

AD-A173 240

ESTIMATION AND CONTROL OF DISTRIBUTED MODELS FOR
CERTAIN ELASTIC SYSTEMS. (U) OKLAHOMA UNIV NORMAN DEPT
OF MATHEMATICS L M WHITE 01 JUL 85 AFOSR-TR-86-0965
AFOSR-84-0271

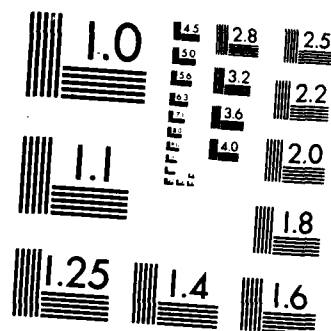
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DOCUMENTATION PAGE

1a. REP UNCLASSIFIED		1b. RESTRICTIVE MARKINGS													
2a. SECURITY CLASSIFICATION AUTHORITY ---		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited													
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 86-0965													
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		7a. NAME OF MONITORING ORGANIZATION AFOSR													
6a. NAME OF PERFORMING ORGANIZATION University of Oklahoma		7b. ADDRESS (City, State and ZIP Code) Bldg 410 Bolling AFB DC 20332-6448													
6b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR 84-0271													
6c. ADDRESS (City, State and ZIP Code) Norman OK 73019		10. SOURCE OF FUNDING NOS. <table border="1"><tr><td>PROGRAM ELEMENT NO. 61102F</td><td>PROJECT NO. 2304</td><td>TASK NO. A1</td><td>WORK UNIT NO.</td></tr></table>		PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304	TASK NO. A1	WORK UNIT NO.								
PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304	TASK NO. A1	WORK UNIT NO.												
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR		11. TITLE (Include Security Classification) Estimation & Control of Distributed Models for certain Elastic Systems Arising in Large Space Structures													
8b. OFFICE SYMBOL (If applicable) NM		12. PERSONAL AUTHOR(S) Professor Luther W. White													
8c. ADDRESS (City, State and ZIP Code) Bldg 410 Bolling AFB DC 20332-6448		14. DATE OF REPORT (Yr., Mo., Day) 7/01/84 to 6/30/85													
13a. TYPE OF REPORT Annual		15. PAGE COUNT 15													
13b. TIME COVERED FROM 7/01/84 TO 6/30/85															
16. SUPPLEMENTARY NOTATION															
17. COSATI CODES <table border="1"><tr><td>FIELD</td><td>GROUP</td><td>SUB. GR.</td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>		FIELD	GROUP	SUB. GR.										18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB. GR.													
19. ABSTRACT (Continue on reverse if necessary and identify by block number) During the period covered by the grant two research papers have been written and three others are in preparation. Titles include: "Identifiability Under Approximations for two-Point Boundary Value Problems", "Estimation of Elastic Parameters in Beams and Certain Plates: H¹ Regularizations", and "Shape Control for Static Beams and Plates."															
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐		21. ABSTRACT SECURITY CLASSIFICATION E													
22a. NAME OF RESPONSIBLE INDIVIDUAL JOHN P. THOMAS, CAPT, USAF		22b. TELEPHONE NUMBER (Include Area Code) 202/767-5028													
		22c. OFFICE SYMBOL NM													

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ANNUAL TECHNICAL REPORT

July 1, 1984 - July 1, 1985

AFOSR-TR- 86 - 0965

Estimation and Control of Distributed Models for Certain Elastic Systems Arising in Large Space Structures

AFOSR Grant No. AFOSR-84-0271

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MATTHEW J. KATZ
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1. Summary.

During the reporting period progress was made toward the goal of developing efficient and accurate estimation and control algorithms for elastic structures composed of beams and plates. Specifically, results were obtained that determine necessary and sufficient conditions for uniqueness in discrete versions of second order elliptic estimation problems. Moreover, theory and algorithms were developed for the estimation of elastic coefficients for static and dynamic models of beams and plates, the control of the shape of static beams and plates by means of actuators of small support, and the optimal placement of actuators to control the shape of beams and plates. These efforts have produced computer codes on which experimentation is currently being conducted to test methods and algorithms.

2. Research Objectives.

The research objective of this project is to study the estimation and control of elastic systems composed of beams and plates in order to develop efficient and accurate control and estimation algorithms. In estimation basic to this goal is to develop an understanding of properties of the parameter to state mappings, an approximation theory associated with particular models of interest, and the suitability of different minimization algorithms for efficient codes for various problems. In control of prime importance is to determine properties of optimal controls and feedback, best location based on design of the actuators and the geometry and elastic properties of the body, and suitable algorithms and codes. Toward these objectives the work during the past year has centered primarily on the estimation and control of static problems although codes for the estimation of elastic and damping parameters in dynamic plate models were also completed.

In considering the parameter to state mapping certainly an important property is injectivity. The injectivity and stability of this mapping is called identifiability. These properties are of course quite important for computational purposes and development of stable algorithms. Typically for distributed systems identifiability is rather difficult to establish. For applications the finite dimensional approximating models are closer to the computer models. An object is to study identifiability for these simpler models. Second order two point-boundary value problems were analyzed for various cases to obtain necessary and sufficient conditions for identifiability.

A second objective was to develop estimation algorithms for the estimation of elastic parameters in plates. Typically for a static problem a model of the form

$$(1) \quad \Delta(a_0 \Delta u) - \nabla \cdot (a_1 \nabla u) + a_2 u = f \quad \text{in } \Omega$$

for Ω a bounded domain in $\mathbb{R}^2$ with appropriate boundary conditions was considered. The estimation problem may be stated as follows. Given an observation z taken in an observation space Z of the deformation find a parameter, for example $\hat{a}_0$, from within an admissible set Q_{ad} in a parameter space Q that minimizes a fit-to-data functional

$$(2) \quad J(a_0) = \|Cu(a_0) - z\|_Z^2 + \varepsilon \|a_0\|_Q^2$$

over the set Q_{ad} . Here C denotes an operator that takes the solutions u in X to the states $Cu(a_0)$ in Z . To solve this problem it is important to determine properties of the optimal estimators such as regularity and stability with respect to constraints and observations. The understanding of these properties aid in developing approximation theories for algorithms and codes.

A third objective over the last year was to develop control algorithms for the design of plates and beams of specified shape. Since the motivating problem was that of designing a mirror deformed to a specified shape and curvature, the control functional chosen for the first static examples was one related to the H^2 -norm of the deformation. More precisely, given a bounded domain $\Omega \subset \mathbb{R}^2$ with the plate model

$$(3) \quad \Delta(a \Delta u) = \sum_{i=1}^{\omega} \beta_i \phi_i \quad \text{in } \Omega$$

with appropriate boundary conditions and ϕ_i in $H^{-2}(\Omega)$, find $\hat{\beta} = (\hat{\beta}_1, \dots, \hat{\beta}_{\omega})$ in $\mathbb{R}^{\omega}$ that minimizes

$$(4) \quad J(\beta) = \int_{\Omega} (a\Delta(u(\beta) - z))^2 dx + \varepsilon \sum_{i=1}^{\omega} \beta_i^2$$

for $\beta \in \mathbb{R}^{\omega}$. The ϕ_i determine the control mechanism and may represent control at a point, distributed along a curve, or over a subset of Ω of positive measure. Further, for the case of point control for design, a problem of interest is to determine optimal location of the actuators. These problems represent a first step and are to be followed by the consideration of the control of dynamic models.

3. Status of the Research.

In this section the status of the research outlined in Section 2 is indicated. In studying identifiability, research has concentrated on the study of identifiability for discrete models that arise as approximations of the distributed models. The discrete model is obtained once basis functions have been chosen to approximate the solution of the partial differential equation and the coefficient to be estimated. In work with Professor K. Kunisch a series of concrete cases was considered using a variety of basis functions. This research has dealt with second order elliptic two point boundary value problems, [1] in Section 4 and extended previous work for the parabolic equation entitled "Parameter Identifiability under Approximation" to appear in the Quarterly of Applied Mathematics. The results obtained in [1] give necessary and sufficient conditions in terms of rank conditions that may be calculated once basis functions for approximating subspaces have been specified. The techniques developed carry over directly to the fourth order elliptic problems.

The investigations into estimation for plates and beams have had several results. In this study to approximate the estimation problem one must translate the constraints that determine Q_{ad} in a meaningful fashion to those for the finite dimensional problem. This may be accomplished in certain instances by considering regularized problems $\epsilon > 0$ in (2). In addition, by absorbing constraints by introducing Lagrange multipliers one may deduce regularity properties of the optimal estimators. These properties are useful in translating the constraints and in applying the penalty method to approximating problems. These results are presented in [2,3,5,6] in Section 4. Further work has been carried out and codes have been written for dynamic problems which are currently being tested.

Concerning the control of plates and beams, the initial stages of research have been motivated by the problem of controlling the shape of a mirror. The model is that of a linear fourth order static plate equation (3). This problem may be viewed as a design problem. The work on this problem is outlined in Section 3 and appears in [4].

The controller ϕ_i may be a function in $L^2(\Omega)$ with support contained in a ball centered at x_i or may be a Dirac delta function δ_{x_i} with mass at x_i . Problem (3)-(4) has been coded in these cases with good results for simply supported boundary conditions. Cases with other boundary conditions are currently under study.

For the case $\phi_i = \delta_{x_i}$ the optimal vector β is in fact a function of X , the actuator locations. Hence, mappings $\Omega^\omega \rightarrow \mathbb{R}^\omega$ and $\Omega^\omega \rightarrow \mathbb{R}$ may be defined by $X \mapsto \beta(X)$, where $\beta(X)$ represents the solution of (2), and $X \mapsto j(X) = J(\beta(X))$, respectively. The question may be considered: Given a compact subset F of Ω and a desired deformation z is there an optimal set of actuator locations. This problem is analyzed in [4] by considering continuity and differentiability properties of the mappings defined above. Moreover, in [4] numerical results are given for the simple case of locating one actuator on a beam. Further, cases are under study with more actuators and for a plate. Finally, these results are being extended to include dynamic models of plates and beams.

4. Publications July 1, 1984 - July 1, 1985

1. Identifiability under Approximations for Two-Point Boundary Value Problems, with K. Kunisch, to appear SIAM J. Cont. and Opt.
2. Estimation of Elastic Parameters in Beams and Certain Plates: H^1 Regularization, submitted to J. Opt. Thy. Appl.
3. Estimation of Elastic Coefficients in Beams: Penalization, submitted to J. Math. Anal. Appl.
4. Shape Control for Static Beams and Plates, in preparation.
5. Estimation of Damping and Elastic Parameters in Plates, in preparation.
6. Estimation of Elastic Parameters for a General Static Plates, in preparation.

5. List of Professional personnel.

(i) Paul Rye, Master of Science in Mechanical Engineering, June 1985.

(ii) Professor Karl Kunisch, Consultant.

6. Interactions.

(1) Talks:

Estimation of Parameters in Beams and Plates, ICASE at NASA Langley Research Center, June 1985.

Shape Control of Plates and Optimal Location of Actuators, Analysis Seminar, University of Oklahoma, Dept. of Mathematics, Spring 1985.

(ii) Consulted with professor Tinsley Odem, TICOM, University of Texas, Dept. of Engineering Mechanics.

7. New Discoveries.

Research in the first year of this project has produced codes for the following:

- (i) Estimation of elastic parameters for beams and plates for static models.
- (ii) Estimation of elastic and damping parameters for dynamic models of plates.
- (iii) Shape control for a plate.
- (iv) Optimal location of actuators for the control of a beam.

- (v) Codes for the estimation of boundary conditions and diffusion coefficients for steady state and transient diffusion equations.

New results include the following:

- (i) Necessary and sufficient conditions for approximate identifiability.
- (ii) Regularity properties for optimal estimators for beams and plates.
- (iii) Differentiability and continuity properties of the optimal control with respect to actuator locations for static plate and beam shape control problems.
- (iv) Approximation theory for regularized and constrained static beam and plate estimation problems.

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