully turbulent flow is further downstream than for the higher Reynolds number case. Since no other data is available to determine if the fourth data point is bad, the beginning of transition for both Reynolds number cases

occurs between $x = 83$ mm and 91 mm. More data points need to be taken to better define the location of transition.



Figure 3.11. Plate, Oil Test O-1, $M = 6$, $Re = 1.69 \times 10^7 / m$

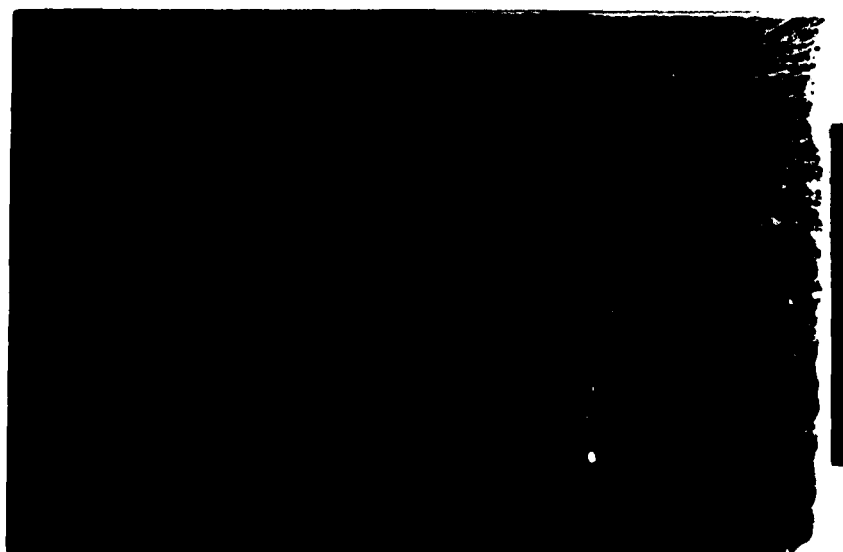


Figure 3.12. Plate, Oil Test O-2, $M = 5.26$, $Re = 2.23 \times 10^7 / m$

O

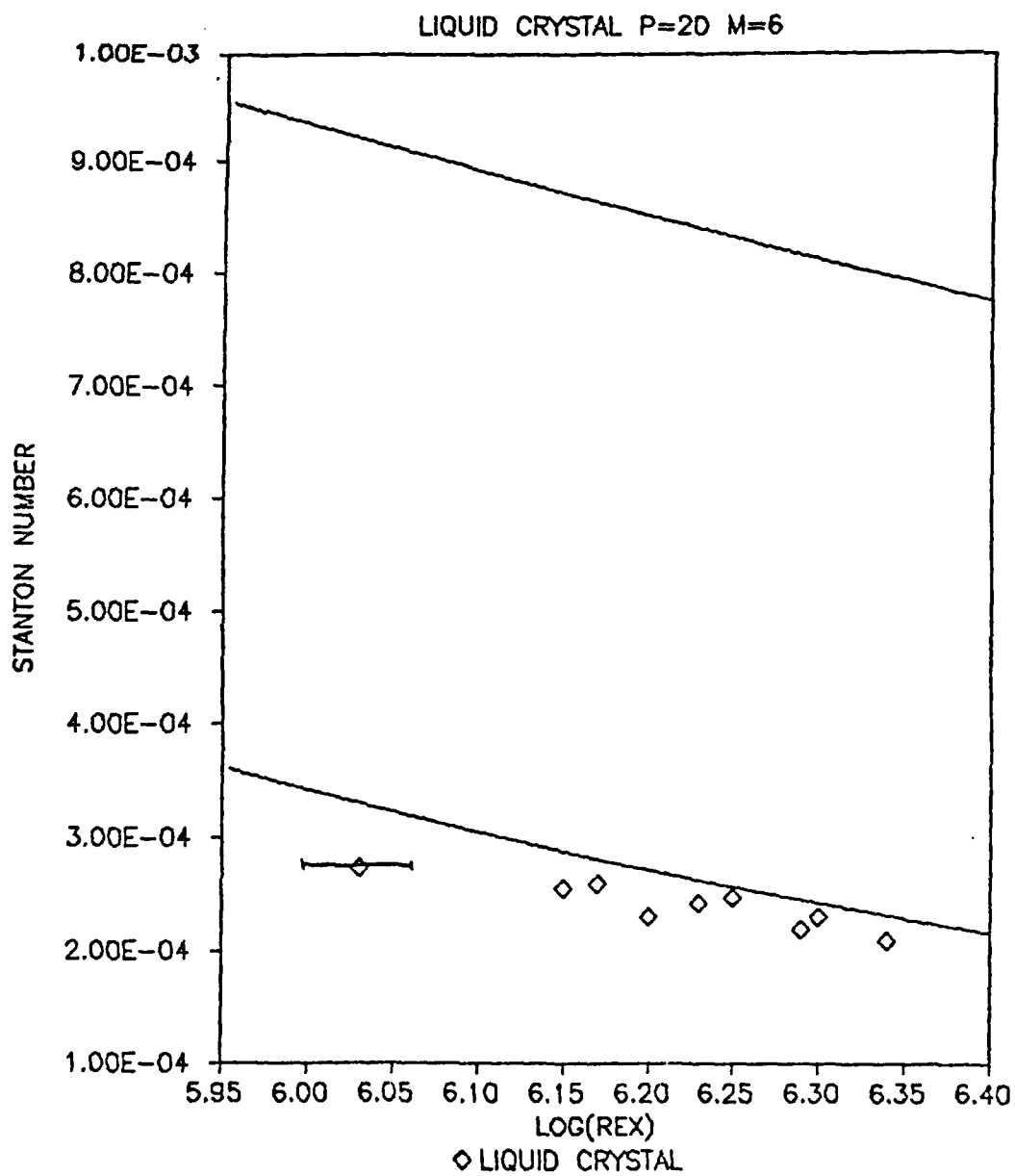


Figure 3.13. Plate, Liquid Crystal L-1, $Re = 1.69 \times 10^7$
 $M = 6$, $Po = 20$ bar

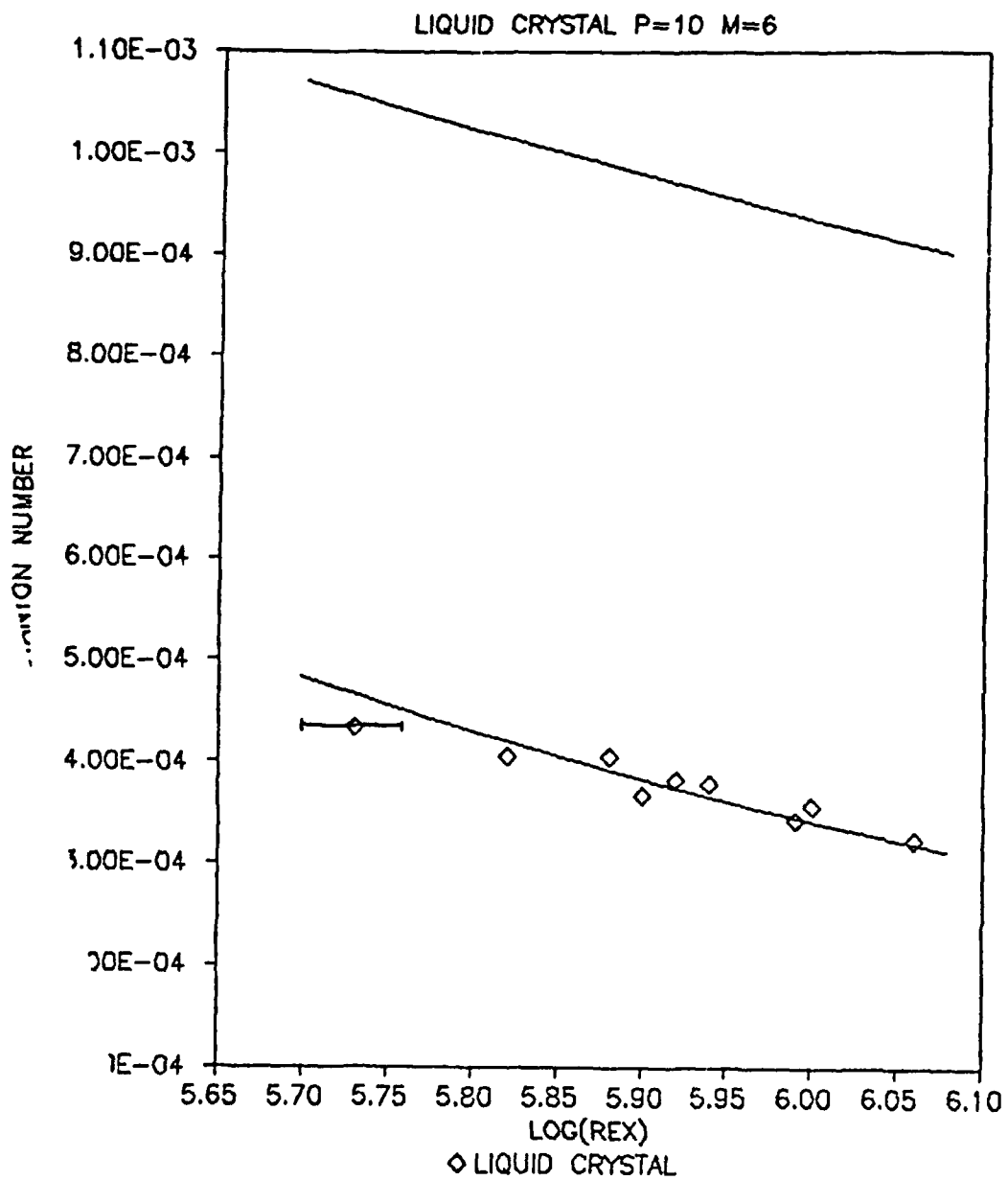


Figure 3.14. Plate, Liquid Crystal L-2, $Re = .844 \times 10^7 / m$
 $M = 6$, $P_o = 10$ bar

Table 3.6. Coefficient of Resistance Calibration Data
For 4 mm Thin Film Gage

		Gage Resistances (ohms)					
T °C		20.5	30.0	41.2	49.0	62.65	69.65
Gage No.							
1		46.26	47.37	48.43	49.28	50.75	51.45
2		50.84	52.04	53.27	54.20	55.85	56.66
3		51.59	52.80	53.27	54.91	56.90	57.33
4		46.06	47.20	48.24	49.15	50.58	51.39
5		47.52	48.62	49.80	50.67	52.20	53.01
6		51.83	53.26	54.29	55.37	56.94	57.90
7		54.41	55.63	56.89	59.84	59.48	60.36
8		47.34	48.39	49.46	50.30	51.72	52.47
9		59.24	60.60	62.05	63.11	64.98	65.96
10		51.72	52.97	54.30	55.27	56.99	57.88
11		57.30	58.58	59.90	60.88	62.62	63.50
12		51.18	52.36	53.52	54.41	55.96	56.77
13		49.09	50.17	51.28	52.12	53.58	54.33
14		45.29	46.27	47.27	48.01	49.34	50.00
15		46.39	47.36	48.37	49.12	50.46	51.11
16		47.60	48.77	49.67	50.52	51.86	52.60
17		54.73	55.87	57.04	57.90	59.44	60.21
18		50.31	51.38	52.53	53.24	54.64	55.37
19		57.41	59.27	59.81	61.05	62.36	63.40
20							*
21		55.41	57.42	57.61	58.82	59.98	@
22		57.19	58.35	59.37	60.24	61.66	**
23		40.05	43.38	41.87	45.01	46.27	***
24		51.95	53.06	54.13	54.92	56.36	57.08
4-25		56.81	58.22	59.11	60.18	61.00	62.38
4-26							@
4-27		44.12	44.99	45.89	46.46	47.62	48.17

@ Infinite resistance, gage unusable

* From Figure 3.18, the second data point looks incorrect, so was omitted from the α calculation
R

** From Figure 3.19, the second data point looks incorrect, so was omitted from the α calculation
R

*** From Figure 3.19, the first and third data points look incorrect, so were omitted from the α calculation
R

Table 3.7. Coefficient of Resistance
For 4mm Thin Film Gage

Gage No.	α R +3 x 10 (/°K)	A0 (ohms)	A1 (ohms/°K)	F
1	2.27	.0359	.1051	.9995
2	2.32	.0210	.1180	.9998
3	2.32	.0001	.1197	.9955
4	2.33	.0313	.1071	.9992
5	2.34	.0058	.1111	.9998
6	2.33	.0865	.1206	.9973
7	2.21	.0211	.1202	.9998
8	2.19	.0130	.1037	.9997
9	2.30	.0175	.1360	.9999
10	2.41	.0180	.1247	.9999
11	2.19	.0269	.1255	.9998
12	2.20	.0348	.1127	.9996
13	2.16	.0213	.1060	.9998
14	2.10	.0239	.0953	.9998
15	2.06	.0179	.0957	.9998
16	2.10	.0748	.0998	.9981
17	2.03	.0299	.1109	.9998
18	2.02	.0559	.1018	.9995
19	2.10	.0026	.1208	.9967
20				
21	2.03	-.0053	.1124	.9952
22	1.84	.0537	.1052	.9991
23	2.05	-.0196	.0889	.9997
24	1.99	.0487	.1034	.9995
4-25	1.85	.1785	.1051	.9823 *
4-26				
4-27	1.85	.0338	.0818	.9995

* Electrical connection is bad, gage cannot be used

Table 3.8. Coefficient of Resistance Calibration Data
For 2 mm Thin Film Gage

Gage Resistances (ohms)							
T °C	20.5	30.0	41.2	49.0	62.65	69.65	
Gage No.							
1	51.35	53.00	53.73	54.70	56.27	56.85	*
2	45.72	46.79	47.87	48.66	50.09	50.87	
3	47.53	48.59	49.79	50.62	52.15	52.94	
4	50.35	51.94	52.82	54.01	55.34	56.44	*
5	62.76	64.25	65.59	66.73	68.64	69.66	
6	55.90	57.60	58.52	59.72	61.35	62.43	
7	45.71	47.12	47.90	49.94	50.31	51.17	**
8	49.57	50.75	51.88	52.77	54.35	55.14	
9	51.01	52.20	53.40	54.32	55.96	56.78	
10	57.51	58.99	60.21	61.36	63.09	64.13	
11	61.34	62.84	64.27	65.40	67.40	68.39	
12	55.65	57.13	58.29	59.36	61.08	62.01	
13	50.91	52.14	53.38	54.30	55.97	56.79	
14	48.72	49.89	51.04	51.93	53.46	54.27	
15	50.18	51.46	52.49	53.41	54.87	55.69	
16	50.12	51.30	52.50	53.37	54.97	55.78	
17	47.54	48.59	49.75	50.50	51.99	52.73	
18	45.85	46.93	48.00	48.80	50.24	50.97	
19	47.88	49.02	50.20	52.04	52.59	53.37	***
20	49.01	50.14	51.24	52.08	53.56	54.33	
21	40.35	41.34	42.29	43.03	44.31	44.97	
22	41.80	42.78	43.78	44.52	45.83	46.49	
23	50.49	51.77	52.85	53.80	55.31	56.13	
24	52.43	53.63	54.76	55.62	57.15	57.96	
2-25	44.88	45.82	46.71	47.47	48.71	49.36	
2-26	41.88	42.78	43.70	44.42	45.66	46.30	
2-27	48.70	49.61	50.57	51.23	52.51	53.13	

* From Figure 3.21, the second data point looks incorrect, so was omitted from the α calculation
R

** From Figure 3.22, the fourth data point looks incorrect, so was omitted from the α calculation
R

*** From Figure 3.24, the fourth data point looks incorrect, so was omitted from the α calculation
R

Table 3.9. Coefficient of Resistance
For 2 mm Thin Film Gage

Gage No.	α R +3 $\times 10$ (/°K)	A0 (ohms)	A1 (ohms/°K)	F
1	2.21	.0392	.1136	.9984
2	2.27	.0225	.1036	.9996
3	2.31	-.0027	.1097	.9999
4	2.43	.0078	.1222	.9970
5	2.21	.0421	.1388	.9992
6	2.29	.1543	.1280	.9937
7	2.35	.1199	.1075	.9943
8	2.27	.0279	.1125	.9995
9	2.29	.0153	.1168	.9997
10	2.30	.0658	.1322	.9985
11	2.32	.0397	.1424	.9995
12	2.28	.0922	.1271	.9983
13	2.34	.0309	.1190	.9997
14	2.30	.0340	.1119	.9996
15	2.19	.0872	.1100	.9984
16	2.28	.0288	.1143	.9997
17	2.21	.0178	.1050	.9998
18	2.25	.0312	.1034	.9996
19	2.32	.0034	.1110	.9998
20	2.19	.0360	.1072	.9996
21	2.31	.0363	.0931	.9996
22	2.27	.0288	.0948	.9998
23	2.24	.0755	.1130	.9988
24	2.12	.0505	.1111	.9994
2-25	2.02	.0156	.0905	.9994
2-26	2.14	.0062	.0895	.9997
2-27	1.84	.0163	.0897	.9997

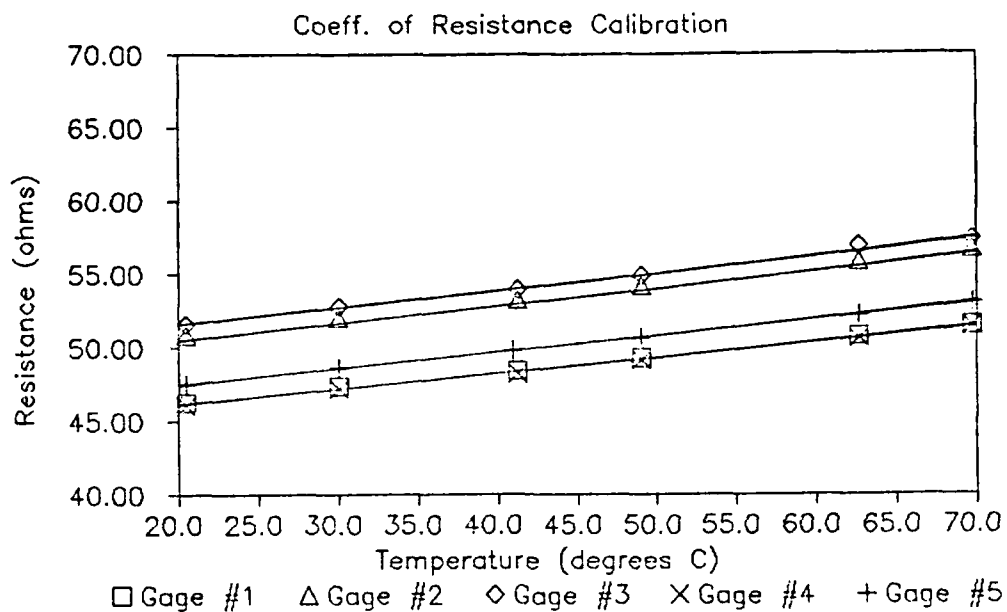


**Figure 3.15. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 1 through 5**

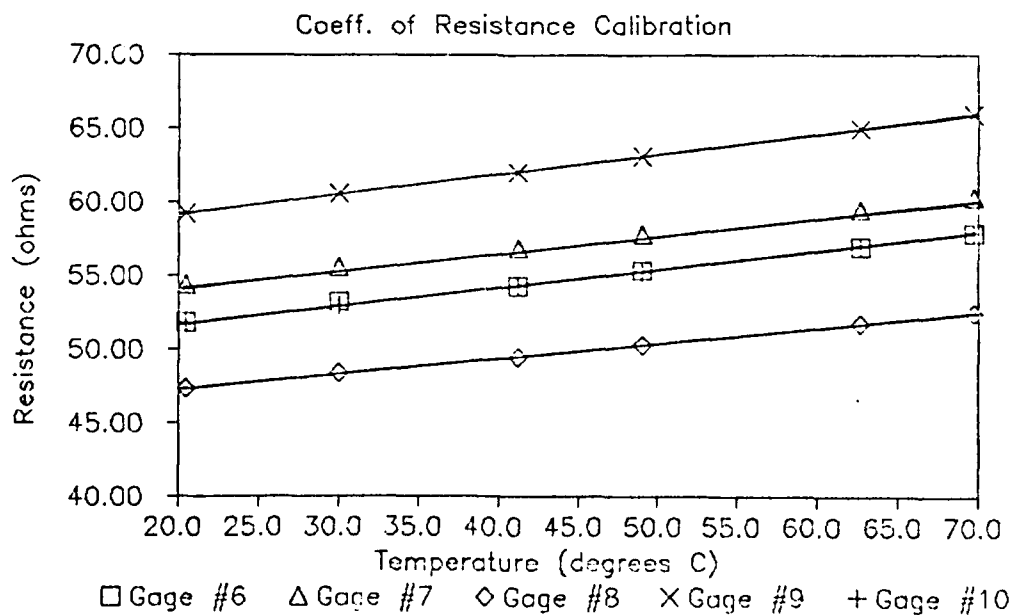


**Figure 3.16. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 6 through 10**

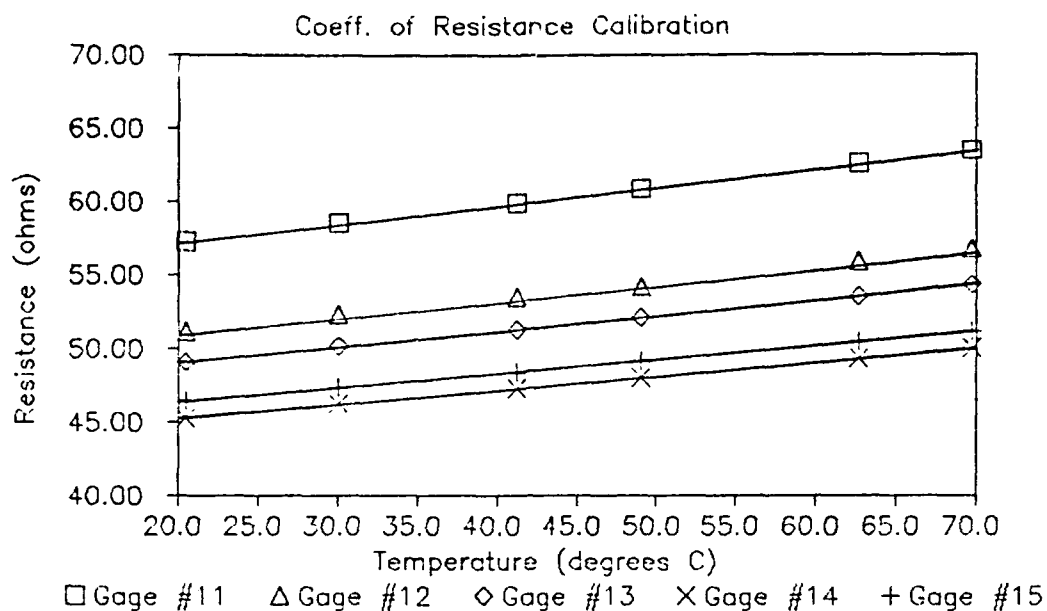


**Figure 3.17. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 11 through 15**

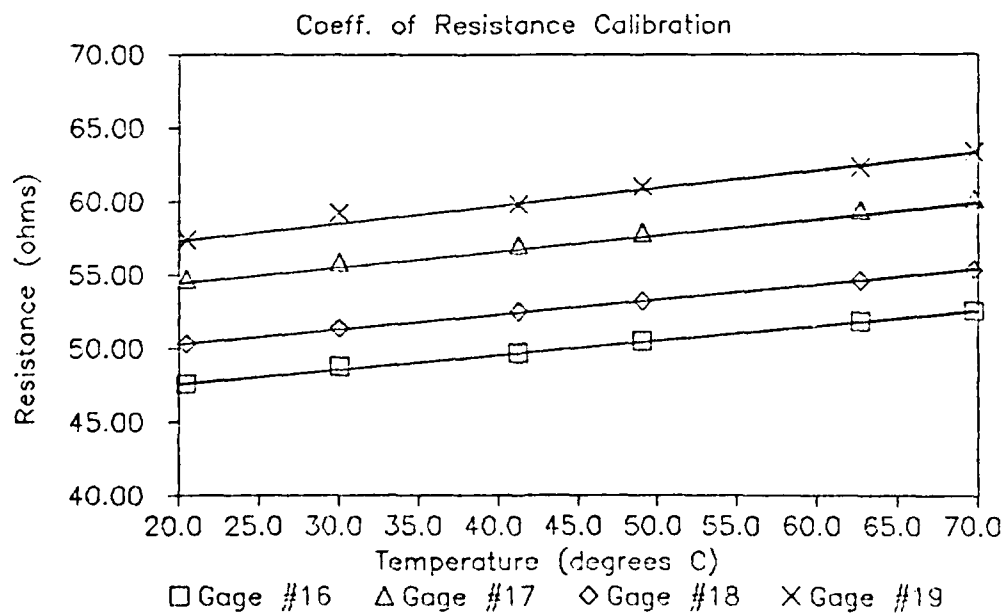


**Figure 3.18. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 16 through 19**

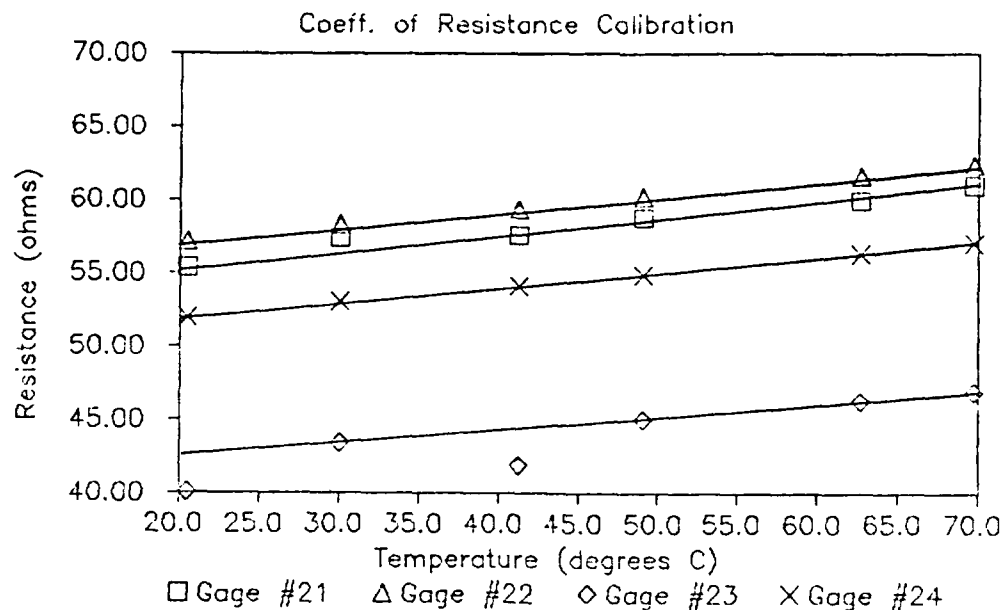


**Figure 3.19. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 21 through 24**

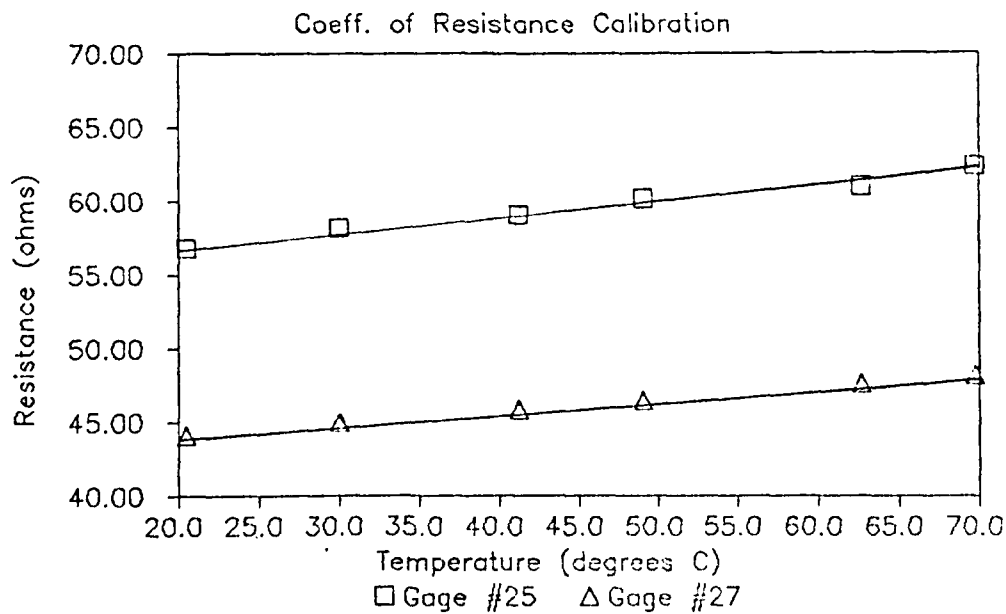


**Figure 3.20. Film Coefficient of Resistance Calibration
For 4mm Gage: Gages 25 through 27**

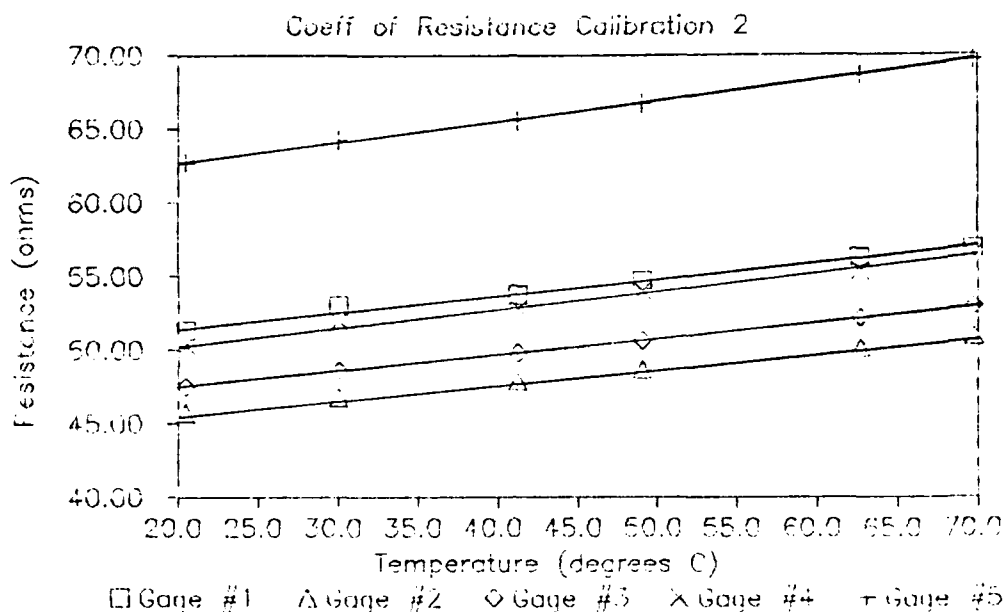


**Figure 3.21. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 1 through 5**

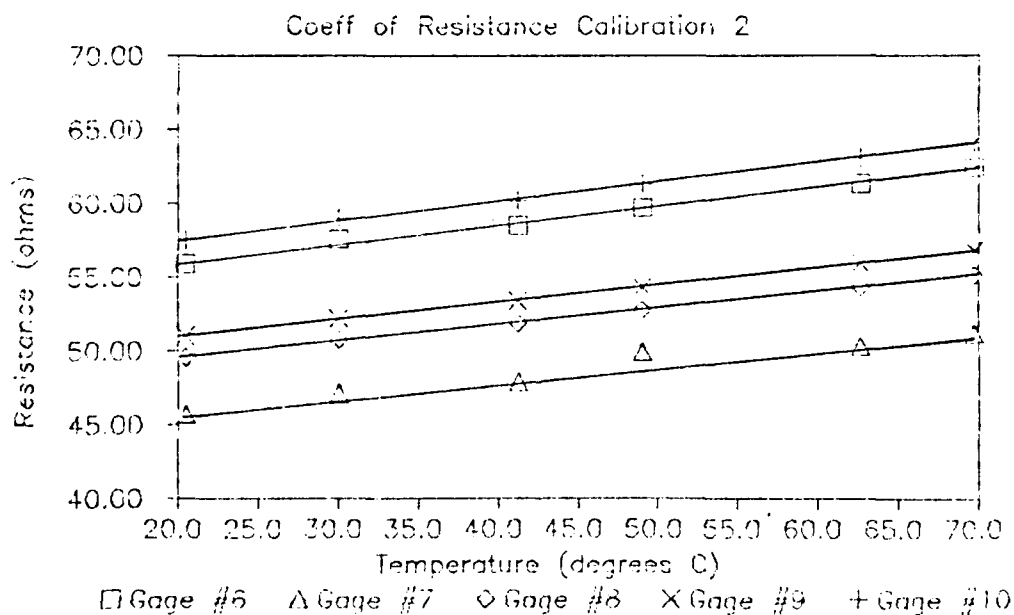


**Figure 3.22. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 6 through 10**

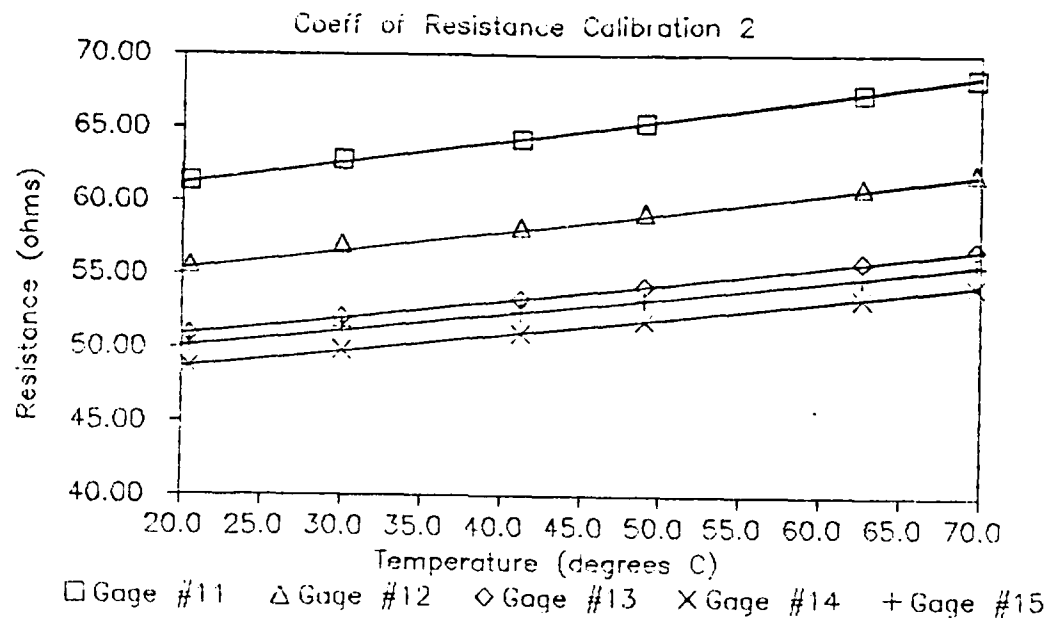


**Figure 3.23. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 11 through 15**

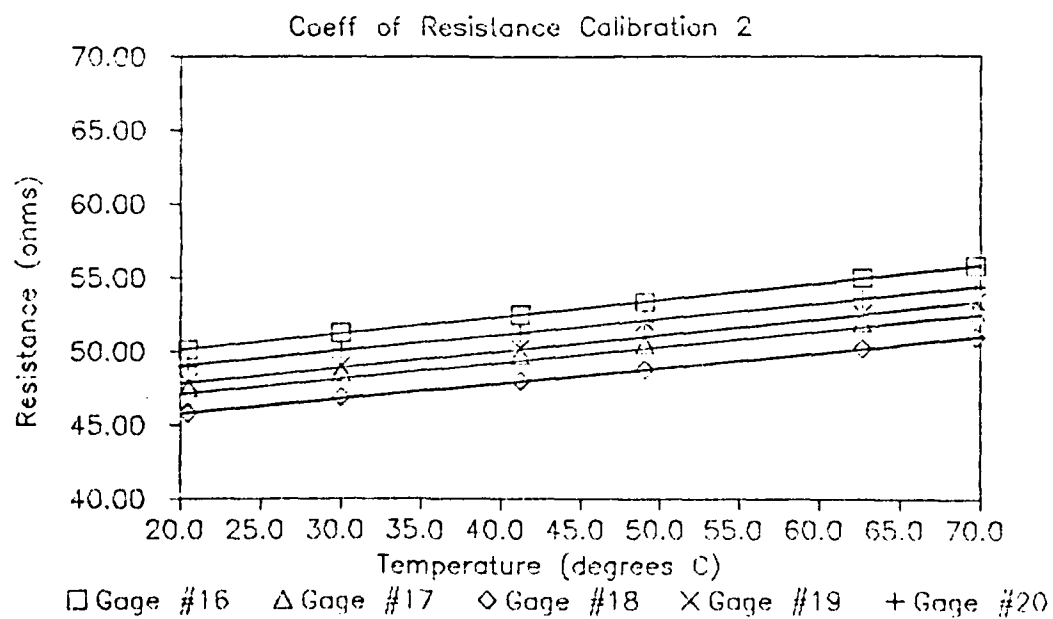


**Figure 3.24. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 16 through 20**

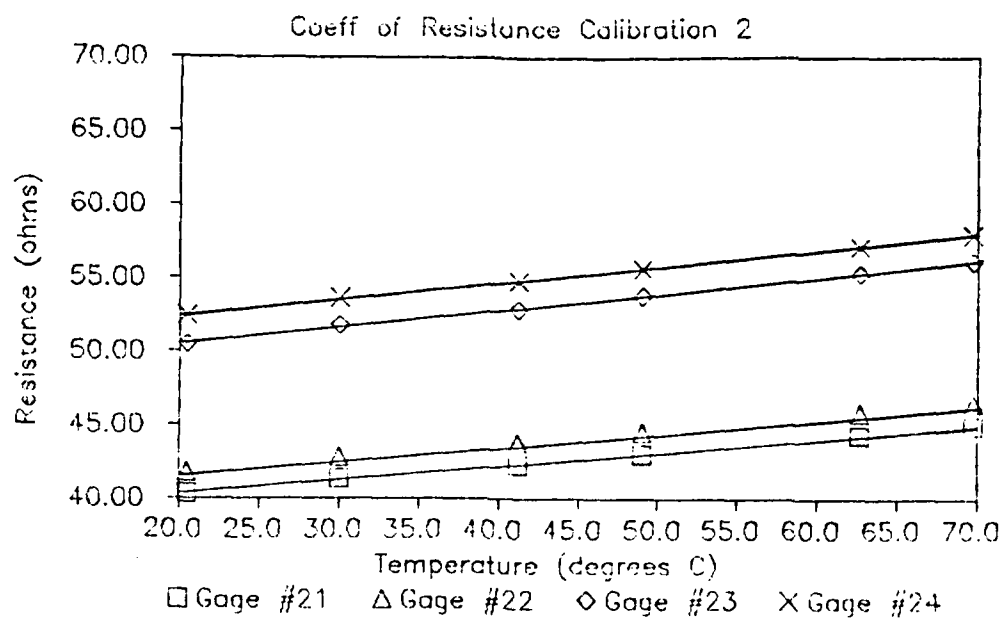


**Figure 3.25. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 21 through 24**

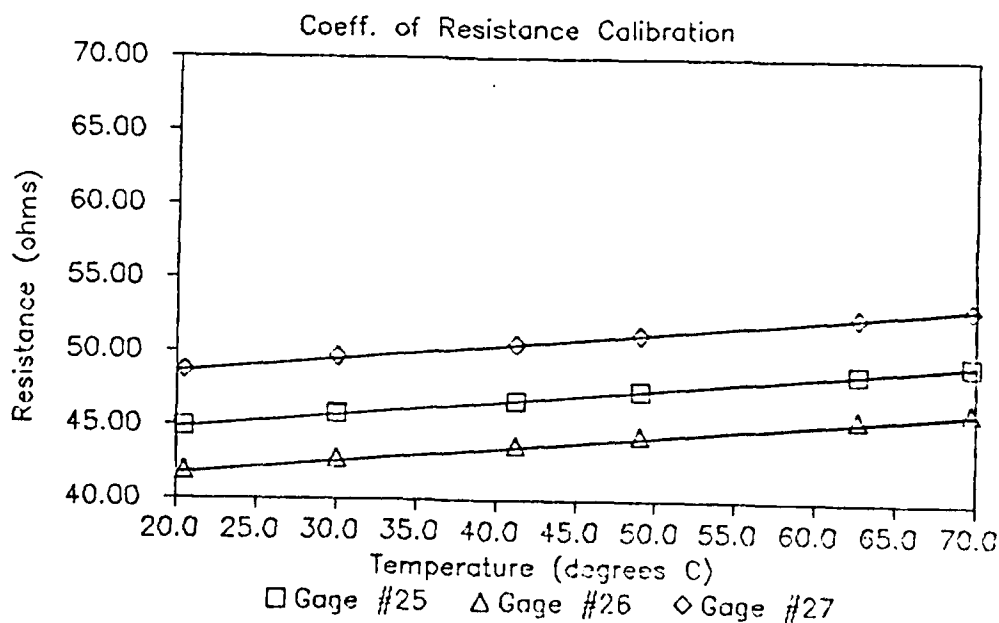


**Figure 3.26. Film Coefficient of Resistance Calibration
For 2mm Gage: Gages 25 through 27**



Figure 3.27. Data Acquisition System for Heat Transfer Test

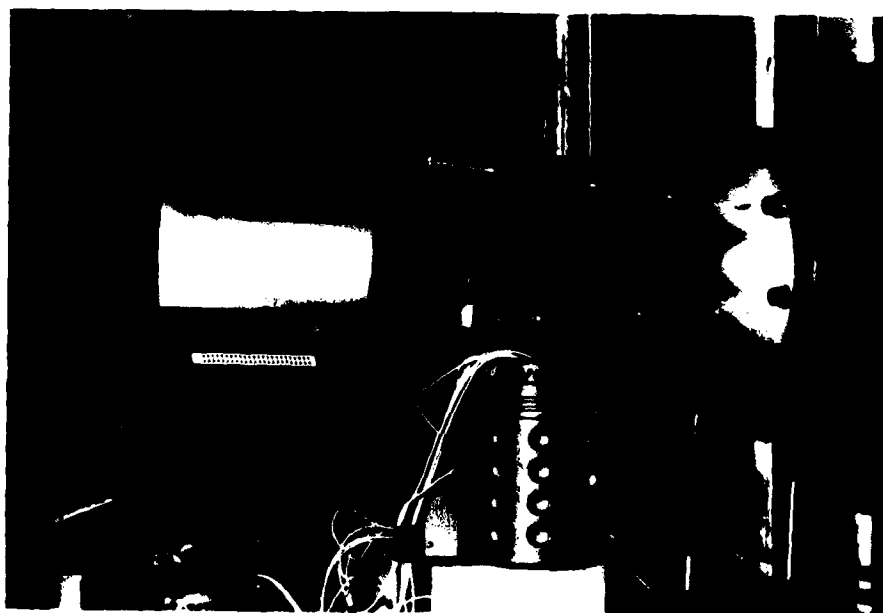


Figure 3.28. Heat Transfer Model Installed in Tunnel

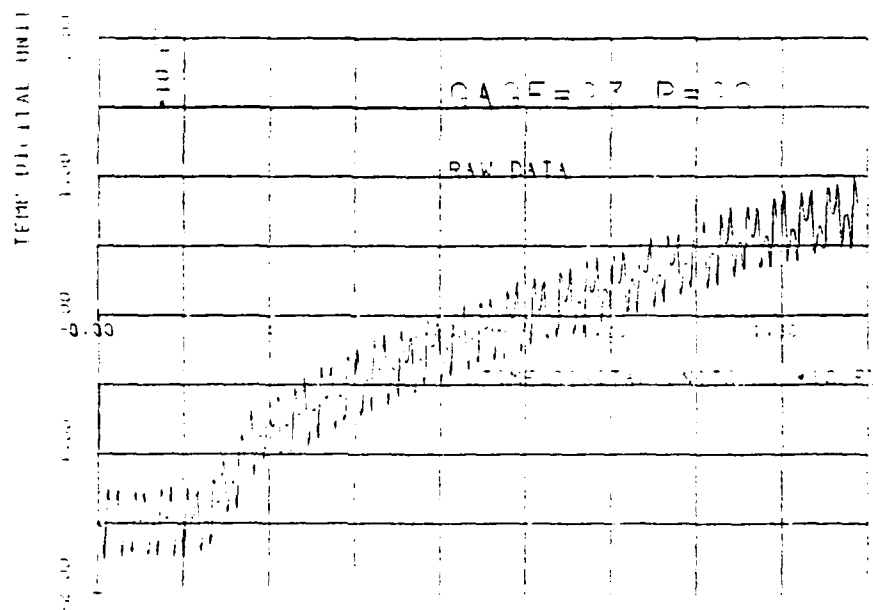


Figure 3.29. Temperature versus Time, $Re = 1.69 \times 10^7$ /m
Raw Data, Gage No 23

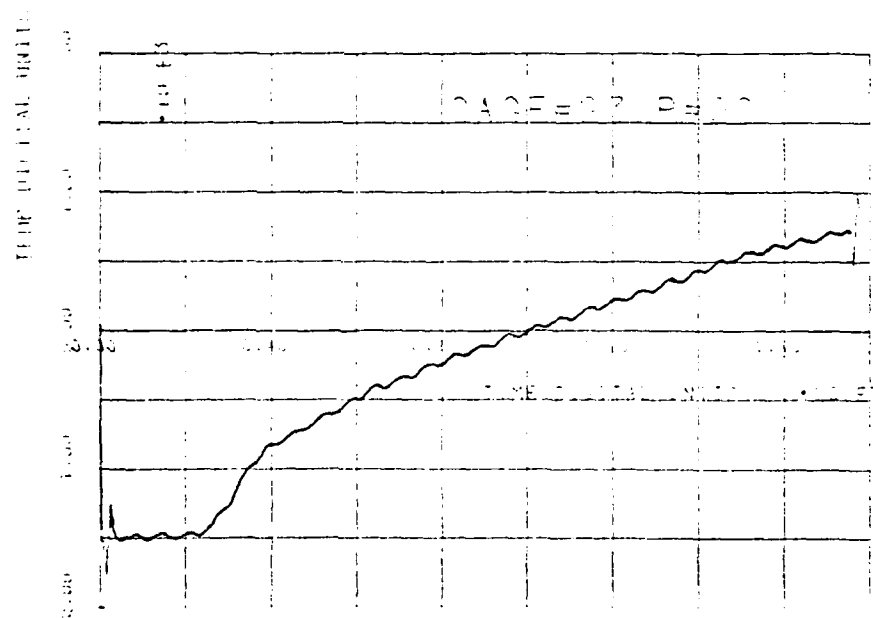


Figure 3.30. Temperature versus Time, $Re=1.69 \times 10^7$ /m
Averaged Data, Gage No 23

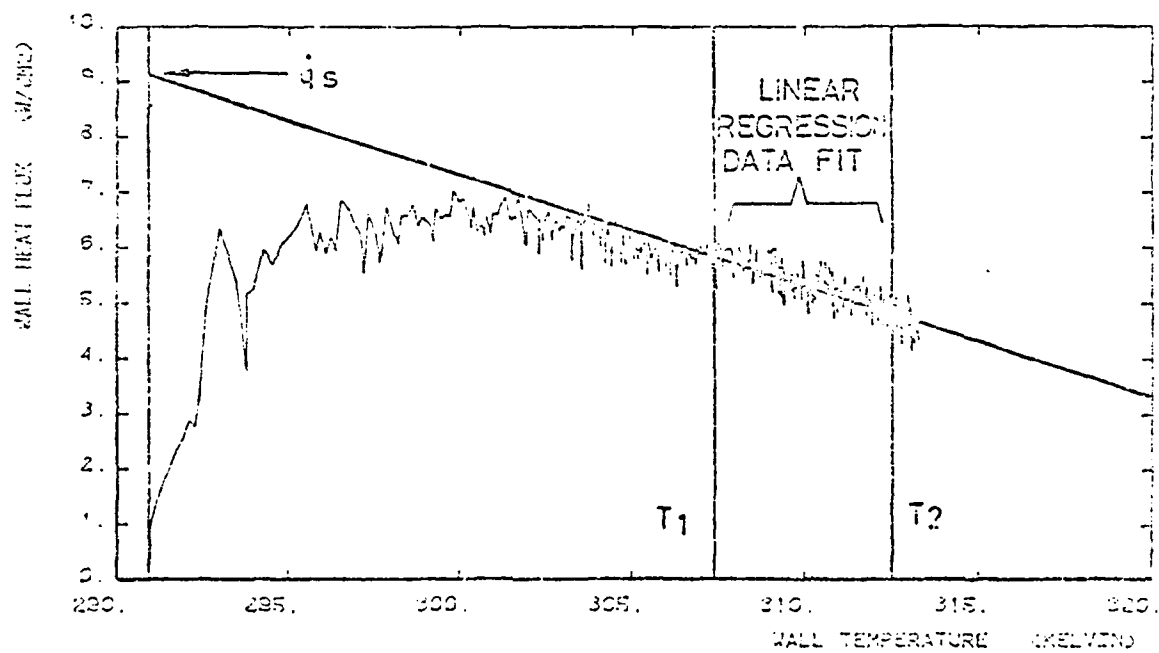


Figure 3.31. Sample of Heat Flux versus Temperature Trace

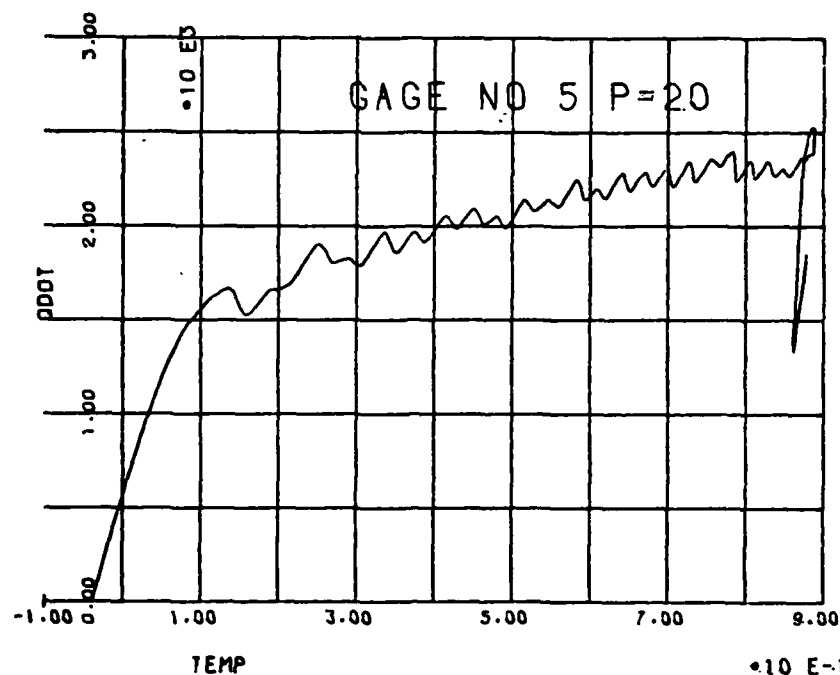


Figure 3.32. Heat Flux versus Temperature, $Re=1.69 \times 10^7$ /m
Gage No 5, Direct Method

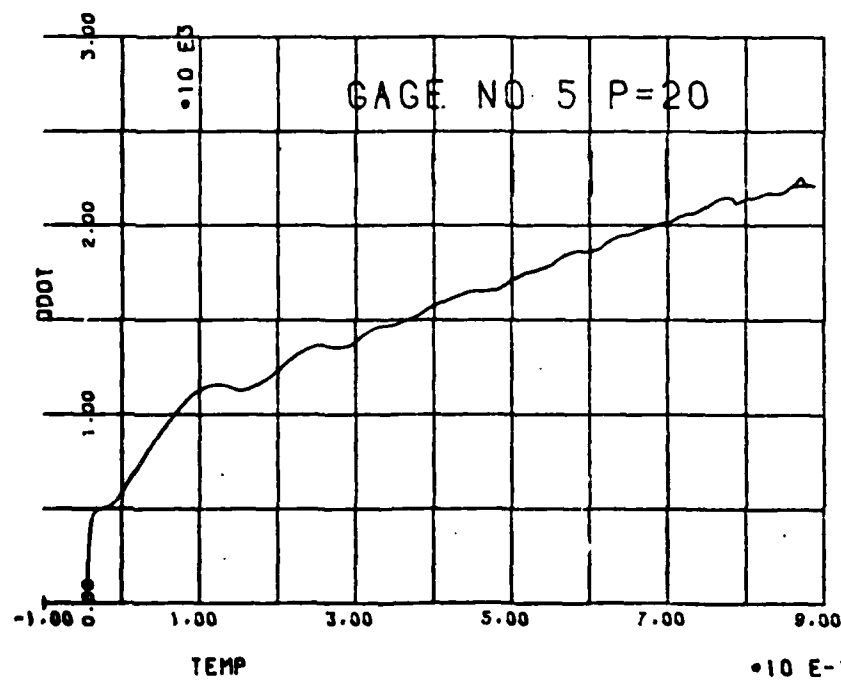


Figure 3.33. Heat Flux versus Temperature, $Re=1.69 \times 10^7$ /m
Gage No 5, Indirect Method

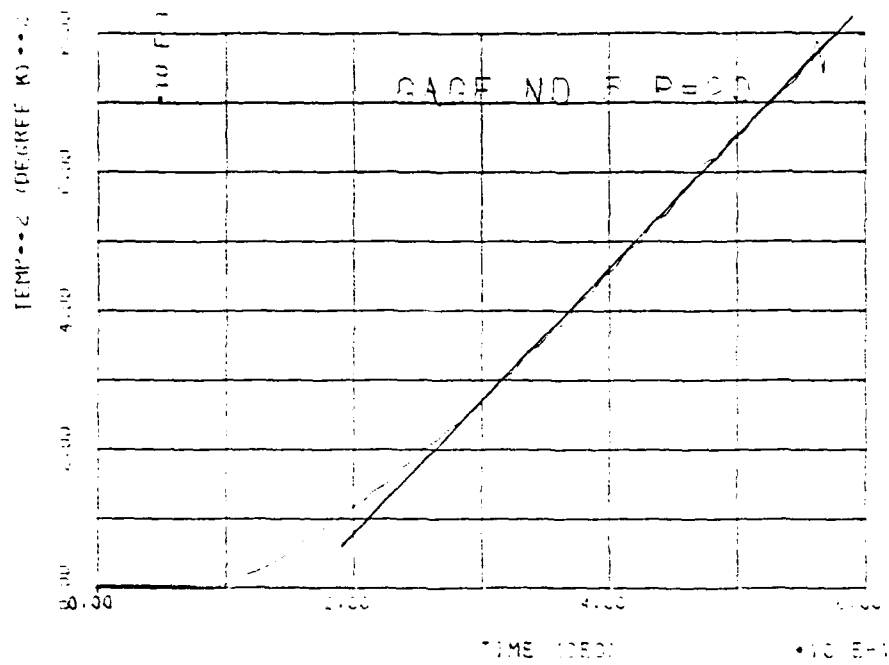


Figure 3.34. Constant Heat Flux Assumption Verification

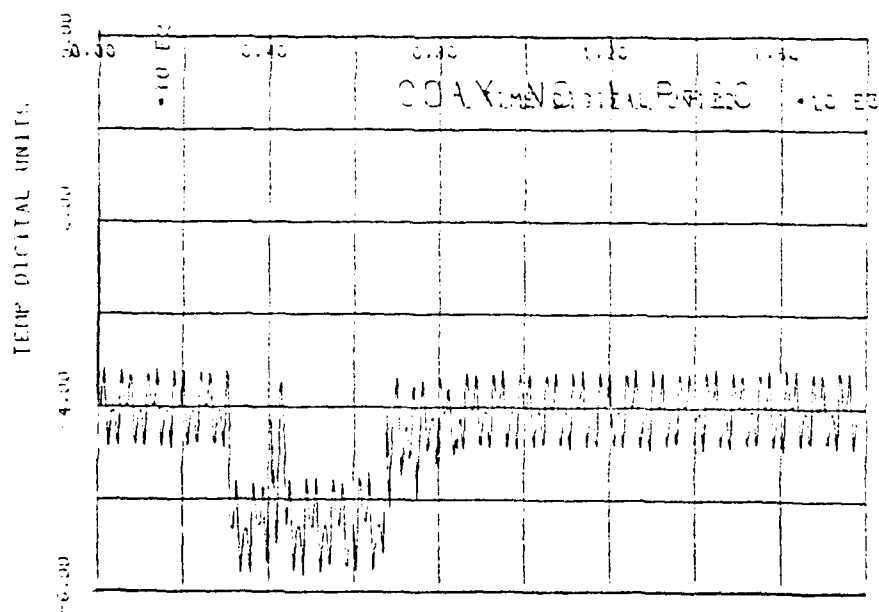


Figure 3.35. Coax Gage #1, $Re=1.69 \times 10^7 /m$, $P_o = 20$ bar
Temperature versus Time

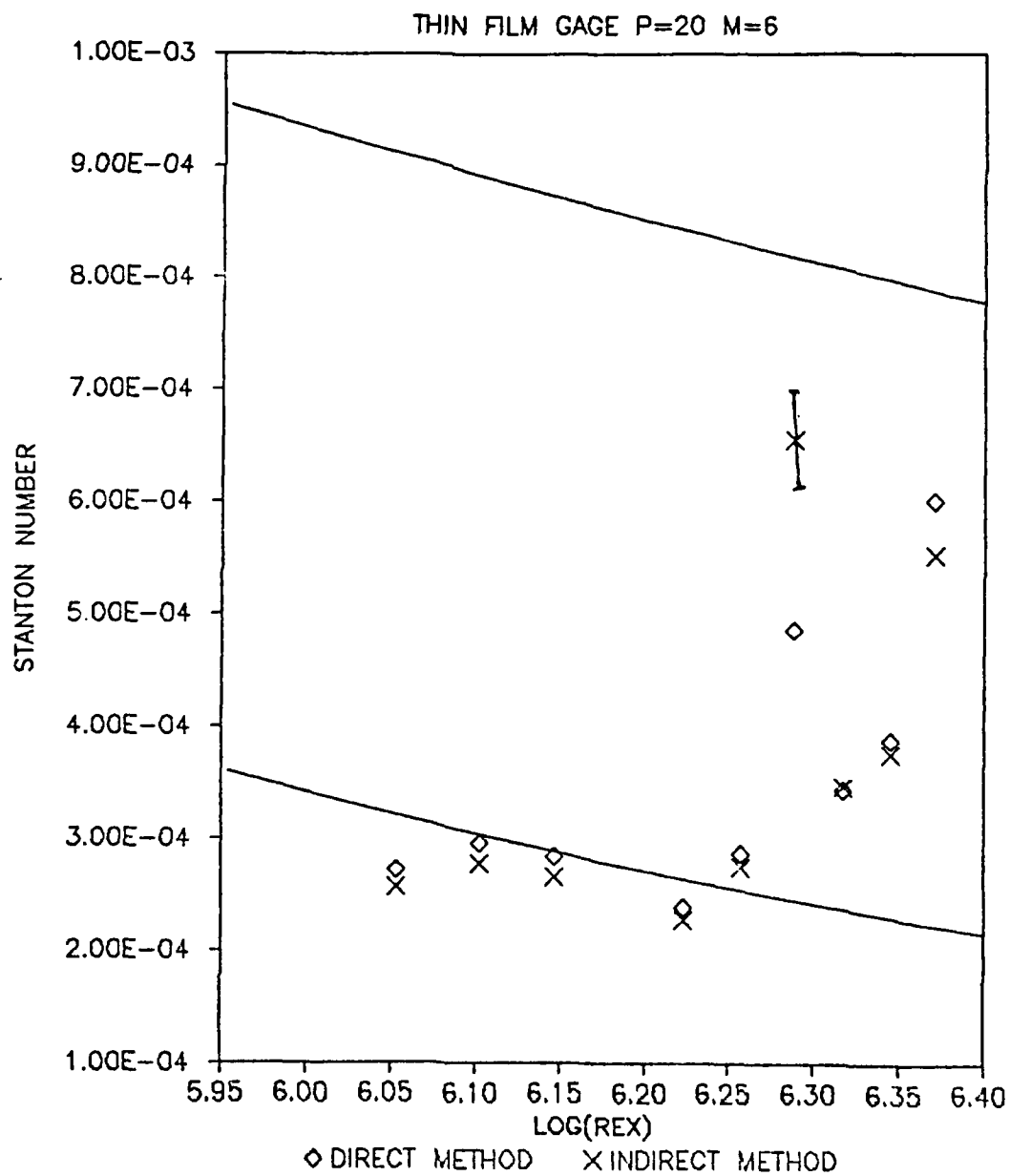


Figure 3.36. Plate, Heat Transfer H-1, M=6, Re=1.69x10⁷ /m
Stanton Number versus Log(Re_x)

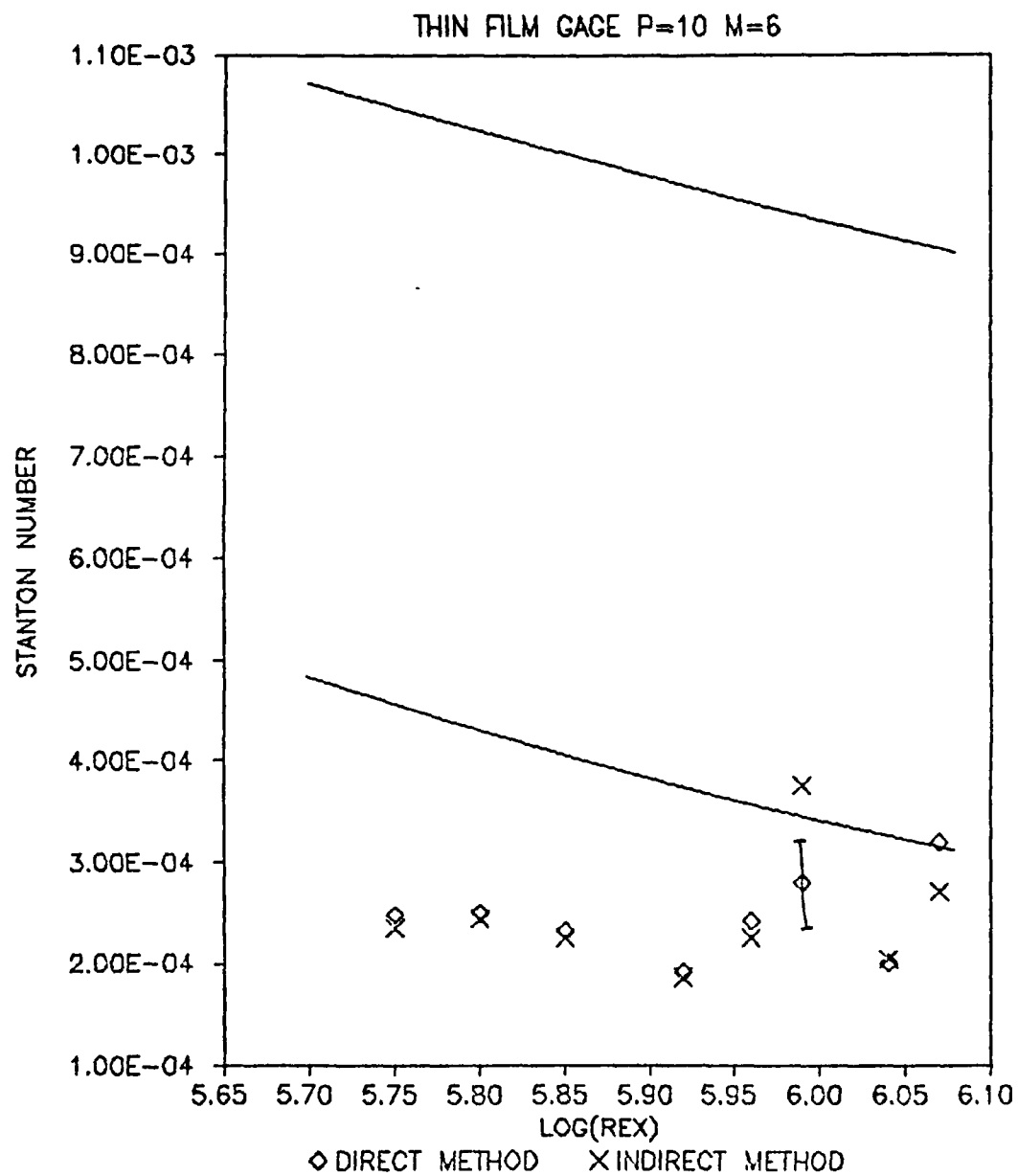


Figure 3.37. Plate, Heat Transfer H-2, M=6, Re=.844x10⁷ /m
Stanton Number versus Log(Re_x)

3.8 Flow Interaction

At Mach numbers above 4, the most severe problems of atmospheric flight are associated with viscous-inviscid interactions (33:177). Cruise vehicles and lifting re-entry vehicles have highly complex, three-dimensional configurations in which exist many regions of high compression that can cause boundary layers to separate. The flow reattachment that follows this separation, causes a rise in heat rates that can far exceed those for an attached boundary layer. A possibly more serious viscous interaction problem is the impingement of shock waves generated by the forebody resulting in local heat rates that may be many times larger than stagnation point values. However, unlike stagnation point heating where the location is obvious, the problem with complex configurations is to determine where the high heat rates are likely to occur, as well as their magnitude.

This study will examine one viscous interaction problem area, that of flow interaction past a blunt fin. The interaction of the bow shock of a fin with the boundary layer on a plate can cause widespread separation and vorticity of a three-dimensional nature. Adjacent to the fin, parallel, finely spaced embedded vortices may form and cause striation heating. The embedded vortices produce highly localized heating rates that are significantly greater than heating in adjacent undisturbed areas. Oil

flow visualization tests were conducted to examine the general structure of the flow interaction region. Sublimation tests were conducted to determine the global heating of the interaction area. The information obtained by first employing sublimation, rather than heat transfer gages, can be used to better determine where to locate those thin film or coax gages in future tests.

3.8.1 Topological Rules

It has been postulated by Peake and Tobak (41) that all cases of three dimensional flow separation and reattachment can be described mathematically. Using

"the assumption of continuous vector fields of skin friction lines and external flow streamlines, when coupled with simple laws of topology, provides a flow grammar"

which enables one to create plausible flow structures. The elementary singular points of the flow field: nodes, spiral nodes (foci), and saddlepoints, can be categorized mathematically. Therefore,

"the types of singular point, their number, and the rules governing the relations between them, can be said to characterize the pattern."

Figure 3.38 shows the structure of these elementary singular points; the solid lines represent the skin friction lines while the dashed lines represent the surface vortex lines which are orthogonal at every point to the system of skin friction lines. By utilizing these ideas, mean flow characteristics and flow mechanisms can be deduced when wind tunnel results give insufficient information.

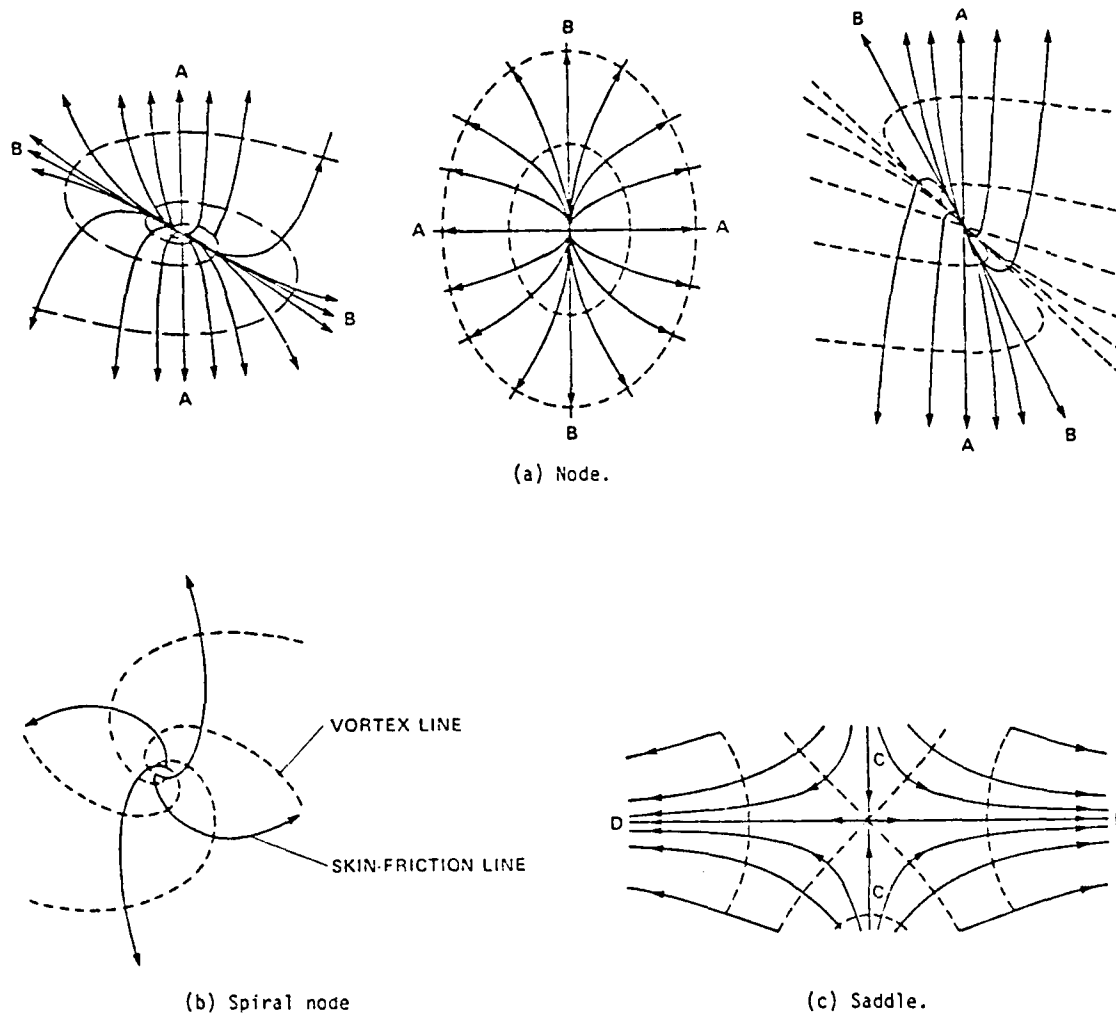


Figure 3.38. Singular Points

Peake and Tobak (41:1-5) list five special topological rules that apply to flow past wings and bodies at angle of attack. Of these five rules, only three could apply to the flow field in this study and are listed here:

1. Skin friction lines on a three dimensional body:

$$\sum_N - \sum_S = 2 \quad (3.49)$$

2. Skin friction lines on a three-dimensional body B connected simply without gaps to a plane wall P that either extends to infinity both upstream and downstream or is the surface of a torus:

$$\left[\sum_N - \sum_S \right]_{P+B} = 0 \quad (3.50)$$

3. Streamlines on a two dimensional plane cutting a three dimensional body:

$$\left[\sum_N + 1/2 \sum_{N'} \right] - \left[\sum_S + 1/2 \sum_{S'} \right] = -1 \quad (3.51)$$

However, in terms of this study, more information is required before the method of topology can be really useful. Namely, the flow on the fin and the flow on the plate near the trailing edge of the fin needs to be studied to get an approximate picture of the entire three dimensional flow pattern before this method of using topological rules can be utilized.

3.8.2 Flow Ahead of the Fin

3.8.2.1 Flow Structure

High speed flow over a blunt fin on a surface results in a very complex, inviscid-viscid interaction flow field. The bow shock from the fin causes the boundary layer to separate from the surface ahead of the fin, resulting in a separated flow region composed of horseshoe vortices near the surface and a lambda-type shock pattern in the plane of symmetry ahead of the fin. Experiments completed by Kaufmann, Korkegi, and Morton (26:5) reveal that the separated flow is very unsteady. Shadowgraph photographs were blurry until the exposure time was decreased to times on the order of microseconds. This is attributed to a "pulsating, scavenging action of the horseshoe vortices" which results in large oscillations of the impinging shock wave.

Korkegi states that the characteristics of skewed shock induced separated regions (32:599) do not exhibit any basic differences between laminar and turbulent flow for comparable extents of separation. (However, the adverse pressure gradients needed for extensive separation, are much larger for turbulent than for laminar flow, and the detail of the flow structure differs because of turbulent mixing in one case and molecular mixing in the other.) Therefore, it is possible to define the structure of the separated flow region, resulting from the interaction of a skewed shock

wave with either a laminar or turbulent boundary layer. This reasoning, which is developed from separated regions induced by skewed shocks, is extended to separated regions induced by bow shocks.

One possible interaction flow field of a separated region induced by a fin bow shock is shown in Figure 3.39 (54:47). This flow field is proposed by Winkelman for turbulent flows and is similar to two dimensional separated flows ahead of steps or ramps. The pressure rise across the fin bow shock presents a strong adverse pressure gradient to the boundary layer flow on the flat plate surface, causing the boundary layer to separate from the surface and form a region of reverse flow adjacent to the plate surface. The effects of the pressure rise are propagated upstream through the reverse flow region. The reverse flow forms several horseshoe vortices in which the flow quickly curves downstream, away from the plane of symmetry (26:4). A nodal point of attachment is postulated to exist between the primary and secondary separation lines, or separated regions. The nodal point is indicated to be produced by adjacent regions of recirculating vortex flow. The reverse flow in the vortices is fed by the separated stream flow and spirals downstream very rapidly. Severe heating rates are

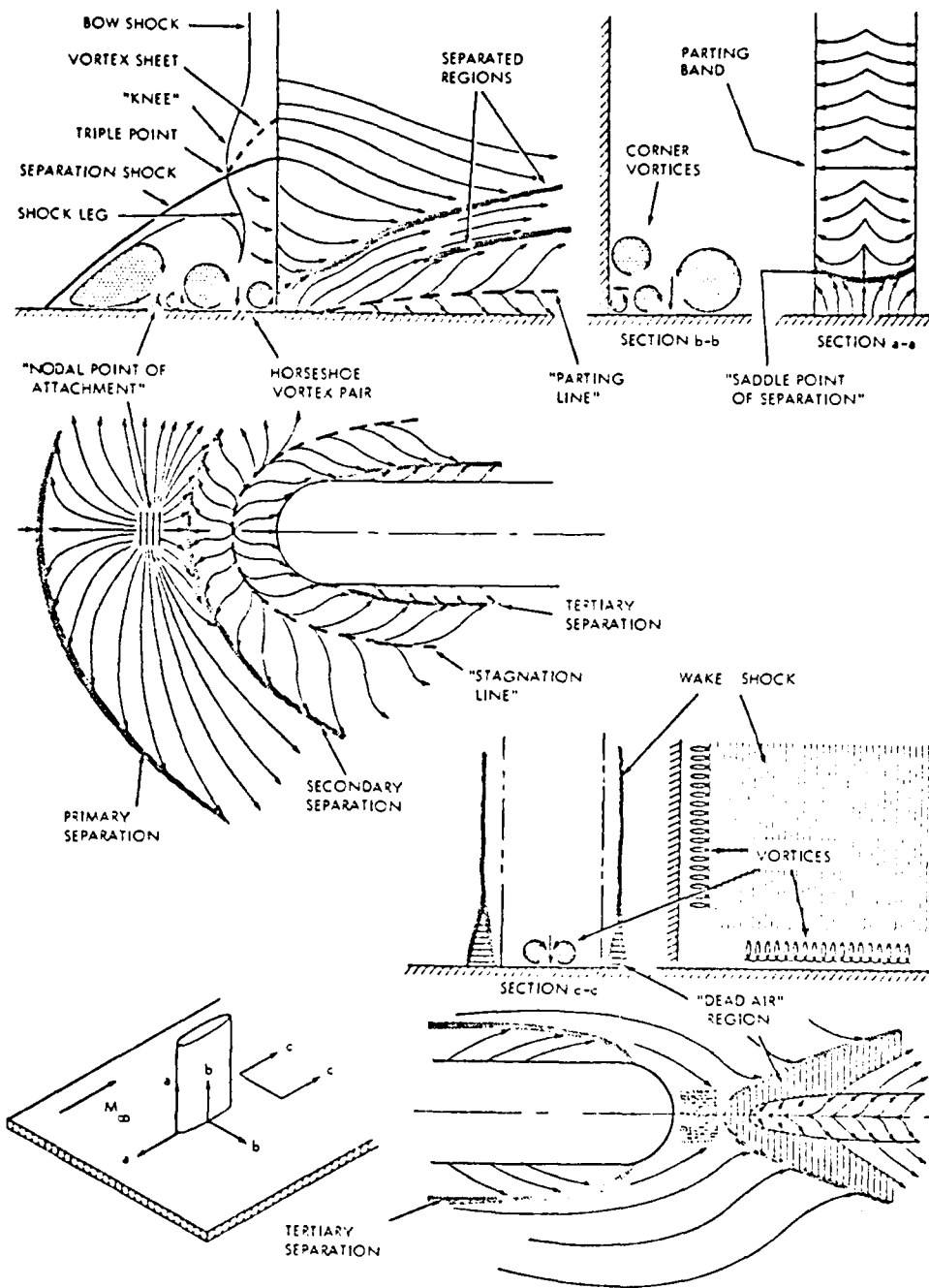


Figure 3.39. Flow Interaction Model

caused by these vortices bringing high energy air flow into close proximity with the surface. This has been shown experimentally in that the heat transfer rates are shown to peak at reattachment regions following a region of separation. In a skewed shock interaction (32:599) in laminar flow, Figure 3.40, Korkegi shows that the two peaks in the heat transfer rate are associated with the two reattachment lines, denoted by R, in the flow interaction region.

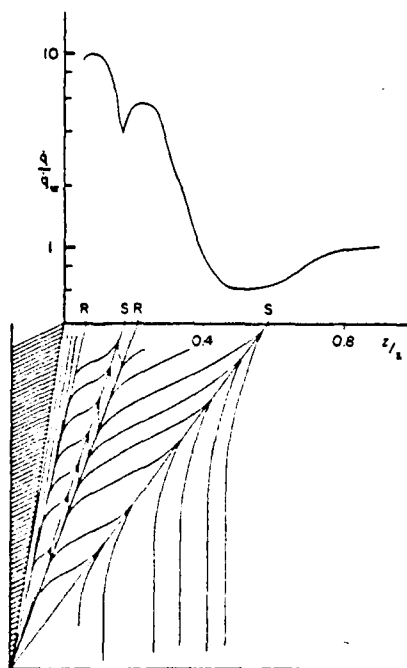


Figure 3.40. Spanwise Heat Transfer for Laminar Flow
 $M = 20$, Wedge Angle = 10°

3.8.2.2 Separation Distance

The separation distance upstream of a fin leading edge is found to scale with fin diameter for conditions when the

fin is slender, is at zero degree incidence, and its diameter is larger than the boundary layer thickness (55:31). As in two dimensional flow, this distance is considerably greater for laminar than for turbulent flow (30:361). For laminar flow, this separation distance is Mach number and Reynold's number dependent. Limited data indicate that the extent of entirely laminar separation ahead of a fin increases with increasing Reynold's number. Laminar separation lengths exceeding six diameters have been observed (55:24) for both Mach 3 and Mach 5 flows.

However, separation is a strong disturbance to a laminar boundary layer and causes earlier transition than for attached undisturbed boundary layers. In this case, transition can occur either ahead of the separated location (turbulent separation) or in the free shear layer bounding the reverse flow region ahead of the fin (transitional separation). The extent of transitional separation diminishes toward the turbulent value as the separation location approaches the undisturbed boundary layer transition location, as can be seen in Figure 3.41 (26:25).

For turbulent flow, the separation distance appears to be insensitive to Reynold's number. The magnitude of the separation distance is usually twice the diameter of the fin, for cases when the fin diameter is larger than the boundary layer thickness.

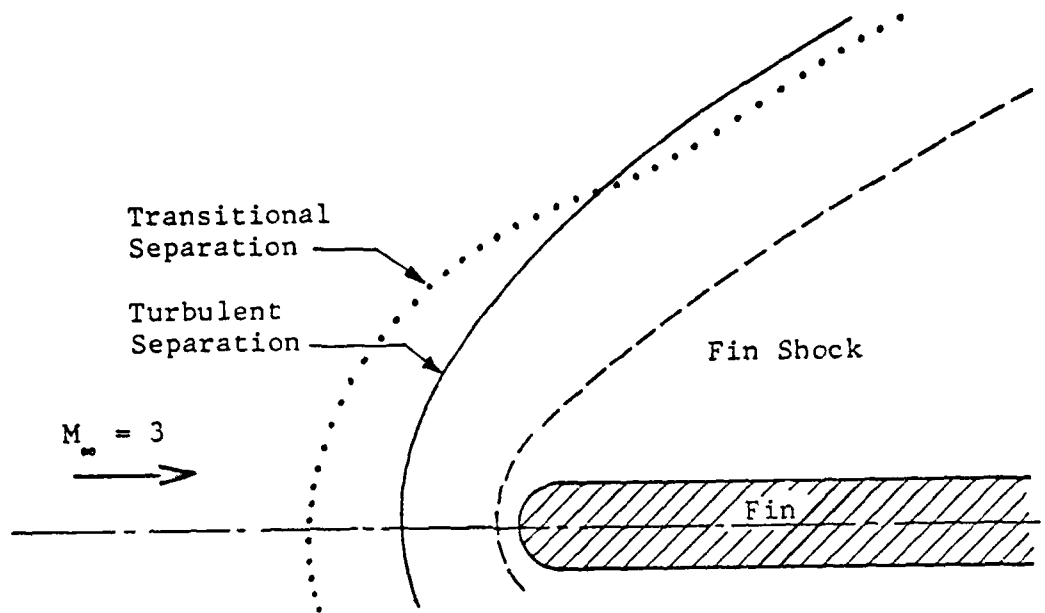


Figure 3.41. Transitional and Turbulent Separation Line Shapes

The effect of leading edge sweep on the extent of the interaction has been investigated by Price and Stalling (43) for supersonic turbulent flow. They find that for sweep angles from 0° to 30° there is a sharp decrease in extent of separation, but no change in the first peak pressure. For larger sweep angles, both decrease until they are barely perceptible at 75° .

For a fin incidence angle of 10° (6:698), Reynold's number affects both the scale of the separation and the surface streamline patterns aft of the separation. Placing

the fin at an incidence of 10° also resulted in a region of increased heat transfer which originated near the leading edge and extended the length of the fin.

Price and Stalling (43) have studied the effect of fin height on turbulent flows past protuberances. Their results were a little surprising in that they found no difference in leading edge separation distances when the fin height varied for lengths smaller than the boundary layer height to many times the extent of the boundary layer.

3.8.3 Embedded Vortices

Striation heating is attributed to streamwise vortex formations embedded in the boundary layer which produce locally enhanced heating levels. Persen (28:1) has suggested that the formation of the embedded vortex system

"is connected with the stability of curved flow, i.e., flows where the streamlines are non-parallel, exhibit a curvature, a pressure gradient and a velocity gradient."

No agreement exists on a universal explanation of regularly spaced streamwise embedded vortices, but several mechanisms have been identified by researchers. One of these mechanisms produce what is known as Goertler vortices. Schlichting (46:526,532-535) describes these streamwise vortices which can occur in boundary layer flow over concave walls. In such boundary layers, the fluid particles near the wall experience a lower centrifugal force than those nearer the boundary layer edge. Such flows are inherently unstable and provide a mechanism for transition from laminar

to turbulent boundary layer flow. When a critical Reynold's number is exceeded, small disturbances will amplify and form a system of streamwise vortices within the boundary layer, as shown in Figure 3.42.

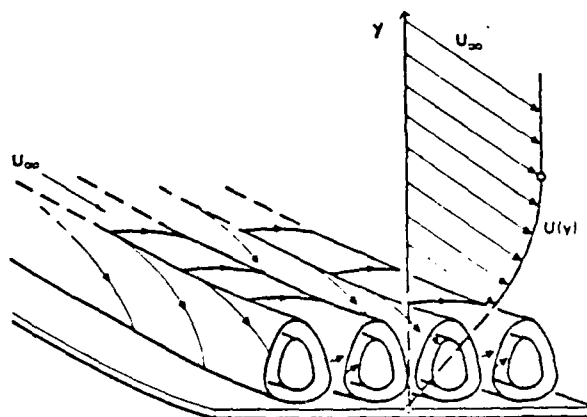


Figure 3.42. Goertler Vortices

It has been suggested by Kipp and Helms (28:3) that a separation bubble resulting from a shock - boundary layer interaction can cause a concave turning of the boundary layer flow, thereby initiating Goertler vortices.

3.9 Results of the Flat Plate with Blunt Fin Study

Table 3.10 shows the test conditions for all the photographs included in this report for tests completed on the flat plate with the blunt fin. Appendix S shows the test conditions and VKI photograph catalog number for every test completed.

Table 3.10. Flat Plate with Fin Test Conditions

Test	P o (Bars)	Re -7 (x10) (/m)	Fin Incidence Ø (degrees)	Fin Sweep Ω (degrees)	Fin Height h (mm)
WO-1, WS-1	10	0.844	15	0	6
WO-2, WS-2	10	0.844	15	30	6
WO-3, WS-3	20	1.69	15	30	6
WO-4, WS-4	10	0.844	15	60	8
WO-5, WS-5	10	0.844	15	75	8
WO-6, WS-6	10	0.844	40	0	6
WO-7, WS-7	10	0.844	40	30	6
WO-8, WS-8	10	0.844	40	60	8
WO-9, WS-9	10	0.844	40	75	8
WO-10, WS-10	10	0.844	40	0	15
WO-11	20	1.69	0	0	6
WO-12	20	1.69	0	0	15
WO-13	20	1.69	15	0	6
WO-14	10	0.844	15	0	15

where WO - Oil flow test
 WS - Sublimation flow test

Table 3.11 shows the fin heights nondimensionalized with the undisturbed boundary layer height which was shown in Table 3.5.

Table 3.11. Nondimensional Fin Height

	h=6mm	h=8mm	h=15mm
Re=1.69x10 ⁷ /m, Po=20 bar	12	16	30
Re=.844x10 ⁷ /m, Po=10 bar	20	26	50

3.9.1 Shadowgraph Results

The shadowgraph results were very blurry and did not give very much detailed information. For future studies on a flat plate model, it may be better to use single-pass shadowgraph/schlieren. Since the model is essentially two dimensional, the photographs are too blurry due to the "double" exposure of the double-pass system. Another factor contributing to the blurriness of the photographs was the time of exposure. Previous studies have shown that the three dimensional flow interaction is so unsteady that it requires exposure times on the order of microseconds. The photographs in this study had an exposure time of 10 milliseconds.

However, the photographs did show that the shock originating from the separation region did not impinge on the fins of 0° sweep and 6 mm in height. It is possible that the flow structure on the plate could change if the fin was made taller and a shock impingement on the fin occurred.

3.9.2 Oil Flow and Sublimation Results

Several photographs of the results from oil flow and sublimation tests are shown in Figures 3.43 through 3.66. The oil flow photographs were taken after the tunnel was stopped and accurately represent the oil flow pattern in the tunnel during the tests except for some oil splattering and spreading of accumulation lines.

In the interaction oil flow photographs of the fins with zero sweep, there appears to be a "bump" present in the secondary separation line, as shown for example in Figure 3.48. This bump is a real phenomena, as its appearance is repeatable. This bump is not really evident in the sublimation photographs. No similar bump was found in a review of the limited references of blunt fin interaction in laminar flow. This bump does not appear to be caused by fin height, as can be seen in a comparison of Figures 3.43 and 3.44. Since a sign of early transition caused by the separated flow occurs on the primary separation line, (refer to Figure 3.41 and section 3.8.2.2), this bump does not indicate early transition since it occurs on the secondary separation line. The bump moves downstream (becomes less noticeable) as the Reynolds number increases, as can be seen in Figures 3.44 and 3.48. It is evident only on the expansion side of the fin when the fin is at incidence. The cause of the bump could be an interaction between the vortices on either side of the secondary separation line, the bump being produced when the vortex closer to the fin is stronger than the other vortex and dominates over the weaker vortex's flow area.

The separation distance ahead of the fin, l , non-dimensionalized with the fin diameter, d , is plotted against the Reynold's number based on the distance from the plate leading edge to the separation line, in Figures 3.67 through

3.69. The figures show the l/d distance for a fin incidence of 0, 15, and 40°. The separation distance is measured from the fin leading edge to the middle of the primary separation line or region. The errors in these measurements vary according to the amount of oil spreading that occurred on the primary separation line. For the short fin height at 0 and 15° incidence, the error corresponds to ± 2 l/d or 4%. At 40° incidence and for all the $h=15$ mm tests, the error corresponds to ± 3 l/d or 5%. A trend that is apparent is that the separation distance increases as the sweep angle is decreased. This is in agreement with the results found by Price and Stallings (43). A Reynolds number dependence on separation distance could not be determined from the oil flow photographs.

There appears to be no effect of fin height on the separation distance for zero sweep as shown in Figures 3.67 and 3.68, for incidence angles of 0 and 15°. This is also shown in the oil flow photographs, Figures 3.42 and 3.44, for 0° incidence. Another effect of fin height can also be seen in these figures. The secondary separation line is much blunter ahead of the fin leading edge for the taller fin. This bluntness can also be seen in Figure 3.48. For a fin incidence angle of 40°, there is a strong effect of fin height for the heights studied in this test, the separation distance increased dramatically when the fin height was increased from 6 to 15 mm, as can be seen in Figure 3.69.

The oil flow/sublimation photographs, Figures 3.45/3.46 and 3.59/3.60, compare the effect of fin height for a fin of 40° incidence and zero sweep. The flow patterns are also very different. In the $h = 6$ mm case, four reattachment regions are apparent. However, if the height of the fin is increased, there are only three reattachment regions. It is hypothesized that the flow passing over the fin combines with the vortices forming the second reattachment region causing an instability, thereby causing an additional vortex pair to form another reattachment region.

Changing the Reynolds number appears to have some effect on the flow patterns, other than increasing the separation distance. The main difference is the change of the "bump" and this can be seen by comparing the oil flow photographs, Figures 3.44 and 3.48, and 3.47 and 3.50. Slight differences in the flow structure can be seen in the oil flow photographs but the corresponding sublimation photographs do not show the same detail.

Kipp and Helms (28:8) show an example of striation heating caused by embedded vortices on the side fuselage of a Space Shuttle orbiter model during a thermal mapping wind tunnel test. (Kipp and Helms reference their Reynolds number to a characteristic length.) The striations were not present at Mach 10, $Re_{\infty} = 2.4 \times 10^6$ and $\alpha = 20^\circ$, but the streaks appeared when the angle of attack was increased to 40° . Striations also appeared at Mach 6, $Re_{\infty} = 2.7 \times 10^6$ and

$\alpha = 40^\circ$ and increasing the Reynold's number at Mach 6 to $Re_\infty = 7.3 \times 10^6$ resulted in a larger number of striations. These streaks from Kipp and Helms (28) look very similar to the streaks shown in Figure 3.66 which were produced in a similar test regime of Mach 6, $Re_\infty = 8.66 \times 10^6$, fin sweep angle of 75° and a fin incidence angle of 40° . Figure 3.64 also shows that the streaks begin to appear when the fin sweep is 60° . However, when the fin is at an incidence angle of 15° , there is no indication of striation heating at sweep angles of 60 or 75° , as can be seen in Figures 3.56 and 3.58.

3.9.3 Proposed Flow Interaction Models

Possible models for the flow interaction region have been developed from the oil flow and sublimation photographs. The details of the models are dependent upon the fin incidence and fin sweep. The models are sketched in Figures 3.70 through 3.74. The models are characterized by a nodal point of attachment and counter-rotating vortex pairs that create the alternating regions of separation and reattachment.

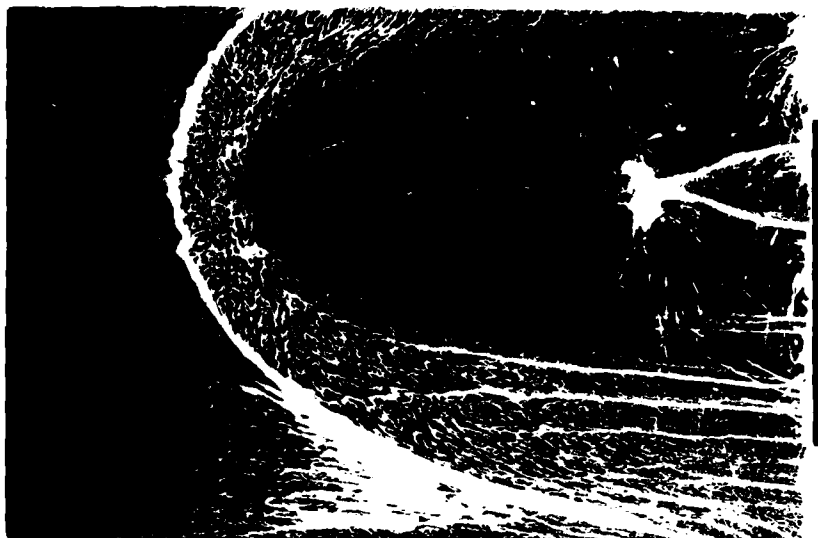


Figure 3.43. Fin, Oil Flow Test WO-11, $M=6$, $Re=1.69 \times 10^7$,
 $\theta = 0^\circ$, $\Omega = 0^\circ$, $h/\delta = 20$, $h = 6$ mm



Figure 3.44. Fin, Oil Flow Test WO-12, $M=6$, $Re=1.69 \times 10^7$,
 $\theta = 0^\circ$, $\Omega = 0^\circ$, $h/\delta = 50$, $h = 15$ mm



Figure 3.45. Fin, Oil Flow Test WO-10, M=6, Re=.844x10⁷ /m
 $\theta = 40^\circ$, $\Omega = 0^\circ$, $h/\delta = 30$, $h = 15$ mm



Figure 3.46. Fin, Sublimation Test WS-10, M=6, Re=.844x10⁷ /m
 $\theta = 40^\circ$, $\Omega = 0^\circ$, $h/\delta = 30$, $h = 15$ mm

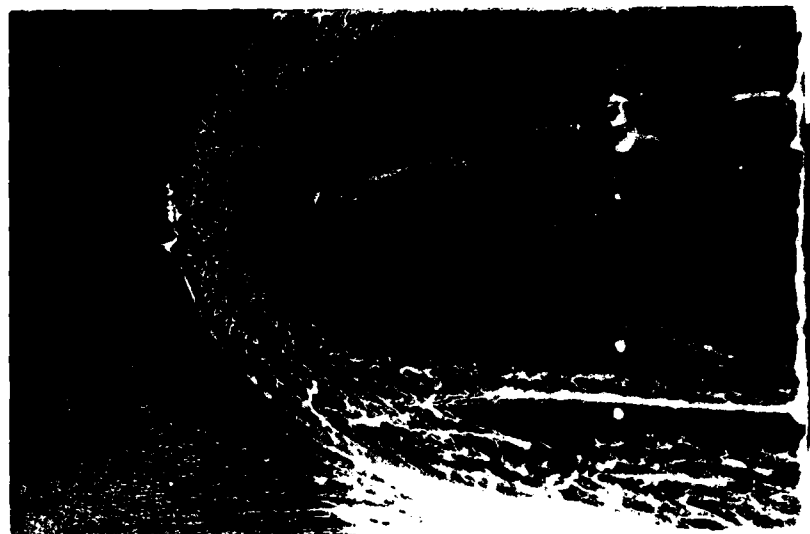


Figure 3.47. Fin, Oil Flow Test WO-13, $M=6$, $Re=1.69 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 0^\circ$, $h/\delta = 20$, $h = 6$ mm

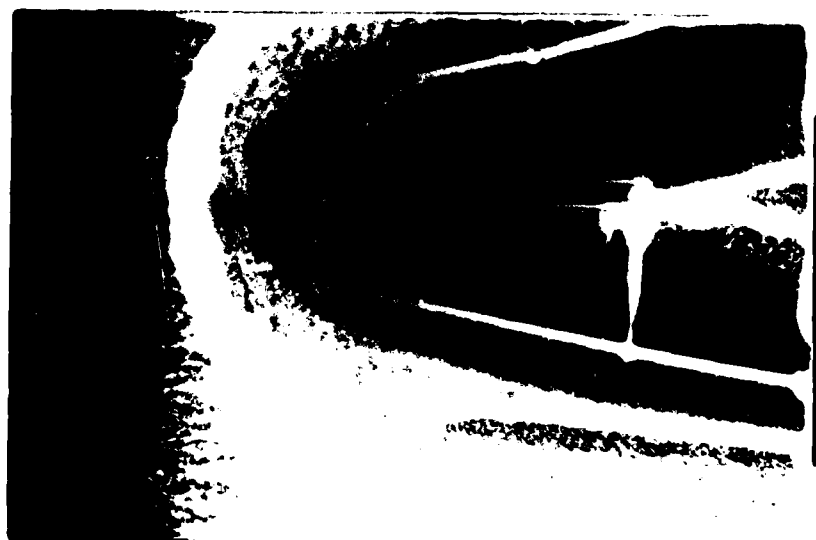


Figure 3.48. Fin, Oil Flow Test WO-14, $M=6$, $Re=.844 \times 10^7$
 $\theta = 0^\circ$, $\Omega = 0^\circ$, $h/\delta = 30$, $h = 15$ mm

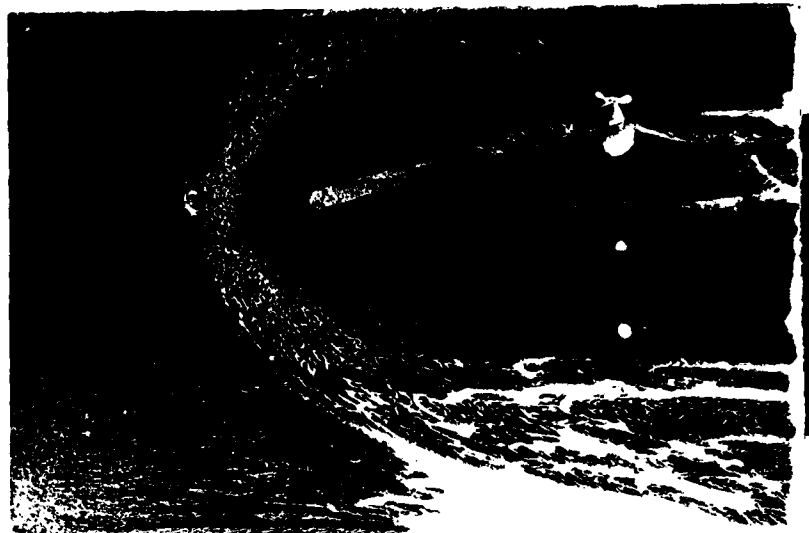


Figure 3.49. Fin, Oil Flow Test WO-1, $M=6$, $Re=.844 \times 10^7$ /m
 $\theta = 15^\circ$, $\Omega = 0^\circ$, $h/\delta = 12$, $h = 6$ mm



Figure 3.50. Fin, Sublimation Test WS-1, $M=6$, $Re=.844 \times 10^7$ /m
 $\theta = 15^\circ$, $\Omega = 0^\circ$, $h/\delta = 12$, $h = 6$ mm

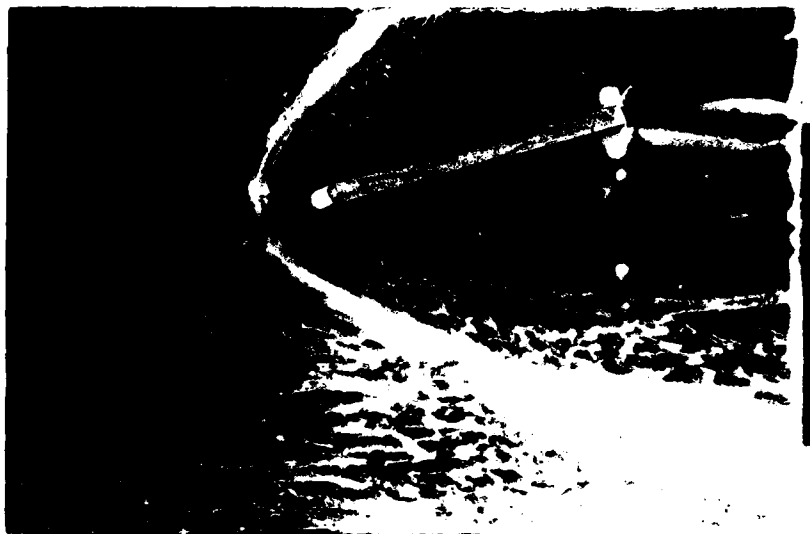


Figure 3.51. Fin, Oil Flow Test WO-2, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 30^\circ$, $h/\delta = 12$, $h = 6$ mm



Figure 3.52. Fin, Sublimation Test WS-2, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 30^\circ$, $h/\delta = 12$, $h = 6$ mm

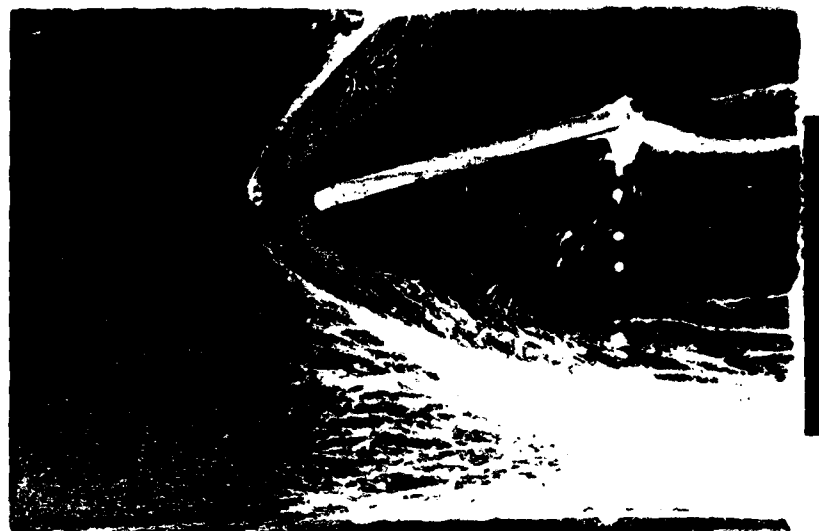


Figure 3.53. Fin, Oil Flow Test WO-3, $M=6$, $Re=1.69 \times 10^7$ /m
 $\theta = 15^\circ$, $\Omega = 30^\circ$, $h/\delta = 20$, $h = 6$ mm



Figure 3.54. Fin, Sublimation Test WS-3, $M=6$, $Re=1.69 \times 10^7$ /m
 $\theta = 15^\circ$, $\Omega = 30^\circ$, $h/\delta = 20$, $h = 6$ mm



Figure 3.55. Fin, Oil Flow Test WO-4, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 60^\circ$, $h/\delta = 16$, $h = 8$ mm



Figure 3.56. Fin, Sublimation Test W5-4, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 60^\circ$, $h/\delta = 16$, $h = 8$ mm



Figure 3.57. Fin, Oil Flow Test WO-5, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 75^\circ$, $h/\delta = 16$, $h = 8$ mm



Figure 3.58. Fin, Sublimation Test WS-5, $M=6$, $Re=.844 \times 10^7$
 $\theta = 15^\circ$, $\Omega = 75^\circ$, $h/\delta = 16$, $h = 8$ mm



Figure 3.59. Fin, Oil Flow Test WO-6, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 0^\circ$, $h/\delta = 12$, $h = 6$ mm



Figure 3.60. Fin, Sublimation Test WS-6, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 0^\circ$, $h/\delta = 12$, $h = 6$ mm



Figure 3.61. Fin, Oil Flow Test WO-7, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 30^\circ$, $h/\delta = 12$, $h = 6 \text{ mm}$



Figure 3.62. Fin, Sublimation Test WS-7, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 30^\circ$, $h/\delta = 12$, $h = 6 \text{ mm}$



Figure 3.63. Fin, Oil Flow Test WO-8, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 60^\circ$, $h/\delta = 16$, $h = 8$ mm



Figure 3.64. Fin, Sublimation Test WS-8, $M=6$, $Re=.844 \times 10^7$
 $\theta = 40^\circ$, $\Omega = 60^\circ$, $h/\delta = 15$, $h = 8$ mm



Figure 3.65. Fin, Oil Flow Test WO-9, $M=6$, $Re=.844 \times 10^7$ /m
 $\theta = 40^\circ$, $\Omega = 75^\circ$, $h/\delta = 16$, $h = 8$ mm

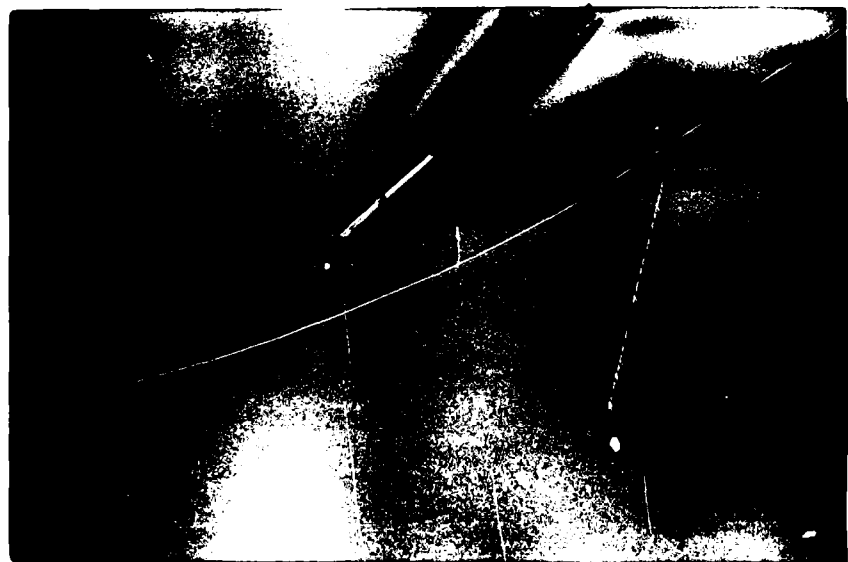
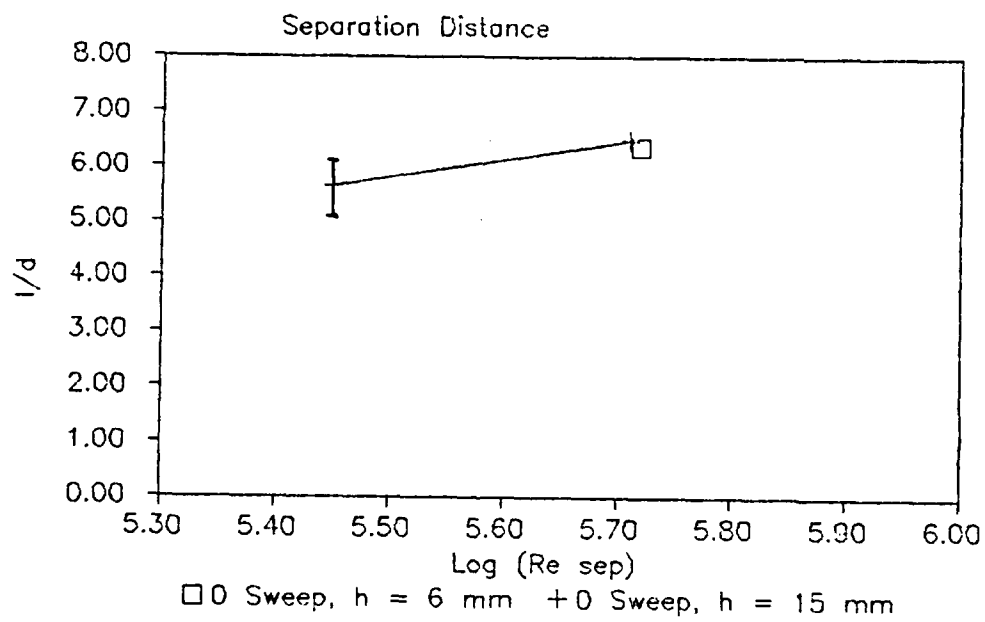


Figure 3.66. Fin, Sublimation Test WS-9, $M=6$, $Re=.844 \times 10^7$ /m
 $\theta = 40^\circ$, $\Omega = 75^\circ$, $h/\delta = 16$, $h = 8$ mm



Sweep Angles

Figure 3.67. Separation Distance, $\theta = 0^\circ$

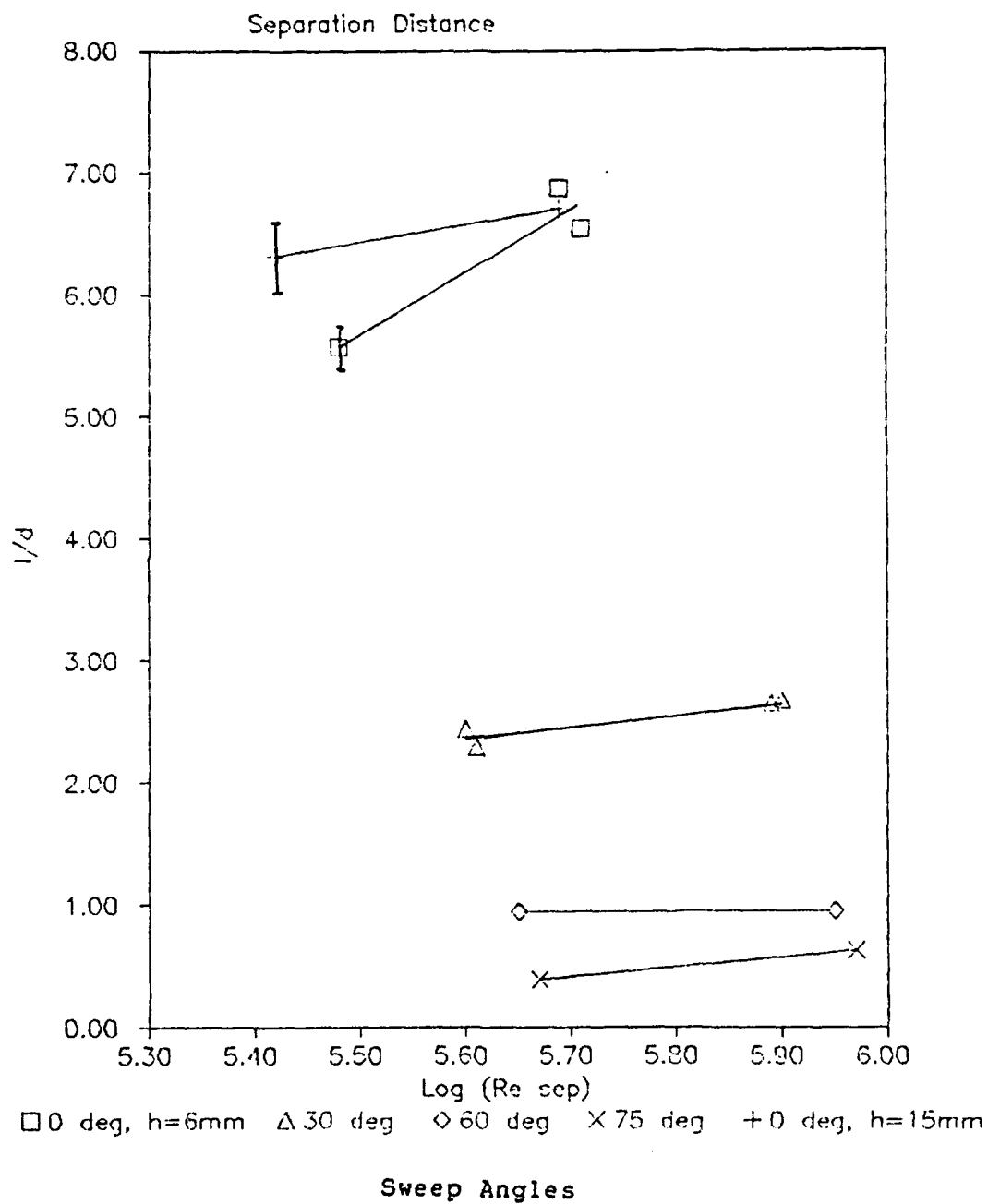


Figure 3.68. Separation Distance, $\theta = 15^\circ$

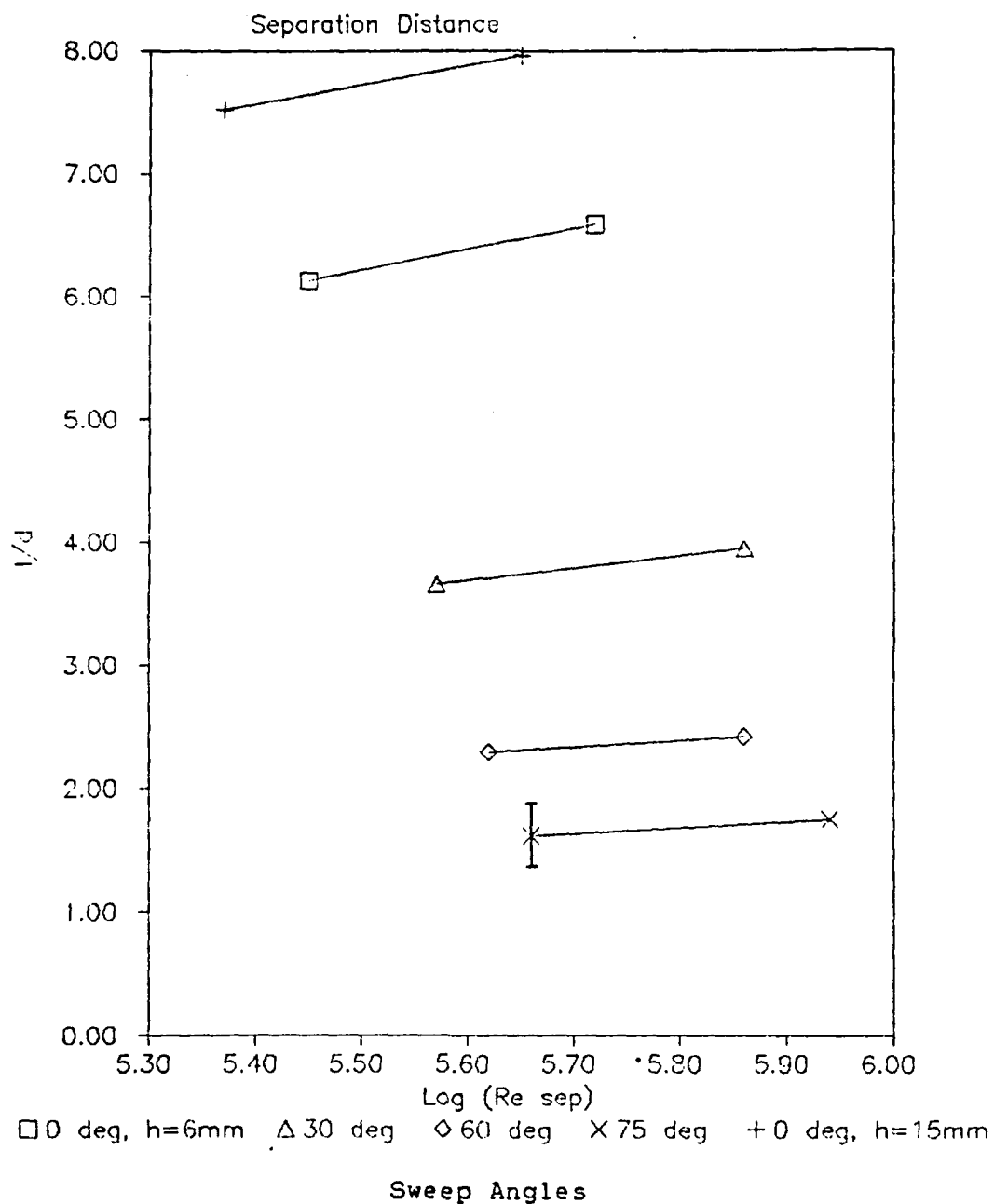


Figure 3.69. Separation Distance, $\theta = 40^\circ$

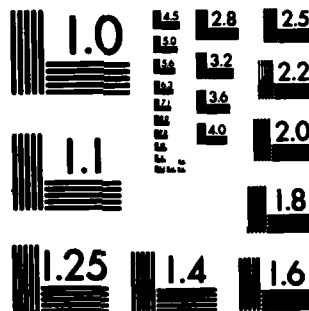
AD-A172 409

NUMERICAL SOLUTION OF LAMINAR AND TURBULENT BOUNDARY
LAYER EQUATIONS INCL. (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... A J CHEN
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

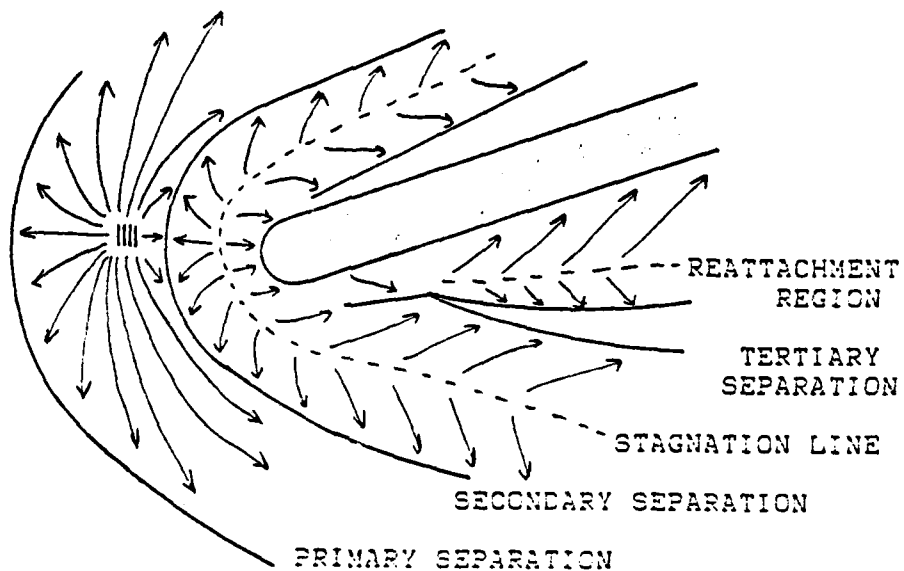


Figure 3.70. Flow Interaction Model, $\theta = 15^\circ$,
 $\Omega = 0, 30, 60^\circ$

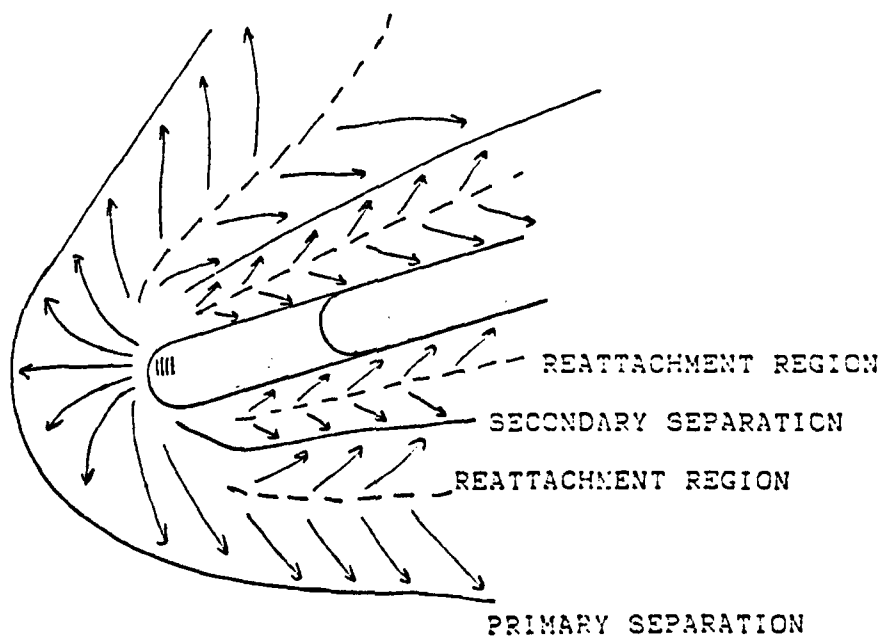


Figure 3.71. Flow Interaction Model, $\theta = 15^\circ$, $\Omega = 75^\circ$

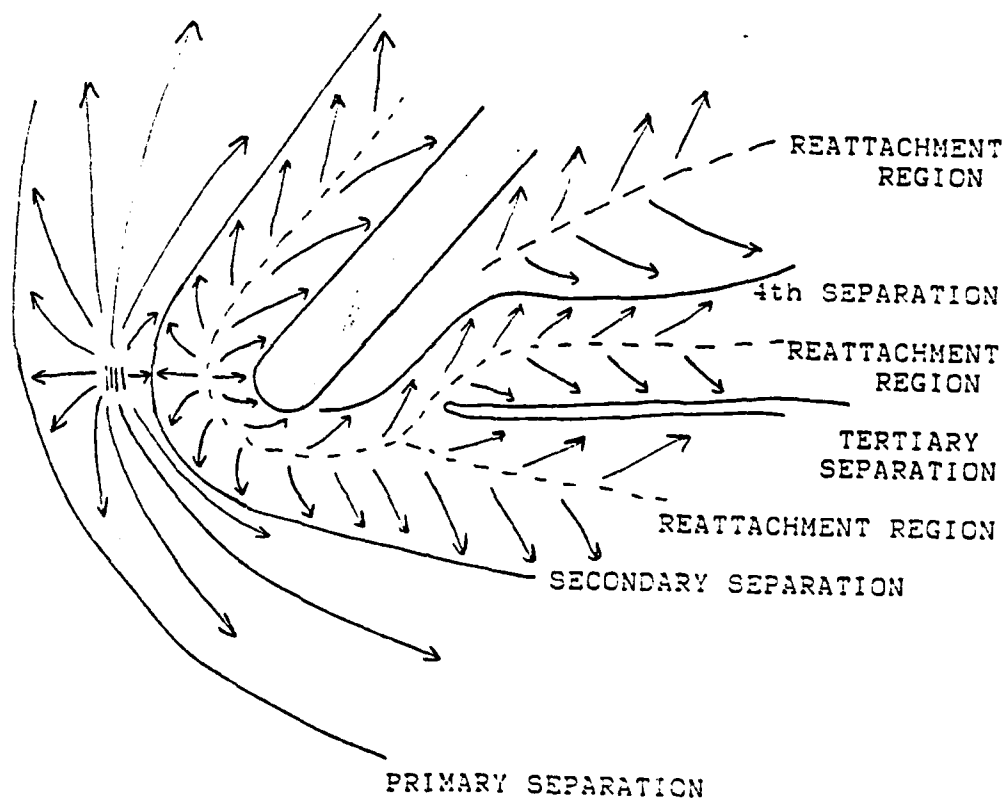


Figure 3.72. Flow Interaction Model, $\theta = 40^\circ$, $\Omega = 0, 30^\circ$

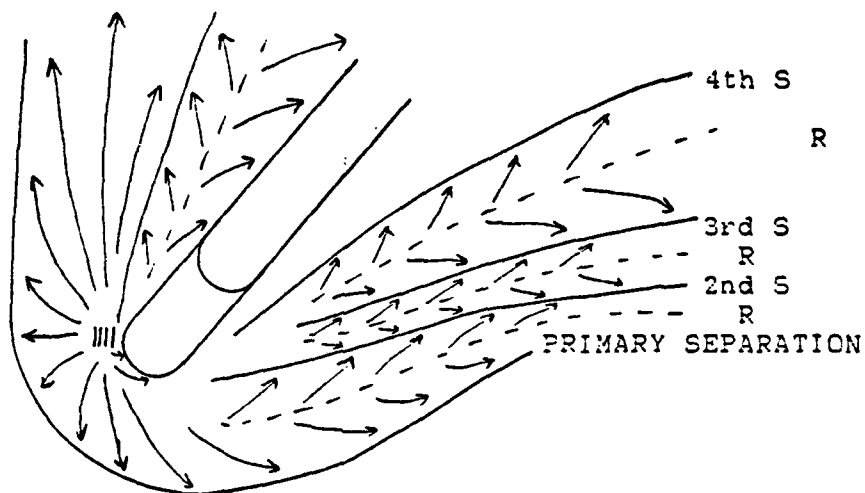


Figure 3.73. Flow Interaction Model, $\varnothing = 40^\circ$, $\Omega = 60^\circ$

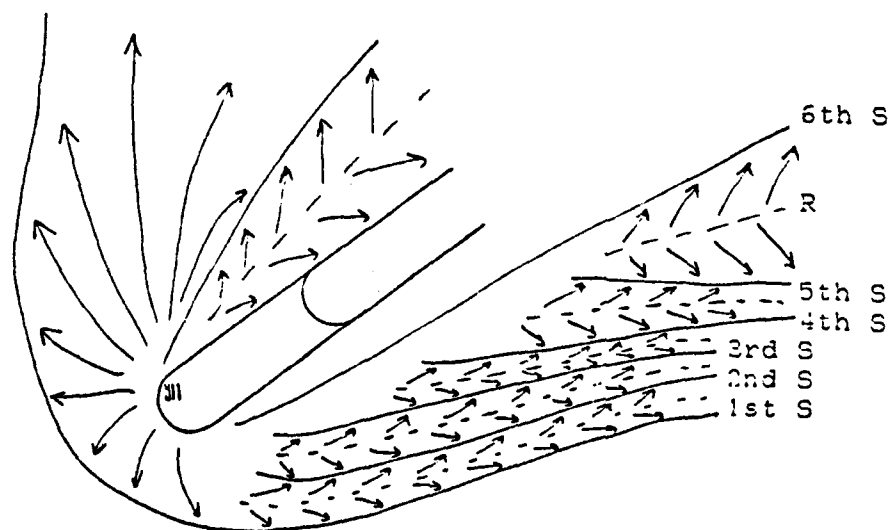


Figure 3.74. Flow Interaction Model, $\varnothing = 40^\circ$, $\Omega = 75^\circ$

IV. Overall Results

The experimental flat plate data obtained from this study are compared to the results of the thesis computer program. The results from the numerical code are for the fully laminar case. The comparisons are completed for two Reynolds numbers, $.844 \times 10^7 / m$ and $1.69 \times 10^7 / m$, and are shown in Figures 3.75 and 3.76, respectively.

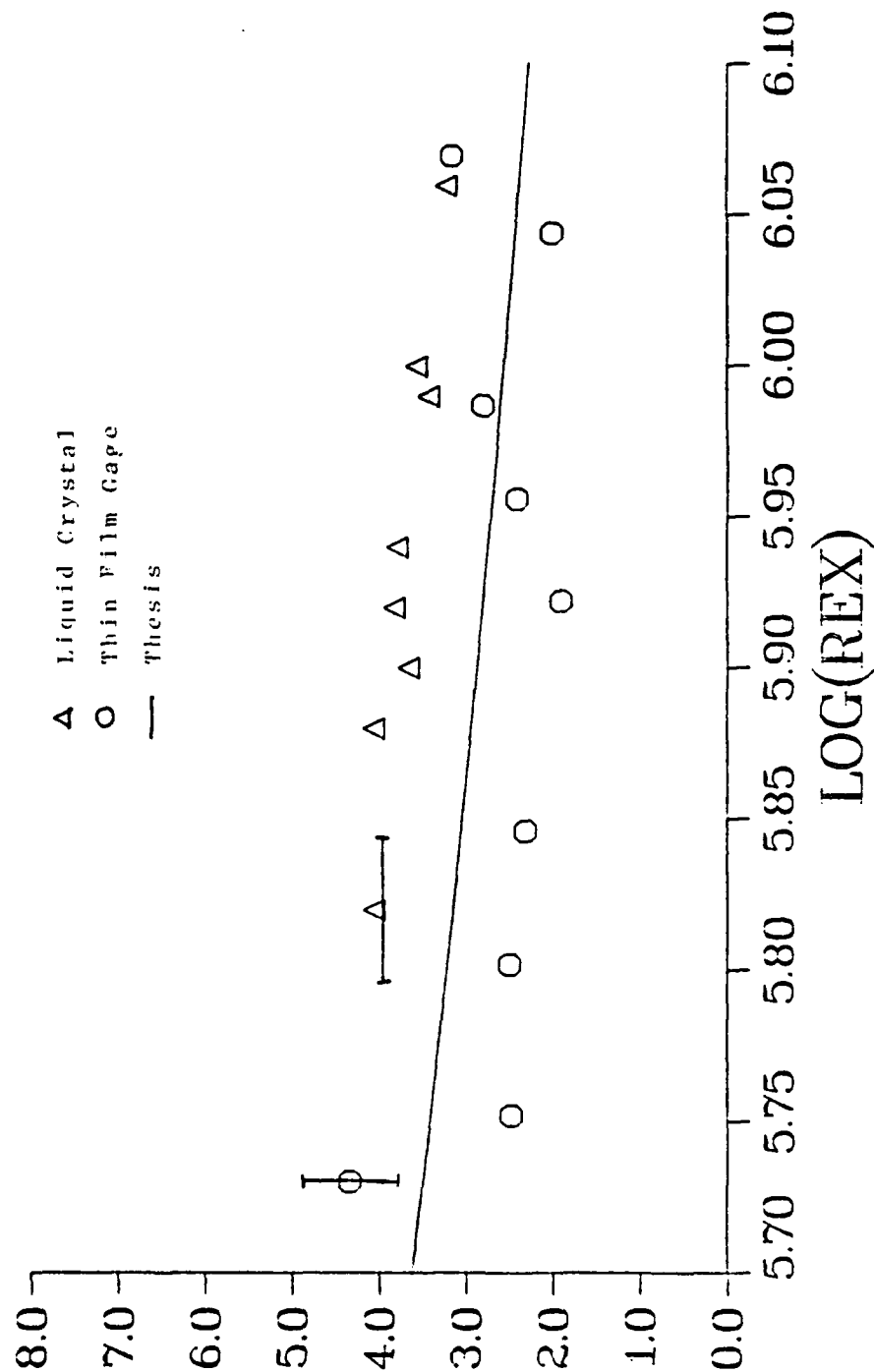
The lower Reynolds number comparison, Figure 3.75, shows the experimental data differing 20% with the numerical results. The liquid crystals being 20% high and the thin film gage being 20% low. The results from the higher Reynolds number case, Figure 3.76, agrees much better with the numerical code. This is in part due to the higher signal that is recorded at the higher Reynolds number.

VKI M=6 P=10

σ_{10}^* IS

Figure 3.75. Experimental and Numerical Comparison

Mach = 6, P = 10 bar, Re = $.844 \times 10^7$ /m



VKI M=6 P=20

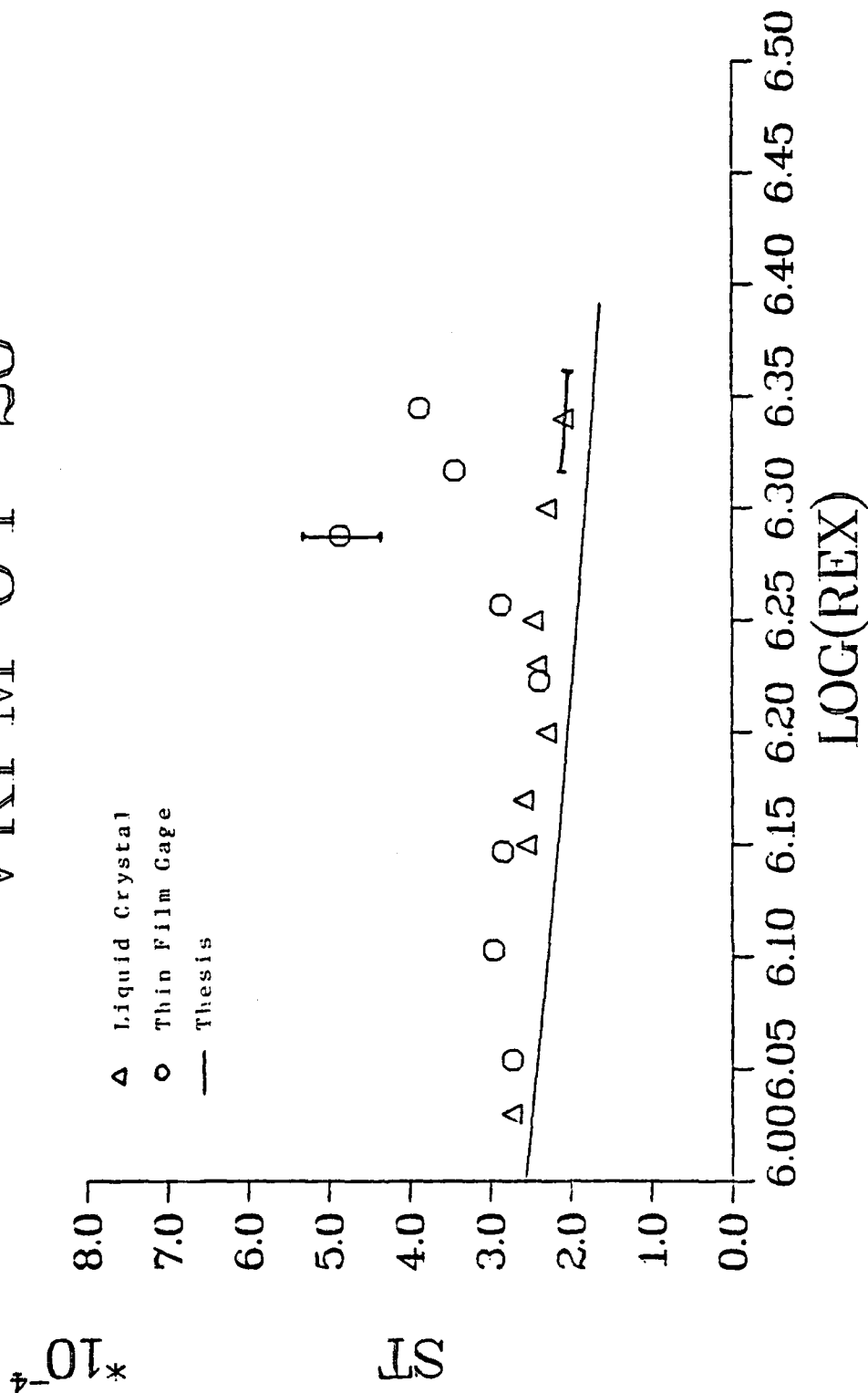


Figure 3.76. Experimental and Numerical Comparison

Mach = 6, P = 20 bar, Re = 1.69×10^7 /m

V. Overall Conclusions

The modification of using the Thomas algorithm in solving the momentum and energy equations instead of using the optimized successive over-relaxation method improved the efficiency of the program for large grids or when high accuracy is required. The steady eddy viscosity turbulence models; Van Driest and Chapman for the inner region, and mixing length and Clauser for the outer region, when compared in a compressible case, showed very little difference in the heat transfer rates (Stanton number).

The liquid crystal results agreed with theory very well, but the method did not show any indication of transition. This is probably due to the poor quality of the color photographs that were used to record the information. The thin film gage data were quite low with respect to theoretical results, and this is due in part to the short testing time.

A large amount of useful information was obtained with the oil flow and sublimation photographs of the flow interaction region of the fin on the flat plate. It was found that the separation distance upstream of the fin increased as the sweep angle is decreased. Striation heating appeared in two cases that were examined. The flow regime that produced the striation heating was Mach 6 flow, $Re = .844 \times 10^7 / m$, fin with 60° or 75° sweep and 40°

incidence. Several flow interaction models were suggested for the different test cases.

The heat transfer results obtained from the numerical technique and experimental technique were compared. Agreement to within 20% exists for the lower Reynolds number comparison. At the higher Reynolds number cases, the agreement between numerical data and experimental data are very good.

VI. Recommendations

Several recommendations for future study have been realized during the course of this project. They are:

1. Modify the steady turbulence model to an unsteady model using the method of Lorber (38).
2. Generate a grid to account for the increase in the boundary layer due to turbulence.
3. Add to the program an adaptive grid method optimized for minimizing truncation error.
4. Extend the code to axisymmetric problems, ultimately to three dimensional cases.
5. Modify the code to solve the momentum and continuity equations in a coupled manner, such as the Davis-Coupled Scheme, to accelerate convergence of the lagged coefficients (3:339).
6. Place the fin closer to the leading edge of the plate (to obtain fully laminar flow) to determine if the "bump" seen in the oil flow photographs is caused by transition or another phenomena.
7. Continue the study of the three dimensional flow interaction region with different shaped fins, ie., taller, wider, sharp edged, using flow visualization and schlieren with shorter exposure times.
8. Expand the flow visualization area of study to include

the fin and the plate area near the trailing edge of the fin to develop a flow interaction model using topological rules (41).

9. Continue the study of the three dimensional flow interaction region using liquid crystals and digital image processing.
10. Obtain quantitative data on the three dimensional flow interaction region using coax gages or thin film gages.
11. Study the heat transfer rates on the fin.
12. Design a new pressure rake to permit calibration of the wind tunnel farther than 143 mm from the nozzle exit.

Appendix A

Time and Mass-Weighted Averaging

A combination of conventional time averaging and mass-weighted averaging (7:49-55) has been used in the development of the governing equations for turbulent boundary layers. The time averaged variables are defined as follows:

$$\bar{\rho} = \bar{\rho} + \rho'' \quad (\text{A.1})$$

$$\bar{P} = \bar{P} + P'' \quad (\text{A.2})$$

where $\bar{\rho}$ and $\bar{P}$ are the time averages of the bulk density and pressure, respectively, and ρ'' and P'' are the superimposed density and pressure fluctuations, respectively. The time average or "mean" of any quantity $q(t)$ is defined by

$$\bar{q}(t) = \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_0^{t+\Delta t} q(t) dt \quad (\text{A.3})$$

For a fluctuating quantity $q''(t)$, the mean is zero by definition, ie.,

$$\bar{q''(t)} = 0 \quad (\text{A.4})$$

therefore,

$$\bar{\rho''} = 0 \quad (\text{A.5})$$

$$\bar{P''} = 0 \quad (\text{A.6})$$

The mass-weighted variables are defined as follows:

$$u = \tilde{u} + u' \quad (A.7)$$

$$v = \tilde{v} + v' \quad (A.8)$$

$$H = \tilde{H} + H' \quad (A.9)$$

$$T = \tilde{T} + T' \quad (A.10)$$

A mass-weighted mean quantity is defined as:

$$\tilde{q} = \frac{\bar{\rho}q}{\bar{\rho}} \quad (A.11)$$

where the bar denotes conventional time averaging and the tilde denotes mass-weighted averaging. A consequence of combining the time and mass-weighted averaging definitions for the product ρq :

$$\rho q = (\bar{\rho} + \rho'')(\tilde{q} + q') = \bar{\rho}\tilde{q} + \bar{\rho}q' + \rho''\tilde{q} + \rho''q' \quad (A.12)$$

is when the time average is taken, eqn (A.12) becomes

$$\bar{\rho}q = \bar{\rho}\tilde{q} + \bar{\rho}q' + \bar{\rho}''q' \quad \text{since } \bar{\rho}'' = 0 \quad (A.13)$$

$$\bar{\rho}q = \bar{\rho}\tilde{q} + \bar{\rho}q' \quad \text{since } \bar{\rho} = \bar{\rho} + \bar{\rho}'' \quad (A.14)$$

By definition, from equation (A.11), $\tilde{\rho}q = \bar{\rho}\tilde{q}$, therefore

$$\bar{\rho}q' = 0 \quad (A.15)$$

Therefore, by analogy let

$$\bar{\rho}u' = 0 \quad (A.16)$$

$$\bar{\rho}v' = 0 \quad (A.17)$$

$$\bar{\rho}H' = 0 \quad (A.18)$$

$$\bar{\rho}T' = 0 \quad (A.19)$$

Total enthalpy can also be expressed in another form including the static enthalpy.

$$\phi H = h + 1/2 \phi u^2 \quad (A.20)$$

If the total enthalpy, eqn (A.20), is mass averaged,

$$\bar{\phi H} + \phi H' = \bar{\phi h} + \phi h' + 1/2 \phi (\bar{u u} + 2\bar{u u'} + u' u') \quad (A.21)$$

Taking the mean of equation (A.21) gives

$$\bar{\phi H} = \bar{\phi h} + 1/2 \bar{\phi u u} + 1/2 \bar{\phi u' u'} \quad (A.22)$$

$$H = h + 1/2 \bar{u u} + 1/2 \frac{\bar{\phi u' u'}}{\bar{\phi}} \quad (A.23)$$

Since $H = \bar{H} + H'$, using the definition: $H = h + 1/2 u^2$

$$H = h + h' + 1/2 (\bar{u} + u')^2 \quad (A.24)$$

$$H + H' = h + h' + 1/2 \bar{u u} + \bar{u u'} + 1/2 u' u' \quad (A.25)$$

Solving for H' and substituting eqn (A.23) into eqn (A.25):

$$H' = h' + \bar{u u'} + 1/2 u' u' - 1/2 \frac{\bar{\phi u' u'}}{\bar{\phi}} \quad (A.26)$$

This result will be used later in Appendix B.

Averaging of the Boundary Layer Equations

The boundary layer equations are given as follows. The equation numbers correspond to the number given originally to the same equation in the main body of the report.

continuity

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi u}{\partial x} + \frac{\partial \phi v}{\partial y} = 0 \quad (2.1)$$

x-momentum

$$\frac{\partial \rho^\# u^\#}{\partial t^\#} + \frac{\partial \rho^\# u^\# u^\#}{\partial x^\#} + \frac{\partial \rho^\# u^\# v^\#}{\partial y^\#} = - \frac{\partial P^\#}{\partial x^\#} + \frac{\partial}{\partial y^\#} \left[\mu^\# \frac{\partial u^\#}{\partial y^\#} \right] \quad (2.2)$$

y-momentum

$$\frac{\partial P^\#}{\partial y^\#} = 0 \quad (2.3)$$

energy

$$\begin{aligned} \frac{\partial \rho^\# H^\#}{\partial t^\#} + \frac{\partial \rho^\# u^\# H^\#}{\partial x^\#} + \frac{\partial \rho^\# v^\# H^\#}{\partial y^\#} = \frac{\partial P^\#}{\partial t^\#} + \frac{\partial}{\partial y^\#} \left[k^\# \frac{\partial T^\#}{\partial y^\#} \right] \\ + \frac{\partial}{\partial y^\#} \left[\mu^\# u^\# \frac{\partial u^\#}{\partial y^\#} \right] \end{aligned} \quad (2.4)$$

Apply mass-weighted averaging on the boundary layer eqns (2.1), (2.2), (2.3), and (2.4).

Continuity

The continuity equation (2.1) after mass-weighted averaging becomes

$$\frac{\partial \bar{\rho}^\#}{\partial t^\#} + \frac{\partial \bar{\rho}^\# (\bar{u}^\# + \bar{u}^{\#'})}{\partial x^\#} + \frac{\partial \bar{\rho}^\# (\bar{v}^\# + \bar{v}^{\#'})}{\partial y^\#} = 0 \quad (A.27)$$

Taking the mean of the equation,

$$\frac{\partial \bar{\rho}^\#}{\partial t^\#} + \frac{\partial \bar{\rho}^\# \bar{u}^\#}{\partial x^\#} + \frac{\partial \bar{\rho}^\# \bar{u}^{\#'}}{\partial x^\#} + \frac{\partial \bar{\rho}^\# \bar{v}^\#}{\partial y^\#} + \frac{\partial \bar{\rho}^\# \bar{v}^{\#'}}{\partial y^\#} = 0 \quad (A.28)$$

But, noting that

$$\overline{\bar{\rho}^\# \bar{u}^{\#'}} = 0 \quad \text{and} \quad \overline{\bar{\rho}^\# \bar{v}^{\#'}} = 0 \quad (A.29)$$

Which results finally in the time and mass-weighted averaged continuity equation:

$$\frac{\partial \bar{\rho}^\#}{\partial t^\#} + \frac{\partial \bar{\rho}^\# \bar{u}^\#}{\partial x^\#} + \frac{\partial \bar{\rho}^\# \bar{v}^\#}{\partial y^\#} = 0 \quad (A.30)$$

x-momentum

The x-momentum equation (2.2) after mass-weighted averaging becomes

$$\begin{aligned} & \frac{\partial \rho \# (u \# + u \#')}{\partial t \#} + \frac{\partial \rho \# (u \# u \# + 2 u \# u \#' + u \# u \#')}{\partial x \#} + \frac{\partial \rho \# (u \# + u \#') (v \# + v \#')}{\partial y \#} \\ &= - \frac{\partial P \#}{\partial x \#} + \frac{\partial}{\partial y \#} \left[\mu \# \frac{\partial (u \# + u \#')}{\partial y \#} \right] \end{aligned} \quad (A.31)$$

Taking the mean of the equation,

$$\begin{aligned} & \frac{\partial \rho \# u \#}{\partial t \#} + \frac{\partial \rho \# u \#'}{\partial t \#} + \frac{\partial \rho \# u \# u \#}{\partial x \#} + \frac{\partial 2 \rho \# u \# u \#'}{\partial x \#} + \frac{\partial \rho \# u \# u \#'}{\partial x \#} + \frac{\partial \rho \# u \# v \#}{\partial y \#} \\ &+ \frac{\partial \rho \# u \# v \#'}{\partial y \#} + \frac{\partial \rho \# u \# v \#'}{\partial y \#} + \frac{\partial \rho \# u \# v \#'}{\partial y \#} = - \frac{\partial P \#}{\partial x \#} \\ &+ \frac{\partial}{\partial y \#} \left[\mu \# \frac{\partial}{\partial y \#} u \# (1 + u \#'/u \#) \right] \end{aligned} \quad (A.32)$$

From eqns (A.29) and (A.32) this results finally in the time and mass-weighted averaged x-momentum equation:

$$\begin{aligned} & \frac{\partial \rho \# u \#}{\partial t \#} + \frac{\partial \rho \# u \# u \#}{\partial x \#} + \frac{\partial \rho \# u \# u \#'}{\partial x \#} + \frac{\partial \rho \# u \# v \#}{\partial y \#} + \frac{\partial \rho \# u \# v \#'}{\partial y \#} = - \frac{\partial P \#}{\partial x \#} \\ &+ \frac{\partial}{\partial y \#} \left[\mu \# \frac{\partial}{\partial y \#} u \# (1 + u \#'/u \#) \right] \end{aligned} \quad (A.33)$$

Y-momentum

Taking the mass-weighted average of the Y-momentum eqn (2.3) has no meaning since the pressure is only averaged in time. Therefore taking the time average of the pressure,

$$\overline{\frac{\partial(P\# + P\#')}{\partial Y\#}} = 0 \quad (\text{A.34})$$

and realizing that, $\overline{P\#'} = 0$, results in the time averaged Y-momentum equation:

$$\overline{\frac{\partial P\#}{\partial Y\#}} = 0 \quad (2.14)$$

Energy

$$\begin{aligned} \frac{\partial \rho\#H\#}{\partial t\#} + \frac{\partial \rho\#u\#H\#}{\partial x\#} + \frac{\partial \rho\#v\#H\#}{\partial Y\#} = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[k\# \frac{\partial T\#}{\partial Y\#} \right] \\ + \frac{\partial}{\partial Y\#} \left[\mu\#u\# \frac{\partial u\#}{\partial Y\#} \right] \end{aligned} \quad (2.4)$$

Before the energy equation (2.4) is mass-averaged, the right hand side of the equality sign can be rewritten in a more convenient form using $h\# = cp\#T\#$,

$$\text{LHS of (2.4)} = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial h\#}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\mu\# \frac{\partial (1/2 u\#u\#)}{\partial Y\#} \right] \quad (\text{A.35})$$

Since $h\# = H\# - 1/2 u\#u\#$:

$$\begin{aligned} \text{LHS of (2.4)} = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial H\#}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\mu\# \frac{\partial (1/2 u\#u\#)}{\partial Y\#} \right] \\ - \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial (1/2 u\#u\#)}{\partial Y\#} \right] \end{aligned} \quad (\text{A.36})$$

Rearranging of eqn (A.36) results in a more convenient form of the energy equation:

$$\begin{aligned} \frac{\partial \rho H}{\partial t} + \frac{\partial \rho u H}{\partial x} + \frac{\partial \rho v H}{\partial y} = \frac{\partial P}{\partial t} + \frac{\partial}{\partial y} \left[\frac{k}{c_p} \frac{\partial H}{\partial y} \right] \\ + \frac{\partial}{\partial y} \left[(\mu - k/c_p) \frac{\partial (1/2 u u)}{\partial y} \right] \quad (A.37) \end{aligned}$$

Taking the mass-weighted average of equation (A.37) gives,

$$\begin{aligned} \frac{\partial \bar{\rho H}}{\partial t} + \frac{\partial \bar{\rho u H}}{\partial x} + \frac{\partial \bar{\rho v H}}{\partial y} = \frac{\partial \bar{P}}{\partial t} \\ + \frac{\partial}{\partial y} \left[\frac{k}{c_p} \frac{\partial \bar{H}}{\partial y} (1 + \bar{H}'/H) \right] \\ + \frac{\partial}{\partial y} \left[(\mu - k/c_p) \frac{\partial (1/2 \bar{u u} + \bar{u u}' + 1/2 \bar{u}' u')}{\partial y} \right] \quad (A.38) \end{aligned}$$

The mean is then taken of equation (A.38) to obtain

$$\begin{aligned} \frac{\partial \bar{\rho H}}{\partial t} + \frac{\partial \bar{\rho u H}}{\partial x} + \frac{\partial \bar{\rho v H}}{\partial y} = \frac{\partial \bar{P}}{\partial t} \\ + \frac{\partial}{\partial y} \left[\frac{k}{c_p} \frac{\partial \bar{H}}{\partial y} (1 + \bar{H}'/H) \right] \end{aligned}$$

$$+ \frac{\partial}{\partial Y\#} \left[\frac{(\mu\# - k\#/cp\#)}{\partial Y\#} \overline{\left(\frac{1}{2} u\#u\# + u\#u\#' + \frac{1}{2} u\#'u\#' \right)} \right] \quad (A.39)$$

Combining eqn (A.39) with eqns (A.16), (A.17) and (A.18) results in the time and mass-weighted averaged energy equation:

$$\begin{aligned} \frac{\partial \rho\#H\#}{\partial t\#} + \frac{\partial \rho\#u\#H\#}{\partial x\#} + \frac{\partial \rho\#u\#'H\#}{\partial x\#} + \frac{\partial \rho\#v\#H\#}{\partial Y\#} + \frac{\partial \rho\#v\#'H\#}{\partial Y\#} = \frac{\partial P\#}{\partial t\#} \\ + \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial H\#(1+H\#'/H\#)}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\frac{(\mu\# - k\#/cp\#)}{\partial Y\#} \overline{\left(\frac{1}{2} u\#u\# \right)} \right] \\ + \frac{\partial}{\partial Y\#} \left[\frac{(\mu\# - k\#/cp\#)}{\partial Y\#} \overline{\left(u\#u\#' + \frac{1}{2} u\#'u\#' \right)} \right] \quad (A.40) \end{aligned}$$

Comparison with Original Equations

The resulting x-momentum equation (A.33) has three terms that make it different than eqn (2.2). These terms are:

$$\frac{\partial \rho\#u\#'u\#}{\partial x\#}, \quad \frac{\partial \rho\#u\#'v\#}{\partial Y\#}, \quad \text{and} \quad \frac{\partial}{\partial Y\#} \left[\mu\# \frac{\partial}{\partial Y\#} \overline{u\#(u\#'/u\#)} \right] \quad (A.41)$$

The energy equation (A.40) has four terms that make it different than equation (A.37). These terms are:

$$\frac{\partial \rho\#u\#'H\#}{\partial x\#}, \quad \frac{\partial \rho\#v\#'H\#}{\partial Y\#}, \quad \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial}{\partial Y\#} \overline{H\#(H\#'/H\#)} \right]$$

$$\frac{\partial}{\partial Y\#} \left[(\mu\# - k\#/cp\#) \frac{\partial}{\partial Y\#} \left(\overline{u\#u\#'} + 1/2 \overline{u\#'u\#'} \right) \right] \quad (A.42)$$

An order of magnitude analysis needs to be completed to determine which of these extra seven terms could be neglected.

Appendix B

Substitution of the Closure Equations

After an order of magnitude analysis is completed on the time and mass-weighted averaged energy equation, an extra term exists which is different than the original form of the energy equation. That term is

$$\frac{\overline{\rho v' H'}}{\Delta Y} \quad (B.1)$$

However, a closure equation is given in Anderson (3:225) which says

$$\overline{\rho c_p v' T'} = - \mu \frac{c_p}{T} \frac{\Delta T}{\Delta Y} \quad (B.2)$$

From a previous analysis completed in Appendix A, it is known that

$$H' = h' + \overline{u u'} + \frac{1}{2} \overline{u' u'} - \frac{1}{2} \frac{\overline{\rho u' u'}}{\rho} \quad (A.26)$$

Substituting eqn (A.26) into the term $\overline{\rho v' H'}$, one obtains

$$\overline{\rho v' H'} = \overline{\rho v' h'} + \overline{\rho v' u u'} \quad (B.3)$$

$$\overline{\rho v' H'} = \overline{\rho c_p v' T'} + \overline{u \rho u' v'} \quad (B.4)$$

Substituting the closure equations for the two terms gives

$$\overline{\rho v' H'} = - \mu \frac{c_p}{T} \frac{\Delta T}{\Delta Y} - \overline{u} \left[\mu \frac{\Delta u}{\Delta Y} \right] \quad (B.5)$$

Putting eqn (B.5) into a form with the total enthalpy,

$$\overline{\phi_{v\#} H\#}' = - \frac{\mu\#}{Pr} \frac{\partial H\#}{\partial Y\#} + \frac{\mu\#}{Pr} \frac{1}{2} \frac{\partial u\#^2}{\partial Y\#} - u\# \frac{\mu\#}{T} \frac{\partial u\#}{\partial Y\#} \quad (B.6)$$

$$\overline{\phi_{v\#} H\#}' = - \frac{\mu\#}{Pr} \frac{\partial H\#}{\partial Y\#} - \frac{\mu\#}{T} (1 - 1/Pr) \frac{1}{2} \frac{\partial u\#^2}{\partial Y\#} \quad (B.7)$$

The energy equation, as previously developed, has the form:

$$\frac{\partial \phi_{H\#}}{\partial t\#} + \frac{\partial \phi_{u\#H\#}}{\partial x\#} + \frac{\partial \phi_{v\#H\#}}{\partial Y\#} + \overline{\phi_{v\#} H\#}' = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[\frac{k\#}{cp\#} \frac{\partial H\#}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\left[\mu\# - \frac{k\#}{cp\#} \right] \frac{\partial}{\partial Y\#} \left(\frac{1}{2} u\#^2 \right) \right] \quad (2.15)$$

Rewriting the energy eqn (2.15) using the definition that

$$\frac{k\#}{cp\#} = \frac{\mu\#}{Pr} \quad (B.8)$$

The energy equation (2.15) becomes

$$\frac{\partial \phi_{H\#}}{\partial t\#} + \frac{\partial \phi_{u\#H\#}}{\partial x\#} + \frac{\partial \phi_{v\#H\#}}{\partial Y\#} + \overline{\phi_{v\#} H\#}' = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[\frac{\mu\#}{Pr} \frac{\partial H\#}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\mu\# (1 - 1/Pr) \frac{\partial}{\partial Y\#} \left(\frac{1}{2} u\#^2 \right) \right] \quad (B.9)$$

Substituting eqn (B.7) into the energy equation (B.9) gives

$$\frac{\partial \phi_{H\#}}{\partial t\#} + \frac{\partial \phi_{u\#H\#}}{\partial x\#} + \frac{\partial \phi_{v\#H\#}}{\partial Y\#} = \frac{\partial P\#}{\partial t\#} + \frac{\partial}{\partial Y\#} \left[\left[\frac{\mu\#}{Pr} + \frac{\mu\#}{T} \right] \frac{\partial H\#}{\partial Y\#} \right] + \frac{\partial}{\partial Y\#} \left[\left[\mu\# (1 - 1/Pr) + \frac{\mu\#}{T} (1 - 1/Pr) \right] \frac{\partial}{\partial Y\#} \left(\frac{1}{2} u\#^2 \right) \right] \quad (2.23)$$

Appendix C

Turbulent Boundary Layer Code

```

PROGRAM UNCOMBL(INPUT,OUTPUT,TAPES,TAPE6=OUTPUT,TAPE7)
COMMON/RESTR/ TIME,X(61),Y(61,59),U(61,0:60),V(61,59),
&              H(61,0:60),TW(61),W(61,59)
DIMENSION A(18362)
EQUIVALENCE (A,TIME)
DIMENSION DX(61),DY(61,59),DYXI(61,59),DYB(61,59),DYF(61,59),
&          ETAXI(61,59),RHO(59),T(59),DYRE(61,59),
&          UNM1(61,59),HNM1(61,59),VNM1(61,59),UIM1(59),
&          HIM1(59),UIM2(59),HIM2(59),Q(61),HHREF(61),
&          YP(61,59),AA(60),O(60),E(60),BB(60),CC(60)
DIMENSION AAH(60),DH(60),EH(60),BBH(60),CCH(60)
DIMENSION TAU(61),CF(61),HC(61)
DIMENSION MUT(59),MUHT(59),MUKT(59)
REAL MUT,MUHT,MUKT,MUCLS
REAL LAM,MU,MUINF,L,MINF,MUW,MUEDG,ME,LL,MUTURB,MUSTAR
REAL LLI,LLO
REAL MITT,CHAP,VO,YPLUS
INTEGER RST,CNT,MODEL
LOGICAL IFSOR
DATA IDIM,JDIM/61,59/
DATA IMAX,JMAX,NT,CNT,KT,NTW,RST/61,30,020,010,10,005,0/
DATA MINF,PINF,TINF/.01,2116,2,530./
C DATA MINF,PINF,TINF/14.24,50343,48.6/
C DATA DT,CP,RG,L,PRL,GAM,EPS/200.,6006.,1715.,28.95,1.,1.4.,00001/
C DATA DT,CP,RG,L,PRL,GAM,EPS/200.,6006.,1715.,1.25,72,1.4.,00001/
C DATA ME,PE,TE,DPDT,TW1,TW2/.01,2116,2,530.,0,0,530.,530./
DATA ME,PE,TE,DPDT,TW1,TW2/6.271,11.06,228.,0,0,530.,530./
OPEN(UNIT=8,FILE='RESTART',FORM='UNFORMATTED')
OPEN(UNIT=9,FILE='WALLQ')
OPEN(UNIT=10,FILE='FIELD')
OPEN(UNIT=11,FILE='GRID')
REWIND 11
REWIND 9
REWIND 10
OPEN(UNIT=13,FILE='ANSWER')
OPEN(UNIT=12,FILE='STANREX')
REWIND 12
REWIND 13
REWIND 7

C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C THIS PROGRAM SOLVES THE UNSTEADY BOUNDARY LAYER EQUATIONS
C ALSO IT SOLVES FOR THE HEAT RATE AT THE WALL AND DETERMINES
C A HEAT TRANSFER COEFFICIENT
C
C
C SYMBOLS:
C
C APLUS      VAN DRIEST DAMPING LENGTH CONSTANT
C CF         SKIN FRICTION COEFFICIENT
C CNT        THE NUMBER OF TIME STEPS BETWEEN PRINTOUT
C CP         COEFFICIENT OF HEAT (FT-LB/SLUG/R)
C DOX        X DISTANCE BETWEEN I AND I+1 NODES
C DEL        BOUNDARY LAYER THICKNESS
C DDEL       NONDIM TURB BOUNDARY LAYER THICKNESS IN TRANSFORMED
C DOELP      NONDIM TURB BOUNDARY LAYER IN PHYSICAL THICKNESS
C DSTAR      NONDIM TURB DISPLACEMENT THICKNESS
C DX         DERIVATIVE OF X WRT XI
C DY         DERIVATIVE OF Y WRT ETA
C DYXI       DERIVATIVE OF Y WRT XI
C DYB        Y DIFFERENCE BETWEEN J AND J-1
C DYC        CENTRAL Y DIFFERENCE
C DYF        Y DIFFERENCE BETWEEN J+1 AND J
C DT         DELTA TIME
C EPS        CONVERGENCE EPSILON

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C	EMAX	MAX ERROR
C	EXTENT	PARAMETER THAT MEASURES THE EXTENT OF TRANSITION
C	FSOR	SAFETY FACTOR ON OPTIMUM SOR ACCELERATION PARAMETER
C	FUDGE	PARAMETER THAT MODIFIES THE AMOUNT OF
C	GAM	TURBULENCE IN THE MODEL DURING TRANSITION
C	GRDFC	RATIO OF SPECIFIC HEATS (GAMMA)
C	H	SCALE FACTOR ON Y FOR GRID
C	HNMI	TOTAL ENTHALPY
C	HIM1	TOTAL ENTHALPY AT OLD TIME LEVEL
C	HIM2	ENTHALPY AT I-1 LOCATION
C	HCI1)	ENTHALPY AT I-2 LOCATION
C	HREF	HEAT COEFFICIENT(BTU/FT ² /S/R)
C	I	REFERENCE HEAT COEFFICIENT(BTU/FT ² /S/R)
C	IMAX	COUNTER IN XI DIRECTION
C	IFSOR	MAX NUMBER OF STEPS IN XI DIRECTION
C	ITRANS	LOGICAL FLAG FOR SOR(T) OR QUADDIAGONAL
C	ITURB	TRANSITION TO TURBULENCE OCCURS AT I=ITRANS
C	J	TRANSITION TO TURBULENCE ENDS AT I=ITURB
C	JMAX	FLOW IS FULLY TURBULENT
C	K	COUNTER IN ETA DIRECTION
C	KT	MAX NUMBER OF STEPS IN ETA DIRECTION
C	L	COUNTER IN ITERATIONS
C	LL	MAX NUMBER OF ITERATIONS
C	MINF	LENGTH OF PLATE (FEET)
C	MODEL=1	PARAMETER OF LENGTH
C	MODEL=2	MACH AT INFINITY
C	KMODL<= 0	VAN DRIEST INNER MODEL
C	KMODL>0	CHAPMAN INNER MODEL
C	MU	CLAUSER'S OUTER MODEL
C	MUINF	MIXING LENGTH OUTER MODEL BASED ON BL THICKNESS
C	MUTURB	VISCOSITY, NONDIMENSIONAL
C	MUCLS	VISCOSITY AT INFINITY(SLUGS/FT/S)
C	MUT(J)	TURBULENT VISCOSITY, NONDIMENSIONAL AT
C	MUHT(J)	CLAUSERS OUTER TURB VISCOSITY, NONDIMENSIONAL
C	MUKT(J)	LAMINAR+TURB VISCOSITY, NONDIMENSIONAL
C	N	LAMINAR VISC/PRL+TURB VISC/PRT, NONDIMENSIONAL
C	NT	LAMINAR VISC(PRL-1)/PRL + TURB VISC(PRT-1)/PRT NONDIM
C	NTW	COUNTER IN TIME
C	PRESS	MAX NUMBER OF TIME STEPS
C	PRL	NUMBER OF TIME STEPS TO CALCULATE OMEGA
C	PRT	NONDIMENSIONALIZED EDGE PRESSURE
C	PE	PRANDTL NUMBER
C	Q	TURBULENT PRANDTL NUMBER
C	RHO	PRESSURE AT THE EDGE OF B.L.(LB/FT ²)
C	RM	HEAT RATE AT THE WALL(BTU/FT ² /S)
C	RE	DENSITY
C	RST	DENSITY TIMES VISCOSITY
C	RINF	REYNOLD'S NUMBER
C	T	RESTART INDICATOR
C	TAU	DENSITY AT INFINITY CONDITIONS (SLUG/FT ³)*3)
C	TTAU	TEMPERATURE
C	TIME	SKIN FRICTION(LB/FT ²)
C	TW	SHEAR STRESS
C	TE	TOTAL TIME ACCUMULATED
C	TINF	TEMPERATURE AT THE WALL(R)
C	TOINF	TEMPERATURE AT THE EDGE OF B.L.(R)
C	U	TEMPERATURE AT INFINITY (DEG RANKINE)
C	UCLAUS	TOTAL TEMPERATURE AT INFINITY (DEG RANKINE)
C	UNMI	VELOCITY IN X DIRECTION
C	UE	TURB CHARACTERISTIC VELOCITY OUTER LAYER BY CLAUSER
C	UFE	VELOCITY IN X DIRECTION AT OLD TIME LEVEL
C	UIM1	VELOCITY AT THE EDGE OF B.L.(FT/S)
C	UIM2	INFINITY TO EDGE VELOCITY RATIO
C		VELOCITY VECTOR AT I-1 LOCATION
C		VELOCITY VECTOR AT I-2 LOCATION

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C      UINF      VELOCITY AT INFINITY CONDITION (FT/SEC)
C      V         VELOCITY IN Y DIRECTION
C      VK        VON KARMEN UNIVERSAL CONSTANT
C      VNM1      VELOCITY IN Y DIRECTION AT OLD TIME LEVEL
C      W         OPTIMIZATION OMEGA
C      XBAR      PARAMETER
C      XTRANS    INITIAL POSITION OF TRANSITION TO TURBULENCE
C      XTURB     FINAL POSITION OF TRANSITION TO TURBULENCE
C              FLOW IS FULLY TURBULENT
C      Y         TRANSFORMED NONDIM NORMAL COORDINATE
C      YP        NONDIM NORMAL PHYSICAL COORDINATE
C      YPLUS     PARAMETER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
DATA MODEL,XTRANS,XTURB,EXTENT/1.,.25,.5,.05/
DATA PRT,VK,YPLSMX/.9,.4,500000./
COSJ=COS(3.14159/FLOAT(JMAX))
WRITE(6,*)'IDIM,JDIM='IDIM,JDIM
WRITE(6,*)'IMAX,JMAX='
READ(5,*)IMAX,JMAX,GRDFC
WRITE(6,*)'NUMERICAL YXI METRIC AFTER IMET='
READ(5,*)IMET
WRITE(6,*)'L(FT)='
READ(5,*)L
WRITE(6,*)'MINF,PINF,TINF='
READ(5,*)MINF,PINF,TINF
WRITE(6,*)'ME,PE,TE='
READ(5,*)ME,PE,TE
WRITE(6,*)'TW1,TW2,XTW2='
READ(5,*)TW1,TW2,XTW2
WRITE(6,*)'ITEQ=0 NO RADIATION EQUILIBRIUM WALL'
WRITE(6,*)'ITEQ='
READ(5,*)ITEQ
WRITE(6,*)'IPRINT=0 NO LISTING ON FILE FIELD'
WRITE(6,*)'IPRT='
READ(5,*)IPRT
WRITE(6,*)'NT,CNT,KT,RST='
READ(5,*)NT,CNT,KT,RST
WRITE(6,*)'DT,EPS='
READ(5,*)DT,EPS
WRITE(10,*)'THE INPUT FOR THIS RUN : '
WRITE(10,*)'NT,CNT,KT,RST = ',NT,CNT,KT,RST
WRITE(10,*)'DT,EPS = ',DT,EPS
JMM1=JMAX-1
GGM1=GAM/(G-1.)
C      MUINF=(TINF*.5*2.27E-8)/(TINF+198.6)
      MUINF=TINF*.5*2.32E-8/(1.+220./(TINF*10.**(9./TINF)))
      AINF=SQRT(GAM*RG*TINF)
      UINF=MINF*AINF
      RINF=PINF/(RG*TINF)
      TOINF=TINF*(1+(GAM-1.)*.5*MINF**2)
      RE=RINF*UINF*L/MUINF
      AE=SQRT(GAM*RG*TE)
      UE=ME*AE
      DO 11 J=1,JMAX
      MUTURB=0.
11  CONTINUE
C      HUEDG=(TE*.5*2.27E-8)/(TE+198.6)
      MUEDG=TE*.5*2.32E-8/(1.+220./(TE*10.**(9./TE)))
      REE=(PE/(RG*TE))*UE*L/MUEDG
      WRITE(9,*)'RE='RE
      WRITE(9,*)'RINF,UINF,MUINF=',RINF,UINF,MUINF
      WRITE(9,*)'MINF,AINF,TINF,PINF=',MINF,AINF,TINF,PINF
      PRESS=PE/(RINF*UINF**2)

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PRESSO=PRESS
WRITE(9,*)'ME,PE,TE,AE,UE=',ME,PE,TE,AE,UE
HREF=.332*CP*SQRT(MUINF*RINF*UINF)/PRL*(2./3.)/778.3
WRITE(9,*)'HREF=',HREF
WRITE(13,*)'I,J,Y+,U+,MU,MUTURB,FUDGE,MITT,LLI,LL0,DELP,YP(I,J)'
DATA ALPHA,TIME/14,0./
C REFERENCE CAN BE BASED ON FREESTREAM OR EDGE CONDITIONS!!!!
C
C DA IS THE 2 DEG/SEC PITCH RATE NONDIMENSIONALIZED TIME
DA=2*L/UINF
C
C TS IS THE NONDIMENSIONAL TIME TO PITCH 11 DEG(14DEG-3DEG)
TS=11/DA
NTS=TS + 5*UINF/L
NNTS=NNTS + TS
C
C GENERATE A GRID AND INITIAL CONDITIONS
C
CALL FIRST(IMAX,JMAX,REE,CP,TW,TE,UE,UINF,PRESS,GGM1,X,Y,U,V,H,
&PRL,XTW2,TW1,TW2,IMET,GRDFC)
DO 10 I=1,IMAX
  U(I,0)=0.
  H(I,0)=0.0
  U(I,JMAX+1)=0.0
  H(I,JMAX+1)=0.0
10 CONTINUE
C
C IF RESTARTING THE PROBLEM, READ THE INITIAL GUESS FROM TAPE 8
C
REWIND 8
IF(RST.EQ.1)READ(8)A
WRITE(6,*)'ALPHA,TIME='
READ(5,*)ALPHA,TIME
WRITE(6,*)'RC,DA='
READ(5,*)RC,DA
WRITE(6,*)'ITERPO=0 DO NOT INTERPOLATE FOR EDGE CONDITIONS'
WRITE(6,*)'ITERPO='
READ(5,*)ITERPO
IF(ITERPO.NE.0)CALL TERPO(ALPHA,UINF,RINF,CP,UE,PE,HE)
PRESS=PE/(RINF*UINF**2)
DO 1888 I=1,IMAX
  U(I,JMAX)=UE/UINF
  H(I,JMAX)=CP*TE/UINF**2+U(I,JMAX)**2*.5
1888 CONTINUE
PRINT*, 'MODEL,XTRANS,XTURB,EXTENT='
READ(5,*)KMODL,XTRANS,XTURB,EXTENT
MODEL=ABS(KMODL)
IF (MODEL.EQ. 1) THEN
  WRITE(10,*) 'TURBULENCE MODEL CONTAINS THE VAN DRIEST MODEL'
  WRITE(9,*) 'TURBULENCE MODEL CONTAINS THE VAN DRIEST MODEL'
  WRITE(13,*) 'TURBULENCE MODEL CONTAINS THE VAN DRIEST MODEL'
ENDIF
IF (MODEL.EQ. 2) THEN
  WRITE(10,*) 'TURBULENCE MODEL CONTAINS THE CHAPMAN MODEL'
  WRITE(9,*) 'TURBULENCE MODEL CONTAINS THE CHAPMAN MODEL'
  WRITE(13,*) 'TURBULENCE MODEL CONTAINS THE CHAPMAN MODEL'
ENDIF
PRINT*, 'PRL,PRT,VON KARMEN CONSTANT,YPLUS MAX FOR SWITCH='
READ(5,*)PRL,PRT,VK,YPLSMX
PRL1=(PRL-1.)/PRL
PRT1=(PRT-1.)/PRT
PRINT*, 'IFSOR FLAG(F OR T),NO TIME STEPS OPT W=,FSOR?'
READ(5,*)IFSOR,NTW,FSOR
COSJ=COSJ*FSOR
WRITE(12,*)'X,REX,REXL,ST,STREXA,STREXB,CF,CFREXA,CFREXB'

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DA=DA*L/UINF
RC=RC*UINF/L
IF (ABS(DA).GT.1.E-16) THEN
    TS=11./DA
    NTS=TS+5.*UINF/L
    NNTS=NTS+TS
END IF

C
C
C
C
CALCULATE DERIVATIVES XI WRT X, ETA WRT Y, AND ETA WRT X
XI WRT X EQUALS 1/DX, ETA WRT Y EQUALS 1/DY, AND
ETA WRT X EQUALS -DYXI/(DX*DY)

    ILOOK=1
    ILOOK2=1
    ITRANS=IMAX+1
    ITURB=IMAX+2
    DO 30 I=2,IMAX
        IM1=I-1
        IM2=I-2
        IP=I+1
        IF (I.EQ.IMAX) THEN
            DX(I)=X(I)-X(IM1)
        ELSE
            DX(I)=(X(IP)-X(IM1))/2.
        ENDIF
        IF ((X(I).GE.XTRANS).AND.(ILOOK.EQ.1)) THEN
            ITRANS=I
            ILOOK=2
        ENDIF
        IF ((X(I).GE.XTURB).AND.(ILOOK2.EQ.1)) THEN
            ITURB=I
            ILOOK2=2
        ENDIF
        DYXI(I,1)=0.0
        DO 32 J=2,UMM1
            JP=J+1
            JM1=J-1
            JM2=J-2
            DY(I,J)=(Y(I,JP)-Y(I,JM1))/2
            IF (I.GT.IMET) THEN
                IF (I.EQ.IMAX) THEN
                    DYXI(I,J)=Y(I,J)-Y(IM1,J)
                ELSE
                    DYXI(I,J)=(Y(IP,J)-Y(IM1,J))*0.5
                    DYXI(I,J)=1.5*Y(I,J)-2.*Y(IM1,J)+.5*Y(IM2,J)
                ENDIF
            ELSE
                DYXI(I,J)=Y(I,J)*.5*DX(I)/X(I)
            ENDIF
            DYB(I,J)=Y(I,J)-Y(I,JM1)
            DYC=(Y(I,JP)-Y(I,JM1))/2.
            DYF(I,J)=Y(I,JP)-Y(I,J)
            ETAX(I,J)=-DYXI(I,J)/(DX(I)*DY(I,J))
            DYRE(I,J)=1./((DYC*RE)
        32 CONTINUE
        DYF(I,1)=Y(I,2)-Y(I,1)
        DY(I,1)=DYF(I,1)
        DYF(I,UMAX)=DYF(I,UMAX-1)
30 CONTINUE
C
C
C
C
PUT INFORMATION INTO THE OLD TIME LEVEL INITIALLY

DO 40 I=1,IMAX
DO 40 J=1,UMAX
    UNM1(I,J)=U(I,J)

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      VNM1(I,J)=V(I,J)
      HNM1(I,J)=H(I,J)
      IF(RST.EQ.0) W(I,J)=1.0
40    CONTINUE
C
      IF(RST.EQ.0) THEN
      DO 43 I=2,IMAX
      DO 43 J=1,11
      W(I,J)=1.6-(J-1)*.06
43    CONTINUE
      END IF
C
C
      START THE TIME LOOP
C
      DO 500 N=1,NT
      TIME=TIME+DT
C
      IF(TIME.LE.0.)GO TO 2001
      IF(TIME.GT.(NNTS+DT))GO TO 2001
      THIS SECTION PITCHES THE PLATE FROM 14 DEG TO 3 DEG AND BACK TO 14
C
      IF (TIME.LE.TS)ALPHA=ALPHA - DA*DT
C
      IF(ALPHA.LE.3.0)ALPHA=3.0
      IF(TIME.GT.TS.AND.TIME.LE.NTS)ALPHA=3.0
      IF(TIME.GT.NTS.AND.TIME.LE.NNTS)ALPHA=ALPHA + DA*DT
      IF(ALPHA.GE.14.)ALPHA=14.0
      IF(TIME.GT.NNTS)ALPHA=14.
      IF(ITERPO.NE.0)CALL TERPO(ALPHA,UINF,RINF,CP,UE,PE,HE)
      PRESS=PE/(RINF*UINF**2)
      DO 2000 I=1,IMAX
      U(I,JMAX)=UE/UINF
      H(I,JMAX)=CP*TE/UINF**2+U(I,JMAX)**2*.5
2000  CONTINUE
      DPDT=(PRESS - PRESS0)
      PRESS0=PRESS
2001  CONTINUE
      IF(N.EQ.1)GO TO 2003
      IF(ITEQ.EQ.0)GO TO 2003
      DO 2002 I=2,IMAX
      IF(X(I).GT.XTW2)THEN
      IF(Q(I).GT.0.)THEN
      TEQ=(Q(I)/(778.3*.85*4.761E-13))**.25
      ELSE
      TEQ=0.
      END IF
      DTRC=DT/RC
      IF(DTRC.GT.34.)DTRC=34.
      EXPRC=EXP(-DTRC)
      TW(I)=TW(I)*EXPRC+(1.-EXPRC)*TEQ
      HW=CP*TW(I)/UINF**2+U(I,1)**2/2.
      H(I,1)=HW
      END IF
2002  CONTINUE
2003  CONTINUE
C
      START BACK AT BEGINNING OF GRID--REINITIALIZE VECTORS
C
      DO 510 J=1,JMAX
      UIM1(J)=0.0
      HIM1(J)=0.0
510  CONTINUE
C
C
      START I LOOP--MARCH IN XI DIRECTION
C
      FUDGE=0.

```

```

DO 400 I=2,IMAX
C TURB MODEL THAT DOES NOT CHANGE WITH N OR I
  IF (I .GE. ITRANS) THEN
    IF (I .GE. ITURB) THEN
      FUDGE=1.0
    ELSE
      XBAR=(X(I)-XTRANS)/EXTENT
      FUDGE=1.0 - 1.0/(EXP(.412*XBAR**2))
    ENDIF
  ENDIF
C
  DELM=0.0
C PUT NEW INFORMATION INTO THE VELOCITY AND ENTHALPY VECTORS AT THE
C I-1 AND I-2 LOCATIONS
C
  DO 410 J=1,JMAX
    UIM2(J)=UIM1(J)
    UIM1(J)=U(I-1,J)
    HIM2(J)=HIM1(J)
    HIM1(J)=H(I-1,J)
410 CONTINUE
C
C DEFINE CONSTANTS TO DETERMINE THE COEFFICIENTS FOR THE U AND V
C VELOCITY EQUATIONS AND THE ENTHALPY EQUATION
C
  IF(I.EQ.2)THEN
    CIJ1=1.
    CIM1=1.
    CIM2=0.
    CVIJ=-.5
    CVIM1=.5
    CVIM2=0.
  ELSE
    CIJ1=1.5
    CIM1=2.0
    CIM2=-.5
    CVIJ=-.75
    CVIM1=1.
    CVIM2=-.25
  ENDIF
C
C START THE ITERATION LOOP
C
  DO 300 K=1,KT
    EMAX=0.0
    EUMAX=0.
    EVMAX=0.
    EHMAX=0.
C
C CALCULATE NEW VALUES FOR TEMPERATURE,DENSITY, AND VISCOSITY USING
C THE NEW VALUES OF U, V, AND H (CALCULATED AT THE OLD ITERATION LEVEL)
C
C CALCULATE TURB MODEL PARAM THAT DO NOT CHANGE WITH J
  IF(I.GT. ITRANS)THEN
    UFE=UINF/UE
    YP(I,1)=0.
    DSTAR=0.
    DO 309 J=2,JMAX
      YP(I,J)=YP(I,J-1)+(1./RHO(J)+1./RHO(J-1))* .5*(Y(I,J)-Y(I,J-1))
      DSTAR=DSTAR+((1.-U(I,J)*UFE)/RHO(J)+(1.-U(I,J-1)*UFE)/RHO(J-1))*
        &.5*(Y(I,J)-Y(I,J-1))
309 CONTINUE
C FIND B.L. THICKNESS
  DO 306 J=2,JMAX
    IF(U(I,J).GE.(0.99*U(I,JMAX)))THEN

```

```

JM1=J-1
DUMX=(.99*U(I,JMAX)-U(I,JM1))/(U(I,J)-U(I,JM1))
DDEL=Y(I,JM1)+(Y(I,J)-Y(I,JM1))*DUMX
DDELP=YP(I,JM1)+(YP(I,J)-YP(I,JM1))*DUMX
GO TO 307
END IF
306 CONTINUE
307 CONTINUE
LLO=.089*DDELP
UCLAUS=.0168/UFE*DSTAR
IOUT=0
END IF
DO 310 J=1,JMAX
  T(J)=H(I,J)-U(I,J)**2/2.
  TEMP=T(J)*UINF**2/CP
  RHO(J)=GGM1*PRESS/T(J)
  MU=TEMP*.5*2.32E-8/(1.+220./(TEMP*10.*(9./TEMP)))/MUINF
  IF (I.GT. ITRANS) THEN
    IF (J.EQ. 1) THEN
      MUW=MU
      MUTURB=0.
      GOTO 312
    ENDIF
    IF (J.EQ. JMAX) GOTO 312
  C CEBECI AND SMITH
    USTAR=SQRT(MUW*RHO(1)*U(I,2)/(Y(I,2)*RE*RHO(J)))
    YPLUS=USTAR*RHO(J)*YP(I,J)*RE/MU
    IF (YPLUS.GE. 15000.) THEN
      DD=1.0
      GOTO 311
    ENDIF
    IF (MODEL.EQ. 1) THEN
      VD=YPLUS/26.0
      DD=(1.0-1.0/(EXP(VD)))**2
    ENDIF
    IF (MODEL.EQ. 2) THEN
      CHAP=YPLUS/111.0
      DD=1.0-2.*COS(CHAP)*1.0/EXP(CHAP)+1.0/EXP(2.0*CHAP)
    DD=SQRT(DD)
    ENDIF
  311 CONTINUE
    LLI=VK*YP(I,J)*SQRT(DD)
    IF (IOUT.EQ.1) GO TO 312
    JP=J+1
    IF (KMODL.GT.0) THEN
      IF (LLO.LT.LLI) THEN
        LL=LLO
      ELSE
        IF (YPLUS.GT.YPLSMX) THEN
          IOUT=1
        END IF
        LL=LLI
      END IF
      MUTURB=RE*RHO(J)**2*LL**2*
      & ABS(U(I,JP)-U(I,J-1))/(2.0*DY(I,J))
    ELSE
      LL=LLI
      MUTURB=RE*RHO(J)**2*LL**2*
      & ABS(U(I,JP)-U(I,J-1))/(2.0*DY(I,J))
    MUCLS=RE*RHO(J)*UCLAUS
    IF (MUTURB.LT.MUCLS.AND. YPLUS.GT.YPLSMX) THEN
      IOUT=1
    END IF
    IF (IOUT.EQ.0.AND. (MUTURB.GT.MUCLS)) THEN
      IOUT=1

```

```

      MUTURB=MUCLS
      END IF
312  CONTINUE      MITT=1.0/((1.+5.5*(YP(I,J)/DDELP)**6)
      TURB=MITT*FUDGE*MUTURB
      MUT(J)=MU+TURB
      MUHT(J)=MU/PRL+TURB/PRT
      MUKT(J)=MU*PRL1+TURB*PRT1
      IF (N.EQ. NT) THEN
        UPLUS=U(I,J)/USTAR
        WRITE(13,*) I,J,YPLUS,UPLUS,MU,MUTURB,FUDGE,MITT,LLI,LLD,
        $DDELP,YP(I,J)
      ENDIF
      ELSE
        MUT(J)=MU
        MUHT(J)=MU/PRL
        MUKT(J)=MU*PRL1
      END IF
310  CONTINUE
C
C      START J LOOP--MARCHING IN ETA DIRECTION, SOLVING FOR U
C
      DO 200 J=JMM1,2,-1
        JM1=J-1
        JM2=J-2
        JP=J+1
        IF(IFSOR) THEN
          UOLD=U(I,J)
          HOLD=H(I,J)
          IF(K.EQ.1) THEN
            WOPT=1.0
          ELSE
            WOPT=W(I,J)
          ENDIF
        END IF
C
C      SETUP CONSTANTS TO DETERMINE COEFFICIENTS IN U, V, AND H EQUATIONS
C
C
      DRMF=(RHO(JP)*MUT(JP)+RHO(J)*MUT(J))/DYF(I,J)*.5
      DRMB=(RHO(JM1)*MUT(JM1)+RHO(J)*MUT(J))/DYB(I,J)*.5
      CON=ETAX(I,J)*U(I,J) + V(I,J)/DY(I,J)
      IF(CON.LT.0.0) THEN
        IF(J.EQ.JMM1) THEN
          CIJ2=-1.0
          CJM1=-1.0
          CJM2=0.0
        ELSE
          CIJ2=-1.5
          CJM1=-2.0
          CJM2=.5
        ENDIF
        DRM1=DRMF
        DRM2=DRMB
        J1=JP
        J2=J+2
        J3=JM1
      ELSE
        IF(J.EQ.2) THEN
          CIJ2=1.
          CJM1=1.
          CJM2=0.
        ELSE

```



```

      CIJ2=1.5
      CJM1=2.0
      CJM2=-.5
    ENDIF
    DRM1=DRMB
    DRM2=DRMF
    J1=JM1
    J2=JM2
    J3=JP
    ENDIF
C   SOLVE FOR U VELOCITY
      CIJ=1. + CIJ1*DT/DX(I)*U(I,J) + CIJ2*DT*CON
      & + DT*DYRE(I,J)*(DRMF+DRMB)
      CONJ1=CJM1*DT*CON + DT*DYRE(I,J)*DRM1
      CONJ2=CJM2*DT*CON
      CONIM=DT/DX(I)*U(I,J)
      CONVS=DT*DYRE(I,J)*DRM2
      IF(IFSOR)THEN
        US=(UNM1(I,J) + CIM1*CONIM*UIM1(J) + CIM2*CONIM*UIM2(J) +
      & CONJ1*U(I,J1) + CONJ2*U(I,J2) + CONVS*U(I,J3))/CIJ
        U(I,J)=WOPT*US + (1.-WOPT)*U(I,J)
      ELSE
        E(J)=(UNM1(I,J)+CIM1*CONIM*UIM1(J)+CIM2*CONIM*UIM2(J))
        D(J)=CIJ
        CC(J)=-CONJ2
        IF (CON .GE. 0.0) THEN
          AA(J)=-CONVS
          BB(J)=-CONJ1
          E(J)=E(J)-CC(J)*U(I,J2)
          CC(J)=0.0
        ELSE
          AA(J)=-CONJ1
          BB(J)=-CONVS
        ENDIF
      ENDIF
      ENDIF
C   C   C
C   SOLVE FOR ENTHALPY
      CON=ETAX(I,J)*U(I,J) + V(I,J)/DY(I,J)
      DRHF=(RHO(JP)*MUHT(JP)+RHO(J)*MUHT(J))/DYF(I,J)*.5
      DRHB=(RHO(JM1)*MUHT(JM1)+RHO(J)*MUHT(J))/DYB(I,J)*.5
      DRKF=(RHO(JP)*MUKT(JP)+RHO(J)*MUKT(J))/DYF(I,J)*.5
      DRKB=(RHO(JM1)*MUKT(JM1)+RHO(J)*MUKT(J))/DYB(I,J)*.5
      IF(CON.LT.0.0)THEN
        IF(J.EQ.JMM1) THEN
          CIJ2=-1.0
          CJM1=-1.0
          CJM2=0.0
        ELSE
          CIJ2=-1.5
          CJM1=-2.0
          CJM2=.5
        ENDIF
        DRH1=DRHF
        DRH2=DRHB
        J1=JP
        J2=J+2
        J3=JM1
      ELSE
        IF(J.EQ.2)THEN
          CIJ2=1.
          CJM1=1.
          CJM2=0.
        ELSE
          CIJ2=1.5

```



```

C      CALL THOMAS(H,I,JMAX,EH,AAH,DH,CCH,BBH,EHMAX)
C      END IF
C
C      COMPUTE THE ERROR FOR ENTHALPY AND U VELOCITY
C      IF(EUMAX.GE.EMAX)EMAX=EUMAX
C      IF(EHMAX.GE.EMAX)EMAX=EHMAX
C      CALCULATE V VELOCITY
C      DO 250 J=2,JMM1
C          JM1=J-1
C          VOLD=V(I,J)
C          D1=DY(I,J)/DX(I)
C          D2=DYXI(I,J)/DX(I)
C          VCN1J=CV1J*D1 + D2
C          VCNJ1=CV1J*D1 - D2
C          V(I,J)=V(I,JM1) + VCN1J*U(I,J) + CVIM1*D1*UIM1(J) +
C          & CVIM2*D1*UIM2(J) + VCNJ1*U(I,JM1) + CVIM1*D1*UIM1(JM1)
C          & + CVIM2*D1*UIM2(JM1)
C          V(I,J)=V(I,JM1)+((DYXI(I,J)+DYXI(I,JM1))* .5*(U(I,J)-U(I,JM1)) +
C          & (Y(I,J)-Y(I,JM1))*(CV1J*(U(I,J)+U(I,JM1)) +
C          & CVIM1*(UIM1(J)+UIM1(JM1)) +
C          & CVIM2*(UIM2(J)+UIM2(JM1)))/DX(I)
C          EV=ABS(V(I,J)-VOLD)
C          IF(EV.GE.EVMAX)EVMAX=EV
C          IF(EVMAX.GE.EMAX)EMAX=EVMAX
250      CONTINUE
C      CHECK CONVERGENCE
C      IF(EMAX.LE.EPS) GO TO 420
C      IF(K.EQ.KT)THEN
C          WRITE(10,6)
C      ENDIF
C      END ITERATION LOOP
300      CONTINUE
420      CONTINUE
C          DO 275 J=1,JMAX
C              U(1,J)=U(2,J)
C              H(1,J)=H(2,J)
C              V(1,J)=V(2,J)
275      CONTINUE
C          DO 435 J=2,JMM1
C              DEL=ABS(U(I,J) - UNM1(I,J))
C              IF(DEL.GT.DELM)DELM=DEL
C              DEL=ABS(H(I,J) - HNM1(I,J))
C              IF(DEL.GT.DELM)DELM=DEL
435      CONTINUE
C          MOVE THE RESULTS INTO THE OLD TIME LEVEL
C          DO 430 J=1,JMAX
C              UNM1(I,J)=U(I,J)
C              HNM1(I,J)=H(I,J)
C              VNM1(I,J)=V(I,J)
430      CONTINUE
C      CALCULATE THE HEAT RATE AT THE WALL
C      FDELY=(-3.*Y(I,1) + 4.*Y(I,2) - Y(I,3))/2.
C      CONDUCTIVITY IS EQUAL TO MUW*CP/PR=K
C      MUW=TW(I)**.5*2.32E-8/(1.+220./(TW(I)*10** (9./TW(I))))

```

```

      Q(I)=MUW/PRL*RHO(1)/FDELY*(-3.*T(1) + 4.*T(2) - T(3))/2.*
      &      UINF**2/L /778.3
C TAW SHOULD USUALLY BE BASED ON EDGE CONDITIONS FORMALLY
C BUT CAN BE BASED ON FREESTREAM IF YOU ARE CONSISTENT!!!!
      IF(I.GT.ITRANS)THEN
        REC=FUDGE*(PRL)**(1./3.)*(1.-FUDGE)*SQRT(PRL)
      ELSE
        REC=SQRT(PRL)
      END IF
      REC=1.
      TAW=TE*(1.+REC*(GAM-1.)*.5*ME**2)
      HC(I)=Q(I)/(TAW- TW(I))
      HHREF(I)=HC(I)/HREF*SQRT(X(I)*L)
      TAU(I)=MUW*RHO(1)*UINF/L/FDELY*(-3*U(I,1)+4*U(I,2)-U(I,3))*1.5
      CF(I)=2.*TAU(I)/(RINF*UINF**2)

C
C
C
C
      CALCULATE THE OPTIMIZATION FACTOR OMEGA EVERY NTW TIME STEPS
      IF(N.EQ.(N/NTW)*NTW)THEN
        DO 519 J=2,JMM1
          W(I,J)=1.85
519      CONTINUE

C
C
C
C
      COMPUTE OMEGA
      DO 530 J=2,JMM1
        DRM=RHO(J)*MUT(J)/DY(I,J)
        DIV=1./DT + 1.5*(U(I,J)/DX(I) + ABS(U(I,J)*ETAX(I,J) +
      &      V(I,J)/DY(I,J))) + DYRE(I,J)* DRM*2.
      &      LAM=(-2.*SQRT(ABS(ABS(ETAX(I,J)*U(I,J) + V(I,J)/DY(I,J)) +
      &      DYRE(I,J)*DRM) *
      &      (DYRE(I,J)*DRM))))/DIV*CDJSJ
        IF(LAM.GT.0.97)LAM=0.97
        WN=2./(1.+SQRT(1.-LAM**2))
        IF(WN.LT.W(I,J))W(I,J)=WN
530      CONTINUE
      ENDIF
      END I LOOP
400      CONTINUE

C
C
C
C
      WRITE OUT SOLUTION AT A TIME STEP
      IF(N.EQ.(N/CNT)*CNT)THEN
        REWIND 8
        WRITE(8)A
        WRITE(10,1)N,TIME
        WRITE(6,*)'DELT=',DELM,'K=',K,'U,V,H ERR=',EUMAX,EVMAX,EHMAX
        WRITE(10,*)'DELT=',DELM,'K=',K,'U,V,H ERR=',EUMAX,EVMAX,EHMAX
        DO 440 I=1,IMAX
          WRITE(10,2)I,X(I)
          IF(IPRT.EQ.0)GO TO 451
          WRITE(10,3)
          DO 450 J=1,JMAX
            T(J)=H(I,J)-U(I,J)**2/2.
            TEMP=T(J)*UINF**2/CP
            RHO(J)=GGM1*PRESS/T(J)
            WRITE(10,4)J,Y(I,J),U(I,J),H(I,J),T(J),TEMP,V(I,J),RHO(J)
450          CONTINUE
          451 CONTINUE
          IF(I.EQ.1) GO TO 440
          WRITE(10,5)Q(I),HHREF(I)
          TIME=TIME+L/UINF
C CF RATIO TO BLASIU CAN BE BASED ON EDGE OR FREESTREAM RE

```

```

      CFCF=CF(I)/.664*SQRT(REE*X(I))
      REX=RE*X(I)
      REXLOG=ALOG10(REX)
      CFREXA=CF(I)*SQRT(REX)
      CFREXB=CF(I)*REX**.8
      ST=HC(I)*778.3/(RINF*UINF*CP)
      STREXA=ST*SQRT(REX)
      STREXB=ST*REX**.8
      WRITE(9,7)TIMEP,ALPHA,X(I),HHREF(I),TW(I),CFCF
      WRITE(12,77)X(I),REX,REXLOG,ST,STREXA,STREXB,CF(I),CFREXA,CFREXB
440    CONTINUE
      ENDIF
      END TIME LOOP
C
500    CONTINUE
C
      COMPUTE PHYSICAL Y VALUES
      DO 1001 J=1,JMAX
        YP(1,J)=0.0
        WRITE(11,8)X(I),Y(1,J),YP(1,J)
1001    CONTINUE
      DO 650 I=2,IMAX
        DO 460 J=1,JMAX
          T(J)=H(I,J) - U(I,J)**2/2
          RHO(J)=GGM1*PRESS/T(J)
460    CONTINUE
C
          YP(I,1)=0.0
          WRITE(11,8)X(I),Y(I,1),YP(I,1)
          DO 470 J=2,JMAX
            YP(I,J)=YP(I,J-1) + (1/RHO(J) + 1/RHO(J-1))/2*(Y(I,J)-Y(I,J-1))
            WRITE(11,8)X(I),Y(I,J),YP(I,J)
470    CONTINUE
650    CONTINUE
8      FORMAT(3(2X,E13.5))
C
C      THIS SECTION FIGURES OUT THE THEORETICAL NONISOTHERMAL VALUES
C      FOR H/HREF
C
C CAUTION! TSTAR BASED ON TW1! DO YOU WANT CONSTANT REFERENCE OR ACTUAL TW?
      TAW=TE*(1.+SQRT(PRL)*(GAM-1)*.5*ME**2)
      TSTAR=TE+.5*(TW1-TE)+.22*(TAW-TE)
      RSTAR=PE/(RG*TSTAR)
      MUSTAR=(2.32E-8*TSTAR**.5/(1.+220./(TSTAR*10.**(.9./TSTAR))))
      HISO=.332*CP*SQRT(MUSTAR*RSTAR*UE)/PRL**(2./3.)*SQRT(X(I)*L)/HREF
      WRITE(6,*)'HISO=',HISO
      WRITE(9,*)'HISO=',HISO
      DO 1000 I=1,IMAX
        IF (I .GE. ITRANS) THEN
          XBAR=(X(I)-XTRANS)/EXTENT
          IF (I .GE. ITURB) THEN
            FUDGE=1.0
          ELSE
            FUDGE=1.0-1.0/(EXP(.412*XBAR**2))
          ENDIF
          HT=HISO
          IF(X(I).GE.XTW2)THEN
            IF(ABS(XTW2-X(I)).LT.1E-3)THEN
              DO 1002 J=1,4
                XX=X(I)+ DX(I)/FLOAT(5)*FLOAT(J)
                TAW=TE*(1.+SQRT(PPL)*(GAM-1)*.5*ME**2)
                TSTAR=TE+.5*(TW1-TE)+.22*(TAW-TE)
                RSTAR=PE/(RG*TSTAR)
                MUSTAR=(2.32E-8*TSTAR**.5/(1.+220./(TSTAR*10.**(.9./TSTAR))))
                HISO=.332*CP*SQRT(MUSTAR*RSTAR*UE)/PRL**(2./3.)*SQRT(X(I)*L)/HREF

```

```

      PHI=((TAW-TW1)-(1.-(XTW2/XX)**.75)**(-1./3.)*
      & (TW2-TW1))/(.9*TOINF-TW2)
      HT=HISO*PHI
      WRITE(9,9)XX,HT
1002  CONTINUE
      ELSE
      XX=X(I)
      TAW=TE*(1.+SQRT(PRL)*(GAM-1.)*.5*ME**2)
      TSTAR=TE+.5*(TW1-TE)+.22*(TAW-TE)
      TAW=TE*(1.+SQRT(PRL)*(GAM-1.)*.5*ME**2)
      RSTAR=PE/(RG*TSTAR)
      MUSTAR=(2.32E-8*TSTAR**.5/(1.+220./(TSTAR*10.**(.9./TSTAR))))
      HISO=.332*CP*SQRT(MUSTAR*RSTAR*UE)/PRL**(.2./3.)*SQRT(X(I)*L)/HREF
      PHI=((TAW-TW1)-(1.-(XTW2/XX)**.75)**(-1./3.)*
      & (TW2-TW1))/(.9*TOINF-TW2)
      HT=HISO*PHI
      WRITE(9,9)X(I),HT
      ENDIF
      ELSE
      WRITE(9,9)X(I),HT
      ENDIF
1000  CONTINUE
      REWIND 8
      WRITE(8)A
1  FORMAT(///,2X,'N=',I6,5X,'TIME=',F15,6,1X,'SEC')
2  FORMAT(//,2X,'I=',I6,5X,'X-LOCATION=',F13,5)
3  FORMAT(/,4X,'J',5X,'Y-LOC',5X,'U',5X,'H',5X,'T',5X,'TEMP(R)',
      & 5X,'V',5X,'RHO',/)
4  FORMAT(1X,I3,7F13,5)
5  FORMAT(/,2X,'HEAT RATE (Q)=' ,F13,6,5X,'H/HREF=' ,F13,6)
6  FORMAT(/, 'THIS SOLUTION HAS NOT CONVERGED WITH KT ITERATIONS',/)
7  FORMAT(7(2X,F13,5))
9  FORMAT(34X,2(2X,F13,5))
77  FORMAT(9(1X,E13,5))
600  CONTINUE
      CLOSE(8)
      CLOSE(10)
      CLOSE(12)
      STOP
      END
      SUBROUTINE FIRST(IMAX,UMAX,RE,CP,TW,TE,UE,UINF,PRESS,GGM1,
      & X,Y,U,V,H,PR,XTW2,TW1,TW2,IMET,GRDFC)
C  THIS SUBROUTINE TAKES THE BLASIUS SOLUTION, MODIFIES IT BY
C  THE DENSITY, AND FORMS A GRID. IT ALSO DETERMINES INITIAL
C  CONDITIONS
      DIMENSION X(61),Y(61,59),DY(61),U(61,0:60),H(61,0:60),RHO(59)
      DIMENSION V(61,59),YN(59),YOLD(59),TW(61)
C  THIS IS A BLASIUS PROFILE OPTIMIZED FOR RE=2.E6. THIS MUST
C  BE MODIFIED FOR MY REYNOLD'S NUMBER. ALSO, I AM BASING THIS
C  GRID ON THE THERMAL BOUNDARY LAYER THICKNESS SO MODIFY Y WITH
C  THE PRANDTL NUMBER
      DATA YOLD/0.,.000248,.0005,.000758,.001024,.001316,.00166,
      & .002,.002672,.003402,.004342,.005548,.007099,
      & .009092,.01165,.014945,.019174,.024609,.031593,
      & .040568,.0521,.06692,.085963,.11043,.14187,.18225,
      & .23414,.30076,.3862,.496/
      DX=1./(IMAX-1.)
      DO 100 I=1,IMAX
      X(I)=DX*(I-1)
      DO 120 J=1,UMAX
      Y(I,J)=YOLD(J)*SQRT(2.E6)/SQRT(RE)*SQRT(X(I))/PR
C120  CONTINUE
C100  CONTINUE
C  C READ GRIDIN

```

```

      DO 100 I=1,IMAX
      DO 200 J=1,JMAX
      READ(7,9999)X(I),Y(I,J)
9999  FORMAT(2E13.5)
200  CONTINUE
100  CONTINUE
C    TO USE ANALYTICAL METRIC FOR YXI FOR I=1 TO IMET
C    Y POINTS AT I=IMET ARE SCALED BY SQRT(X)
C    THIS WILL NOT GUARANTEE A SMOOTH GRID PAST I=IMET
      DO 110 I=1,IMET
      DO 110 J=1,JMAX
      Y(I,J)=Y(IMET,J)*SQRT(X(I)/X(IMET))
110  CONTINUE
C
C    DEFINE THE WALL TEMPERATURES-- IF TW1=TW2 NO TEMP STEP
C                                     IF TW1<TW2 TEMP STEP
C
      DO 11 I=1,IMAX
      IF(X(I).LE.XTW2)THEN
        TW(I)=TW1
      ELSE
        TW(I)=TW2
      ENDIF
11  CONTINUE
      HE=CP*TE/UINF**2 + (UE/UINF)**2/2.
      DO 150 I=2,IMAX
      HW=CP*TW(I)/UINF**2
      HSUB=HE - HW
      DO 130 J=1,JMAX
      DELTA=5.2/SQRT(RE)*SQRT(X(I))
      DELTAT=DELTA
      IF(Y(I,J).LE.DELTAT)THEN
        U(I,J)=UE/UINF*(1.5*Y(I,J)/DELTA - .5*(Y(I,J)/DELTA)**3.)
      ELSE
        U(I,J)=UE/UINF
      ENDIF
      V(I,J)=0.0
      IF(Y(I,J).LE.DELTAT)THEN
        H(I,J)=HW + HSUB*(1.5*Y(I,J)/DELTAT - .5*(Y(I,J)/DELTAT)**3)
      ELSE
        H(I,J)=HE
      ENDIF
      TEMP=H(I,J) - U(I,J)**2/2.
      RHO(J)=GGM1*PRESS/TEMP
130  CONTINUE
      DO 145 J=2,JMAX
      Y(I,J)=Y(I,J)*GRDFC
145  CONTINUE
150  CONTINUE
      DO 160 J=1,JMAX
      U(1,J)=U(2,J)
      V(1,J)=0.0
      H(1,J)=H(2,J)
160  CONTINUE
      RETURN
      END
      SUBROUTINE TERPO(ALPHA,UINF,RINF,CP,UE,PE,HE)
C
C    THIS SUBROUTINE INTERPOLATES TO GET VELOCITY,PRESSURE,TEMPERATURE,
C    AND ENTHALPY AT A NEW ANGLE
C
      DIMENSION EVEL(14),ETEMP(14),EPRESS(14)
      DATA EVEL/0.,4843.7962,4843.7962,4834.5371,4823.7297,4811.1933,
&          4796.8408,4780.5741,4762.3358,4742.0933,4719.8062,
&          4695.4522,4669.00,4639.60/

```

```

DATA ETEMP/O.,65.62,65.62,73.08,81.78,91.84,103.33,116.30,130.80,
& 146.83,164.39,183.49,204.12,227.99/
DATA EPRESS/O.,1.3063,1.3063,1.7262,2.2334,2.8322,3.5250,4.3135,
& 5.1985,6.1801,7.2581,8.4319,9.7010,11.060/

```

```

C
  IALPH=INT(ALPHA)
  UE=EVEL(IALPH) + (ALPHA-IALPH) * (EVEL(IALPH+1) - EVEL(IALPH))
  TE=ETEMP(IALPH) + (ALPHA-IALPH) * (ETEMP(IALPH+1) - ETEMP(IALPH))
  HE=CP*TE + UE**2/2
  PE=EPRESS(IALPH)+(ALPHA-IALPH)*(EPRESS(IALPH+1) - EPRESS(IALPH))
  RETURN
END

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
  SUBROUTINE THOMAS(U,I,JMAX,E,AA,D,CC,BB,EUMAX)
  DIMENSION U(61,0:60),E(60),AA(60),D(60),CC(60),BB(60)
  JMM1=JMAX-1
  DO 530 J=2,JMM1
    JM=J-1
    D(J)=D(J)-BB(J)*AA(JM)/D(JM)
    AA(J)=AA(J)-BB(J)*CC(JM)/D(JM)
    E(J)=E(J)-BB(J)*E(JM)/D(JM)
  530 CONTINUE
C
C  NOW SOLVE FOR U OR H
C
  UOLD=U(I,JMM1)
  U(I,JMM1)=(E(JMM1)-AA(JMM1)*U(I,JMAX))/D(JMM1)
  EU=ABS(U(I,JMM1)-UOLD)
  IF (EU .GE. EUMAX) THEN
    EUMAX=EU
  ENDIF
  DO 540 J=JMAX-2,2,-1
    UOLD=U(I,J)
    JP=J+1
    JP2=J+2
    U(I,J)=(E(J)-AA(J)*U(I,JP)-CC(J)*U(I,JP2))/D(J)
    EU=ABS(U(I,J)-UOLD)
    IF (EU .GE. EUMAX) THEN
      EUMAX=EU
    ENDIF
  540 CONTINUE
  RETURN
END

```


Appendix D

Mach Number Iteration Program

```
100 PRINT "What is P2 in mm Hg?"
101 GAM=1.4
102 INPUT P2M
103 PRINT "What is the number of divisions of P1?"
104 INPUT P1D
105 P1=(P2M*760)/101 - P2M
106 PRINT "p1 = " P1 " mm of mercury"
107 PRINT "What is the number of divisions of P1?"
108 INPUT P1D
109 PRINT
110 DP="0.2/P2/1000"
111 P1D=P1D-DP " mm of mercury"
112 P1=P1D/P1D
113 M=0.000 " initial guess
114 A=(GAM+1)/(2*(1-GAM))
115 B=(1/(M*2+(GAM-1)/2))/(.5*(GAM-1))
116 C=(GAM-1)/(GAM+1)
117 MACH=(A*(C+(1/(P1*(GAM-1)+B*(GAM))))).5
118 DIFF=ABS(MACH-M)
119 IF DIFF <.001 THEN PRINT "p1 = " P1
120 IF DIFF <.001 THEN PRINT "p1/p1 = " P1
121 IF DIFF <.001 THEN PRINT "mach=" MACH ELSE M=MACH:GOTO 114
122 PRINT "Is there any more pressure data?"
123 PRINT "Type a 1 if yes, 0 if no"
124 INPUT ANS
125 IF ANS = 1 GOTO 100
```

Appendix E

Calibration of the VKI H-3 Wind Tunnel

Purpose

To determine the size of the flat plate model to construct, the scope of the Mach 6 uniform free jet must be determined. The first time the tunnel was calibrated was after its installation in 1970 (29). That previous calibration showed the existence of a small centerline disturbance and the slow streamwise growth of the outer shear layer.

This test will examine the streamwise distribution from the nozzle exit, taking measurements every 50 mm. Also the uniformity of the flow at each streamwise position will be examined by rotating the rake 30 degrees at a time from a vertical position until the entire cross-sectional area is examined.

3.3.2 Theory

To determine the uniformity of the flow, a distribution of pitot pressure is measured along various cross sections of the test chamber. The pressure in the reservoir is also monitored throughout the test. The ratio of the reservoir pressure, P_{o1} , and the test chamber total pressure, P_{o2} , gives the Mach number of the flow coming out of the nozzle exit.

The pitot pressure in the test chamber is actually the total pressure behind a normal shock. The normal shock is

present due to the presence of the pitot probe itself. Isentropic flow relations and normal shock theory can be used to find the initial Mach number (35:148). The ratio of total pressures can be written in terms of the initial Mach number.

$$\frac{P_{o1}}{P_{o2}} = \left[\frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} \right]^{1/(\gamma-1)} \left[\frac{1 + .5(\gamma-1) M_1^2}{.5(\gamma+1) M_1^2} \right]^{\gamma/(\gamma-1)} \quad (E.1)$$

This equation can be rewritten, solving for M_1 , so that

$$M_1^2 = \frac{(\gamma+1)}{2\gamma} \left[\frac{(\gamma-1)}{(\gamma+1)} + \left(\frac{P_{o1}}{P_{o2}} \right)^{\frac{\gamma-1}{\gamma}} \frac{1}{[.5(\gamma+1)]} \left[\frac{1}{M_1^2} + .5(\gamma-1) \right]^{\frac{\gamma}{\gamma-1}} \right] \quad (E.2)$$

An iterative procedure must be used to solve for M_1 since it appears on both sides of the equation. Using an initial guess for M_1 in the equation as 5.999, the solution for the actual M_1 given the total pressure ratios will be obtained very quickly. The Mach number output is compared with the Mach number input, and then the output is used as the next iteration's input. The iteration continues until the difference between the input and output Mach numbers is less than 0.001. A computer program which does this iteration is included in Appendix D.

Description of Equipment

A pitot rake with eleven pitot tubes was used to measure the total pressure in the test chamber. The

detailed dimensions of the pitot rake are given in Figure E.1. The pitot rake tubes originally extended 50 mm ahead of the metal plate support, but due to tunnel blockage problems, the probes were shortened until only 10 mm of the pitot tube extended ahead of the plate support. The shortening of the probes had no effect on the results, but this is discussed in the section on error analysis and repeatability. The probe centerlines are 13 mm apart.

The pitot probes were connected to a Scanl-Valve pressure transducer. The probes were read sequentially during each run, beginning with the number 1 probe and advancing to the number 11 probe. The data was recorded on a Graphispot recorder on advancing graph paper. The total pressure for the reservoir was measured by a pressure transducer and also recorded on a Graphispot recorder. The data acquisition system is shown in Figure E.2.

The installed pressure rake is shown in Figure E.3. This is the position the rake is in during the startup sequence of the tunnel. After the tunnel has started, the rake is injected into the free jet. Figure E.4 shows the position of the rake after injection. In this particular figure, the rake is at an orientation of 120° . Note there is a portion of the diffuser nozzle cut to permit the injection of the rake. Due to tunnel blockage problems, a

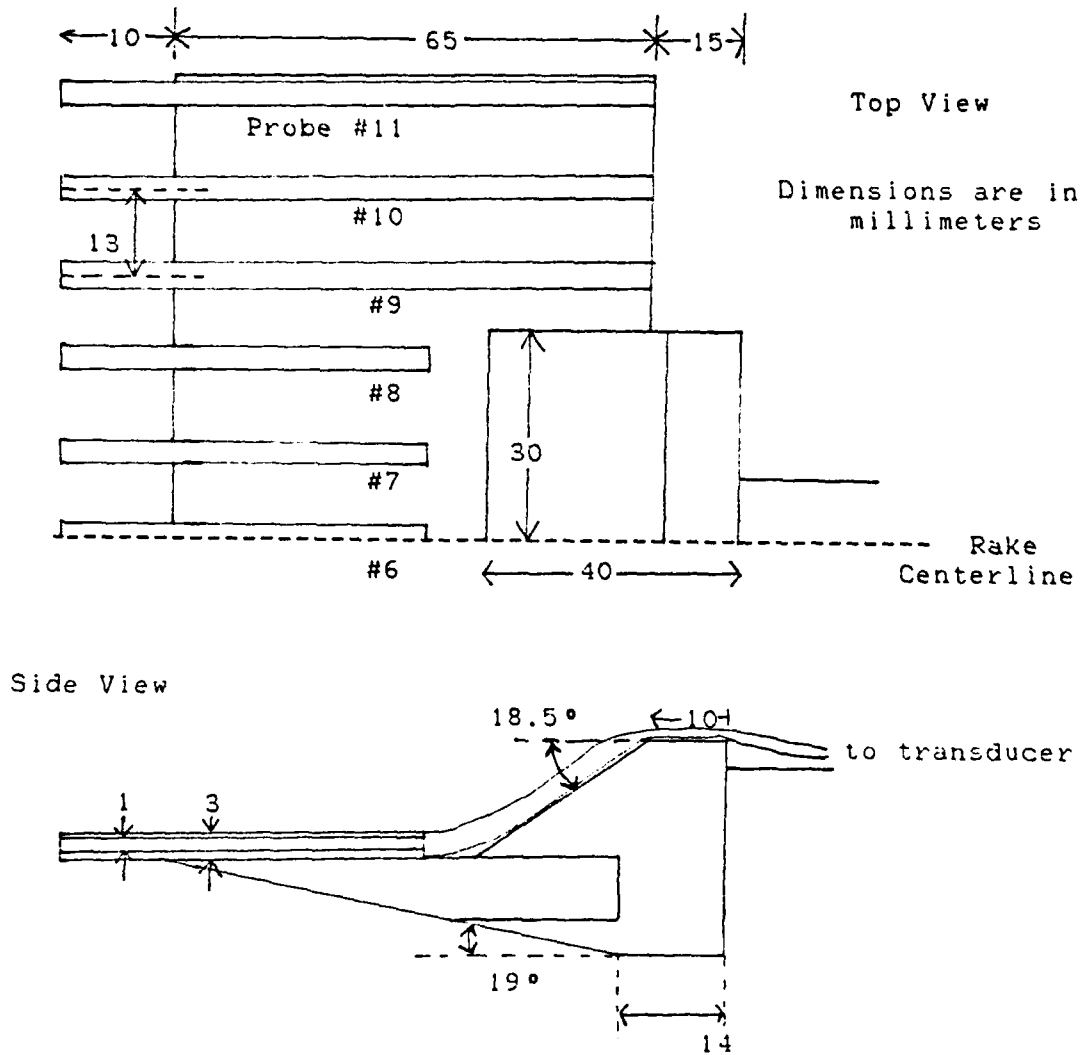


Figure E.1. Pitot Pressure Rake

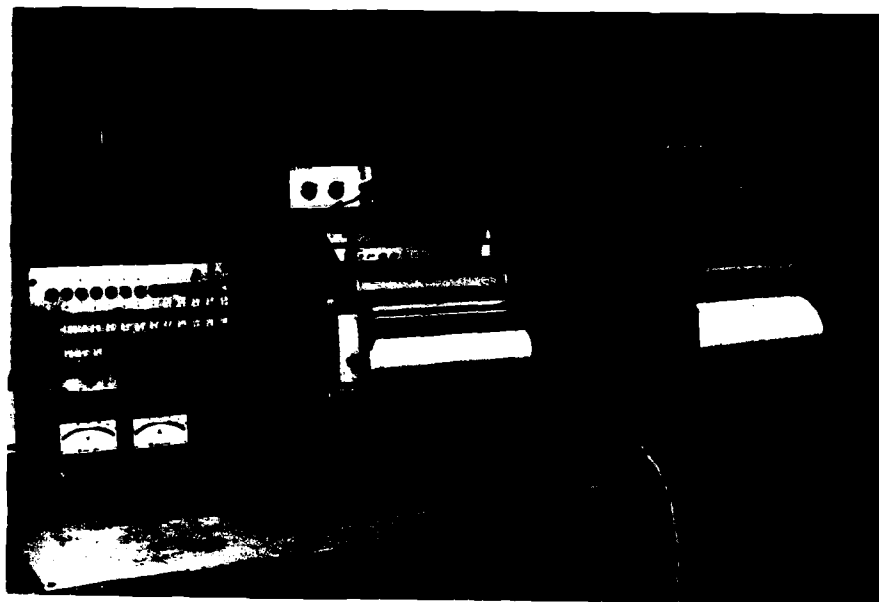


Figure E.2. Data Acquisition System for Calibration



Figure E.3. Pitot Rake Installed



Figure E.4 Pitot Rake in Injected Position

metal piece was constructed and attached to the rake support to replace the cut portion of the diffuser during the test. After injection, the additional piece fits closely to the diffuser.

The reservoir pressure transducer was calibrated against a manometric balance equipped with precision weights. During tests, the transducer was referenced against atmospheric pressure. The pitot rake transducer was calibrated against a vacuum pump and a mercury manometer. During tests, it also was referenced against atmospheric pressure. The atmospheric pressure was read from a mercury manometer.

The orientation of the injected rake is 0° when the rake is vertical and probe number 1 is on the top. The injected rake is at 90° when it is horizontal and probe number 1 is at the left most position when looking forward toward the reservoir. The other angles, 30° , 60° , 120° , and 150° are defined accordingly.

Results

Data was taken in the streamwise direction up to 143.5 mm from the nozzle exit. The previous studies (1; 14; 15; 53) at VKI had used models that were 296 mm in length and 110 mm in width, so it had originally been intended to calibrate the tunnel as far as 300 mm behind the nozzle exit. However, the tunnel would block after injection of the pressure rake at $x = 150$ mm. After several changes to the wind tunnel and model, the farthest distance the calibration tests were eventually run at was 143.5 mm behind the nozzle exit. At this distance, the data set could be fairly well completed at all the rake orientations.

Tunnel Blockage

The tunnel blockage phenomena is linked to the complexity of the flow which has to pass through the diffuser. The complex flow is composed of the wake of the model, the shock waves formed from the model, and also the distance from the nozzle exit and the diffuser nozzle entrance. It appeared that the tunnel blockage occurrence was primarily a function of the location of the diffuser,

I but still a function of rake orientation and reservoir stagnation pressure.

When the rake was 153.5 mm behind the nozzle exit, blockage occurred at $P_o = 10$ and 20 bars. At this time, the tests at the 90° rake orientation had already been completed. It was at this point that the pressure probes were shortened from 50 mm to 10 mm in length. That way the rake support could be moved closer toward the nozzle exit, and therefore the diffuser nozzle entrance could also be moved closer to the nozzle exit. The tunnel ran for perhaps a second after injection of the rake at 153.5 mm with $P_o = 20$ bars, and then blocked. The rake and diffuser nozzle were moved forward 10 mm and the tunnel ran successfully. However, at 143.5 mm, blockage still occurred when the rake was at 60° and 90° for $P_o = 10$ bars, so no data could be taken at those conditions. However, the tunnel ran for all the orientations at $P_o = 20$ bars at $x=143.5$ mm.

C Although blockage occurred with the rake at 90° , $x = 153.5$ mm and $P_o = 20$ bars, at the end of the calibration tests it was found that the tunnel did not block when the rake was at 150° and $P_o = 20$ bars. It was promising to see that some of the rake orientations were less disturbing to the flow and did not cause tunnel blockage. Unfortunately, time did not permit an extensive study on the tunnel blockage phenomena, so the exact cause of blockage could not be determined.

Error Analysis and Repeatability

The error analysis for the Mach number calculations was completed by examining the equation of total pressure ratio, equation (E.1). The natural logarithm was taken of the entire equation and then each term was partially differentiated with respect to P_{o1} , P_{o2} , or M_1 . The resulting equation is

$$\frac{dP_{o1}}{P_{o1}} - \frac{dP_{o2}}{P_{o2}} = A \frac{dM_1}{M_1} \quad (E.3)$$

where

$$A = \frac{4\gamma M_1^2 / [(\gamma-1)(\gamma+1)]}{[2\gamma M_1^2 / (\gamma+1) - (\gamma-1)/(\gamma+1)]} + \frac{\gamma M_1^2}{[1 + .5(\gamma-1)M_1^2]} + \frac{\gamma(\gamma+1)M_1^2 / (\gamma-1)}{[.5(\gamma+1)M_1^2]} \quad (E.4)$$

If $M_1 = 6.0$ and $\gamma = 1.4$, then $A = 4.17$. The error in Mach number is then

$$\frac{dM_1}{M_1} = \left[\left[\frac{1}{4.17} \frac{dP_{o1}}{P_{o1}} \right]^2 + \left[\frac{1}{4.17} \frac{dP_{o2}}{P_{o2}} \right]^2 \right]^{1/2} \quad (E.5)$$

The same type of analysis is completed on the equations used to calculate P_{o1} and P_{o2} from the graphisplot recorder data, accounting for errors of ± 0.2 mm in the atmospheric pressure readings. P_{o1} varied 1.3 division during a test

while P_{o2} could be read to within 0.3 division and varied 0.3 divisions during a test. This corresponds to errors of

$$dP_{ATM}/P_{ATM} = 0.000263$$

$$dP_{o1}/P_{o1} = 0.0158 \quad \text{for } P_{o1} = 10 \text{ bars}$$

$$dP_{o2}/P_{o2} = 0.0042 \quad \text{for } P_{o1} = 10 \text{ bars}$$

$$dM_1/M_1 = 0.0039 \quad \text{for } P_{o1} = 10 \text{ bars}$$

$$dP_{o1}/P_{o1} = 0.0073 \quad \text{for } P_{o1} = 20 \text{ bars}$$

$$dP_{o2}/P_{o2} = 0.0070 \quad \text{for } P_{o1} = 20 \text{ bars}$$

$$dM_1/M_1 = 0.0024 \quad \text{for } P_{o1} = 20 \text{ bars}$$

(E.6)

The errors in the calibration tests are listed in Table E.1.

Table E.1. Measurement Errors in Calibration

	dM	$d(P_{o2}/P_{o1})$
$P_{o1} = 10 \text{ bars}$	± 0.024	± 0.00035
$P_{o1} = 20 \text{ bars}$	± 0.015	± 0.00022

Several runs were repeated to verify that shortening the probe lengths had no effect on the data taken later. This is shown in Figures E.5 through E.7. Figure E.7 shows results for the probe shortening and also repetition of a test with the same probe length. All the tests are repeatable to within the measurement errors.

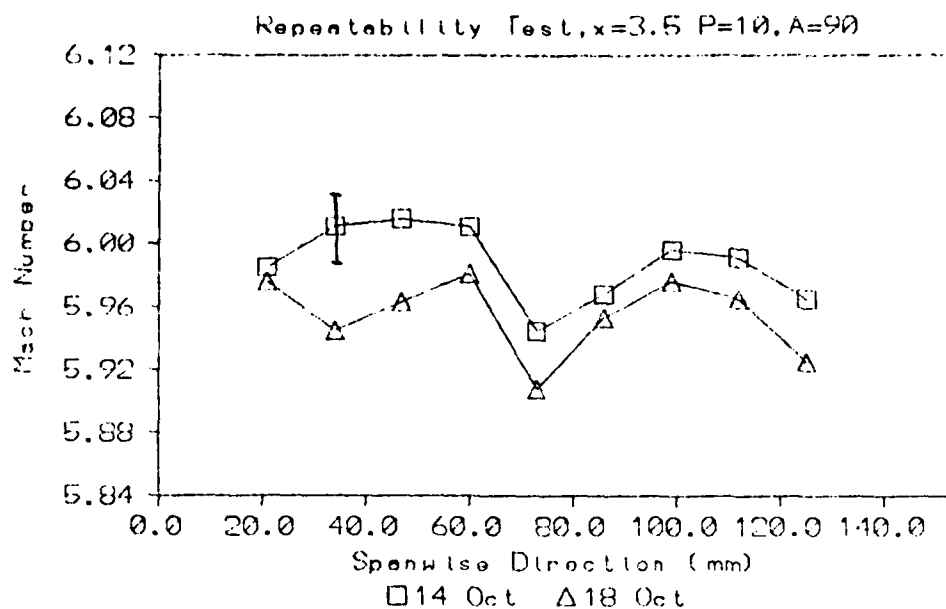


Figure E.5. Repeatability, $x = 3.5$ mm,
 $P_o = 10$ bar, $A = 90^\circ$

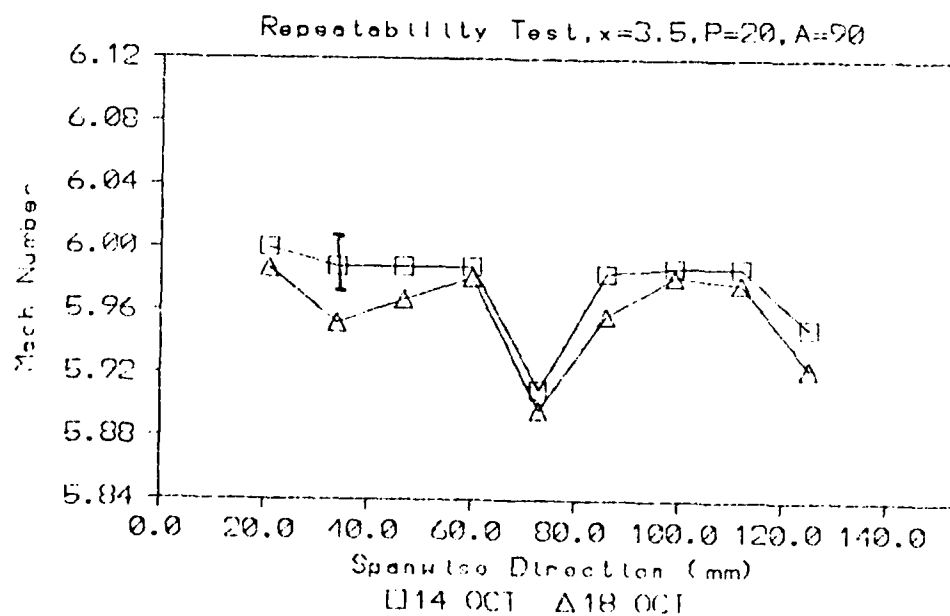


Figure E.6. Repeatability, $x = 3.5$ mm,
 $P_o = 20$ bar, $A = 90^\circ$

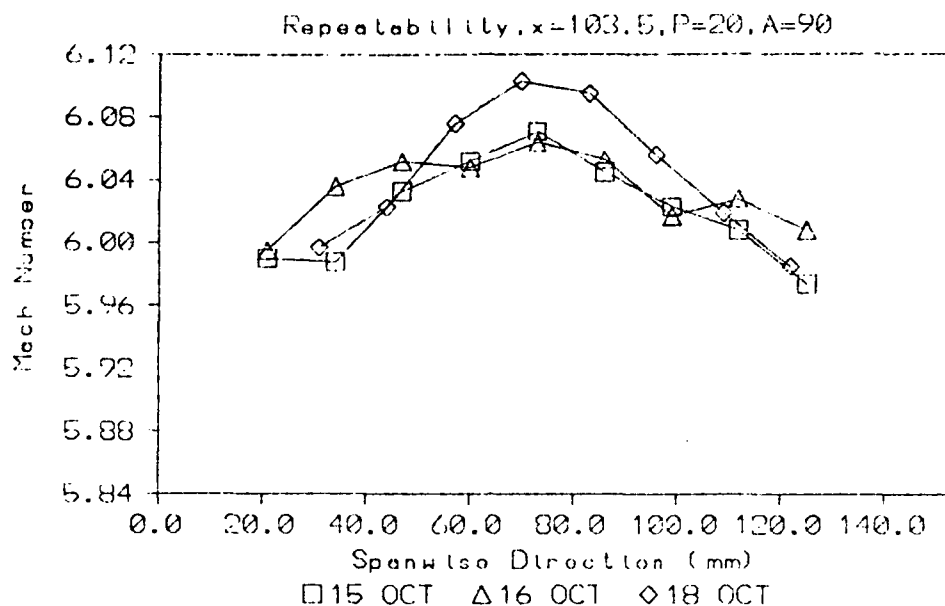


Figure E.7. Repeatability, $x = 103.5$ mm,
 $P_o = 20$ bar, $A = 90^\circ$

Streamwise Flow

The streamwise flow at a radial distance of 39 mm from the centerline is examined by plotting the data from probes 3 and 9 versus streamwise direction, x . The trend of the streamwise flow is shown in Figures E.8 and E.9. For each angle, there are two sets of data, for viewing purposes the number 3 probe was plotted slightly ahead of the probe 9 data, although the probes were actually at the same x location. The trend is a slow rise in Mach number as the distance from the nozzle increases from a mean Mach number of 5.97 to a mean Mach number of 6.00. This is not unexpected as the flow exiting out of the nozzle continues

to expand slightly in the open test section, thereby slightly increasing the flow velocity and Mach number.

The centerline streamwise flow is examined by plotting the data from probe number 6 versus x . It can be seen in Figures E.10 and E.11. The trend of a rise in Mach number is also seen as in the $r = 39$ mm case, but the rise in Mach number is more evident on the centerline. The rise is sudden between $x = 3.5$ mm and 53.5 mm, but then the Mach number increases very slowly, perhaps even becoming constant near Mach = 6.05 after $x = 143.5$ mm.

Cross-sectional Variation

The Mach number is fairly constant over the cross section of the test section with variations of approximately 2 % for the flow at $P_o = 10$ bar and 2.2% for the flow at $P_o = 20$ bar at the center of the flow. This can be seen from Figures E.12 through E.19. The centerline disturbance will be examined in a later section. The flow has been determined to be axisymmetric and after taking into account the measurement error, there are no significant trends due to the angular orientation of the rake.

Comparing the plots at $P_{o1} = 10$ bars, and $P_{o1} = 20$ bars reveals no significant difference between the plots when taking the measurement error into account. This means there are no Reynolds numbers effects for the uniform flow at Mach 6 in the range of Reynolds numbers that have been studied.

Shear Layer

The ratio P_{o2}/P_{o1} is plotted in Figures E.20 through E.23 to illustrate the growth of the shear layer when $P_{o1} = 20$ bars. The boundaries of the shear layer/uniform flow region are shown in Table E.2. The errors in these interpretations are approximately ± 2 mm.

Using the data from Table E.2, the region of uniform flow is sketched in Figure 3.2. The boundary beginning at the nozzle exit is interpolated to 143.5 mm behind the nozzle exit. After $x = 143.5$ mm, the region of uniform flow could be extrapolated to 300 mm behind the nozzle exit. However, the region of uniform flow resulting from the extrapolation appears unrealistic. It is difficult to believe that the shear layer would grow as fast as the extrapolation indicated. Therefore, this seems to indicate that there is not enough information available to draw a conclusion about the extent of the growth of the shear layer farther than the 143 mm after the nozzle exit.

Table E.2. Definition of Uniform Flow Regions

x location (mm)	left edge of shear layer (mm)	right edge of shear layer (mm)	diameter of uniform flow region (mm)
3.5	17 ± 2	134 ± 2	117 ± 2
53.5	16 ± 2	135 ± 2	118 ± 2
103.5	22 ± 2	133 ± 2	111 ± 2
143.5	27 ± 2	125 ± 2	98 ± 2

Centerline Disturbance

A centerline disturbance was evident in every test. This disturbance can be seen in Figures E.12 through E.19. The disturbance was, at worst, approximately 2.2% of the mean flow. The centerline disturbance only affected the center probe at the nozzle exit, but this disturbed region expanded to affect the three center probes (a width of at least 26 mm) at $x = 143.5$ mm from the nozzle exit. The centerline disturbance was negative at the exit and then became a positive disturbance at $x = 53.5$ mm. The centerline flow behaves as if it were a separate jet itself - increasing in Mach number as it expands inside the larger uniform jet of Mach 6.

Conclusions of Calibration

This study shows the presence of a slow growing shear layer and centerline disturbance. These results are similar to those found by an earlier study (29). The extent of the uniform flow has been obtained and the Mach number variations have been shown to be within 2.2% in this uniform flow region.

The flow can be considered to be a uniform axisymmetric jet of Mach 6. The uniform flow has been determined to be approximately 117 ± 2 mm in diameter at the nozzle exit and remains relatively constant until a distance of 103.5 mm downstream of the nozzle exit. The uniform flow then decreases to approximately 98 ± 2 mm in diameter at 143.5 mm

downstream of the nozzle exit. More information needs to be obtained farther downstream of the nozzle exit to determine the extent of the uniform flow. This would entail, not only conducting more tests, but designing a new pressure rake to avoid the occurrence of tunnel blockage.

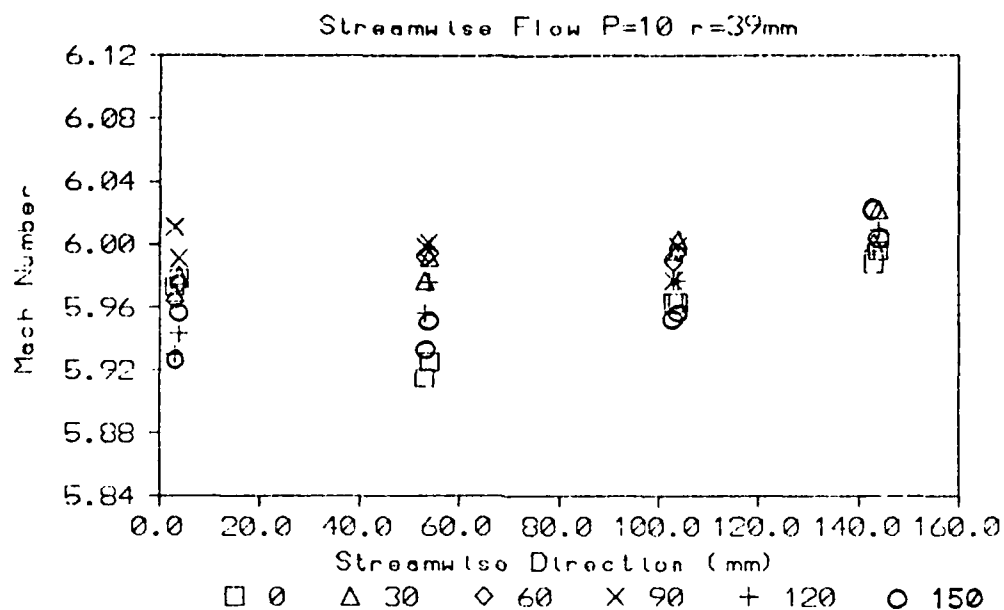


Figure E.8. Streamwise Flow, $r = 39$ mm, $P_o = 10$ bar

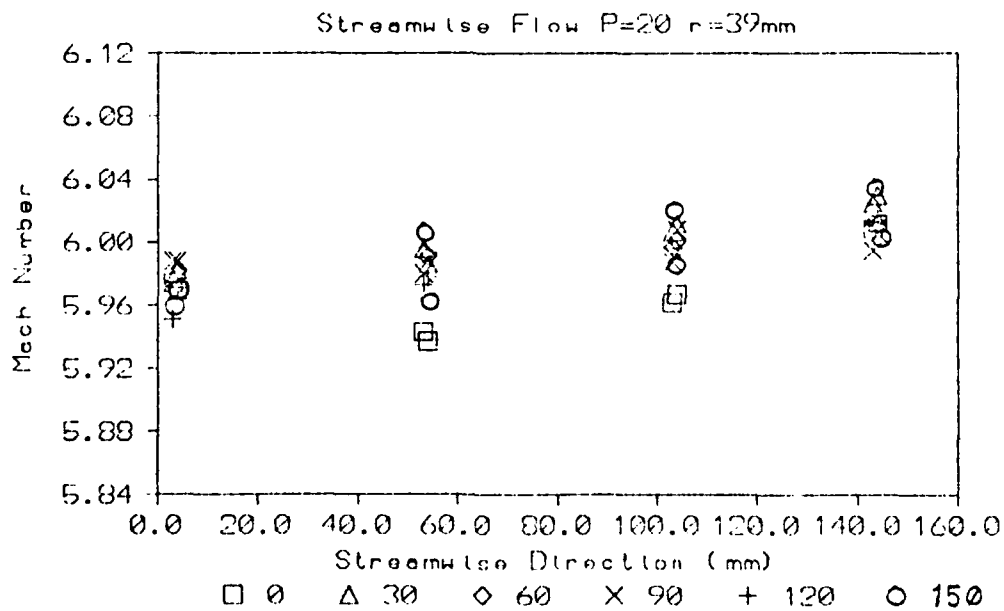


Figure E.9. Streamwise Flow, $r = 39$ mm, $P_o = 20$ bar

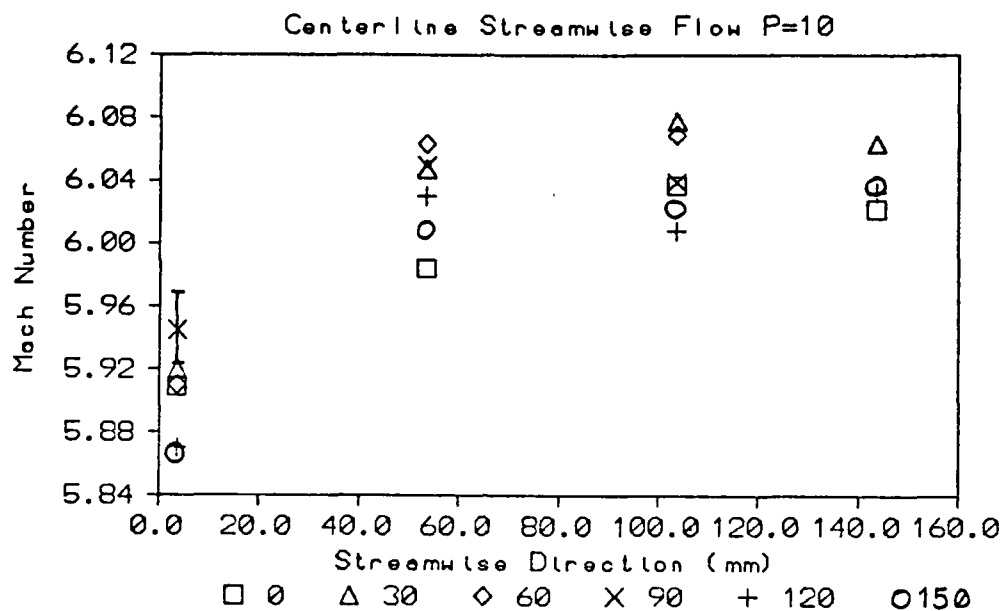


Figure E.10. Centerline Streamwise Flow, $P_o = 10$ bar

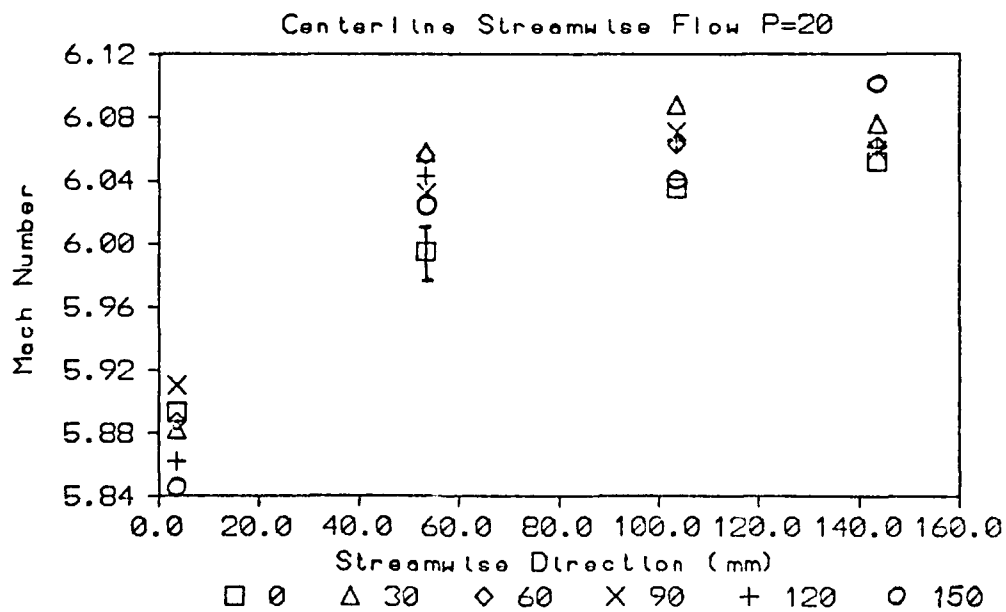


Figure E.11. Centerline Streamwise Flow, $P_o = 20$ bar

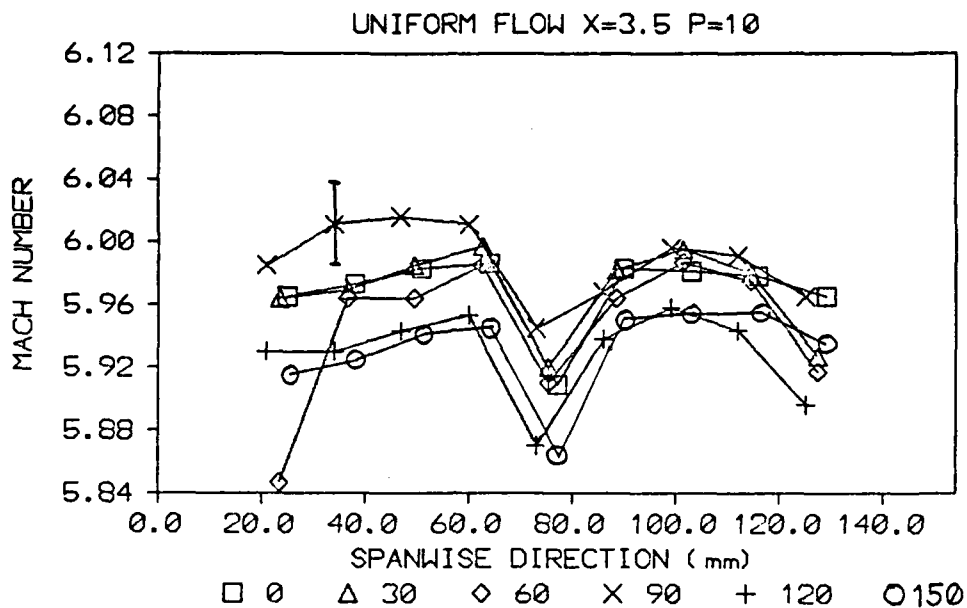


Figure E.12. Uniform Flow, $x = 3.5$ mm, $P_o = 10$ bar

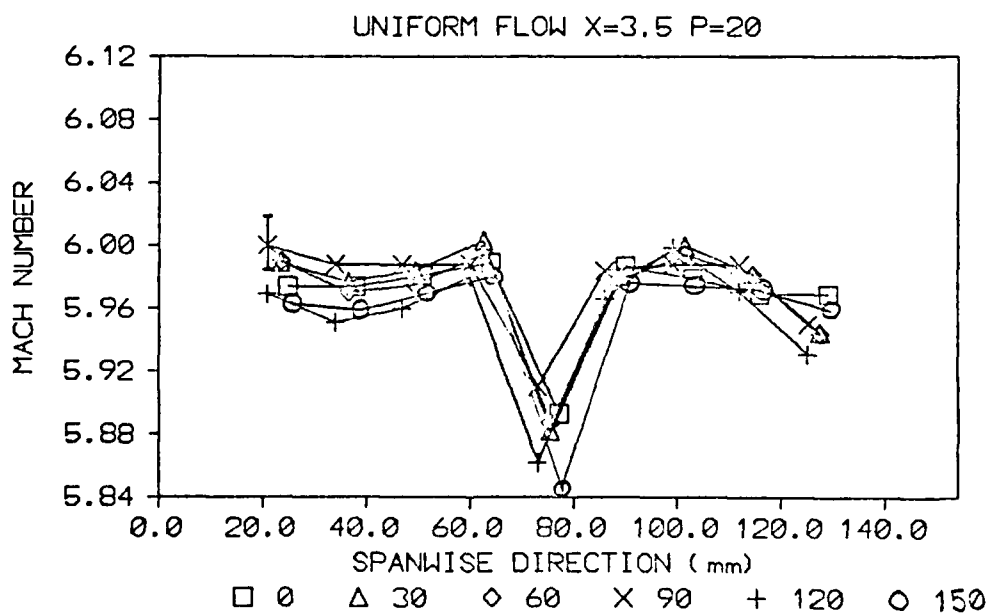


Figure E.13. Uniform Flow, $x = 3.5$ mm, $P_o = 20$ bar

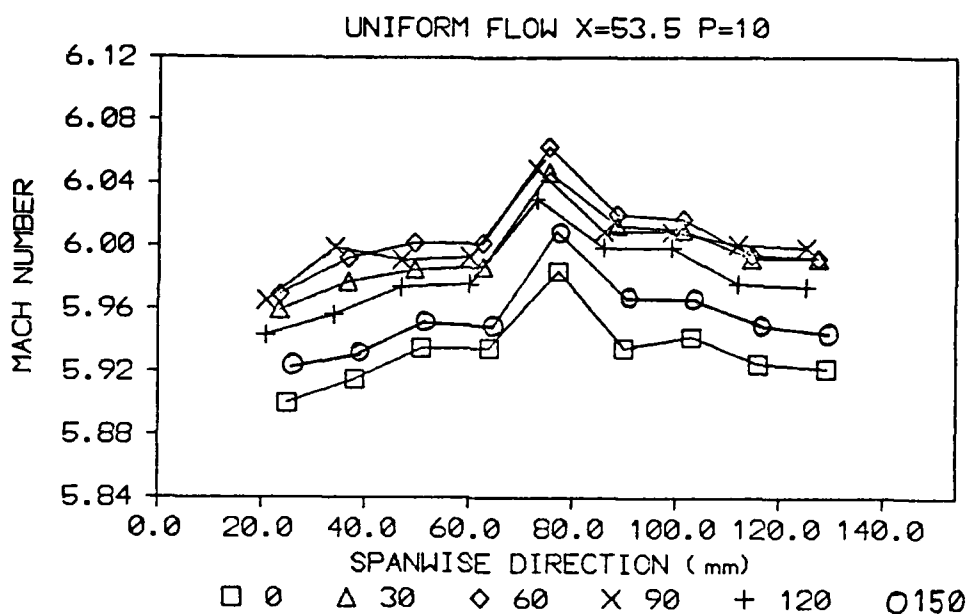


Figure E.14. Uniform Flow, $x = 53.5$ mm, $P_o = 10$ bar

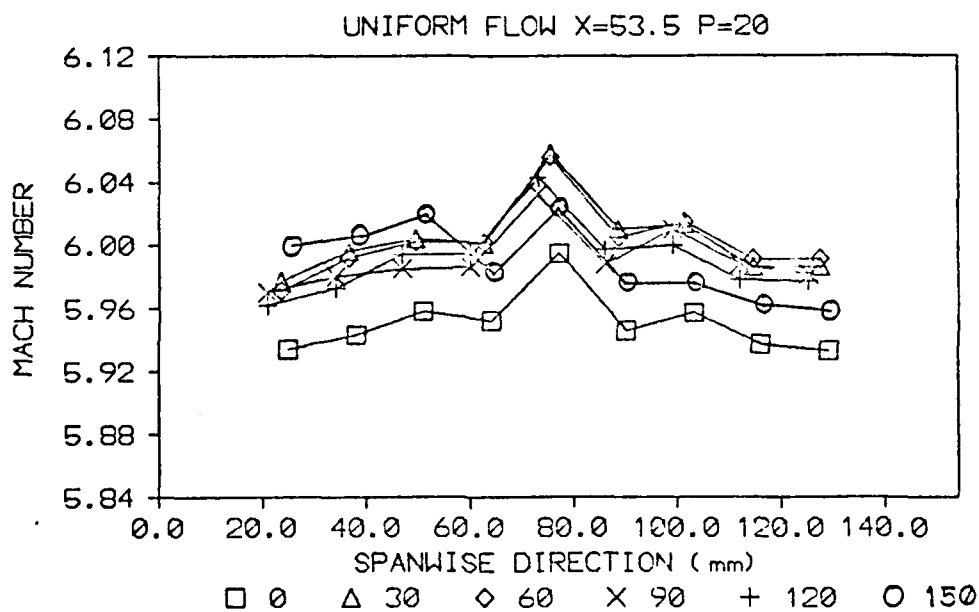


Figure E.15. Uniform Flow, $x = 53.5$ mm, $P_o = 20$ bar

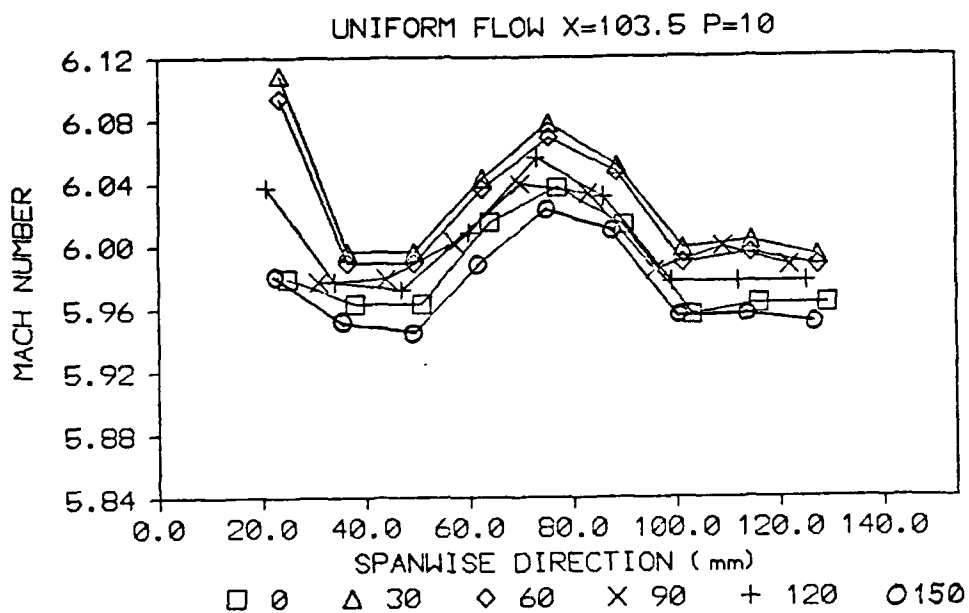


Figure E.16. Uniform Flow, $x = 103.5$ mm, $P_o = 10$ bar

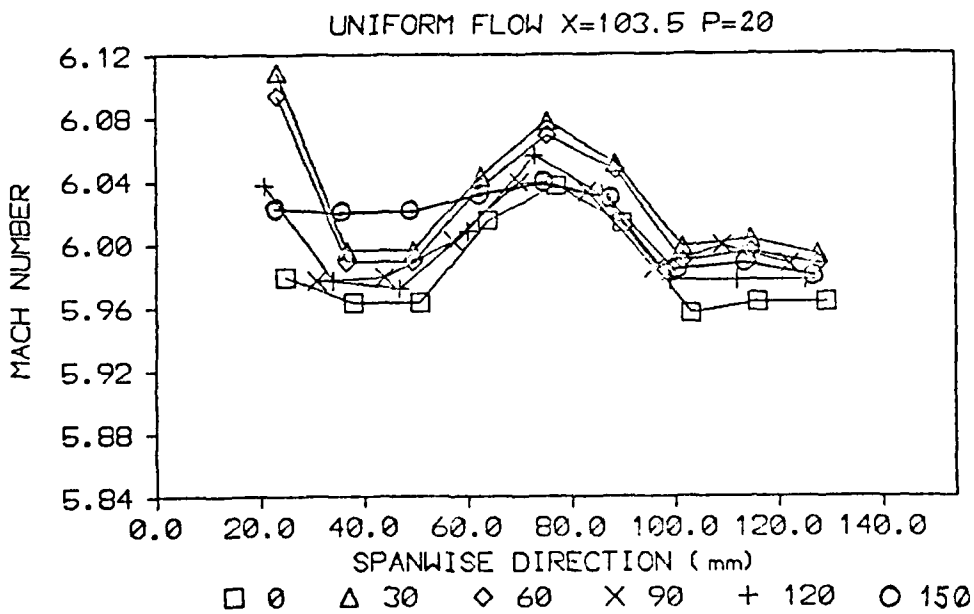


Figure E.17. Uniform Flow, $x = 103.5$ mm, $P_o = 20$ bar

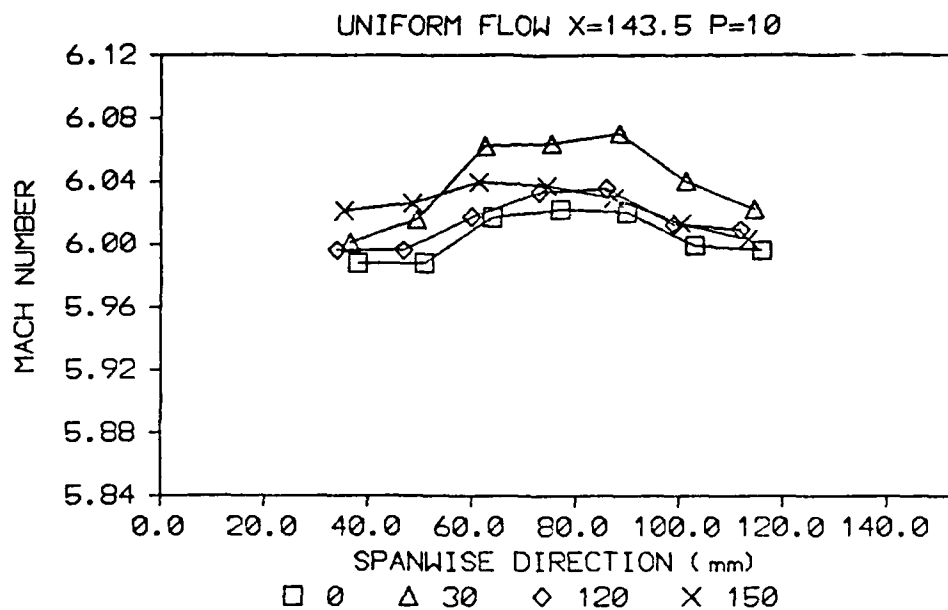


Figure E.18. Uniform Flow, $x = 143.5$ mm, $P_o = 10$ bar

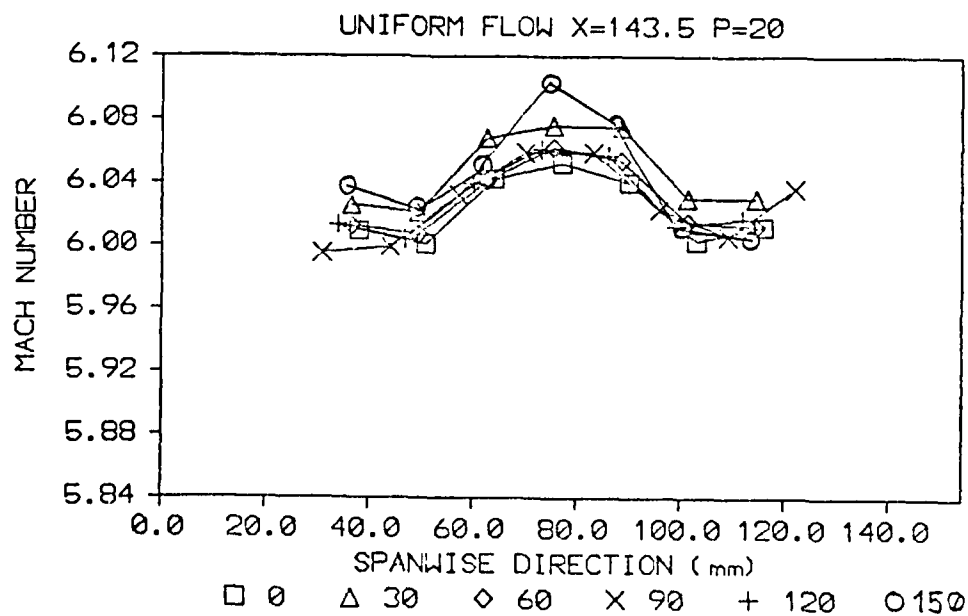


Figure E.19. Uniform Flow, $x = 143.5$ mm, $P_o = 20$ bar

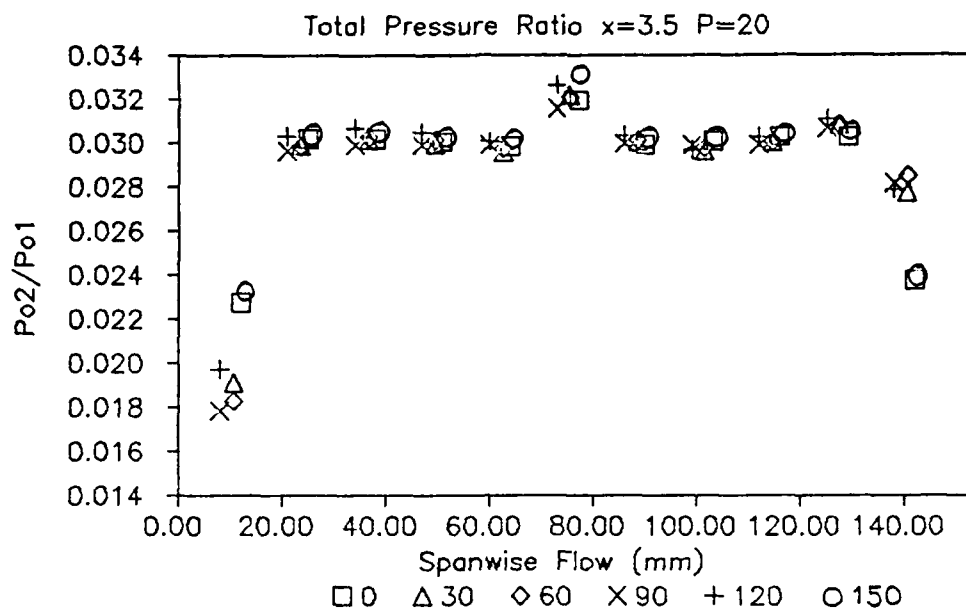


Figure E.20. Total Pressure Ratio,
 $x = 3.5$ mm, $P_o = 20$ bar

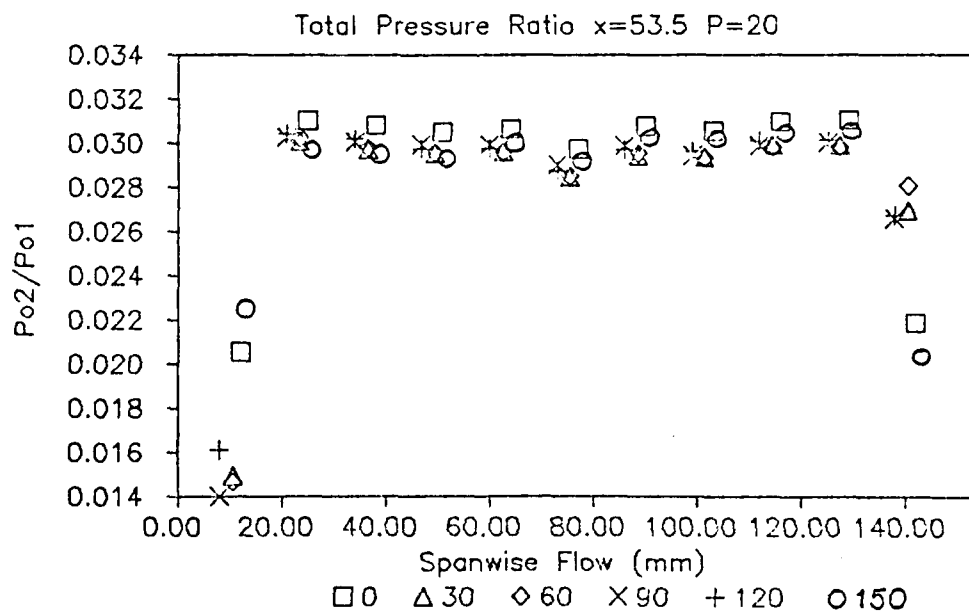


Figure E.21. Total Pressure Ratio,
 $x = 53.5$ mm, $P_o = 20$ bar

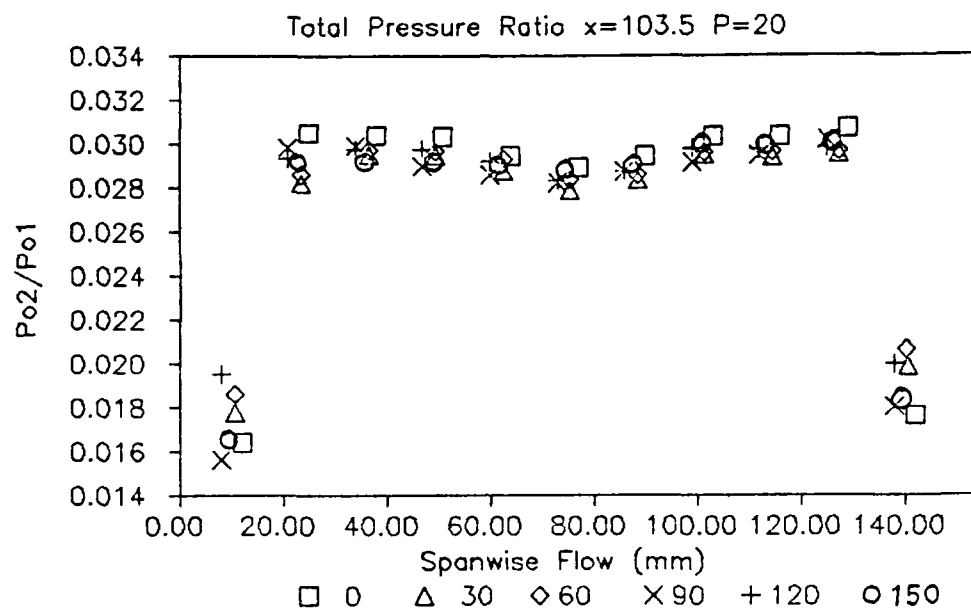


Figure E.22. Total Pressure Ratio,
 $x = 103.5$ mm, $P_o = 20$ bar

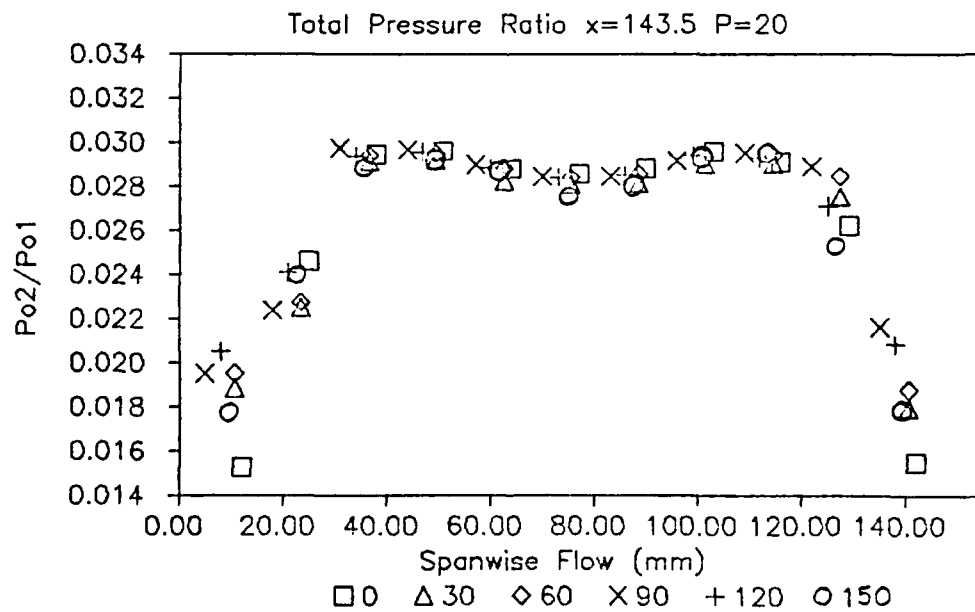


Figure E.23. Total Pressure Ratio,
 $x = 143.5$ mm, $P_o = 20$ bar

Appendix F

Preparation of the Liquid Crystal Model (19:25,26; 14:37,38)

The encapsulated liquid crystals are sold as a water-based slurry. After shaking the bottle vigorously, the slurry is then diluted by addition of distilled water (50 to 150% dilution, depending on the batch of liquid crystals). After thorough mixing (several hours) with a small propeller mixer, the slurry is filtered through a thin layer of cotton. If the liquid crystals were not diluted enough, most of the liquid crystals will be trapped in the cotton, and the filtered solution will be mostly composed of water.

The model used for the liquid crystal technique should have a low thermal conductivity so that the heat is not conducted away too rapidly. The surface of the material has to be painted in dull black. This is to prevent light reflections from the underlaying model surface which would overwhelm the intensity changes due to temperature change. This is especially critical if the DIP system is used for the color/temperature correlation. Before applying the coat of liquid crystals, the surface must be degreased with a substance that will not dissolve the paint on the model.

The liquid crystals are then sprayed with an air brush onto the model's surface in very thin layers. Each layer should be at the limit between droplets and a completely wet film. The first layer should be allowed to dry thoroughly

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in the surrounding air. It is the basis for all subsequent layers and has to provide the bonding to the model. The other layers can be dried quickly with a hair dryer. The hair dryer will also act as a heater, so the quality of the liquid crystal layer can be easily inspected. The most critical color is the red, and if it shows up properly, the model is ready.

A damaged coat of liquid crystals can be removed under running water by wiping with a soft cloth. The remaining particles of liquid crystals can be easily removed with a methanol soaked cloth. The gelatin of the microcapsules is destroyed by methanol.

Appendix G

Digital Image Processing

The use of the digital image processing (DIP) makes the liquid crystal technique more quantitative by taking the subjectiveness out of determining the temperature from a color picture. The system works directly from a video frame. It works with a black and white version of the frame and according to the gray intensity of a point (pixel), a "false color" is assigned to it. The VKI system distinguishes between 256 shades of gray from black (intensity 0) to white (intensity 255).

The critical link between the DIP and the liquid crystal technique is the calibration of the liquid crystals. The calibration gives the correlation between the "false color" (intensity) and the temperature.

The use of a video camera complicates this system somewhat. In order to photograph the model, a bright light must be used to illuminate the model. However, the variations in the intensity of this bright light overwhelms the intensity variations due to temperature changes. A solution to this problem is to film the model (with no temperature changes) when illuminated by the bright light. This picture represents the intensity variations due only to the lighting. The intensities of this "reference" frame are then subtracted appropriately from the calibration and test video frames.

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The correlation between temperature and intensity, after correcting for the reference frame intensity, can now be used, but with a precaution. The relation of intensity to temperature is multi-valued, i.e., there are two different colors for one intensity. Hence, the analysis requires a simultaneous inspection of the video image and the intensity curve of the corrected picture. Once the surface temperatures are obtained, the heat transfer rate and Stanton number can be determined.

Several computer programs are used in the processing of the liquid crystal video frames and are located in the VKI PDP computer under the account name CREUSOT. Some explanations concerning the computer programs are located in several references (14:12-14; 12:35-39).

Appendix H

Electrical Analog Circuits

The surface heat flux is deduced from temperature versus time signals using electrical analog circuits. Each of these circuits consists of a series of resistances and capacitances in which voltages and currents behave in ways similar to temperatures and heat fluxes, respectively.

The following is a comparison of the thermal and electrical equations, and is extracted from Schultz and Jones (48:37,38). Further behavior of the analog circuit is described and is extracted from Ligrani, Camci, and Grady (37:16-20).

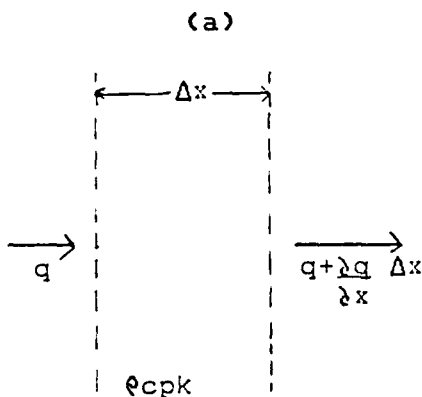


Figure H.1a.
Thermal Circuit

Rate of gain of energy in
element Δx

$$= - \frac{\lambda q}{\Delta x} \Delta x$$

which by conservation of
energy

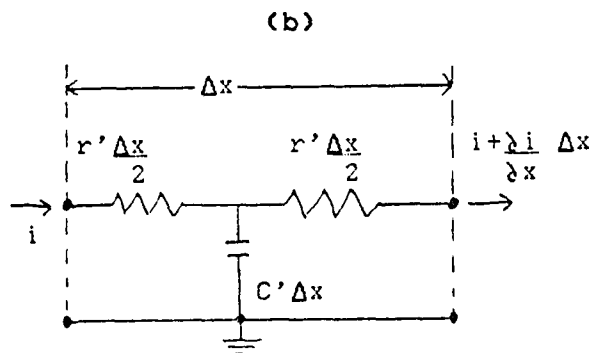


Figure H.1b.
Electrical Circuit

Rate of gain of charge in
element Δx

$$= - \frac{\lambda i}{\Delta x} \Delta x \quad (H.1)$$

which by conservation of
charge

$$= \rho_{cp} \Delta x \frac{\partial T}{\partial t} = C' \Delta x \frac{\partial V}{\partial t} \quad (H.2)$$

Thus

$$\frac{\partial q}{\partial x} = - \rho_{cp} \frac{\partial T}{\partial t}$$

Thus

$$\frac{\partial i}{\partial x} = - C' \frac{\partial V}{\partial t} \quad (H.3)$$

The conduction equation is:

$$q = - k \frac{\partial T}{\partial x}$$

Ohm's law is:

$$i = - \frac{1}{r' \Delta x} \frac{\Delta V}{\Delta x} \Delta x = - \frac{1}{r'} \frac{\partial V}{\partial x} \quad (H.4)$$

Combining the above equations and the diffusion equation is obtained

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho_{cp}}{k} \frac{\partial T}{\partial t}$$

Combining the above equations and the transmission line equation is obtained

$$\frac{\partial^2 V}{\partial x^2} = r' C' \frac{\partial V}{\partial t} \quad (H.5)$$

It is evident from equations (H.5a) and (H.5b), that the heat diffusion equation is analogous to the electrical transmission line equation. Taking the Laplace transform of equation (H.5b),

$$\frac{\partial^2 V^*}{\partial x^2} = r' C' [s V^* + V(t=\infty) - V(t=0)] \quad (H.6)$$

where $V(t=\infty) = 0$.

The initial condition is

$$V^* = 0 \quad \text{at } t = 0 \quad (H.7)$$

and the boundary conditions are

$$V^* = 0 \quad \text{at } x = \infty \quad (H.8)$$

$$- \frac{1}{r'} \frac{\partial V^*}{\partial x} = i^*_{\text{input}} \quad \text{at } x = 0 \quad (H.9)$$

Applying the initial condition to equation (H.6) and solving the differential equation obtained in V^* and x gives:

$$V^* = A \exp [x (r' C' s)^{1/2}] + B \exp [-x (r' C' s)^{1/2}] \quad (H.10)$$

where A and B are constants. The boundary condition (H.8) gives A=0, and after using the boundary condition (H.9),

$$\frac{\partial V^*}{\partial x} = -(r' C' s)^{1/2} B \exp [-x (r' C' s)^{1/2}] = - i^*_{\text{input}} r' \quad (H.11)$$

or alternatively,

$$B = i^*_{\text{input}} [r' / (C' s)]^{1/2} \quad (H.12)$$

So when $x = 0$, using equations (H.10) and (H.12), the voltage ΔV at the input of the analog circuit, shown in Figure H.2, is given by

$$\Delta V^* = i^*_{\text{input}} [r' / (C' s)]^{1/2} \quad (H.13)$$

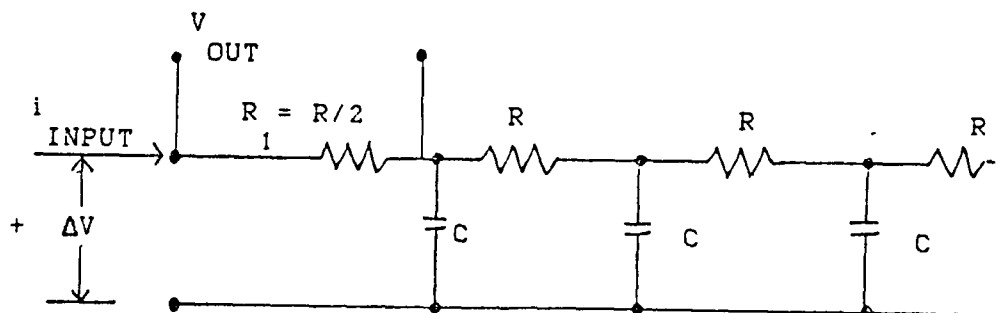


Figure H.2 Electrical Analog Using Equal Sections Representing a Homogeneous Heat Conductor

Using equation (3.26) which is repeated here

$$\frac{\Delta V}{\alpha \frac{V_{REF}}{R}} = T_s \quad (H.14)$$

the circuit input is now the thin film output,

$$\Delta V_s = \alpha \frac{V_{REF}}{R} T_{s*} \quad (H.15)$$

Substituting equation (H.15) into (H.13) and rearranging,

$$i_{*input} = (sC'/r')^{1/2} \alpha \frac{V_{REF}}{R} T_{s*} \quad (H.16)$$

Now the heat conduction equation (15:57-60) analogous to equation (H.13) is

$$q_s = (\rho c p k s)^{1/2} T_{s*} \quad (H.17)$$

So substituting into equation (H.17) for T_{s*} using equation (H.16) gives

$$q_s = (\rho c p k)^{1/2} (r'/C')^{1/2} (\alpha \frac{V_{REF}}{R})^{-1} i_{*input} \quad (H.18)$$

For the analog termination shown in Figure H.2, equation (H.18) after inversion then becomes

$$q_s = (\rho c p k)^{1/2} (r'/C')^{1/2} (\alpha \frac{V_{REF}}{R})^{-1} \frac{V_{OUT}}{R} \quad (H.19)$$

For the substrate heat transfer behavior, the values of q_s obtained from equation (3.32), which is repeated here

$$q_s = \frac{1}{2} (\rho c p k \pi / t)^{1/2} \frac{\Delta V}{\alpha \frac{V_{REF}}{R}} \quad (H.20)$$

and equation (H.18) must be the same provided that q_s is a constant for $t > 0$ and that $T_a = T_{REF}$. Thus,

$$q_s = (\rho c p k \pi / t)^{1/2} \frac{\Delta V}{2 \alpha V_{REF} R} = (\rho c p k)^{1/2} (r' / C')^{1/2} \frac{V_{OUT}}{\alpha V_{REF} R} \quad (H.21)$$

If a calibration of circuits is then made with a step in heat flux, a calibration coefficient for the analog time may then be defined using

$$\frac{A^*}{\beta} = (r' / C')^{1/2} (1/R)^{1/2} = \frac{\Delta V}{2 V_{OUT}} (\pi / t)^{1/2} \quad (H.22)$$

This single time dependent calibration coefficient thus defines the characteristics of the analog circuit in relation to heat flow. Heat fluxes for gages operating when $T_a = T_{REF}$ are obtained by substituting ΔV from equation (H.22) into equation (H.21) to give the equation

$$q_s = \frac{V_{OUT}}{V_{REF}} \frac{(\rho c p k)^{1/2}}{\alpha} \frac{A^*}{\beta} \quad (H.23)$$

which is then used for heat transfer data reduction with the temperature correction on α , if required.

Circuit Considerations

The actual values of r' and C' in equation (H.18) required in an analog circuit are dependent on many factors. Any arrangement of resistive and capacitive components represents semi-infinite heat transfer behavior provided that the value of $r'C'$ (which is analogous to $\rho c p k$) is

constant with time. Two performance criteria by which circuits are judged are

- a) response time - the time required for the analog output to fully account for an alteration in heat transfer rate, ie., voltage input signal, and
- b) operation time - the time after the first input to the analog that the output signal represents a particular heat transfer profile to a stated accuracy.

The aim in analog circuit design is to minimize response time and maximize operation time.

The T-section analog circuit, shown in Figure H.2, has a short response time. For a circuit with all stages of identical component values, this response time is RC . The operation time for which this circuit reproduces a step input in heat transfer to within 1.4% is given by

$$0.2n^2(RC) \quad (H.24)$$

where n is the number of stages. When long operating times are required, arithmetically increasing component values are used, as shown in Figure H.3. For the purpose of calculating operating times from equation (H.24), h T-section analog stages of arithmetically increasing values of capacitance and resistance are equivalent to

$$h(h+1)/2 \quad (H.25)$$

identical stages. When using arithmetically increasing components, the first two stages must be identical if the

frequency response of the circuit is to be acceptable. Figure H.3 shows the analog circuit used in this study, with 24 stages, $R = 4099$ ohms, $C = 20 \times 10^{-9}$ farads, an operating time of 1.4 seconds and a response time of 8.2×10^{-5} seconds.

Appendix I

Calibration of Thin Film Gages

The equation used for heat flux measurements is equation (3.36), but the equation will be repeated here for completeness.

$$q_s = \frac{VOUT}{VREF} \frac{(\epsilon_{cpk})^{1/2}}{\alpha_R} \frac{A^*}{\beta} [1 + \alpha (T_a - T_{REF})] \quad (I.1)$$

The determination of different parts of equation (I.1); $VREF$, α_R , $(\epsilon_{cpk})^{1/2}$, and A^*/β , will be discussed in detail. The $[1 + \alpha (T_a - T_{REF})]$ correction term has been previously discussed. These details are extracted from Ligrani, Camci, and Grady (37:24-33).

Output Voltage: VOUT

The output voltage from the analog circuit, VOUT, is different from the other terms in equation (I.1) because it is dependent on the heat flux to a particular gage during specific flow conditions. The other terms in equation (I.1) are determined from sensor or measurement chain calibration characteristics.

Temperature Coefficient of Resistance : α_R

The temperature coefficient of resistance, α_R , of a thin film gage depends on a variety of factors including the film material, the size of the film metal deposit, and its shape. Therefore each gage film must be calibrated

independently since it is impossible to make each film exactly the same.

The temperature coefficient of resistance is defined by

$$R = R_{REF} [1 + \alpha (T - T_{REF})] \quad (I.2)$$

This can be rewritten

$$\frac{R - R_{REF}}{T - T_{REF}} = \alpha \quad (I.3)$$

where T_{REF} is some reference temperature. The magnitude of $(R - R_{REF}) / (T - T_{REF})$ is found by plotting resistance versus temperature for a given gage and then calculating the slope. This can also be done mathematically by using a least squares algorithm to find the slope of the line between all the data points. The program used for this calculation is listed in Appendix J. R_{REF} is determined by measuring the resistance of the gage and its connecting wires just before input to an analog circuit bridge.

The calibration may be performed in an oil bath. A multimeter is used to measure the film resistances at a range of constant oil bath temperatures. The range of temperatures to calibrate against is determined by the range of temperatures that the gages will be used. The value of α may be obtained from the experimental readings using a least squares program.

Experience has shown that maintaining constant oil bath temperature during readings is very critical. This is easiest to accomplish when the oil bath is being cooled

between readings. If constant bath temperatures are maintained during readings, then the accuracy of the calibration is governed by the accuracies of the multimeter and thermometer.

1/2

Substrate Thermal Product: (θ_{cpk})

1/2

The substrate thermal product is (θ_{cpk}). For quartz substrates, the value of the thermal product is well known between ambient and 100°C. However, the thermal product for machinable ceramic substrates are not well defined. The value of the thermal product depends on the history of the gage construction, in particular the high temperature baking details of the ceramic production process. Therefore, when the gages are completed, each ceramic piece must be calibrated.

The thermal product of ceramic will vary with test temperature, so calibration must be completed under the same conditions for which the gages will be used. An arrangement for calibration would consist of a quartz gage and a sample ceramic gage on a flat test plate in the VKI CT-2 tunnel at the same distance downstream of the leading edge. The sample ceramic gage must have been made at the same time as the actual test gage so as to have undergone the same production process. The heat flow is then calculated from the quartz gage readings. The value of q_s obtained is substituted into equation (I.1) to give

$$(\rho_{cpk})_{\text{ceramic}}^{1/2} = q \frac{V_{REF}}{V_{OUT}} \alpha \frac{1}{R A^*/\beta} \frac{1}{[1 + \alpha (T_a - T_{REF})] R} \quad (I.4)$$

Calibration Constant for the Electrical Analog: A^*/β

Once the electrical analog circuits are set up to process the signals from thin film gages they are ready to be calibrated. The purpose of the analog circuit calibration is to obtain A^*/β in equation (I.1). Recall that the calibration constant is defined from Appendix H as

$$\frac{A^*}{\beta} = \frac{\Delta V}{2V_{OUT}} (\pi/t)^{1/2} \quad (I.5)$$

If a parabolic voltage signal is inserted into the input of the analog circuit, then the output is a step in voltage, V_{OUT} . In actual practice, V_{OUT} is measured in digital data units in order to calibrate both the analog circuit as well as the other parts of the measurement chain before the computer: amplifier, filter, multiplexer, analog/digital converter. Therefore, equation (I.5) becomes

$$\frac{A^*}{\beta} = \frac{\Delta V}{2N_{OUT}} (\pi/t)^{1/2} \quad (I.6)$$

and during an actual measurement, equation (I.1) is changed to

$$q_s = \frac{N_{OUT} (\rho_{cpk})}{V_{REF} \alpha} \frac{A^*}{\beta} \frac{1}{R} [1 + \alpha (T_a - T_{REF})] \quad (I.7)$$

where N_{OUT} is V_{OUT} measured in digital data units.

Appendix J

Coefficient of Resistance Program

```
100 'This program calculates the thin film gage coefficient
110 ' of resistance using a least squares method
120 '
130 DIM X(7),Y(30,7),T(30,7),T(7)
140 LPRINT"no 30.5 30.0 41.0 49.0 62.65 69.65 80 81 82 83"
150 LPRINT
160 LPRINT
170 'Set temperatures
180 X(1)=30.5
190 X(2)=30.0
200 X(3)=41.0
210 X(4)=49.0
220 X(5)=62.65
230 X(6)=69.65
240 'Set the number of temperature/resistance data points
250 TMAX=6
260 FOR I=1 TO TMAX
270 T(I)=X(I)-X(1)
280 NEXT I
290 PRINT "how many thin film gages are there?"
300 INPUT N
310 'Input data points
320 FOR G=1 TO N
330 PRINT "type in data"
340 FOR I=1 TO TMAX
350 PRINT "resistance data for gage #, at temp # ",G,I
360 INPUT Y(G,I)
370 NEXT I
380 'The data will be displayed to check for correctness
390 PRINT "the data you input is:"
400 FOR I=1 TO TMAX
410 PRINT I,X(I),Y(G,I)
420 NEXT I
430 PRINT "is the data correct?"
440 PRINT "type 1 if yes, 2 if no"
450 INPUT ANG
460 IF ANG = 1 THEN GOTO 560
470 'If one of the data points is wrong, input the correction
480 PRINT "type in the data number that is wrong"
490 INPUT M
500 PRINT "type in the correct resistance data"
```

```

510 INPUT Y(J,N)
520 GOTO 590
530 'Begin the computation of the thin film coefficient
540 ' of resistance, using a least squares
550 ' algorithm
560 SX=0
570 SY=0
580 SX2=0
590 SY2=0
600 SXY=0
610 FOR I=1 TO TMAX
620 R(J,I)=Y(J,I)-Y(J,1)
630 NEXT I
640 FOR I=1 TO TMAX
650 SX=SX+R(J,I)
660 SY=SY+R(J,I)
670 SX2=SX2+R(J,I)*R(J,I)
680 SY2=SY2+R(J,I)*R(J,I)
690 SXY=SXY+R(J,I)*R(J,I)
700 NEXT I
710 DEN=SX2-SX*SX/TMAX
720 A1=(SXY-SX*SY/TMAX)/DEN
730 AC=(SY-A1*SX)/TMAX
740 F=A1-A1*(SX2-SX*SX/TMAX)/(SY2-SY*SY/TMAX)
750 AP=A1/Y(J,1)
760 PRINT J;AC;A1;F;AP
770 NEXT J

```

Appendix K

Read Data Program

```
PROGRAM READAVE
C  READING OF DATA FROM DATA ACQUISITION SYSTEM
C
    LOGICAL*1 NAME4(15),YES
    DIMENSION LXX(768),LXXX(256)
    COMMON IVEC(12,1780),x(50),y(50),time(1780),IV(12,500),T(500)
1   ,V(12,500)
    CHARACTER*1 ANSW
    CHARACTER*20 C3
C
    ISTEP=12
    NPTS=1780
    IV4=1
    TYPE 98765
98765 FORMAT(/,' DATA FILE NAME : ', $)
    CALL GETSTR (5,NAME4,14)
    CALL ASSIGN(4,NAME4)
    DEFINE FILE 4(103,256,U,IV4)
110  FORMAT(A20)
55   FORMAT(/,' WHAT IS THE OUTPUT FILE NAME? ', $)
    TYPE 55
    ACCEPT 110,C3
    OPEN (UNIT=10,NAME=C3,TYPE='NEW')
C
C READ THE DATA FROM THE DIRECT ACCESS FILE
C
    IV4=2
    IJ=0
    DO 900 IXY=1,28
        J=0
        DO 901 I=1,3
            READ(4'IV4)LXXX
            DO 901 II=1,256
                J=J+1
                LXX(J)=LXXX(II)
901    CONTINUE
        DO 902 IJJ=1,64
            IJ=IJ+1
            DO 903 K=1,12
                JJ=K+(IJJ-1)*ISTEP
                IVEC(K,IJ)=LXX(JJ)
                time(ij)=ij
903    CONTINUE
        IF (IJ .EQ. 1780) GO TO 909
902    CONTINUE
900    CONTINUE
909    CONTINUE
```

```

C
C SAVE ONLY EVERY FOURTH POINT IN ANOTHER FILE
C
      do 400 n=1,12
        J=3
        K=0
        do 410 i=1,npts
          J=J+1
          IF (J .EQ. 4) THEN
            K=K+1
            T(K)=time(i)
            V(N,K)=ivec(n,i)
            J=0
          ENDIF
        410   continue
      400   continue
C
C OPTION TO SMOOTH THE DATA
C
      130   FORMAT(I3)
      120   FORMAT(I1)
      NPTS=445
      5     FORMAT(/,' DO YOU WANT TO smooth THE DATA? YES=1,NO=2 ', $)
      TYPE 5
      ACCEPT 120, IANS
      IF (IANS .EQ. 2) GO TO 999
      type 200
      format(/,' enter number of points to fit: ', $)
      accept *, IFITT
      is=ifitt/2 + 1
      iend=npts-is+1
      DO 201 N=1,12
        do 210 i=is,iend
          SUM=0.0
          do 220 j=1,ifitt
            k=i-is+j
            SUM=SUM+V(N,K)
          220   continue
          V(N,I)=SUM/IFITT
        210   continue
      201   CONTINUE
C
999     CONTINUE
C
C WRITE TO DATA FILE TO REDUCE
C
      DO 600 N=1,12
        DO 500 I=1,NPTS
          WRITE(10,*) T(I),V(N,I)
        500   CONTINUE
      600   CONTINUE
      stop
      end

```

```

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      subroutine lstsq(yfit,ifit)
      COMMON IVEC(12,1780),x(50),y(50),time(1780),IV(12,500),T(500)
      mdpt=ifit/2+1
      a=0.0
      b=0.0
      c=0.0
      d=0.0
      e=0.0
      f=0.0
      g=0.0
      do 300 i=1,ifit
        a=a+x(i)
        b=b+x(i)**2
        c=c+x(i)**3
        d=d+x(i)**4
        e=e+y(i)
        f=f+(x(i)*y(i))
        g=g+(x(i)*x(i)*y(i))
300    continue
      a1=(a*e-ifit*f)/(a*a-ifit*b)
      a0=(e-a1*a)/ifit
      yfit=a0+a1*x(mdpt)
      return
      END

```

Appendix L

Input Data Files

```

12  2  10  445  1
1.0  1.0  1.0  1.0
0.0  0.0  291.9  1950.0  6.0  10.0  293.65
0.0  0.00234  0.0058  0.1111  47.52
0.0  0.00221  0.0211  0.1202  54.41
0.0  0.00230  0.0175  0.1360  59.24
0.0  0.00219  0.0269  0.1255  57.30
0.0  0.00216  0.0213  0.1060  49.09
0.0  0.00206  0.0179  0.0957  46.39
0.0  0.00203  0.0299  0.1109  54.73
0.0  0.00210  0.0026  0.1208  57.41
0.0  0.00203  -.0053  0.1124  55.41
0.0  0.00205  -.0196  0.0889  40.05
9.35e-5  0.752  47.7  -273  -964
1.175e-4  0.751  54.8  -166  -1305
8.91e-5  0.752  59.3  -19  -1706
1.197e-4  0.750  57.7  -310  -2048
8.62e-5  0.754  49.6  117  -1131
1.049e-4  0.752  47.1  334  -967
2.117e-4  0.749  77.4  72  -1068
7.65e-5  0.751  60.2  -63  -2048
8.04e-5  0.752  56.7  -162  -1097
8.18e-5  0.748  41.5  365  -1632

```

```

12  2  10  445  1
1.0  1.0  1.0  1.0
0.0  0.0  291.9  1950.0  6.0  20.0  293.65
0.0  0.00234  0.0058  0.1111  47.52
0.0  0.00221  0.0211  0.1202  54.41
0.0  0.00230  0.0175  0.1360  59.24
0.0  0.00219  0.0269  0.1255  57.30
0.0  0.00216  0.0213  0.1060  49.09
0.0  0.00206  0.0179  0.0957  46.39
0.0  0.00203  0.0299  0.1109  54.73
0.0  0.00210  0.0026  0.1208  57.41
0.0  0.00203  -.0053  0.1124  55.41
0.0  0.00205  -.0196  0.0889  40.05
9.35e-5  0.752  47.8  -273  -1273
1.175e-4  0.752  54.8  -166  -994
8.91e-5  0.754  59.4  -19  -1169
1.097e-4  0.751  57.8  -310  -2048
8.62e-5  0.754  49.7  117  -1093
1.049e-4  0.753  47.1  334  -611
2.117e-4  0.750  77.4  72  -888
7.65e-5  0.752  60.3  -63  -1743
8.04e-5  0.752  56.8  -162  -566
8.18e-5  0.749  41.5  365  -1497

```

Appendix M

Data Conversion Program

```
PROGRAM READDAT
C THIS PROGRAM READS THE DATA OBTAINED FROM THE MEDIUM SPEED
C OR HIGH SPEED DATA ACQUISITION SYSTEM
C THEN IT CONVERTS THE DIGITAL DATA UNITS INTO THE
C PHYSICAL DATA UNITS
COMMON ALPHAR(16), CONV(16), DCAL(16), IVEC(16,2000), ZERO(16)
1  ,VC(2,2000), VB(2,2000), Q(16,2000), RCAL(16), CONVC(2)
2  ,VT(16,2000), TIME(2000), DATA(130), RT(16,2000), CONVB(2)
3  ,A0(16), A1(16), VREF(16), RREF(16), dvt(16,2000), FZERO(16), R(16)
character*20 c2,c3,c7,c8,c9
4  FORMAT(/, ' WHAT IS THE NAME OF THE INPUT DATA FILE? ', $)
5  FORMAT(/, ' WHAT IS THE NAME OF THE INFO FILE? ', $)
6  FORMAT(/, ' WHAT IS THE NAME OF THE COAX OUTPUT FILE? ', $)
7  FORMAT(/, ' WHAT IS THE NAME OF THE THIN OUTPUT FILE? ', $)
110 FORMAT(A20)
TYPE 5
ACCEPT 110, C3
OPEN(UNIT=3, NAME=C3, STATUS='OLD')
TYPE 4
ACCEPT 110, C2
OPEN(UNIT=2, NAME=C2, STATUS='OLD')
OPEN(UNIT=7, NAME=C7, STATUS='NEW')
OPEN(UNIT=8, NAME=C8, STATUS='NEW')
TYPE 7
ACCEPT 110, C9
OPEN(UNIT=9, NAME=C9, STATUS='NEW')
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C ALPHAR RESISTANCE COEFFICIENT (/DEGREE K)
C A0 INTERCEPT IN ALPHAR CURVE
C A1 SLOPE OF ALPHAR CURVE
C CONV(N) CONVERSION BETWEEN DIGITAL UNITS TO OHMS
C FOR THE CHANNEL CONNECTED TO THE THIN FILM GAGE
C (OHMS/DIGITAL UNITS)
C CONVB CONVERSION BETWEEN DIGITAL UNITS TO VOLTAGE
C FOR THE CHANNEL CONNECTED TO BACKFACE COAX
C CONVC CONVERSION BETWEEN DIGITAL UNITS TO VOLTAGE
C FOR THE CHANNEL CONNECTED TO COAX GAGE
C WITH REFERENCE JUNCTION IN AN ICE BATH
C DCAL(N) OUTPUT IN DATA UNITS FROM CIRCUIT DURING CALIBRATION
C DELTAV CHANGE IN PARABOLA VOLTAGE
C IVEC(N, J) OUTPUT IN DIGITAL UNITS FROM CIRCUIT DURING TESTS
C NCH NUMBER OF DATA CHANNELS
C NCOAX NUMBER OF COAX GAGES
C NTC CHANNEL NUMBER OF FIRST COAX GAGE
C NTHIN NUMBER OF THIN FILM GAGES
C NTT CHANNEL NUMBER OF FIRST THIN FILM GAGE
C NBF NUMBER OF BACK FACE GAGES
```



```

C NTB      CHANNEL NUMBER OF FIRST BACK FACE GAGE
C PO       TUNNEL STAGNATION PRESSURE (BAR)
C Q        HEAT TRANSFER RATE (J/M**2 SEC = WATTS/M**2)
C RCK      SUBSTRATE THERMAL PRODUCT (SQRT(RHO CP K))
C          (J/M**2 DEGREE K SEC**1/2)
C RCAL     RESISTANCE AT AMBIENT TEMP AT TIME OF
C          ALPHAR CALIBRATION
C RREF     RESISTANCE OF GAGE AT AMBIENT TEMP AT TIME OF
C          ACTUAL TEST
C TA       AMBIENT TEMPERATURE AT TEST TIME (DEGREE K)
C TIM      TIME OF THE CHANGE IN PARABOLA VOLTAGE
C TISO     WALL TEMPERATURE AT TIME=0 (DEGREE K)
C TCAL     AMBIENT TEMPERATURE AT TIME OF ALPHAR
C          CALIBRATION (DEGREE K)
C VREF(N)  VOLTAGE OF GAGE AT BEGINNING OF THE TEST AT TIME=0
C XM       MACH NUMBER
C

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

```

C READ IN ELECTRICAL ANALOG CIRCUIT INFO
      READ(3,*) NCH,NCOAX,NTHIN,NPTS,CASE
      READ(3,*) CONV(1),CONVC(2),CONVB(1),CONVB(2)
      READ(3,*) DELTAV,TIM,TA,RCK,XM,PO,TCAL
      DO 600 N=1,NTHIN
        READ(3,*) DCAL(N),ALPHAR(N),A0(N),A1(N),rcal(n)
      600 continue
      do 601 n=1,nthin
        read (3,*) CONV(N),VREF(N),RREF(N),ZERO(N),FZERO(N)
      601 CONTINUE
      PI=3.14159
C

```

```

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

```

C SET UP DATA ARRAYS
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 155 N=1,NCH
        DO 156 I=1,NPTS
          READ(2,*) time(i),IVEC(N,I)
          time(i)=time(i)*.00032
        156 CONTINUE
      155 CONTINUE
C

```

```

C
CCCCCCCCCCCC

```

```

C CONVERT TO PHYSICAL DATA UNITS
CCCCCCCCCCCC
C

```

```

      800 FORMAT(' DO YOU WANT TO CONVERT THE THIN FILM GAGE DATA INTO')
      810 FORMAT(' (1)OHMS TO NUMERICALLY INTEGRATE?, OR')
      820 FORMAT(' (2)HEAT TRANSFER RATES SINCE ALREADY INTEGRATED ')
      821 FORMAT(' BY ELECTRICAL ANALOG CIRCUITS?')
      830 FORMAT(' OR DO YOU WANT TO WORK WITH THE (3)COAX DATA?')
      840 FORMAT(' OR DO YOU WANT TO (9)QUIT')
      90 PRINT 800
      PRINT 810

```

```

      PRINT 820
      PRINT 821
      PRINT 830
      PRINT 840
10  FORMAT(A1)
      accept *,ians
      IF (ians .EQ. 1) CALL STHIN(NTHIN,NPTS,CASE,XM,PO,TA,
1   RCK,TISO,TCAL)
      IF (ians .EQ. 2) CALL ATHIN(NTHIN,DELTAV,TIM,RCK,TA,TCAL,PI
1   ,NPTS,CASE,XM,PO,TISO)
      IF (ians .EQ. 3) CALL SCOAX(NCOAX,NPTS,NBF,XM,PO,TISO)
      IF (ians .EQ. 9) GO TO 860
CCCCCCCCCCCCCCCCCCCC
      PRINT 850
      accept *,ians
850  FORMAT(' DO YOU WANT TO LOOK AT THE OPTIONS AGAIN? (1)yES (2)nO')
      IF (ians .EQ. 1) GO TO 90
860  CONTINUE
      STOP
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE SCOAX(NCOAX,NPTS,NBF,XM,PO,TISO)
      COMMON ALPHAR(16),CONV(16),DCAL(16),IVEC(16,2000),ZERO(16)
1   ,VC(2,2000),VB(2,2000),Q(16,2000),RCAL(16),CONVC(2)
2   ,VT(16,2000),TIME(2000),DATA(130),RT(16,2000),CONVB(2)
3   ,A0(16),A1(16),VREF(16),RREF(16),dvt(16,2000),FZERO(16),R(16)
      dimension temp(2,2000)
C COAX GAGE DATA CONVERSION FROM VOLTAGE TO TEMPERATURE
C IS TAKEN FROM THE CALIBRATION TABLES AND CONVERTED INTO AN
C EQUATION USING A LEAST SQUARES METHOD
C TEMP DEGREE C = (MILLIVOLTS + 0.036) / 0.061
195  FORMAT(3F10.4)
      DO 210 N=1,NCOAX
          DO 200 J=1,NPTS
              VC(N,J)=IVEC(N+10,J)*CONVC(N)
              TEMP(N,J)=(VC(N,J)+0.036)/0.061 + 273.15
200   CONTINUE
210  CONTINUE
C      DO 220 J=1,NPTS
C          BTEMP(1,J)=(VB(1,J) + 0.036)/0.061 + 273.15
C          BTEMP(NBF,J)=(VB(NBF,J) + 0.036)/0.061 + 273.15
C 220 CONTINUE
      WRITE(8,*) XM,PO
      WRITE(8,*) NCOAX,NPTS
      DO 300 J=1,NPTS
          WRITE(8,*) TIME(J),TEMP(1,J),TEMP(1,1)
300  CONTINUE
      DO 400 J=1,NPTS
          WRITE(8,*) TIME(J),TEMP(2,J),TEMP(NBF,1)
400  CONTINUE

```

```

98 RETURN
END

```

```

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

```

```

SUBROUTINE STHIN(NTHIN,NPTS,CASE,XM,PO,TA,rck,TISO,tcal)
COMMON ALPHAR(16),CONV(16),DCAL(16),IVEC(16,2000),ZERO(16)
1 ,VC(2,2000),VB(2,2000),Q(16,2000),RCAL(16),CONVC(2)
2 ,VT(16,2000),TIME(2000),DATA(130),RT(16,2000),CONVB(2)
3 ,AO(16),A1(16),VREF(16),RREF(16),dvt(16,2000),FZERO(16),R(16)

```

```

C THIN FILM GAGE CONVERSION TO OHMS FOR

```

```

C NUMERICAL INTEGRATION

```

```

DO 700 N=1,NTHIN
R(N)=(FZERO(N)-ZERO(N))*CONV(N)
DO 710 J=1,NPTS
RT(N,J)=(IVEC(N,J)-FZERO(N))*CONV(N)

```

```

710 CONTINUE

```

```

700 CONTINUE

```

```

WRITE(9,*) XM,PO,TA,RCK,TCAL
WRITE(9,*) NTHIN,NPTS,CASE
DO 203 N=1,NTHIN
WRITE(9,*) ALPHAR(N),VREF(N),RREF(N)

```

```

203 continue

```

```

do 204 n=1,nthin
write(9,*) AO(N),A1(N),RCAL(N),R(N)

```

```

204 CONTINUE

```

```

DO 200 I=1,NTHIN
DO 210 J=1,NPTS
WRITE(9,*) TIME(J),RT(I,J)

```

```

210 CONTINUE

```

```

200 CONTINUE

```

```

97 RETURN

```

```

END

```

```

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCC
C

```

```

SUBROUTINE ATHIN(NTHIN,DELTAV,TIM,RCK,TA,TCAL,PI,NPTS,CASE
1 ,XM,PO,TISO)
COMMON ALPHAR(16),CONV(16),DCAL(16),IVEC(16,2000),ZERO(16)
1 ,VC(2,2000),VB(2,2000),Q(16,2000),RCAL(16),CONVC(2)
2 ,VT(16,2000),TIME(2000),DATA(130),RT(16,2000),CONVB(2)
3 ,AO(16),A1(16),VREF(16),RREF(16),dvt(16,2000),FZERO(16),R(16)

```

```

C THIN FILM GAGE CONVERSION TO HEAT TRANSFER DATA

```

```

C AFTER ELECTRICAL ANALOG INTEGRATION

```

```

ASTAR=0
CON=0
PRINT*, 'THE VALUE OF ASTAR SHOULD BE AROUND .0001'
DO 500 I=1,NTHIN
ASTAR=DELTAV*SQRT(PI/TIM)/(2*DCAL(I))
TYPE*, ASTAR
CON=ASTAR*(1+ALPHAR(I)*(TA-TCAL))/(VREF(I)*ALPHAR(I))

```

```

      DO 510 J=1,NPTS
        Q(I,J)=DVT(I,J)*(RCK)*CON
510   CONTINUE
      ASTAR=0
      CON=0
500   CONTINUE
      WRITE(7,*) XM,PO,RCK
      WRITE(7,*) NTHIN,NPTS,CASE
      DO 200 I=1,NTHIN
        DO 210 J=1,NPTS
          WRITE(7,*) TIME(J),Q(I,J)
210   CONTINUE
200   CONTINUE
99    RETURN
      END

```

Appendix N

Numerical Integration Program

```
PROGRAM THIN
C THIS PROGRAM NUMERICALLY INTEGRATES THIN FILM VOLTAGE DATA INTO
C HEAT TRANSFER RATES
C TWO REDUCTION METHODS ARE AVAILABLE:
C 1. AEDC REDUCTION METHOD
C   - USES SEMI-INFINITE SLAB ASSUMPTION
C   - INDIRECT METHOD, SMOOTHS THE DATA
C 2. SHULTZ AND JONES METHOD
C   - USES SEMI-INFINITE SLAB ASSUMPTION
C   - DIRECT METHOD
COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
1  ,AO(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
2  ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),DTISO(16),TW(16)
CHARACTER*20 C2,C7,C10,C8
4 FORMAT(/,' WHAT IS THE NAME OF THE INPUT FILE? ', $)
5 FORMAT(/,' WHAT IS THE NAME OF THE OUTPUT FILE? ', $)
6 FORMAT(/,' WHAT IS THE NAME OF THE PLOT FILE? ', $)
7 FORMAT(/,' WHAT IS THE NAME OF THE TS**2 PLOT FILE? ', $)
110 FORMAT(A20)
TYPE 4
ACCEPT 110,C2
OPEN(UNIT=2,NAME=C2,STATUS='OLD')
TYPE 6
ACCEPT 110,C7
OPEN(UNIT=7,NAME=C7,STATUS='NEW')
TYPE 5
ACCEPT 110,C10
OPEN(UNIT=10,NAME=C10,STATUS='NEW')
TYPE 7
ACCEPT 110,C3
OPEN(UNIT=8,NAME=C8,STATUS='NEW')
C ***** NOMENCLATURE *****
C AR      FILM TEMPERATURE COEFFICIENT OF RESISTANCE
C DV      DATA IN OHMS
C VREF    REFERENCE VOLTAGE
C TA      AMBIENT TEMPERATURE DURING THE TEST
C TCAL    REFERENCE TEMPERATURE USED WHEN FINDING AR
C RCAL    REFERENCE RESISTANCE AT AMBIENT TEMP WHEN
C          FINDING AR
C Q        CUMULATIVE HEATING (WATTS SEC/M**2)
C QDOT    HEAT TRANSFER RATE (WATTS/M**2)
C RCK     SQRT(RHO CP K), THERMAL PRODUCT
C XM      MACH NUMBER
C
READ(2,*) XM,PO,TA,RCK,TCAL
READ(2,*) NTHIN,NPTS,CASE
```

```

DO 55 N=1,NTHIN
  READ(2,*) AR(N),VREF(N),RREF(N)
55 CONTINUE
DO 56 N=1,NTHIN
  READ(2,*) A0(N),A1(N),RCAL(N),R(N)
56 CONTINUE
DO 200 I=1,NTHIN
  DO 210 J=1,NPTS
    READ(2,*) TIME(J),RT(I,J)
210 CONTINUE
200 CONTINUE
WRITE(10,*) XM,PO
WRITE(10,*) NTHIN,NPTS,CASE
CALL GAGEDATA(NTHIN,NPTS,TA,XM,PO,RCK,TCAL)
STOP
END

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
  SUBROUTINE GAGEDATA(NTHIN,NPTS,TA,XM,PO,RCK,TCAL)
    COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
    1  ,A0(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
    2  ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),DTISO(16),TW(16)
    7  CONTINUE
    TYPE 104
104 FORMAT(' DO YOU WISH TO (1)SMOOTH DATA  ',
  & ' (9)EXIT , (2)REDUCE DATA'//)
    ACCEPT *,IANS
    IF (IANS .EQ. 2) GO TO 20
    IF (IANS .EQ. 9) GO TO 99
C ***** SMOOTH THE DATA *****
    TYPE 106
106 FORMAT(' ENTER (ODD) NO. OF POINTS TO FIT,
  & ' 13 FORMAT'//)
    ACCEPT *,IFITT
    IS=IFITT/2 + 1
    IEND=NPTS-IS+1
    DO 299 L=1,NTHIN
      DO 9 I=IS,IEND
        DO 8 J=1,IFITT
          K=I-IS+J
          X(J)=TIME(K)
          Y(J)=RT(L,K)
        8 CONTINUE
        CALL LSTSQR(RT(L,I),DYDX,IFITT)
      9 CONTINUE
299 CONTINUE
    GO TO 7
C ***** REDUCE THE DATA *****
    20 CONTINUE
    5  FORMAT(' WHAT IS ISTART? ')
    TYPE 5
    ACCEPT*, ISTART

```

```

C ***** CHANGE TO TEMP AND VOLTS
  DO 297 N=1,NTHIN
    TW(N)=TCAL+(RREF(N)-RCAL(N)-AO(N))/A1(N)
    DO 296 J=1,NPTS
      TEMP(N,J)=(RT(N,J)-AO(N))/A1(N)
      DV(N,J)=RT(N,J)*VREF(N)/RREF(N)
      Q(N,J)=0.0
      QDOT(N,J)=0.0
296   CONTINUE
      DVFAKE(N)=R(N)*VREF(N)/RREF(N)
297   CONTINUE
    DO 211 N=1,NTHIN
      WRITE(10,*) TW(N)
211   CONTINUE
    TIME(ISTART)=TSTART
    DO 3 I=ISTART,NPTS
      TIME(I)=TIME(I)-TSTART
3     CONTINUE
    TYPE 300
300  FORMAT(' CHOOSE THE METHOD OF DATA REDUCTION '//
&      (1)AEDC ,(3)AEDC WITH TEMP, OR (2)SHULTZ AND JONES '/')
    ACCEPT *,MET
    IF (MET .EQ. 1) THEN
      CALL AEDC(CASE,NTHIN,NPTS,TA,TCAL,RCK,XM,PO,ISTART)
    ENDIF
    IF (MET .EQ. 2) THEN
      CALL SJ(CASE,NTHIN,NPTS,TA,TCAL,RCK,XM,PO,ISTART)
    ENDIF
    IF (MET .EQ. 3) THEN
      CALL TAEDC(CASE,NTHIN,NPTS,TA,TCAL,RCK,XM,PO,ISTART)
    ENDIF

C
C SMOOTH THE RESULTING DATA
C
  TYPE 106
  ACCEPT *,IFITT
  IS=IFITT/2 + 1
  IEND=NPTS-IS+1
  DO 499 L=1,NTHIN
    DO 49 I=IS,IEND
      DO 48 J=1,IFITT
        K=I-IS+J
        X(J)=K
        Y(J)=TEMP(L,K)
48      CONTINUE
        CALL LSTSQR(TEMP(L,I),DYDX,IFITT)
49      CONTINUE
499   CONTINUE
    DO 399 L=1,NTHIN
      DO 39 I=IS,IEND
        DO 38 J=1,IFITT
          K=I-IS+J
          X(J)=K

```

```

        Y(J)=QDOT(L,K)
38      CONTINUE
        CALL LSTSQR(QDOT(L,I),DYDX,IFITT)
39      CONTINUE
399    CONTINUE
        DO 331 N=1,NTHIN
            DO 221 I=1,NPTS
                WRITE(7,*) TEMP(N,I),QDOT(N,I)
                WRITE(10,*) TEMP(N,I),QDOT(N,I)
221      CONTINUE
331    CONTINUE
        ZERO=0.0
        WRITE(8,*) ZERO,ZERO
        DO 338 I=1,NPTS
            WRITE(8,*) TIME(I),(TEMP(1,I))**2
338    CONTINUE
99      RETURN
        END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
        SUBROUTINE AEDC(CASE,NTHIN,NPTS,TA,TREF,RCPK,XM,PO,ISTART)
        COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
        1      ,A0(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
        2      ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),TISO(16),TW(16)
C ***** NOMENCLATURE *****
C RCPK      THERMAL PARAMETER
C           R = DENSITY
C           CP = SPECIFIC HEAT CAPACITY
C           K = CONDUCTIVITY
C Q         CUMULATIVE HEATING
C QDOT      HEAT TRANSFER RATE
C
96      FORMAT(' AEDC REDUCTION METHOD ')
        TYPE 96
        TYPE 97
97      FORMAT(' ***** WORKING ***** ')
        DO 299 N=1,NTHIN
            DO 2 J=ISTART+1,NPTS
                J2=J-1
                SUM=0.
                DO 1 I=ISTART,J2
                    SUM=SUM+((DV(N,I+1)-DV(N,ISTART))*(DV(N,I)
&                -DV(N,ISTART))/(SQRT(TIME(J)-TIME(I))
&                +SQRT(TIME(J)-TIME(I+1))))
1          CONTINUE
                Q(N,J)=SQRT(1.0/3.14159)*(TIME(J)-TIME(J-1))
&                *RCPK*SUM*(1+AR(N)*(TA-TREF))/
&                (AR(N)*(VREF(N)+DVFAKE(N)))
2          CONTINUE
                Q(N,ISTART)=Q(N,ISTART+1)
                IE=NPTS-8
                IS=ISTART+8

```



```

      DO 20 I=IS,IE
        QDOT(N,I)=(-2.*Q(N,I-8)-Q(N,I-4)+Q(N,I+4)+2.*Q(N,I+8))
&          /((TIME(I)-TIME(I-1))*40.)
20    CONTINUE
      DO 663 I=1,8
        QDOT(N,I+ISTART-1)=QDOT(N,IS)
        QDOT(N,I+IE)=QDOT(N,IE)
663    CONTINUE
299    CONTINUE
      RETURN
      END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE TAEDC(CASE,NTHIN,NPTS,TA,TREF,RCPK,XM,PO,ISTART)
      COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
      1    ,A0(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
      2    ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),TISO(16),TW(16)
C ***** NOMENCLATURE *****
C RCPK      THERMAL PARAMETER
C           R = DENSITY
C           CP = SPECIFIC HEAT CAPACITY
C           K = CONDUCTIVITY
C Q         CUMULATIVE HEATING
C QDOT      HEAT TRANSFER RATE
C
96    FORMAT(' TAEDC REDUCTION METHOD ')
      TYPE 96
      TYPE 97
97    FORMAT(' ***** WORKING ***** ')
      DO 299 N=1,NTHIN
        DO 2 J=ISTART+1,NPTS
          J2=J-1
          SUM=0.
          DO 1 I=ISTART,J2
            SUM=SUM+((TEMP(N,I+1)-TEMP(N,ISTART))+{TEMP(N,I)
&              -TEMP(N,ISTART)})/(SQRT(TIME(J)-TIME(I))
&              +SQRT(TIME(J)-TIME(I+1.)))
1        CONTINUE
          Q(N,J)=SQRT(1.0/3.14159)*(TIME(J)-TIME(J-1))
&          *RCPK*SUM
2        CONTINUE
          Q(N,ISTART)=Q(N,ISTART+1)
          IE=NPTS-8
          IS=ISTART+8
          DO 20 I=IS,IE
            QDOT(N,I)=(-2.*Q(N,I-8)-Q(N,I-4)+Q(N,I+4)+2.*Q(N,I+8))
&              /((TIME(I)-TIME(I-1))*40.)
20      CONTINUE

```

```

        DO 663 I=1,8
            QDOT(N,I+ISTART-1)=QDOT(N,IS)
            QDOT(N,I+IE)=QDOT(N,IE)
663    CONTINUE
299    CONTINUE
        RETURN
        END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
        SUBROUTINE SJ(CASE,NTHIN,NPTS,TA,TREF,RCPK,XM,PO,ISTART)
        COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
        1      ,AD(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
        2      ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),TISO(16),TW(16)
C ***** NOMENCLATURE *****
C RCPK      THERMAL PARAMETER
C           R = DENSITY
C           CP = SPECIFIC HEAT CAPACITY
C           K = CONDUCTIVITY
96  FORMAT(' SHULTZ AND JONES REDUCTION METHOD ')
        TYPE 96
        TYPE 97
97  FORMAT(' ***** WORKING ***** ')
        IEND=NPTS
        J1=ISTART+1
        DO 299 N=1,NTHIN
            DO 2 J=J1,IEND
                J2=J-1
                SUM=0.
                DO 1 I=ISTART+1,J2
                    SUM=SUM+(((DV(N,I)-DV(N,I-1))
&                        )/(SQRT(TIME(J)-TIME(I))
&                        +SQRT(TIME(J)-TIME(I-1))))
1      CONTINUE
                QDOT(N,J)=RCPK*SQRT(1.0/3.14159)
&          *2.0*SUM*(1+AR(N)*(TA-TREF))/
&          (AR(N)*(VREF(N)+DVFAKE(N)))
2      CONTINUE
                QDOT(N,ISTART)=QDOT(N,ISTART+1)
101  FORMAT(I5)
299  CONTINUE
        RETURN
        END
C

```

AD-A172 489

NUMERICAL SOLUTION OF LAMINAR AND TURBULENT BOUNDARY
LAYER EQUATIONS INCL. (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... A J CHEN
MAR 86 F/G 28/4

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11-86



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C

```
      SUBROUTINE LSTSQR(YFIT,DYDX,IFIT)
      COMMON AR(16),VREF(16),TIME(2000),DV(16,2000),RREF(16)
      1      ,A0(16),A1(16),RCAL(16),TEMP(16,2000),Q(16,2000),DVFAKE(16)
      2      ,QDOT(16,2000),X(40),Y(40),R(16),RT(16,2000),TISO(16),TW(16)
C ** FIRST ORDER LEAST SQUARES FIT OF IFIT POINTS.
C **   DYDX=F(X(MIDPT))
      MIDPT=IFIT/2 + 1
      A=0.
      B=0.
      C=0.
      D=0.
      E=0.
      F=0.
      G=0.
      DO 2 I=1,IFIT
          A=A+X(I)
          B=B+X(I)**2
          C=C+X(I)**3
          D=D+X(I)**4
          E=E+Y(I)
          F=F+(X(I)*Y(I))
          G=G+(X(I)*X(I)*Y(I))
      2  CONTINUE
      B1=(A*E-IFIT*F)/(A*A-IFIT*B)
      B0=(E-B1*A)/IFIT
      YFIT=B0+B1*X(MIDPT)
      DYDX=B1
      RETURN
      END
```

Appendix O

Temperature Reconstruction Program

```
PROGRAM RECON
C THIS PROGRAM RECONSTRUCTS THE TEMPERATURE DATA VERSUS TIME
C FROM THE HEAT TRANSFER VERSUS TIME DATA
C THE OUTPUT IS THEN HEAT TRANSFER VERSUS TEMP
  DIMENSION Q(16,2000),TEMP(16,2000),T(2000)
  CHARACTER*20 C7,C10
110 FORMAT(A20)
  5 FORMAT(/,' WHAT IS THE NAME OF THE INPUT FILE? ', $)
  6 FORMAT(/,' WHAT IS THE NAME OF THE OUTPUT FILE? ', $)
  TYPE 5
  ACCEPT 110,C7
  OPEN(UNIT=7,NAME=C7,STATUS='OLD')
  TYPE 6
  ACCEPT 110,C10
  OPEN(UNIT=10,NAME=C10,STATUS='NEW')
  PI=3.14159
C RCK IS THE CERAMIC THERMAL PRODUCT
C RCK = SQRT(RCPK) UNITS OF J/M**2 K SEC**1/2
C READ IN THE Q VERSUS TIME DATA
  READ(8,*) XM,PO,RCK,TISO
  READ(8,*) NTHIN,NPTS,CASE
  DO 500 N=1,NTHIN
    DO 510 J=1,NPTS
      READ(7,*) T(J),Q(N,J)
510    CONTINUE
500  CONTINUE
C RECONSTRUCT THE TEMPERATURE
  DO 700 N=1,NTHIN
    DO 600 M=2,NPTS
      SUM=0.0
      TEMP(N,M)=0.0
      QDIFF=0.0
      JM=M-1
      DO 610 J=1,JM
        QDIFF=Q(N,J+1)+Q(N,J-1)-2*Q(N,J)
        SUM=SUM+SQRT(T(J)-T(J-1))*(M-J)**(3./2.)*(QDIFF)
610    CONTINUE
      TEMP(N,M)=4.*SUM/(3.*RCK*SQRT(PI))
      TEMP(N,M)=TEMP(N,M)+TISO
600  CONTINUE
700  CONTINUE
C WRITE OUT THE Q VERSUS TEMPERATURE DATA
  WRITE(10,*) XM,PO,TISO
  WRITE(10,*) NTHIN,NPTS,CASE
  DO 400 N=1,NTHIN
    TEMP(N,1)=TEMP(N,2)
    DO 410 J=1,NPTS
      WRITE(10,*) TEMP(N,M),Q(N,M)
410    CONTINUE
400  CONTINUE
CCCCCCCCC
  STOP
  END
```

Appendix P

Isothermal Heat Transfer Program

```
PROGRAM QDOTISO
C THIS PROGRAM DOES A LEAST SQUARES FIT OF THE
C HEAT TRANSFER RATE VERSUS TEMPERATURE
C CURVE BETWEEN THE TEMPERATURES T1 AND T2
C
  DIMENSION XLOC(16),ST(16),H(16),QISO(16),REX(16)
1  ,TW(16),TEMP(16,2000),Q(16,2000)
  REAL MUFPS,MUINF
  CHARACTER*20 C8,C10,C12
110 FORMAT(A20)
  4  FORMAT(/,' WHAT IS THE INPUT FILE NAME?', $)
  TYPE 4
  ACCEPT 110,C8
  OPEN(UNIT=8,NAME=C8,STATUS='OLD')
  5  FORMAT(/,' WHAT IS THE OUTPUT FILE NAME?', $)
  TYPE 5
  ACCEPT 110,C10
  OPEN(UNIT=10,NAME=C10,STATUS='NEW')
C READ IN THE DATA
  READ(8,*) XM,PBAR
  READ(8,*) NTHIN,NPTS,CASE
  DO 10 N=1,NTHIN
    READ(8,*) TW(N)
  10 CONTINUE
  DO 500 N=1,NTHIN
    DO 510 J=1,NPTS
      READ(8,*) TEMP(N,J),Q(N,J)
  510 CONTINUE
500 CONTINUE
  TISO=288.95
  TSTAG=473.15
  PSTAG=PBAR*1.0E5
  PR=.72
  GC=287.0
  GAM=1.4
  FACT=1+.5*(GAM-1)*XM**2
  TINF=TSTAG/FACT
  PINF=PSTAG/FACT**((GAM/(GAM-1)))
  RINF=PINF/(GC*TINF)
  AINF=SQRT(GAM*GC*TINF)
  UINF=AINF*XM
C CALCULATE MU IN FPS, THEN CONVERT TO MKS
  TF=(9./5.)*(TINF-273.15) + 32.0
  TR=459.67 + TF
  MUFPS=SQRT(TR)*2.32E-8/((1+(220./(TR*10.**((9./TR))))))
  MUINF=MUFPS*3.281*14.59
```

```

      CP=GC*GAM/(GAM-1)
      RREY=UINF*RINF/MUINF
C
      TYPE*,TINF,RINF,UINF,AINF,CP,MUINF,PINF,RREY
C
C LEAST SQUARES ALGORITHM
C
      DO 520 N=1,NTHIN
        TYPE*, ' WORKING ON GAGE #',N
        I=0
        SX=0
        SY=0
        SX2=0
        SY2=0
        SXY=0
        DEN=0
        A1=0
        A0=0
        QISO(N)=0
        DO 530 J=300,440
          I=I+1
          SX=SX+TEMP(N,J)
          SY=SY+Q(N,J)
          SX2=SX2+TEMP(N,J)**2
          SY2=SY2+Q(N,J)**2
          SXY=SXY+Q(N,J)*TEMP(N,J)
530      CONTINUE
        DEN=SX2-SX*SX/I
        A1=(SXY-SX*SY/I)/DEN
        A0=(SY-A1*SX)/I
        F=A1**2*(SX2-SX**2/I)/(SY2-SY**2/I)
        QISO(N)=A0
C CALCULATE ST FROM ISOTHERMAL Q
        TAW=TINF*(1+SQRT(PR)*(GAM-1)*.5*XM**2)
        H(N)=QISO(N)/(TAW-TISO)
        ST(N)=H(N)/(CP*RINF*UINF)
525      CONTINUE
        TYPE*,QISO(N)
520      CONTINUE
C
C CALCULATE X POSITION OF EACH GAGE AND
C THE LOCAL REYNOLDS NUMBER OF EACH GAGE
C
CCCCC FOR CASE NUMBER 1
C
      IF (CASE.EQ.1) THEN
        DO 505 N=1,NTHIN
          XLOC(N)=(2*N+3)*4.0 +7.0+40.0
          REX(N)=RINF*UINF*XLOC(N)/MUINF/1000
505      CONTINUE
        ENDIF
C

```


CCCCC FOR CASE NUMBER 2

C

IF (CASE .EQ. 2) THEN

DO 506 N=1,8

XLOC(N)=(2*N+2)*4.0 + 7.0 + 40.0

REX(N)=RINF*UINF*XLOC(N)/MUINF/1000

506 CONTINUE

XLOC(9)=22*4.0 + 7.0 + 40.0

XLOC(10)=24*4.0 + 7.0 + 40.0

REX(9)=RINF*UINF*XLOC(9)/MUINF/1000

REX(10)=RINF*UINF*XLOC(10)/MUINF/1000

ENDIF

C

CCCCC FOR CASE NUMBER 3

C

IF (CASE .EQ. 3) THEN

XLOC(1)=50.0

XLOC(2)=86.0

DO 508 N=1,NTHIN

REX(N)=RINF*UINF*XLOC(N)/MUINF/1000

508 CONTINUE

ENDIF

C

CCCCCCCCCCCCCCCCC

C

C PRINT OUT THE RESULTS TO SEE

DO 507 N=1,NTHIN

WRITE(10,*) N,XLOC(N),LOG10(REX(N)),ST(N),H(N)

1 ,QISO(N),REX(N)

507 CONTINUE

STOP

END

Appendix G

Temperature Calibration for Coax Gage

DEG F	0	1	2	3	4	5	6	7	8	9	10	DEG F
THERMOELECTRIC VOLTAGE IN ABSOLUTE MILLIVOLTS												
-400	-0.030	-0.032	-0.033	-0.034	-0.035	-0.036	-0.037	-0.038	-0.039	-0.040	-0.041	-400
-380	-0.050	-0.052	-0.054	-0.056	-0.058	-0.060	-0.062	-0.064	-0.066	-0.068	-0.070	-380
-360	-0.070	-0.072	-0.074	-0.076	-0.078	-0.080	-0.082	-0.084	-0.086	-0.088	-0.090	-360
-340	-0.090	-0.092	-0.094	-0.096	-0.098	-0.100	-0.102	-0.104	-0.106	-0.108	-0.110	-340
-320	-0.110	-0.112	-0.114	-0.116	-0.118	-0.120	-0.122	-0.124	-0.126	-0.128	-0.130	-320
-300	-0.130	-0.132	-0.134	-0.136	-0.138	-0.140	-0.142	-0.144	-0.146	-0.148	-0.150	-300
-280	-0.150	-0.152	-0.154	-0.156	-0.158	-0.160	-0.162	-0.164	-0.166	-0.168	-0.170	-280
-260	-0.170	-0.172	-0.174	-0.176	-0.178	-0.180	-0.182	-0.184	-0.186	-0.188	-0.190	-260
-240	-0.190	-0.192	-0.194	-0.196	-0.198	-0.200	-0.202	-0.204	-0.206	-0.208	-0.210	-240
-220	-0.210	-0.212	-0.214	-0.216	-0.218	-0.220	-0.222	-0.224	-0.226	-0.228	-0.230	-220
-200	-0.230	-0.232	-0.234	-0.236	-0.238	-0.240	-0.242	-0.244	-0.246	-0.248	-0.250	-200
-180	-0.250	-0.252	-0.254	-0.256	-0.258	-0.260	-0.262	-0.264	-0.266	-0.268	-0.270	-180
-160	-0.270	-0.272	-0.274	-0.276	-0.278	-0.280	-0.282	-0.284	-0.286	-0.288	-0.290	-160
-140	-0.290	-0.292	-0.294	-0.296	-0.298	-0.300	-0.302	-0.304	-0.306	-0.308	-0.310	-140
-120	-0.310	-0.312	-0.314	-0.316	-0.318	-0.320	-0.322	-0.324	-0.326	-0.328	-0.330	-120
-100	-0.330	-0.332	-0.334	-0.336	-0.338	-0.340	-0.342	-0.344	-0.346	-0.348	-0.350	-100
-80	-0.350	-0.352	-0.354	-0.356	-0.358	-0.360	-0.362	-0.364	-0.366	-0.368	-0.370	-80
-60	-0.370	-0.372	-0.374	-0.376	-0.378	-0.380	-0.382	-0.384	-0.386	-0.388	-0.390	-60
-40	-0.390	-0.392	-0.394	-0.396	-0.398	-0.400	-0.402	-0.404	-0.406	-0.408	-0.410	-40
-20	-0.410	-0.412	-0.414	-0.416	-0.418	-0.420	-0.422	-0.424	-0.426	-0.428	-0.430	-20
0	-0.430	-0.432	-0.434	-0.436	-0.438	-0.440	-0.442	-0.444	-0.446	-0.448	-0.450	0
20	-0.450	-0.452	-0.454	-0.456	-0.458	-0.460	-0.462	-0.464	-0.466	-0.468	-0.470	20
40	-0.470	-0.472	-0.474	-0.476	-0.478	-0.480	-0.482	-0.484	-0.486	-0.488	-0.490	40
60	-0.490	-0.492	-0.494	-0.496	-0.498	-0.500	-0.502	-0.504	-0.506	-0.508	-0.510	60
80	-0.510	-0.512	-0.514	-0.516	-0.518	-0.520	-0.522	-0.524	-0.526	-0.528	-0.530	80
100	-0.530	-0.532	-0.534	-0.536	-0.538	-0.540	-0.542	-0.544	-0.546	-0.548	-0.550	100
120	-0.550	-0.552	-0.554	-0.556	-0.558	-0.560	-0.562	-0.564	-0.566	-0.568	-0.570	120
140	-0.570	-0.572	-0.574	-0.576	-0.578	-0.580	-0.582	-0.584	-0.586	-0.588	-0.590	140
160	-0.590	-0.592	-0.594	-0.596	-0.598	-0.600	-0.602	-0.604	-0.606	-0.608	-0.610	160
180	-0.610	-0.612	-0.614	-0.616	-0.618	-0.620	-0.622	-0.624	-0.626	-0.628	-0.630	180
200	-0.630	-0.632	-0.634	-0.636	-0.638	-0.640	-0.642	-0.644	-0.646	-0.648	-0.650	200
220	-0.650	-0.652	-0.654	-0.656	-0.658	-0.660	-0.662	-0.664	-0.666	-0.668	-0.670	220
240	-0.670	-0.672	-0.674	-0.676	-0.678	-0.680	-0.682	-0.684	-0.686	-0.688	-0.690	240
260	-0.690	-0.692	-0.694	-0.696	-0.698	-0.700	-0.702	-0.704	-0.706	-0.708	-0.710	260
280	-0.710	-0.712	-0.714	-0.716	-0.718	-0.720	-0.722	-0.724	-0.726	-0.728	-0.730	280
300	-0.730	-0.732	-0.734	-0.736	-0.738	-0.740	-0.742	-0.744	-0.746	-0.748	-0.750	300
320	-0.750	-0.752	-0.754	-0.756	-0.758	-0.760	-0.762	-0.764	-0.766	-0.768	-0.770	320
340	-0.770	-0.772	-0.774	-0.776	-0.778	-0.780	-0.782	-0.784	-0.786	-0.788	-0.790	340
360	-0.790	-0.792	-0.794	-0.796	-0.798	-0.800	-0.802	-0.804	-0.806	-0.808	-0.810	360
380	-0.810	-0.812	-0.814	-0.816	-0.818	-0.820	-0.822	-0.824	-0.826	-0.828	-0.830	380
400	-0.830	-0.832	-0.834	-0.836	-0.838	-0.840	-0.842	-0.844	-0.846	-0.848	-0.850	400
420	-0.850	-0.852	-0.854	-0.856	-0.858	-0.860	-0.862	-0.864	-0.866	-0.868	-0.870	420
440	-0.870	-0.872	-0.874	-0.876	-0.878	-0.880	-0.882	-0.884	-0.886	-0.888	-0.890	440
460	-0.890	-0.892	-0.894	-0.896	-0.898	-0.900	-0.902	-0.904	-0.906	-0.908	-0.910	460
480	-0.910	-0.912	-0.914	-0.916	-0.918	-0.920	-0.922	-0.924	-0.926	-0.928	-0.930	480
500	-0.930	-0.932	-0.934	-0.936	-0.938	-0.940	-0.942	-0.944	-0.946	-0.948	-0.950	500
520	-0.950	-0.952	-0.954	-0.956	-0.958	-0.960	-0.962	-0.964	-0.966	-0.968	-0.970	520
540	-0.970	-0.972	-0.974	-0.976	-0.978	-0.980	-0.982	-0.984	-0.986	-0.988	-0.990	540
560	-0.990	-0.992	-0.994	-0.996	-0.998	-1.000	-1.002	-1.004	-1.006	-1.008	-1.010	560
580	-1.010	-1.012	-1.014	-1.016	-1.018	-1.020	-1.022	-1.024	-1.026	-1.028	-1.030	580
600	-1.030	-1.032	-1.034	-1.036	-1.038	-1.040	-1.042	-1.044	-1.046	-1.048	-1.050	600
620	-1.050	-1.052	-1.054	-1.056	-1.058	-1.060	-1.062	-1.064	-1.066	-1.068	-1.070	620
640	-1.070	-1.072	-1.074	-1.076	-1.078	-1.080	-1.082	-1.084	-1.086	-1.088	-1.090	640
660	-1.090	-1.092	-1.094	-1.096	-1.098	-1.100	-1.102	-1.104	-1.106	-1.108	-1.110	660
680	-1.110	-1.112	-1.114	-1.116	-1.118	-1.120	-1.122	-1.124	-1.126	-1.128	-1.130	680
700	-1.130	-1.132	-1.134	-1.136	-1.138	-1.140	-1.142	-1.144	-1.146	-1.148	-1.150	700
720	-1.150	-1.152	-1.154	-1.156	-1.158	-1.160	-1.162	-1.164	-1.166	-1.168	-1.170	720
740	-1.170	-1.172	-1.174	-1.176	-1.178	-1.180	-1.182	-1.184	-1.186	-1.188	-1.190	740
760	-1.190	-1.192	-1.194	-1.196	-1.198	-1.200	-1.202	-1.204	-1.206	-1.208	-1.210	760
780	-1.210	-1.212	-1.214	-1.216	-1.218	-1.220	-1.222	-1.224	-1.226	-1.228	-1.230	780
800	-1.230	-1.232	-1.234	-1.236	-1.238	-1.240	-1.242	-1.244	-1.246	-1.248	-1.250	800
820	-1.250	-1.252	-1.254	-1.256	-1.258	-1.260	-1.262	-1.264	-1.266	-1.268	-1.270	820
840	-1.270	-1.272	-1.274	-1.276	-1.278	-1.280	-1.282	-1.284	-1.286	-1.288	-1.290	840
860	-1.290	-1.292	-1.294	-1.296	-1.298	-1.300	-1.302	-1.304	-1.306	-1.308	-1.310	860
880	-1.310	-1.312	-1.314	-1.316	-1.318	-1.320	-1.322	-1.324	-1.326	-1.328	-1.330	880
900	-1.330	-1.332	-1.334	-1.336	-1.338	-1.340	-1.342	-1.344	-1.346	-1.348	-1.350	900
920	-1.350	-1.352	-1.354	-1.356	-1.358	-1.360	-1.362	-1.364	-1.366	-1.368	-1.370	920
940	-1.370	-1.372	-1.374	-1.376	-1.378	-1.380	-1.382	-1.384	-1.386	-1.388	-1.390	940
960	-1.390	-1.392	-1.394	-1.396	-1.398	-1.400	-1.402	-1.404	-1.406	-1.408	-1.410	960
980	-1.410	-1.412	-1.414	-1.416	-1.418	-1.420	-1.422	-1.424	-1.426	-1.428	-1.430	980
1000	-1.430	-1.432	-1.434	-1.436	-1.438	-1.440	-1.442	-1.444	-1.446	-1.448	-1.450	1000

TYPE



NEW REFERENCE TABLES
SUPERSEDES N.B.S. CIRCULAR #504

Reference Junction at 32°F

DEG F 0 1 2 3 4 5 6 7 8 9 10 DEG F
* CONVERTED FROM DEGREES CELSIUS 1946-1.

—CONTINUED
TABLE X

NICKEL-CHROMIUM vs. COPPER-NICKEL
(Chromel-Constantan)

TYPE E

Temperature in Degrees F

Reference Junction at 32°F

DEG F	0	1	2	3	4	5	6	7	8	9	10	DEG F
THERMOELECTRIC VOLTAGE IN ABSOLUTE MILLIVOLTS												
450	15.029	15.071	15.112	15.150	15.189	15.226	15.262	15.298	15.333	15.368	15.402	450
460	15.249	15.291	15.333	15.375	15.417	15.459	15.501	15.543	15.585	15.627	15.668	460
470	15.670	15.712	15.755	15.797	15.839	15.881	15.923	15.965	16.007	16.049	16.091	470
480	16.093	16.135	16.178	16.220	16.262	16.304	16.347	16.389	16.431	16.473	16.515	480
490	16.517	16.559	16.602	16.644	16.687	16.729	16.772	16.814	16.857	16.899	16.942	490
500	16.982	17.024	17.067	17.109	17.152	17.194	17.237	17.279	17.322	17.364	17.407	500
510	17.449	17.492	17.534	17.577	17.619	17.662	17.704	17.747	17.789	17.832	17.874	510
520	17.917	17.959	18.002	18.044	18.087	18.129	18.172	18.214	18.257	18.299	18.342	520
530	18.384	18.427	18.469	18.512	18.554	18.597	18.639	18.682	18.724	18.767	18.809	530
540	18.852	18.895	18.937	18.980	19.022	19.065	19.107	19.150	19.192	19.235	19.277	540
550	19.319	19.362	19.404	19.447	19.489	19.532	19.574	19.617	19.659	19.702	19.744	550
560	19.787	19.830	19.872	19.915	19.957	20.000	20.042	20.085	20.127	20.170	20.212	560
570	20.254	20.297	20.339	20.382	20.424	20.467	20.509	20.552	20.594	20.637	20.679	570
580	20.722	20.764	20.807	20.849	20.892	20.934	20.977	21.019	21.062	21.104	21.147	580
590	21.189	21.232	21.274	21.317	21.359	21.402	21.444	21.487	21.529	21.572	21.614	590
600	21.657	21.699	21.742	21.784	21.827	21.869	21.912	21.954	21.997	22.039	22.082	600
610	22.124	22.167	22.209	22.252	22.294	22.337	22.379	22.422	22.464	22.507	22.549	610
620	22.592	22.634	22.677	22.719	22.762	22.804	22.847	22.889	22.932	22.974	23.017	620
630	23.059	23.102	23.144	23.187	23.229	23.272	23.314	23.357	23.399	23.442	23.484	630
640	23.527	23.570	23.612	23.655	23.697	23.740	23.782	23.825	23.867	23.910	23.952	640
650	23.995	24.037	24.080	24.122	24.165	24.207	24.250	24.292	24.335	24.377	24.420	650
660	24.462	24.504	24.547	24.589	24.632	24.674	24.717	24.759	24.802	24.844	24.887	660
670	24.929	24.972	25.014	25.057	25.099	25.142	25.184	25.227	25.269	25.312	25.354	670
680	25.397	25.440	25.482	25.525	25.567	25.610	25.652	25.695	25.737	25.780	25.822	680
690	25.865	25.907	25.950	25.992	26.035	26.077	26.120	26.162	26.205	26.247	26.290	690
700	26.332	26.375	26.417	26.460	26.502	26.545	26.587	26.630	26.672	26.715	26.757	700
710	26.799	26.842	26.884	26.927	26.969	27.012	27.054	27.097	27.139	27.182	27.224	710
720	27.267	27.309	27.352	27.394	27.437	27.479	27.522	27.564	27.607	27.649	27.692	720
730	27.734	27.777	27.819	27.862	27.904	27.947	27.989	28.032	28.074	28.117	28.159	730
740	28.202	28.245	28.287	28.330	28.372	28.415	28.457	28.500	28.542	28.585	28.627	740
750	28.670	28.712	28.755	28.797	28.840	28.882	28.925	28.967	29.010	29.052	29.095	750
760	29.137	29.180	29.222	29.265	29.307	29.350	29.392	29.435	29.477	29.520	29.562	760
770	29.605	29.648	29.690	29.733	29.775	29.818	29.860	29.903	29.945	29.988	30.030	770
780	30.073	30.115	30.158	30.200	30.243	30.285	30.328	30.370	30.413	30.455	30.498	780
790	30.540	30.583	30.625	30.668	30.710	30.753	30.795	30.838	30.880	30.923	30.965	790
800	31.008	31.051	31.093	31.136	31.178	31.221	31.263	31.306	31.348	31.391	31.433	800
810	31.476	31.518	31.561	31.603	31.646	31.688	31.731	31.773	31.816	31.858	31.901	810
820	31.943	31.986	32.028	32.071	32.113	32.156	32.198	32.241	32.283	32.326	32.368	820
830	32.411	32.453	32.496	32.538	32.581	32.623	32.666	32.708	32.751	32.793	32.836	830
840	32.878	32.920	32.963	33.005	33.048	33.090	33.133	33.175	33.218	33.260	33.303	840
850	33.345	33.388	33.430	33.473	33.515	33.558	33.600	33.643	33.685	33.728	33.770	850
860	33.813	33.856	33.898	33.941	33.983	34.026	34.068	34.111	34.153	34.196	34.238	860
870	34.281	34.323	34.366	34.408	34.451	34.493	34.536	34.578	34.621	34.663	34.706	870
880	34.748	34.791	34.833	34.876	34.918	34.961	35.003	35.046	35.088	35.131	35.173	880
890	35.216	35.258	35.301	35.343	35.386	35.428	35.471	35.513	35.556	35.598	35.641	890
900	35.683	35.726	35.768	35.811	35.853	35.896	35.938	35.981	36.023	36.066	36.108	900
910	36.151	36.194	36.236	36.279	36.321	36.364	36.406	36.449	36.491	36.534	36.576	910
920	36.619	36.662	36.704	36.747	36.789	36.832	36.874	36.917	36.959	37.002	37.044	920
930	37.087	37.130	37.172	37.215	37.257	37.300	37.342	37.385	37.427	37.470	37.512	930
940	37.555	37.597	37.640	37.682	37.725	37.767	37.810	37.852	37.895	37.937	37.980	940
950	38.022	38.065	38.107	38.150	38.192	38.235	38.277	38.320	38.362	38.405	38.447	950
960	38.490	38.533	38.575	38.618	38.660	38.703	38.745	38.788	38.830	38.873	38.915	960
970	38.958	39.001	39.043	39.086	39.128	39.171	39.213	39.256	39.298	39.341	39.383	970
980	39.426	39.469	39.511	39.554	39.596	39.639	39.681	39.724	39.766	39.809	39.851	980
990	39.894	39.937	39.979	40.022	40.064	40.107	40.149	40.192	40.234	40.277	40.319	990
1000	40.362	40.404	40.447	40.489	40.532	40.574	40.617	40.659	40.702	40.744	40.787	1000
1010	40.829	40.872	40.914	40.957	40.999	41.042	41.084	41.127	41.169	41.212	41.254	1010
1020	41.297	41.340	41.382	41.425	41.467	41.510	41.552	41.595	41.637	41.680	41.722	1020
1030	41.765	41.808	41.850	41.893	41.935	41.978	42.020	42.063	42.105	42.148	42.190	1030
1040	42.233	42.275	42.318	42.360	42.403	42.445	42.488	42.530	42.573	42.615	42.658	1040
1050	42.699	42.742	42.784	42.827	42.869	42.912	42.954	42.997	43.039	43.082	43.124	1050
1060	43.167	43.209	43.252	43.294	43.337	43.379	43.422	43.464	43.507	43.549	43.592	1060
1070	43.634	43.677	43.719	43.762	43.804	43.847	43.889	43.932	43.974	44.017	44.059	1070
1080	44.102	44.145	44.187	44.230	44.272	44.315	44.357	44.400	44.442	44.485	44.527	1080
1090	44.570	44.613	44.655	44.698	44.740	44.783	44.825	44.868	44.910	44.953	44.995	1090
1100	45.038	45.081	45.123	45.166	45.208	45.251	45.293	45.336	45.378	45.421	45.463	1100
1110	45.506	45.549	45.591	45.634	45.676	45.719	45.761	45.804	45.846	45.889	45.931	1110
1120	45.974	46.017	46.059	46.102	46.144	46.187	46.229	46.272	46.314	46.357	46.399	1120
1130	46.442	46.484	46.527	46.569	46.612	46.654	46.697	46.739	46.782	46.824	46.867	1130
1140	46.909	46.951	46.994	47.036	47.079	47.121	47.164	47.206	47.249	47.291	47.334	1140
1150	47.376	47.419	47.461	47.504	47.546	47.589	47.631	47.674	47.716	47.759	47.801	1150
1160	47.844	47.887	47.929	47.972	48.014	48.057	48.099	48.142	48.184	48.227	48.269	1160
1170	48.312	48.354	48.397	48.439	48.482	48.524	48.567	48.609	48.652	48.694	48.737	1170
1180	48.779	48.821	48.864	48.906	48.949	48.991	49.034	49.076	49.119	49.161	49.204	1180
1190	49.246	49.289	49.331	49.374	49.416	49.459	49.501	49.544	49.586	49.629	49.671	1190
1200	49.714	49.757	49.799	49.842	49.884	49.927	49.969	50.012	50.054	50.097	50.139	1200
1210	50.182	50.224	50.267	50.309	50.352	50.394	50.437	50.479	50.522	50.564	50.607	1210
1220	50.649	50.692	50.734	50.777	50.819	50.862	50.904	50.947	50.989	51.032	51.074	1220
1230	51.117	51.159	51.202	51.244	51.287	51.329	51.372	51.414	51.457	51.499	51.542	1230
1240	51.584	51.627	51.66969									

—CONTINUED
TABLE X

NICKEL-CHROMIUM vs. COPPER-NICKEL
(Chromel-Constantan)

TYPE E

Temperature in Degrees F

Reference Junction at 32°F

TYPE



NEW REFERENCE TABLES
SUPERSEDES NBS CIRCULAR #501

DEG F	0	1	2	3	4	5	6	7	8	9	10	DEG F
THERMOELECTRIC VOLTAGE IN ABSOLUTE MILLIVOLTS												
1400	57.873	57.917	57.961	58.005	58.049	58.093	58.137	58.181	58.225	58.269	58.313	1400
1410	58.312	58.356	58.400	58.444	58.487	58.531	58.575	58.619	58.663	58.707	58.750	1410
1420	58.750	58.794	58.838	58.882	58.926	58.969	59.013	59.057	59.101	59.144	59.188	1420
1430	59.188	59.232	59.276	59.319	59.363	59.407	59.451	59.494	59.538	59.582	59.626	1430
1440	59.626	59.669	59.713	59.757	59.800	59.844	59.888	59.932	59.975	60.019	60.063	1440
1450	60.063	60.106	60.150	60.194	60.237	60.281	60.325	60.368	60.412	60.455	60.499	1450
1460	60.499	60.543	60.586	60.630	60.674	60.717	60.761	60.804	60.848	60.892	60.935	1460
1470	60.935	60.979	61.022	61.066	61.109	61.153	61.197	61.240	61.284	61.327	61.371	1470
1480	61.371	61.415	61.458	61.501	61.545	61.588	61.632	61.675	61.719	61.762	61.806	1480
1490	61.806	61.849	61.893	61.936	61.980	62.023	62.067	62.110	62.154	62.197	62.240	1490
1500	62.240	62.284	62.327	62.371	62.414	62.458	62.501	62.545	62.588	62.632	62.675	1500
1510	62.675	62.718	62.762	62.805	62.849	62.892	62.936	62.979	63.023	63.066	63.109	1510
1520	63.109	63.152	63.196	63.239	63.282	63.326	63.369	63.412	63.456	63.499	63.542	1520
1530	63.542	63.585	63.629	63.672	63.715	63.759	63.802	63.846	63.889	63.932	63.975	1530
1540	63.975	64.018	64.061	64.105	64.148	64.191	64.235	64.278	64.321	64.365	64.408	1540
1550	64.408	64.451	64.495	64.538	64.581	64.625	64.668	64.711	64.755	64.798	64.841	1550
1560	64.841	64.884	64.928	64.971	65.014	65.058	65.101	65.145	65.188	65.231	65.275	1560
1570	65.275	65.318	65.361	65.405	65.448	65.491	65.535	65.578	65.621	65.665	65.708	1570
1580	65.708	65.751	65.794	65.838	65.881	65.924	65.968	66.011	66.054	66.098	66.141	1580
1590	66.141	66.184	66.228	66.271	66.314	66.358	66.401	66.444	66.488	66.531	66.574	1590
1600	66.574	66.617	66.661	66.704	66.747	66.790	66.834	66.877	66.920	66.963	67.006	1600
1610	67.006	67.049	67.092	67.136	67.179	67.222	67.265	67.308	67.352	67.395	67.438	1610
1620	67.438	67.481	67.524	67.567	67.610	67.653	67.696	67.739	67.782	67.825	67.868	1620
1630	67.868	67.911	67.954	67.997	68.040	68.083	68.126	68.169	68.212	68.255	68.298	1630
1640	68.298	68.341	68.384	68.427	68.470	68.513	68.556	68.599	68.642	68.685	68.728	1640
1650	68.728	68.771	68.814	68.857	68.900	68.943	68.986	69.029	69.072	69.115	69.158	1650
1660	69.158	69.201	69.244	69.287	69.330	69.373	69.416	69.459	69.502	69.545	69.588	1660
1670	69.588	69.631	69.674	69.717	69.760	69.803	69.846	69.889	69.932	69.975	70.018	1670
1680	70.018	70.061	70.104	70.147	70.190	70.233	70.276	70.319	70.362	70.405	70.448	1680
1690	70.448	70.491	70.534	70.577	70.620	70.663	70.706	70.749	70.792	70.835	70.878	1690
1700	70.878	70.921	70.964	71.007	71.050	71.093	71.136	71.179	71.222	71.265	71.308	1700
1710	71.308	71.351	71.394	71.437	71.480	71.523	71.566	71.609	71.652	71.695	71.738	1710
1720	71.738	71.781	71.824	71.867	71.910	71.953	71.996	72.039	72.082	72.125	72.168	1720
1730	72.168	72.211	72.254	72.297	72.340	72.383	72.426	72.469	72.512	72.555	72.598	1730
1740	72.598	72.641	72.684	72.727	72.770	72.813	72.856	72.899	72.942	72.985	73.028	1740
1750	73.028	73.071	73.114	73.157	73.200	73.243	73.286	73.329	73.372	73.415	73.458	1750
1760	73.458	73.501	73.544	73.587	73.630	73.673	73.716	73.759	73.802	73.845	73.888	1760
1770	73.888	73.931	73.974	74.017	74.060	74.103	74.146	74.189	74.232	74.275	74.318	1770
1780	74.318	74.361	74.404	74.447	74.490	74.533	74.576	74.619	74.662	74.705	74.748	1780
1790	74.748	74.791	74.834	74.877	74.920	74.963	75.006	75.049	75.092	75.135	75.178	1790
1800	75.178	75.221	75.264	75.307	75.350	75.393	75.436	75.479	75.522	75.565	75.608	1800
1810	75.608	75.651	75.694	75.737	75.780	75.823	75.866	75.909	75.952	75.995	76.038	1810
1820	76.038	76.081	76.124	76.167	76.210	76.253	76.296	76.339	76.382	76.425	76.468	1820
1830	76.468	76.511	76.554	76.597	76.640	76.683	76.726	76.769	76.812	76.855	76.898	1830

DEG F 0 1 2 3 4 5 6 7 8 9 10 DEG F
* CONVERTED FROM DEGREES CELSIUS 1962.

TABLE XI

COPPER vs. COPPER-NICKEL
(Copper-Constantan)

TYPE T

Temperature in Degrees F

Reference Junction at 32°F

TYPE



NEW REFERENCE TABLES
SUPERSEDES NBS CIRCULAR #501

DEG F	0	1	2	3	4	5	6	7	8	9	10	DEG F
THERMOELECTRIC VOLTAGE IN ABSOLUTE MILLIVOLTS												
-450	-6.254	-6.255	-6.256	-6.257	-6.258							-450
-440	-6.240	-6.242	-6.243	-6.245	-6.247	-6.248	-6.250	-6.251	-6.253	-6.255	-6.256	-440
-430	-6.217	-6.220	-6.223	-6.225	-6.227	-6.230	-6.232	-6.234	-6.236	-6.238	-6.240	-430
-420	-6.187	-6.191	-6.194	-6.197	-6.200	-6.203	-6.206	-6.209	-6.212	-6.215	-6.217	-420
-410	-6.150	-6.154	-6.158	-6.162	-6.166	-6.170	-6.173	-6.177	-6.181	-6.184	-6.187	-410
-400	-6.105	-6.110	-6.115	-6.119	-6.124	-6.128	-6.133	-6.137	-6.142	-6.146	-6.150	-400
-390	-6.053	-6.059	-6.064	-6.069	-6.075	-6.080	-6.085	-6.090	-6.095	-6.100	-6.105	-390
-380	-6.000	-6.006	-6.012	-6.017	-6.023	-6.028	-6.033	-6.038	-6.043	-6.048	-6.053	-380
-370	-5.950	-5.957	-5.963	-5.970	-5.975	-5.981	-5.986	-5.991	-5.996	-6.002	-6.007	-370
-360	-5.900	-5.907	-5.914	-5.920	-5.926	-5.931	-5.937	-5.942	-5.947	-5.953	-5.958	-360
-350	-5.850	-5.857	-5.864	-5.870	-5.876	-5.881	-5.887	-5.892	-5.897	-5.903	-5.908	-350
-340	-5.800	-5.807	-5.814	-5.820	-5.826	-5.831	-5.837	-5.842	-5.847	-5.853	-5.858	-340
-330	-5.750	-5.757	-5.764	-5.770	-5.776	-5.781	-5.787	-5.792	-5.797	-5.803	-5.808	-330
-320	-5.700	-5.707	-5.714	-5.720	-5.726	-5.731	-5.737	-5.742	-5.747	-5.753	-5.758	-320
-310	-5.650	-5.657	-5.664	-5.670	-5.676	-5.681	-5.687	-5.692	-5.697	-5.703	-5.708	-310
-300	-5.600	-5.607	-5.614	-5.620	-5.626	-5.631	-5.637	-5.642	-5.647	-5.653	-5.658	-300
-290	-5.550	-5.557	-5.564	-5.570	-5.576	-5.581	-5.587	-5.592	-5.597	-5.603	-5.608	-290
-280	-5.500	-5.507	-5.514	-5.520	-5.526	-5.531	-5.537	-5.542	-5.547	-5.553	-5.558	-280
-270	-5.450	-5.457	-5.464	-5.470	-5.476	-5.481	-5.487	-5.492	-5.497	-5.503	-5.508	-270
-260	-5.400	-5.407	-5.414	-5.420	-5.426	-5.431	-5.437	-5.442	-5.447	-5.453	-5.458	-260
-250	-5.350	-5.357	-5.364	-5.370	-5.376	-5.381	-5.387	-5.392	-5.397	-5.403	-5.408	-250
-240	-5.300	-5.307	-5.314	-5.320	-5.326	-5.331	-5.337	-5.342	-5.347	-5.353	-5.358	-240
-230	-5.250	-5.257	-5.264	-5.270	-5.276	-5.281	-5.287	-5.292	-5.297	-5.303	-5.308	-230
-220	-5.200	-5.207	-5.214	-5.220	-5.226	-5.231	-5.237	-5.242	-5.247	-5.253	-5.258	-220
-210	-5.150	-5.157	-5.164	-5.170	-5.176	-5.181	-5.187	-5.192	-5.197	-5.203	-5.208	-210
-200	-5.100	-5.107	-5.114	-5.120	-5.126	-5.131	-5.137	-5.142	-5.147	-5.153	-5.158	-200

DEG F 0 1 2 3 4 5 6 7 8 9 10 DEG F
* CONVERTED FROM DEGREES CELSIUS 1962.

Coax Gage Data Reduction Program

274

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      SUBROUTINE COAXDATA
      COMMON H(2,2000),TBF(2,2000),QDOT(2,2000)
      1      ,TIME(2000),TW(2,2000),Q(2,2000),X(40),Y(40)
      CCCCCCCC READ IN THE DATA
      READ(8,*) XM,PO,TISO
      READ(8,*) NCOAX,NPTS
      DO 120 N=1,NCOAX
        DO 125 J=1,NPTS
          READ(8,*) TIME(J),TW(N,J),TBF(N,J)
125      CONTINUE
120      CONTINUE
      TSTAG=473.15
      7      CONTINUE
      TYPE 104
104      FORMAT(' DO YOU WISH TO (1)SMOOTH TW .',/
      & '(9)EXIT OR, (2)REDUCE DATA'/)
      ACCEPT *, IANSWER
105      FORMAT(A1)
      IF (IANSWER .EQ. 2) GO TO 20
      IF (IANSWER .EQ. 9) GO TO 99
      C ***** SMOOTH THE DATA *****
      PRINT 106
106      FORMAT(' ENTER (ODD) NO. OF POINTS TO FIT,
      & ' I3 FORMAT'/)
      ACCEPT *, IFITT
      IS=IFITT/2 + 1
      IEND=NPTS-IS+1
      DO 299 N=1,NCOAX
        DO 9 I=IS,IEND
          DO 8 J=1,IFITT
            K=I-IS+J
            X(J)=TIME(K)
            Y(J)=TW(L,K)
      8      CONTINUE
            CALL LSTSQR(TW(L,I),DYDX,IFITT)
      9      CONTINUE
299      CONTINUE
      GO TO 7
      C ***** REDUCE THE DATA *****
      20      CONTINUE
      PRINT 300
300      FORMAT('CHOOSE THE METHOD OF DATA REDUCTION .',/
      & '(A)EDC OR (N)SWC .',/
      ACCEPT *, MET
      IF (MET .EQ. 1HA) THEN
        CALL AEDC(NCOAX,NPTS,TSTAG)
      ENDIF
      IF (MET .EQ. 1HN) THEN
        CALL NSWC(NCOAX,NPTS,TSTAG)
      ENDIF
      99      RETURN
      END

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

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C
      SUBROUTINE AEDC(NCOAX,NPTS,TSTAG)
      COMMON H(2,2000),TBF(2,2000),QDOT(2,2000)
      1      ,TIME(2000),TW(2,2000),Q(2,2000),X(40),Y(40)
C ***** NOMENCLATURE *****
C RCPK      THERMAL PARAMETER      BTU**2/FT**4 SEC DEGREE R**2
C           R = DENSITY
C           CP = SPECIFIC HEAT CAPACITY
C           K = CONDUCTIVITY
C Q         CUMULATIVE HEATING      BTU/FT**2
C
      CASE=3
      96  FORMAT('COAX AEDC REDUCTION METHOD ')
      PRINT 96
      PRINT 97
      97  FORMAT(' ***** WORKING ***** ')
      WRITE(10,*) XM,PO,TISO
      WRITE(10,*) NCOAX,NPTS,CASE
      ISTART=1
      IEND=NPTS
      RCPK=0.1673
C CHANGE FROM BTU**2/FT**4 SEC DEGREE R**2 TO METRIC UNITS OF
C      J**2/ M**4 SEC DEGREE K**2
      RCPK=RCPK*(1.055E3)**2/((.3048)**4*(9./5.))**2)
      DO 299 N=1,NCOAX
      Q(N,ISTART)=0.
      J1=ISTART+1
      DO 2 J=J1,IEND
      J2=J-1
      SUM=0.
      DO 1 I=ISTART,J2
      SUM=SUM+((TW(N,I+1)-TW(N,ISTART)+TW(N,I)
      &          -TW(N,ISTART))/(SQRT(TIME(J)-TIME(I))
      &          +SQRT(TIME(J)-TIME(I+1))))
      1  CONTINUE
      Q(N,J)=SQRT(RCPK/3.14159)*(TIME(J)-TIME(J-1))
      &      *SUM
      2  CONTINUE
      IS=9
      IE=IEND-8
      DO 20 I=9,IE
      QDOT(N,I)=(-2.*Q(N,I-8)-Q(N,I-4)+Q(N,I+4)+2.*Q(N,I+8))
      &      /(((TIME(I)-TIME(I-1))*40.))
      TAW=.9*TSTAG
      H(N,I)=QDOT(N,I)/(TAW-TW(N,I))
      WRITE(7,*) TIME(I),TBF(N,I),TW(N,I),QDOT(N,I),H(N,I)
      20  CONTINUE
      DO 663 I=1,8
      QDOT(N,I)=QDOT(N,9)
      QDOT(N,I+IE)=QDOT(N,IE)
      663  CONTINUE

```

```

        DO 201 J=1,NPTS
            WRITE(10,*) TW(N,J),Q(N,J)
201    CONTINUE
299    CONTINUE
        RETURN
        END
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
        SUBROUTINE NSWC(NCOAX,NPTS,TSTAG)
            DIMENSION TT(2,10)
            COMMON H(2,2000),TBF(2,2000),QDOT(2,2000)
            1    ,TIME(2000),TW(2,2000),Q(2,2000),X(40),Y(40)
            CASE=3
            PRINT 96
            96    FORMAT(' COAX NSWC REDUCTION METHOD ')
            PRINT 97
            97    FORMAT('***** WORKING *****')
C ***** NOMENCLATURE *****
C    THICK      LENGTH OF COAX GAGE (FT)
C    CP         HEAT CAPACITY (BTU / LB DEGREE R)
C    RHO        DENSITY
C              =.322*1728 = 556.42 LB/FT**3
C              (= .322 LB/IN**3)
C    COND       CONDUCTIVITY
C              =.000267*12 = .003204 BTU/FT SEC DEGREE R
C              (= .000267 BTU/IN SEC DEGREE R)
C              (= .000267*12*3600 = 11.53 BTU/FT HR DEGREE R)
C    THET       STABILITY PARAMETER
C ***** PRELIMINARY CALCULATIONS *****
            WRITE(10,*) XM,PO,TISO
            WRITE(10,*) NCOAX,NPTS,CASE
            DO 299 N=1,NCOAX
                NODES=10
                NODEM1=NODES-1
                HIOT=TIME(2)-TIME(1)
                THICK=0.03125
                DX=THICK/NODEM1
C CHANGE CP TO J/ KG DEGREE K
                CP=0.094*1.055E3/(.4536*(9./5.))
C CHANGE RHO TO KG/ M**3
                RHO=0.322*1728.0*.4536/(.3048)**3
C CHANGE COND TO J/M SEC DEGREE K
                COND=0.267E-3 * 12.0 * 1.055E3/(.3048*(9./5.))
                CONDUX=COND/DX
                THETOT=COND/(CP*RHO*DX**2)
                M=0
                ADJUST=TBF(N,1)-TW(N,1)
                DO 52 I=1,NPTS
                    TBF(N,I)=TBF(N,I)-ADJUST
52    CONTINUE

```



```

DO 102 I=1,NODES
  TT(1,I)=TW(N,1)
102  CONTINUE
    L1=1
    L2=2
C LOOP TIME INTERVALS
  DO 302 IHI=2,NPTS
C THET MUST BE LESS THAN 0.50 FOR PROCEDURE TO
C BE NUMERICALLY STABLE
    THET=TIME(IHI)-TIME(IHI-1)
    DTLO=THET/THETOT
    NTLO=HIDT/DTLO
    IF (NTLO .GE. 0) GO TO 32
    NTLO=IABS(NTLO)
32   CONTINUE
    NTLO=NTLO+1
    DTLO=HIDT/NTLO
    THET=THETOT*DTLO
    B=1.0-(2.0*THET)
C LOOP TIME SUBINTERVALS
    DO 252 ITLO=1,NTLO
      DO 202 INODE=2,NODEM1
        TT(L2,INODE)=THET*(TT(L1,INODE+1)
&      +TT(L1,INODE-1))+B*TT(L1,INODE)
202   CONTINUE
        TFAC=FLOAT(ITLO)/FLOAT(NTLO)
        TT(L2,1)=TW(N,IHI-1) + (TW(N,IHI)
&      TW(N,IHI-1))*TFAC
        TT(L2,NODES)=TBF(N,IHI-1)+(TBF(N,I
&      - TBF(N,IHI-1))*TFAC
        LHOLD=L1
        L1=L2
        L2=LHOLD
252   CONTINUE
C NEW WALL TEMPERATURES NOW AT L1
C AT THE INPUT RECORDED TIME, TIME(IHI)
    QDOT(N,IHI)=-(CONDUX/2.)*(-3.*TT(L1
&      4.*TT(L1,2)-TT(L1,3))
    TAW=.9*TSTAG
    H(N,IHI)=QDOT(N,IHI)/(TAW-TW(N,IHI)
    WRITE(7,*) TIME(IHI),TBF(N,IHI),TW
&      QDOT(N,IHI),H(N,IHI)
302  CONTINUE

```

```

      DO 201 I=1,NPTS
        WRITE(10,*) TW(N,I),Q(N,I)
201    CONTINUE
299    CONTINUE
      RETURN
      END

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE LSTSQR(YFIT,DYDX,IFIT)
      COMMON H(2,2000),TBF(2,2000),QDOT(2,2000)
1      ,TIME(2000),TW(2,2000),Q(2,2000),X(40),Y(40)
C ** FIRST ORDER LEAST SQUARES FIT OF IFIT POINTS,
C **   DYDX=F(X(MIDPT))
      MIDPT=IFIT/2 + 1
      A=0.
      B=0.
      C=0.
      D=0.
      E=0.
      F=0.
      G=0.
      DO 2 I=1,IFIT
        A=A+X(I)
        B=B+X(I)**2
        C=C+X(I)**3
        D=D+X(I)**4
        E=E+Y(I)
        F=F+(X(I)*Y(I))
        G=G+(X(I)*X(I)*Y(I))
2      CONTINUE
      B1=(A*E-IFIT*F)/(A*A-IFIT*B)
      A1=(E-B1*A)/IFIT
      YFIT=A1+B1*X(MIDPT)
      DYDX=B1
      RETURN
      END

```

Appendix S

Test Parameter/Photo Catalog Number

Photo No.	Photo Description	Test
85-2334	Calibration, rake in uninjected position	
85-2335	Calibration, rake in injected position	
85-2336	Calibration, instrumentation	
85-2356	Oil flow, plexiglas flat plate, P=20 bar, $\alpha=0^\circ$	
85-2357	Oil flow, plexiglas flat plate, P=10 bar, $\alpha=0^\circ$	
85-2358	Oil flow, plexiglas flat plate, P=20 bar, $\alpha=-5.5^\circ$	
85-2359	Oil flow, plexiglas flat plate, P=10 bar, $\alpha=-5.5^\circ$	
85-2361	Oil flow, aluminum flat plate, P=20 bar, $\alpha=-5.5^\circ$	O-2
85-2362	Oil flow, aluminum flat plate, P=20 bar, $\alpha=-5.5^\circ$	
85-2364	Oil flow, aluminum flat plate, P=20 bar, $\alpha=0^\circ$	
85-2365	Oil flow, aluminum flat plate, P=20 bar, $\alpha=0^\circ$	
85-2366	Oil flow, aluminum flat plate, P=10 bar, $\alpha=0^\circ$	
85-2481	Oil flow, aluminum flat plate, P=20 bar, $\alpha=0^\circ$	O-1
85-2482	Oil flow, aluminum flat plate, P=20 bar, $\alpha=0^\circ$	
85-2483	Oil flow, aluminum flat plate, P=10 bar, $\alpha=0^\circ$	
85-Z	Double pass schlieren, P=20 bar, $\alpha=-5.5^\circ$, $\varnothing=40^\circ$, $\Omega=60^\circ$	
85-Y	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=40^\circ$, $\Omega=60^\circ$	
85-X	Double pass schlieren, P=20 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=60^\circ$	
85-V	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=60^\circ$	
85-U	Double pass schlieren, P=20 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=0^\circ$	
85-T	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=0^\circ$	
85-S	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=30^\circ$	
85-Q	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=40^\circ$, $\Omega=30^\circ$	
85-P	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=40^\circ$, $\Omega=75^\circ$	
85-N	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=40^\circ$, $\Omega=75^\circ$	
85-M	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-K	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-J	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=15^\circ$, $\Omega=30^\circ$	
85-I	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=30^\circ$	
85-H	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=60^\circ$	
85-G	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=15^\circ$, $\Omega=60^\circ$	
85-F	Double pass schlieren, P=10 bar, $\alpha=0^\circ$, $\varnothing=15^\circ$, $\Omega=75^\circ$	
85-E	Double pass schlieren, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=75^\circ$	
85-2486	Sublimation, P=20 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-2487	Sublimation, P=20 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-2488	Sublimation, P=10 bar, $\alpha=-5.5^\circ$, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-2489	Sublimation, P=10 bar, $\varnothing=15^\circ$, $\Omega=0^\circ$	WS-1
85-2490	Sublimation, P=20 bar, $\varnothing=15^\circ$, $\Omega=0^\circ$	
85-2491	Sublimation, P=20 bar, $\varnothing=15^\circ$, $\Omega=30^\circ$	WS-3

85-2492	Sublimation, P=10 bar, $\theta=15^\circ$, $\Omega=30^\circ$	WS-2
85-2497	Sublimation, P=10 bar, $\theta=15^\circ$, $\Omega=60^\circ$	WS-4
85-2498	Sublimation, P=20 bar, $\theta=15^\circ$, $\Omega=60^\circ$	
85-2499	Sublimation, P=10 bar, $\theta=15^\circ$, $\Omega=75^\circ$	WS-5
85-2507	Sublimation, P=20 bar, $\theta=40^\circ$, $\Omega=0^\circ$	
85-2508	Sublimation, P=20 bar, $\theta=40^\circ$, $\Omega=0^\circ$	
85-2509	Sublimation, P=10 bar, $\theta=40^\circ$, $\Omega=0^\circ$	WS-6
85-2510	Sublimation, P=10 bar, $\theta=40^\circ$, $\Omega=30^\circ$	WS-7
85-2511	Sublimation, P=10 bar, $\theta=40^\circ$, $\Omega=60^\circ$	WS-8
85-2512	Sublimation, P=10 bar, $\theta=40^\circ$, $\Omega=75^\circ$	WS-9
85-2513	Sublimation, P=10 bar, $\theta=0^\circ$, $\Omega=0^\circ$	
86-59	Sublimation, P=20 bar, $\theta=15^\circ$, $\Omega=0^\circ$, h=15mm	
86-78	Sublimation, P=20 bar, $\theta=0^\circ$, $\Omega=0^\circ$, h=15mm	
86-79	Sublimation, P=10 bar, $\theta=0^\circ$, $\Omega=0^\circ$, h=15mm	
86-80	Sublimation, P=20 bar, $\theta=40^\circ$, $\Omega=0^\circ$, h=15mm	
86-81	Sublimation, P=10 bar, $\theta=40^\circ$, $\Omega=0^\circ$, h=15mm	WS-10

85-2514	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=0^\circ$, $\Omega=0^\circ$	WO-11
85-2515	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=0^\circ$	
85-2518	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=0^\circ$ *	WO-1
85-2521	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=0^\circ$	WO-13
85-2522	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=30^\circ$	WO-2
85-2527	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=30^\circ$	WO-3
85-2528	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=30^\circ$	
85-2529	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=30^\circ$	
85-2530	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=60^\circ$	
85-2533	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=60^\circ$	WO-4
85-2534	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=75^\circ$	WO-5
85-2535	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=75^\circ$	
85-2536	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=0^\circ$	
85-2537	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=0^\circ$	WO-6
85-2538	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=30^\circ$	
85-2539	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=30^\circ$	WO-7
85-2540	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=60^\circ$	
85-2541	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=60^\circ$	WO-8
85-2542	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=75^\circ$	
85-2543	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=75^\circ$	WO-9
86-53	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=0^\circ$, h=15mm	
86-54	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=40^\circ$, $\Omega=0^\circ$, h=15mm	WO-10
86-55	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=0^\circ$, $\Omega=0^\circ$, h=15mm	WO-12
86-56	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=0^\circ$, $\Omega=0^\circ$, h=15mm	WO-14
86-57	Oil flow, P=20 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=0^\circ$, h=15mm	
86-58	Oil flow, P=10 bar, $\alpha=0^\circ$, $\theta=15^\circ$, $\Omega=0^\circ$, h=15mm	

* Started using motor oil

86-264	Heat transfer model installed in tunnel
86-265	Heat transfer data acquisition set-up
86-266	Heat transfer data acquisition set-up

Color
Slide
No

Photograph Description

1	Liquid Crystal, L-2, P=10, time = 4.08 seconds
2	Liquid Crystal, L-2, P=10, time = 5.54 seconds
3	Liquid Crystal, L-2, P=10, time = 6.00 seconds
4	Liquid Crystal, L-2, P=10, time = 6.45 seconds
5	Liquid Crystal, L-2, P=10, time = 6.91 seconds
6	Liquid Crystal, L-2, P=10, time = 7.38 seconds
7	Liquid Crystal, L-2, P=10, time = 7.84 seconds
8	Liquid Crystal, L-2, P=10, time = 8.30 seconds
9	Liquid Crystal, L-2, P=10, time = 8.76 seconds
10	Liquid Crystal, L-2, P=10, time = 9.23 seconds
11	Liquid Crystal, L-2, P=10, time = 9.69 seconds
12	Liquid Crystal, L-2, P=10, time = 10.15 seconds
13	Liquid Crystal, L-2, P=10, time = 10.41 seconds
14	Liquid Crystal, L-1, P=20, time = 3.03 seconds
15	Liquid Crystal, L-1, P=20, time = 3.42 seconds
16	Liquid Crystal, L-1, P=20, time = 3.85 seconds
17	Liquid Crystal, L-1, P=20, time = 4.28 seconds
18	Liquid Crystal, L-1, P=20, time = 4.70 seconds
19	Liquid Crystal, L-1, P=20, time = 5.13 seconds
20	Liquid Crystal, L-1, P=20, time = 5.56 seconds
21	Liquid Crystal, L-1, P=20, time = 5.98 seconds
32	Liquid Crystal Calibration, Temperature = 24.2 °C
34	Liquid Crystal Calibration, Temperature = 23.4 °C
35	Liquid Crystal Calibration, Temperature = 24.8 °C

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Vita

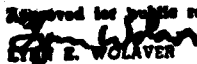
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This study combines the techniques of computational fluid dynamics and experimentation.

An unsteady laminar boundary layer code was shown to be more efficient when modified to solve the momentum and energy equations using Thomas' algorithm instead of optimized successive overrelaxation (SOR). A steady eddy viscosity turbulence model was then added to the code: two models for the inner region, Van Driest and Chapman, and two models for the outer region, a mixing length and Clauser were used. The computer code was verified against theoretical and experimental data from the literature. The code shows excellent results for laminar flow and each of the models showed very good agreement with turbulent experimental data.

The experimental study was conducted in a Mach 6 blowdown wind tunnel. Oil flow, sublimation, liquid crystals, and thin film gages were used to examine the flow structure and heat transfer on a flat plate. The heat transfer rates from these gages were lower than from the liquid crystals. The viscid-inviscid interaction flow region of flow past a blunt fin mounted on the flat plate was examined using the same flow visualization methods and possible flow interaction models are presented. Striation heating appeared when the fin incidence was 40° and the sweep was 60 or 75° .

The numerical and experimental techniques of this study were then merged. The heat transfer results from the flat plate experiment were compared with results from the computer code and were found to be in good agreement. (The)

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