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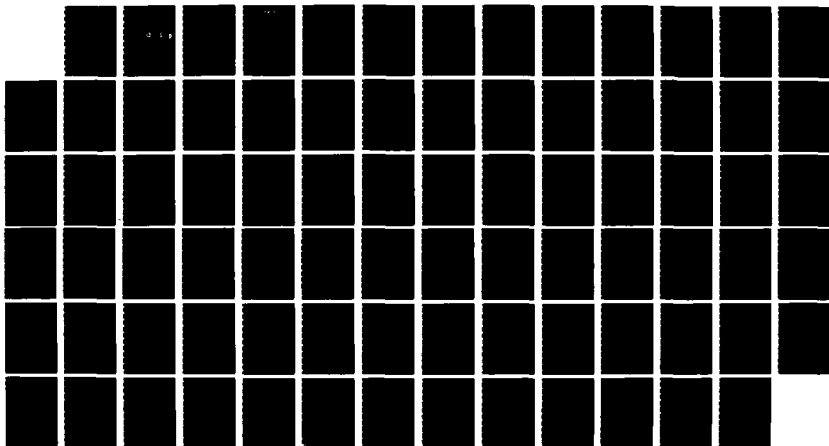
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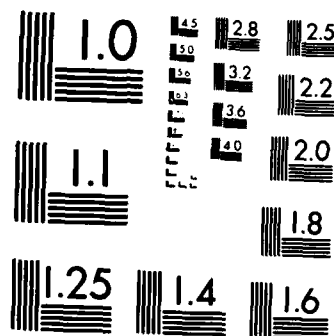
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ANALYSIS OF ALERT RATE CHANGES AND PROTECTION ASSOCIATED WITH ACTIVE BCAS THREAT CRITERIA

(Institute for Defense Analyses Paper P-1512)

Irvin W. Kay



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FINAL REPORT

Prepared for
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16. Abstract Protection afforded by active BCAS is analyzed in terms of the possible violation of a prescribed minimum allowable miss distance standard. Part of the analysis is based on a theoretical distribution of aircraft velocities at low altitudes in a terminal area. This velocity distribution is taken from a mathematical model of air traffic similar to one used previously to fit the Mitre Los Angeles Standard Traffic Model. It is shown that the present version of the mathematical traffic model fits data derived from the Houston Intercontinental Airport environment. That data was originally extracted from ARTS tapes and processed by Mitre to determine, among other things, the effect of various BCAS alert logic parameter choices on alert rates that would have occurred at Houston if the aircraft involved had been equipped with BCAS. This paper demonstrates that the mathematical model can be used to predict accurately changes in the Houston alert rates due to changes in BCAS alert logic parameters. <i>Requires BCAS (Procedural Collision Avoidance System), Air traffic control terminal area.</i>			
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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
AREA				
m ²	square meters	0.5	square centimeters	cm ²
ft ²	square feet	0.09	square meters	m ²
yd ²	square yards	0.8	square meters	m ²
mi ²	square miles	2.5	square kilometers	km ²
acres	acres	0.4	hectares	ha
MASS (weight)				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
VOLUME				
teaspoon	teaspoons	5	milliliters	ml
Tablespoon	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.96	liters	l
gal	gallons	3.8	liters	l
cu ft	cubic feet	0.03	cubic meters	m ³
yd ³	cubic yards	0.76	cubic meters	m ³
TEMPERATURE (exact)				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

* 1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Mon., Publ. 286, Units of Weight and Measure, Price \$2.25, SO Catalog No. C13.10 286.

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
AREA				
cm ²	square centimeters	0.16	square inches	in ²
m ²	square meters	1.2	square yards	yd ²
km ²	square kilometers	0.4	square miles	mi ²
ha	hectares (10,000 m ²)	2.5	acres	acres
MASS (weight)				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	short tons
VOLUME				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m ³	cubic meters	35	cubic feet	ft ³
m ³	cubic meters	1.3	cubic yards	yd ³
TEMPERATURE (exact)				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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I. INTRODUCTION

The purpose of this investigation is to analyze the protection afforded by active Beacon Collision Avoidance System (BCAS) alert logics in view of certain anticipated compromises that must be made in the logic parameters. It has been argued that such compromises are necessary in order to prevent the alert rate that the system would generate in congested traffic areas from becoming too high.

An attempt has also been made in this investigation to use real air traffic data to test the assumptions on which the analysis is based. However, scope limitations have made it necessary to rely upon data that already exist in accessible form. The source chosen for this purpose is a briefing presented in February 1980 (Ref. 1) by the Mitre Corporation, which is currently responsible for developing active BCAS alert logics for the FAA. Reasons for choosing this particular data source are discussed in Chapter III.

Not surprisingly, in the data, which are derived from Automated Radar Terminal System (ARTS) tapes containing 65 hours of radar recording at Houston International Airport, there is no instance of a mid-air collision.* Because such events are indeed rare, it is not feasible to use the incidence of actual collisions as a measure of BCAS protection in a test involving real air traffic. However, if it is conceded that some minimum separation between aircraft is necessary for safe flight, then a minimum miss distance standard can be imposed and its

*According to Ref. 2, fatalities due to mid-air collisions are only 0.2 percent of all passenger fatalities due to transportation accidents in general.

maintenance adopted as the basis for empirically assessing the amount of protection that a collision avoidance system can furnish. This principle, the use of which was already implied in Ref. 1, in fact defines what is meant here by the protection that a given version of BCAS alert logic will provide.

A tool that is fundamental in achieving the stated goals of this investigation is a mathematical model of the statistical distributions of air traffic density and velocity over the Houston terminal region. A similar analytical characterization was originally devised (Ref. 3) to fit the same statistical distributions in the Los Angeles Standard Traffic Model (Ref. 4), which consists of data synthetically generated to simulate traffic conditions that were predicted for the Los Angeles Basin in 1982. Chapter III and Appendix A furnish details concerning the consistency of this traffic model with various empirical results taken from Ref. 1.

The evaluation of active BCAS protection occurs in Chapter IV. Among the questions considered there are: (1) basic factors that should be taken into account in the selection of active BCAS alert logic parameter values; (2) for a given choice of parameters, if no alert occurs, the probability that a prescribed miss distance standard will be violated; (3) if an alert does occur, the probability that it is unnecessary, i.e., that the miss distance standard would not be violated if the alert were ignored; (4) if an alert occurs, the probability distribution of the time available for the aircraft to react and perform an avoidance maneuver; (5) loss of protection due to the use of slant range in the horizontal alert algorithm instead of horizontal range.

Implicit in the discussion of protection in Chapter IV is the likelihood that an aircraft pilot's perception of the quality of alerts generated by the BCAS logic may be as important as the actual quantity. That is, it seems reasonable to suppose

that no one would object to frequent alerts if they appear to be justified by circumstances; however, if the overwhelming majority of alerts are viewed as unnecessary, a pilot may reject the system even when the alert rate is moderate. It also seems reasonable to expect an even greater loss of confidence in the system if it should produce many unnecessary alerts and also, on occasion, fail to produce one that would have been perceived as justified.

The most important conclusions resulting from the present investigation are summarized in Chapter V. Although a number of such conclusions were reached, the primary objective has been to formulate analytical procedures that may be useful in selecting active BCAS logic parameters. For example, with the aid of such procedures it may be possible in the future to incorporate into the system adaptive BCAS parameter selection (desensitization), tailored both to the observed traffic distribution and each individual aircraft according to its aerodynamic or other relevant characteristics.

II. DESCRIPTION OF ACTIVE BCAS*

A. GENERAL CHARACTERISTICS

Active BCAS (Beacon Collision Avoidance System) is a cooperative airborne collision avoidance system of the beacon-transponder type. It will, in principle, protect a user against any other aircraft equipped with an Air Traffic Control Radar Beacon System (ATCRBS) transponder and altitude encoder. It will also protect against future participants in the Discrete Address Beacon System (DABS) that is intended to replace ATCRBS.

In its operation BCAS relies upon measurements of an intruder's relative range and altitude. It also requires the intruder's range and altitude rates, which it obtains by tracking.

BCAS generates the measured data by transmitting the same ATCRBS interrogation signals that are usually transmitted by ground installations. An intruder will respond in the usual manner with altitude (and identity) encoded reply signals. The signal's delay time provides the range and its encoded message the altitude.

This data is processed to determine whether a threat exists. Then the BCAS conflict resolution logic determines what commands or advisories are needed for protection of the aircraft.

*The information contained in this chapter was extracted from Ref. 5 and Ref. 6 and reinterpreted slightly to emphasize a point of view that is appropriate to the present investigation. The alphanumeric variable and parameter names used in this and subsequent chapters are those given in Table 2-1, Ref. 5.

B. THREAT CRITERIA

BCAS employs two types of threat criteria: (1) horizontal, which depends on the intruder's relative range and range rate; (2) vertical, which depends on the intruder's relative altitude and altitude rate. A threat is recognized, and an alert is generated, only if both criteria are satisfied simultaneously and if, by means of a linear projection of the altitude rate, it is estimated that the vertical separation of the aircraft will be less than a prescribed threshold, called ALIM, when the aircraft reach their position of closest approach.

The horizontal threat criterion depends upon two fixed parameters, RDTHR and H1, and two prescribed, but flexible, parameters: TRTHR, designated here by τ , and DMOD, designated here by R_0 . The vertical threat criterion depends upon a single prescribed flexible parameter, TVTHR, which, however, is always assigned the same value as TRTHR, and a fixed parameter ZTHR.

For a horizontal separation R and separation rate $\dot{R}$, if the aircraft are closing at a rate that is greater than the fixed parameter RDTHR, the horizontal threat criterion is the condition

$$R + \tau \dot{R} \leq R_0 .$$

For

$$-RDTHR \leq -\dot{R} \leq RDTHR$$

$\dot{R}$ is replaced by $-RDTHR$. Otherwise, for aircraft that are separating, the criterion becomes

$$\dot{R} R < H1 .$$

The vertical criterion is satisfied if either of two conditions are met. For a relative altitude A and altitude rate $\dot{A}$, the first condition is

$$A + \tau \dot{A} \leq 0 ,$$

and the second is

$$A \leq ZTHR.$$

C. DESENSITIZATION

Because of traffic conditions, if nothing were done to alter the threat logic, it is expected that an aircraft entering terminal air space would experience increasingly larger BCAS alert rates. In addition, the close proximity of aircraft in normal terminal flight patterns would tend to cause unnecessary and disruptive alerts.

Therefore, a provision has been made in the system design concept to make the threat logic more restrictive by altering the flexible threat criterion parameters when BCAS operates in certain predetermined zones. This process, known as desensitization, will be implemented automatically by means of radar signal transmissions from certain strategically located ground-based radar beacon transponders (RBXs). An RBX may also provide certain useful information, such as the local altitude relative to sea level.

At the time that this report was prepared, five different levels of desensitization could be imposed. The following rules were then current for aircraft below 10,000 ft. In level 1 the BCAS alert logic is inhibited, as is its interrogation of RBXs. In level 2 the alert logic is inhibited but RBX interrogation is permitted. In level 3 the value of DMOD (R_0) is set at 0.1 nmi, τ (TRTHR and TVTHR) is set at 18 sec when the intruder is equipped with BCAS and 20 sec when it is not, and the value of ALIM is set at 340 ft. In level 4 the value of DMOD is set at 0.3 nmi, τ is set at 25 sec, and the value of ALIM is set at 340 ft. In level 5 the value of DMOD is set at 1.0 nmi, τ is set at 30 sec, and the value of ALIM is set at 440 ft.

All aircraft above 10,000 ft are in level 5. For such aircraft, however, altitude thresholds with larger values replace ALIM because of increased altimeter error.

III. AIR TRAFFIC STATISTICS

A. DATA

From the outset of this investigation it was felt that some empirical data would be needed to support the conclusions reached by analysis that otherwise must depend largely upon a set of unsubstantiated assumptions. Unfortunately, most of the presently available air traffic data are, at least in part, synthetic in nature.

For example, NAFEC* has performed a series of experiments (Refs. 7-9) to determine alert rates and the effect of BCAS on pilots and air traffic control operations. These experiments have involved real people; however, all are based on computer generated traffic data designed to simulate conditions that are thought to be specific to certain airports.

Other potentially useful sources of data also have an artificial character. Among these is the source that undoubtedly furnishes the most detailed air traffic data that is presently available--the Los Angeles Standard Traffic Model (Ref. 4). None of the data that comprise that model were actually derived from direct observation of real air traffic.

There are some references (Refs. 1, 10, 11) that do, at least, summarize certain real air traffic statistics. Of these, Ref. 1 contains the most extensive statistical information. For this reason this report relies upon Ref. 1 for its empirical data base. Various statistical summaries from that source are used here to test the foregoing theoretical analysis as a means of verifying indirectly the assumptions on which the analysis depends.

*National Aviation Facilities Experimental Center.

Reference 1 contains processed and reduced data extracted from ARTS tapes covering a 65-hour period of radar observation at Houston Intercontinental Airport. The processing, by computer, involved the application of BCAS alert logic, for various choices of algorithm parameters, to the Houston traffic in order to determine how the choice of parameters might affect the alert rate and the protection that the logic affords.

Of particular interest for present purposes are the results contained in Table 1, which gives the number of alerts that occurred for several combinations of tau and DMOD values for two different altitude threshold standards (ALIM). Table 1 consists of the relevant part of a similar but more extensive table appearing in Ref. 1.

TABLE 1. DISTRIBUTION OF ALERTS (PARTIAL DATA)

$DMOD(R_o)$	ALIM	Tau	Positive Commands
0.3 nmi	370 ft	25 sec	181
0.3 nmi	340 ft	25 sec	173
0.1 nmi	370 ft	20 sec	83
0.1 nmi	340 ft	20 sec	81
0.1 nmi	370 ft	18 sec	73
0.1 nmi	340 ft	18 sec	69

Reference 1 also contains some useful statistical data in the form of histograms showing the distribution of speeds and separations of aircraft in conflict. In addition, L. Zarrelli kindly supplied some histograms that did not appear in the original presentation. One of these is also relevant to the purposes of this report: a histogram giving the observed distribution of closing speeds of aircraft in conflict. These histograms are reproduced in Appendix A (Figs. A-1, A-3 and A-4).

B. THE THEORETICAL TRAFFIC MODEL

Reference 3 indicates how the distribution of low-altitude air traffic density about a point of reference in the Los Angeles Basin and the distribution of aircraft velocities, defined by the Los Angeles Standard Traffic Model data (Ref. 4), can be represented analytically. The mathematical expressions used for this purpose, although simple, fit the data with an accuracy that is consistent with the statistical fluctuations of the several traffic samples provided in Ref. 4.

This mathematical model of air traffic can be formulated as follows. The horizontal coordinates of an aircraft picked at random, located by means of a cartesian coordinate system with its origin at the traffic center, are independent, identically distributed Gaussian random variables.* Each coordinate has a zero mean and the same standard deviation σ_R . Similarly, the cartesian components of the aircraft velocity vector are also independent, identically distributed Gaussian random variables with zero mean and the same standard deviation σ_0 .

One consequence of these assumptions is that the aircraft density is Rayleigh distributed in range R from the traffic center. That is, the probability density P_R of aircraft distribution over range is given by

$$P_R = \frac{R}{\sigma_R^2} e^{-\frac{R^2}{2\sigma_R^2}}.$$

*Actually, it was found that the distribution is bimodal. However, since the fraction of aircraft in one mode is small compared to that in the other, the single mode approximation described here is still quite accurate.

Another consequence is that the separation R between a pair of aircraft selected at random is also Rayleigh distributed with

$$\hat{\sigma}_R = \sigma_R \sqrt{2} ,$$

replacing σ_R in the definition of P_R . A third consequence is that the components of the relative velocity between the two aircraft are also independent, identically distributed Gaussian random variables with mean zero but with a standard deviation

$$\sigma_v = \sigma_o \sqrt{2} .$$

From this it follows that the closing speed is Rayleigh distributed, i.e., with a probability density P_v given by

$$P_v = \frac{v}{\sigma_v^2} e^{-\frac{v^2}{2\sigma_v^2}} .$$

It was determined in Ref. 3 that a good fit is obtained for the Los Angeles Standard Traffic Model by setting

$$\sigma_R = 20 \text{ nmi}$$

and

$$\sigma_o = 122 \text{ ft/sec (72.2 kn)} .$$

Since no detailed information is available for the distribution of traffic and aircraft velocities at Houston it is not possible to verify directly a similar fit of the model to the observed Houston traffic. However, the statistical summaries provided by Ref. 1 are available, at least, to test the compatibility of such a model with the underlying data recorded on the ARTS tapes.

C. APPLICATION OF THE THEORETICAL TRAFFIC MODEL TO THE HOUSTON DATA

In order to apply the theoretical traffic model to the Houston data given in Ref. 1, it is necessary to calculate the appropriate alert probabilities in terms of the assumed values of τ and R_0 (DMOD). This is done in Appendix A.

There it is shown that the four quantities τ , R_0 , $\hat{\sigma}_R$, and σ_v can be subsumed into two independent dimensionless parameters ρ and κ , defined by

$$\rho = \frac{R_0}{\sigma_v \tau}, \quad \kappa = \frac{\sigma_v \tau}{\hat{\sigma}_R}.$$

An inspection of these equations reveals that (1) ρ is independent of the aircraft density; (2) κ is independent of DMOD; (3) a change in τ is equivalent to a change in the aircraft velocity distribution; (4) the product $\rho\kappa$ is independent of the aircraft velocity distribution and τ .

Figure A-2 illustrates the result of one type of calculation done in Appendix A. It provides curves of the probability $P(\kappa, \rho)$ that two co-altitude aircraft chosen at random will be in conflict, i.e., will satisfy the horizontal alert condition (Eq 1 in Chapter IV, Section A), as a function of ρ for various values of κ . It is assumed that the alert rate, or the number of alerts occurring during the period covered by the Ref. 1 ARTS tapes, is proportional to this probability for the appropriate values of ρ and κ .

In order to fit the theory to the histograms from Ref. 1 it is assumed that virtually all conflicts occur at altitudes below 10,000 ft where the theoretical traffic model is applicable. It is also implicit in the analysis that the histograms are based on a random sample from the traffic. This is only approximately true for the histogram depicting the distribution of speeds of aircraft in conflict, since the fact that

each such aircraft is one of a pair for which the alert condition is satisfied tends to bias the sample against low-speed aircraft.

Figures A-1, A-3 and A-4 illustrate the results of other calculations done in Appendix A, to determine the theoretical distribution that best fits each histogram. The calculations indicate that the σ_0 value of 72.2 kn appropriate to the Los Angeles Standard Traffic model is too small. The figures contain curves based on σ_0 values of 83.3 kn, which would correspond to 99 percent confidence that no aircraft would violate the 250 kn speed limit, and 106 kn, which appears to provide the best fit. The theoretical distributions are essentially independent of the aircraft density variation, since varying κ from 0.01 to 0.1 produces only insignificant modifications in the curves.

Using the probability $P(\kappa, \rho)$ illustrated by the curves of Fig. A-2, Appendix A tests the ability of the theory to predict the alert changes due to changes in τ and DMOD indicated in Table 1. This is done for the three possible aircraft velocity distributions characterized by σ_0 values of 72.2 kn, 83.3 kn, and 106 kn. The results are given in Table 2.

TABLE 2. ALERT REDUCTION DUE TO CHANGES IN τ AND DMOD

Change in τ From 25 sec to (sec)	ALIM (ft)	Actual % Change in Alerts	Predicted % Change in Alerts			Error (%)			σ_0 (kn)
			72.2	83.3	106	72.2	83.3	106	
20	370	54.1	55.1	53.2	49.9	1.0	0.9	4.2	
20	340	53.2	55.1	53.2	49.9	1.9	0.0	3.3	
18	370	59.7	62.4	61.2	58.7	2.7	1.5	1.0	
18	340	60.1	62.4	61.2	58.7	2.3	1.1	1.4	

According to Table 2 the best predictions occur for $\sigma_0 = 83.3$ kn. However, the prediction error is small for all three σ_0 values, including 106 kn, which provides the best theoretical fit to the histogram data. Thus, it appears that predictions of the change in the alert rate due to changes in the values of τ and DMCD is insensitive to the aircraft velocity distribution.

Also, as pointed out in Appendix A, the change is virtually independent of the aircraft density distribution. This observation leads to the natural conjecture that changes in τ and DMOD would have the same effect on fractional changes in the alert rate at any other airport.

IV. ALERT LOGIC PROTECTION

A. PARAMETER SELECTION

For the purposes of the foregoing analysis it is assumed that collision alerts are triggered by the idealized horizontal algorithm embodied in the condition

$$R + \tau \dot{R} \leq R_0, \quad (1)$$

where R is the separation between encountering aircraft, $\dot{R}$ is the separation rate, and τ and R_0 (termed DMOD in BCAS parlance), are prescribed positive parameters.

As shown in Ref. 12, the algorithm (1) will only furnish protection against collisions under all circumstances when the DMOD parameter R_0 is at least large enough to satisfy the relation

$$R_0 \geq \epsilon_R + \tau \epsilon_{\dot{R}} + \frac{1}{2} U \tau^2.$$

In this relation ϵ_R is a bound on the horizontal separation measurement error, $\epsilon_{\dot{R}}$ is a bound on the horizontal separation rate measurement error, U is a bound on the maximum anticipated relative acceleration of the encountering aircraft, and τ is the effective time required to respond to an alert and maneuver to a safe vertical separation. If a guaranteed minimum miss distance requirement is to be imposed as well, the miss distance standard D must also be added to the right side of the inequality.

Allowance for protection against acceleration is sometimes discounted because it is usually assumed that aircraft only accelerate during turning maneuvers and that only ten percent of aircraft in a terminal region are expected to be turning at any

given time. However, for some aircraft an allowance for acceleration may always be necessary.

In performing a vertical avoidance maneuver after an alert, acceleration must be applied vertically, but it may not be possible for some aircraft to create an acceleration vector that does not also have a horizontal component. When this is the case, that component must be taken into account in the horizontal collision algorithm like any other horizontal acceleration.

The danger of ignoring this effect is similar to that in the situation discussed in Ref. 14, wherein a fast aircraft is overtaking a slower one in a tail chase and the avoidance maneuver is supposed to be horizontal. There it was assumed that acceleration takes place normal to the flight path throughout the maneuver, so that the aircraft maintains constant speed and turns with a constant angular rate. It was found that, because forward velocity along the original flight path decreases, if the slower aircraft is the one attempting to perform the avoidance maneuver a collision may be unavoidable no matter how large the estimated time to collision may be.

A parameter similar to DMOD should be defined for the vertical collision algorithm, for which BCAS presently employs an unmodified tau criterion, since, at the very least, altimeter measurement errors, particularly the error in estimating altitude rates, are far from negligible. Moreover, when there is a non-zero altitude rate the existence of a vertical acceleration is, if anything, more likely than the existence of a horizontal acceleration in general.

In the selection of collision algorithm parameters it seems logical to choose τ first because, unless R_0 is at least as large as the minimum allowable miss distance D , the protection afforded by DMOD depends upon τ . On the other hand, if the value of τ is reduced without changing the DMOD value the protection afforded by DMOD will appear to be greater.

However, this will be true only for those aircraft that can respond to an alert quickly enough to make use of the information that it provides. On the other hand, reducing DMOD without changing τ reduces the protection for all aircraft, although it is also true that the protection afforded by any DMOD value is greater for the aircraft that can respond more quickly to an alert.

Some compromise in protection will undoubtedly occur even for level 5*, for which a DMOD value of 1 nmi and a τ value of 30 sec have been selected. For example, even if no allowance is made for measurement error or a minimum miss distance, the acceleration bound U for which the term $1/2 U \tau^2$ contributing to R_0 is equal to 1 nmi is just 13.5 ft/sec² or about 0.42 g. If a not unreasonable allowance of 1,000 ft is made for the total measurement error contribution and 1,000 ft for the miss distance, the system will protect against accelerations up to about 0.3 g. For aircraft that can respond quickly enough to accommodate a τ value of 25 sec, the protection against acceleration without taking into account measurement error or a minimum miss distance is about 0.6 g. With a total measurement error plus miss distance allowance of 2,000 ft, the protection would still take into account accelerations up to 0.4 g.**

The parameter τ should be a sum of three terms: (1) the data updating interval which, because of BCAS logic rules (Ref. 5), if an alert is not missed, will be 2 or 3 secs; (2) the pilot-aircraft response time which, according to the results of

*Reference 4 defines BCAS protection levels 1-5 that result from desensitization of the system in air terminal regions, ranging from no CAS operation at level 1 to maximum protection at level 5 (see p. 7).

**In fact, if 1,000 ft is an appropriate allowance for measurement error when τ is 30 sec, the measurement error term would lessen when τ is 25 sec; therefore, the protection is actually somewhat greater than this estimate indicates.

a NAFEC experiment (Ref. 15) is not likely to exceed 8 sec, and (3) the time t_e required for a vertical avoidance maneuver.

In estimating t_e it will be assumed that the avoidance maneuver begins with a constant vertical acceleration (8 ft/sec^2 , according to Ref. 5) which is applied until a limiting altitude rate v_1 is reached. Reference 5 prescribes 16.67 ft/sec for v_1 in a climbing maneuver and 25 ft/sec in a descending maneuver.

The assumption of a constant vertical acceleration may be incorrect. For example, if the motion is like that of a horizontal turn in which the angular turning rate is constant, although the magnitude of the acceleration may have a constant value a , its vertical component would be a function of time, namely, $a \cos \frac{a}{v}t$, where v is the aircraft's speed when it begins the maneuver. The estimate for t_e should be modified appropriately if the vertical acceleration is not constant, e.g., by assuming that a is an equivalent average value for an acceleration that may be considerably larger at its peak.

The choice of t_e must be at least consistent with the case of a co-altitude aircraft encounter with no relative altitude rate. In the case of non co-altitude encounters for which the aircraft are closing vertically the choice of t_e depends upon the maximum altitude rate expected for an intruder. As this expected rate for an intruder approaches the appropriate avoidance maneuver limit, 16.67 ft/sec or 25 ft/sec , specified for the BCAS logic, the required value of t_e becomes arbitrarily large unless both aircraft perform cooperative maneuvers.

For the co-altitude case a vertical acceleration a occurs for a time t_0 given by

$$t_0 = \frac{v_1}{a} ;$$

i.e., acceleration occurs for 2.1 sec in a climbing maneuver and 3.1 sec in a descending maneuver. Since the value of t_e will surely be greater than either of these times, the limiting

speed of 16.67 ft/sec or 25 ft/sec will always be reached before the maneuver is completed. The vertical separation z when the maneuver is completed is given by

$$z = v_1 t_e - \frac{v_1^2}{2a}.$$

When the level 5 value of τ , i.e., 30 sec, is used t_e is 19 sec. Then calculations of z show that a vertical separation of 299 ft will be achieved in a climbing maneuver and 436 ft in a descending maneuver. For a τ of 25 sec t_e is 14 sec and the separations will be 166 ft and 236 ft respectively.

For a final separation of just 160 ft, in a climbing maneuver it would be unnecessary to use a value greater than 10 sec for t_e . This would correspond to a τ value of only 21 sec.

B. ALGORITHM SENSITIVITY TO PARAMETER CHANGES

The quantity R_0 is essentially an allowance for measurement error and any relative acceleration due, in general, to possible turning maneuvers by the encountering aircraft.* The quantity τ is an allowance for the amount of time that would be required for an aircraft to perform a vertical avoidance maneuver after an alert.*

If these interpretations of R_0 and τ actually govern their choice their values will be such as to guarantee a successful avoidance maneuver in any encounter for which the assumed bounds on acceleration and escape maneuver time are satisfied. However, if for some values of either parameter the condition (1) is not adequate, it is also possible to revise the algorithm to ensure that a safe avoidance maneuver can be accomplished by increasing the value of the other parameter sufficiently. That is, if τ is insufficient by an amount $\Delta\tau$, for example, for a particular R the corrective effect would be the same if, instead of increasing τ

*Cf. Ref. 12 and Ref. 13.

by $\Delta\tau$, R_0 were changed to $R_0 - \Delta\tau \dot{R}$. This would amount to an increase of $-\Delta\tau \dot{R}$ in DMOD (or, conversely, the change in τ would be $-\frac{\Delta R_0}{\dot{R}}$) when the aircraft are closing. Therefore, the equivalent change in DMOD would be large for large closing speeds, small for small closing speeds, and it would actually be necessary to decrease the value of DMOD in order to obtain the equivalent effect when the encountering aircraft are separating. This suggests that it is dangerous to draw conclusions about the effectiveness of a particular choice of R_0 and τ when the evidence is based on a statistical sample of events, whether simulated or real, unless extreme caution is exercised in the interpretation of the events.

C. THE RISK WHEN THERE IS NO ALERT

As observed in Ref. 4, since midair collisions are quite rare it will be easier in practice to evaluate a collision avoidance system in terms of the near misses, that is, violations of prescribed separation standards, that the CAS prevents rather than actual collisions that it might have prevented. Although the usual interpretation of (1) (Refs. 12 and 13) refers to the avoidance of a collision rather than a violation of a horizontal separation standard, it is possible, nevertheless, by making R_0 sufficiently large, to include in the algorithm a limit on the minimum miss distance permitted in an encounter.

In order to examine the relation between the collision algorithm associated with (1) and the minimum miss distance standard, it is convenient to use a coordinate system of the type introduced by J. Holt in Ref. 12 for describing the hazard region defined by (1). The origin of the coordinate system is located at one of two encountering aircraft with a coordinate axis parallel to the relative velocity vector of the other. The other aircraft is located by polar coordinates relative to this axis, which is assumed to be oriented in a direction opposite that of the relative velocity. Assuming a constant

relative velocity of magnitude V , the separation rate is given in terms of the angular coordinate θ by

$$\dot{R} = -V \cos \theta. \quad (2)$$

A cross range velocity of magnitude V_θ is also implied, given by

$$V_\theta = V \sin \theta. \quad (3)$$

The condition (1) is satisfied, then, at all points within the cardioid region bounded by the curve

$$R = R_0 + \tau V \cos \theta, \quad (4)$$

illustrated in Fig. 1.

In the analyses that follow, for the most part, it is to be understood that R_0 has had removed from it the measurement error allowance, and either linear flight is assumed or the acceleration allowance has also been removed. The error allowance R_{oe} has the form

$$R_{oe} = \epsilon_R + \tau \epsilon_R^*,$$

where ϵ_R is a bound on the error in measuring R and ϵ_R^* is a bound on the error in measuring $\dot{R}$. The acceleration allowance R_{oa} has the form

$$R_{oa} = \frac{1}{2} U \tau^2,$$

where U is a bound on the magnitude of the possible relative acceleration.

In addition to polar coordinates it is also convenient to use cartesian coordinates defined, as usual, by

$$x = R \cos \theta, \quad y = R \sin \theta.$$

For linear flight, by definition, the intruding aircraft located at (x,y) moves in the direction of decreasing x and so that y remains constant. Thus, the minimum miss distance is always equal to y .

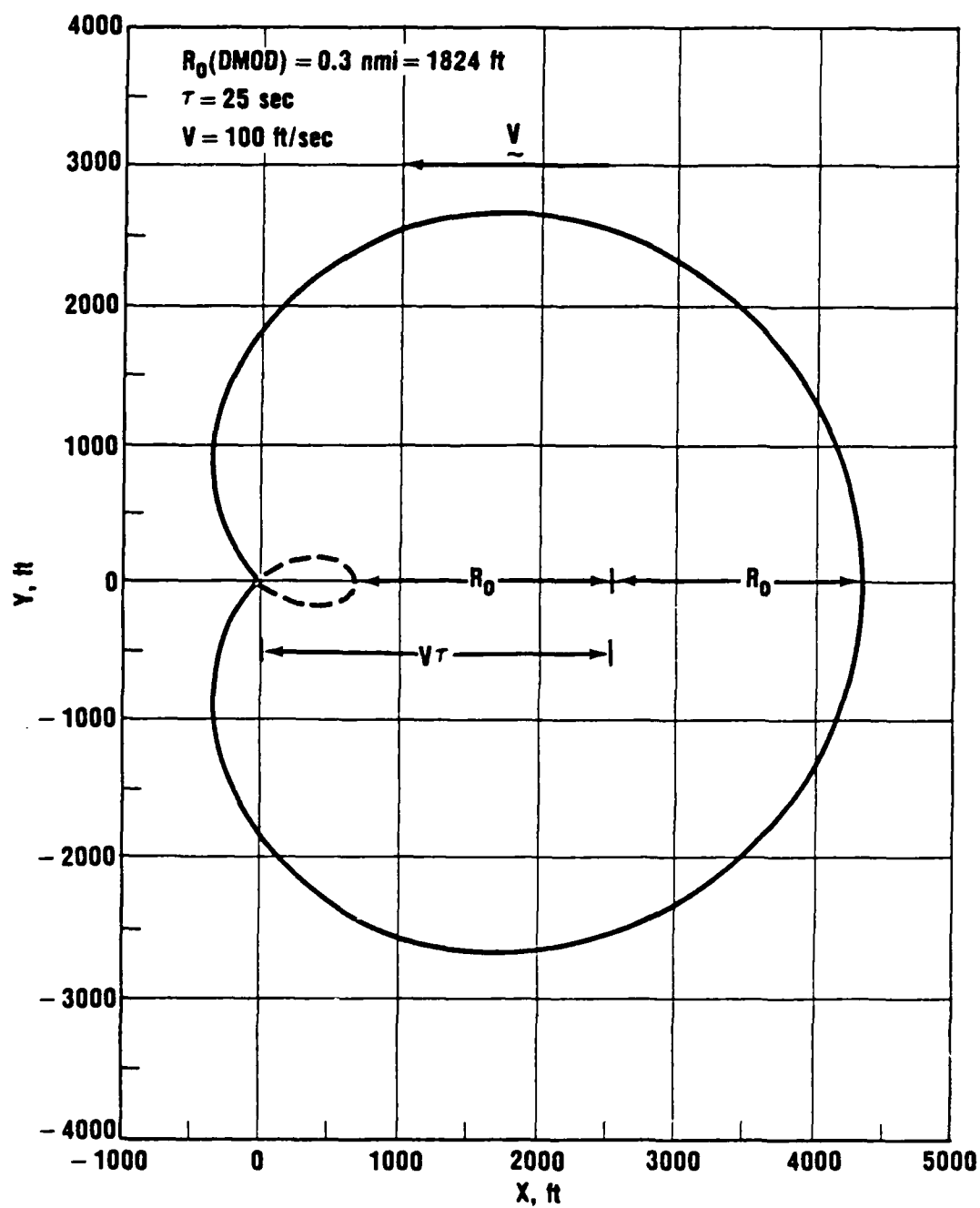


FIGURE 1. The hazard region defined by a collision algorithm.

In discussing levels of safety afforded by the condition (1) perhaps the most basic question that should be addressed is what separation is assured if no alert at all occurs during an encounter. In terms of the cardioid illustrated in Fig. 1, this can happen only when the ordinate y associated with the intruder's position is larger than the maximum ordinate y_m of points on the curve determined by (4).

The maximum ordinate y_m is a solution of the equation

$$\frac{dy}{dx} = \frac{\frac{dR}{d\theta} \sin \theta + R \cos \theta}{\frac{dR}{d\theta} \cos \theta - R \sin \theta} = 0,$$

which is, more explicitly, equivalent to

$$-\tau V \sin^2 \theta + (R_0 + \tau V \cos \theta) \cos \theta = 0. \quad (5)$$

The equation (5) is equivalent to

$$\cos^2 \theta + \frac{R_0}{2\tau V} \cos \theta - \frac{1}{2} = 0$$

for which possible solutions are

$$\cos \theta = \frac{-R_0 \pm \sqrt{R_0^2 + 8\tau^2 V^2}}{4\tau V}. \quad (6)$$

Then, since

$$y_m = R \sin \theta = (R_0 + \tau V \cos \theta) \sin \theta,$$

on substituting from (6) it can be seen that y_m is given by the greater of

$$y_{\pm} = \frac{1}{4} \left(3 R_0 \pm \sqrt{R_0^2 + 8\tau^2 V^2} \right) \left(\frac{1}{2} + \frac{R_0}{R_0 \pm \sqrt{R_0^2 + 8\tau^2 V^2}} \right)^{\frac{1}{2}}$$

i.e.,

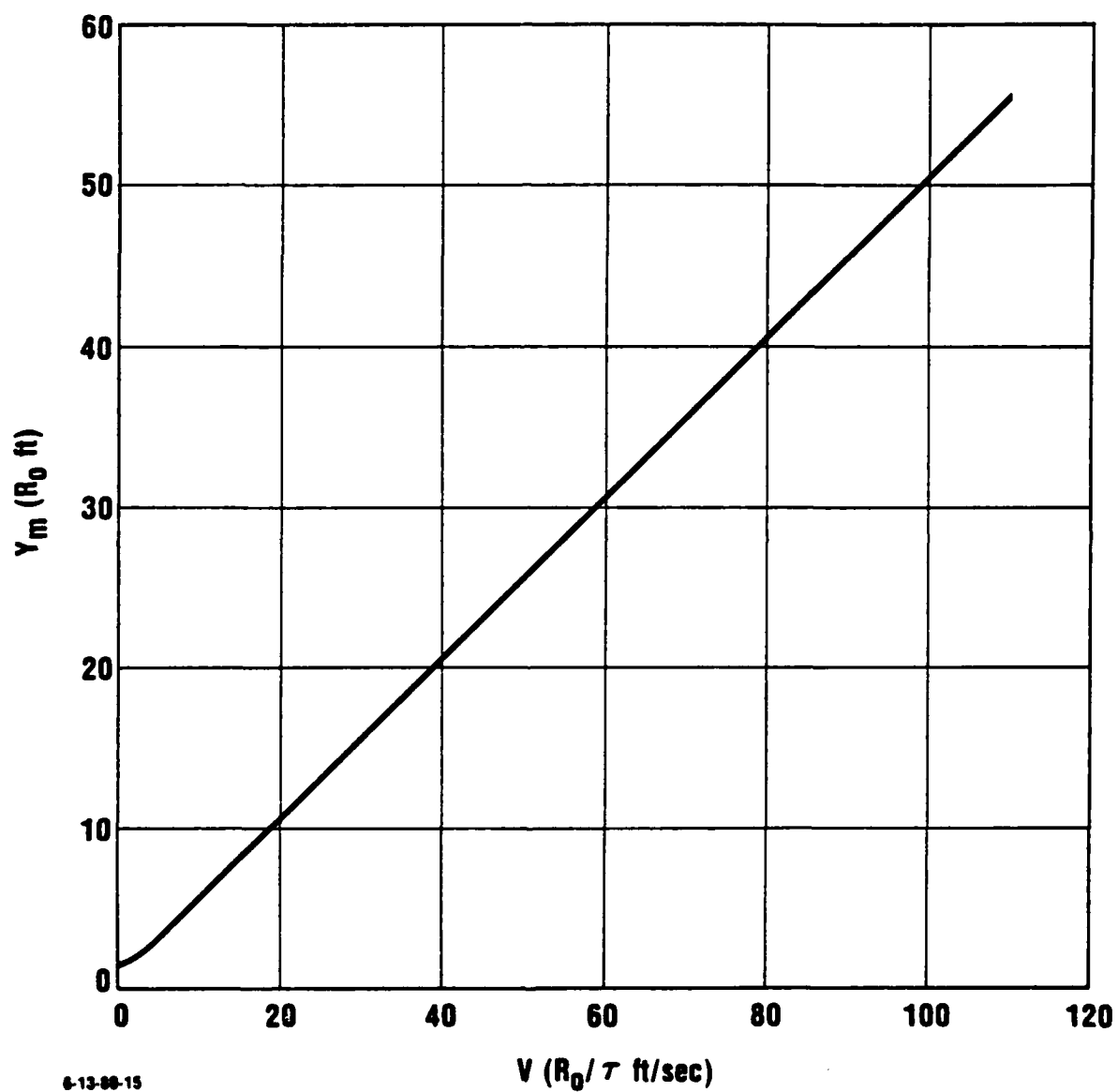
$$y_m = \frac{1}{4} \left(3R_o + \sqrt{R_o^2 + 8\tau^2 V^2} \right) \left(\frac{1}{2} + \frac{R_o}{R_o + \sqrt{R_o^2 + 8\tau^2 V^2}} \right)^{\frac{1}{2}} \quad (7)$$

It can be seen from (7) that y_m is an increasing function of V as V varies from 0 to ∞ . Thus, while y_m can be arbitrarily large, the smallest value that it can have is R_o . Therefore, if there is no alert a minimum miss distance of at least R_o is guaranteed.

The minimum miss distance y_m given by (7) as a function of velocity V can be illustrated by a single curve, shown in Fig. 2, if y_m is measured in units of R_o ft and V is measured in units of R_o/τ ft/sec. It can be seen by inspection of Fig. 2 that the curve is essentially linear for all values of V above 5 and deviates only slightly from linear for values between 0 and 5.

The curve illustrates the fact that even when R_o is quite small, say 5 to 10 ft, although it is very possible that a normal separation standard such as 1,000 ft might be violated, it is still likely, at least, that the miss distance will exceed 100 ft. For example, if R_o is 5 ft and τ is 25 sec a V of 40 corresponds to a relative velocity of 8 ft/sec. According to Fig. 2 the value of y_m for V equal to 40 is about 20.7, which is equivalent to more than 103 ft. In other words, as long as the relative velocity is at least 8 ft/sec the miss distance will be greater than 103 ft. For a τ of 20 sec the velocity corresponding to 40 would be 10 ft/sec; since this is greater than 8 ft/sec, a miss distance exceeding 103 ft would be slightly less likely.

A probability distribution for the minimum miss distance would give a more precise idea of the risk associated with a collision algorithm when an alert condition does not occur in



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FIGURE 2. The minimum distance versus relative velocity when no alert occurs.

an encounter. This probability distribution depends upon the knowledge of a probability distribution for the relative velocity, which may be a property of the local air traffic.

For the Los Angeles Standard Traffic model (Ref. 4), in the lower altitudes, each velocity component of an aircraft appears to be normally distributed with a mean of zero and a standard deviation of about 122 ft/sec. This would imply a similar distribution for each component of the relative velocity vector between two aircraft, but with a standard deviation of $122 \times \sqrt{2} = 173$ ft/sec. The relative velocity magnitude V would then be Rayleigh distributed with a standard deviation of 173 ft/sec. Thus,

$$P(V \leq V_o) = \frac{1}{\sigma^2} \int_0^{V_o} e^{-\frac{v^2}{2\sigma^2}} v dv = 1 - e^{-\frac{V_o^2}{2\sigma^2}}, \quad (8)$$

where, in this case,

$$\sigma = 173 \text{ ft/sec.}$$

Equation (8) used in conjunction with (7) provides the probability distribution $P(y_m \leq D)$ of interest.

The complimentary probability

$$P(y_m > D) = 1 - P(y_m \leq D)$$

provides a lower bound on the probability that the miss distance will be at least D if there is no alert. Figure 3 contains curves of this bound for normalized values σ of σ_v (measured in units of R_o/τ ft/sec) and D (measured in units of R_o ft) covering the CAS avoidance algorithm parameters that are likely to occur at altitudes below 10,000 ft. The value

$$\sigma = 2.37$$

for one curve shown in Fig. 3 corresponds to a σ_v of 173 ft/sec when R_o is 0.3 nm1 and τ is 25 sec.

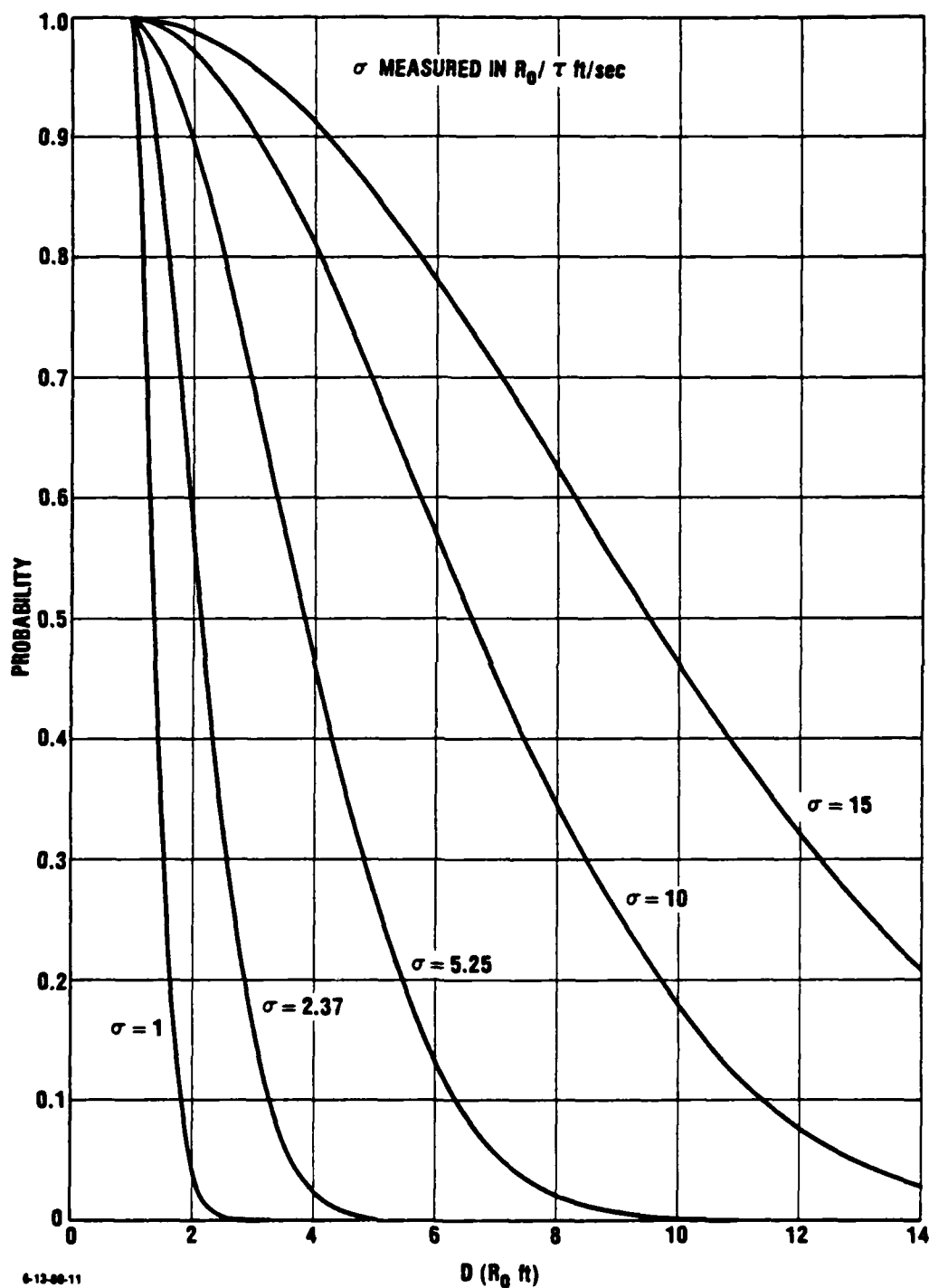


FIGURE 3. Lower bound on probability of a miss distance of at least D if no alert.

If the R_0 value of 0.3 nmi is assumed to cover measurement errors for which a total allowance of 1,000 ft is to be made, the curve labelled with the normalized value

$$\sigma = 5.25$$

provides the appropriate probability bound. The measurement error allowance of 1,000 ft might, for example, result from a range error of 150 ft and a range rate error of slightly more than 20 kn. An examination of the curve, which would then correspond to a reduced value for R_0 of 824 ft, shows that the probability of at least a 1,000 ft miss distance is 98.8 percent or better.

D. THE RISK WHEN AN ALERT OCCURS

Except for the case of pop-up targets, an alert occurs when an intruder is on or near the boundary of the cardioid shown in Fig. 1. A reasonable measure of safety provided by the CAS algorithm (1) might then be the amount of time t_D available for maneuver after such an alert before the separation R between the encountering aircraft is reduced to some miss distance D chosen as a minimum separation standard.

In calculating t_D it may be observed that, because of (2),

$$y = R \sqrt{1 - \frac{\dot{R}^2}{V^2}}, \quad (9)$$

so that when an alert occurs, implying at that instant the relation

$$R + \tau \dot{R} = R_0 \quad (1')$$

in accordance with (1),

$$y = R \sqrt{1 - \frac{(R - R_0)^2}{\tau^2 V^2}}. \quad (10)$$

When the aircraft separation is D

$$x = x_D = \sqrt{D^2 - y^2}, \quad (11)$$

since it is assumed that the flight is linear and, therefore, that y is constant. It follows from (10) and (11) that

$$x_D = \sqrt{D^2 - R^2} \left[1 - \frac{(R-R_0)^2}{\tau^2 V^2} \right]. \quad (12)$$

If x_A is the horizontal distance of the intruder to the point of closest approach at the time when the alert occurs, then the time t_A between the alert and the closest approach is given by

$$t_A = \frac{x_A}{V} = \frac{R \cos \theta}{V}. \quad (13)$$

Then, because of (1') and (2), it follows that

$$t_A = \frac{R(R-R_0)}{\tau V^2}. \quad (14)$$

The definitions of t_A and x_D imply that

$$t_D = t_A - \frac{x_D}{V}.$$

Thus, because of (14) and (12) it follows that

$$t_D = \frac{R(R-R_0)}{\tau V^2} - \frac{\sqrt{D^2 - R^2} \left[1 - \frac{(R-R_0)^2}{\tau^2 V^2} \right]}{V} \quad (15)$$

It may happen, however, that when the alert occurs

$$y \geq D, \quad (16)$$

in which case t_D has no meaning since the separation will always be at least D . Because of (9) the condition (16) is equivalent to

$$V \geq \frac{|\dot{R}|R}{\sqrt{R^2 - D^2}}.$$

When the aircraft are closing, which is the only non-trivial case, this condition is

$$V \geq \frac{-\dot{R}R}{\sqrt{R^2 - D^2}} = \frac{R(R - R_0)}{\tau \sqrt{R^2 - D^2}}, \quad (17)$$

because of (1').

The relation (17) defines $V_{\max}$, which is the largest value that V can have when there is any risk that the separation standard can be violated at all. The minimum value $V_{\min}$ of V is, of course, the magnitude $-\dot{R}$ of the separation rate; i.e.,

$$V_{\min} = \frac{R - R_0}{\tau}. \quad (18)$$

According to (17) and (18), then,

$$V_{\max} = \frac{R}{\sqrt{R^2 - D^2}} V_{\min}. \quad (19)$$

From (15) it follows that the corresponding times $t_{D\min}$ and $t_{D\max}$ to violation of the separation standard are given by

$$t_{D\min} = \frac{R - D}{R - R_0} \tau, \quad (20)$$

$$t_{D\max} = t_{D\min} \left(\frac{R + D}{R} \right). \quad (21)$$

The quantity $t_{D\min}$ given by (20) is the least possible amount of time available for maneuvering before the separation standard is violated after an alert. It corresponds to the case

of a head-on collision. The quantity t_{Dmax} given by (21) is the greatest amount of time available for maneuvering when the alert is necessary, i.e., if the separation standard will, in fact, be violated unless an avoidance maneuver is performed. It corresponds to the case in which the miss distance will be exactly equal to the separation standard. Figures 4a and b illustrate the functional dependence of t_{Dmin} and t_{Dmax} on the separation R for various values of D .

The solid line curves in Fig. 4a correspond to the parameters of the previous example in which R_0 is 0.3 nmi (with a 1,000 ft measurement error), τ is 25 sec and D is 1,000 ft. Since in Fig. 4a t_D is normalized to units of τ sec and R and D to units of R_0 ft, for this case the normalized value of D is 1.2136 as indicated.

For R very large compared to R_0 and D , (20) and (21) show that the time available to perform a maneuver to ensure the required separation is τ , which is the same as the time provided by the CAS algorithm to avoid an actual collision. This, of course, is not a particularly surprising result.

If R_0 is larger than D , it has already been seen that the separation requirement is guaranteed when no alert occurs. On the other hand, if an alert does occur, according to (20), as the range R approaches R_0 the time available to perform a maneuver that guarantees the required separation increases, and, in fact, becomes arbitrarily large.* If R_0 is less than D it follows from (20) and (21) that the time available to maneuver before the separation standard is violated is always less than τ unless the separation is sufficiently large, i.e., unless

$$R > \frac{D^2}{R_0} . \quad (22)$$

*Such an occurrence may still not produce absolute safety (cf. Sec. A).

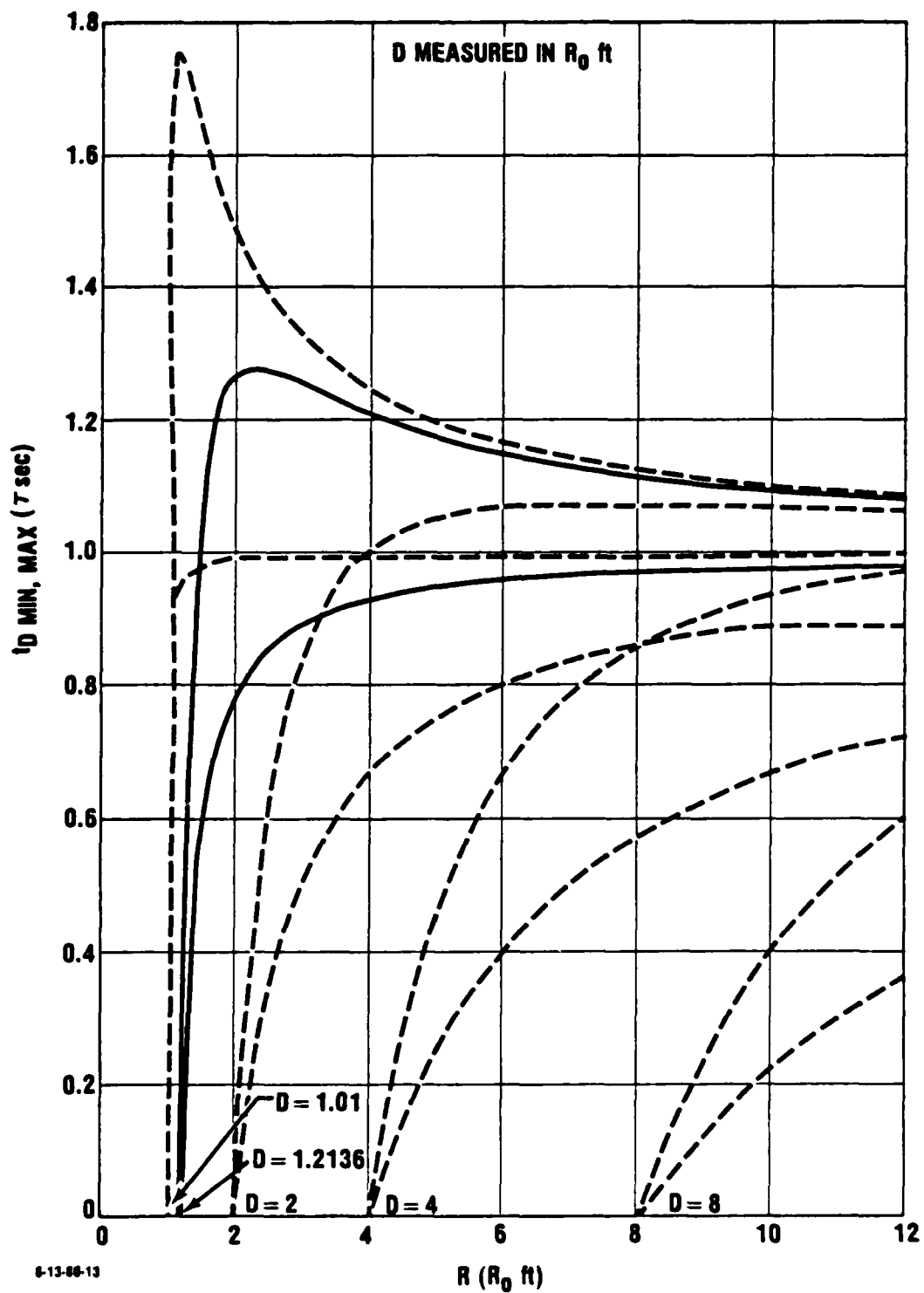


FIGURE 4a. Minimum and maximum times to separation equal to miss distance standard.

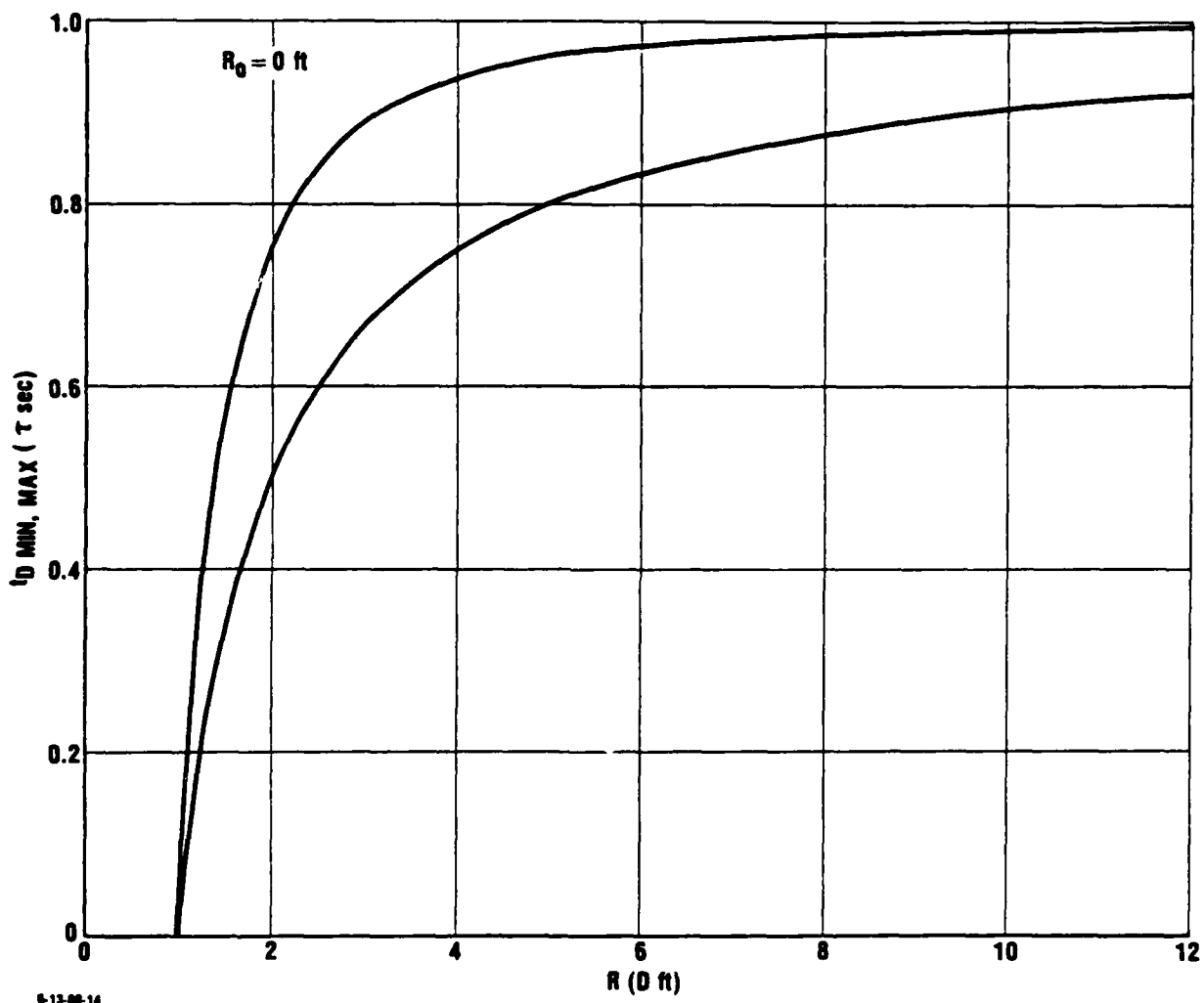


FIGURE 4b. Minimum and maximum times to separation equal to miss distance standard.

The probability distribution of t_D after an alert has occurred can be used to assess the risk of a CAS algorithm. This probability distribution is directly related to the distribution of the relative cross-range velocity component V_θ , which, consistent with the assumptions stated earlier, will be taken to be a normally distributed random variable with mean zero.

The standard deviation σ for V_θ is $\sqrt{2}$ times that associated with a velocity component of a single aircraft. Hence, for example, for V_θ in the Los Angeles Standard Traffic Model σ would be 173 ft/sec.

From (2) and (3) it follows that

$$V^2 = \dot{R}^2 + V_\theta^2. \quad (23)$$

Thus, according to (1), when an alert occurs

$$V^2 = \frac{(R-R_0)^2}{\tau^2} + V_\theta^2. \quad (24)$$

Then substituting from (24) into (15) leads to the result

$$t_D = \left[\frac{R(R-R_0) - \sqrt{D^2(R-R_0)^2 + (D^2-R^2)V_\theta^2\tau^2}}{(R-R_0)^2 + V_\theta^2\tau^2} \right] \tau. \quad (25)$$

The following argument shows that when the aircraft are closing t_D is an increasing function of $|V_\theta|$. Define v by

$$|V_\theta|\tau = (R-R_0)v. \quad (26)$$

Then (25) can be written

$$t_D = \frac{R - \sqrt{D^2 + (D^2-R^2)v^2}}{(R-R_0)(1+v^2)} = \frac{R^2 - D^2}{(R-R_0)[R + \sqrt{D^2 + (D^2-R^2)v^2}]} \quad (27)$$

For closing aircraft, when there is an alert, $R > R_0$ according to (1). Also, if the separation standard has not already been violated $R > D$. Then it follows by inspection of (27) that t_D is an increasing function of v and hence, by (26), an increasing function of $|V_\theta|$ as stated.

This fact, combined with (18), (19) and (24), provides the range of V_θ values for which a maneuver is necessary, after an alert, in order to maintain the desired separation. That is, the miss distance will be at least D if and only if V_θ lies outside the range defined by

$$0 \leq |V_\theta| \leq \frac{(R-R_0)D}{\tau \sqrt{R^2-D^2}} = \bar{V}_\theta. \quad (28)$$

The probability that V_θ is in the range defined by (28) is given by

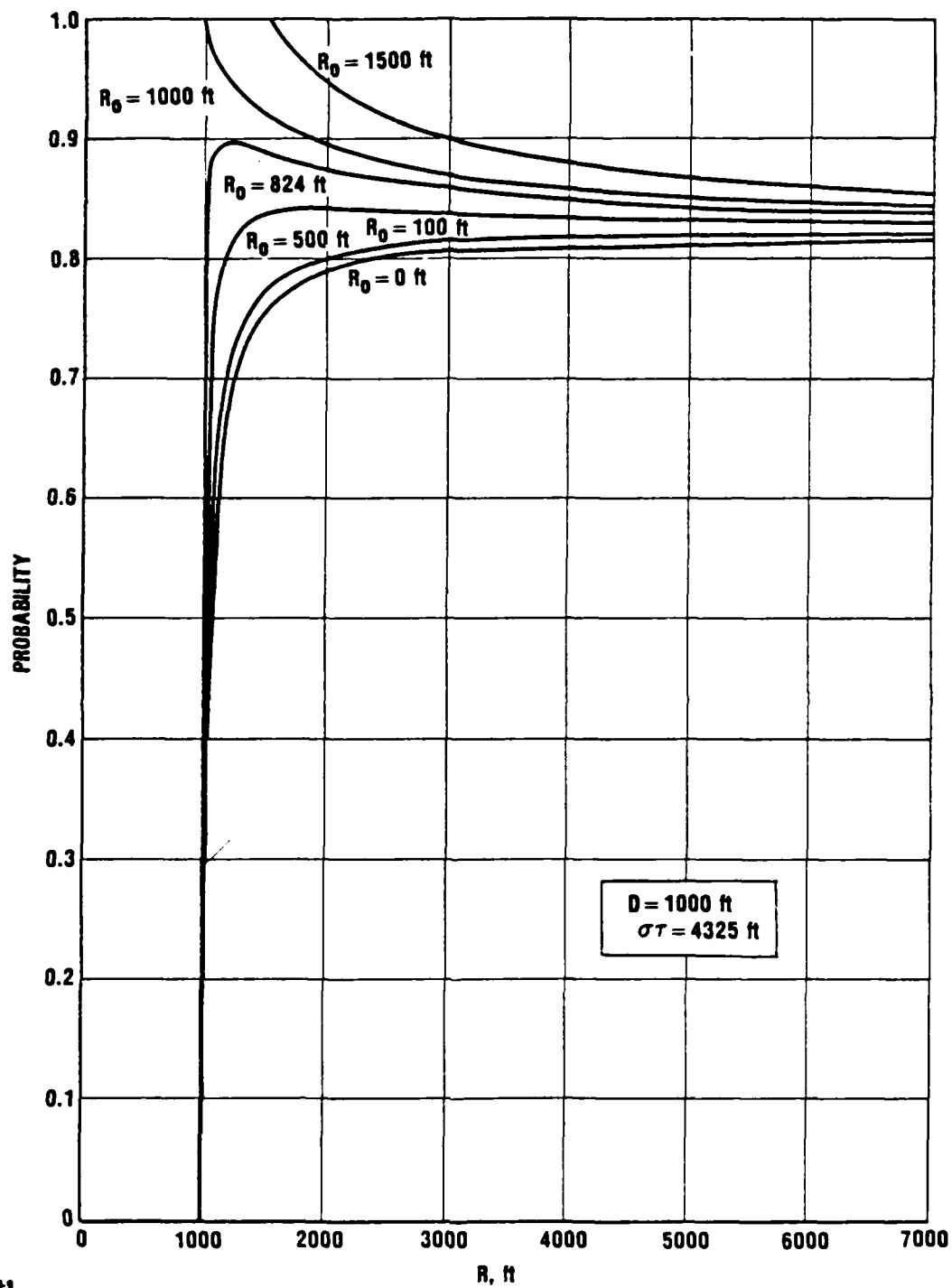
$$P(0 \leq |V_\theta| \leq \bar{V}_\theta) = \frac{2}{\sigma\sqrt{2\pi}} \int_0^{\bar{V}_\theta} e^{-\frac{V_\theta^2}{2\sigma^2}} dV_\theta. \quad (29)$$

The probability P_U that the alert, having occurred, is unnecessary is the complement:

$$P_U = 1 - P(0 \leq |V_\theta| \leq \bar{V}_\theta). \quad (30)$$

The probability P_U as a function of the separation R that exists when an alert occurs is illustrated in Fig. 5 for various values of the DMOD parameter R_0 . For these cases the miss distance standard D is 1,000 ft and the quantity σ is 4,325 ft, which corresponds to a σ of 173 ft/sec and a τ of 25 sec.

Figure 5 indicates that for aircraft separations beyond 0.5 nmi or so, reducing DMOD has a relatively small effect on reducing the probability that an alert is unnecessary and hence on reducing the number of unnecessary alerts. On the other hand,



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FIGURE 5. Probability that an alert, having occurred, is unnecessary.

reducing DMOD has a substantial effect on the reduction of unnecessary alerts only for separations less than 1,500 ft or so.*

This phenomenon has an interesting parallel, according to the remark in Section IV-B concerning the change ΔR_0 in DMOD that is equivalent to a change $\Delta \tau$ in τ . The point was made that $\Delta \tau$ is $-\frac{\Delta R_0}{R}$, which is equal to $\frac{\Delta R_0}{R-R_0} \tau$ when an alert occurs, implying that the equivalent change in τ is largest when the separation R is near the value R_0 of DMOD.

Given that an alert has occurred, a measure of the algorithm safety is the probability distribution $P(0 \leq t_D \leq t)$ of the available maneuvering time t_D . If $\bar{V}_\theta$ in (29) is replaced by the more general $|V_\theta|$, the probability distribution for t_D can be obtained in a straightforward way from (29) with the aid of (26) and (27). Curves illustrating this probability distribution are presented in Fig. 6 for various values of the DMOD parameter R_0 ft and values of the separation R ft for which DMOD has a significant effect on the number of unnecessary alerts, again with assumed values of 1,000 ft for the miss distance standard D and 4,325 ft for σ .

It can be seen that each probability distribution curve in Fig. 6 starts from 0 at t_{Dmin} sec, given by (20), rises to a value $1-P_U$ given by (29), at t_{Dmax} sec, given by (21), and then becomes horizontal. Between t_{Dmin} and t_{Dmax} the curves are convex; hence a lower bound on the probability distribution for t_D can be obtained by replacing the curve in this interval by a straight line, which is indicated by a dashed line in Fig. 6. Clearly, this lower bound can be constructed by inspection of the curves in Figs. 4 and 5 with no need for further calculation other than the obvious changes in scale that are required for consistency with the measurement units adopted in Fig. 6.

*The implication is that any additional alerts suppressed by the reduction of DMOD must have been necessary, i.e., their suppression will result in a violation of the miss distance standard.

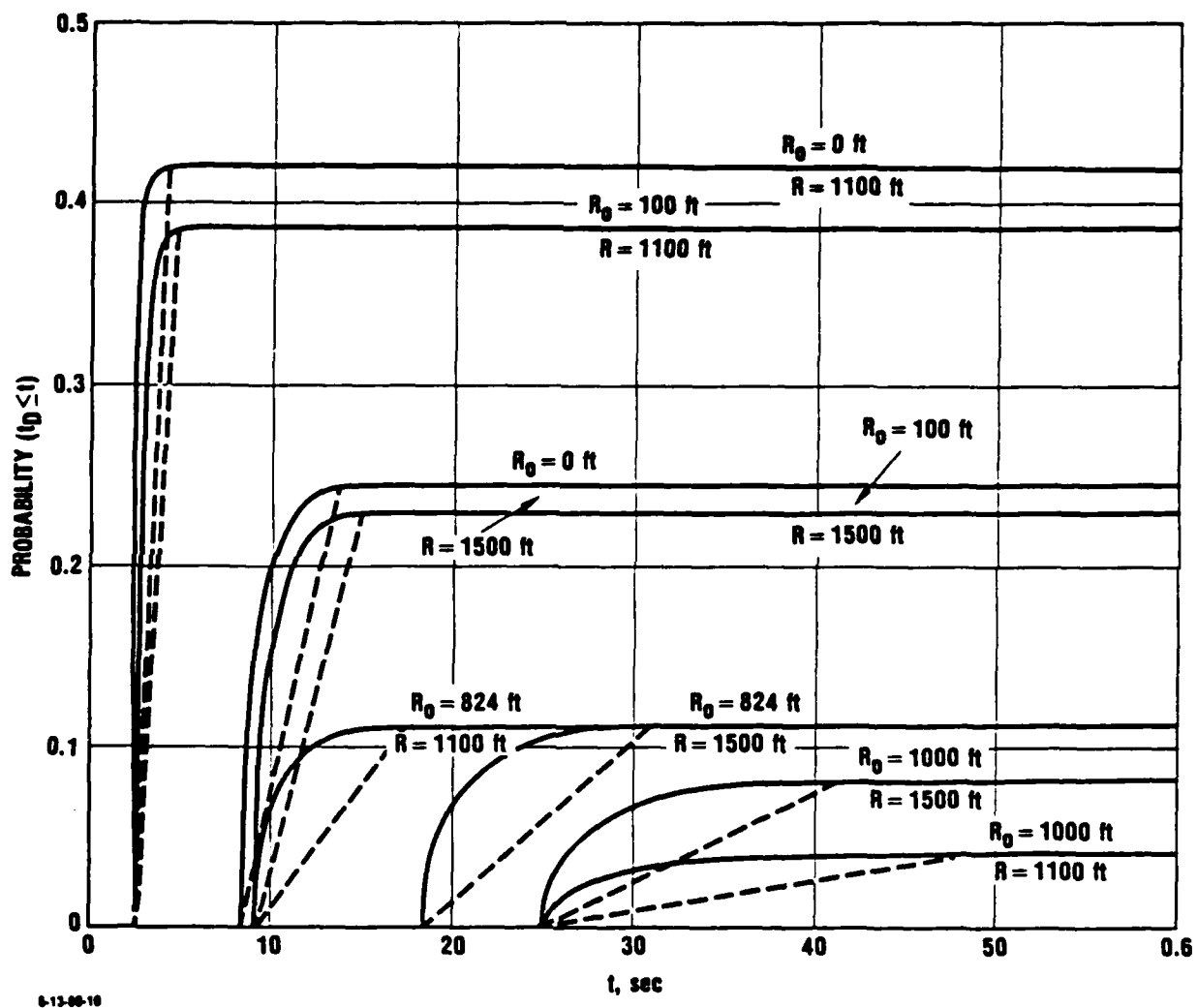


FIGURE 6. Probability distribution for the time to a separation equal to the miss distance standard when an alert occurs.

E. DISCUSSION OF SAFETY VERSUS UNNECESSARY ALERTS

An examination of Figs. 4, 5 and 6 reveals some paradoxical relationships between the aircraft separation R at the time of an alert, the value of the effective DMOD parameter R_0 , the miss distance standard D , the probability that the alert is unnecessary, and the probable amount of time available for a maneuver when the alert is necessary. These relationships not only pertain to the utility of the collision algorithm in providing adequate maneuver time, but they may also affect the aircraft pilot's subjective evaluation of the reliability of the BCAS.

According to Fig. 5, for example, when the effective DMOD is greater than or equal to the miss distance standard the probability that an alert, having occurred, is unnecessary is always high. Moreover, the smaller the aircraft separation at the time of the alert, the more likely it is that the alert is unnecessary. To some extent this is also the case for DMOD values that are less than the miss distance standard but near it.

In addition, Fig. 4 indicates that, when an alert is necessary, for DMOD values below but near the miss distance standard, the maneuver time provided by the algorithm may be greater for smaller separations at the time of the alert than for larger. In such a case, logically, the encounter should be judged more dangerous when the aircraft separation is large than when it is small, a fact that appears to violate intuition and therefore may be confusing to the pilot.

Figure 6 is consistent with these observations. However, it also indicates that for small values of the effective DMOD the collision algorithm is not only almost worthless unless the separation between the conflicting aircraft is large, but may actually introduce additional subjective dangers. For example, the curve labeled $R_0 = 0$ ft, $R = 1,100$ ft shows that, for the assumed parameter values, the a priori probability that an alert,

having occurred, is unnecessary is nearly 60 percent. This is for a separation that is within 10 percent of the allowable miss distance. On the other hand, if the alert should happen to be necessary the same curve indicates that the maneuver time made available by the collision algorithm will never be more than 4.5 sec and the probability is quite high that it will be less than 3 sec.* Therefore, in this case the collision algorithm is totally counterproductive: it produces a lot of false alerts and never provides adequate maneuver time when it generates a true alert. For DMOD values greater than zero, but small compared with the miss distance standard, the same phenomenon can be observed.

Figure 7, which gives the probability that an alert is necessary as a function of τ for a miss distance standard of 1,000 ft, indicates that the unnecessary alert problem is relieved rapidly with decreasing τ for τ values smaller than 25 sec. However, Fig. 4 and Fig. 6 taken together show that the maneuver time provided by the algorithm decreases in the same proportion as τ .

F. NON CO-ALTITUDE ENCOUNTERS

1. The Effect of Using Slant Range Instead of Horizontal Range in the Horizontal Collision Algorithm

When two encountering aircraft are at different altitudes their measured separation is actually a slant range R_S given by

$$R_S = \sqrt{R^2 + z^2}, \quad (30)$$

* It should be remembered that the probability distributions shown in Fig. 6 are a priori. Once it is granted that an alert is necessary, the probability scale changes so that, in fact, the horizontal line for each curve is positioned at a probability level equal to one.

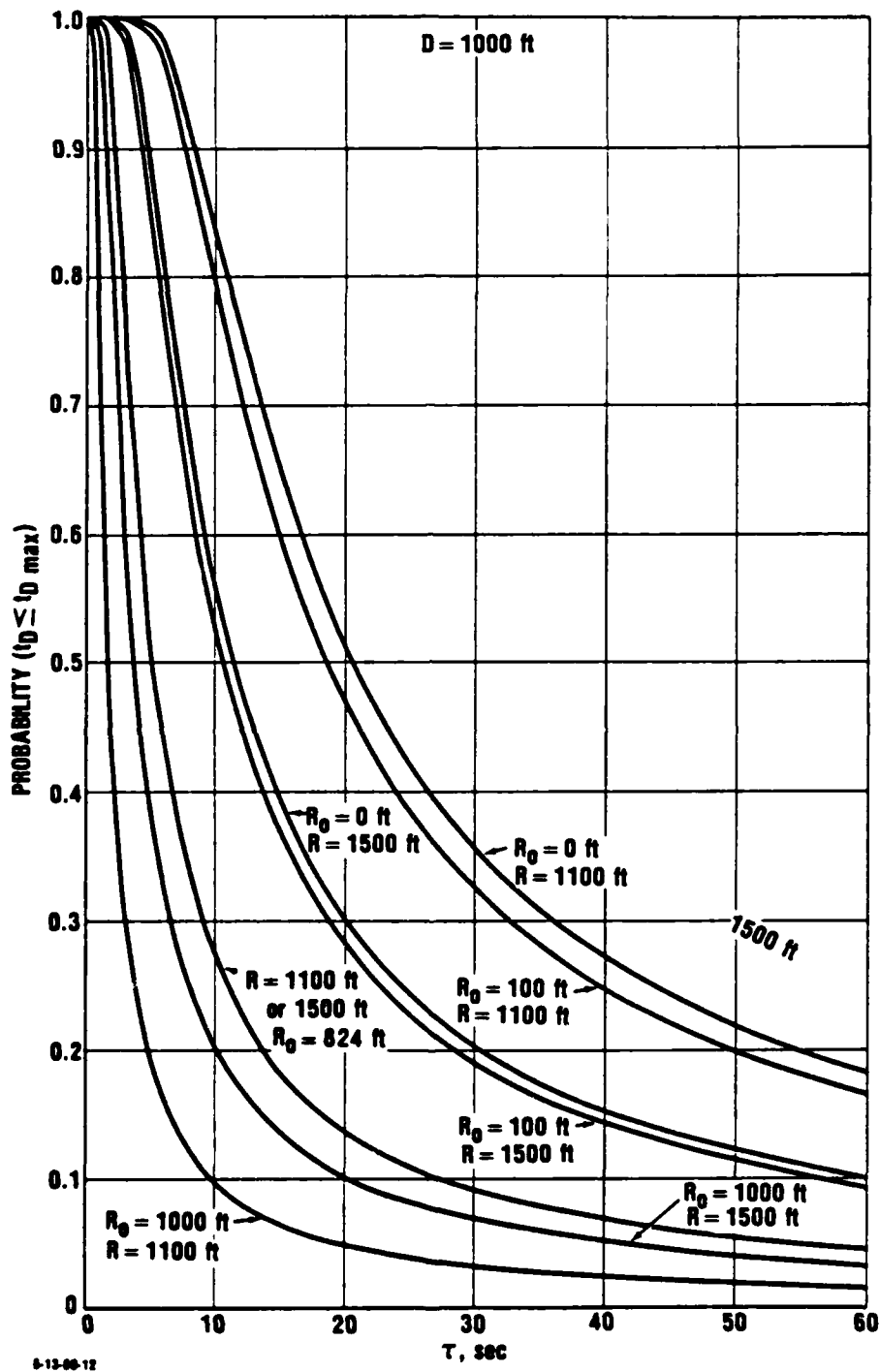


FIGURE 7. Probability that an alert, having occurred, is necessary versus tau.

where z is the relative altitude of one with respect to the other and R is the horizontal separation. On the other hand, the collision algorithm (1) is a hazard criterion that is based on the horizontal separation and separation rate of the aircraft. Thus, using R_S in place of R (and $\dot{R}_S$ in place of $\dot{R}$) is tantamount to incorporating additional measurement errors in the criterion.

The effect of this additional error is compensated to some extent by the fact that there already exists some vertical separation between the encountering aircraft, and, therefore, less maneuver time is needed to achieve the specified vertical separation after the BCAS generates a positive command. However, there is no comparable effect to mitigate the influence of the additional error on increasing the chance that the collision algorithm will fail to provide a necessary alert. Thus, it seems useful to examine some of the implications of ignoring the difference between R_S and R when encountering aircraft are not co-altitude.

First of all, it should be recognized that the effective error induced in the separation rate $\dot{R}$ by treating the slant range as if it were the horizontal separation has a special functional dependence on R and $\dot{R}$. From (30) it follows that, for a constant relative altitude z ,

$$\dot{R}_S = \frac{R\dot{R}}{R_S} \quad (31)$$

Thus, the effective error ϵ_{RS} in the separation and $\epsilon_{\dot{R}S}$ in the separation rate due to using slant range in place of the horizontal separation are given by

$$\begin{aligned} \epsilon_{RS} &= R - R_S, \\ \epsilon_{\dot{R}S} &= \dot{R} - \dot{R}_S = \dot{R}\left(1 - \frac{R}{R_S}\right). \end{aligned} \quad (32)$$

When the aircraft are closing, $\dot{R}$ is negative; hence, according to (32), ϵ_{RS} and $\dot{\epsilon}_{RS}$ are both negative. If the measured slant range R_S is used in (1) in place of R , an alert occurs when

$$R - \epsilon_{RS} + \tau (\dot{R} - \dot{\epsilon}_{RS}) < R_0,$$

i.e., when

$$R + \tau \dot{R} < R_0 + \epsilon_{RS} + \tau \dot{\epsilon}_{RS}. \quad (33)$$

The implication of (33) is that use of the slant range in place of the horizontal separation in effect reduces the DMOD parameter by the magnitude of the total effective error ϵ_S defined by

$$\epsilon_S = \epsilon_{RS} + \tau \dot{\epsilon}_{RS}. \quad (34)$$

The error ϵ_S is inherently more serious than real measurement errors because it always reduces the protection provided by DMOD, whereas measurement errors may actually increase the protection on occasion.

The hazard region diagram of J. Holt, an example of which for co-altitude aircraft is illustrated in Fig. 1, can furnish some insight concerning the sensitivity of the protection provided by the collision algorithm to altitude differences between encountering aircraft. The diagram in the case of non co-altitude aircraft represents the region defined by (1) when R_S is substituted for R and $\dot{R}_S$ for $\dot{R}$.

The result of this substitution and further substitutions from (2) and (31) is the equation

$$R_S = \frac{\tau VR}{R_S} \cos \theta + R_0 \quad (35)$$

for the hazard region boundary. The curve is given by (35) in the same polar coordinate system used previously for the case

of co-altitude aircraft. The quantity R_S is given by (30), with R interpreted as the radial coordinate and z as a fixed parameter. In (35) the quantities τ , V and R_0 are also fixed parameters, and θ is the angular coordinate.

For each of the limiting cases $V = 0$ and $R_0 = 0$ the hazard region boundary reduces to a circle. For $V = 0$ the equation for the boundary is

$$R^2 = R_0^2 - z^2; \quad (36)$$

for $R_0 = 0$ it is

$$R^2 - \tau VR \cos \theta + z^2 = 0. \quad (37)$$

According to (36), the circle corresponding to $V = 0$ is centered at the origin of the coordinate system and has a radius equal to $\sqrt{R_0^2 - z^2}$. According to (37), the circle corresponding to $R_0 = 0$ is centered on the polar axis a distance $\frac{\tau V}{2}$ to the right of the origin of the coordinate system and has a radius equal to $\sqrt{(\frac{\tau V}{2})^2 - z^2}$.

In either case all protection disappears if the circle radius is zero. Thus, when the relative velocity is zero there will be no protection if the difference in aircraft altitudes is greater than or equal to DMOD.* This is also true for the case of zero DMOD* if the aircraft altitude difference is greater than or equal to the quantity $\frac{\tau V}{2}$.

Thus, in the previous example wherein the DMOD was assumed to be 824 ft and τ was assumed to be 25 sec, all protection vanishes when the altitude difference is no smaller than 824 ft. If DMOD is zero and the relative velocity is 50 ft/sec, the

*In this context the reference is to the effective DMOD, which is the remainder after subtracting the total measurement error $\epsilon_R + \tau \epsilon_{\dot{R}}$.

protection would not exist for aircraft whose altitude difference is as large as 625 ft.

Figures 8, 9, 10 and 11 illustrate, in terms of hazard region diagrams, the effect of altitude differences on protection by the collision algorithm for various values of $V\tau$ and altitude difference z measured in units of R_0 ft. As might be expected from consideration of the two extreme cases, $V = 0$ and $R_0 = 0$, much protection is lost when the altitude difference is a significant fraction of DMOD and the product $V\tau$ is small compared to R_0 .

2. Projected Vertical Miss Distance

In the case of non co-altitude aircraft encounters the active BCAS logic (Ref. 5) requires that the relative altitude rate of the encountering aircraft be projected ahead so as to obtain an estimate of the vertical miss distance at the time of minimum horizontal separation. The time used for this purpose is the ratio $-\frac{\dot{R}}{R}$ (referred to as "true tau") unless the ratio exceeds a pre-selected constant, called TVPCMD, in which event it is that constant that is used.

It would, perhaps, be more useful to obtain the vertical miss distance that occurs at the time when it is estimated that the horizontal separation standard will be violated. This is the time t_D given by (15). Since t_D depends upon the magnitude V of the relative velocity and that quantity is not known in general, it would be necessary to use, instead, a value taken from the time interval between t_{Dmin} , given by (20), and t_{Dmax} , given by (21), however. The safest value for this purpose is t_{Dmax} , of course.

The use of t_{Dmax} in place of true tau, instead of the constant TVPCMD, when the horizontal separation rate is small also ensures a low probability of missing a threat. The alert rate will tend to be even smaller because t_{Dmax} is always less than true tau. This can be seen as follows.

Z AND HORIZONTAL COORDINATES MEASURED IN UNITS OF R_0 FT

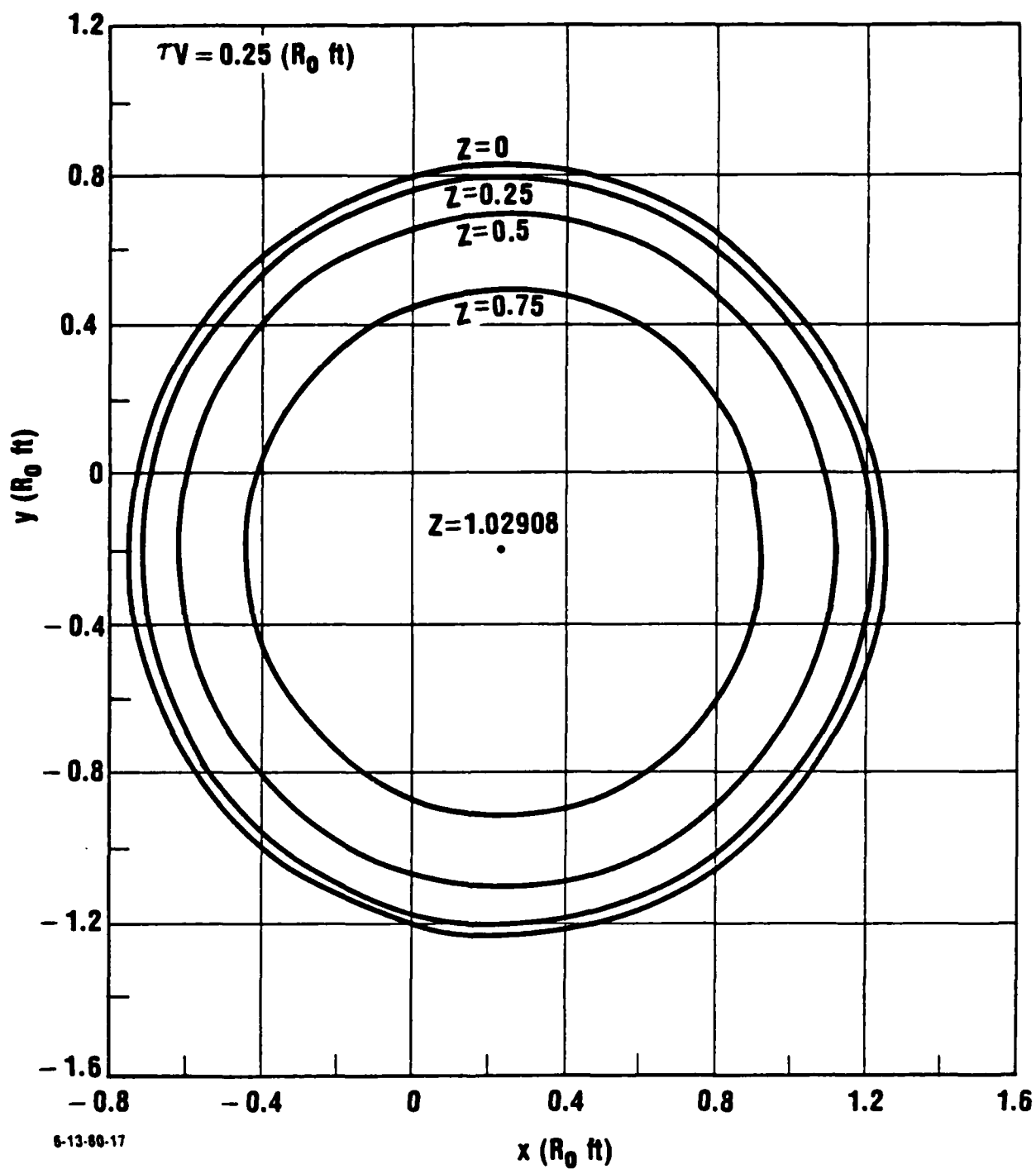
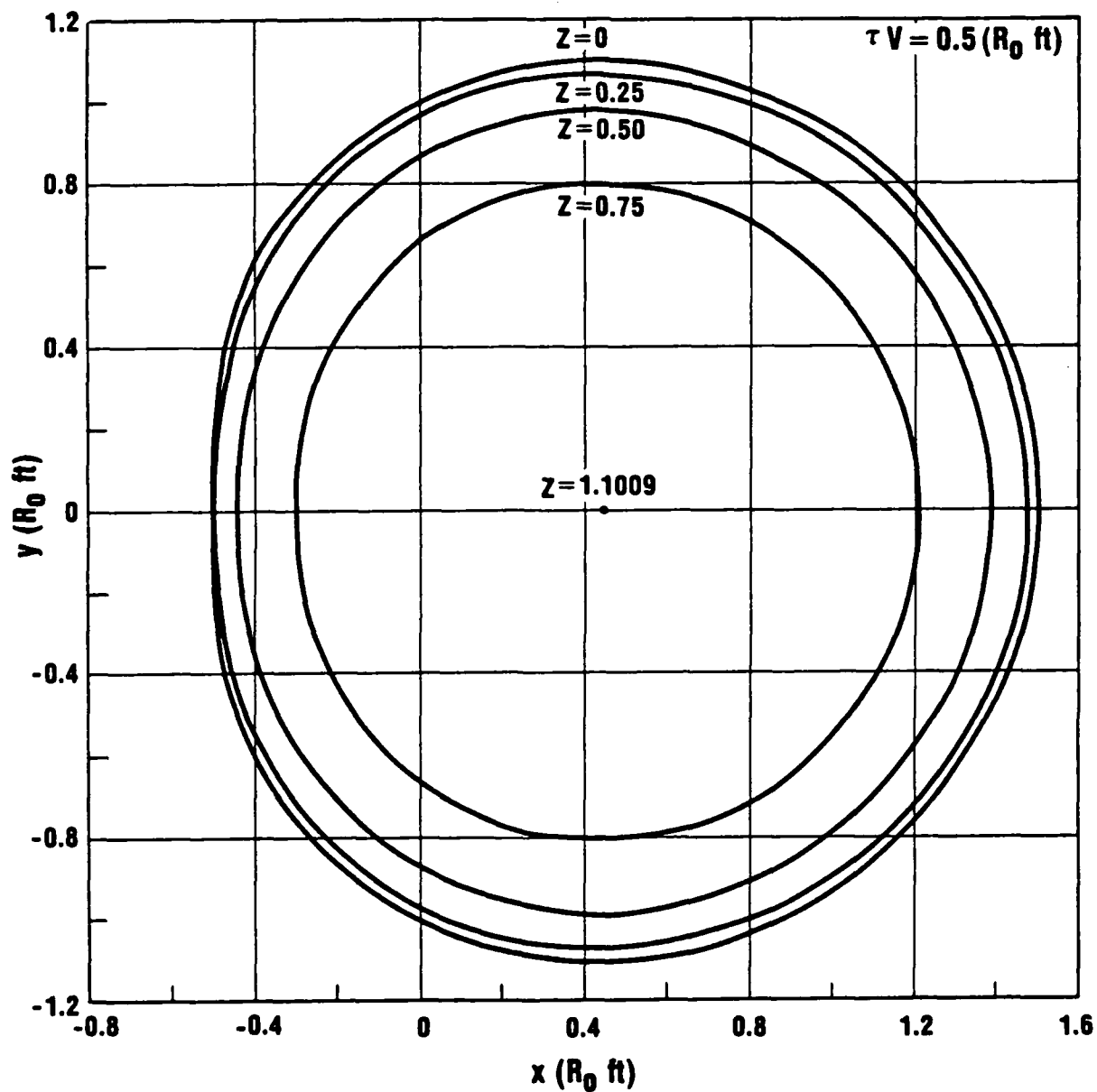


FIGURE 8. The hazard region defined by a collision algorithm when slant range is used instead of horizontal range.

Z AND HORIZONTAL COORDINATES MEASURED IN UNITS OF R_0 FT



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FIGURE 9. The hazard region defined by a collision algorithm when slant range is used instead of horizontal range.

Z AND HORIZONTAL COORDINATES MEASURED IN UNITS OF R_0 FT

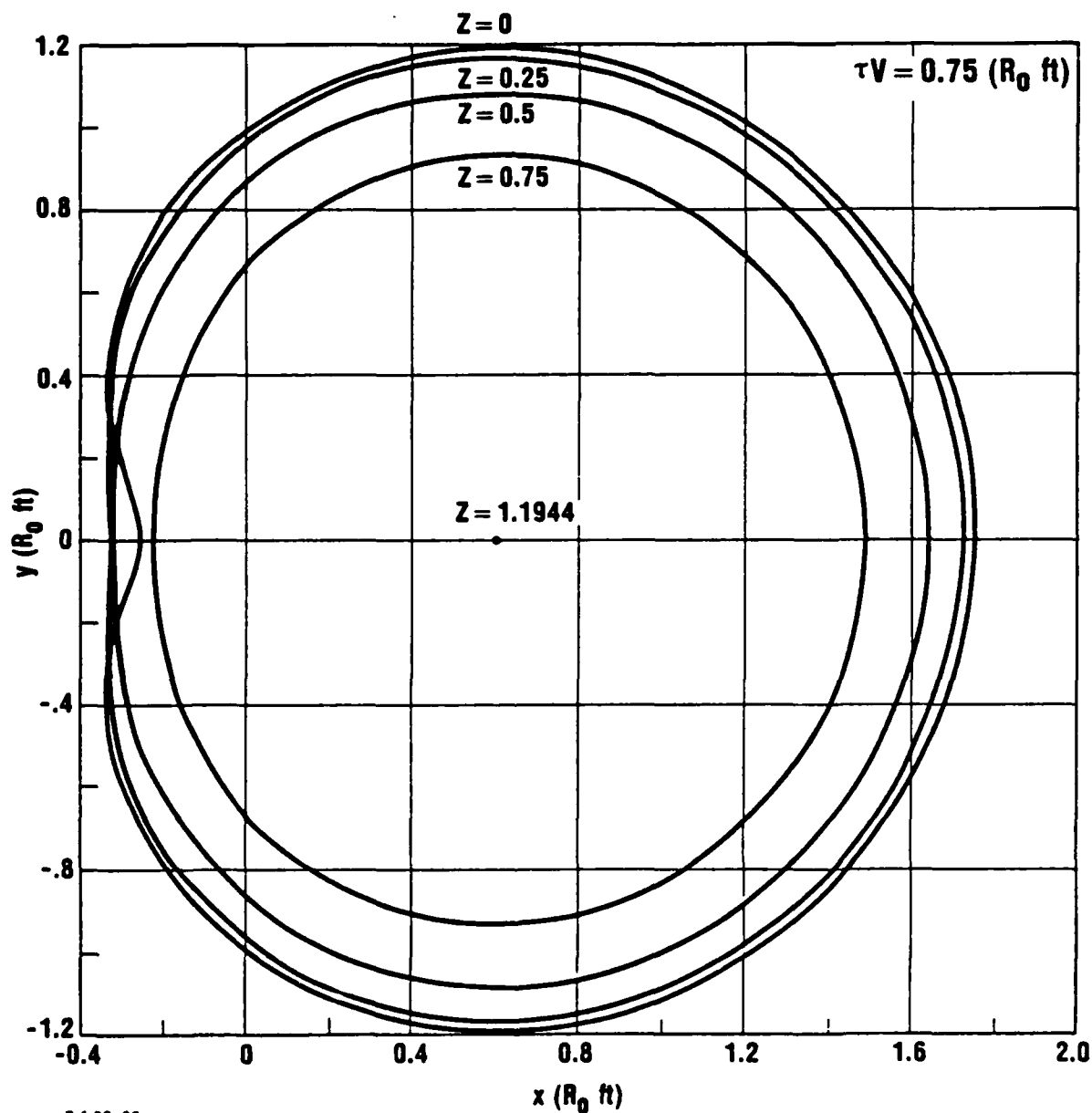
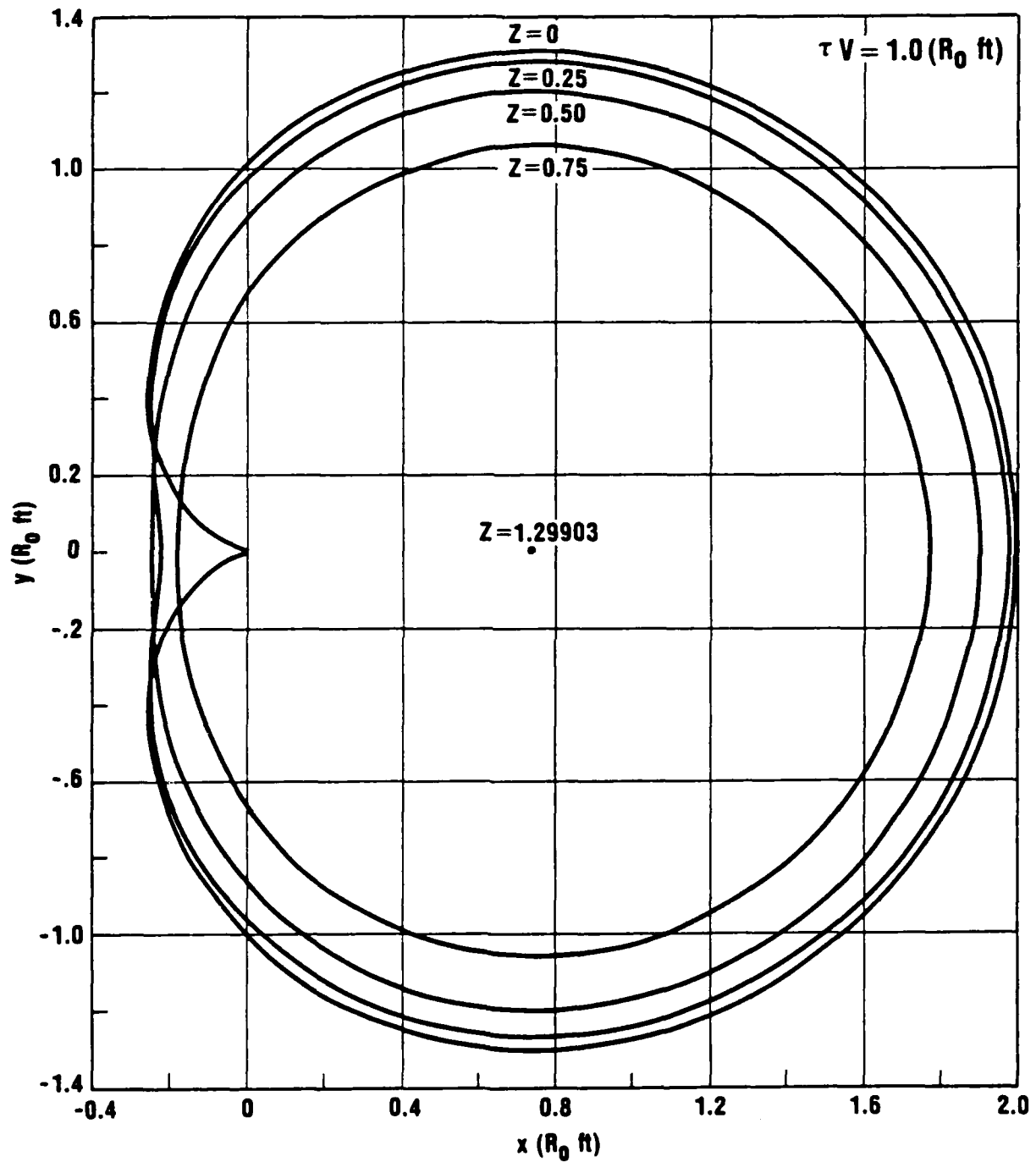


FIGURE 10. The hazard region defined by a collision algorithm when slant range is used instead of horizontal range.

Z AND HORIZONTAL COORDINATES MEASURED IN UNITS OF R_0 FT



7-1-80-97

FIGURE 11. The hazard region defined by a collision algorithm when slant range is used instead of horizontal range.

According to (1) at the time of a horizontal alert

$$-\frac{R}{\tau} = \frac{R\tau}{R-R_0} \quad (38)$$

On the other hand, according to (20) and (21)

$$t_{Dmax} = \frac{R^2 - D^2}{R(R-R_0)} \tau. \quad (39)$$

Now, it is certainly true that

$$R - \frac{D^2}{R} < R. \quad (40)$$

Then, upon multiplying (40) by τ and dividing by $R-R_0$ it will be seen to follow from (38) and (39) that

$$t_{Dmax} < -\frac{R}{R}, \quad (41)$$

as asserted.

G. EFFECTIVE HAZARD REGION DEFINITION

Reference 12 contains the logical basis for selecting collision algorithm parameters when the only available data is that assumed for active BCAS, namely, vertical and horizontal separations and separation rates. There it is demonstrated for horizontal criteria that the modified tau algorithm (1) is the most efficient one possible in the sense that it is both necessary and sufficient to protect against all encounters for which a priori bounds on relative acceleration and measurement errors and a required time to perform an escape maneuver are assumed.

When the parameters R_0 and τ appearing in (1) are chosen properly there is no need for an additional minimum range criterion.* Conversely, the exclusion of any part of the

* Cf. Ref. 13.

hazard region defined by (1) introduces the risk that a necessary alert will be missed.

The same argument that supports the use of the horizontal algorithm (1) in connection with horizontal separation data also applies to a similar algorithm, such as that employed in active BCAS logic, in connection with vertical separation data. Therefore, a vertical collision algorithm should have the same form as (1), with a DMOD parameter analogous to R_0 to account for measurement error and vertical acceleration. Because of its meaning the value of τ should be the same as that used in the horizontal alert criterion.

In the present version of active BCAS logic (Ref. 5) both the horizontal and vertical collision algorithms deviate in certain respects from these precepts. In view of the principles established with mathematical precision by Ref. 12, it is difficult to understand how these deviations can be justified.

The active BCAS vertical algorithm has the form

$$\begin{aligned} z + \tau \dot{z} &< 0, \quad \dot{z} < 0 \\ z &< z_0, \quad \dot{z} > 0 \end{aligned} \tag{42}$$

where z is the vertical separation, $\dot{z}$ is the vertical separation rate, and z_0 is a fixed minimum vertical separation threshold. Using (42) instead of a modified tau criterion is equivalent to using, instead of (1), a horizontal algorithm with zero DMOD along with a minimum range criterion. This will not only result in missed necessary alerts on some occasions but will also generate unnecessary alerts on others.*

*Actually, the active BCAS logic does provide for the removal of some, but not all, of the unnecessary hazard space when there is a positive vertical separation rate.

In the case of the horizontal algorithm, the active BCAS logic removes part of the hazard space when there is a positive horizontal separation rate.* According to Ref. 5 this is done in order to reduce the amount of time that an alert condition remains in effect when the encountering aircraft are separating. The implication is that the risk is less, or becomes less in due course, during an alert that occurs when encountering aircraft are, or appear to be, separating rather than closing.

However, it is possible to dispute this assumption on two accounts. The algorithm (1) provides a hazard region even when there is a positive separation rate $\dot{R}$ because of the presence of the DMOD parameter R_0 , which was introduced in order to compensate for measurement error and some relative acceleration. Therefore, the assumption may be incorrect because (a) due to measurement error, the aircraft may actually be closing even though they appear to be separating, and (b) if the aircraft are initially separating slowly while traveling on nearly parallel flight paths a turn by one of them after the alert should have occurred could rapidly change the separation rate from positive to negative.

In fact, neglecting an alert, or prematurely eliminating an alert condition, may be more hazardous in the region of positive separation rates than elsewhere because in that region the separation R is necessarily small. This will be the case if the effective DMOD parameter R_0 is less than the miss distance standard D and the separation R is near D , as Figs. 4 and 5 indicate.

*Cf. Chapter II, Section B.

V. SUMMARY OF CONCLUSIONS

A. VALIDITY OF A MATHEMATICAL MODEL OF AIR TRAFFIC

A mathematical model for low-altitude (below 10,000 ft) air traffic, originally formulated to fit the Los Angeles Standard Traffic Model, has been tested against empirically-derived traffic statistics for the Houston Intercontinental air terminal region. According to the theory, the horizontal cartesian coordinates of an aircraft, relative to some geographical center, are identically distributed, independent Gaussian random variables with mean zero and a common standard deviation; the same statistical assumptions are postulated for the aircraft's velocity components.

The theoretical model is consistent with processed empirical data derived by the Mitre Corporation from ARTS tapes containing radar records of 65 hours of air traffic around the Houston terminal. Mitre processed the data to expose and study mid-air conflicts that would be defined by active BCAS alert logic at various parameter settings. Results that are compared here with the theory are of two types. One is in the form of histograms showing, for aircraft in conflict: (1) the distribution of absolute speeds, (2) the distribution of closing speeds, (3) the distribution of separations. The other is the reduction that occurs in the number of alerts when BCAS logic parameters are reduced by certain specified amounts.

The theoretical traffic model fits the histogram data best when the velocity component standard deviation is taken to be 106 kn. The accuracy of the fit appears to be independent of the model's traffic density standard deviation.

The theory also predicts the alert reductions that were observed by Mitre when BCAS logic parameters were changed. The predictions are virtually independent of the models' traffic density standard deviation and are accurate for three different values of its velocity component standard deviation that were tried: 72.2 kn (the Los Angeles Standard Traffic Model value), 83.3 kn (the 99 percent confidence level that no aircraft violates the 250 kn speed limit), and 106 kn (the value that provides the best fit to the Mitre histogram data). This leads to the conjecture that alert rate changes due to changes in BCAS logic parameters are essentially independent of the air traffic environment.

B. BCAS ALERT LOGIC PROTECTION

1. Alert Logic Parameter Selection

a. Some allowance for acceleration should be made in selecting the DMOD parameter because, aside from the possibility of turns, certain aircraft may not be able to perform vertical avoidance maneuvers without introducing an additional horizontal velocity component into the forward motion.

b. A DMOD parameter should be included in the vertical tau alert criterion because of altitude measurement errors and vertical acceleration.

c. In desensitization a reduction of τ is preferable, in principle, to a reduction in DMOD because reducing DMOD reduces protection for all aircraft, whereas reducing τ may not affect aircraft that can react quickly to an alert.

d. If reasonable τ values (see Fig. 3) are maintained when alert logic parameters are reduced, the probability of missing an alert that is necessary to avoid violating an adequate minimum miss distance standard will be low.

e. The projected vertical miss distances rule presently used in active BCAS logic relies upon the substitution of a

pre-selected constant, TVPCMD, for true tau, which is the separation divided by the closing rate, when the value of true tau exceeds that constant. A better way to deal with large values of true tau (i.e., small separation rates) might be to use t_{Dmax} , given by (21), in place of true tau.

2. Protection And The Alert Rate

a. For a given minimum miss distance standard D and a separation R when there is an alert, either the alert is unnecessary or the amount of available maneuver time lies between t_{Dmin} , given by (20), and t_{Dmax} , given by (21).

b. When the effective DMOD is greater than or near the minimum miss distance standard, the smaller the aircraft separation at the time of an alert, the more likely it is that the alert is unnecessary.

c. When the effective DMOD is smaller than but near the minimum miss distance standard the maneuver time available after an alert may be greater for smaller separations of the aircraft in conflict than for larger. This may violate a pilot's intuition and tend to reduce his acceptance of the system unless he is forewarned of the paradox.

d. The possibility of pilot disenchantment with the system is enhanced by the fact that the percentage of unnecessary alerts will always be high. In fact, this is true even for small values of DMOD, for which the available maneuver time after a necessary alert will generally be too small to be useful. This will be true unless the separation is large, indicating a head-on or near head-on collision. Therefore, it may be better to turn the system off than to desensitize it to very small DMOD values.

3. Dubious Compromises of Protection

a. The use of slant range in place of a horizontal range estimate may seriously compromise protection because the effective error, unlike random measurement errors, always reduces protection.

b. When aircraft are apparently separating instead of closing, reducing the hazard region defined by BCAS alert logic, which is the current practice, is dangerous because of measurement error and possible unforeseen accelerations and the fact that in such a region the aircraft separation is necessarily small.

REFERENCES

1. L. B. Zarrelli and J. A. Grupe, "BCAS Parameter Study," Mitre Briefing, 22 February 1980.
2. R. A. Rucker, "Aviation Collision Risk in the U.S. vs Airway Lateral Separation Standards -- A Historical Perspective," Mitre Report No. MTR-7030, September 1975.
3. I. W. Kay, "Analysis of Collision Avoidance Threat Logic That Uses Bearing Data," IDA Study S-500, September 1978.
4. S. Cohen and F. Maginnis, "Statistical Summary of the 1982 Los Angeles Basin Standard Traffic Model," Mitre Corporation Report No. MTR-6387, April 1973.
5. N. Broste et al., "Active BCAS Collision Avoidance System -- Collision Avoidance Algorithms," Mitre Report No. MTR-79W00110, Rev. 1, January 11, 1980.
6. "Engineering Requirement for the Active Beacon Collision Avoidance System, Appendix A (Proposal Technical Characteristics for the Active Beacon Collision Avoidance System)", FAA Report No. FAA-ER-250-2, July 18, 1979.
7. B. Billman et al., "Air Traffic Control/Full Beacon Collision Avoidance System -- Chicago Simulation," FAA Final Report No. FAA-RD-79-16, April 1979.
8. B. Billman et al., "Air Traffic Control/Full Beacon Collision Avoidance System -- Knoxville Simulation," FAA Final Report No. FAA-RD-79-25, August 1979.
9. A. D. Adkins et al., "Air Traffic Control/Active Beacon Collision Avoidance System Knoxville Simulation," Final Report (FAA Report No. FAA-NA-79-54), October 31, 1979.
10. N. Broste et al., "Preliminary Evaluation of Active Beacon Collision Avoidance System Performance (Simulated): Protection and Alarms," Mitre Report No. MTR-79W00135, April 1979.

11. W. H. Harman, *"Air Traffic Density and Distribution Measurements,"* Lincoln Laboratory Report No. ATC-80, May 1979.
12. J. Holt et al., *"Computer/Simulation Study of Air-Derived Separation Assurance Systems in Multiple Aircraft Environment,"* Collins Radio Company, Report No. RD-68-34 (Third Interim Report), July 1968.
13. J. J. Bagnall and I. W. Kay, *"Review and Analysis of Some Collision Avoidance Algorithms With Particular Reference to ANTC-117,"* IDA Study S-450, June 1975.
14. I. W. Kay, *"Horizontal Midair Collision Avoidance Maneuvers in Two-Aircraft Encounters,"* IDA Paper P-1348, August 1978.
15. B. Billman et al., *"Modeling Pilot Response Delays to Beacon Collision Avoidance System Commands,"* FAA Report No. FAA-RD-79-74 (Final Report), October 1979.

APPENDIX A

ANALYSIS AND APPLICATION OF A THEORETICAL
MODEL OF AIR TRAFFIC STATISTICS

APPENDIX A

ANALYSIS AND APPLICATION OF A THEORETICAL MODEL OF AIR TRAFFIC STATISTICS

A. VELOCITY DISTRIBUTIONS

As observed in Ref. A-1, low altitude aircraft velocities in the Los Angeles Standard Traffic Model (Ref. A-2) appear to be normally distributed with no directional bias. More specifically, the two horizontal velocity components associated with every aircraft appear to be independent random variables with identical Gaussian probability distributions, each with zero mean and a standard deviation of 122 ft/sec (72.2 kn). This implies that the aircraft speeds are Rayleigh distributed, i.e., have a probability density of the form

$$P_S = \frac{S}{\sigma_o^2} e^{-\frac{S^2}{2\sigma_o^2}}, \quad (A-1)$$

where S is the aircraft speed and in this case the parameter σ_o has the value 122 ft/sec or 72.2 kn.

It also follows that the components of the relative velocity between two aircraft selected at random are independent, identically distributed Gaussian random variables with zero mean but with a standard deviation σ_v of $\sqrt{2}$ times 122 ft/sec, or 173 ft/sec. Thus, the aircraft separation rate, because it is equal to the radial component of their relative velocity, is a Gaussian distributed random variable with zero mean and a standard deviation of 173 ft/sec.

Reference A-3 contains a figure with a histogram depicting the percentage distribution of speeds of aircraft in conflict in the region around Houston Intercontinental Airport. Actually, the figure contains two such histograms, for one of which the definition of a conflict is based on a fixed set of parameters in the BCAS alert logic, while for the other the parameters were permitted to vary in accordance with the local desensitization zone rules. Only the data generated by the fixed parameter logic is considered here.

It is assumed here that the number of aircraft in conflict above 10,000 ft is negligible during the 65-hour period of ARTS tape data analyzed by the Mitre Corporation for the program summarized in Ref. A-3. This is tantamount to an assumption that restricting the data to aircraft in conflict is equivalent to taking a sample of low-altitude aircraft, albeit one that is somewhat biased against low speeds.

With these assumptions the histogram supplied by Ref. A-3 can be used to ascertain whether a probability density of the form (A-1), deduced originally from the Los Angeles Standard Traffic Model, will fit the Houston traffic environment as well. For this purpose it should be observed that the probability distribution corresponding to the density defined by (A-1) is given by

$$P(S < x) = \int_0^x P_S dS = 1 - e^{-\frac{x^2}{2\sigma_0^2}}. \quad (A-2)$$

Figure A-1 reproduces the aircraft speed histogram of Ref. A-3 superposed on three probability distribution curves calculated by means of (A-2). The dashed-line curve corresponds to the Los Angeles Standard Traffic Model σ_0 value of 72.2 kn (122 ft/sec). The solid-line curve corresponds to a σ_0 value of 83.3 kn, which would imply a 99 percent confidence level that no aircraft violates the speed limit of 250 kn. The

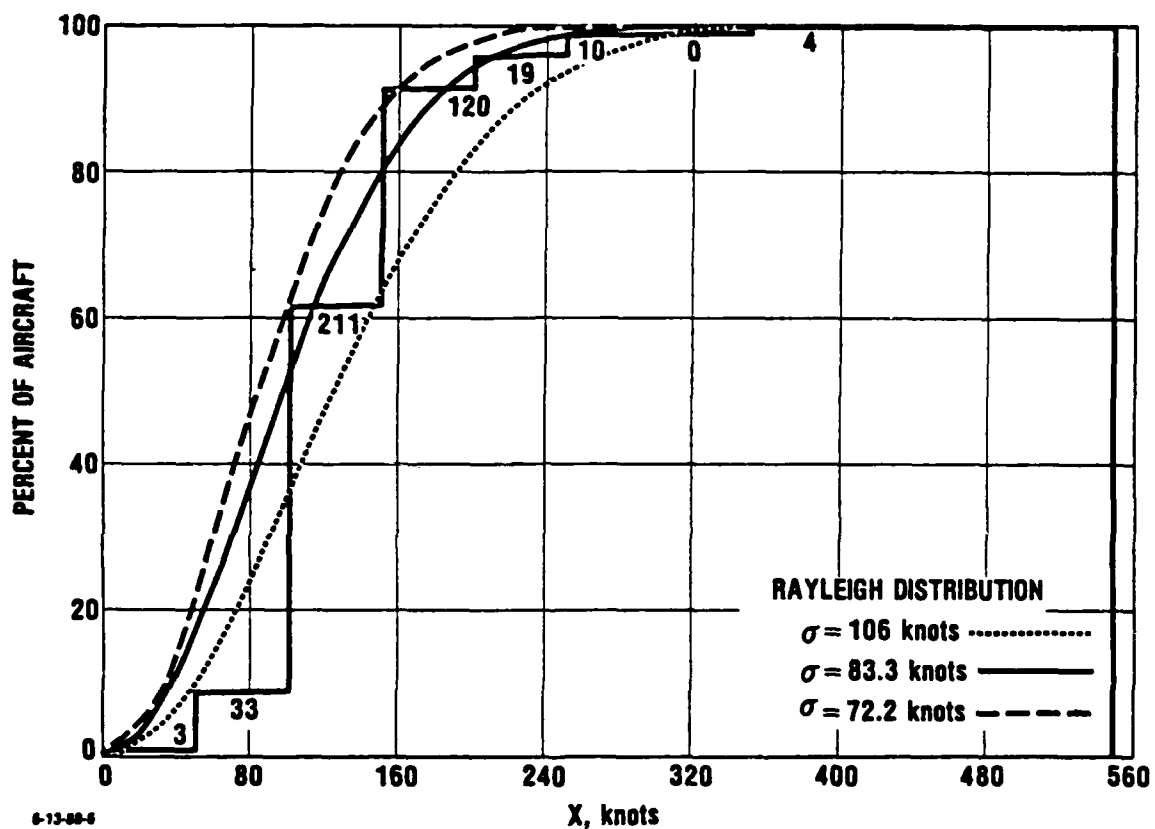


FIGURE A-1. Observed percent of aircraft with positive commands having speeds less than X.

remaining curve corresponds to a σ_0 value of 106 kn and is included for reasons that will become apparent later.

It is evident from the figure that the Los Angeles Traffic Model distribution does not fit the histogram as well as the other two. Although neither of those could be regarded as incompatible with the empirical data, it is clear that a distribution corresponding to a σ_0 value between 83.3 and 106 kn would fit best.

B. TRAFFIC DENSITY DISTRIBUTION AND THE ALERT PROBABILITY

Reference A-3 does not provide sufficient data to infer the horizontal or vertical distribution of aircraft in the Houston terminal region. However, as in the case of velocities, statistics from the Los Angeles Standard Traffic Model can be assumed.

According to Ref. A-1 low-altitude aircraft in the model are distributed about a traffic center in such a manner that the horizontal coordinates relative to the center are independent, identically distributed Gaussian random variables with mean zero and a standard deviation σ_R .* The value of σ_R in the Los Angeles model is 20 nmi.

It follows that the traffic is Rayleigh distributed in range from the center and uniformly in direction. It also follows that the separation R between two aircraft chosen at random is Rayleigh distributed with the probability density

$$P_R = \frac{R}{\hat{\sigma}_R^2} e^{-\frac{R^2}{2\hat{\sigma}_R^2}}, \quad (A-3)$$

where

$$\hat{\sigma}_R = \sqrt{2} \sigma_R .$$

*Actually, the distribution given by Ref. A-1 is bimodal, but one mode is so dominant that the other can be neglected without affecting any results obtained here.

This, together with the assumed Gaussian distribution for the separation rate $\dot{R}$, can be used to calculate the horizontal alert probability. In terms of τ and the DMOD parameter R_0 the condition to be imposed for this purpose is

$$R + \tau \dot{R} \leq R_0^*.$$

The probability that two aircraft selected at random will be in a horizontal alert condition can be expressed, in accordance with (A-3), by

$$\begin{aligned} P(R_0, \sigma_v, \tau, \hat{\sigma}_R) &= \frac{1}{\sqrt{2\pi} \sigma_v \hat{\sigma}_R} \int_0^\infty e^{-\frac{R^2}{2\hat{\sigma}_R^2}} R \int_{-\infty}^{\frac{R_0 - R}{\tau}} e^{-\frac{\dot{R}^2}{2\sigma_v^2}} d\dot{R} dR \quad (A-4) \\ &= \frac{1}{\sqrt{2\pi} \sigma_v} \left[\int_{-\infty}^{\frac{R_0}{\tau}} e^{-\frac{\dot{R}^2}{2\sigma_v^2}} d\dot{R} - \frac{1}{\tau} \int_0^\infty e^{-\frac{R^2}{2\hat{\sigma}_R^2} - \frac{(R - R_0)^2}{2(\sigma_v \tau)^2}} dR \right]. \end{aligned}$$

This expression can be transformed, by standard means, into one involving only elementary functions and normal distribution functions that are easy to evaluate numerically.

First, however, it is advantageous to simplify (A-4) by changing variables so that the parameters R_0 , σ_v , τ , and $\hat{\sigma}_R$ are replaced by dimensionless quantities. It is found, in fact, that the introduction of quantities ρ and κ defined by

$$\begin{aligned} \kappa &= \frac{\sigma_v \tau}{\hat{\sigma}_R}, \\ \rho &= \frac{R_0}{\sigma_v \tau} \end{aligned} \quad (A-5)$$

*The two other horizontal threat criterion conditions given on p. 5 should have a negligible effect on any conclusions based on the analysis of this appendix.

reduces the number of independent parameters from four to two. The result is an expression equivalent to (A-4):

$$P(R_O, \sigma_V, \tau, \sigma_R) = P(\kappa, \rho) = \frac{1}{\sqrt{2\pi}} \left[I_1(\rho) - I_2(\kappa, \rho) \right], \quad (A-6)$$

where

$$I_1(\rho) = \int_{-\infty}^{\rho} e^{-\frac{u^2}{2}} du \quad (A-7)$$

and

$$I_2(\kappa, \rho) = \int_0^{\infty} e^{-\frac{1}{2} [\kappa^2 w^2 + (w-\rho)^2]} dw.$$

The function $I_2(\kappa, \rho)$ in (A-7) can be simplified further by completing the square of the exponent in the integrand; i.e.,

$$I_2(\kappa, \rho) = e^{-\frac{\rho^2 \kappa^2}{2(\kappa^2+1)}} \int_0^{\infty} e^{-\frac{(\kappa^2+1)}{2} \left(w - \frac{\rho}{\kappa^2+1} \right)^2} dw \quad (A-8)$$

$$= \frac{e^{-\frac{\rho^2 \kappa^2}{2(\kappa^2+1)}}}{\sqrt{\kappa^2+1}} \int_{-\frac{\rho}{\sqrt{\kappa^2+1}}}^{\infty} e^{-\frac{u^2}{2}} du.$$

Figure A-2 provides curves depicting the probability $P(\kappa, \rho)$ as a function of ρ for various values of κ . The range of κ and ρ in the figure should be sufficient to cover all combinations of values of the original parameters, R_O , σ_V , τ , $\hat{\sigma}_R$ that are likely to be of interest for analyses of BCAS. The adequacy of the range of κ and ρ can be verified by substituting numerically typical values for R_O , σ_V , τ , and $\hat{\sigma}_R$ into (A-5); e.g.,

$$R_O = 1 \text{ nmi},$$

$$\sigma_V = 100 \text{ kn},$$

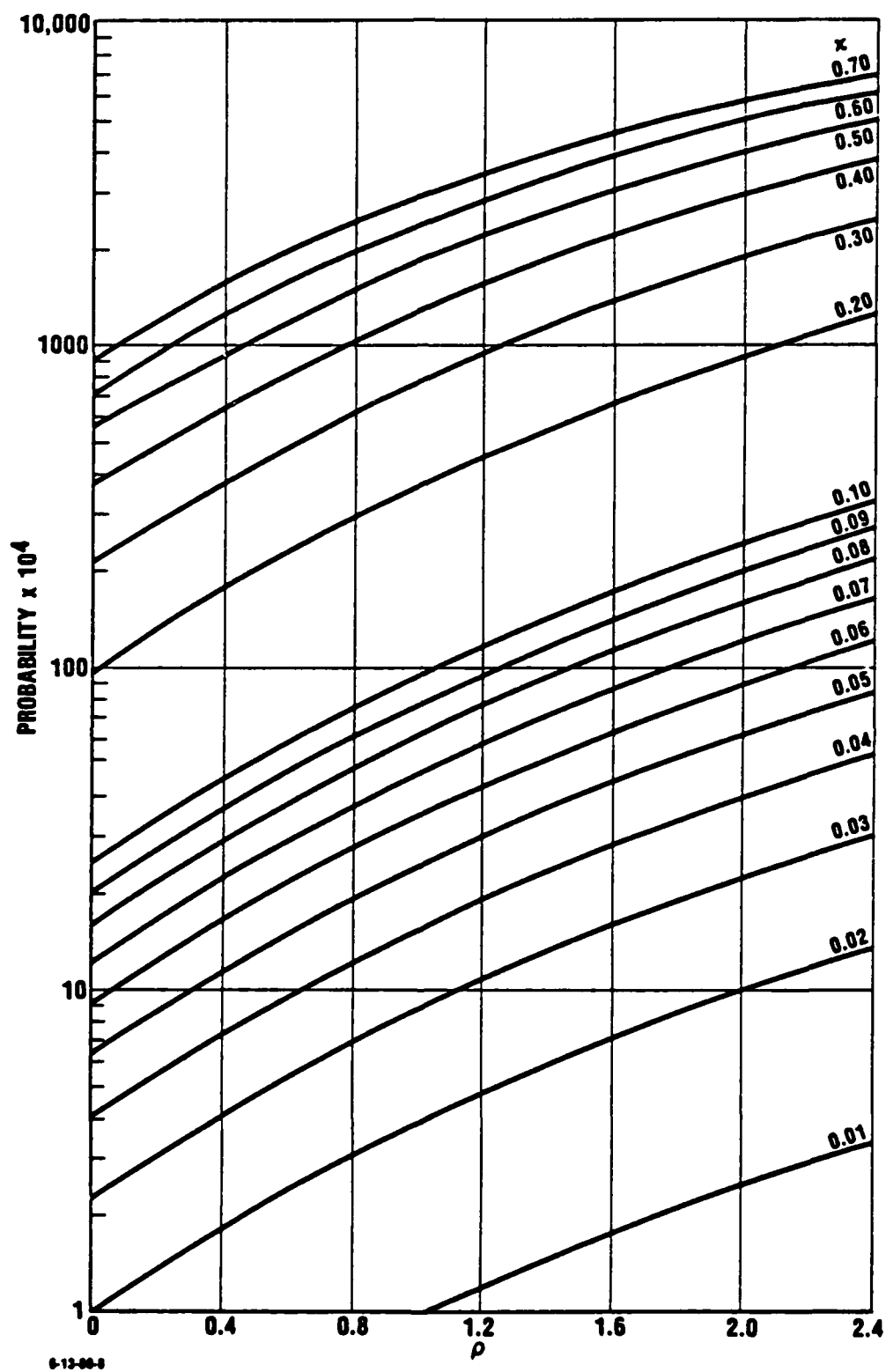


FIGURE A-2. Probability that two aircraft selected at random will satisfy a horizontal alert condition.

$$\begin{aligned}\tau &= 30 \text{ sec,} \\ \hat{\sigma}_R &= 20 \times \sqrt{2} \text{ nmi (as in the Los Angeles} \\ &\quad \text{Standard Traffic Model)} \\ &= 28.3 \text{ nmi}\end{aligned}$$

leads to

$$\kappa = \frac{100 \times 30}{3600 \times 28.3} = 0.0295$$

and

$$\rho = 1 \div \left(\frac{100}{3600} \times 30 \right) = 1.2.$$

According to (A-5), if the DMOD parameter R_0 is changed by a factor α the quantity ρ is changed by the same factor but κ is unaffected. However, if τ is changed by a factor α the quantity κ is changed by the same factor and ρ by the factor $1/\alpha$; in fact, the product $\kappa\rho$ is independent of τ and σ_v .

C. OBSERVED EFFECT OF PARAMETER CHANGES IN THE ALERT LOGIC

Reference A-3 gives estimates of the percentage change in the number of alerts for the Houston traffic data due to percentage changes in τ and in DMOD (R_0). One would expect that the probability depicted by the curves in Fig. A-2 will be proportional to the number of alerts over any given time period and therefore that the same percentage changes should be observed in that probability. If so, it might be expected that the results reported in Ref. A-3, when compared with similar changes induced in the probability depicted in Fig. A-2, would provide a means for estimating the quantities $\hat{\sigma}_R$ and σ_v for the Houston traffic.

Reference A-3 reports:

- (1) decreasing τ from 25 to 20 sec reduces the number of alerts by 31.5 percent;
- (2) decreasing DMOD (R_0) from 0.3 nmi to 0.1 nmi reduces the number of alerts by 25 percent. However, in another section

Ref. A-3 states that for a 370 ft altitude separation standard (designated as ALIM) for $R_0 = 0.3$ nmi and $\tau = 25$ sec there are 181 alerts, and for the same altitude parameter, when R_0 is reduced to 0.1 nmi and τ to 20 sec, the number of alerts is reduced to 83.

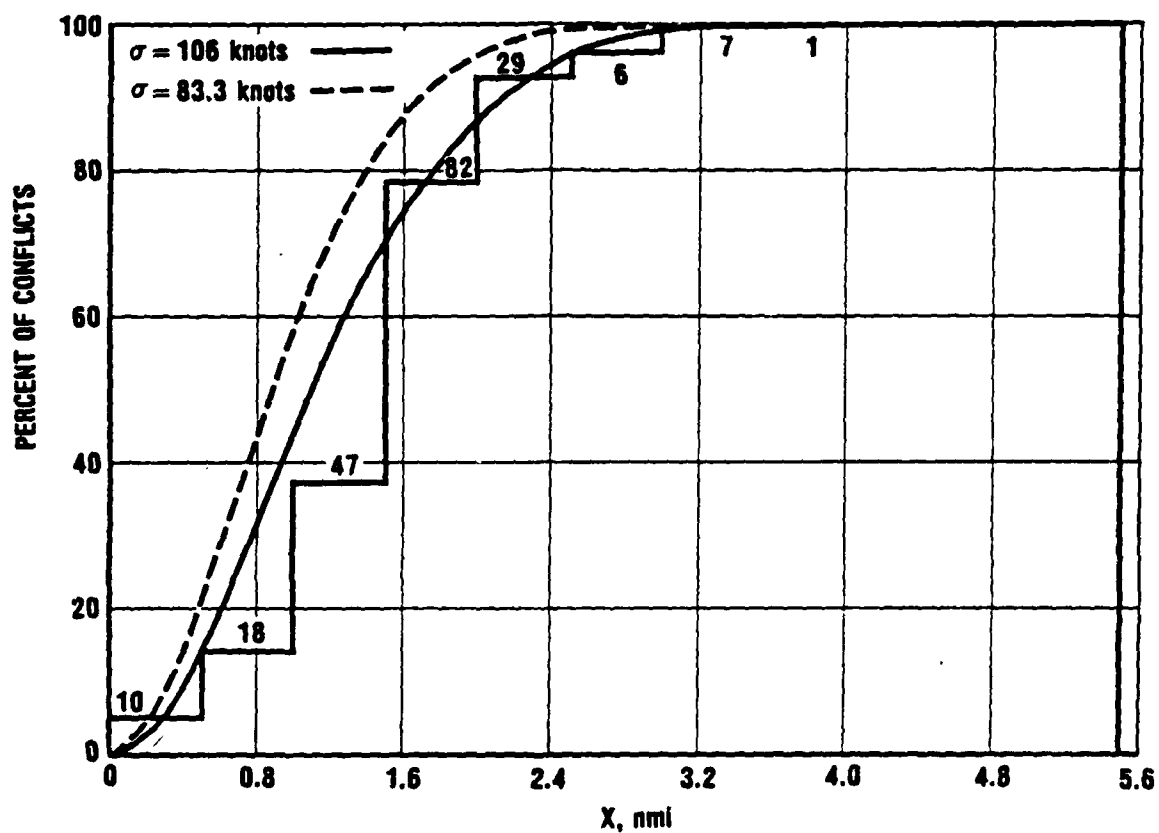
The partial percentage reductions in alerts reported by Ref. A-3 are incompatible with the reported total reduction from 181 to 83, which is equivalent to a total reduction of 54.1 percent. The combined reported partial reductions imply a total reduction of $[1 - (1-0.25)(1-0.315)] \times 100$, or 48.6 percent. This discrepancy may be due to a typographical error, e.g., if 25 percent were inadvertently written in place of 33.1 percent.

A similar discrepancy occurs for the other total reduction figures given by Ref. A-3 for the case of a 340 ft altitude separation standard, namely, a reduction from 173 to 81. In this case the total reduction is 53.2 percent rather than the implied percentage reduction of 48.6 percent.

Because of these discrepancies and the fact that the data concerning the effect of reducing the parameters separately is not given explicitly in terms of actual alert numbers by Ref. A-3, those reported reductions will be ignored here. The explicit data concerning the combined effect of reducing both τ and DMOD will be used, however, and are displayed in Table A-2.

D. APPLICATION OF THE THEORETICAL MODEL TO THE HOUSTON DATA

Reference A-3 also provides some additional data that can be used to fit the theoretical air traffic model embodied in (A-6) to the observed statistics for Houston. This data is another histogram, reproduced in Fig. A-3, that depicts the distribution of aircraft separations for the conflicts observed during the 65 hours covered by the Houston ARTS tapes.



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FIGURE A-3. Percent of conflicts with separation at beginning of conflict less than X.

Finally, L. Zarrelli has kindly provided another histogram that depicts the distribution of closing speeds for the conflicts. It is reproduced here in Fig. A-4.

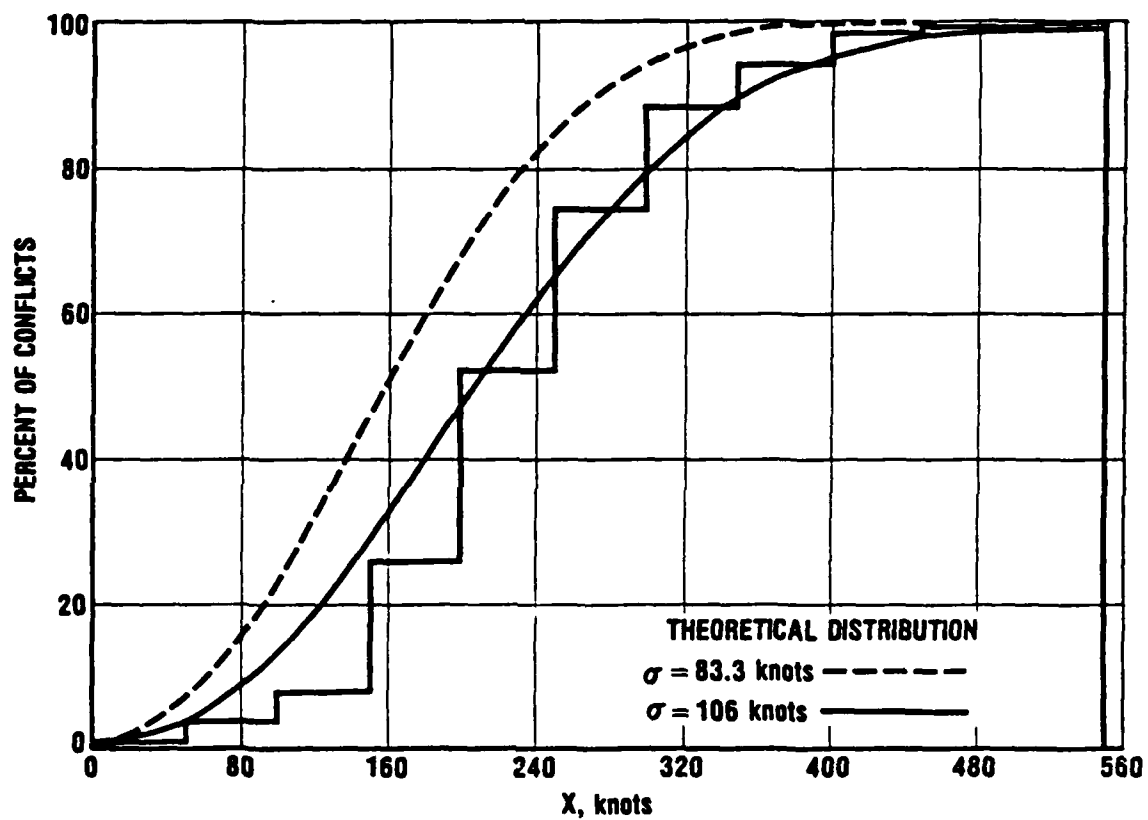
For the data contained in both histograms all conflicts were defined by level 4 alert parameter values: a τ value of 25 sec and a DMOD (R_0) value of 0.3 nmi. Similar data, for which the conflicts were determined by the application of desensitization rules according to the previously defined local zones in which they occurred, were also provided by Ref. A-3 and L. Zarrelli. However, because of the mixing of parameter values these data are not suited to the purposes of this report.

The theoretical model of air traffic statistics can be tested by using it to predict the percentage changes that took place in the number of alerts when certain alert logic parameter changes were made in the data processing experiment reported by Ref. A-3. For this purpose it should be remembered that the quantity σ_v appearing in (A-5) is $\sqrt{2}$ times the σ_0 of the Rayleigh distributions. Thus, the 72.2 kn value of σ_0 becomes 102.1 kn and the 83.3 kn value becomes 117.8 kn.

The quantity κ in (A-5) requires a knowledge of $\hat{\sigma}_R$, which is associated with the density distribution of aircraft. However, after some numerical experimentation it can be seen that the predicted reductions in alerts are virtually independent of $\hat{\sigma}_R$.^{*} Therefore, an arbitrarily chosen value of 20 nmi for $\hat{\sigma}_R$ seems as good as any other within physically reasonable limits.

The paired values of ρ and κ of interest, calculated by means of (A-5), are those appearing in Table A-1 along with the corresponding probability calculations given by (A-6). Table A-2 gives the corresponding alert changes observed empirically, according to Ref. A-3. Table A-3 is a comparison of predicted with observed results.

^{*}No attempt has been made in the present investigation to explain this remarkable fact, which is not an obvious consequence of the general theory.



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FIGURE A-4. Observed percent of conflicts with closing speeds less than X.

TABLE A-1. PREDICTED CHANGES IN THE ALERT PROBABILITY BECAUSE OF CHANGES IN ALERT LOGIC PARAMETERS

τ (sec)	R_0 (nmf)	σ_0 (Kn)	ρ	κ	Prob. $\times 10^4$	% Change
25	0.3	72.2	0.423	0.0355	5.90	-
20	0.1	72.2	0.176	0.0284	2.65	55.1
18	0.1	72.2	0.196	0.0256	2.22	62.4
25	0.3	83.3	0.367	0.0409	7.24	-
20	0.1	83.3	0.153	0.0327	3.89	53.2
18	0.1	83.3	0.170	0.0294	2.81	61.2
25	0.3	106	0.288	0.0520	10.4	-
20	0.1	106	0.120	0.0416	5.21	49.9
18	0.1	106	0.133	0.0374	4.30	58.7

TABLE A-2. DISTRIBUTION OF ALERTS (PARTIAL DATA)

DMOD(R_0)	ALIM	TAU	Positive Commands
0.3 nmi	370 ft	25 sec	181
0.3 nmi	340 ft	25 sec	173
0.1 nmi	370 ft	20 sec	83
0.1 nmi	340 ft	20 sec	81
0.1 nmi	370 ft	18 sec	73
0.1 nmi	340 ft	18 sec	69

TABLE A-3. ALERT REDUCTION DUE TO CHANGES IN τ AND DMOD

Change in τ From 25 secs to (sec)	Alim (ft)	Actual % Change in Alerts	Predicted % Change in Alerts			Error %			σ_0 (kn)
			72.2	83.3	106	72.2	83.3	106	
20	370	54.1	55.1	53.2	49.9	1.0	0.9	4.2	
20	340	53.2	55.1	53.2	49.9	1.9	0.0	3.3	
18	370	59.7	62.4	61.2	58.7	2.7	1.5	1.0	
18	340	60.1	62.4	61.2	58.7	2.3	1.1	1.4	

According to these results, for example, if $\sigma_0 = 72.2$ kn the predicted reduction in alerts would be $100 \times \left(1 - \frac{2.65}{5.9}\right)$ percent or 55.1 percent. Applied to the 181 aircraft in conflict for an altitude standard of 370 ft this would yield a predicted reduction to 81.3 aircraft, a result that differs from what was observed by 1.7 aircraft. This is an error of less than one percent.

The σ_0 value of 83.3 kn is apparently the most consistent with the Houston data; it leads to the most accurate prediction of the effect of alert logic parameter changes on the reduction of alerts. However, all of the σ_0 values considered lead to predictions that are in excellent agreement with the observed results. Thus, it appears that the effect of parameter changes on the reduction of alerts is not very sensitive to the air traffic statistics.

The remaining data, the aircraft closing speed and separation statistics, can be used to test the theoretical traffic model and σ_0 values. For this purpose it is necessary to calculate the appropriate probabilities. These can be defined as follows. $P_v(x)$ is the conditional probability that two aircraft, known to be in conflict, have a closing speed that is less than x kn, and $P_R(x)$ is the conditional probability that their separation at the beginning of the conflict is less than x nmi.

Dividing the joint probability $P_{vC}(x)$ that two aircraft are in conflict and have a closing speed less than x by $P(\kappa, \rho)$, the probability that the two aircraft are in conflict, yields $P_v(x)$. A similar calculation using $P_{RC}(x)$, the joint probability that two aircraft are in conflict and have a separation less than x at the beginning of the conflict, in place of $P_{vC}(x)$, yields $P_R(x)$.

The joint probabilities can be written at once using the horizontal collision algorithm condition and the assumed theoretical traffic distributions. That is,

$$P_{VC}(x) = \frac{1}{\hat{\sigma}_R^2 \sigma_V \sqrt{2\pi}} \int_{-x}^{\frac{R_0}{\tau}} \int_0^{R_0 - \tau \dot{R}} e^{-\frac{\dot{R}^2}{2\sigma_V^2}} e^{-\frac{R^2}{2\hat{\sigma}_R^2}} R dR d\dot{R};$$

$$P_{RC}(x) = \frac{1}{\hat{\sigma}_R^2 \sigma_V \sqrt{2\pi}} \int_0^x R \int_{-\infty}^{\frac{R_0 - R}{\tau}} e^{-\frac{R^2}{2\hat{\sigma}_R^2}} e^{-\frac{\dot{R}^2}{2\sigma_V^2}} d\dot{R} dR. \quad (A-9)$$

These integrals can be transformed by means of the same procedure used in deriving (A-6) from (A-4). The results are

$$P_{VC}(x) = P_C - 1 + \frac{1}{\sqrt{2\pi}} \left(\int_{\frac{x}{\sigma_V}}^{\infty} e^{-\frac{u^2}{2}} du + \frac{e^{-\frac{\rho^2 \kappa^2}{2(\kappa^2+1)}}}{\sqrt{\kappa^2+1}} \int_{\frac{\rho \kappa^2}{\sqrt{\kappa^2+1}} + \frac{x \sqrt{\kappa^2+1}}{\sigma_V}}^{\infty} e^{-\frac{u^2}{2}} du \right)$$

(A-10)

and

$$P_{RC}(x) = P_C - e^{-\frac{1}{2} \left(\frac{x}{\sigma_V \tau} \right)^2 \kappa^2}$$

$$+ \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{1}{2} \left(\frac{x}{\sigma_V \tau} \right)^2 \kappa^2} \int_{\rho - \frac{x}{\sigma_V \tau}}^{\infty} e^{-\frac{u^2}{2}} du + \frac{e^{-\frac{\rho^2 \kappa^2}{2(\kappa^2+1)}}}{\sqrt{\kappa^2+1}} \int_{\frac{x \sqrt{\kappa^2+1}}{\sigma_V \tau} - \frac{\rho}{\sqrt{\kappa^2+1}}}^{\infty} e^{-\frac{u^2}{2}} du \right),$$

(A-11)

where P_C is given by (A-6). The required probabilities are then given by

$$P_V(x) = \frac{P_{VC}(x)}{P_C} , \quad (A-12)$$

$$P_R(x) = \frac{P_{RC}(x)}{P_C} .$$

Histograms, constructed by Mitre from the Houston data, showing the observed closing speed and separation distributions of aircraft in conflict are reproduced in Fig. A-4 and A-3. Included in those figures are theoretical curves calculated using $R_0 = 0.3$ nmi, $\tau = 25$ second, $\hat{\sigma}_R = 20$ nmi* in (A-10) with values of σ_0 equal to 83.3 kn (dashed line) and 106 kn (solid line). In both cases the σ_0 value of 83.3 kn produces curves that lie above the corresponding histogram. Conversely, the theoretical distributions based on a σ_0 value of 106 kn fit their corresponding histograms reasonably well in both cases.

*Some numerical experimentation, again, demonstrates that the theoretical distributions are virtually independent of $\hat{\sigma}_R$.

REFERENCES

- A-1. I. W. Kay, *"Analysis of Collision Avoidance Threat Logic That Uses Bearing Data,"* IDA Study S-500, September 1978.
- A-2. S. Cohen and F. Maginnis, *"Statistical Summary of The 1982 Los Angeles Basin Standard Traffic Model,"* Mitre Corporation Report No. MTR-6387, April 1973.
- A-3. L. B. Zarrelli and J. A. Grupe, *"BCAS Parameter Study,"* Mitre Briefing, 22 February 1980.

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