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GEOMETRIC PROBLEMS IN ADAPTIVE CONTROL(U) HARVARD UNIV
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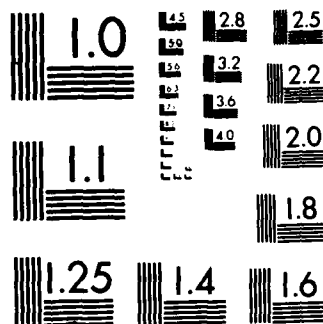
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Summary

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Adaptive control has become practical in recent years because of the increased use of VLSI technology in implementing feedback control. The work described here has lead to the first proofs of convergence for some adaptive algorithms for stabilizing linear time invariant, but unknown, systems. It has also clarified robustness issues associated with this class of adaptive control algorithms. With the use of geometrical methods it has been possible to establish the impossibility of achieving several types of adaptive behavior. New directions for expanding the field of adaptive control have been explored.

Introduction

The overall goal of research in adaptive control is to investigate how, and to what extent, it is possible to design controllers which will work even if the system which is being controlled or observed is unknown, changed or encounters new obstacles. This involves an ever increasing amount of computer control and techniques which, in other contexts, might be thought of as being artificially intelligent.

In the last few years there has been considerable progress on one particular problem in adaptive control. This is the problem of building a "universal" stabilizer for linear time invariant systems. Progress in this area has centered around the idea that, in an interesting set of cases, it is possible to "try out" a set of gains, at least one of which is known to be stabilizing, and to detect when a suitable gain has been found. All this is done by an autonomous controller governed by ordinary differential equations and coupled to the original system to be stabilized through its inputs and outputs. The results in this area now seem to have reached a certain maturity in the sense that the a priori conditions which are imposed seem natural and the results have been simulated in a variety of circumstances.

Self-Tuning Regulations

Self-tuning regulators are governing devices which can adapt to changes in the plant or system which is to be governed while achieving some desired control objective. The development of a coherent yet powerful approach to the design of such adaptive controllers has been and continues to be one of the major research goals in systems engineering. Typical applications would include the design of autopilots, say for jet aircraft or helicopters, which can adapt to variations in the system parameters, such as attitude or altitude.

Philosophically there have been two approaches to adaptive control: The "indirect method" which applies statistical methods in order to overcome parameter uncertainty and the "direct method" which is capable of learning enough about the system "on-line" to control the closed-loop input-output behavior.

Regarding "indirect methods", work at Harvard has analyzed the minimum amount of inherent complexity in statistical procedures due to the complicated geometry of the parameterization of many familiar classes of systems. Such necessary conditions for the consistency of statistical identification schemes are based on geometric approach to variational problems on manifolds, developed in the first quarter of this century.

The design of "direct" adaptive controllers can be illustrated by the following example: Given

(1) $\dot{y} = y + bu, \quad u, y \in \mathbb{R}^2, \quad b \in \mathbb{R} - \{0\}$ find a compensator

(2) $\dot{k} = f(k, y)$

and a feedback law

(3) $u = g(k, y)$

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such that the overall "closed-loop" system (1)-(3) satisfies

$$\lim_{t \rightarrow \infty} k(t) \text{ exists, } \lim_{t \rightarrow \infty} y(t) = 0$$

for all initial data (y_0, k_0) .

The point here is that b is an unknown parameter. If b were a known positive real number, choosing $k > \frac{1}{b}$ and $u = -ky$ would stabilize (1). If b were known only to be positive then choosing (2) so that k would eventually become larger than $\frac{1}{b}$ would stabilize (1). Indeed

$$(2) \quad k' = y^2$$

will work, having chosen $\mu = -\text{sign}^*(b)k$. The stability analysis rests largely on the fact that, after such feedback, $\dot{y}$ is approximately μy where $\mu < 0$. Extending a more complicated but fundamental controller (2)-(3) introduced by R. Nussbaum, it has been possible to design an adaptive controller

$$(2)' \quad k' = y^2$$

$$(3)' \quad u = s(k)ky$$

where $s(k)$ switches sign slowly but sufficiently often so that the Ce'saro mean of $s(k)k$ has $\liminf = -\infty$ and $\limsup = +\infty$. This gives stability for y despite the fact that to first order $\dot{y}$ is $\mu(t)y$ with μ either positive or negative. Surprisingly, $(2)' - (3)'$ "learns" the sign of b eventually rendering $\mu(t)$ negative.

The simplicity of $(2)' - (3)'$ has made it possible to extend the scope of our adaptive controller. Briefly, in joint work with J.C. Willems we have shown $(2)', (3)'$ is self-tuning for (1) whenever (1) is replaced by

(1a) a minimum phase system

$$\dot{x} = Ax + bu$$

$$y = cx$$

with $cb \neq 0$; or

$$(1b) \dot{x} = Ax + bu \quad x, b, c \in \text{a Hilbert space}$$

$$y = cx$$

where A is perhaps unbounded but has pure point spectrum, (1b) is minimum phase, and $c, b \neq 0$.

Aside from including delay-systems and distributed systems, that (2) - (3)' works even for infinite-dimensional systems stresses the major advantage of our controller: Being an output-feedback strategy one needs no estimate on the order of the system. Indeed, one use of "minimum phase" is an adaptive version of the classical root-locus plots for output feedback. Present work involves extending the scope of this adaptive control to include nonlinear systems with stable zero dynamics. This can be done for low-order systems and possibly some aspects of nonlinear adaptive control theory are within sight.

Self-Tuning and Classical Theory

It is possible to draw on classical control theory and modern stability theory in the design of self-tuning regulator for minimum phase systems. This controller differs from earlier controllers in that it does not use either of the standard constructions which

- (i) use an explicit identification subroutine; or
- (ii) do parameter adjustment for a universal observer/augmented error model.

Instead, this controller is a classical direct output feedback gain whose value is adjusted by the inherent stability or instability of the plant. The design is simple, enabling one to give a fairly straightforward stability proof, and points to similar design philosophies for classes of more complicated systems. The first

main result is:

Theorem: Suppose

$$\begin{aligned}\dot{x} &= Ax + bu \\ y &= cx\end{aligned}\tag{1}$$

is a minimum phase system with $cb \neq 0$. Consider the (adaptive) controller

$$\dot{k} = y^2, \quad u = s(k)y\tag{2}$$

where the Ce'saro mean $C(k)$ of $s(k)k$ satisfies

$$\begin{aligned}\lim_{k \rightarrow \infty} \sup C(k) &= +\infty \\ \lim_{k \rightarrow -\infty} \inf C(k) &= -\infty\end{aligned}$$

Then, (1)-(2) is self-tuning, i.e. for all initial data (x_0, k_0)

- (i) $\lim_{t \rightarrow \infty} x_t = 0$
- (ii) $\lim_{t \rightarrow \infty} k_t$ exists.

We emphasize that (2) represents direct output-to-input feedback and that consequently we require no knowledge of (an upper bound of) the dimension of the state space of (1). This eliminates the problem of "unmodelled dynamics", but also hints at the possibility of direct adaptive control for infinite dimensional systems. This would include delay-time systems (e.g. systems where transmission is required over long distances) or flexible and articulated space structures.

The design of (2) is quite intuitive. The pole-zero configuration of (1) is depicted in Figure 1, where 0's represent zeros and x's represent poles.

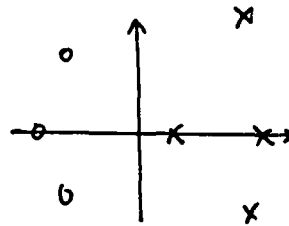


Figure 1. Pole-zero configuration of (1).

We note that, if (A,b,c) were known, then by choosing $u = \text{sign}(cb)ky$ with $k \gg 0$ the root-loci for figure 1 would tend to the zeros with one infinite branch moving to $-\infty$ along the real axis. This is depicted in Figure 2.

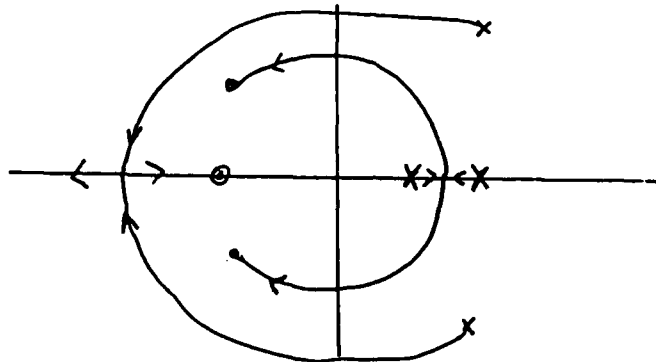


Figure 2.

In the case where (A,b,c) are unknown our strategy is to tune k according to (2), thereby increasing k as long as y is unstable. The feedback law

$$u = s(k)ky$$

switches sign sufficiently often to offset our lack of knowledge of $\text{sign}(cb)$. The stability proof is given in detail in the references. A similar philosophy has been pursued for the cases where

$$cb = 0, cAb = 0, \dots, cA^{n-1}b > 0$$

asking only for an upper bound on n^* .

Adaptive Control and Least Squares

In his recent thesis A. Bloch has investigated total least squares questions from a geometrical point of view and has resolved the issues of existence and uniqueness of minima, maxima and other stationary points in a definitive way. Recall that total least squares refers to fitting a line (plane ...) to a set of points using the criteria of minimizing the sum of the squares of the Euclidean distances between the points and the line, rather than the sum of the squares of the y-axis deviations. In many cases total least squares is to be preferred to ordinary least squares fitting and in system identification it is particularly attractive. Bloch has formulated this problem, following earlier work of Byrnes and Willems, as a minimization problem in a Grassmannian space. Having a correct global version of this problem he was able to relate it to a recent work in symplectic geometry and to illustrate its significance in this context.

Recent Developments

Recently our work at Harvard on adaptive control has turned to problems involving adaptation in a broader setting. Among the remaining problems which call for adaptation is a class which we might think of as multimode problems. As contrasted with the universal stabilizer situation discussed above, these problems have a more pronounced discrete character. An example is an electric circuit which contains diodes. Such circuits may be governed by different numbers of differential equations depending on the voltage across the diodes. Mechanical systems involving the common nonlinearity, dead zone, are also of this type. The regulation of such systems is not well understood; even the stability of an autonomous system of this type may be difficult to check. It is clear that differential equation methods must, in such cases, be supplemented with combinatorial analysis. In a recent paper we have begun the study of such systems.

Work is underway to attempt a classification of systems of this type. It is possible to associate with each such system a certain convex polytope and a corresponding partially ordered set. The dynamics on each element of this set is governed by a set of ordinary differential equations and jump conditions are given which describe the transitions. It is expected that one can identify the qualitative features of such systems and describe a control philosophy which effectively adapts to the circumstances the system finds itself in.

The interface between subsystems which we ordinarily find useful is conceptualized by a block diagram. Historically this was based on an analog computer circuit diagram. In robotic manipulation the interface between "subsystems" is more complex in that when two rigid bodies are interfaced there is a force vector and a torque vector which characterizes the interface -- all together six scalar quantities must be balanced. In designing a control system to achieve a suitable coupling between bodies the positioning and orientation problems which must be solved involve, again, six variables. By doing a mathematical analysis of this process we were able to show that rigid-rigid interfaces have a certain intrinsic complexity associated with them which is, in a robotic context, needlessly complex and that by going to a passively adaptive fluid membrane-rigid interface the complexity can be reduced to a manageable level.

Mathematical theory has been tested by building fluid membrane surfaces and measuring their characteristics as interfaces with rigid bodies. These experimental results have verified the analysis to such a large extent that (with support from NSF) more elaborate systems are being built.

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DEGREES AWARDED

Anthony Bloch, Ph.D. 1985, "Completely Integrable Hamiltonian Systems and Total Least Squares Estimation".
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Nikolaos Papageorgiou, Ph.D., 1983, "Nonsmooth and Multivalued Applications in Optimization".
Bijoy Ghosh, Ph.D. 1983, "Simultaneous Stabilization and Pole Placement of a Multimode Linear Dynamical System".
Nicholas Gunther, Ph.D. 1982, "Hamiltonian Mechanics and Optimal Control".

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