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OPTIMAL ASSEMBLY OF SYSTEMS USING SCHUR-FUNCTIONS AND
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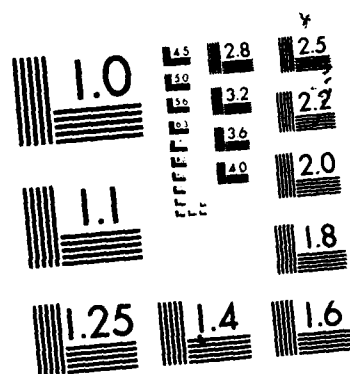⊫ 56
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Optimal Assembly Of Systems
Using Schur-functions And Majorization

By

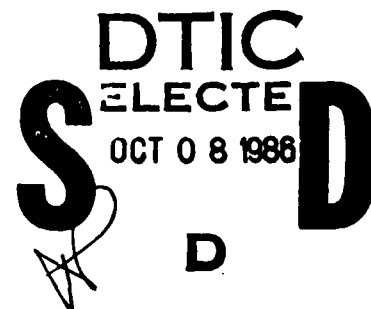Emad El-Neweihi†, Frank Proschan†and Jayaram Sethuraman‡

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University of Illinois at Chicago
Department of Mathematics, Statistics and Computer Science
Chicago, Illinois 60680

and

Florida State University
Department of Statistics
Tallahassee, Florida 32306

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Abstract

A general assembly of n systems from k types of components is considered. The techniques of majorization and Schur-function are utilized to pinpoint the optimal assembly under several criteria. Earlier results of Derman, Lieberman and Ross(1972) and El-Neweihi, Proschan and Sethuraman(1986) are generalized.

1. Introduction

In this paper we consider the optimal assembly of n systems from components of k types. Special cases of such a problem have been studied earlier in the literature. El-Neweihi, Proschan and Sethuraman(1986) studied the case of a single type of components. Derman, Leiberman and Ross(1972) considered the case where each system consisted of one component of each of k types. We generalize the ideas of both of these papers to the case where the systems may consist of varying numbers of components from more than one type.

An assembly of the n systems corresponds to a partitioning **A** of the components to the different systems. For more details see Section 2. When the components act independently, we show in sections 2 and 3 that an intuitively motivated partitioning **A*** provides the optimal assembly under many different criteria.

In Section 3, we allow each system to have dependent components, and under some general conditions on the reliability function we show that the same partitioning **A*** provides an optimal assembly.

The results of this paper are based on the well known techniques of Schur- funtions and majorization. This makes them clear and simple and at the same stime more general than in the papers cited.

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2. Reliabilities of general assembly
with independent components

In this section and the next section we will consider the assembly of n systems from k types of components. To be more specific we shall assume that the jth system is a series system of m_{ij} components of type i, where all the components act independently, $1 \leq i \leq k$, $1 \leq j \leq n$. The matrix $\mathbf{M} = (m_{ij})$ will be called type-enumerator matrix of the n systems, and for each j, $1 \leq j \leq n$, the vector $\mathbf{m}_j = (m_{1j}, m_{2j}, \ldots, m_{kj})$ will be called the type-enumerator vector of system j. We will make the following important assumption about the type-enumerator vectors:

(2-1) $$\mathbf{m}_1 \geq \mathbf{m}_2 \geq \cdots \geq \mathbf{m}_n,$$

where the inequality sign between two vectors stands for coordinate-wise inequality.

By allowing some m_{ij}'s to be equal to zero we can accomodate systems with fewer than k types of components. Let $M_i = \sum_{j=1}^{n} m_{ij}$, $1 \leq i \leq k$. We need M_i components of type i, $1 \leq i \leq k$ in order to assemble n systems with type-enumerator matrix $\mathbf{M}$. For this, we will only need to partition the M_i components of type i, into n subsets $A_{i1}, A_{i2}, \ldots, A_{in}$ of sizes $m_{i1}, m_{i2}, \ldots, m_{in}$, $1 \leq i \leq k$ and construct the jth system as a series system with the m_{ij} components of type i in A_{ij}, $1 \leq i \leq k$, $1 \leq j \leq n$. Each partitioning $\mathbf{A} = (A_{ij})$ of this type leads to a different assembly and there are $\prod_{i=1}^{k} \binom{M_i}{m_{i1}, m_{i2}, \ldots, m_{in}}$ such partitionings.

With no loss of generality we may assume that we can re-order the components of type i (if necessary) so that their reliabilities satisfy

$$p_{i1} \leq p_{i2} \leq \ldots \leq p_{iM_i}, \qquad \text{for} \quad 1 \leq i \leq k.$$

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The reliability of the jth system assembled from the partitioning $\mathbf{A}$, like the one described above, is given by

$$(2\text{-}2) \qquad\qquad R_j(\mathbf{A}) = \prod_{i=1}^{k} \prod_{s \in A_{ij}} p_{is},$$

for $1 \leq j \leq n$. Let $X_j(\mathbf{A}) = \log R_j(\mathbf{A})$, $1 \leq j \leq n$. Then $\mathbf{R}(\mathbf{A}) = (R_1(\mathbf{A}), R_2(\mathbf{A}), \ldots, R_n(\mathbf{A}))$ and $\mathbf{X}(\mathbf{A}) = (X_1(\mathbf{A}), X_2(\mathbf{A}), \ldots, X_n(\mathbf{A}))$ are the reliability vector and the log-reliability vector of the n systems assembled from the partitioning $\mathbf{A}$.

Consider the special partitioning $\mathbf{A}^*$ defined as follows. The components of type i are partitioned into sets $A_{i1}^*, A_{i2}^*, \ldots, A_{in}^*$ of sizes $m_{i1}, m_{i2}, \ldots, m_{in}$, respectively, with $A_{i1}^* = \{1, 2 \ldots, m_{i1}\}$, $A_{i2}^* = \{m_{i1} + 1, m_{i1} + 2, \ldots, m_{i1} + m_{i2}\}$, etc., $1 \leq i \leq k$. Notice that A_{i1}^* consists of components of type i with the m_{i1} lowest reliabilities, A_{i2}^* consists of components of type i with the m_{i2} next lowest reliabilities, ..., and A_{in}^* consists of components of type i with the m_{in} largest relibilities. Let $\mathbf{R}(\mathbf{A}^*)$ and $\mathbf{X}(\mathbf{A}^*)$ be the reliability vector and the log-reliability vector of the n systems assembled from $\mathbf{A}^*$.

Theorem 2.1. Consider the general problem of assembling n series systems with k types of components with a type-enumerator matrix $\mathbf{M}$ satifying (2-1). Let $\mathbf{A}$ be a general partitioning and $\mathbf{A}^*$ be the specific partitioning described as above. Let $\mathbf{R}(\underline{\mathbf{A}})\,(\mathbf{X}(\mathbf{A}))$ and $\mathbf{R}(\mathbf{A}^*)\,(\mathbf{X}(\mathbf{A}^*))$ be the vector of reliabilities (log-reliabilities) of the n systems assembled according to the partitionings $\mathbf{A}$ and $\mathbf{A}^*$ respectively. Then

$$\mathbf{X}(\mathbf{A}^*) \overset{m}{\geq} \mathbf{X}(\mathbf{A})$$

where $\overset{m}{\geq}$ stands for majorization.

Proof. It is clear that

$$X_1(\mathbf{A}^*) \leq X_2(\mathbf{A}^*) \leq \ldots \leq X_n(\mathbf{A}^*).$$

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Let $X_{(1)}(\mathbf{A}) \leq X_{(2)}(\mathbf{A}) \leq \ldots \leq X_{(n)}(\mathbf{A})$ be an increasing rearrangement of $X_1(\mathbf{A})$, $X_2(\mathbf{A}), \ldots, X_n(\mathbf{A})$. Notice that $X_n(\mathbf{A}^*)$ is the sum of the m_{in} largest log-reliabilities of the components of type i for $1 \leq i \leq k$, and thus exceeds $X_{(n)}(\mathbf{A})$ which is the sum of the log- reliabilities of m_i' components of type i with $m_i' \geq m_{in}$ for $1 \leq i \leq k$. A similar argument shows that $X_n(\mathbf{A}^*) + X_{n-1}(\mathbf{A}^*) \geq X_{(n)}(\mathbf{A}) + X_{(n-1)}(\mathbf{A})$. Continuing in this fashion, we find that

$$X_n(\mathbf{A}^*) + X_{n-1}(\mathbf{A}^*) + \cdots + X_j(\mathbf{A}^*)$$
$$\geq X_{(n)}(\mathbf{A}) + X_{(n-1)}(\mathbf{A}) + \cdots + X_{(j)}(\mathbf{A}), \qquad 2 \leq j \leq n.$$

Also,

$$\sum_{j=1}^{n} X_j(\mathbf{A}^*) = \sum_{i=1}^{k} \sum_{s=1}^{M_i} \log p_{i_s} = \sum_{j=1}^{n} X_j(\mathbf{A}).$$

Hence

$$\mathbf{X}(\mathbf{A}^*) \overset{m}{\geq} \mathbf{X}(\mathbf{A}). \diamond$$

Special cases of the general setup of assembly of systems described in this section and of Theorem 2.1 have appeared earlier in the literature. The use of majorization allows us to show later in this section, in a clear and simple fashion, that the partitioning $\mathbf{A}^*$ leads to the optimal assembly under several different criteria.

El-Neweihi, Proschan and Sethuraman(1986) set $k = 1$, that is considering only one type of components and studied the optimal assembly of n systems. Theorem 2.1 above reduces to Theorem 2.1 of their paper.

Derman, Leirberman and Ross (1972) considered the case where the elements of the type-enumerator matrix $\mathbf{M}$ are all equal to unity. In this case we have n components of type i for $1 \leq i \leq n$ and there are $(n!)^k$ possible partitions. They use a different approach to show that the partitioning $\mathbf{A}^*$ is optimal under some of the criteria listed below.

We now show that the partitioning $\mathbf{A}^*$ found in Theorem 2.1 provides the optimal assembly under several different criteria. We list these below as remarks.

Remark 2.2. Let $N(\mathbf{A})$ be the number of functioning systems among the n systems assembled from the partitioning $\mathbf{A}$. Then $E(N(\mathbf{A}))$, the expected number of functioning systems is maximized for the partitioning $\mathbf{A}^*$. This follows from the fact that $E(N(\mathbf{A})) = \sum_{j=1}^{n} \exp(X_j(\mathbf{A}))$ is a Schur-convex function of $\mathbf{X}(\mathbf{A})$ and is maximized at $\mathbf{A}^*$.

Remark 2.3. Suppose that we allow only those systems to be assembled whose reliabilities are at least 1/4. This is a condition that will be very often met in practice. Then the variance of $N(\mathbf{A})$, the number of working systems is minimized at $\mathbf{A}^*$. To see this we note that

$$V(N(\mathbf{A})) = \sum_{j=1}^{n} e^{X_j(\mathbf{A})}(1 - e^{X_j(\mathbf{A})})$$

and the function $e^x(1 - e^x)$ is concave for $x \geq \frac{1}{4}$. Thus $V(N(\mathbf{A}))$ is a Schur-concave function of $\mathbf{X}(\mathbf{A})$ in the region $\mathbf{X}(\mathbf{A}) \geq (\frac{1}{4}, \frac{1}{4}, \ldots, \frac{1}{4})$ and is minimized at $\mathbf{A}^*$.

Remark 2.4. We can strengthen Remark 2.2 as follows. The number of working systems $N(\mathbf{A}^*)$ under $\mathbf{A}^*$ is stochastically larger than $N(\mathbf{A})$ for any other partitioning $\mathbf{A}$. This follows from Theorem 2.1 above and Theorem 2.2 of Pledger and Proschan(1971). This result can be translated as follows: Consider a new system $S(\mathbf{A})$ which is an r-out-of-n system whose components are the n assembled systems according to partitioning $\mathbf{A}$, $1 \leq r \leq n$. Then the reliability of $S(\mathbf{A})$ is maximized when it is constructed from the partitioning $\mathbf{A}^*$.

Remark 2.5. Let $Y_j(\mathbf{A})$ be the expected number of working components in the jth system assembled from the partitioning $\mathbf{A}$, $1 \leq j \leq n$, and let $\mathbf{Y}(\mathbf{A}) = (Y_1(\mathbf{A}), \ldots, Y_n(\mathbf{A}))$. Assume further that $m_{i1} = m_{i2} = \cdots = m_{in}$ for $1 \leq i \leq k$. Then $\mathbf{Y}(\mathbf{A}^*) \overset{m}{\geq} \mathbf{Y}(\mathbf{A})$ for all

A. This follows from the relation

$$Y_j(\mathbf{A}) = \sum_{i=1}^{k} \sum_{s \in A_{ij}} p_{is}$$

and an argument similar to the one in the proof of Theorem 2.1. We can now state that $\mathbf{A}^*$ maximizes any Schur-convex function of $\mathbf{Y}(\mathbf{A})$ and minimizes any Schur-concave function of $\mathbf{Y}(\mathbf{A})$. Examples of such functions are $\left(\sum_{j=1}^{n} Y_j^p(\mathbf{A})\right)^{1/p}$, $p \geq 1$ and $\prod_{j=1}^{n} Y_j(\mathbf{A})$ respectively.

3. Life times of general assembly
with independent components

In this section we extend the results of the previous section on optimal assembly of n systems with k types of components and type-enumerator matrix $\mathbf{M}$ satisfying (2-1) to include a time element. Let the life-times of the M_i components of type i be $T_{i1}, T_{i2}, \ldots, T_{iM_i}$ and suppose that

$$(3\text{-}1) \qquad T_{i1} \overset{st}{\le} T_{i2} \overset{st}{\le} \cdots \overset{st}{\le} T_{iM_i},$$

for $1 \le i \le k$. Let $\mathbf{A}$ be a partitioning of the components as described in the previous section and let $T_1(\mathbf{A}), T_2(\mathbf{A}), (\mathbf{A}), \ldots, T_n(\mathbf{A})$ be the life-times of the n assembled systems and let

$$\mathbf{T}(\mathbf{A}) = (T_1(\mathbf{A}), T_2(\mathbf{A}), \ldots, T_n(\mathbf{A}))$$

be the life-time vector corresponding to the partitioning $\mathbf{A}$. We assume that the components act independently, as in the previous section, and we therefore have, for each time $t \ge 0$,

$$(3\text{-}2) \qquad P\big(T_j(\mathbf{A}) > t\big) = \prod_{i=1}^{k} \prod_{s \in A_{ij}} P(T_{is} > t),$$

$1 \le j \le n$, which is analogous to (2-2). Condition (3-1) states that the ordering among $\{P(T_{is} > t),\ 1 \le s \le M_i,\ 1 \le i \le k\}$ is the same for all t. The optimal partitioning $\mathbf{A}^*$ of the previous section which dependeds on this ordering remains invariant for all t. Thus we obtain immediately from Theorem 2.1 and Remark 2.4 that

$$(3\text{-}3) \qquad \begin{aligned} &\big(\log P(T_1(\mathbf{A}^*) > t), \ldots, \log P(T_n(\mathbf{A}^*) > t)\big) \\ &\qquad \overset{m}{\ge} \big(\log P(T_1(\mathbf{A}) > t), \ldots, \log P(T_n(\mathbf{A}) > t)\big) \end{aligned}$$

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and

$$(3\text{-}4) \qquad \sum_{j=1}^{n} I(T_j(\mathbf{A}^*) \geq t) \overset{st}{\geq} \sum_{j=1}^{n} I(T_j(\mathbf{A}) \geq t)$$

for all t and all partitionings $\mathbf{A}$. From (3-4) it follows that

$$(3\text{-}5) \qquad T_{(r)}(\mathbf{A}^*) \overset{st}{\geq} T_{(r)}(\mathbf{A})$$

for $1 \leq r \leq n$ and all partitionings $\mathbf{A}$, where $T_{(r)}(\mathbf{A})$ is the rth order statistic of $(T_1(\mathbf{A})$, $T_2(\mathbf{A}), \ldots, T_n(\mathbf{A}))$. As a further consequence, we obtain

$$(3\text{-}6) \qquad \sum_{r=1}^{n} E\big(g_r(T_{(r)}(\mathbf{A}^*))\big) \geq \sum_{r=1}^{n} E\big(g_r(T_{(r)}(\mathbf{A}))\big)$$

whenever $g_1, \ldots, g_n$ are increasing functions. In particular, if $g_r(x) = x$ for $1 \leq r \leq n$, we obtain

$$(3\text{-}7) \qquad E\left(\sum_{j=1}^{n} T_j(\mathbf{A}^*)\right) \geq E\left(\sum_{j=1}^{n} T_j(\mathbf{A})\right)$$

which shows that a system consisting of using the n assembled systems one at a time in succession has maximum expected life if assembled from the partitioning $\mathbf{A}^*$. We can extend the result on individual order statistics in (3-5) to a result for the whole vector of order statistics by strengthening condition (3-1) as follows. Assume that the component life-times have proportional hazard functions and satisfy (3-1), i.e.,

$$(3\text{-}8) \qquad P(T_{i_s} > t) = \exp(-\lambda_{i_s} H(t))$$

where $H(t)$ is a hazard function and

$$\lambda_{i1} \geq \lambda_{i2} \geq \cdots \geq \lambda_{iM_i},$$

for $1 \leq i \leq k$. For any partitioning $\mathbf{A}$, let

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$$\lambda_j(\mathbf{A}) = \sum_{i=1}^{k} \sum_{s \in A_{ij}} \lambda_{is}, \quad 1 \le j \le n,$$

and let $\Lambda(\mathbf{A}) = (\lambda_1(\mathbf{A}), \lambda_2(\mathbf{A}), \ldots, \lambda_n(\mathbf{A}))$. It easily follows that $\Lambda(\mathbf{A}^*) \overset{m}{\ge} \Lambda(\mathbf{A})$ for any partitioning $\mathbf{A}$. Futhermore $(T_1(\mathbf{A}), \ldots, T_n(\mathbf{A}))$ are independent random variables with proportional hazards and $P(T_j(\mathbf{A}) > t) = \exp(-\lambda_j(\mathbf{A}) H(t))$, $1 \le j \le n$. From Theorem 3.4 of Proschan and Sethuraman(1976) it follows that

$$(3\text{-}9) \qquad (T_{(1)}(\mathbf{A}^*), T_{(2)}(\mathbf{A}^*), \ldots, T_{(n)}(\mathbf{A}^*)) \overset{st}{\ge} (T_{(1)}(\mathbf{A}), T_{(2)}(\mathbf{A}), \ldots, T_{(n)}(\mathbf{A}))$$

for all partitionings $\mathbf{A}$. This is a stronger result than (3-5). A consequence of (3-9) is

$$(3\text{-}10) \qquad \sum_{j=1}^{n} T_j(\mathbf{A}^*) \overset{st}{\ge} \sum_{j=1}^{n} T_j(\mathbf{A})$$

which is stronger than (3-7).

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4. Assembly of systems with one component of each type

In the previous two sections we studied the assembly of n series systems from k types of components. In this section we study the assembly of more general systems. We will assume that all the elements of the type-enumerator matrix $\mathbf{M}$ are equal to 1. We make this assumption for reasons of simplicity. Thus we have n components of type i for $1 \leq i \leq k$, and we need to construct n systems, each of which needs one component of each type. As before this can be done by using a partitioning $\mathbf{A} = (A_{ij})$, where A_{ij} is the singleton $\{a_{ij}\}$ and, for each i, $1 \leq i \leq k$, $(a_{i1}, \ldots, a_{in})$ is a permutation of $(1, \ldots, n)$. Thus the jth system will consist of components $a_{1j}, a_{2j}, \ldots, a_{kj}$. The reliability of this system will be assumed to be given by $R_j(\mathbf{A}) = R(c_{1j}, c_{2j}, \ldots, c_{kj})$ where c_{ij} is an attribute associated with component a_{ij}. This attribute may be the actual reliability of the component or some concommitant of it. Without loss of generality we may assume that

$$(4\text{-}1) \qquad c_{i1} \leq c_{i2} \leq \cdots \leq c_{ik}, \quad 1 \leq i \leq k$$

The use of a general function R as above allows us to consider more general systems than series systems. We will assume that the function R satisfies the following two conditions which are generally satisfied by reliability functions:

$$(4\text{-}2) \qquad R(c_1, \ldots, c_k) \quad \text{is nondecreasing in each coordinate,}$$

and for $(c_1, \ldots, c_k)$, $(d_1, \ldots, d_k)$,

$$(4\text{-}3) \quad R(c_1, \ldots, c_k) + R(d_1, \ldots, d_k) \leq R(c_1 \vee d_1, \ldots, c_k \vee d_k) + R(c_1 \wedge d_1, \ldots, c_k \wedge d_k),$$

where $c \vee d = \max(c, d)$ and $c \wedge d = \min(c, d)$. Such reliability functions occur, for instance, when $R(c_1, \ldots, c_k) = P(Y_1 \leq c_1, \ldots, Y_k \leq c_k)$, where $Y_1, \ldots, Y_k$ represent some (possibly

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dependent) random variables(damages) associated with the k components and the system fails as soon as some Y_i exceeeds c_i(threshold). Let $\mathbf{R}(\mathbf{A}) = (R_1(\mathbf{A}),\ldots,R_n(\mathbf{A}))$ be the vector of reliabilities of the n systems assembled from the partitioning $\mathbf{A}$. Let $\mathbf{A}^*$ be the partitioning mentioned in the previous two sections, which in view of (4-1) corresponds to $A_{ij}^* = \{j\}$, $1 \le i \le k$, $1 \le j \le n$. Theorem 4.1 below shows that $\mathbf{R}(\mathbf{A}^*)$ maximizes $\mathbf{R}(\mathbf{A})$ in the weak mojorization sense. We draw several interesting conclusions from this fact in the remarks that follow.

Theorem 4.1. Let the attribute $\{c_{ij}\}$ satisfy (4-1) and let the reliability function R satisfy (4-2) and (4-3). Then

$$(4\text{-}4) \qquad\qquad \mathbf{R}(\mathbf{A}^*) \overset{w.m.}{\ge} \mathbf{R}(\mathbf{A})$$

for all partitionings $\mathbf{A}$, where $\overset{w.m.}{\ge}$ stands for weak majorization.

Proof. From the monotonicity of R it follows that

$$R_1(\mathbf{A}^*) \le R_2(\mathbf{A}^*) \le \cdots \le R_n(\mathbf{A}^*).$$

Let $R_{(1)}(\mathbf{A}), R_{(2)}(\mathbf{A}),\ldots,R_{(n)}(\mathbf{A})$ be an increasing re-arrengement of $R_1(\mathbf{A}), R_2(\mathbf{A}),\ldots,R_n(\mathbf{A})$. We have to prove that

$$(4\text{-}5) \qquad\qquad \sum_{j=r}^{n} R_j(\mathbf{A}^*) \ge \sum_{j=r}^{n} R_j(\mathbf{A})$$

for $1 \le r \le n$. Note that

$$R_n(\mathbf{A}^*) = R(c_{1n},\ldots,c_{kn})$$

and

$$R_{(n)}(\mathbf{A}) = R(c_{1a_{1l}},\ldots,c_{ka_{kl}})$$

for some l with $1 \le l \le n$. From the monotonicity of R, it follows that $R_n(\mathbf{A}^*) \ge R_{(n)}(\mathbf{A})$. Again, from the monotonicity of R and from (4-3),

$$R_{(n)}(\mathbf{A}) + R_{(n-1)}(\mathbf{A}) = R(c_{1a_{1l}},\ldots,c_{ka_{kl}}) + R(c_{1a_{1m}},\ldots,c_{ka_{km}}) \text{ for some } 1 \le l, m \le n$$

$$\le R(c_{1a_{1l}} \vee c_{1a_{1m}},\ldots,c_{ka_{kl}} \vee c_{ka_{km}}) + R(c_{1a_{1l}} \wedge c_{1a_{1m}},\ldots,c_{ka_{kl}} \wedge c_{ka_{km}})$$

$$\le R_{n-1}(\mathbf{A}^*) + R_n(\mathbf{A}^*).$$

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Inequality (4-5) is established in a similar fashion for all r with $1 \leq r \leq n$. $\Diamond$

An immediate consequence of Theorem 4.1 is the following. Let ϕ be a Schur-convex function of n arguments which is non-decreasing in each argument. Then

$$(4\text{-}6) \qquad \qquad \phi(\mathbf{R}(\mathbf{A}^*)) \geq \phi(\mathbf{R}(\mathbf{A}))$$

for all partitionings $\mathbf{A}$. The opposite inequality is true if ϕ is Schur- concave and non-increasing. This fact is used in the following remarks to point out that in many ways the assembly from $\mathbf{A}^*$ is optimal. Let $N(\mathbf{A})$ be the number of working systems among the n systems assembled from partitioning $\mathbf{A}$.

Remark 4.2. $E(N(\mathbf{A}^*)) \geq E(N(\mathbf{A}))$. This follows from fact that $E(N(\mathbf{A}^*)) = \sum_{j=1}^{n} R_j(\mathbf{A}^*)$ $\geq \sum_{j=1}^{n} R_j(\mathbf{A}) = E(N(\mathbf{A}))$.

For the remaining remarks below we assume that the n assembled systems are stochastically independent.

Remark 4.3. Suppose that $R_j(\mathbf{A}) \geq \frac{1}{2}$ and $R_j(\mathbf{A}^*) \geq \frac{1}{2}$, $1 \leq j \leq n$. Then $Var(N(\mathbf{A}^*)) \leq Var(N(\mathbf{A}))$. Observe that the function $\phi(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} x_j(1 - x_j)$ is a Schur-concave function which is nonincreasing in the region $x_1 \geq \frac{1}{2}, x_2 \geq \frac{1}{2}, \ldots, x_n \geq \frac{1}{2}$. Now $Var(N(\mathbf{A}^*)) = \sum_{j=1}^{n} R_j(\mathbf{A}^*)(1 - R_j(\mathbf{A}^*)) \leq \sum_{j=1}^{n} R_j(\mathbf{A})(1 - R_j(\mathbf{A})) = Var(N(\mathbf{A}))$.

Remark 4.4. $P(N(\mathbf{A}^*) \geq 1) \geq P(N(\mathbf{A}) \geq 1)$. Again the function $\psi(x_1, x_2, \ldots, x_n) = 1 - \prod_{j=1}^{n}(1 - x_j)$, where $0 \leq x_j \leq 1$, $1 \leq j \leq n$, is Schur-convex and non-decreasing. Therefore $P(N(\mathbf{A}^*) \geq 1) = 1 - \prod_{j=1}^{n}(1 - R_j(\mathbf{A}^*)) \geq 1 - \prod_{j=1}^{n}(1 - R_j(\mathbf{A})) = P(N(\mathbf{A}) \geq 1)$.

Remark 4.5. Let $2 \leq k \leq n$, and assume $R_j(\mathbf{A}) \geq \frac{k-1}{n}, R_j(\mathbf{A}^*) \geq \frac{k-1}{n}$, $1 \leq j \leq n$. Then $P(N(\mathbf{A}^*) \geq k) \geq P(N(\mathbf{A}) \geq k)$. This follows immediately from Theorem 4.1 above and Theorem 1.1 of Boland and Proschan(1983).

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In the above discussion we assumed that the each of the n systems required one component from each of the k types. We can incorporate the case where fewer than k types are needed for some systems by allowing the type- enumerator matrix to consist of ones and zeros only. This together with condition (2-1) implies that when a system contains one component from each of m types, $1 \leq m \leq k$, then these are types $1, 2, \ldots, m$. the reliability of such a system will depend on the attributes $c_1, \ldots, c_m$ of the components. We will assume that this reliability is equal to

$$R(c_1, c_2, \ldots, c_m, \infty, \ldots, \infty)$$

where R is a function of k arguments satisfying (4-2) and (4-3). We can now introduce fictitious extra components whose attributes are ∞ of the appropriate types so that the optimal assembly problem for the present case reduces to the optimal assembly problem of n systems from k types of components with a type-enumerator matrix consisting of all ones.

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A general assembly of n systems for k types of components is considered. The techniques of majorization and schur-functions are utilized to pinpoint the optimal assembly under several criteria. Earlier results of Derman, Lieberman and Ross (1972) and El-Neweihi, Proschan and Sethuraman (1986) are generalized.

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