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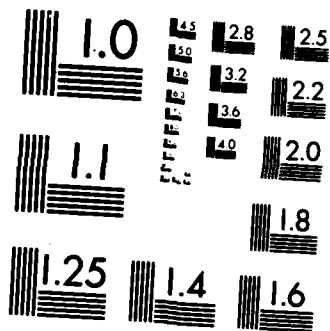
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RESEARCH MEMORANDUM

COMPUTATION OF RANGE AND HEADINGS ALONG GREAT CIRCLE PATHS

Stuart G. Dunn

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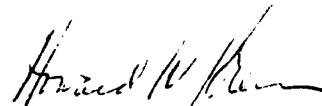
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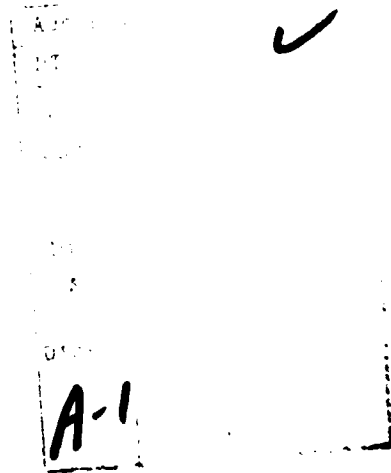
1. Enclosure (1) is forwarded as a matter of possible interest.
2. The algorithm and code discussed in this Research Memorandum were used to determine distances and headings for cargo aircraft as part of a model for assessing the Navy's requirements for organic airlift. The algorithm and code may be of interest to other research requiring great circle distances between points on the globe.



Howard W. Kreiner
Director
Logistics Program

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COMPUTATION OF RANGE AND HEADINGS ALONG GREAT CIRCLE PATHS

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ABSTRACT

This research memorandum presents formulas for computing range and headings along great circle paths and a Fortran subroutine for implementing the formulas.

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INTRODUCTION

When a great circle path is traveled between two points on the earth's surface, the heading generally varies in time. Formulas are presented below for the initial and final headings on the shortest great circle path between an origin, O , and a destination, D , whose positions are specified by latitude and longitude. A formula is also given for the great circle distance between O and D .

In appendix A, a Fortran computer routine is provided for implementing these formulas for great circle headings and distances.

RESULTS

Assume that the latitude and longitude for the origin and destination are given as

$$O = (LAT_O, LNG_O)$$

$$D = (LAT_D, LNG_D) .$$

It is assumed that all latitudes lie in the range $[-90^\circ, +90^\circ]$, positive being to the north. Likewise all longitudes lie in the range $[-180^\circ, +180^\circ]$, positive being eastwards. One computes the quantities

$$\Delta LAT = LAT_D - LAT_O$$

$$\Delta LNG = LNG_D - LNG_O$$

$$LAT_{ave} = 1/2 (LAT_D + LAT_O) .$$

The shortest great circle distance between O and D may be found from the formula

$$d = 60 \cos^{-1} \left[\sin(LAT_O) \sin(LAT_D) + \cos(LAT_O) \cos(LAT_D) \cos(\Delta LNG) \right] . \quad (1)$$

In equation 1, the arc cosine function is assumed to take on values in the range $[0^\circ, 180^\circ]$, and the distance, d , is calculated in nautical miles.

Before presenting the general formulas for initial heading H_O and final heading H_D , several special cases must be considered that correspond to travel along a meridian. These cases arise when $\Delta LNG = 0^\circ$ or $\pm 360^\circ$, when $\Delta LNG = \pm 180^\circ$, or when O or D is at the north or south pole.

When O (for example) is the north pole, its longitude is indeterminate and the case should be treated as having $\Delta LNG = 0^\circ$. Likewise, for the south pole and for D . If $\Delta LNG = 0^\circ$ or $\pm 360^\circ$, the rules are:

- If $\Delta LAT > 0^\circ$, $H_O = H_D = 0^\circ$.
- If $\Delta LAT < 0^\circ$, $H_O = H_D = 180^\circ$.
- If $\Delta LAT = 0^\circ$, no movement is required, and the headings are indeterminate.

On the other hand, if $\Delta LNG = \pm 180^\circ$, travel over the north or south pole is generally required. Therefore, the rules are:

- If $LAT_{ave} > 0^\circ$, the path crosses the north pole, so that $H_O = 0^\circ$ and $H_D = 180^\circ$.
- If $LAT_{ave} < 0^\circ$, the path crosses the south pole, so that $H_O = 180^\circ$ and $H_D = 0^\circ$.

When $LAT_{ave} = 0^\circ$, the headings are indeterminate, since O and D are on diametrically opposite sides of the earth. For any initial heading H_O , a great circle path may be found to D , and it will have $H_D = H_O$.

When the foregoing special cases do not apply, the initial heading may be found from the formula

$$H_O = \text{Tan}^{-1} \left[\frac{\cos(\Delta LAT/2)}{\tan(\Delta LNG/2) \sin(LAT_{ave})} \right] - \text{Tan}^{-1} \left[\frac{\sin(\Delta LAT/2)}{\tan(\Delta LNG/2) \cos(LAT_{ave})} \right] + 180^\circ - 180^\circ \begin{cases} \text{sign}(\tan(\Delta LNG/2)) & \text{if } LAT_{ave} \geq 0^\circ \\ 0 & \text{if } LAT_{ave} \leq 0^\circ \end{cases} \quad (2)$$

In equation 2, Tan^{-1} represents the principal determination of the arc tangent function, taking on values in the interval $(-90^\circ, 90^\circ)$. The final

heading is given by

$$H_D = -\text{Tan}^{-1} \left[\frac{\cos(\Delta LAT/2)}{\tan(\Delta LNG/2) \sin(LAT_{ave})} \right] - \text{Tan}^{-1} \left[\frac{\sin(\Delta LAT/2)}{\tan(\Delta LNG/2) \cos(LAT_{ave})} \right] \\ + 180^\circ - 180^\circ \begin{cases} 0 & \text{if } LAT_{ave} \geq 0^\circ \\ \text{sign}(\tan(\Delta LNG/2)) & \text{if } LAT_{ave} \leq 0^\circ \end{cases} \quad (3)$$

DERIVATION OF HEADING FORMULAS

The geometry used to derive equations 1 through 3 is shown in figure 1. Two cases are illustrated: an "eastbound" case, in which $\Delta LNG \in (0^\circ, 180^\circ)$ or equivalently $\Delta LNG \in (-360^\circ, -180^\circ)$, and a "westbound" case, in which $\Delta LNG \in (-180^\circ, 0^\circ)$ or $\Delta LNG \in (180^\circ, 360^\circ)$. In both cases, the path to be traversed is marked with arrows. Meridians have been indicated from the north pole to O and D . The lengths of these segments of meridians, a and b , and the length, c , of the great circle path will be measured as angles subtended by the arcs at the center of the earth. The internal angles A , B , and C of the spherical triangles have been indicated.

In Case I, the eastbound case, we have

$$H_O = A \\ H_D = 180^\circ - B \quad (4)$$

while in Case II, the westbound case,

$$H_O = 360^\circ - A \\ H_D = 180^\circ + B \quad (5)$$

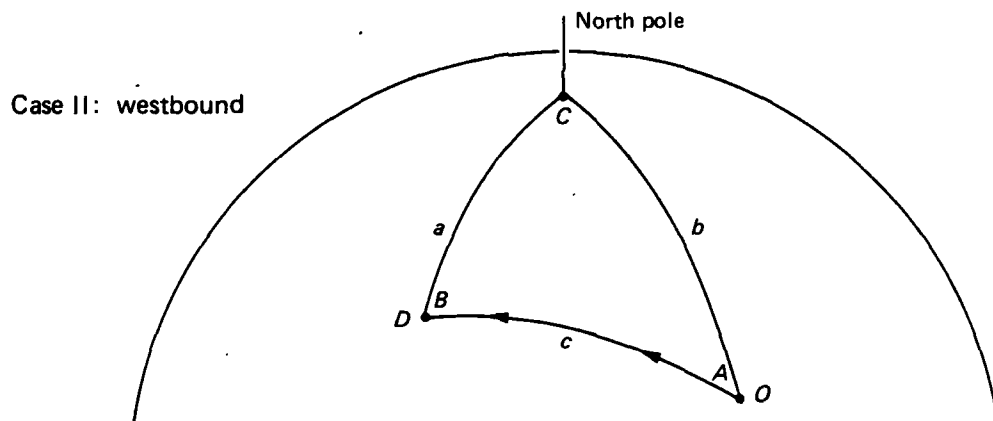
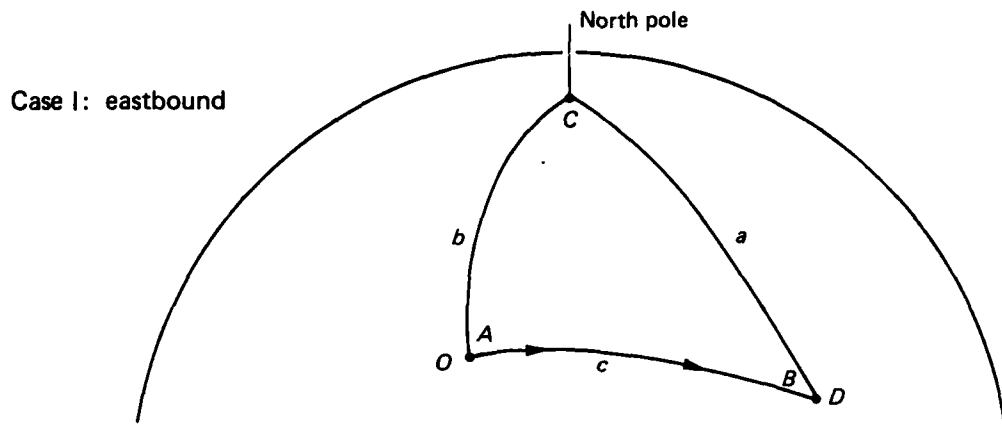


FIG. 1: SPHERICAL TRIANGLES USED IN DERIVATION

The angle C , which is measured at the north pole, may be written in the form

$$C = \begin{cases} |\Delta LNG| & \text{if } |\Delta LNG| < 180^\circ \\ 360^\circ - |\Delta LNG| & \text{if } |\Delta LNG| > 180^\circ \end{cases} \quad (6)$$

so that C is always positive and less than 180° .

Equation 1 for the great circle distance is easily obtained from the law of cosines for spherical triangles:¹

$$\cos(c) = \cos(a) \cos(b) + \sin(a) \sin(b) \cos(C) .$$

When a and b are expressed in terms of LAT_D and LAT_O , and equation 6 is used, the result is

$$c = \cos^{-1} \left[\sin(LAT_O) \sin(LAT_D) + \cos(LAT_O) \cos(LAT_D) \cos(\Delta LNG) \right] .$$

One degree of arc subtended at the earth's center corresponds to a distance of 60 n.mi. on the earth's surface, so that equation 1 immediately follows.

It is clear from equations 4 and 5 that the desired headings may be found once A and B are known. The spherical triangles in figure 1 may be solved to give the angles A and B in terms of a , b , and C , using Napier's analogies and standard rules for determining quadrants. The appropriate formulas are

$$\frac{\sin(a - b)/2}{\sin(a + b)/2} = \frac{\tan(A - B)/2}{\cot C/2} \quad (7)$$

$$\frac{\cos(a - b)/2}{\cos(a + b)/2} = \frac{\tan(A + B)/2}{\cot C/2} \quad (8)$$

The quantities $a - b$ and $a + b$ may easily be related to LAT_{ave} and ΔLAT . Furthermore, equation 6 relates C to $|\Delta LNG|$. Thus, equations 7 and 8 may

1. Taken from Chemical Rubber Co., *CRC Standard Mathematical Tables*, 20th edition, 1972, pp. 200-201.

be manipulated to give

$$\frac{A - B}{2} = \tan^{-1} \left[\frac{-\sin(\Delta LAT/2)}{\tan|\Delta LNG/2| \sin(LAT_{ave})} \right] \quad (9)$$

$$\frac{A + B}{2} = \tan^{-1} \left[\frac{\cos(\Delta LAT/2)}{\tan|\Delta LNG/2| \sin(LAT_{ave})} \right] \quad (10)$$

Note that the inverse tangent functions appearing in equations 9 and 10 are not necessarily the principal determination of arc tangent. Another way of stating this is to say that $(A - B)/2$ and $(A + B)/2$ are determined by these equations, apart from possible addition or subtraction of 180° .

The task now is to resolve the quadrants in which the angles lie. The standard rules for doing so are stated in *CRC Standard Mathematical Tables* as:

RULES FOR DETERMINING THE QUADRANT OF A CALCULATED PART OF AN OBLIQUE SPHERICAL TRIANGLE

- (a) If $A > B > C$, then $a > b > c$.
- (b) A side (angle) which differs more from 90° than does another side (angle) is in the same quadrant as its opposite angle (side).
- (c) Half the sum of any two sides and half the sum of the opposite angles are in the same quadrant.

Rule (c) implies that $(A + B)/2$ is in the same quadrant as $(a + b)/2 = 90^\circ - LAT_{ave}$. Thus, one has

$$0^\circ < 1/2(A + B) \leq 90^\circ \quad \text{if } LAT_{ave} \geq 0^\circ$$

$$90^\circ < 1/2(A + B) < 180^\circ \quad \text{if } LAT_{ave} < 0^\circ$$

Consequently, equation 10 gives

$$\frac{A+B}{2} = \text{Tan}^{-1} \left[\frac{\cos(\Delta\text{LAT}/2)}{\tan|\Delta\text{LNG}/2|(LAT_{ave})} \right] + \begin{cases} 0^\circ & \text{if } LAT_{ave} \geq 0^\circ \\ 180^\circ & \text{if } LAT_{ave} < 0^\circ \end{cases} \quad (11)$$

where, as noted earlier, the principal determination of arc tangent, Tan^{-1} , takes on values between -90° and $+90^\circ$.

Notice that by construction, A and B both lie in the interval $(0^\circ, 180^\circ)$. (The limiting cases in which A or B equals 0° or 180° have been handled separately.) It follows that

$$(A - B)/2 \in (-90^\circ, 90^\circ) .$$

As a result, the inverse tangent function in equation 9 must be identified as Tan^{-1} . Equations 9 and 11 may then be combined to obtain expressions for A and B . When these are inserted into equations 4 and 5, cases I and II may be combined into the expressions for H_O and H_D in equations 2 and 3.

By examining various limits of equation 2, it is straightforward to check that:

- H_O is continuous at $\Delta\text{LNG} = 0^\circ$, for $\Delta\text{LAT} \neq 0^\circ$.
- H_O is continuous at $\Delta\text{LAT} = 0^\circ$, for $\Delta\text{LNG} \neq 0^\circ$.

The heading may take on any value between 0° and 360° if ΔLAT and ΔLNG are allowed to go to zero simultaneously. This is the mathematical realization of the earlier observation that heading is indeterminate in this limit.

Similarly, one finds that:

- H_O is continuous at $LAT_{ave} = 0$, for $\Delta\text{LNG} \neq \pm 180^\circ$.
- H_O is continuous at $\Delta\text{LNG} = \pm 180^\circ$, for $LAT_{ave} \neq 0^\circ$.

Again, a singularity is present at the point $LAT_{ave} = 0^\circ$, $\Delta\text{LNG} = \pm 180^\circ$. Any value of H_O may be obtained as this point is approached, depending on the limiting relation between LAT_{ave} and ΔLNG . This reflects an indeterminacy in heading mentioned above.

APPENDIX A
LISTING OF THE FORTRAN SUBROUTINE GRCRCL

APPENDIX A

LISTING OF THE FORTRAN SUBROUTINE GRCRCL

```

C      SUBROUTINE GRCRCL( LATC, LONG, LATD, LONGD, DIST, HEADING )
C      COMPUTE THE DISTANCE (DIST (NM)) AND INITIAL HEADING (HEADING
C      (DEG)) FROM POSITION LATC-LONG (DECIMAL DEGREES) TO POSITION
C      LATD-LONGD.
C
C      REAL LATC,LONG,LATD,LONGD,DIST,HEADING
C
C      REAL*8 PI,DEGRAD,EPS,HALF
C
C      PARAMETER (PI = 3.14159265358979300)
C      PARAMETER (DEGRAD = PI/180.000)
C      PARAMETER (EPS = 1.00-7)
C      PARAMETER (HALF = 5.00-1)
C
C      REAL*8 DLATC,DLONG,DLATD,DLONGD,HOLLON,HOLLAT
C      REAL*8 AVGLAT,RADHDG,TEMP1,TEMP2
C      REAL*8 OABS,OSIN,OCOS,OTAN,OASIN,OACOS,DATAN
C
C      DLATC = DEGRAD*DBLE(LATC)
C      DLONG = DEGRAD*DBLE(LONG)
C      DLATD = DEGRAD*DBLE(LATD)
C      DLONGD = DEGRAD*DBLE(LONGD)
C
C      HOLLON = (DLONG - DLONGD)*HALF
10  IF (HOLLON .LE. -PI*HALF) THEN
      HOLLON = HOLLON + PI*HALF
      GO TO 10
    ENDIF
20  IF (HOLLON .GT. PI*HALF) THEN
      HOLLON = HOLLON - PI*HALF
      GO TO 20
    ENDIF
      HOLLAT = (DLATD - DLATC)*HALF
      AVGLAT = (DLATD + DLATC)*HALF
C
C      DIST = 60.000*OACOS(OSIN(OCOS(DLATD))*OSIN(OCOS(DLATC))*
C      OABS(DLATD))*OCOS(DLONG-DLONGD))/DEGRAD
C
C      SPECIAL CASES.
C
C      IF ((OABS(HOLLON).LE.EPS).OR.(OABS(HOLLON).GE.PI*HALF-EPS))
C      THEN
C        HEADING = PI/DEGRAD
C        IF (AVGLAT.GE.0.000) HEADING = 0.000
C        RETURN
C      ENDIF
C      IF ( OABS(HOLLAT).GE.PI*HALF-EPS) THEN
C        DIST = 0.000
C        HEADING = 0.000
C        RETURN
C      ENDIF

```

C
C
C

REGULAR CASE.

```
TEMP1 = DATAN(DCOS(HOLLAT)/(COTAN(HOLLON)*OSIN(AVGLAT)))  
TEMP2 = DATAN(SIN(HOLLAT)/(COTAN(HOLLON)*DCOS(AVGLAT)))  
RADMOG = TEMP1-TEMP2+PI  
IF (AVGLAT .GE. 0.000) RADMOG = RADMOG - DSIGN(PI,HOLLON)  
HEADNG = RADMOG/DEGRAD
```

C

RETURN
END

END

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