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The State-of-the-Art Parabolic Equation Approximation as Applied to Underwater Acoustic Propagation With Discussions on Intensive Computations

An Invited Paper Presented at the
108th Meeting of the Acoustical Society of America,
8-12 October 1984, Minneapolis, Minnesota

Ding Lee
Surface Ship Sonar Department

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Naval Underwater Systems Center
Newport, Rhode Island / New London, Connecticut

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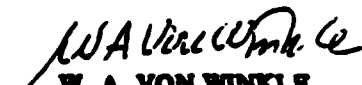
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PREFACE

This document was prepared under NUSC Project No. A65020, "Finite-Difference Solutions to Acoustic Wave Propagation." Principal Investigator, Dr. D. Lee (Code 3332). The sponsoring activity was the Naval Material Command, Program Manager, CAPT D. F. Parrish (NAVMAT 08L), Program Element 61152N, Navy Subproject/Task ZR00000101, Inhouse Laboratory Independent Research.

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W. A. VON WINKLE
Associate Technical Director of Technology

The author of this document is located at the
New London Laboratory, Naval Underwater Systems Center,
New London, Connecticut 06320.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block numbers) This document contains the slide presentation entitled "The State-of-the-Art Parabolic Equation Approximation as Applied to Underwater Acoustic Propagation With Discussions on Intensive Computations," given at the 108th meeting of the Acoustical Society of America, 8-12 October 1984, in Minneapolis, Minnesota. The Parabolic Equation (PE) has applications in many different scientific fields such as electromagnetics, optics theory, quantum		

20. Continued:

mechanics, plasma physics, seismology, underwater acoustics, and others. The subject of this presentation is PE approximation as applied to underwater acoustic wave propagation. A review will be given on past contributions, recent developments will be highlighted, and, looking ahead, we will discuss what the PE method can do in order to stimulate future research and development, as well as applications. Intensive computations with respect to the PE implementation will also be discussed.

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The PE approximation begins with the expression

$$\phi(r, z) = u(r, z) v(r), \quad (2)$$

where $v(r)$ is strongly dependent on the range variable r while $u(r, z)$ is weakly dependent on r .

$$\begin{aligned} \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + k_0^2 n^2(r, z) \phi &= 0 \\ \phi(r, z) &= u(r, z) v(r) \\ \left[v_{rr} + \frac{1}{r} v_r \right] u + \left[u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r \right) u_r + u_{zz} + k_0^2 n^2(r, z) \right] v &= 0 \\ v_{rr} + \frac{1}{r} v_r + k_0^2 v &= 0 \\ u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r \right) u_r + u_{zz} + k_0^2 (n^2(r, z) - 1) u &= 0 \\ \hline v(r) &= H_0^{(1)}(k_0 r) = \sqrt{\frac{2}{\pi k_0 r}} e^{i(k_0 r - \frac{\pi}{4})} \\ u_{rr} + (2ik_0) u_r + u_{zz} + k_0^2 (n^2(r, z) - 1) u &= 0 \\ \hline u_r &= \frac{i}{2} k_0 (n^2(r, z) - 1) u + \frac{i}{2k_0} u_{zz} \end{aligned}$$

Standard PE

VUGRAPH 2

Substituting Eq. (2) into Eq. (1) gives

$$\left[v_{rr} + \frac{1}{r} v_r \right] u + \left[u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r \right) u_r + u_{zz} + k_0^2 n^2(r, z) \right] v = 0. \quad (3)$$

Set the first term of Eq. (3) in brackets equal to $-k_0^2 v$ and the second term in brackets equal to $k_0^2 u$, we obtain two equations:

$$v_{rr} + \frac{1}{r} v_r + k_0^2 v = 0, \quad (4)$$

and

$$u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r\right) u_r + u_{zz} + k_0^2 (n^2(r, z) - 1) u = 0. \quad (5)$$

Considering only the outgoing wave in the range direction, we see that the solution of Eq. (4) is the zeroth order Hankel function of the first kind, $H_0^{(1)}(k_0 r)$.

Applying the farfield approximation, $k_0 r \gg 1$, to the argument of $H_0^{(1)}(k_0 r)$, we find that

$$v(r) = H_0^{(1)}(k_0 r) \approx \sqrt{\frac{2}{\pi k_0 r}} e^{i(k_0 r - \frac{\pi}{4})}. \quad (6)$$

Using Eq. (6) to simplify the coefficient $(1/r + (2/v) v_r)$, in Eq. 5., we find

$$u_{rr} + (2ik_0) u_r + u_{zz} + k_0^2 (n^2(r, z) - 1) u = 0. \quad (7)$$

Dropping u_{rr} , based on the paraxial approximation, $|u_{rr}| \ll |2ik_0 u_r|$, we find

$$u_r = \frac{i}{2} k_0 (n^2(r, z) - 1) u + \frac{i}{2k_0} u_{zz}. \quad (8)$$

This is the first parabolic wave equation derived by Tappert and is referred to as the STANDARD PE.

I would like to take another approach to Eq. (8), which you will find useful in observing some important physical properties.

$$\begin{aligned}
 &u_{rr} + 2ik_0 u_r + u_{zz} + k_0^2(n^2(r,z) - 1)u = 0 \\
 &\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) \left(\frac{\partial}{\partial r} + ik_0 + ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0 \\
 &\text{[NO SCATTERING]} \\
 &\text{One-way outgoing wave: } \left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0 \\
 &\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} = 1 + \frac{1}{2} \left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right] \\
 &u_r = \frac{1}{2} k_0 (n^2(r,z) - 1) u + \frac{1}{2k_0} u_{zz}
 \end{aligned}$$

VUGRAPH 3

If we neglect the scattering effects, Eq. (7) can be expressed in an operator form

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) \left(\frac{\partial}{\partial r} + ik_0 + ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0. \quad (9)$$

Note that Eqs. (7) and (9) are the same if and only if there is no scattering.

Again, considering only the one-way outgoing wave, we deal with the solution

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}}\right) u = 0. \quad (10)$$

Making use of the slowly varying property of $n(r,z)$ upon r , for the time being, we approximate the square-root operator by

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2} \left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]. \quad (11)$$

We refer to this as the small angle approximation. Substituting Eq. (11) into Eq. (10), we obtain the standard PE, that is,

$$u_r = \frac{i}{2} k_0 (n^2(r, z) - 1) u + \frac{i}{2k_0} u_{zz}.$$

The derivation is left for the audience. It is important to note that at this point the standard PE, based on the PE approximation, obeys the following limitations:

LIMITATIONS

1. Farfield Approximation, $k_0 r \gg 1$.
2. $n(r, z)$ Slowly Varying in r .
3. One-Way Outgoing Wave.
4. No Scattering.
5. A Particular Square-Root Approximation.

VUGRAPH 4

Within these limitations, the standard PE is a very good mathematical model for long range, low frequency propagation. At this stage, the only effective solution algorithm for the standard PE was the split-step Fourier algorithm by Tappert and Hardin.³

Before I mention some earlier important developments, I want to mention the "Workshop on Wave Propagation and Underwater Acoustics," held in Mystic, Connecticut, in November 1974. This workshop was not limited to PE, but, after attending this workshop, a comprehensive article on PE was written by Frederick D. Tappert ("The Parabolic Approximation Method,"²), which is a chapter of the book cited in vugraph 5.

The Parabolic Equation Approximation in
Wave Propagation & Underwater Acoustics

F. D. Tappert
ed. J. B. Keller and J. S. Papadakis

Lecture Notes in Physics, Springer-Verlag (1977)

VUGRAPH 5

To this date, the Tappert article is still the most comprehensive article on PE.

In 1977, there was another workshop⁴ in Woods Hole, Massachusetts, where Tappert gave another paper on the application of his split-step algorithm. Not many people are familiar with this paper.

Selected Applications of the Parabolic Equation Method
in Underwater Acoustics

F. D. Tappert

International Workshop on Low-Frequency Propagation
& Noise, Vol. 2, Woods Hole, Mass. pp. 155-194, (1974).

VUGRAPH 6

Since 1974, the interest in the PE was on the rise, but little was done. However, there were a number of notable contributions.

The first practical results of applying the PE were published by C. W. Spofford.⁵ Then a few papers⁶⁻¹² were published in relation to the normal mode method and its application. These are

A Synopsis of the AESD Workshop on Acoustic-Propagation Modeling by Non-Ray-Tracing Techniques, C. W. Spofford, AESD TN-73-05 (1973)

Eikonal Approximation and the Parabolic Equation, D. R. Palmer, J. Acoust. Soc. Am., 60(2), 343-354 (1976)

Relation Between the Solutions of the Helmholtz and Parabolic Equation for Sound Propagation, J. A. DeSanto, J. Acoust. Soc. Am., 62(2), 295-297 (1977)

A Correction to the Parabolic Equation, J. A. DeSanto, J. S. Perkins, and R. B. Baer, J. Acoust. Soc. Am., 64(6), 1664-1666 (1978)

On the Parabolic Approximation to the Reduced Wave Equation, G. A. Kriegsmann & E. W. Larsen, SIAM J. Appl. Math., 34(1), 201-204 (1978)

Helmholtz Equation as an Initial Value Problem with Approximation to Acoustic Propagation, R. M. Fitzgerald, J. Acoust. Soc. Am. 57(4), 839-842, (1975)

Propagation of Normal Mode in the Parabolic Approximation, S. T. McDaniel, J. Acoust. Soc. Am., 57(2), 307-311 (1975)

Parabolic Approximation for Underwater Sound Propagation, S. T. McDaniel, J. Acoust. Soc. Am., 58(6), 1178-1185 (1975)

VUGRAPH 7

Researchers are curious and interested in the relationship between the PE and the Helmholtz equation. Fitzgerald¹⁰ analyzed the PE in terms of normal mode theory. Dave Palmer⁶ observed that the normal mode formalism was difficult in ordering the geometric optics path-length parameter because of mode-coupling. The PE removes this difficulty.

Then, during approximately the same time period, a few interesting developments happened, one was the computer code.

The Use of the Parabolic Equation Method in Sound Propagation Modeling,

F. B. Jensen and H. Krol

SACLANTCEN MEMO SM-72 (1975)

The AESD Parabolic Equation Model,

H. K. Brock

NORDA TN-12 (1978)

VUGRAPH 8

Jensen¹³ had a PE package called PAREQ in his laboratory performing a variety of research and applications; so too did Brock.¹⁴ These computer models used the split-step algorithm to solve the standard PE. G. Gartrell¹⁵ wrote a split-step code on the IBM 370/168.

However, in solving the standard PE, a number of users found a phase error. DeSanto, Perkins, and Baer⁸ discussed this phase error and introduced a correction to the parabolic approximation. Notable was a technique introduced by Brock, Buchal, and Spofford¹⁶ to modify the sound-speed profile to improve the accuracy of the PE.

Modifying the Sound-Speed Profile to Improve the Accuracy of the Parabolic Equation Technique

H. K. Brock, R. N. Buchal, and C. W. Spofford

J. Acoust. Soc. Am., 62(3), pp. 543-552 (1977)

VUGRAPH 9

Interest was also increasing in the application of the PE to solve real problems. The PE models in various laboratories were all based on the use of the split-step Fourier algorithm with an artificial bottom treatment.

In order to apply the PE to solve real problems, the model would be required to have many capabilities. The natural question is what can PE do?

Can PE offer these capabilities. These questions stirred up research and development interest. I ask the question in a different way -- what can we do to improve the PE capability?

First, to improve the PE capability, Lee, Papadakis, and Prieser^{17,18} initiated the numerical solution of the parabolic wave equation so that under shallow water or strong bottom interaction environments the numerical technique can handle the bottom boundary condition.

**Numerical Solution of the Parabolic Wave Equation:
An Ordinary-Differential-Equation Approach**

D. Lee and J. S. Papadakis

J. Acoust. Soc. Am., 68, pp. 1482-1488 (1980)

**Generalized Adams Methods for Solving Underwater Wave
Propagation Problems**

D. Lee and S. Preiser

J. Comp. & Math. with Appls., 7(2), pp. 195-202 (1981)

Finite-Difference Solution to the Parabolic Wave Equation

D. Lee, G. Botseas, and J. S. Papadakis

J. Acoust. Soc. Am., 70(3), pp. 795-800 (1981)

VUGRAPH 10

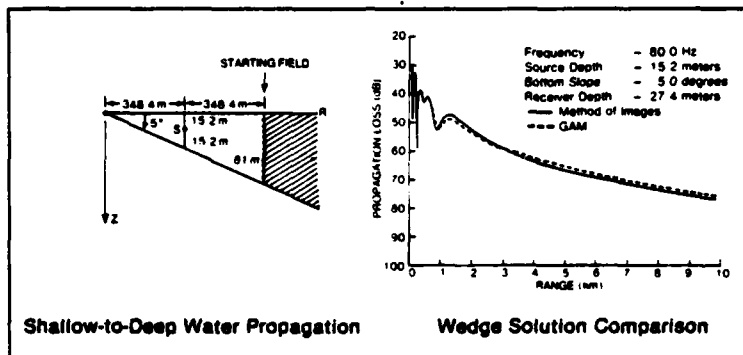
Lee and Papadakis introduced the approach of using an ordinary differential equation (ODE) to determine a special kind of bottom boundary (rigid) within the framework of the PE.

This bottom boundary treatment was incorporated into the ODE and finite difference models. At that time, we did not have a better implementation of the ODE solution, but we concentrated our efforts on developing a very basic, general purpose finite-difference scheme,¹⁹ which could be implemented into computer code.²⁰ This scheme is known today as the Implicit Finite Difference (IFD) model and is an implicit Crank-Nicolson

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method, which is unconditionally stable. The IFD model is used quite often to solve the standard PE.

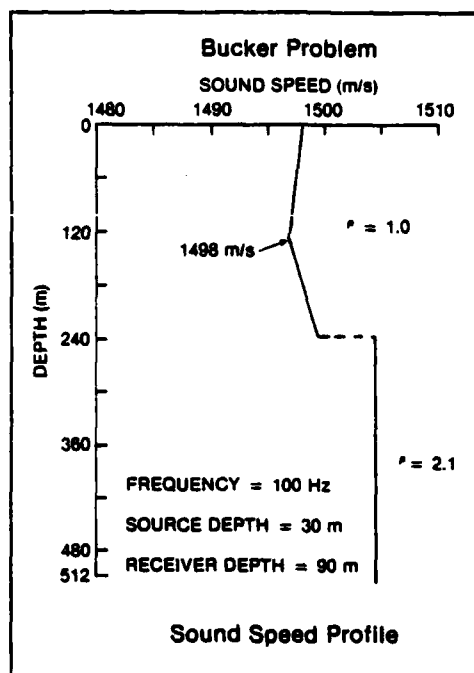
We used the ODE model to solve a wedge problem consisting of range versus propagation loss. The results turned out surprisingly well.



VUGRAPH 11

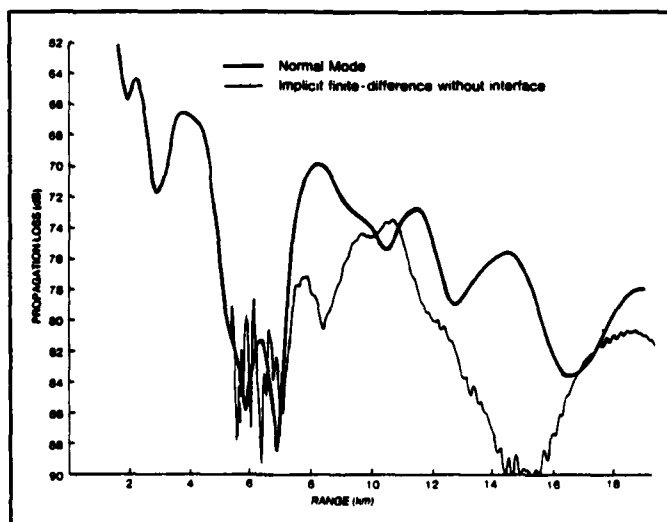
After we showed that our bottom boundary treatment worked, we were asked many questions pertaining to our IFD model's capabilities. One of the first practical questions was a problem proposed by H. P. Bucker. He asked: Your IFD model can handle bottom boundary conditions -- can it handle the interface condition?

The problem did not seem to be a particularly difficult one, however, at that time, no existing PE code could do it.



VUGRAPH 12

The problem was solved easily by the normal mode solution. The normal mode solution was used as a reference solution for comparison of transmission loss predictions.



VUGRAPH 13

Handling the interface wasn't difficult, but it required some thought as to the best approach. Thanks to many valuable discussions with Dr. Suzanne T. McDaniel, we worked out a finite-difference treatment for the horizontal interface.²¹ The horizontal interface development is documented in the article outlined in vugraph 14.

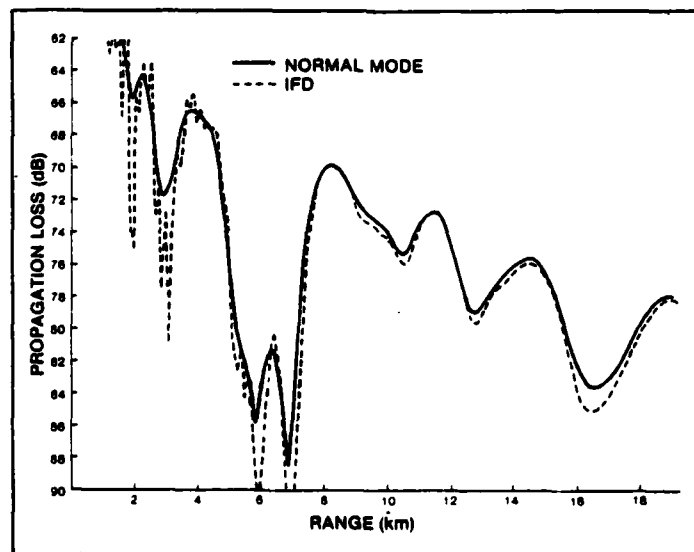
A Finite-Difference Treatment of Interface Conditions for the Parabolic Wave Equation: The Horizontal Interface

S. T. McDaniel and Ding Lee

J. Acoust. Soc. Am., 71(4), pp. 855-858 (1982)

VUGRAPH 14

We then incorporated the horizontal interface conditions into the finite difference code and ran the Bucker problem on the VAX 11/780 computer at NUSC.



VUGRAPH 15

As vugraph 15 shows, without the interface treatment, the results from both IFD and split-step do not agree with our normal mode reference solution results, but after the finite-difference treatment, the results are in reasonable agreement.

Naturally, it is logical to extend the finite-difference technique to handle the irregular interface condition.²² Dr. McDaniel and I have worked this out and I shall talk about this a little later.

While the capabilities of the PE were being developed, use of the PE model was increasing. During this period of development, NORDA sponsored a workshop, the "NORDA Parabolic Equation Workshop."²³ Many interesting, realistic problems were introduced at the workshop and a comprehensive report was published.

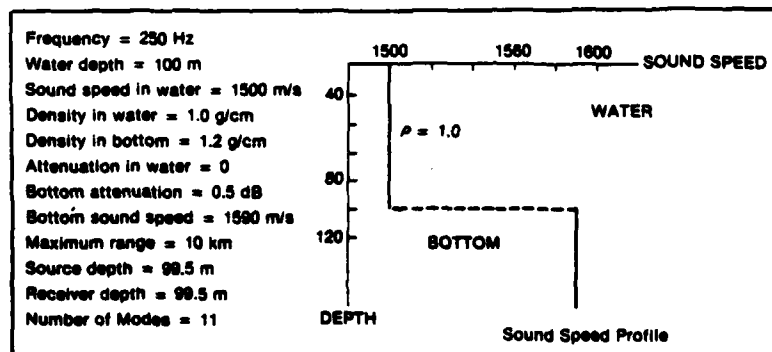
NORDA Parabolic Equation Workshop

James A. Davis, DeWayne White, and Raymond C. Cavanagh

NORDA TN-143 (1981)

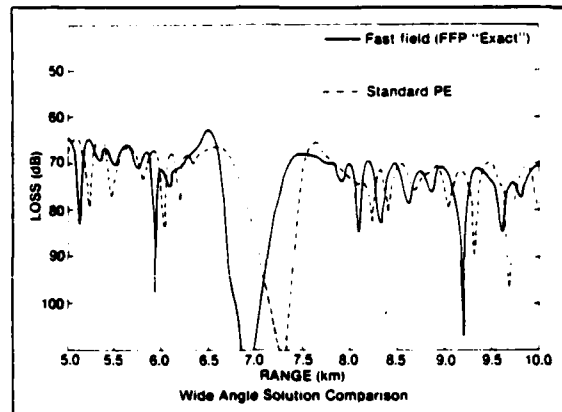
VUGRAPH 16

One of the problems introduced was that of wide angle propagation.



VUGRAPH 17

This problem was solved satisfactorily by both the normal mode and the fast field program (FFP). We then used the FFP solution as a benchmark reference solution.



VUGRAPH 18

Note, in vugraph 18, that the dashed-dotted line was the solution produced by the standard PE (both the split-step and the IFD), the disagreement is clear. It was due to the size of the angle of propagation. An important capability required to produce agreement is the wide angle capability.

Let me briefly describe the mathematical development of the wide angle capability. Recall the one-way outgoing wave equation, that is,

$$\left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \right) u = 0,$$

where we made the approximation

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2} \left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]$$

to obtain the standard PE

$$u_r = \frac{1}{2} k_0 (n^2(r, z) - 1) u + \frac{1}{2k_0} u_{zz}.$$

$$\text{One-way outgoing wave: } \left(\frac{\partial}{\partial r} + ik_0 - ik_0 \sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \right) u = 0$$

$$\sqrt{1 + (n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2} \left[(n^2 - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]$$

$$u_r = \frac{1}{2} k_0 (n^2(r, z) - 1) u + \frac{1}{2k_0} u_{zz}$$

VUGRAPH 19

To be general, we approximate the square-root operator by a rational function approximation as follows:

$$\sqrt{1 + \left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx \frac{1 + p \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + q \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]} \quad (12)$$

If $p = 1/2$ and $q = 0$, we see clearly it reduces to the small angle approximation, thus, resulting in the standard PE.

For a special selection, $p = 3/4$ and $q = 1/4$, the right-hand side of Eq. (12) becomes

$$\sqrt{1 + \left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} \approx \frac{1 + \frac{3}{4} \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + \frac{1}{4} \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]} \quad (13)$$

which is the rational function approximation of the square-root operator by John F. Claerbout.²⁵ We choose to keep p and q arbitrary so that we can determine p and q to suit our needs.

If we use Eq. (12) for the square-root operator and substitute it into the one-way outgoing wave equation, we find a pseudopartial differential equation, that is,

$$\frac{\partial}{\partial r} u = \left(-ik_0 + ik_0 \frac{1 + p \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + q \left[\left(n^2(r,z) - 1\right) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]} \right) u, \quad (14)$$

which is the Wide Angle Wave Equation.

A Rational Function Approximation

$$\sqrt{1 + (n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} = \frac{1 + p \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + q \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}$$

$$\sqrt{1 + (n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2}} = \frac{1 + \frac{1}{4} \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + \frac{1}{4} \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}$$

Jon F. Claerbout approximation

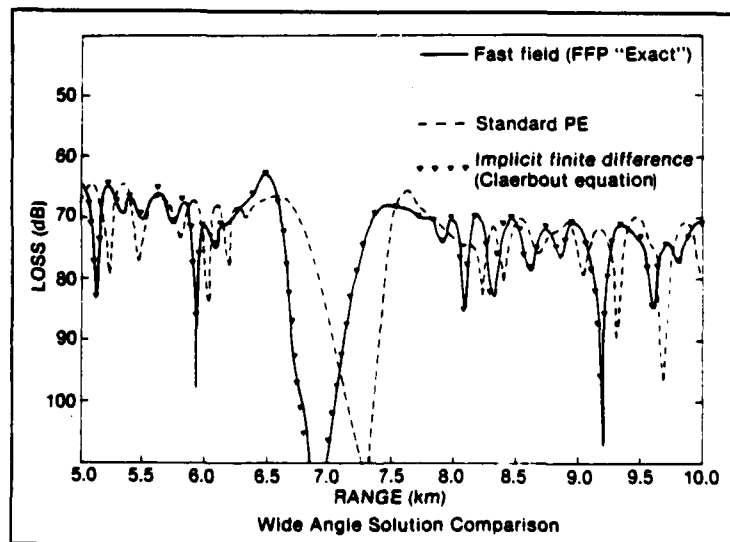
Wide Angle Wave Equation

$$\frac{\partial}{\partial r} u = \left(-ik_0 + ik_0 \frac{1 + p \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]}{1 + q \left[(n^2(r,z) - 1) + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right]} \right) u$$

VUGRAPH 20

We tend to refer to it as the Wide Angle PE, but in a sense this is deceptive. The PE is our key equation. When $p = 1/2$ and $q = 0$, the standard PE is a special case. Therefore, when we refer to the Wide Angle PE, we are actually referring to the pseudopartial differential equation (14).

With the wide angle capability, you see that the IFD produced agreeable results when compared with the benchmark FFP solution.



VUGRAPH 21

There have been a number of authors²⁴⁻²⁷ who have contributed to the theoretical development of the wide angle capability.

Fundamentals of Geophysical Data Processing with
Applications to Petroleum Prospecting

Jon F. Claerbout

McGraw-Hill (1976)

High Angle PE

Robert R. Greene

NORDA Parabolic Equation Workshop (1981)

IFD: Wide Angle Capability

George Botseas, Ding Lee, and Kenneth E. Gilbert

NUSC TR#6905 (1983)

Extension of the Parabolic Equation Model for High-Angle
Bottom-Interacting Paths

L. B. Dozier and C. W. Spofford

SAI Technical Report SAI-78-712-WA (1977)

VUGRAPH 22

Greene used the term "rational parabolic" as opposed to "parabolic." He also applied the rational function approximation to the square-root operator. This rational function approximate technique was applied earlier by Claerbout.²⁵

A number of other people also examined and developed the wide angle capability. At NUSC, we incorporated this into our IFD code and a comprehensive report was published.²⁴ In the seismology field, Berkhout used the continued fraction to approximate the square root operator.²⁸

Wave Field Extrapolation Technique in Seismic
Migration

A. J. Berkhout

Geophysics, Vol. 46, #12, pp. 1838-1856 (1981)

VUGRAPH 23

It is interesting to note that Berkhout's first order continued fraction coincides exactly with Tappert's standard small angle PE. His second order continued fraction coincides with the Claerbout approximation. Roughly speaking, the small angle PE can accommodate propagation angles up to 15°, and wide angle PE (IFD) can accommodate propagation angles up to 40°.

David Thomson²⁹ was the first to apply the Split-step algorithm to handle the wide angle.

A Wide Angle Split-Step Algorithm for the Parabolic Equation

David J. Thomson and N. R. Chapman

J. Acoust. Soc. Am., 74(6), pp. 1848-1854 (1983)

VUGRAPH 24

Recently, one of my colleagues, Donald St. Mary³⁰ of the University of Massachusetts, developed a very high angle PE using a higher order rational approximation that can accommodate a propagation angle greater than 45°. This is good news in the area of shear wave propagation.

Formulation and Discretization of a Very Wide Angle Parabolic Equation

Donald F. St. Mary and Ding Lee (1984)

VUGRAPH 25

All the authors I have mentioned thus far have made contributions to the wide angle capability in this period of time. Two researchers, Estes and Fain,³¹ started earlier examining the wide angle formulation using the Taylor series expansion.

**Numerical Technique for Computing the Wide Angle
Acoustic Field in an Ocean with Range-Dependant
Velocity Profiles**

L. E. Estes and G. Fain

J. Acoust. Soc. Am., 62(1), pp. 38-43 (1977)

VUGRAPH 26

Continuous use of the PE for research and application purposes showed reasonable success, however, under unusual environments, it was not fully developed to handle everything. SACLANT and NORDA called our attention to the fact that in the irregular sloping interface situation, if the depth partition points do not fall on the interface boundary, inaccuracy will occur. In numerical analysis language, this is called numerical reflections. The present computer code cannot handle the situation without some modification because the irregular interface conditions treatment is not included in the present code. In addition to the progress made by McDaniel and me, recent progress has been made by Jaeger³² to treat the interface condition by using the irregular interface condition developed by McDaniel and me.

**A Computer Program for Solving the Parabolic Equation
Using an Implicit Finite-Difference Solution Method
Incorporating Exact Interface Conditions**

Larry Ernest Jaeger

Naval Postgraduate School, MS Thesis (1983)

**A Finite-Difference Treatment of Interface Conditions for the
Parabolic Wave Equation: The Irregular Interface**

Ding Lee and S. T. McDaniel

J. Acoust. Soc. Am., 73(5), pp. 1441-1447 (1983)

VUGRAPH 27

Independently from the direct application of the irregular interface condition, Jules deG Gribble³³ made an important improvement on the finite difference model.

Jules allows variable mesh spacing and deals with interfaces that do not lie on a mesh point. He extended the finite difference treatment of interfaces to the parabolic wave equation. Jules' main effort was centered at the employment of variable vertical mesh size in conjunction with a high level system of ODE solver.

**Extending the Finite Difference Treatment of Interfaces
When Using the Parabolic Wave Equation**

Jules deGribble

J. Acoust. Soc. Am., 76(1), pp. 217-221 (1984)

VUGRAPH 28

Up to this point, it seems that there is enough PE capability for research studies as well as for applications. Three types of PE models exist.

EXISTING PARABOLIC EQUATION MODELS

1. Split-Step Fourier Algorithm Model
2. Implicit Finite-Difference Model
3. Ordinary-Differential-Equation Model

VUGRAPH 29

Little has been done to compare the available computer models. The only published literature was given by Kewley³⁴ who discussed practical solutions of the PE model for underwater acoustic wave propagation.

Practical Solutions of the Parabolic Equation Model for
Underwater Acoustic Wave Propagation
in Computational Techniques & Applications Conference

D. J. Kewley, L. T. Sin Fai Lamn, and G. Gartrell
ed. J. Noye, Sydney

North-Holland, Amsterdam (1983)

VUGRAPH 30

Among these three models, IFD is the more general purpose. The ODE solution has great potential, but is not yet fully developed. Applications of the PE model indicates that it is doing well as a research code, especially the IFD code, because of its accurate computation of the wave field. However, a number of users are actually using these codes for applications.

In solving either the small angle PE or the wide angle wave equation, we deal with one input parameter, the reference wavenumber k_0 . Application of the above equations to solve real problems requires a clever selection of the reference wavenumber k_0 . This selection was ignored by the model developers and the users. For the user, in practice, he does not usually have the knowledge to select the best k_0 ; therefore, users have, in most cases, ignored the selection of k_0 . The model developer built in the automatic selection of k_0 according to physical experiences. Inappropriate k_0 will lead to an evident phase error. Pierce recently reemphasized the importance of k_0 selection and introduced a formula to determine the range of k_0 based on the Rayleigh quotient. Some numerical experiments have been carried out at the Naval Underwater Systems Center, New London Laboratory; results show some phase shift effects dependent on k_0 variations. Tappert and Lee joined Pierce in studying the natural selection of k_0 .

**The Natural Reference Wavenumber for Parabolic
Approximations in Ocean Acoustics**

Allan D. Pierce

J. Comp. & Math. with Appis. (To be published) (1984)

VUGRAPH 31

It is interesting to note that L. Ngheim-Phu and Tappert³⁶ developed a PE box that can be put on shipboard for fast prediction.

**A High-speed, Compact, and Interactive Parabolic Equation
Solution GENERator System (PESOGN)**

Lan Ngheim-Phu and F. D. Tappert

J. Acoust. Soc. Am., 75(S1), p.s26 (1984)

VUGRAPH 32

The interest in the PE solution has continuously increased. An interesting development was the Yale University Workshop.³⁷ Three days were devoted to discussions by invited speakers about the state-of-the-art of computational ocean acoustics.

COMPUTATIONAL OCEAN ACOUSTICS WORKSHOP

YALE UNIVERSITY

Sponsors: ONR (Math. Dept.)
Yale U. (Computer Science Dept.)
NUSC (Independent Research)

Special Issue: J. Comp. & Math. with Applications

Book: Pergamon Press
Editors: Martin H. Schultz and Ding Lee

VUGRAPH 33

Besides the book published by the Pergamon Press, there was a technical document outlining the progress in the development and application of the PE.

Recent Progress in the Development and Application of the Parabolic Equation

Ed. Paul D. Scully-Power and Ding Lee

NUSC TD7145 (1984)

VUGRAPH 34

Most contributions we have mentioned so far deal with the solution of linear PE. McDonald and Kuperman³⁸ developed a two-dimensional (range and depth) formula for the propagation of nonlinear acoustic pulses and weak shocks in a refracting medium. The equation they developed is the nonlinear time domain counterpart of the linear frequency domain PE.

Time Domain Solution of the Parabolic Equation Including

Nonlinearity, E. B. McDonald and W. A. Kuperman, To appear in J. Comp. & Math. with Appls. (1985)

VUGRAPH 35

This technical document³⁹ contains some of the interesting topics Tappert and I discussed over a period of two summers. These include

- Range Refraction Corrected PE.(ref 40)
- Density Effects in PE.(ref 39)
- Time Domain PE.
- High Frequency PE (hybrid ray-PE). (ref 39)
- Backscattering (in range) PE.(ref 41)
- Rough Boundaries in PE.(ref 42)
- 3-Dimensional PDE.(ref 43)
- Stable Explicit Schemes for Solving 3-Dimensional (2-Dimensional) PE
- Shear Waves in PE.
- Match Interface Conditions Between Fluid and Elastic Media.
- Large Angle PE.
- Currents in PE.
- Doppler Effects in PE.
- Moving Sources and Receivers, or Time-Variables.
- The Application of Multi-Array Processors to Solve the Parabolic Wave Equation
- Reciprocity in the Time Domain and the PE Method.

VUGRAPH 36

As for the computation of long range propagation, it is on a very large scale. Ideally, we would like to have in principle

LARGE SCALE COMPUTATIONS

1. Accuracy
2. Speed
3. Minimal computer storage requirements

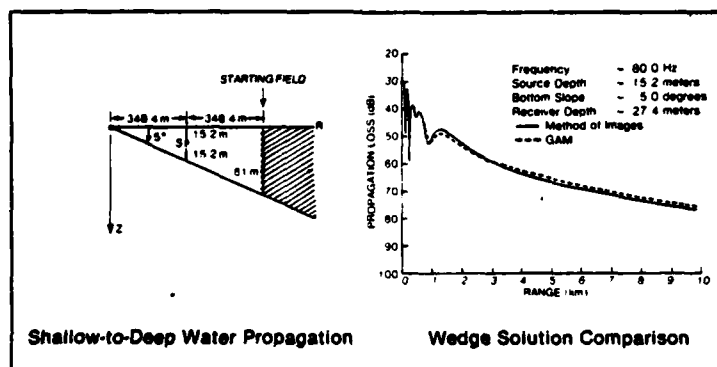
VUGRAPH 37

Let me use the IFD as a simple example. Using the IFD to solve the parabolic wave equation, either small angle or wide angle, requires the solution of a system of equations of the form

$$Au^{n+1} = Bu^n + u^n + u^{n+1}$$

VUGRAPH 38

Since both matrices A and B are tridiagonal, a special recursive formula is applied to solve the system at minimal cost. Not counting the overhead, the tridiagonal solver requires $6N$ operations. The solution looks so simple and economical that it does not seem to require intensive computations even for long range propagation problems. However, this is not generally true. Let us recall our earlier solution to the wedge problem of a rigid bottom boundary condition. Our early solution to this problem was applying the Generalized Adams ODE method, which produced the accurate solution.



VUGRAPH 39

For the sloping bottom boundary, in order to solve the system satisfactorily, we adopted a variable dimension procedure. We started at approximately 348 m for a 0.15 m depth increment and ended up to solve an initial system of equations of order 400. Due to the memory storage limit, calculations were not allowed to go up to a maximum range of 10 km, which means we have to solve a system of equations of order 5740. Recently, we improved the technique and relaxed the storage requirement, thus, increased the speed.

The parabolic wave equation can be solved by a first order Generalized Adams method.⁴⁴

PE	$u_r = a(k_0, r, z)u + b(k_0, r, z)u_{zz}$
ODE	$u_r = A(k_0, r, z)u + g(k_0, r, z, u_0)$
GAB	$u^{m+1} = e^{Ah}u^n + h(Ah) - 1(e^{Ah} - I)g_n$
EXP	$e^{Ah} \approx (I - Ah)^{-1}$
Improved GAB	$(I - Ah)u^{n+1} = u^n + hg_n$

VUGRAPH 40

Since Matrix A increases dimension at every range step, e^{Ah} has to be updated at every range increment. It costs N^3 operations for the inversion and cost N^2 memory storages because we calculate the matrix exponential by the rational Pade approximation, which requires the inversion of an $N \times N$ matrix. Even though the Matrix A is tridiagonal, the inverse destroys the tridiagonal property and fills up the storage. You can see the cost. By a careful study, for a reasonable choice of the step size, we can calculate the new wave field by avoiding this expensive matrix exponential calculation and solving a system of equations.

This system is again tridiagonal, we reduce N^3 operations to $6N$ operations; moreover, we reduce a N^2 memory storage requirement to only $3N$ storages. By coincidence, this problem can also be solved nicely by the IFD. Now, one can again, by taking advantage of using hardware, improve the efficiency of the computation. You shall hear this from other talks.

CONCLUSIONS

Now you have heard of the many important contributions toward the application of PE approximation in solving ocean acoustic problems and have also heard of some interesting PE developments. You may note that there are many not yet well-developed PE approximations, many more than existing PE capabilities. However, I would like to call your attention to the fact that NOT every problem can be solved by the PE approximation in an efficient manner. If the physical conditions fall within the limitation of the PE approximation, PE approximation is an efficient method to apply.

NOT every problem can be solved by
the Parabolic Equation Approximation

VUGRAPH 41

I welcome your comments and participation in solving these problems and applying your solutions to realistic problems.

Thank you.

LIST OF REFERENCES

1. F. D. Tappert and R. H. Hardin, "Computer Simulation of Long-Range Ocean Acoustic Propagation Using the Parabolic Equation Method," Eighth International Congress on Acoustics, London, p. 452, 1974.
2. F. D. Tappert, "The Parabolic Equation Approximation Method," in Wave Propagation and Underwater Acoustics, edited by J. B. Keller and J. S. Papadakis, Lecture Notes in Physics, vol. 70, Springer-Verlag, Heidelberg, 1977.
3. R. H. Hardin and F. D. Tappert, "Applications of the Split-Step Fourier Method to the Numerical Solution of Nonlinear and Variable Coefficient Wave Equations," SIAM Review 15, p. 423, 1973.
4. F. D. Tappert, "Selected Applications of the Parabolic Equation Method in Underwater Acoustics," in International Workshop on Low-Frequency Propagation & Noise, vol. 2, Woods Hole, Massachusetts, pp. 155-194, 1974.
5. C. W. Spofford, "A Synopsis of the AESD Workshop on Acoustic-Propagation Modeling by Non-Ray-Tracing Techniques," AESD Technical Report AESD TN-73,05, 1973.
6. D. R. Palmer, "Eikonal Approximation and the Parabolic Equation," Journal of the Acoustical Society of America, vol. 60, no. 2, pp. 343-354, 1976.
7. J. A. DeSanto, "Relation Between the Solutions of the Helmholtz and Parabolic Equations for Sound Propagation," Journal of the Acoustical Society of America, vol. 62, no. 2, pp. 295-297, 1977.
8. J. A. Santo, J. S. Perkins, and R. B. Baer, "A Correction to the Parabolic Approximation," Journal of the Acoustical Society of America, vol. 64, no. 6, pp. 1664-1666, 1978.
9. G. A. Kriegsmann and E. W. Larsen, "On the Parabolic Approximation to the Reduced Wave Equation," SIAM J. Appl. Math., vol. 34, no. 1, pp. 201-204, 1978.
10. R. M. Fitzgerald, "Helmholtz Equation as an Initial Value Problem With Approximation to Acoustic Propagation," Journal of the Acoustical Society of America, vol. 57(4), pp. 839-842, 1975.
11. S. T. McDaniel, "Propagation of Normal Mode in the Parabolic Approximation," Journal of the Acoustical Society of America, vol. 57, no. 2, pp. 307-311, 1975.
12. S. T. McDaniel, "Parabolic Approximations for Underwater Sound Propagation," Journal of the Acoustical Society of America, vol. 58, no. 6, pp. 1178-1185, 1975.
13. F. B. Jensen and H. Krol, "The Use of the Parabolic Equation Method in Sound Propagation Modeling," SACLANTCEN Memo SM-72, 1975.

14. H. K. Brock, "The AESD Parabolic Equation Model," NORDA TN-12, 1978.
15. G. Gartrell, "A Parabolic Equation Propagation Loss Model," Weapons Systems Research Laboratory, WSRL-0034-TR, South Australia, 1978.
16. H. K. Brock, R. N. Buchal, and C. W. Spofford, "Modifying the Sound-Speed Profile to Improve the Accuracy of the Parabolic Equation Technique," Journal of the Acoustical Society of America, vol. 62, no. 3, pp. 543-555, 1977.
17. D. Lee and J. S. Papadakis, "Numerical Solutions of the Parabolic Wave Equation: An Ordinary-Differential-Equation Approach," Journal of the Acoustical Society of America, vol. 68, pp. 1482-1488, 1980.
18. D. Lee and S. Prieser, "Generalized Adams Methods for Solving Underwater Wave Propagation Problems," Journal of Comp. & Math. With Applications, vol. 7, no. 2, pp. 195-202, 1981.
19. D. Lee, G. Botseas, and J. S. Papadakis, "Finite-Difference Solution to the Parabolic Wave Equation," Journal of the Acoustical Society of America, vol. 70, no. 3, pp. 795-800, 1981.
20. Ding Lee and George Botseas, "IFD: An Implicit Finite Difference Computer Model for Solving the Parabolic Equation," NUSC Technical Report 6659, Naval Underwater Systems Center, 27 May 1982.
21. S. T. McDaniel and D. Lee, "A Finite-Difference Treatment of Interface Conditions for the Parabolic Wave Equation: The Horizontal Interface," Journal of the Acoustical Society of America, vol. 71, no. 4, pp. 855-858, 1982.
22. D. Lee and S. T. McDaniel, "A Finite-Difference Treatment of Interface Conditions for the Parabolic Wave Equation: The Irregular Interface," Journal of the Acoustical Society of America, vol. 72, no. 3 and vol. 73, no. 5, pp. 1441-1447, 1983.
23. James A. Davis, DeWayne White, and Raymond C. Cavanagh, "NORDA Parabolic Equation Workshop," NORDA TN-143, 1981.
24. G. Botseas, D. Lee, and K. E. Gilbert, "IFD: Wide Angle Capability," NUSC Technical Report 6905, Naval Underwater Systems Center, 1983.
25. J. F. Claerbout, Fundamentals of Geophysical Data Processing with Applications to Petroleum Prospecting, McGraw-Hill Book Company, Inc., NY, 1976.
26. Robert R. Greene, "High Angle PE," in NORDA Parabolic Equation Workshop, p. 80, 1981.
27. L. B. Dozier and C. W. Spofford, "Extension of the Parabolic Equation Model for High-Angle Bottom-Interacting Paths," SAI Technical Report SAI-78-712-WA, 1977.

28. A. J. Berkhout, "Wave Field Extrapolation Techniques in Seismic Migration," Geophysics, vol. 46, no. 12, 1638-1656, 1981.
29. David J. Thomson and N. R. Chapman, "A Wide-Angle Split-Step Algorithm for the parabolic Equation," Journal of the Acoustical Society of America, vol. 74, no. 6, pp. 1848-1854, 1983.
30. D. F. St. Mary and D. Lee, "Formulation and Discretization of a Very Wide Angle Parabolic Equation," (in preparation).
31. L. E. Estes and G. Fain, "Numerical Technique for Computing the Wide Angle Acoustic Field in an Ocean with Range-Dependent Velocity Profiles," Journal of the Acoustical Society of America, vol. 62, no. 1, pp. 38-43, 1977.
32. Larry Ernest Jaeger, "A Computer Program for Solving the Parabolic Wave Equation Using an Implicit Finite-Difference Solution Method Incorporating Exact Interface Conditions," Naval Postgraduate School, MS Thesis, 1983.
33. Jules deG Gribble, "Extending the Finite Difference Treatment of Interfaces When Using the Parabolic Wave Equation," Journal of the Acoustical Society of America, 1984 (to be published).
34. D. J. Kewley, L. T. Sin Fai Lam, and G. Gartrell, "Practical Solutions of the Parabolic Equation Model for Underwater Acoustic Wave Propagation," in Computational Techniques and Applications, ed. J. Noye, Sydney, North-Holland, Amsterdam, 1983.
35. A. D. Pierce, "The Natural Reference Wavenumber for Parabolic Approximations in Ocean Acoustics," Journal of Comp. & Math. with Applications (in preparation).
36. Lan Ngheim-Phu and F. D. Tappert, "A High-Speed Compact and Interactive Parabolic Equation SOLution GENeration System (PERSOGEN), Journal of the Acoustical Society of America, vol. 75, no. S1, p. S26, 1984.
37. M. H. Schultz and D. Lee, editors, Computational Ocean Acoustics. Pergamon Press (to be published, 1985).
38. E. B. McDonald and W. A. Kuperman, "Time Domain Solution of the Parabolic Equation Including Nonlinearity," Journal of Comp. & Math. With Applications (to be published, 1985).
39. Paul D. Scully-Power and Ding Lee, Recent Progress in the Development and Application of the Parabolic Equation, MUSC Technical Document 7145, Naval Underwater Systems Center, 7 May 1984.
40. F. D. Tappert and D. Lee, "A Range Refraction Parabolic Equation," Journal of the Acoustical Society of America (to be published, 1984).
41. D. Lee, Y. Saad, and S. T. McDaniel, "Numerical Computations of Sound Scatterings in the Ocean," (in preparation, 1984).

42. L. B. Dozier, "PERUSE: A Numerical Treatment of ROugh Surface Scattering for the Parabolic Wave Equation," Journal of the Acoustical Society of America, vol. 75, no. 5, pp. 1415-1432, 1984.
43. D. Lee and W. L. Siegmann, "A Mathematical Model for the 3-Dimensional Ocean Sound Propagation," Journal of Mathematical Modelling (to be published, 1985).
44. D. Lee, K. R. Jackson, and S. Preiser, "An Improved Implementation of Generalized Adams Methods for Underwater Wave Propagation Problems," Journal of Comp. & Math. With Applications (to be published, 1984).