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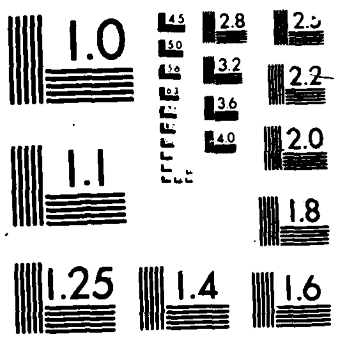
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RESEARCH MEMORANDUM

NOMOGRAM FOR OPTIMAL SEARCH

Henry R. Richardson

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A Division of CNA Hudson Institute

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This research memorandum presents a nomogram for optimal search that can be used to rapidly estimate the amount of search effort required to achieve specified levels of detection probability. The nomogram can also be used to determine how the search effort should be distributed in order to approximate an optimal allocation.

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1. Enclosure (1) is forwarded as a matter of possible interest.

2. This research memorandum presents a nomogram for optimal search. It can be used to estimate the amount of search effort required to achieve specified levels of detection probability. The nomogram can also be used to determine how the search effort should be distributed in order to approximate an optimal allocation. Applications include deep ocean search and surface surveillance.

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ABSTRACT

This research memorandum presents a nomogram for optimal search that can be used to rapidly estimate the amount of search effort required to achieve specified levels of detection probability. The nomogram can also be used to determine how the search effort should be distributed in order to approximate an optimal allocation.

TABLE OF CONTENTS

INTRODUCTION

This research memorandum presents a nomogram for optimal search. The nomogram is intended to provide a rapid method for estimating the amount of search effort required to achieve specified levels of detection probability and to indicate the manner in which the search effort should be distributed in order to approximate an optimal allocation.

The next section introduces the basic assumptions and the third section describes the nomogram. Examples illustrating the use of the nomogram are given in the fourth section and mathematical details are presented in the appendix.

ASSUMPTIONS

Figure 1 shows the search geometry. The target location is assumed to have a bivariate normal probability distribution, and the (x,y)-coordinate system is assumed to be aligned with the distribution's principal axes. The standard deviations along the principal axes are given by σ_x and σ_y, respectively, and the contours of equal probability density are described by the formula

$$(x/\sigma_x)^2 + (y/\sigma_y)^2 = r^2 . \tag{1}$$

The parameter r in equation 1 is referred to as the "generalized radius," because if $\sigma_x = \sigma_y = 1$, then the contours are circles with radius r. In general, however, the contours are ellipses.

The standard deviations σ_x and σ_y are derived from an analysis of the information pertaining to the target's loss. It is useful to note that if x_p and y_p are the half-axis lengths for a p-probability containment ellipse, then σ_x and σ_y can be found from the equations

$$\sigma_x^2 = x_p^2 / |2 \ln (1-p)| ,$$

and

$$\sigma_y^2 = y_p^2 / |2 \ln (1-p)| ,$$

where ln denotes the natural logarithm.

The author wishes to thank Lloyd W. Koenig for his careful review of this work.

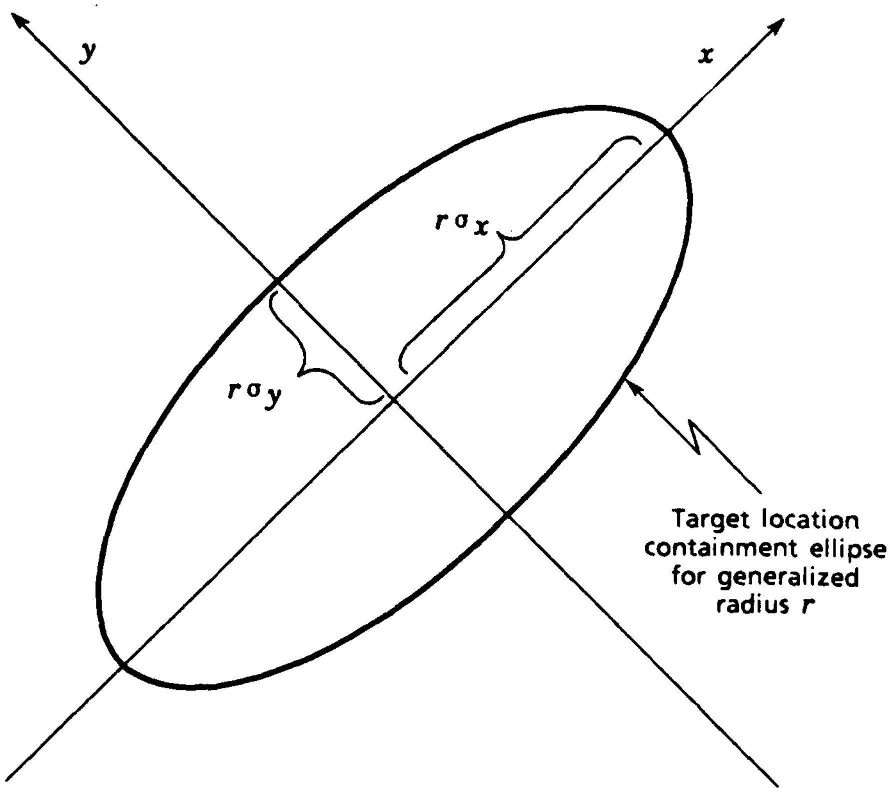

FIG. 1: SEARCH GEOMETRY

The following notations are used:

$$SEP = \text{search effectiveness probability}$$

$$W = \text{search sweep width}$$

$$V = \text{search speed}$$

$$R = WV = \text{search sweep rate}$$

$$A = \pi\sigma_x\sigma_y \ = \ \text{area enclosed by the "1 } \sigma \text{ ellipse"}^{1}$$

$$T \ = \ \text{search time}$$

$$L = VT \ = \ \text{search track length}$$

$$S \ = \ \text{search track spacing.}$$

SEP is the probability that the target will be detected by the allocation of search effort under consideration and is the measure of effectiveness maximized by an optimal search plan.

THE OPTIMAL SEARCH NOMOGRAM

The nomogram for optimal search is given in figure 2. The nomogram has three vertical scales. The left-most scale is two sided with *SEP* on the left-hand side and "normalized search effort" RT/A on the right-hand side. The right-most scale gives the generalized radius r defined by equation 1, and the center scale gives the optimal search coverage factor K_r^* (defined in the appendix) for a point with generalized radius r.

The coverage factor K at a given point is the expected number of times the target is "covered" by the search effort applied to the vicinity of the point in question. For parallel path search, K is given by

$$K = \frac{W}{S} \ , \tag{2}$$

where, as defined above, W is sweep width and S is track spacing. Alternatively, if the search covers the vicinity of the point in a more or less uniform manner, then K can be computed by the equation

$$K = \frac{WL}{B} \ , \tag{3}$$

where for a small box centered on the point in question, B is the area of the box, W is sweep width, and L is track length.

1. The 1 σ ellipse is the contour where $r = 1$ in equation 1.

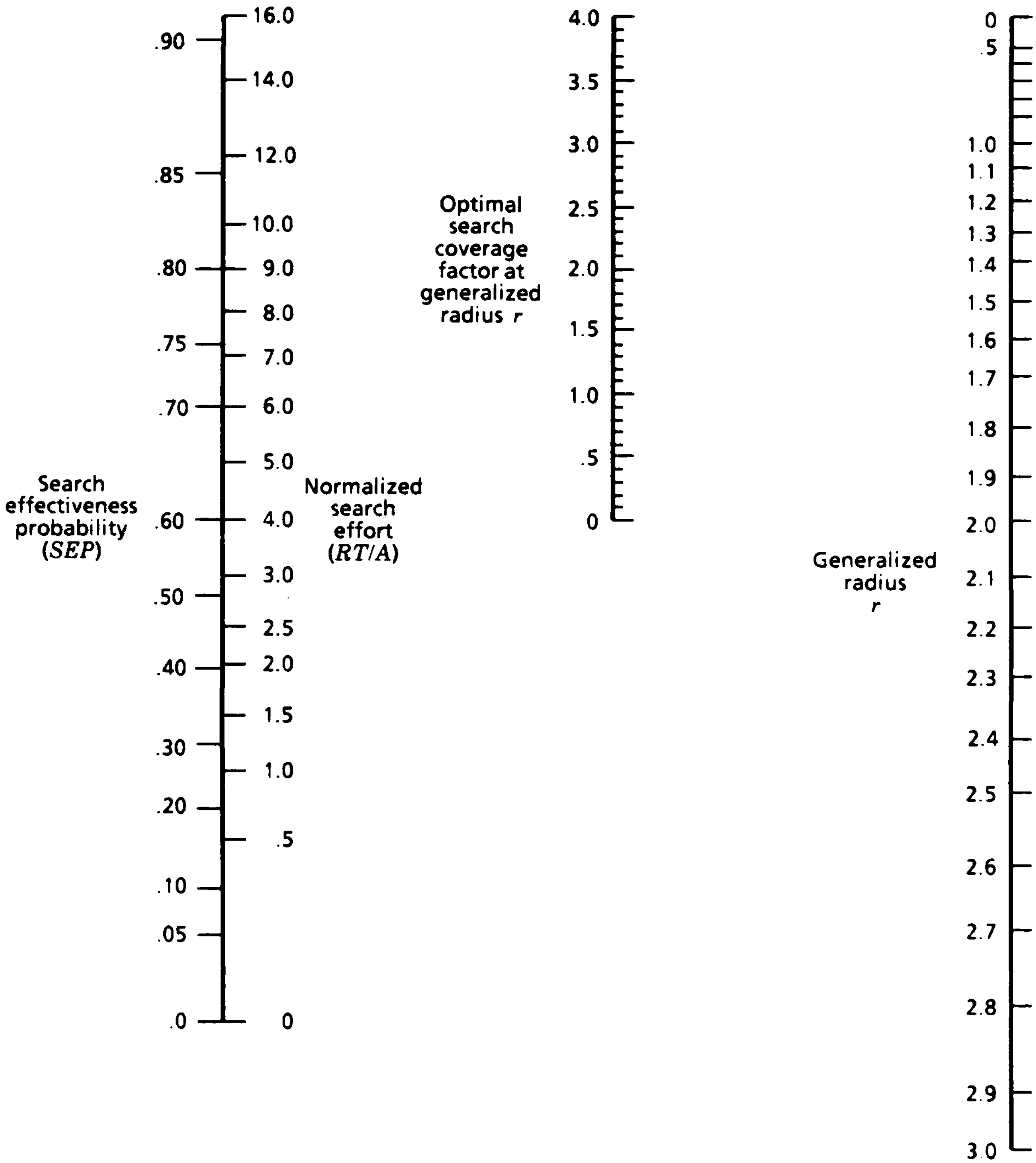

FIG. 2: NOMOGRAM FOR OPTIMAL SEARCH

EXAMPLES

One example of the use of the optimal search nomogram is motivated by deep ocean search and illustrated in figure 3, where W = 100 feet = .0167 n.mi., V = 3 knots, and the target location probability distribution is circular normal with $\sigma_x = \sigma_y = 1$ n.mi.

The first problem is to determine the amount of search time required to reach various levels of *SEP*. In order to solve this problem, choose a value of *SEP* on the left side of the left axis and read across the axis to obtain a value for the normalized search effort RT/A. For example, if $SEP = .7$, then $RT/A = 6$. In the example, the area of the 1 σ circle is 3.14 n.mi.^{2}, and because

$$RT/A = 6 \, , \tag{4}$$

with $R = WV$, search time T can be calculated as follows:

$$T = \frac{6A}{R} = \frac{(6)(3.14)}{(.0167)(3)} = 376 \text{ hours} \, . \tag{5}$$

If instead an *SEP* of .8 is desired, a similar calculation yields a required search time of T = 564 hours. Therefore a 33-percent increase in *SEP* requires a 50-percent increase in search time – a manifestation of the law of diminishing returns.

Continuing with example 1, suppose that the search planner wishes to determine the farthest contour that must be searched in order to achieve $SEP = .8$. This is obtained by aligning .8 on the *SEP* scale with 0 on the coverage factor scale and reading the resulting generalized radius on the right-most scale. This process is indicated by the line drawn in figure 3. The reasoning behind this construction is that the farthest search contour corresponds to the points where search coverage first becomes 0. From this process the conclusion is reached that no search should be carried out beyond a generalized radius of 2.45. In this case this generalized radius equates to a true radius of 2.45 n.mi. because $\sigma_x = \sigma_y = 1$ n.mi.

Another example is illustrated in figure 4. Consider a hypothetical case in which a substantial amount of search has been carried out in the vicinity of the center of the target location probability distribution, but very little search

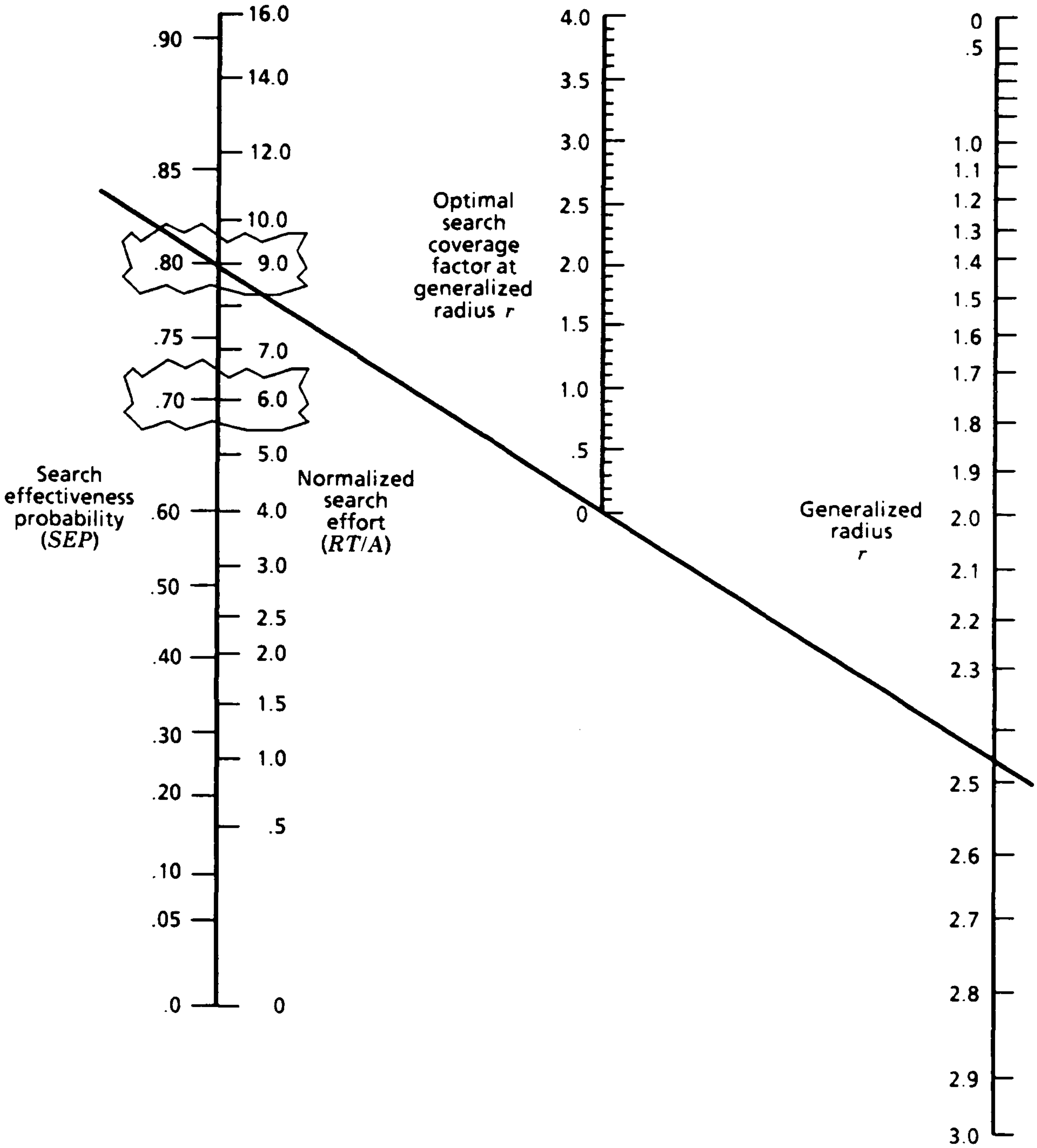

FIG. 3: EXAMPLE 1

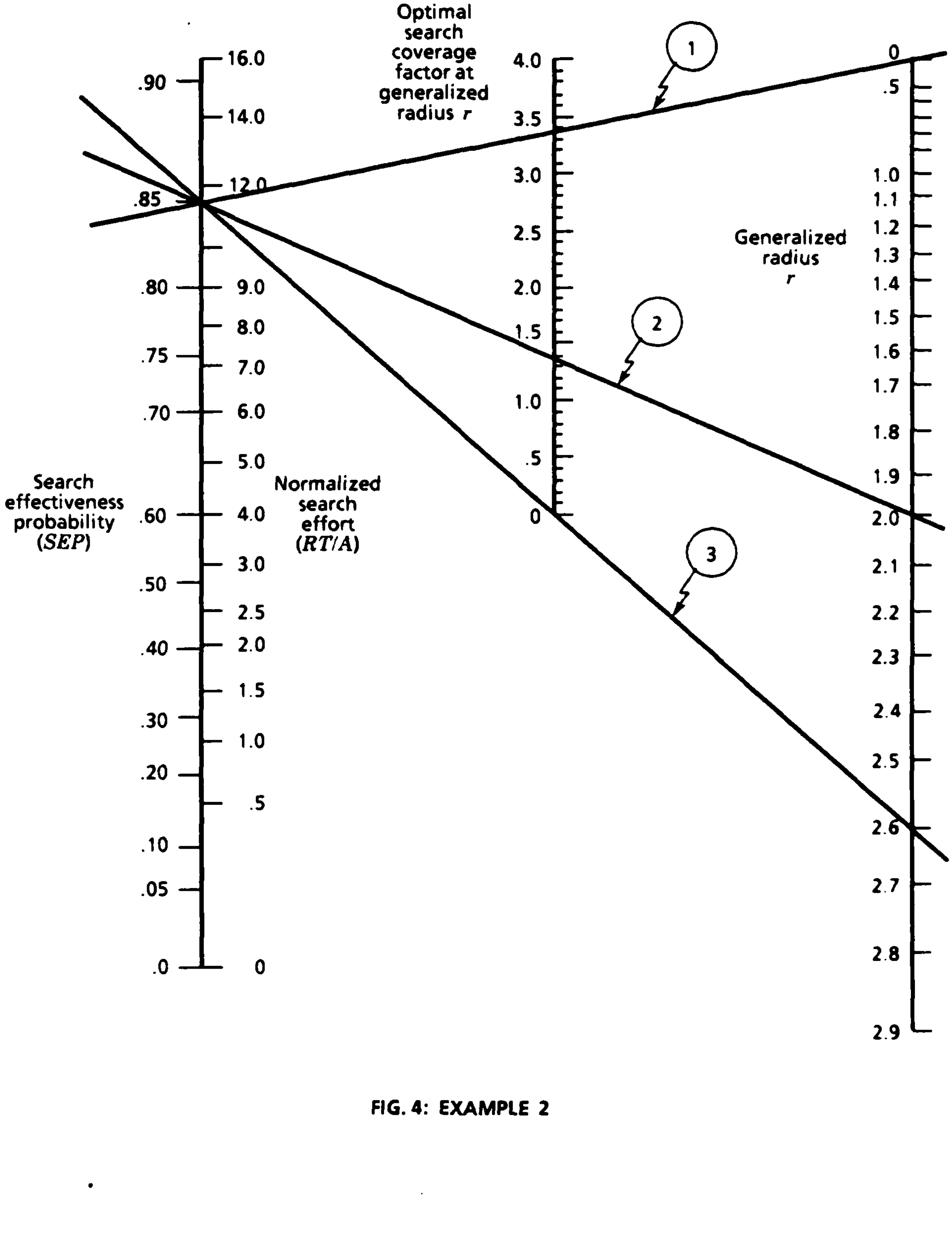

FIG. 4: EXAMPLE 2

has been carried out elsewhere. The problem is to determine how much search to apply to other locations so that the overall allocation of search will approximate an optimal allocation. A similar problem to this arose during the search for the U.S. nuclear submarine *Scorpion* in 1968.

Suppose that the coverage factor at the center of the target location probability distribution has reached the value $K = 3.4$. The center of the target location probability distribution corresponds to generalized radius 0, so 0 is aligned on the right-hand axis with 3.4 on the center axis to find that an *optimal* search achieving a coverage factor of 3.4 at the center of the distribution would obtain an overall $SEP = .85$. This is shown by line number 1 in figure 4. Such an *SEP* would only be attained, of course, if the rest of the search effort were applied in the required optimal manner. To determine what the necessary coverage factors would be at other points in the search region, calculate the generalized radius using equation 1, then align this value on the right-hand scale with *SEP* on the left-hand scale, and read the required coverage factor on the center scale. For example, at a generalized radius of 2.0, the search effort resulting in a coverage factor of approximately 1.4 should be applied. This is shown by line number 2.

To obtain the value of the contour beyond which no search should be conducted, align the value of *SEP* on the left-hand scale with coverage factor 0 on the center scale and read the generalized radius on the right-hand scale. For this particular case, as shown by line number 3, no search should be carried out beyond a generalized radius of 2.6.

The above examples show that the nomogram presented in figure 1 can be used to estimate the amount of search effort needed to obtain various levels of *SEP*. In addition, it can be used to determine the coverage factors that should be obtained at various points in the search region if the desired value of *SEP* is to be obtained with minimum search effort. With a little practice, the optimal search nomogram can be a useful decision aid for search planning and evaluation.

APPENDIX

MATHEMATICAL DETAILS

APPENDIX

MATHEMATICAL DETAILS

This appendix presents the mathematical details used in constructing the optimal search nomogram. The theory underlying the equations presented here is given in reference [A-1] (see in particular section 2.2). A general discussion of nomography is given in reference [A-2].

The target location probability distribution is given by a bivariate normal probability distribution with standard deviations σ_x and σ_y along the principal axes. The coordinate system geometry is shown in figure 1 in the main text.

For convenience, relationships are expressed in terms of dimensionless quantities. Consequently, areas are normalized by the area A of the 1 σ target location probability ellipse. This area is given by

$$A = \pi \sigma_x \sigma_y \, . \tag{A-1}$$

The generalized radius of a point (x,y) is denoted r and is defined by

$$(x/\sigma_x)^2 + (y/\sigma_y)^2 = r^2 \, . \tag{A-2}$$

Use of the theory of optimal search [A-1] shows that no search effort should be applied outside the elliptical contour specified by generalized radius r_{max} and given by

$$r^2_{max} = 2\sqrt{RT/A} \, , \tag{A-3}$$

where R is sweep rate, T is search time, and A is the area of the 1 σ target location probability ellipse.

If K denotes the coverage factor (the expected number of search coverages of the target at a specified point), then the probability of detecting the target, given that it is at the point in question, is given by the uniform random search equation

$$d(K) = 1 - \exp(-K) . \tag{A-4}$$

The optimal allocation of search effort for search time T can be described by giving the desired coverage factor at each point in the search region. The coverage factor is assumed to be constant on contours of equal generalized radius. If K_r^* denotes the coverage factor corresponding to an optimal search at points with generalized radius r, then

$$K_r^* = \begin{cases} \frac{1}{2}(r_{max}^2 - r^2), \text{for } 0 \le r^2 \le r_{max}^2 \\ 0, \text{otherwise} \end{cases} . \tag{A-5}$$

The coverage factor at the center of the probability distribution is easily seen to be

$$K_o^* = \frac{1}{2} r_{max}^2 . \tag{A-6}$$

Consequently, equation A-5 can be rewritten in the form

$$K_r^* = \begin{cases} K_o^* - \frac{1}{2} r^2, \text{for } 0 \le r^2 \le r_{max}^2 \\ 0, \text{otherwise} \end{cases} . \tag{A-7}$$

By combining equation A-3 and A-6, the normalized search effort RT/A is expressed in terms of the coverage factor K_o^* by

$$RT/A = K_o^{*2} . \tag{A-8}$$

Equations A-7 and A-8 are the fundamental equations used in constructing the optimal search nomogram. First observe that equation A-7 is linear in the variables K_r^*, K_o^*, and r^2. Such relationships can be expressed in terms of three vertical scales with suitable spacing. The nonlinear relationship r^2 is expressed by non-uniform gradations on the axis corresponding to the generalized radius.

As shown below, *SEP* as well as normalized search effort *RT/A* can be written in terms of K_o^*. *SEP* and *RT/A* are the variables expressed on the left-most axis of the nomogram, while the generalized radius r is the variable expressed on the right-most axis of the nomogram. The values of K_r^* are the optimal search coverage factors at generalized radius r and are expressed on the center scale of the nomogram.

In order to write *SEP* as a function of K_o^*, the theory of optimal search can be used to show that *SEP* (the probability of detection at time T) is given by the equation

$$SEP = 1 - \left(1 + \frac{r_{max}^2}{2}\right) \exp\left(-\frac{r_{max}^2}{2}\right) . \quad \text{(A-9)}$$

In view of equation A-6, however, the equation for *SEP* can be rewritten in the form

$$SEP = 1 - (1 + K_o^*) \exp(-K_o^*) . \quad \text{(A-10)}$$

Thus, as intended, *SEP* can be expressed in terms of the coverage factor achieved by an optimal search at the center of the probablity distribution.

Although not used in the nomogram, it is worth pointing out for reference that the expected time to detection $\bar{\tau}$ is easily computed by the equation

$$\bar{\tau} = 6 \pi \sigma_x \sigma_y / R . \quad \text{(A-11)}$$

Also for reference, note that the "local effectiveness probability" $LEP(r)$ at a point with generalized radius r is defined to be the probability of detecting the target, given that it lies at the point in question. Thus

$$LEP(r) = d(K_r^*) , \tag{A-12}$$

where the function d is given by equation A-4.

Equation A-12 can be used to add *LEP* to the center scale; this can be valuable in certain search operations in which *LEP* rather than coverage factor is used as the primary method of measuring search effectiveness within the cells of a grid system.

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[A-1] Stone, L. D. *Theory of Optimal Search*. New York: Academic Press, 1975

[A-2] Fasal, John H. *Nomography*. New York: Frederick Ungar Publishing Company, 1968

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