(12)

AFRPL TR-86-054

AD:

AD-A173 756

Final Report
for the period
September 1985 to
August 1986

Identification of Large Space Structures on Orbit

September 1986

Authors:
E. Denman
T. Hasselman
C. T. Sun
J. N. Juang
J. Junkins
F. Udwadia
V. Venkayya
M. Kamat

American Society of Civil Engineers
345 East 47th Street
New York, NY 10017

F04611-85-C-0092

Approved for Public Release

prepared for the: **Air Force Rocket Propulsion Laboratory**

Air Force Space Technology Center
Space Division, Air Force Systems Command
Edwards Air Force Base,
California 93523-5000

NOTICE

When U.S. Government drawings, specifications, or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

FOREWORD

The work described in this report was performed by the American Society of Civil Engineers under Air Force Rocket Propulsion Laboratory (AFRPL) contract number F04611-85-C-0092. The work was performed during the period September 1985 to August 1986. AFRPL Project Manager was Lt Eric Dale.

This technical report has been reviewed and is approved for publication and distribution in accordance with the distribution statement on the cover and on the DD Form 1473.

Eric M. Dale
ERIC M. DALE, 1Lt, USAF
Project Manager

L. Kevin Slimak
L. KEVIN SLIMAK
Chief, Interdisciplinary Space Technology Branch

FOR THE COMMANDER

Robert L. Geisler
ROBERT L. GEISLER
Chief, Propulsion Analysis Division

SECURITY CLASSIFICATION OF THIS PAGE

AD-A173 256

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for Public Release. Distribution is Unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFRPL-TR-86-054
6a NAME OF PERFORMING ORGANIZATION: American Society of Civil Engineers; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Air Force Rocket Propulsion Laboratory
6c ADDRESS (City, State and ZIP Code): 345 East 47th St, New York, NY 10017	7b. ADDRESS (City, State and ZIP Code): AFRPL/DYSS, Edwards Air Force Base, CA 93523-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: F04611-85-C-0092
8c ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS: PROGRAM ELEMENT NO. 62302F; PROJECT NO. 2864; TASK NO 00; WORK UNIT NO DA
11 TITLE (Include Security Classification): IDENTIFICATION OF LARGE SPACE STRUCTURES ON ORBIT (U)	
12. PERSONAL AUTHOR(S): Denman, E., Hasselman, T., Sun, C.T., Juang, J-N., Junkins, J., Udwadia, F., Venkayya, V., and Kamat, M.	
13a. TYPE OF REPORT: Final; 13b. TIME COVERED: FROM 85/9 TO 86/8	14. DATE OF REPORT (Yr., Mo., Day): 86/9; 15. PAGE COUNT: 256
16 SUPPLEMENTARY NOTATION	

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR	Structures, Modeling, Indentification, Algorithms.
22	01		
22	02		

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

The aspects of state of the practice modeling and identification of large space structures are discussed in this report. Methods of modeling a structure as well as identification procedures are presented along with data acquisition and processing requirements. Recommendations for developing algorithms for the task of space structure identification are given.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED [X] SAME AS RPT. [ ] OTIC USERS [ ]	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL: ERIC M. DALE, 1Lt, USAF	22b TELEPHONE NUMBER (Include Area Code): (805) 277-5147; 22c OFFICE SYMBOL: DYSS

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

SECURITY CLASSIFICATION OF THIS PAGE

ACKNOWLEDGEMENT

The Task Committee on Methods for Identification of Large Structures in Space wishes to express its most sincere appreciation to the Air Force Rocket Propulsion Laboratory without whose support this report would not have been a reality. In particular, the committee wishes to acknowledge technical monitors, Dr. Alok Das and Lt. Eric Dale of AFRPL for their valuable assistance.

The committee is grateful to Drs. Roy Craig, Sheldon Rubin and David Young for their valuable comments in reviewing the manuscript. Finally, the assistance of Edward Kippel of ASCE cannot be overlooked. Ed did a superb job in coordinating the overall activity of the committee and in making sure that the committee met all the necessary deadlines.

PREFACE

Propelled by the increased research activity in the area of large space structures, the Structures and Materials Committee of the Aerospace Division of the American Society of Civil Engineers foresaw the need for the establishment of a Task Committee on Methods for Identification of Large Structures in Space. With the support of several prominent experts in the field of identification, Manohar Kamat proposed the formation of such a task committee, which was finally approved by the Executive Committee of the Aerospace Division and by the Management Group C around mid-1983. The charter of the task committee was to develop a state-of-the-art report on methods for identification of large structures in space. Judging by the history of previous such task committees, it was immediately apparent that regardless of the overwhelming initial enthusiasm of all its members, this task committee was unlikely to finish the work they had started unless financial support could be secured. Accordingly, the task committee set out to find an agency that would sponsor its activity. After a great deal of effort, the committee was successful in winning the support of the Air Force Rocket Propulsion Laboratory (AFRPL) at Edwards Air Force Base, California to support the activity of the committee. The support provided by AFRPL was to be used to provide writing stipends for members, travel expenses for committee meetings and other expenses to produce the report.

The writing group of the committee consisted of some of the most highly dedicated individuals any task committee can aspire to have as members. The writing group consisted of Eugene Denman of the Department of Electrical Engineering of the University of Houston (Sections 7, 8, parts of 4 and 9 and the Bibliography), Timothy Hasselman, President of Engineering Mechanics Associates Inc. of Torrance, California (Parts of Sections 1, 3, 4 and 9), C. T. Sun of the School of Aerospace Engineering of Purdue University (Part of Section 3), Jer-Nan Juang of the Structural Dynamics Branch of NASA Langley Research Center (Section 5, Parts of 9), John Junkins of the Aerospace Engineering

Department of Texas A & M University (Section 6, Parts of 1), Firdaus Udwadia of the Civil Engineering Department of University of Southern California (Section 2, Parts of 4), Vipperla Venkayya of the Flight Dynamics Laboratory of Wright-Patterson Air Force Base and Manohar Kamat of the School of Engineering Science and Mechanics of Georgia Institute of Technology (Parts of Section 1, Parts of 9). Jer-Nan Juang, Manohar Kamat and Vipperla Venkayya provided editorial support. In his characteristic indomitable spirit, Eugene Denman took upon himself the finalization of the manuscript using TEX. John Junkins enlisted the assistance of his excellent graduate student, N. Glenn Creamer, for the writing effort of Section 6 and help in the overall editing of this report.

Two individuals on the Task Committee deserve special mention for their contributions beyond the call of duty. Timothy Hasselman took special pains to review the entire manuscript and made extremely valuable suggestions for improvement of the final format. Last, but not least, Jer-Nan Juang undertook an extensive writing effort and served as one of the editors.

The report provides an excellent overview of the subject of identification, with particular emphasis on structures, perhaps lacking in sufficient detail in some portions. There may be some redundancy in the material in the report, but this is to be expected due to the fact that the report was written by a committee. Time did not allow for the editing task to remove all of the redundancies. There may also be some minor points on which different views are expressed by the members, although there was no major disagreement within the committee on the overall scope of the task that remains for the technical community in resolving the issue of identification of large space structures. It is hoped that this report will serve as a valuable starting point for professionals as well as students of structural identification.

TABLE OF CONTENTS

LIST OF FIGURES.

LIST OF TABLES.

SECTION 1. INTRODUCTION.

Many proposed space missions envisioned for the coming decades involve vehicles and structures which certainly qualify as large space structures. **A large space structure will be defined as a structure in space which consist of a large number of structural elements configured in such a way to provide a space platform for ongoing scientific experiments or defense missions. The structure will be constructed of thousands of structural elements and may be highly flexible. Structures with special mission requirements, such as pointing accuracies, orientation control, shape control and target tracking may be classified as large space structures if the structure has a large number of controlled structural modes with stringent tolerance requirements.** Typical design configurations are football field size or larger structures envisioned for communication antennas, astronomical observatories, solar power stations, space defense platforms and manned laboratories such as the space station. In many cases, it is anticipated that these structures will require active control to carry out ongoing maneuvers, suppress vibration, achieve fine pointing, and possibly, maintain a precise shape. A fundamental obstacle to routinely achieving some of the most challenging objectives is the anticipated inability to analytically model, with sufficient precision and confidence, the structural dynamics of these highly flexible structures.

The inability to model the dynamics of large orbiting structures stems from several causes:

a. Due to the launch costs (which are a function of mass), there is an obvious incentive to make these large structures light; This immediately leads to structures incapable of supporting their own weight in a 1 g environment and it follows that ground based testing of fully deployed/erected structures is near-impossible. Researchers will likely have difficulty making accurate predictions of the equilibrium shape and in some cases it will be necessary to measure and correct the equilibrium shape on-orbit.

b. The anticipated use of composite materials to improve strength/weight ratios and to provide passive damping will have a negative spinoff due to increased modeling uncertainty including uncertain time variations in material properties as a consequence of ageing (due both to cycling and exposure to the space environment).

c. Further complications arise due to the combination of very high flexibility and large thermal gradients. For example, the extreme gradients encountered during the recent Solar Array Flight Experiment (SAFE, deployed from the Space Shuttle), resulted in an unanticipated, nonlinear deformation of the entire structure. It is interesting to note that this ten story solar array represents the first large space structure actually deployed in and retrieved from orbit.

d. Finally, parameter variations due to vehicle reconfiguration, docking with other vehicles/structures, and consumption of fuel, etc., cause difficulties in characterizing precisely the dynamics of such systems.

There exists a substantial body of literature on control of large space structures which suggests that the number of modes that must be considered will be significantly greater than $\gg 10$ for model verification and closed loop control design studies. Experience suggests that blindly assuming the validity of a structural model (derived, for example, from the intended vehicle structural design, using NASTRAN or a similar finite element code) is dangerous indeed. This is especially so for first-of-a- kind, extremely flexible structures, which make use of relatively new materials, joint mechanisms, and erection/deployment techniques. Even for simpler structures, one can seldom manufacture the structure with sufficient precision such that even ten natural frequencies and (especially) mode shapes can be confidently predicted. For the class of large structures under discussion, especially those made of composite materials and using difficult-to-model joints, the *a priori* models will have to be validated and verified prior to flight. In addition, it is anticipated that on-orbit adjustments will be required.

One approach to control of large flexible space structures is to attempt adapting the control law in near real time to better control the actual structure (based upon real-time estimation of the closed loop response in comparison to a reference model). While there is little doubt that adaptive controls are useful in controlling poorly known systems, an evaluation of the state-of-the-art in adaptive control indicates that the dimensionality of most large space structure dynamical models is too high to rely solely upon adaptive control techniques to achieve stable, high precision control...unless done in conjunction with system identification techniques such as those discussed herein. On the other hand, existing identification methods will require careful evaluation, modifications and extensions.

1.1. Motivation.

Structural identification research has historically been driven primarily by demands for aerospace structural identification, trouble-shooting vibrating industrial machinery, and suppression of vibration of civil structures due to seismic input. Only during the past five years has identification of flexible space structures emerged as a serious problem. A large majority of the presently available vibration test equipment and measured data analysis methods implicitly rely upon tests in a ground vibration laboratory, or ground-based in situ tests. For example, the family of structural identification methods based upon harmonic excitation must be viewed with caution since it may be difficult to create precise harmonic excitation on orbit. Furthermore, the combination of many very low frequency modes and the absence of environmental damping means that the time required to achieve steady state (even if one can generate the required harmonic excitation) may defeat some of the steady-state response methods. Thus the majority of standard ground vibration testing and identification methods will require special modifications of actuators, sensors, and data analysis methodology.

The present study was motivated by the committee's early assessment that existing methodology is not adequate to solve many of the identification problems implicit in the next generation of orbital missions. Thus, this report is intended to evaluate the existing methods and the direction of recent research in the light of the difficulties mentioned above. The underlying objective is to make the engineering community aware of available methods, and motivate research to extend these methods to better address the special circumstances of large flexible space structures.

1.2. Direction and Scope of the Report.

The concept underlying "system identification" may generally be unfamiliar to civil structural engineers. Parametric system identification involves the determination of certain parameters associated with the system that characterize, either in the physical or modal coordinates, its mass, stiffness and damping properties derived from its output response to a carefully designed input. Nonparametric identification, on the other hand, seeks to identify a broad and sometimes empirical input-output map for the structure. The identification problem is thus the inverse of the analysis problem for dynamical systems with which the structural engineers are quite familiar. However, whereas the analysis problems are most often linear, the identification problems are intrinsically nonlinear.

System identification is being increasingly used by engineers in characterizing structures in earthquake environments, underwater objects, and more recently large flexible space structures. A great deal of methodology for system identification, however, has been developed by control engineers who use a terminology far different from the one used by structural dynamicists, and hence a significant "communication gap" exists between the two groups. The Glossary gives a list of some of the most commonly used terms and their usage by the two groups in conveying the same meaning.

The applications of system identification are numerous and so are the methods. No single method is good for every identification problem. The choice of the method is determined by several factors such as the size and shape of the full-scale object or its model, the type of the input excitation, the frequency and quality of the output signals, just to name a few. Because of the breadth and depth of the subject matter of system identification it is deemed necessary in this report to focus on those methods that are more appropriate for the identification of large space structures and, in the same spirit, attempt to elaborate on the state-of-the- practice rather than the state-of-the-art in these methods. This report thus represents an assessment of these methods and, even though the style is applications-oriented, in view of the scope and current evolution of methodology the discussion is necessarily constrained to selectively treat details and to rely heavily upon references to the published literature for standard material. In this regard an extensive body of references has been included and an attempt has been made to collect the most important papers and books available which bear on this subject. The literature review emphasizes the "modern" literature (e.g. the past twenty years, with special emphasis on the past five or ten years), and primarily the literature available in English. Selective use is made of simple examples to illustrate various points regarding the essential features, strengths and weaknesses of a few approaches. The task of a thorough evaluation of all the methods with a view to determining their merits and demerits is not possible at this time. The practical experience base necessary to perform such an evaluation is not yet available. Such an evaluation study on the class of structures of interest in this report would require a great deal of thought in evolving an appropriate basis for comparison and an even bigger effort to implement the actual evaluation process followed by a final assessment.

While this report represents the committee's joint effort, it is no doubt incomplete (in view of the large volume of ongoing research, and in view of the finite spheres of competence). However, the merit of this report will likely be measured by the extent to which

it helps researchers focus upon the rather diffuse literature and develop the methodology needed for the challenging missions before the structural dynamics, identification and control community.

Section 2 provides the necessary theoretical background on the choice of the models for large space structures, the different identification methods and related numerical procedures

Section 3 discusses issues involved in structural modeling which is a key part of the structural identification process for large space structures. Among the issues discussed are reduced-order plant models for control systems, minimizing the number of parameters to be estimated and maximizing the correspondence between parameters and physical measurements, substructuring to facilitate verification of large models by partitioning, the treatment of nonlinearities, the quantification of nonlinearities and finally experimental considerations as they impact system identification.

Section 4 deals with nonparametric models that perform identification using input-output relations and touches upon techniques used for linear and nonlinear systems in the time and frequency domains.

Sections 5 and 6 deal with parametric models. Section 5 provides a rather extensive discourse on the identification of modal characteristics of flexible structures in the time and frequency domains. Section 6 discusses methods for identification of structural modal parameters and uses several simple examples to bring out the strengths and weaknesses of the methods.

Section 7 deals with some of the problems associated with parameter estimation or system identification of on-orbit space structures. Distinction between continuous and

discrete time algorithms, optimum input test signals, optimum sensor and actuator locations, optimum sampling rates and uncertainty in system modeling and system inputs are emphasized.

Section 8 addresses hardware requirements of identification and the division of the hardware-software directed aspects for realizing the goal of structural identification and control.

Section 9 concludes the report by making recommendations, identifying current issues and needs and also future areas of research that must be pursued in order to make the goal of identification and control of highly flexible space structures realizable.

1.3. The Identification Process.

1.3.1. General Description.

System identification is the process of using the observed input to a system and its observed response (or output) to derive an analytical model of the system which can be used to predict its response to future inputs. System identification is referred to as the inverse problem in system analysis. Instead of using a model of the system to predict how the system will respond to given inputs, the response of the system to known inputs is observed, and used to deduce a model of the system. In practice, system identification is statistical estimation, although not always recognized as such. System properties derived from samples of noisy data vary depending on the particular data sample used. The predictive accuracy of the analytical model is of ultimate concern. The identification process is critical in achieving predictive accuracy, and more importantly, being able to quantify predictive accuracy before the model is placed in service. Figure 1. illustrates the identification process. Part (a) of figure shows the basic schematic diagram, where input and output measurements from an unknown system are processed in an identification operation

to yield information about the nature of the system. Part (b) shows a corresponding logical flow diagram beginning with the physical system, on which experiments are performed to produce input-output measurements, which in turn are used to identify a model of the system.

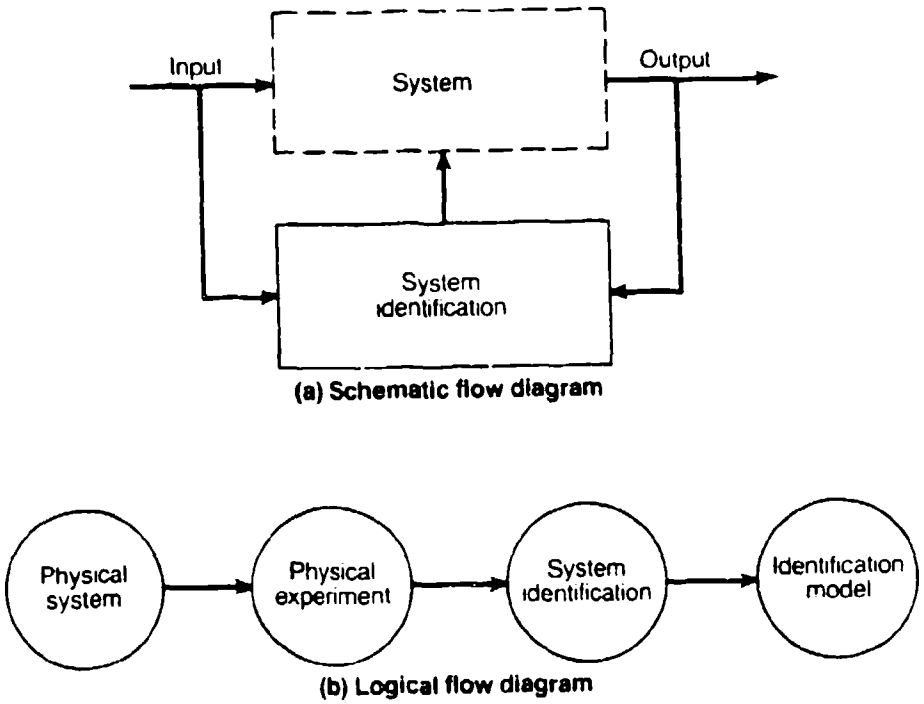


Figure 1. Simplified Illustration of the Identification Process.

Both of these illustrations oversimplify the identification process to the point of obscuring concepts and relationships which are essential for practical implementation. Figure 2 is more revealing. This logical flow diagram is drawn in three-dimensional perspective to allow the interrelationships between logical elements to be shown more clearly. Each element is connected to each of the other elements by two-headed arrows indicating a two-way flow of information. In this illustration, the entire diagram represents the identification

process, with estimation algorithms shown separately as tools to be used in accordance with model verification procedures. In fact, the physical system (or test article) along with the experiments and model may also be considered as tools to be used in accordance with model verification procedures. These procedures are designed to achieve modeling objectives, just as the physical system is designed to achieve mission objectives.

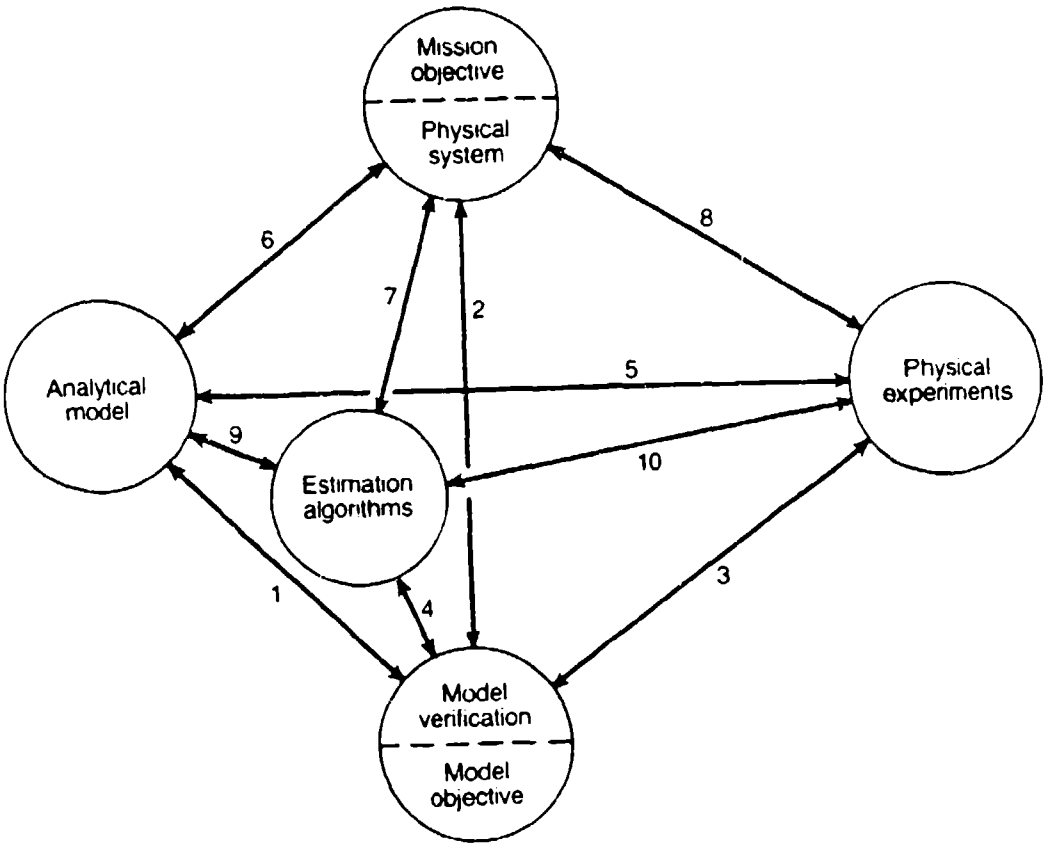


Figure 2. Logical Flow Diagram Illustrating Identification Process for Large Space Structures.

The identification process is better understood by considering each of the paths between major elements of the process. For purposes of discussion, the paths have been numbered in Figure 2. Path 1 represents the flow of information between the analytical

modeling effort and model verification procedures embodied in a model verification plan. Clearly, the plan must be tailored to the specific nature of the model.

The basic structural model is likely to be a finite element model consisting of thousands of finite elements and perhaps tens of thousands of degrees of freedom. The structural model used for control purposes must be greatly reduced. The type of reduction will determine the parameters which must be estimated. This is an input to the formulation of model verification procedures. On the other hand, limitations associated with model verification procedures may limit the type of parameters which can be estimated. A model is only verifiable to the extent that its parameters can be estimated from experimental data. When a model is not verifiable, an alternative model must be sought. This is an input to the modeling effort.

Path 2 represents the flow of information between the physical system and the model verification plan. It is the physical system which must be controlled, and for which structural integrity must be maintained. The location of critical components such as antennae, mirrors, thrusters, etc. will determine those parts of the structure which must be directly controlled. Points of force application such as thrusters, reaction wheels, docking ports, etc., in conjunction with the structural configuration will determine locations of critical stress. These areas will be of particular concern for model verification and constitute important inputs to the verification plan. On the other hand, limitations associated with model verification are likely to influence structural design. For example, it will be desirable to design joints in such a way that nonlinearities are avoided, insofar as possible, so that maximum use can be made of linear modeling and analysis techniques.

Path 3 represents the flow of information between the selection of physical experiments and the model verification plan. Data requirements are dictated by the model parameters and input-output relationships to be estimated. These data requirements affect the design

of experiments incorporated in the test plan. Limitations on testing will obviously affect model verification procedures. For example, structures which are too large and (or) too weak to be tested in a ground vibration test laboratory might have to be tested piecewise (substructuring) and (or) simulated by scale models for purposes of testing. Such factors have a major effect on model verification procedures.

Path 4 represents the interrelationship between model verification procedures and the selection of estimation algorithms. A most important consideration in the selection of estimation algorithms is the ability to evaluate in some sense the quality of the estimates. From a statistical point of view, it is desirable to quantify the statistical significance of the estimates. One will want to choose from alternative estimators those which yield the highest degree of significance, or greatest confidence; intuitively, they will afford the highest degree of predictability. The choice of estimators will in turn affect model verification procedures. For example, data requirements will be different for recursive and non- recursive estimators. In general, test procedures as well as model verification procedures must be commensurate with the data requirements of the estimators.

Path 5 may be thought of in terms of compatibility between the analytical model and experimental procedures. For example, it is often desirable to define the coordinates of a reduced analytical model so that they directly correspond to measurement coordinates. The corresponding analytical mass matrix can then be used to normalize both analytical and experimental modes, and cross-orthogonality computations are straightforward. Such direct correspondence will no doubt be unachievable for on-orbit identification, because of limitations on the number of measurement channels and data processing capacity. Nevertheless, the degree of correspondence must be optimized.

Path 6 relating the analytical model to the physical system involves some of the same considerations as Path 2 between the physical system and model verification procedures.

However, because analytical modeling methods are much further developed than methods for the verification of those models, Path 6 will be less restrictive in terms of the limitations the analytical model and physical system place on each other. Nevertheless, certain modeling objectives (such as substructuring) will imply corresponding design objectives (such as simple interfaces between major structural subassemblies), and certain system requirements (such as high damping) may lead to the use of special materials (such as composites which have anisotropic material properties).

Path 7 is not quite as obvious as some of the other relationships, at least to the extent that estimation algorithms influence the physical system. It is not difficult to think of examples where the system affects the selection of estimation algorithms. Fluid systems such as pumps and heat exchangers, for example, might lend themselves more to nonparametric estimation which identifies input-output relations as opposed to parametric estimation which identifies specific model parameters. Nonparametric estimation might also be more suitable for certain types of joints. Conversely, it is not altogether implausible that the eventual inability to adequately model (i.e. satisfactorily verify candidate models) certain physical phenomena such as fluid turbulence or Coulomb friction, either analytically or empirically, might lead to alternative designs.

Path 8 is rather obvious. The physical system and the service environment for which it is designed clearly dictate the test hardware and the nature of experiments to be performed on that hardware. Some aspects of the space environment are difficult to simulate experimentally, e.g. the thermal environment. This experimental limitation has helped to motivate interest in composite materials which have a net zero coefficient of thermal expansion, so that the effects of thermal deformation are minimized.

The interplay between estimation algorithms and analytical models (Path 9) on the one hand, and physical experiments (Path 10) on the other is also fairly obvious. As a

general rule, the simpler the model, the simpler the estimator, and vice versa. Complex models will require more sophisticated estimators able to discriminate among the more subtle effects of parameter variations and (or) input-output variations. The computational efficiency of estimation algorithms will have a major influence on experiments conducted in space, and on the amount of data which can be recorded and processed. Models which lend themselves to recursive estimation will probably be better suited to on-orbit estimation than those which require powerful nonlinear search algorithms, for example.

1.3.2. Model Development.

Because of their size and complexity, and the difficulty of simulating the environment in which they must function, large space structures will require a tremendous amount of information to be identified. This information, including both analytical and experimental data, will of necessity be accumulated over a period of time, perhaps years. The most efficient and useful means of storing the information is in the form of verified analytical models.

The foregoing section discussed the logical process of creating such a model (i.e. identifying a physical system). This section addresses the problem from a programmatical point of view. When viewed in this way, the process of model development takes on an evolutionary character and provides the program setting for identification. The model development process may be divided into three phases: (a) Initial Definition, (b) Experimental Verification and (c) Final Certification. Initial definition includes the statement of modeling objectives, the formulation of a strategy for achieving them and the construction of a basic analytical model. These tasks are a prerequisite for identification. Certification is included as part of the model development process for large space structures because of the need for quality control on the complex models which will be "flown" with the structure as part of its control system. Initial definition and final certification essentially

constitute the front end and back end, respectively, of the identification process founded upon experimental verification. Each of the three phases are discussed in the subsections which follow.

1.3.2.1. Initial Definition.

Although a model may be expected to undergo many changes during the course of its development, initial definition will prove to be important. Foresight used in creating the model will not only reduce the number of changes ultimately required, but can make the inevitable changes easier to implement. It may not be possible to identify all modeling objectives initially. However, a deliberate effort to define these objectives at the outset, followed by the formulation of a strategy for achieving them, will provide the foundation for the subsequent development of verification procedures. As previously discussed, the definition of modeling objectives should not only address the nature of the physical system and how the model is to be used, but also how the model will be verified, including consideration of experimental techniques, and the estimation algorithms available to process various forms of experimental data.

Structural models used for control purposes must be greatly reduced from the original finite element models, which are likely to contain tens of thousands of degrees of freedom (with corresponding equations of motion). The models used for purposes of identification must be similarly reduced. Not only does the magnitude of the identification effort grow exponentially as a function of the number of degrees of freedom and number of parameters to be estimated, but numerical procedures may break down because of round- off error and ill-conditioning. The chance of identifying a system incorrectly also increases with the size of a model; the larger the number of parameters and (or) input-output relationships to be estimated, the larger the number of measurements required.

Thinking of a system as a mapping from a space of input functions to a space of output functions, one can classify system models as either *parametric* or *non-parametric*. The use of these two types of models is generally governed by the extent of *a priori* information that is available about the physical system. In parametric models one starts with an assumed mathematical description of the system. The identification task then reduces to the determination of the values of one or more of the unknown parameters that are involved in the model's description. A search is conducted in a *finite dimensional parameter space*. If for instance, the system is described by a set of ordinary differential equations, then the coordinates of the parameter space may be taken to be the values of the coefficients and the values of the initial conditions. If the identification is carried out using a test input signal, then the unknown parameters, if any, of that signal could of course increase the dimensionality of that space. Nonparametric identification deals with creating an adequate input-output description of a system by a search in *function space*. Examples of nonparametric models are impulse responses, frequency response functions, covariance functions, Volterra series and Wiener series. Typically, the amount of *a priori* information needed for parametric models is greater than that for nonparametric models. This is offset by the, generally speaking, more difficult task of searching in function space (in principle, an infinite number of parameters) for non-parametric models, as opposed to searching in parameter space for parametric models. Whereas large errors can result if the order of a parametric model is incorrect, nonparametric models have the advantage that explicit specification of the order of the system is not necessary.

1.3.2.2. Experimental Verification.

Experimental verification involves four steps: (1) validating the basic structure of a model, i.e., the equations of motion in symbolic form, (2) estimating the parameters' values or input-output relationships of that model with an acceptable degree of confidence, (3) ensuring that the model is in satisfactory agreement with experimental data, and (4)

evaluating the limitations of the model based on initial modeling assumptions and the degree of success achieved in the first three steps. These steps are typically repeated until the desired results are achieved, or until no further improvement can be made to the model.

While non-parametric models may be employed in isolated cases, e.g. to model component interfaces, it is anticipated that parametric models derived from finite element analysis will comprise the major portion of models used for structural analysis and control of large space structures. Statistical estimation is then relegated to the estimation of parameter values, assuming that the basic structure of the model is correct. Validation of this assumption is generally not an easy task. Nor is it always straightforward. In practice, parameter estimation consisting of steps (2) and (3) is carried out, and the results examined for consistency with a priori information about the model. If the results are inconsistent, either because the parameter estimates are unreasonable, or because the response of the resulting model is in disagreement with some of the measured response, the validity of the assumed model is in doubt, i.e. some aspect of the model may be in error. The identification of modeling errors can be difficult. At the present time, there are no direct means for identifying modeling errors; the analyst must rely upon his or her understanding of the system, the model, and the experimental data in an intuitive process of trial and error. Some estimation algorithms provide more information for use in this process than others.

The estimation of parameters or input-output relationships is conducted by experimentation on the system. One subjects the system to a set of test inputs applied at certain locations, and measures its response at the same or different locations. In general, it is not possible to identify a universally valid model from specific sets of input-output data, for the model may be left deficient in aspects which depend on some information that may be lacking from the specific test inputs used. One therefore aims, more modestly, at deciphering a model from among a class of models, $\underline{M}$, using a class of inputs, $\underline{I}$, applied

at a set of locations, $\underline{L}_i$, and measuring the responses from among a class of outputs, $\underline{Q}$, at a set of locations, $\underline{L}_o$, so that a suitable error criterion, $\underline{R}$, is minimized.

An important aspect of the experimental procedures is the characterization of the noise in measurements of the inputs and the outputs. This knowledge plays an important part not only in the choice of the algorithms used subsequently for the search procedures but also in the choice of the error criterion, $\underline{R}$, which the search procedure attempts to minimize. For measurement errors that affect mainly the measured output of the system, the error criterion is generally related to the difference between the measured response and the model response; for cases where input measurement errors predominate, the criterion often relates to the difference between the measured input and the input that corresponds to the output obtained from the model. The criteria chosen and the a priori knowledge about the measurement noise affect the statistical properties of the parameter estimates obtained. Thus the measurement process, by affecting the error criterion, has a direct bearing on the degree to which a model is verified. By affecting one's confidence (statistically speaking) in the results obtained, it also affects the extent to which the model obtained may be deemed correct, i.e."valid."

Statistical estimation is accompanied by measures of statistical confidence attached to the estimates. In the case of parametric estimation, this information is expressed in the form of a parameter covariance matrix. The diagonal elements of this matrix correspond to the variances of individual parameter estimates, while the off-diagonal terms define the correlation of the estimates. A model may not be completely verified if some of the parameter estimates are highly correlated, even though the corresponding variances may be small. In general, uncorrelated estimates with small coefficients of variation are sought.

In addition to obtaining reasonable estimates of parameter values and (or) input-output relations with a high degree of confidence, it is necessary that the response predicted

by the model be in satisfactory agreement with measured response data. With both conditions satisfied, the model may be considered to be verified. If some of these conditions are not met, the model may be considered partially verified. A partially verified model can be of practical use provided that the unverified parts of the model are identified, and that they do not play an important role in achieving primary modeling objectives.

The predictive accuracy of a model cannot be assessed until the model has been verified. Additional information is, however, required to evaluate predictive accuracy. This additional information includes a definition of model uncertainty as well as a definition of measurement uncertainty (noise) associated with input and output measurements. At the present time such information does not exist, although it could conceivably be derived from past experience. As a practical matter it may be difficult to differentiate between modeling error and measurement error, in which case they would have to be treated together.

1.3.2.3. Final Certification.

Final certification is looked upon as a formal procedure for qualifying a model. It should be conducted independently of the model verification effort to ensure that modeling objectives have been satisfied, that requirements for model "fidelity", the quality of being true to the physical model, and "robustness", the quality of being unperturbable or stable in the sense that it doesn't change over a broad range of input- output conditions, have been satisfied, and that requirements for predictive accuracy have been satisfied. It will probably include independent analysis based on the verified model, as well as a thorough review of the model verification report. Without adequate model verification, certification will not be possible. Verification must provide the answers to the questions posed in certification.

1.4. Classification of Identification Methods.

Numerous surveys and reviews have been written on the subject of structural system identification. The dominant themes have been (a) to organize the rather extensive body of literature within some logical framework, (b) to present case studies or examples which typify particular "methods," and (c) to compile a bibliography. The term "methods" refers sometimes to generic categories, and sometimes to the methods of individual authors. Case studies are problem dependent and vary from method to method, offering little opportunity for the comparison of methods, either generic or individual. Bibliographies tend to reflect either explicitly or implicitly the perceptions which different writers have of system identification.

The intent of this section is not to repeat the work which other reviewers have already done, but rather to build upon that work by helping to interpret some of the perceptions and suggesting a more general framework of classification within which others may be placed. Whereas other reviewers have tended to use hierarchical classification systems presented in terms of technology trees, the approach here is to classify the methods without prioritization. Priority systems may later be imposed to derive technology trees if desired.

The literature on structural system identification tends to fall into three distinct areas or fields:

I. Identification of Input-Output Relationships

II. Identification of Modal Characteristics

III. Identification of Model Parameters

The fields are distinguished from each other by their underlying assumptions, which become progressively more restrictive, and more difficult to satisfy. For example, input-output relationships need not be linear, although linearity is often a good assumption.

Even if linear, they need not lend themselves to modal representation. On the other hand, the identification of modal characteristics not only carries with it the assumption of linear behavior, but also the assumption that modal characteristics exist. The identification of model parameters is based on the assumption that an appropriate model exists, and in addition that the structure of the model, which defines the parameters, is known.

The escalating degree of complexity is evident in the chronological development of each field. The identification of input-output relationships matured during the 1970's with the availability of digital data acquisition and processing equipment; the identification of modal characteristics now appears to be maturing as more powerful micro- processors become available, methods and software are subjected to broader application, and commercial benefits are realized. It is difficult to foresee when the identification of model parameters will mature. Perhaps the motivation provided by the desire to build and control large space structures will prove to be a significant factor in this process.

In addition to the order of complexity among the three fields, an order of dependency is also apparent. Structural model parameters can be estimated either from modal characteristics, input-output relationships, or from time- history data. Modal characteristics can be estimated either from input- output relationships or time-history data, and input-output relationships, as the name suggests, are estimated directly from measured input and output time-histories.

Before proceeding, it is worthwhile to consider some of the reviews which have previously been published. The more recent ones include reviews by Allemang and Brown (1986) with 79 references [1], Hsieh, Kot and Srinivasan (1983) with 63 references [2], Martinez (1981) with 86 references [3], Ibanez (1979) with 132 references [4], and Hart and Yao (1977) with 68 references [5].

The survey paper by Allemang and Brown [1] offers a brief overview of methods used in "Experimental Modal Analysis," (Identification of Modal Characteristics). It is the only one of the foregoing review papers which addresses this field in particular, but represents the many papers currently being generated within the "modal testing community." This community supports the annual International Modal Analysis Conference (IMAC) and a new journal of the Society of Experimental Mechanics called "The International Journal of Analytical and Experimental Modal Analysis."

Hsieh, Kot and Srinivasan [2] present a topical discussion of system identification directed toward nuclear power plant applications. They make no attempt to classify methods or present case studies. They do discuss some of the important issues involved in system identification, such as the identifiability of a system, the significance of *a priori* knowledge about the system, and the reliability and accuracy of system identification techniques. The authors point out that few reported efforts to verify system-identification methods were found, and those used artificially generated data. They conclude with a statement emphasizing the importance of understanding the reliability and limitations of system identification techniques for real nuclear power plant structures before any attempt is made to incorporate these techniques in the design process.

A literature survey by Martinez is included as one chapter in his doctoral dissertation [3]. Here the emphasis is on aerospace applications rather than large civil structures. Martinez' dissertation develops the Extended Kalman Filter Equations in the frequency domain for purposes of parameter estimation. The state space formulation and stochastic process techniques are used to obtain (theoretically) reliable statistical parameter estimates. Martinez' literature survey and his subsequent development of the Kalman Filter approach for parameter estimation in both the time domain and the frequency domain offer one of the more thorough reviews of this approach to system identification found in the current literature. Apropos of on-orbit identification, Martinez points out that for

reduced order models (representative of those which might be used as plant models with control systems of large space structures) the combination of state and parameter estimation, resulting in an optimal filtering solution, is an attractive alternative in that it uses data in the time domain where it is actually measured and recorded. The advantage of this approach is that computations can be made recursively in real time.

Ibanez' review paper [4] is significant in that it offers a reasonably thorough review of the literature in model parameter identification up through 1978. In reviewing the "bewildering number of methods" which have been investigated "to acquire and use experimental data to improve models," Ibanez sets the tone of his review by citing this quotation by Lord Rayleigh (1884):

> *"By a fiction as remarkable as any to be found in law, what has once been published, even though it be in the Russian language, is spoken of as known, and it is too often forgotten that the rediscovery in the library may be a more difficult and uncertain process than the first discovery in the laboratory."*

Ibanez includes thirteen example problems in his review, and summarizes a number of earlier reviews including those by Hart and Yao [5], Berman [6], Collins, Young and Kiefling [7], Gersch [8], Pilkey and Cohen [9], Rodeman and Yao [10], Schiff [11], Young and On [12] and Natke [13]. He presents the technology tree originally proposed by Collins, Young and Kiefling, and extended by Hart and Yao, whose primary branches divide model parameter identification into Time-domain and Frequency-domain methods.

Berman [6], on the other hand, chooses to divide the field into three main branches: (1) "Direct Verification" where a comparison is made between the response of a prior model and that of the actual structure, followed by changes to the prior model until the theoretical and measured response agree to within a specified error criterion; (2) "Direct Modification" (in which category he places his own approach) whereby test data are needed

to make direct modifications to the model without first comparing the response of the two; and (3) "Direct Identification" which yields a model directly without a prior model.

Gersh [8] suggests a scheme for classifying model parameter identification based on dividing the field into three main branches consisting of: (1) Equation Error Methods, (2) Output Error Methods and (3) Maximum Likelihood Methods. He points out that Maximum Likelihood Methods yield a measure of statistical reliability of the parameter estimates.

Schiff [11] suggests yet other systems for classifying identification methods including classification by method of excitation and classification by method of data analysis. The latter is similar to the Time-domain and Frequency-domain technology trees proposed by Collins, Young, Kiefling, Hart and Yao.

It is clear that there are many ways to view this body of knowledge. Different ways accomplish different objectives (and tend to reflect the perception of different writers). What is most important, however, is that all facets of system identification be recognized for the sake of completeness; they can be prioritized later as previously suggested.

Having taken the first step in the classification of identification methods by dividing them into identification of input-output relationships, modal characteristics, and model parameters, one may proceed to further classify the identification of model parameters. Figure 2 provides some insight for doing so. The identification process therein is seen to embrace three elements consisting of an analytical model, physical experiments and estimation algorithms. Methods can therefore be classified according to:

A. The model and its corresponding parameters,

B. The experimental data used in parameter estimation, and

C. The estimation algorithms used to estimate parameter values from experimental data.

Table 1 offers a breakdown within each of these categories. The column on the right hand side of the table suggests a code which may be used in the classification process.

Table 1. Identification of Structural Model Parameters: Classification of Methods.

CATEGORY/SUBCATEGORY	CODE
A. *Model/Parameters*	
• Lumped Parameter Model	LPM
- lumped mass	-M
- damping constraint	-D
- spring stiffness	-K
• Finite Element Model	FEM
- linked FEM parameters (γ, E, I, h, D, etc.)	-L
- submatrix scaling coefficients (mass, stiffness)	-S
- distributed parameters (modal, other)	-D
• Physical Matrix Model (FEM origin not available)	PMM
- matrix elements	-E
- submatrix scaling coefficients	-S
- distributed parameters	-D
• Modal Matrix Model	MMM
- component (substructure) modal matrix elements	-C
- system modal matrix elements	-S
• Equivalent Continuum Model	ECM
- equivalent continuum parameters (EI, GA, etc.)	-C
- discretized equivalent continuum parameters	-D

Most of Table 1 is self explanatory. Under Part A, it is seen that different types of parameters are defined for different types of models, so that it is necessary to specify the type of model and type(s) of parameter(s) which apply. Part B could be coded similarly, although it may not be necessary to distinguish among the types of sampled time-history data or averaged time-history data being used. If so, appropriate codes could easily be added.

Table 1. Identification of Structural Model Parameters: Classification of Methods (continued).

CATEGORY/SUBCATEGORY	CODE
B. *Measurement Data*	
• Sampled Time-History Data	STH
- acceleration, $\ddot{x}(t)$	
- velocity, $\dot{x}(t)$	
- displacement, $x(t)$	
- input, $u(t)$	
• Averaged Time-History Data	ATH
- $\frac{1}{T}\int_0^T \ddot{x}^2(t)\,dt$	
- etc.	
• Characteristic Functions	
- impulse response functions, $h(t)$	IRF
- frequency response functions, $H(f)$	FRF
• Modal Properties	
- real modal properties	RMP
- complex modal properties	CMP
C. *Estimation Algorithm*	
• Direct vs. Iterative	D or I
• Deterministic vs. Statistical	D or S
• Batch vs. Recursive Data Processing	B or R
• Pseudo Inverse vs. Optimization Solution	I or O

The classification of estimation algorithms is treated a little differently. Estimation algorithms are characterized by four properties, each consisting of a pair of alternative descriptors. For example, the first pair consists of "direct" vs. "iterative." Direct algorithms lead to a solution in one step and are used primarily in linear estimation, i.e. when the measurement data are linearly related to the parameters being estimated. Iterative methods are used primarily in nonlinear estimation.

The second pair of descriptors has more to do with the interpretation of the estimator, than its mathematical form. For example, a least- squares method can be interpreted either

deterministically or statistically. The statistical interpretation is significant in evaluating the reliability of the estimates.

Batch data vs. recursive data processing is an important consideration in determining storage requirements and processing time for the estimator. Recursive algorithms offer advantages in ground-based as well as space- based operations.

The pseudo inverse vs. optimization methods differ primarily in their ability to handle constraints. Optimization methods permit greater latitude in handling constraints, particularly inequality constraints which are routinely treated in nonlinear optimization.

It is a fairly simple matter to classify a given method, whether generic or particular, by going through Table 1 and simply checking off the appropriate categories. For example, Martinez' Iterated Extended Kalman Filter method applied in the frequency domain to a finite element model with parameter linking would be classified as (FEM-L/ FRF / I,S,R,I). The model- parameter, data, and estimation algorithm descriptors are separated by slashes; the characteristic properties of the estimation algorithms themselves are separated by commas.

In addition to serving as a guide for the classification of existing model parameter identification methods, Table 1 may be used in the formulation of new methods to the extent that it suggests different combinations of model-parameters, measurement data and estimation algorithms than those presently in use. The selection of alternatives within each of the three categories can be guided by the particular requirements of a given application.

SECTION 2. SYSTEM MODELING: THEORETICAL BACKGROUND.

System identification hinges around the notion of models [14]. Models for large space structures may be thought of in terms of the task that the structure is to perform. Some of the uses are:

- Models for simulation purposes, where knowledge of the system is the intended use.
  - such models may be used for diagnostic purposes (e.g. early fault detection in large space structural systems) and may be helpful in the design of parts of the structure where knowledge of some subsystem is needed.

- Models for prediction purposes, where the aim might be to control the system.
  - attitude control, pointing accuracy, shape control of antennas, etc.

The model selection may be thought of in terms of:

a) The delineation of a set of models, **S**, from among all possible models (e.g. linear/nonlinear, static/dynamic, discrete/continuous, parametric/non-parametric).

b) The choice of a class of models from this set (which can often be transformed into another) and the structural parameterization of these models (e.g. models based on frequency domain characteristics like mode shapes and eigenvalues as opposed to models that use mechanical properties like EI, EA, etc.). The order of the model, if a parametric class of models is chosen, needs to be specified at this level.

c) The complete specification of the model.

2.1. Choice of Models.

The choice of the type of model to be used is controlled largely by the intended use of the model, and the *a priori* information available about the physical system. The model arrived at in c) of course need not be the final model that is chosen for the system, or

the one that may be used, e.g. for control purposes. Some considerations germane to the model selection process are:

- Where are the dominant noise sources in the experimental testing? Can they be distinguished explicitly in the chosen model?
- What kinds of parameterization are needed? Are any of these parameters being measured directly? Can any available *a priori* information be implemented in the model? In certain models, this information may be directly implemented, while in others, this information would only lead to implicit relations. Such implicit relations generally impose a heavy burden on the computing aspects of the identification process. For instance, models that attempt to identify parameters in the stiffness matrix of a structure may use experimental information regarding the lowest fundamental frequency of the system as an implicit relation (constraint).
- Is the complexity of the model (the order of the model and the number of parameters) commensurate with the proposed uses of the model? Can such a complex model be handled within the general experimental and computing environment in which the identification process is required to be performed? Can a maximum bound for the model complexity be assessed, given a certain environment in which the identification process is to occur? This would delimit the set of models from which the most "suitable" one would be sought.
- For the model complexity chosen, is the experimental design sufficient to yield a unique set of system parameters?

The last issues are of special importance to large space structural systems. The problem of finding the order of a system model which is intended for a certain use is indeed a nontrivial problem, especially for large spatially extended structures. The extent of parameterization and the order of the system are often related to each other in the identification of a large structural system. For instance, in the finite-element approach the

number of elements controls the number of parameters in the system, while the number of nodes controls the order of the system. Generally speaking, the number of elements increases with the number of nodes.

Another important aspect of the chosen model deals with the uniqueness of the parameters to be identified. Put alternately, is there more than one set of parameter values that will yield the same input-output relations? If so, then it would be impossible to distinguish the "true" model from the various models that each of these parameter sets yields [15]. Though this problem has not been sufficiently addressed in the literature to date, it is one that is of great engineering significance, for the choice of one model as opposed to its competitors, in a non-unique situation, can cause substantial errors in the determination of parameters of engineering significance, like shear forces and bending moments in structural systems [16]. In general, the degree of non-uniqueness of a model used depends on the *a priori* information about the system, the experimental testing and the error criterion used. Often, though not always, it increases with the system's complexity, [17].

Both parametric and non-parametric techniques are used in the identification of large structural systems, [17]. The non-parametric methods are basically input-output descriptions of the system in the form of generalized impulse responses or transfer functions. For multi-input and multi-output systems, transfer functions are often employed. The parametric models generally involve representation of the system for large space structures through differential equations. While continuum representations for large space structures through the use of partial differential equations are infrequent in practice, discrete spatial formulations through the use of ordinary matrix differential equations are relatively common.

By and large, the models used for large space structures are chosen, in practice, from the set S which can be described by the following adjectives: linear dynamic, time-

invariant, causal, parametric, and either stochastic or deterministic. Therefore, this report will concentrate, though not entirely, on these types of models. Unless otherwise mentioned or contextually obvious, henceforth the set S shall denote this restricted set of models.

Input-Output Description: Consider a finite memory system whose output, $y(t_0)$, at time t_0, depends on its input $u(t)$, for $t < t_0$. Let u_i, for $i = 1, 2, \cdots, N$ be samples taken periodically at times $t_0 - i\Delta t$. The response $y(t_0)$ can be represented approximately by the relation $\hat{y}(t_0) = f(u_1, u_2, \cdots)$. The multidimensional Taylor series expansion then yields

$$\hat{y}(t_0) = \sum_{1}^{N} a_i u_i + \sum_{1}^{N}\sum_{1}^{N} a_{ij} u_i u_j + \sum_{1}^{N}\sum_{1}^{N}\sum_{1}^{N} a_{ijk} u_i u_j u_k + \cdots. \tag{1}$$

This polynomial, called the Kolmogorov-Gabor (KG) polynomial [18], assumes that the system's initial conditions are zero. If a_{ij}, a_{ijk}, etc. are all zero, then the system will, of course, be linear. Denoting $a_i = h_i \Delta t$, and taking limits as N goes to infinity, we get for such a system,

$$y(t) = \int_0^t h_1(\tau) u(t-\tau) d\tau. \tag{2}$$

Similarly denoting $a_{ij} = h_{ij} \Delta t \Delta t$, and letting N go to infinity, it is easy to show that the second term on the right in the KG expansion yields in the limit,

$$\int_0^t \int_0^t h_2(\tau_1, \tau_2) u(t-\tau_1) u(t-\tau_2) d\tau_1 d\tau_2. \tag{3}$$

The KG series can thus, in the limit, be expressed as an integral series which is called the Volterra series. This is the general input- output relation of a nonlinear system whose input is $u(t)$ and whose output is $y(t)$.

$$y(t) = \int_0^t h_1(\tau)u(t-\tau)d\tau + \int_0^t \int_0^t h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 + \int_0^t \int_0^t \int_0^t h_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \cdots . \tag{4}$$

The term $h_1(t)$ can be recognized as the impulse response of a linear system; the kernels of the other integrals, namely $h_2(t_1,t_2)$, $h_3(t_1,t_2,t_3),\cdots$,etc., are called the generalized impulse responses and provide the nonlinear contributions to the complete response of the system. For instance, the kernel $h_2(t_1,t_2)$ provides the "cross-talk effect" between the two *delta-function* inputs at times t_1 and t_2. Notice that the expansion in many ways is analogous to a polynomial expansion for each term is analogous to the number of integrals involved in each term. For physically realizable systems, the kernels have the following properties:

$$h_i(t_1,t_2,\cdots,t_i) = 0 \qquad \text{for } t_j < 0 \qquad j = 1,2,\cdots,i; \tag{5a}$$

$$h_i(t_1,t_2,\cdots,t_i) \to 0 \qquad \text{for } t_j \to \infty \qquad j = 1,2,\cdots,i; \tag{5b}$$

$$h_i \qquad \text{is symmetrizable.} \tag{5c}$$

Thus the Volterra series gives a general relationship between the input and the output of a nonlinear system. For a system modeled by such a Volterra Series, identification of the system would simply mean the determination of the kernels, h_i. However, the determination of these kernels is often a formidable task, in practice. The identification scheme being nonparametric, it is to some extent nonphysical. In most applications the series is terminated after a few terms, [19].

A more computationally tractable approach for nonlinear systems was developed by Wiener by using Gaussian white noise (GWN) for the test signal, [20]. The input-output relation can then be expressed as

$$y(t) = \sum_{1}^{\infty} G_i\{g_i, u(t)\} \tag{6}$$

where the g_i are functions of i variables and are analogous to the impulse kernels for the Volterra series, and the functions G_i have a convenient orthogonality property for white noise signals $u(t)$, i.e.,

$$E[G_i G_j] = 0 \qquad \text{for} \qquad i < j. \tag{7}$$

The first two terms of this series look like

$$G_1\{g_1, u(t)\} = \int_{-\infty}^{\infty} g_1(\tau_1) u(t - \tau_1) d\tau_1, \tag{8a}$$

$$\begin{aligned} G_2\{g_2, u(t)\} = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_2(\tau_1, \tau_2) u(t - \tau_1) u(t - \tau_2) d\tau_1 d\tau_2 \\ &- P\int_{-\infty}^{\infty} g_2(\tau_1, \tau_1) d\tau_1 \end{aligned} \tag{8b}$$

where P is the power level in the GWN. The kernels can now be determined, relatively easily, by the correlational techniques. While this method has been used in the identification of some biological systems, very few applications to large structural systems have been undertaken to date. Reference [20] is probably the only one that has used the method for successfully interpreting the nonlinear response of a large structural system. A study done on the nonlinear identification of soil using a similar technique also indicates that the method may have promise in the mapping of inputs applied at specific locations, when in the nonlinear regime. Such methods may be of particular use in the determination of nonlinear structural responses during docking maneuvers for space vehicles, and indeed

warrant further research. Perhaps the most important restriction of the series representation considered above is that the model is applicable when the input-output relations are single valued.

The digital determination of a function such as $h_2(x_1, x_2)$ of course requires that the function be determined at a finite number of sample points. One can thus "parameterize" the generalized impulse functions obtaining estimates of their values using an appropriate sampling interval, say Δt. The choice of the number of points and the sampling interval, Δt, used to represent these functions depends on an *a priori* knowledge of the system. The Wiener approach has great computational advantage over the Volterra series representation. Yet, the computational effort required for higher order kernels may at times be high. For instance if h_1 is approximated using 10 samples (parameters), h_2 would roughly need 55 parameters (taking into account the symmetry property of the kernel) and h_3 would need about 220. The approach, it should be remembered, is applicable to systems driven by GWN, a circumstance which may be difficult to arrange in the controlled testing of large structures.

For linear systems a commonly used model is the transfer function model. For an output vector $y_k \in \mathbf{R}^p$, and an input vector $u_k \in \mathbf{R}^m$, one can express the input-output relation by

$$y_i(t) = \sum_{1}^{m} \int_0^t h_{ij}(t-\tau) u_j(\tau) d\tau. \tag{9}$$

Taking the z-transform (Fourier or Laplace transform) yields,

$$y_k(z_k) = T(z) u_k(z_k) \tag{10}$$

where

$$y_k^T = (y_{1k}, y_{2k}, \cdots, y_{pk})$$

$$u_k^T = (u_{1k}, u_{2k}, \cdots, u_{mk})$$

with $T(z)$ the transfer function for the system. Despite difficulties often encountered, e.g. the handling of noise, initial states, common (or closely spaced) poles, this form of modeling (or a variant of it) has often been used in structural systems because of the simple interpretation of the elements of the matrix T. Any element T_{ij} of the transfer function may be thought of as a transmittance between the input u_{jk} and the output y_{ik}. There are a few advantages [21] to the use of transfer matrix representation for systems that are thought to belong to the set **M**. They are:

- The transfer matrix representation is unique for a fixed ordering of inputs and outputs.
- The transfer matrix has a simple physical interpretation.
- Canonical forms that are observable and controllable can be derived from these models rather easily.

While the first two points are somewhat obvious, the third point needs a little elaboration. The transmittance matrix equation, when discretized, can also be put in terms of the input in the time domain by defining

$$y_k = \sum_0^\infty H_i u_{k-i-1} \tag{11}$$

where H_i are called the Markov parameters, [21], and are simply the weighting patterns for the various impulse responses between inputs j and responses i. These Markov matrices enable us to construct the $sp \times sm$ Hankel submatrices, H_s^*, defined as:

$$H_s^* = \begin{bmatrix} H_0 & H_1 & \cdots & H_{s-1} \\ H_1 & H_2 & \cdots & H_s \\ \vdots & \vdots & \ddots & \vdots \\ H_{s-1} & H_s & \cdots & H_{2s\ 2} \end{bmatrix}. \tag{12}$$

The advantage of such representations is that they are linked to the state-space representation of such systems in a neat manner which permits observable and controllable canonical forms to be obtained, often directly [15].

While other multi-input-multi-output models are often discussed in system identification literature, e.g. Matrix Factor Description (MFD), their use in system identification may be questionable. Such representations have not yet been used in large structural systems.

Explicit Representations: Large structural systems are often continuous in nature and their dynamics could generally be represented through the use of Newton's laws in terms of partial differential equations (PDE) where the displacement of a infinitesimal mass element of the system, y, is governed by its geometrical position vector, r, relative to an inertial reference frame, its acceleration, the restoring forces (linear or nonlinear) that the rest of the system exerts on the mass element, and the externally-applied forces. This can be expressed as:

$$m(r)\frac{d^2y(r,t)}{dt^2} + F_r(y, \frac{dy}{dt}, r, t) = F(y, \frac{dy}{dt}, r, t) \tag{13}$$

with suitable boundary and initial conditions. The function $m(r)$ represents the mass in a small volume around the coordinate location r. The restoring forces F_r, could be linear (in y, $\dot{y}$) or nonlinear, and the applied force, F, may include dependence on the response of the system, y, possibly indicative of feedback control. While in many situations such representations are helpful in understanding the dynamic behavior of structural systems, it must be remembered that continuum models, like all other models, are only descriptions

of physical systems, and therefore have inherent limits to their applicability. For instance, the validity of the statement that a continuous beam has an infinite number of natural frequencies of vibration may be questionable in view of the very large, yet finite, number of atoms it contains ! Since the PDE representations are generally difficult to handle in all except the simplest structural components, most structural systems are discretized (generally through the use of some technique based on variational methods) and described by a finite number of degrees of freedom. Such models can then be expressed by a system of ordinary differential equations (nonlinear in general) with appropriate initial conditions. In practice, the models that have been used most extensively for large structural systems are those described by linear matrix differential equations of the form:

$$M\frac{d^2x(t)}{dt^2} + D\frac{dx(t)}{dt} + Kx(t) = f(t) \tag{14}$$

with appropriate initial conditions, where M, D, and K are the mass, the damping and the stiffness matrices. The vector $f(t)$ is the forcing vector and often corresponds to the measured input signals to the system. One set of model parameters which would describe such a model would be the unknown elements of the M, K and D matrices. All such unknown parameters could then be collected into a parameter vector θ, the determination of which would be the intent of the identification methods to be used.

The creation of such models, it should be emphasized, is a process that involves a holistic appreciation of the system's behavior and of the behavior intended to be portrayed by the model. It is a process that, rather than being deductive, is primarily inductive and is intuition- and experience- based. To date, there are very few analytical guidelines for the building of models. For instance, the order (generally twice the number of degrees of freedom chosen) of the model used, may be an important determinant of how well the description(model) of the system reflects that part of the system's behavior which is deemed to be of interest. Often, the order of a linear structural model is implicitly

expressed, e.g., by the number of mode shapes of interest. The description of different aspects of a given system's behavior may, and often does, require different order models. Thus the validation of models can only be undertaken in the context of their intended use. This will be pursued further in the discussion of the validation phase of the identification process.

An important aspect of model-building which has not permeated into the large-space-structure identification area is the concept of identifiability. The ability to be able to uniquely determine the parameters of a given model depends upon the parameterization chosen, the error criterion chosen and the "structure" of the model chosen. All these aspects are affected by one's *a priori* knowledge of the system. The larger the number of parameters in a system, intuitively speaking, the better the fit between the model response and the system response that one should expect and the greater the problems with non-uniqueness. The non-uniqueness problem can be best explained by the first order (note that our linear matrix differential equation system can always be put in this form) noise-free system:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{15a}$$

$$y(t) = Cx(t) \tag{15b}$$

where the state vector $x \in R^n$, $u \in R^m$, and $y \in R^p$. Let the matrices A, B, and C be fully parameterized, i.e. there are $n^2 + nm + np$ parameters to be estimated. It is easy to show that the system described by the triplet (A, B, C) is equivalent to an infinite number of systems described by (TAT^{-1}, TB, CT), for any nonsingular matrix T, in the sense that all these systems will have the same input- output relations ! It is thus seen that the input-output measurements determine an *equivalence class of systems* described by the equations above. Since the equivalence class always consists of an infinite number of

elements, the identification procedure is non-unique. If *a priori* knowledge of the system is available, one can improve this situation, and perhaps even avert it completely, by suitably fixing certain elements of the matrices A, B, and C. This leads to the concept of canonical forms*(not to be confused with the Jordan Canonical Form one encounters in the diagonalization of matrices)*. Without going into the details [16], three important truths relating to these canonical forms are listed here:

- For all systems with the same n, m,and p it is not possible to arrive at the same canonical form. A set of canonical forms needs to be used to describe the general systems described by the difference equations above.
- The set of canonical forms is finite.
- The elements of the set are distinct. It is not possible to transform one member of a set into another member.

In a canonical form many of the elements of the matrices may be set to zeros or ones. The remaining elements are then estimated. Canonical forms can likewise be defined for linear systems where both input and output measurement noise are present. This leads to the *innovations concept* of representing systems.

2.2. Identification Methods.

The literature in the area of identification methods is extensive and it would be impossible to do justice to it in a brief report such as this. What is presented here is a tutorial treatment of the methods, so that the reader may be able to put the material that follows into proper perspective [14], [22]. No attempt will be made to cover all the methods available (e.g., methods such as quasilinearization and dynamic programming have not even been touched upon). The aim of this exposition is to help the reader unfamiliar with system identification to gain a coherent picture of the salient techniques of the methods that are currently used in practice.

Assume a set S of models

$$\mathbf{S} = \{S(\theta)|\theta \in D_S\} \tag{16},$$

where θ is the system parameter vector, together with measured data,

$$z^N = \{z(1), z(2), \cdots, z(n)\}^T \tag{17a}$$

where each $z(t)$ represents an input-output pair $y(t), u(t)$ at time t. The measurements $y(t)$ and $u(t)$ are p and m dimensional vectors, where the measurements are defined as

$$y^N = \{y(1), y(2), \cdots, y(n)\}^T \tag{17b}.$$

The identification procedure involves the selection of a proper member $S(\tilde{\theta})$ in the model set S that best 'describes' the data. Thus, the identification method reduces to determining the mapping from z^N to the set S.

$$z^N \rightarrow S(\tilde{\theta}). \tag{18}$$

2.2.1. Prediction and Equation Errors.

The various models described above are all ways of representing relationships between signals that are input to a system and signals that record the response which the system produces. They all have one common feature; they all provide a rule for computing the next output or a prediction of the output from the given past observations. Thus, given the vector z^{t-1} of observations, they provide the vector $y(t)$, given the vector θ. This can be expressed as:

$$\tilde{y}(t, \theta) = Y(\theta, t, z^{t-1}). \tag{19}$$

The prediction equation above is always a deterministic relation. For example, when using a stochastic state-space model, the prediction rule may be provided by the Kalman Filter. This time-varying filter initiated at some time t_0, is driven by the data set z^{t-1}. When solved explicitly, it yields a function like the one described in (19). All the stochastic assumptions involved in the modeling process (noise characteristics,etc.) only serve to arrive at this deterministic predictor function.

Using the predictor equation, one can determine how good the prediction was by computing the error

$$\mathcal{E}(t,\theta) = y(t) - \tilde{y}(t,\theta). \tag{20}$$

This quantity is also called the *prediction error at time t, corresponding to the model $S(\theta)$*. For a simulation model this error quantity is generally called the *output error*. The prediction error is a random variable whose probability density function depends on the value of vector θ. Generally, these errors are assumed to be independent for different t. This assumption, together with (19), is often referred to as the *probabilistic model*.

One reasonable approach to obtain the mapping (18) then would be: using z^N and (19) and (20), select the parameter vector $\tilde{\theta}$ in such a way that the prediction errors, $\mathcal{E}(t,\tilde{\theta}), \quad t = 1,2,3,\cdots,N$ are as small as possible. The question of what is meant by "small" must be based on the *a priori* information of the errors in the measurement process and *a priori* knowledge of the system parameters.

Another error criterion often used in assessing the ability of a model to describe the relationship between the measured signals and the predicted response is called the equation error. It indicates the extent to which the measurements satisfy the model equations. For the system of (14), for example, the equation error would be given by

$$\mathcal{E}_{eq}(t,\theta) = M\ddot{x} + D\dot{x} + Kx - f(u(t)) \tag{21}$$

where x and u correspond to measured quantities. It should be noted that determination of this error quantity generally does not depend upon obtaining the response of the model to the measured inputs. However, the estimates of the error may be heavily contaminated by numerical noise when numerical differentiation is performed on noise corrupted data. In general, the parameter set $\hat{\theta}$ which is estimated using these two definitions of error (output error and equation error) will be different.

The error $\mathcal{E}$ defined above is a vector quantity, in general, and it is analytically useful to introduce a scalar measure $r(t,\theta,\mathcal{E}(t,\theta))$ to evaluate the magnitude of the prediction error $\mathcal{E}(t,\theta)$. After having recorded data up to time N, a natural criterion would be

$$\mathbf{R}(\theta, z^N) = \frac{1}{N}\sum_{t=1}^{N} r(t,\theta,\mathcal{E}(t,\theta)).$$

The determination of the functional form of r then establishes the error criterion that is used.

2.2.2. Measurement Noise, *A priori* Information and Error Criterion.

Noise is defined as unwanted signal or disturbance. These unwanted signals can be thought of as resulting from

- modeling the system incorrectly, and/or,
- modeling errors in the experimental measurements of the input signals and the measured responses of the system or in processing these measurements.

Consider, for instance, a system whose "correct" model is

$$\ddot{y} + ky + \epsilon y^3 = f(t). \tag{22}$$

Had S been chosen to be the set of linear systems, then the modeling error would be expressed by the cubic term above. Note that even though the value of ϵ may be very small compared to k, its effect on the system's output could be profound in certain regimes of response. On the other hand, for inputs of very small amplitude, the numerical value of the modeling error could be very small. If, during an experiment, instead of measuring the output $y(t)$, the signal recorded was $y(t) + n(t)$, then $n(t)$ would be the output measurement error. Both systematic (or bias) and random errors may occur in experimental measurement. Systematic and random errors are introduced by the physical apparatus used in experimental testing and in data reduction, with random errors also associated with environmental effects on the system.

While in concept measurement and modeling errors may be thought of as very different types of errors, in practice the results of identification(estimates of θ, say) are contaminated by both of these two types of errors in a manner which may be often impossible to disentangle. This often adds considerable difficulties in validating the model which is arrived at.

Since measurement errors are always present in an experimental situation, even in the absence of modeling errors, the parameters that are determined will seldom result in an exact match between the measured responses and the model responses. Thus an error criterion is generally chosen and the parameters are obtained by minimizing this error criterion to an acceptable degree. Also, since the experimental data collected is indeed limited, rather than the true values of the parameters, one can only obtain estimates of these parameters. Thus, the problem of the determination of the parameters in a model reduces to one of statistical estimation. One would then ideally want the probability density function (pdf.), $p(\tilde{\theta}; N)$, where θ is the parameter vector to be identified through the use of N data samples. One would intuitively expect that as the number of samples N increases, the pdf. of $\tilde{\theta}$ becomes more peaked around the true value, θ. Due to the

analytical difficulty of obtaining the entire pdf. of $\tilde{\theta}$ and the difficulty in visualizing these multidimensional pdfs., one resorts to using

- the expected value of $\tilde{\theta}$; $E[\tilde{\theta}]$
- the bias of the estimate; $E[\tilde{\theta}] - \theta$
- the covariance of the estimate; $\mathrm{cov}[\theta] = E[\{\tilde{\theta} - E[\theta]\}\{\tilde{\theta} - E[\theta]\}^T]$

It may be noted that if the distribution of θ were a multivariate Gaussian distribution, these parameters would define the pdf. completely.

The estimators $\tilde{\theta}$ used for estimating the parameter, $\theta \in R^j$, depend on the *a priori* information available about the measurement process, the error criterion chosen, and the 'cost', $C(\tilde{\theta}|\theta)$, associated with choosing the value $\tilde{\theta}$ for the estimate when the true value is θ. Three different estimators will be discussed, each associated with a progressively smaller amount of *a priori* information. The error criterion, the measurement and modeling errors, and the *a priori* information interact with each other in a manner that is now discussed.

Bayesian Estimator: *A priori* knowledge required is:

- $p(y^N|\theta)$
- $p(\theta)$
- $C(\tilde{\theta}|\theta)$.

The conditional risk of choosing $\tilde{\theta}(y)$ if the true parameter θ is given by the N-dimensional integral

$$E_{y|\theta}\{C(\tilde{\theta}|\theta)\} = \int C(\tilde{\theta},\theta)p(y^N|\theta)dy^N. \tag{23}$$

The average risk taken over the pdf. of θ then becomes the j-dimensional integral

$$\mathbf{R}(\tilde{\theta}) = \int C(\tilde{\theta}|\theta)p(\theta|y^N)d\theta. \tag{24}$$

The Bayes's estimate $\tilde{\theta}$ is such that,

$$\frac{\partial \mathbf{R}(\tilde{\theta})}{\partial \tilde{\theta}} = 0. \tag{25}$$

Note that $p(\theta|y^N) = p(y^N|\theta)p(\theta)/p(y^N)$.

A suitable error criterion for obtaining the estimates, $\tilde{\theta}$, would then be the minimization of the risk, $\mathbf{R}(\tilde{\theta})$. The Bayesian estimator places the entire problem in a probabilistic framework. Note that it also places the true parameter vector θ in a probabilistic context.

Maximum Likelihood Estimate: When only the first piece of information stated above is available, the true parameter θ can no longer be thought of as a random variable; it simply becomes an unknown. A suitable strategy then would be to determine

$$\max_{\tilde{\theta}} p(\tilde{\theta}|y^N) \tag{26}$$

This is equivalent to maximizing $p(y^N|\theta)$, which is generally referred to as the *Likelihood Function*. Denoting the pdf. of the prediction error by $f(t, \theta, \mathcal{E})$ and assuming that the errors are independent in time, this can be shown to reduce to

$$\mathbf{R}(\theta, y^N) = log\{p(y^N|\theta)\} = \sum_{t=1}^{N} log\, f(t, \theta, \mathcal{E}(t, \theta)). \tag{27}$$

Thus, a suitable error criterion (note that the logarithm is a monotone function), $R(\theta)$, for obtaining the estimates is to minimize the negative of the log likelihood function. For this choice of the error criterion the value of $\tilde{\theta}$ so obtained is called the *Maximum Likelihood Estimate* (MLE). The MLE has some useful properties: asymptotic (large N) normality, asymptotic unbiasedness and asymptotic efficiency. Under various assumptions on the pdf. $f(t, \theta, \mathcal{E})$ various different MLE estimates can be found.

When $f(t, \theta, \mathcal{E})$ is assumed to be Gaussian, the error criterion reduces to:

$$r(t,\theta,\mathcal{E}) = -log\, f(t,\theta,\mathcal{E})$$
$$= \frac{p}{2}log(2\pi) + \frac{1}{2}log\{det\Lambda(t,\theta)\} + \frac{1}{2}\mathcal{E}^T\Lambda^{-1}(t,\theta)\mathcal{E} \tag{28a}$$

where the mean of the distribution of $\mathcal{E}(t,\theta)$ is assumed to be zero, the covariance to be $\Lambda(t,\theta)$ and where p is a unknown constant. When Λ is not a function of θ, the error criterion becomes a quadratic function. The function $r(t,\theta,\mathcal{E})$ then becomes

$$r(t,\theta,\mathcal{E}) = \frac{1}{2}\mathcal{E}^T\Lambda^{-1}\mathcal{E}. \tag{28b}$$

Consider, for simplicity, a single output system (i.e., $p = 1$). Let the vector y^N be obtained from a model linear in the vector θ, i.e.,

$$y^N = U\theta. \tag{29}$$

When the measurements are contaminated by zero-mean gaussian noise, $n(t)$, with a covariance $E[nn^T] = S$ the equation to maximize the log likelihood function (which is now quadratic) is:

$$\frac{\partial}{\partial\tilde{\theta}}\{(y^N - U\tilde{\theta})^T|S|^{-1}(y^N - U\tilde{\theta})\} = 0. \tag{30}$$

(Here the noise vector n is N-dimensional.) This yields the estimate $\tilde{\theta}$ to be

$$\tilde{\theta}_N = \{U^TS^{-1}U\}^{-1}\{U^TS^{-1}y^N\}, \tag{31a}$$

with

$$\text{cov}[\tilde{\theta}] = [U^TS^{-1}U]^{-1}. \tag{31b}$$

This estimate (relation (31a)) is often called the Markov Estimate. Note that it depends linearly on the measurement data. The covariance of the estimate depends on the number of data points and the output measurement noise covariance. It represents the confidence

which may be placed in the estimate that has been obtained and is a crucial piece of information for the validation phase of the identification process. Relations (31a) and (31b) may also be thought of as resulting from a weighted least-squares approach with the weighting matrix taken to be the covariance of the measurements.

Least-Squares Estimator: If no knowledge of the measurement noise is available in the example above, then it would perhaps be appropriate to choose $S = \sigma^2 I$. Using this in expressions (31) yields:

$$\hat{\theta} = [U^T U]^{-1} U y^N \tag{32}$$

and,

$$\mathrm{Cov}(\tilde{\theta}) = \sigma^2 [U^T U]^{-1}. \tag{33}$$

This is exactly the least-squares estimate to the measurement equation

$$y^N = U\alpha + n, \tag{34}$$

where n is the appropriately-defined measurement noise vector. Thus it is seen that the various estimates obtained, as well as their covariances, depend on the *a priori* information available. This *a priori* knowledge also affects the error criteria in the minimization process.

Though it has been shown that the Markov and Least-Squares Estimators follow from the MLE estimator when the distribution of $\mathcal{E}$ is gaussian and independent of the parameter vector θ, these linear estimators can be derived under considerably more general conditions. (Of course, when the estimator is a MLE, the helpful asymptotic properties of the MLE apply.)

Model Nonlinear in the Parameter Vector, θ: In many problems that involve unknown parameters, the measurements do not depend linearly on θ. In such a situation one may try to use an iterative approach based on the results obtained so far. At each iteration the error $\mathcal{E}$ is linearized around the best estimate of $\tilde{\theta}$ from the previous iteration. The method goes as follows.

$$\mathcal{E}(t,\theta) = y(t) - Y(t,\theta) - W\Delta\theta = e(t,\theta) - W\Delta\theta, \tag{35}$$

where W is the sensitivity matrix defined by

$$[W]_{ij} = \frac{\partial Y_i}{\partial \theta_j}$$

and e is evaluated from knowledge of the parameter θ obtained from the previous iteration. The unknown parameter to be updated at each iteration is then $\Delta\theta$. Then

$$\theta_{i+1} = \theta_i + \Delta\theta. \tag{36}$$

If one is using the quadratic error criterion $(e - \Delta\theta)^T S^{-1}(e - \Delta\theta)$ to be minimized, then one again arrives at the same equations (again for a single output system, i.e. $p = 1$) (31a) and (31b) for obtaining the estimates of $\Delta\theta$. Thus, the adjustment $\Delta\theta_{i+1}$ at the $(i+1)th$ iteration is given by

$$\Delta\theta_{i+1} = [W_i^T S^{-1} W_i]^{-1} W_i^T S^{-1} e^N. \tag{37}$$

The success of the technique described above depends heavily on the initial guess of the parameter vector, θ_0, for the first iteration. The method is known to diverge in many instances. Another drawback of the technique is that it requires the sensitivity of the measurements with respect to the parameter vector to be determined for $t = 1, 2, \cdots, N$.

2.2.3. General Numerical Procedures.

The minimization of the error criterion, R, may in general be thought of as an iterative process, where the estimate of θ at the (i+1)th iterate takes the form

$$\tilde{\theta}_{i+1} = \tilde{\theta}_i - \mu_i \Gamma_i^{-1} \mathbf{R}_\theta(z^N, \tilde{\theta}_i) \tag{38}$$

where Γ_i^{-1} is called the gain factor and $\mathbf{R}_\theta$ is the gradient of the error criterion [23] with respect to the parameter vector θ. These quantities of course depend on the amount of data available (value of N). The gradient direction is often modified by the matrix Γ_i. The scaling parameter μ is generally chosen to facilitate convergence. Typical choices for Γ_i are:

$$\bullet\ \Gamma_i = I \qquad \text{(simple gradient algorithm)} \tag{39a}$$

$$\bullet\ \Gamma_i = \{Tr[\mathbf{R}_{\theta,\theta}(z^N, \tilde{\theta}_i)]I\} \qquad \text{(normalized gradient)} \tag{39b}$$

$$\bullet\ \Gamma_i = \{\mathbf{R}_{\theta,\theta}(z^N, \tilde{\theta}_i)\} \qquad \text{(Newton's method)} \tag{39c}$$

$$\bullet\ \Gamma_i = \frac{1}{N}\sum_1^N \Phi(t, \tilde{\theta}_i)\Lambda^{-1}\Phi(t, \tilde{\theta}_i); \qquad \Phi = -\mathcal{E}_\theta(t, \theta) \qquad \text{(Gauss-Newton for (28a))} \tag{39d}$$

Here the subscripts on $\mathbf{R}$ represent differentiation with respect to the subscripted parameters. The iterations are stopped when the criterion $\mathbf{R}$ is smaller than a certain value. It should be noted that the Markov estimators and the Least-Squares estimators of the previous section can be put in this framework. Covariance estimates of the parameter $\tilde{\theta}$

can likewise be obtained in a relatively straightforward manner. They provide information on the confidence which may be placed in the model.

Recursive Methods: A situation that one can visualize in the estimation process applied to large space structures is that the estimates may be required on-line. This may be required perhaps for some on-line decisions regarding control of the system. Alternately, the limited onboard computing facilities available may make the minimization of $\mathbf{R}$ using the relation (38) for each N infeasible due to the large amount of computation that may be required. In such cases a recursive technique may be used to approximate (38). Different recursive schemes [24], [25], resulting in different statistics of the estimates obtained, can be used. One such simple scheme would be to approximate $\theta(N)$ as

$$\tilde{\theta}(N) = \tilde{\theta}(N-1) - \mu(N)[\Gamma(N)]^{-1}\mathbf{R}_{\theta}^{N}[\tilde{\theta}(N-1), z^{N}]. \tag{40}$$

Assuming that $\theta(N-1)$ minimizes $\mathbf{R}(\theta, z^{N})$,

$$\mathbf{R}_{\theta}^{N}[\tilde{\theta}(N-1), z^{N}] = -\frac{1}{N}\Phi[N, \tilde{\theta}(N-1)]\Lambda^{-1}\mathcal{E}[N, \tilde{\theta}(N-1)]. \tag{41}$$

where Φ has been defined in (39).

The values of $\mathcal{E}(N,\theta)$ and $\Phi(N,\theta)$ are generally found by a recursive rule starting from $t = 0$ and they depend on the nature of the error criterion used. These recursive relations are used, then, and the estimate of the parameter, $\tilde{\theta}$, at time t, is employed whenever these recursive relations require the value of θ at time t. Using the last of the relations in (39) leads to a recursive scheme such as

$$\tilde{\theta}(N) = \tilde{\theta}(N-1) + \frac{\mu}{N}[\Gamma(N)]^{-1}\Phi(N)\Lambda^{-1}\mathcal{E}(N, \tilde{\theta}) \tag{42}$$

along with

$$\Gamma(N) = \Gamma(N-1) + \frac{1}{N}\{\Phi(N)\Lambda^{-1}\Phi^T(N) - \Gamma(N-1)\}. \tag{43}$$

Such schemes bear a close relation to Markov Estimators as well as to Stochastic Convergence Methods. Note that the update is linearly dependent on the prediction error, $\mathcal{E}$. Recursive schemes for Least-Squares Estimation can be obtained in a straightforward manner by modifying the corresponding expressions (like (31) and (33)) in a suitable manner.

Another way of approaching recursive estimation is through the use of the Kalman Filter [20]. This filter (as originally conceived) provides a recursive relation for the estimation of the state of a linear system. The estimate is linear in the observations. The problem can be thought of as a Bayesian updating of the state vector of a linear system when both measurement noise and input noise may be present. Under the assumption that the noise is Gaussian, the filter is optimal in the Least- Squares sense. Under other noise conditions it is the optimal-linear filter. Formulations of the Kalman Filter, when applied to the estimation of an "augmented state vector" which includes the parameter vector θ (along with its augmented state equation, $\dot{\theta} = 0$), generally lead to what is called the Extended Kalman Filter, for now the equations for this augmented state vector become nonlinear [22], [25]. A procedure of successive linearization around the new estimate of the augmented state vector may then be carried out. The estimates are obtained, after each such linearization, through the usual Kalman Filter equations. The nonlinear nature of this problem implies that all approaches to obtaining the estimates will be *inherently iterative*. Recursive formulations based on suitable Gaussian assumptions regarding the statistics of the state and the measurement vectors (also called the pseudo Gaussian approach) can also be used for obtaining the conditional means of the state vectors. Minimization of various error criteria (depending on whether one is using MLE, Least- Squares, etc.), under

the constraints imposed by these recursive relations, can then be carried out by gradient search techniques.

2.3. Experimental Testing.

The identification process depends rather heavily on the nature of the experiment(s) done to characterize the physical system. This is perhaps the aspect of structural system identification that is most influenced by some of the macro-level considerations such as costs, available time, mission goals, and so forth. Among the experimental conditions available for adjustment are:

- the boundary conditions of the system, [26]
- the locations at which the measurements are to be taken and the inputs provided [27], [28]
- the times at which the samples are to be obtained [29]
- the extent and nature of filtering of data prior to sampling
- the nature of the input signals (e.g., sine wave tests, GWN, chirp signals)
- the types of sensors and actuators to be used.

Each of these considerations will affect the amount of information that is gathered in the experimental testing of the system. Experimental design obviously depends on:

- the goal of the experiment and the intended application of the results
- the class of models, S, to be used
- the identification method to be used
- the extent of prior knowledge about the system
- the constraints on the operation of the system.

For on-board identification of large space structures, the last point appears to have considerable significance. The testing technique must not render the structure inoperable over considerable periods of time.

SECTION 3. STRUCTURAL MODELING AND EXPERIMENTAL CONSIDERATIONS.

Structural modeling is a key part of structural system identification for large space structures. As discussed in Section 1.3.1, the model must not only represent the physical system, but must also be coordinated with testing, model verification procedures, and estimation algorithms. Section 1.3.2 discussed system identification from a model development point of view, where the first stage of development is the formulation of an analytical model, the second stage involves the refinement and verification of the model, and the final stage addresses certification.

The key modeling issues for model verification and certification are:

(1) Coordinate reduction aimed at achieving low-order plant models for control systems.

(2) Parameter definition aimed at minimizing the number of parameters which must be estimated, and optimizing the correspondence between parameters and physical measurements,

(3) Substructuring to facilitate the verification of large models by partitioning,

(4) The treatment of nonlinearities, and

(5) The quantification of modeling uncertainty for purposes of assessing predictability, and implementing stochastic control.

Of the two types of models previously mentioned (parametric and non-parametric), maximum use will be made of parametric models for large space structures; nonparametric models may be used in special cases where conventional modeling techniques are not suitable. Parametric models may be divided into two classes - discrete and continuous. The former is characterized by ordinary differential equations, the latter by partial differential or difference equations. While some research is being conducted in the use of continuous models, discrete models based on finite element methods are used in practice because of

their general applicability and availability. These methods are implemented in a number of commercially available computer programs such as NASTRAN, ANSYS and EAL.

Equivalent continuum models have been suggested as a means of simplifying the modeling of space structures which are very large in terms of the number of structural elements they comprise, but simple in terms of overall structural configuration and its repetitive pattern of assembly. Typical examples include uniform truss-type beam and truss-type plate or shell assemblies. Equivalent continuum models are so named because they replace complex structural frameworks with continua which are equivalent in some sense, e.g. stress-strain relationships, or kinetic and strain energy. These equivalent continuum models can be discretized for use in conventional matrix structural analysis based on ordinary differential equations. As with actual continuum models, the discretization can be accomplished in either of two ways: with discrete (nodal) coordinates and (equivalent) finite element modeling, or with distributed (modal) coordinates. The nodal coordinate approach should produce results which are similar, if not identical, to results based on superelement modeling, where coordinate reduction and the repetitive pattern of structural assembly are both utilized. The modal coordinate approach is capable of achieving greater accuracy, to the extent that closed form solutions are available for defining the generalized (modal) displacement functions.

Although the equations of motion which govern the dynamic behavior of large space structures in orbit contain inherent nonlinearities due to large rotations, large deformation, nonrigid joints, etc., it is anticipated that for purposes of structural identification, the equations of motion will be cast in linear form. This can be accomplished in a number of ways including linearization, isolation of local nonlinearities, and the replacement of nonlinearities by time-dependent coefficients or parameters.

Substructuring will undoubtedly be employed to model hinged structures with large relative rotations (e.g. solar arrays and antennae) and other structural configurations where major structural assemblies are joined at simple interfaces. Substructuring may also be used to isolate nonlinearities. Substructuring is particularly attractive for structural identification because it allows the identification to be focussed on smaller objects, thereby improving the resolution of the process. Substructuring facilitates coordinate reduction, which is essential for producing the low-order models required by control systems. Coordinate reduction will be just as essential for purposes of structural identification. In this case, the need is for fewer parameters; the number of dynamic degrees of freedom is less critical.

These topics are discussed in the subsections which follow. The intent is not to offer a complete survey of each topic, but rather to reveal their salient features as they relate to the identification of large space structures, and to cite references in the published literature to avail the reader of sufficient background material for a deeper understanding.

3.1. Finite Element Modeling and Coordinate Reduction.

Coordinate reduction is usually accomplished in several stages as a model is developed from the most fundamental finite elements, to superelements or macroelements, to subassemblies, to major structural assemblies and finally to the complete structural system. All of these coordinate reductions have the potential of introducing modeling error. In fact, the initial discretization of a continuous model to a finite element model implies a coordinate reduction from the continuous system, having (theoretically) an infinite number of degrees of freedom per element, to a finite number associated with the selected generalized coordinates or shape functions built into the particular elements (beam elements, plate elements, shell elements, solid elements, etc.). The degree of error resulting from this discretization process depends on the mesh geometry and how well it matches

the geometry of the structure, the coarseness of the mesh, the type and number of shape functions used, etc. It is virtually impossible to assess the magnitude of error introduced by this process except by comparison with alternative solutions.

Having generated the initial finite element model of a structural component using one of the standard finite element modeling programs, the second stage of coordinate reduction, called static reduction or static condensation, is applied. Equations of motion are first partitioned (neglecting damping for the time being):

$$\begin{bmatrix} M_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \end{bmatrix}. \tag{44}$$

In partitioning the equations, two assumptions are made: (1) that there are no external forces applied at the coordinates, x_2, and (2) that the mass associated with these coordinates can be neglected. It then follows that x_2 can be expressed in terms of x_1 as

$$x_2 = -K_{22}^{-1} K_{21} x_1 \tag{45}$$

so that the equations of motion reduce to

$$M_{11}\ddot{x}_1 + (K_{11} - K_{12}K_{22}^{-1}K_{21})x_1 = f_1. \tag{46}$$

At this point, a number of similarly reduced components (in this case superelements, macroelements, etc.) may be assembled into a structural subassembly whose equations of motion are again partitioned

$$\begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} \\ M_{21}^{(1)} & M_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \ddot{x}_1^{(1)} \\ \ddot{x}_2^{(1)} \end{bmatrix} + \begin{bmatrix} K_{11}^{(1)} & K_{12}^{(1)} \\ K_{21}^{(1)} & K_{22}^{(1)} \end{bmatrix} \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} f_1^{(1)} \\ 0 \end{bmatrix}. \tag{47}$$

The partitioning is defined in accordance with the division of $x^{(1)}$ into subsets $x_1^{(1)}$ and $x_2^{(1)}$ where no external forces are applied at the coordinates $x_2^{(1)}$. However, in this case

$M_{12}^{(1)}$, $M_{21}^{(1)}$ and $M_{22}^{(1)}$ are not null. A third stage of coordinate reduction, known as Guyan reduction [30], is applicable in this case. Guyan reduction employs the same transformation of coordinates as static reduction, except that it is applied to the mass matrix as well as the stiffness matrix. In the case of (47), the transformation is

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} I \\ -[K_{22}^{(1)}]^{-1} K_{21}^{(1)} \end{bmatrix} \left[x_1^{(1)} \right]. \tag{48}$$

Equation (48) defines a set of static shape functions for $x_2^{(1)}$ in the sense that a static deformation involving the vector of displacements $x_2^{(1)}$ is obtained for each vector, $x_1^{(1)}$, defined by individually setting each element of $x_1^{(1)}$ to unity while constraining all other elements of $x_1^{(1)}$ to zero. The independent coordinates, $x_1^{(1)}$, of (48) may be thought of as generalized static displacement coordinates. In this sense, Guyan reduction may be interpreted as the consistent mass matrix [31] equivalent of static reduction. Application of the transformation in (48) to (47) gives

$$\left[M_{11}^{(1)} - M_{12}^{(1)}[K_{22}^{(1)}]^{-1} K_{21}^{(1)} - \left([K_{22}^{(1)}]^{-1} K_{21}^{(1)} \right)^T \left(M_{21}^{(1)} - M_{22}^{(1)}[K_{22}^{(1)}]^{-1} K_{21}^{(1)} \right) \right] \ddot{x}_1^{(1)}$$

$$+ \left[K_{11}^{(1)} - K_{12}^{(1)}[K_{22}^{(1)}]^{-1} K_{21}^{(1)} \right] x_1^{(1)} = f_1^{(1)}. \tag{49}$$

The use of Guyan reduction requires some insight in deciding how to partition $x^{(1)}$; modeling errors are minimized by a judicious choice of $x_1^{(1)}$ where, for example, most of the mass is associated with the $x_1^{(1)}$ coordinates. In this case, Guyan reduction approaches that of static reduction. Clearly, the modeling error introduced by the static shape functions implied in (45) and (48) increases with increasing mode number. Misapplication of Guyan reduction can lead to serious modeling errors, e.g. when one attempts to reduce the number of coordinates too far. Intuitively, the degree of permissible reduction varies inversely with

the desired bandwidth of the model. The consistent mass matrix interpretation of Guyan reduction offers useful insight for the partitioning of coordinates.

The model building approach is advanced another step by coupling together a number of subassemblies, each represented by equations in the form of (49). For future reference, these coupled equations may be written

$$M^{(2)}\ddot{x}^{(2)} + K^{(2)}x^{(2)} = f^{(2)}. \tag{50}$$

These would be the equations of a major assembly. Further Guyan reduction may be applied at this stage, in which case (50) would be partioned as

$$\begin{bmatrix} M_{11}^{(2)} & M_{12}^{(2)} \\ M_{21}^{(2)} & M_{22}^{(2)} \end{bmatrix} \begin{bmatrix} \ddot{x}_1^{(2)} \\ x_2^{(2)} \end{bmatrix} + \begin{bmatrix} K_{11}^{(2)} & K_{12}^{(2)} \\ K_{21}^{(2)} & K_{22}^{(2)} \end{bmatrix} \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix} = \begin{bmatrix} f_1^{(2)} \\ 0 \end{bmatrix}. \tag{51}$$

The reduced equations would be of a form analogous to (49) with the superscripts (1) in (49) replaced by superscripts (2).

3.2. Distributed Finite Element Parameters.

The matrices $M^{(2)}$ and $K^{(2)}$ of (50) will have lost physical significance as a result of the various coordinate reductions, and will certainly be too large in dimension (perhaps several hundred) to attempt the estimation of individual matrix elements. Experience has shown, that in order for parameter estimation to be successful, five conditions should be satisfied:

(1) The number of parameters being estimated should be small.

(2) The parameters being estimated should be distributed over the structure in such a way that small changes in the parameter values (e.g. less than 10 %) cause a measurable change in the structural behavior being observed (analogous to controllability).

(3) The measurements of structural behavior should be sufficiently complete to identify the parameters being estimated (analogous to observability).

(4) The parameters being estimated should be representative of the differences between the analytical model and the actual structure, i.e. must span the parameter space required to bring the model into agreement with experimental data.

(5) The changes to elements of the stiffness matrix should not imply nonexistent load paths in the structure, i. e., structural connectivity should be maintained.

The meaning of "small" in item (1) will depend on the estimation algorithm being used, and on the particular application. For example, with one form of Bayesian estimation [32] numerical illconditioning was encountered when the number of parameters exceeded 30, whereas successful runs were made with 10-20 parameters. Using an alternative formulation of the Bayesian estimator [33], 100 parameters were estimated with no apparent illconditioning. This is not meant to imply that one formulation is superior to the other in all cases. The number of parameters being estimated is only one of many factors to be considered when selecting an estimation algorithm.

As pointed out in [34], it is futile to attempt to independently estimate the individual finite element parameters of a typical finite element model. Not only would there be too many of them, ($\gg 100$), but the global structural behavior normally observed in the lower modes (or lower range of frequency response) of a structure is insensitive to even large changes in individual parameters. For example, the stiffness of a single shell element in a typical finite element model of a cylindrical shell might be changed by an order of magnitude with no significant effect on the lower modes. To be effective, the parameters defined for purposes of parameter estimation must be distributed in some fashion over a sufficiently large portion of the structure.

There are many ways to define distributed parameters. Dobbs and Nelson [35] have used parameter linking, a technique frequently used in structural optimization, whereby a parameter (such as thickness) common to a number of finite elements in a region, is chosen for estimation. Changes to the parameter are made uniformly to all of the elements which have this parameter in common. An example of parameter linking, in the case of a planar truss-beam, would be to link all longitudinal elements, all transverse elements, and all diagonal elements, resulting in only three stiffness parameters to be estimated. In general, parameter linking allows the estimation of parameters which appear nonlinearly in the mass or stiffness matrices, e.g. plate thickness.

Another way to define distributed parameters is by submatrix scaling, where the mass and/or stiffness matrix are first expanded into a linear series (sum) of submatrices

$$M = M_0 + \sum_j \alpha_j M_j \tag{52a}$$

$$K = K_0 + \sum_j \beta_j K_j \tag{52b}$$

where α_j and β_j are scalar coefficients. Initially, α_j and β_j may be assigned values of unity, so that the initial values of M and K are given by the simple superposition of all submatrices. Since the finite element matrix assembly process utilizes superposition as a means of generating the mass and stiffness matrices, it should be easy to generate the M_j and K_j in the process of generating M and K.

Sometimes one is given M and K with no opportunity to repeat the modeling process for purposes of generating M_j and K_j. One way to generate meaningful submatrices given this situation is to generate a set of orthogonal vectors, δ_j, such that

$$\delta_j^T M \delta_k = 0 \quad : \qquad j \neq k \tag{53a}$$

$$\delta_j^T K \delta_k = 0 \quad : \qquad j \neq k. \tag{53b}$$

Then

$$M_j = \frac{1}{\bar{M}_j} \left(M \delta_j \delta_j^T M \right) \tag{54a}$$

$$K_j = \frac{1}{\bar{K}_j} \left(K \delta_j \delta_j^T K \right) \tag{54b}$$

where

$$\bar{M}_j = \delta_j^T M \delta_j \quad \text{and} \quad \bar{K}_j = \delta_j^T K \delta_j.$$

Choosing δ_j from the set of undamped normal modes is one possibility. It is also possible to generate static deformation functions which are mutually orthogonal, as was done by Lee and Hasselman [36].

A third type of distributed parameter, used successfully in practical applications of parameter estimation [32], [36], is the modal matrix element. Continuing with the alternating sequence of substructure assembly and coordinate reduction described in Section 3.1, one may apply a modal transformation to (50) or to a reduced version of (50) obtained by applying Guyan reduction to the partitioned (51). Suppose the modal transformation is given by

$$x^{(2)} = {}^{o}\phi \, q \tag{55}$$

where ${}^{o}\phi$ is a truncated modal matrix consisting of the analytical modes corresponding to $M^{(2)}$ and $K^{(2)}$, and q is the reduced set of modal coordinates. The transformed equations are written in the form

$$^{o}m\ddot{q} + {}^{o}k\, q = f_q = {}^{o}\phi^T f^{(2)} \tag{56}$$

where

$$^{o}m = {}^{o}\phi^T M^{(2)} {}^{o}\phi = I \qquad \text{(diag)} \tag{57a}$$

$$^{o}k = {}^{o}\phi^T K^{(2)} {}^{o}\phi = {}^{o}\omega_o^2 \qquad \text{(diag)}. \tag{57b}$$

The "true" equations of motion (including damping) may be written as

$$I\ddot{\eta} + \xi\dot{\eta} + \omega_o^2\eta = f_\eta. \tag{58}$$

Equations (56) and (58) are related by

$$q = \psi\eta \tag{59}$$

$$I = \psi^T m \psi \tag{60a}$$

$$\omega_o^2 = \psi^T k \psi \tag{60b}$$

where

$$m = {}^{o}m + \Delta m = I + \Delta m \tag{61a}$$

$$k = {}^{o}k + \Delta k = {}^{o}\omega_o^2 + \Delta k \tag{61b}$$

and ξ is the modal damping matrix, as yet undefined. The object of parameter estimation would then be to estimate Δm, Δk and ξ.

A physical interpretation of Δm and Δk follows from perturbation analysis. If the true modes ϕ can be expressed as linear combinations of the original analytical modes ${}^{o}\phi$, then

$$\phi = {}^{o}\phi\psi. \tag{62}$$

The cross orthogonality product of analytical modes and true modes is then

$${}^{o}\phi^{T} M^{(2)} \phi = {}^{o}\phi^{T} M^{(2)} {}^{o}\phi\psi = \psi. \tag{63}$$

By linear perturbation analysis it can be shown that

$$\Delta m = (I - \psi) + (I - \psi)^{T} \tag{64a}$$

$$\Delta k = (\omega_o^2 - {}^{o}\omega_o^2) + {}^{o}\omega_o^2 (I - \psi) + (I - \psi)^{T} {}^{o}\omega_o^2 \tag{64b}$$

where I is the identity matrix and both ω_o^2 and ${}^{o}\omega_o^2$ are diagonal matrices. Thus Δm and Δk are defined in terms of the difference between the true eigenvalues and those given by the analytical model, and the cross orthogonality between the true eigenvectors and those of the analytical model, assuming small differences between the two.

The estimation of Δm and Δk, and thereby m and k according to (61), allows ω_o and ψ to be obtained by numerical solution of the eigenproblem:

$$(k - \omega_o^2 m)\psi = 0. \tag{65}$$

The revised modal matrix in physical coordinates is then obtained by the transformation, $\phi = {}^{o}\phi\psi$, (62).

It is recognized that estimates of Δm and Δk may be obtained directly from (64) whenever experimental estimates of ϕ and ω_o are available. This approach was attempted by Garba and Wada [37] and also by Lee and Hasselman [36] with generally unsatisfactory results. The solution of (65) with Δm and Δk obtained in this manner yielded improved frequencies, but the corresponding mode shapes were in some cases worse than those of the original model. The degradation of mode shapes may be due to "noise" in the measured mode shapes. Estimation of m and k by (64) using ψ computed from (63) involves taking small differences of relatively large numbers in (63), making the estimates sensitive to noise. Thus, while (64) offers an insightful interpretation of Δm and Δk, it should not be used as a means of estimating these parameters for purposes of improving the analytical model. On the other hand, they may be used as a basis for evaluating combined modeling and measurement accuracy, as discussed in Section 3.6.

3.3. Equivalent Continuum Modeling.

There are two distinct features in large space structures, i.e., the presence of a large number of structural members and periodic geometries. When full-scale finite elements are used to model this type of structure, the periodic nature of the structure is sometimes not utilized; moreover, the size of the structure can often lead to an expensive computational effort which yields limited information about its dynamic characteristics.

From the practical control engineering and system identification viewpoint, it is the global behavior of large space structures which is if most interest. The need for a model that is capable of describing the global response without detailed deformation in each member is then obvious. The continuum modeling of large periodic structures appears to have provided an answer to this need.

Continuum models employ basically a smearing approach that turns an actually discrete structure into an effective continuous body. In such a transformation, effective properties are obtained and then used to describe the structure. These effective properties may involve a smaller number of effective material and structural parameters than that of the actual structure. The identification problem can then be reduced to identifying the effective properties.

There are three methods which may be employed to construct continuum models for large truss or frame structures. A brief account of these methods is given below.

a. *Effective Modulus Method*

A typical cell of the structure is isolated and studied for its load- deflection characteristics, which are subsequently compared with the stress- strain relations of a continuum. From this comparison the effective moduli are obtained.

Heki and Saka [38] used this method to analyze lattice plates which were effectively represented by anisotropic continuum plates. Nayfeh and Hefzy [39], [40] considered both truss-like and frame-like structures. The procedures employed by Nayfeh and Hefzy are summarized as follows. First, all sets of parallel members in the typical cell are identified. The unidirectional effective continuum properties are derived for each of these sets. Finally, orthogonal transformations are used to determine the contribution of each set to the global effective continuum properties of the structure. The resulting effective continua are usually anisotropic. If the structure possesses symmetries, then the number of effective moduli is reduced.

The effective modulus method transforms a discrete structure into a 2-D or 3-D solid. For beam-like or plate-like structures, the effective beam or plate equations of motion can be derived following the conventional approach available in continuum mechanics.

b. Energy Method

The essence of this method is to construct a continuum that contains the same amount of strain energy and kinetic energy stored in a typical cell of the structure. Sun and Yang [41] derived a 2-D continuum with couple stress to represent an orthogonal gridwork with rigid joints. In [41] the equations of motion were derived using Hamilton's principle on the "smoothed" strain and kinetic energy functions. The stress-strain relations were obtained directly from the strain energy functions.

Noor et al [42] derived effective stiffnesses for beam- and plate-like truss structures using the energy-equivalence concept. The procedures involved using an approximate continuous displacement (or strain) field commonly used in continuum mechanics for developing beam and plate equations to estimate the strain energy and kinetic energy in a typical cell. Later, the same method was employed to study structures with rigid joints for which the representative continuum possesses coupled stresses [43], [44].

c. Direct Structural Method

Many large space structures, although complex in detail, often behave grossly as a beam, plate or a thin shell. The gross dynamic behavior of these structures may be governed by the proper continuum beam, plate or shell equations. If such equations are derived, the original discrete structure may be replaced by the equivalent model.

In using the direct structural method, a continuum model must be chosen *a priori*. A typical cell is then analyzed using conventional methods (analytical or the finite element method) for loading that produces the basic deformations in the continuum model. The continuum beam or plate properties are obtained from these basic load-deformation relations for the typical cell.

Sun et al [45] presented the procedures of the direct structural method for beam-like and plate-like truss and frame structures which are symmetrically constructed with respect to their mid-planes. It was found that Timoshenko- type beam and Mindlin-type plate theories were most suitable for continuum modeling. Later, Sun and Kim [46] extended this method to include truss structures which were not symmetric with respect to their mid-planes. Material damping of structural members can be easily included in the formulation [47]. For simple geometries of the typical cell, explicit expressions of the effective continuum properties can be obtained [48].

Once the continuum model is established, the analysis follows the classical methods for solving Timoshenko beam and Mindlin-type plate equations, which are abundant in the literature. In fact, Abrate and Sun [49] studied large amplitude vibration of truss beams using the continuum model with excellent results.

3.3.1. Extended Timoshenko Beam.

The conventional Timoshenko beam theory was developed using homogenous isotropic materials in which the three basic deformations, i.e., extension, transverse shear and bending, are not coupled. A more general beam theory is needed to model an arbitrary truss beam in which the three basic deformations may be coupled. The force-deformation relations which account for these coupling effects are given by

$$\begin{bmatrix} N \\ Q \\ M \end{bmatrix} = \begin{bmatrix} EA & \eta_{12} & \eta_{13} \\ \eta_{12} & GA & \eta_{23} \\ \eta_{13} & \eta_{23} & EI \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial w}{\partial x} + \psi \\ \frac{\partial \psi}{\partial x} \end{bmatrix} \tag{66}$$

where

N = extensional force

Q = transverse shear force

M = bending moment

u = longitudinal displacement of the mid-plane

w = transverse displacement of the mid-plane

ψ = rotation of cross-section

EA = longitudinal rigidity

GA = transverse shear rigidity

EI = bending rigidity

η_{ij} = coupling coefficients

The stress-strain relations were obtained directly from the strain energy functions. The corresponding equations of motions are

$$\frac{\partial N}{\partial x} + q_x = m\ddot{x} + R\ddot{\psi} \tag{67a}$$

$$\frac{\partial Q}{\partial x} + q_z = m\ddot{w} \tag{67b}$$

$$\frac{\partial M}{\partial x} - Q = R\ddot{u} + \rho I\ddot{\psi} \tag{67c}$$

in which

q_x = externally applied force per unit length in the longitudinal direction

q_z = transverse load per unit length

m = mass per unit length

ρI = mass moment of inertia of cross-section

$R = \int_{-h/2}^{h/2} \rho\, z dz$

where h is the thickness of the beam.

The sign convention is illustrated in Figure 3.

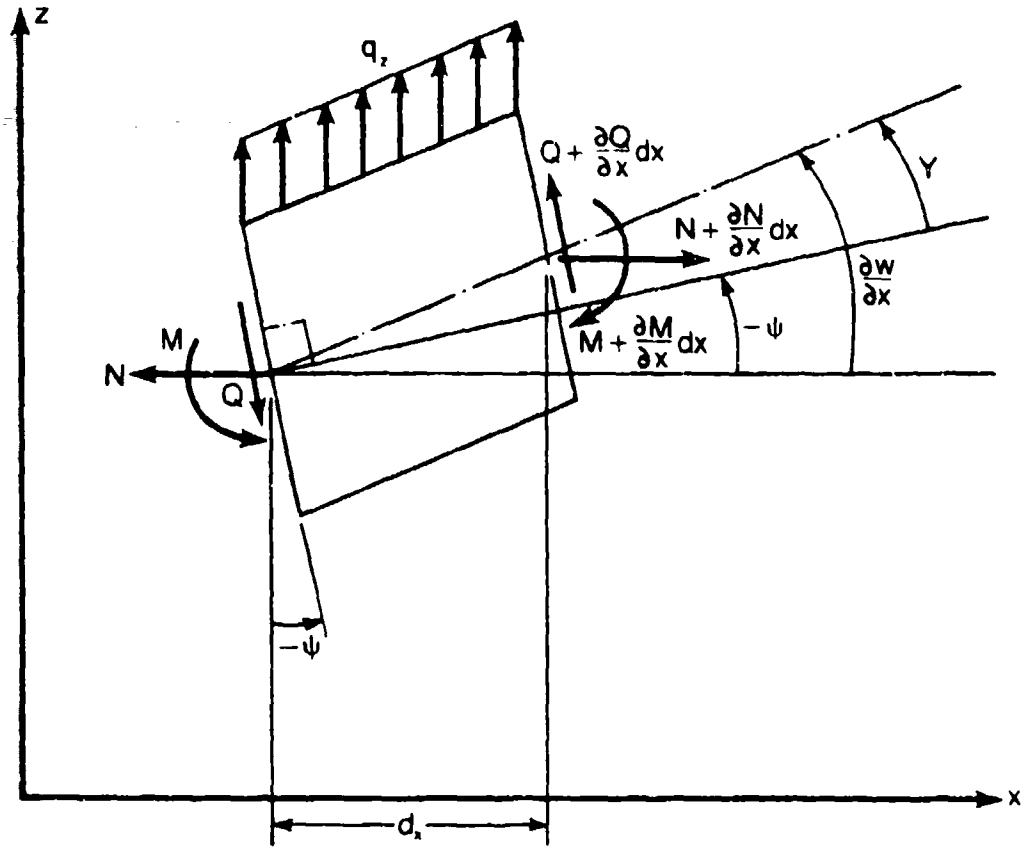


Figure 3. Timoshenko Beam Element.

Substitution of (44) into (45) yields the displacement equations of motion:

$$\begin{bmatrix} \partial_x(EA\partial_x) & \partial_x(\eta_{12}\partial_x) & \partial_x(\eta_{12}+\eta_{13}\partial_x) \\ \partial_x(\eta_{12}\partial_x) & \partial_x(GA\partial_x) & \partial_x(GA+\eta_{23}\partial_x) \\ -\eta_{12}\partial_x+\partial_x(\eta_{13}\partial_x) & -GA\partial_x+\partial_x(\eta_{23}\partial_x) & -GA\partial_x+\partial_x(EI\partial_x) \end{bmatrix} \begin{bmatrix} u \\ w \\ \psi \end{bmatrix}$$

$$= \begin{bmatrix} m & 0 & R \\ 0 & m & 0 \\ R & 0 & \rho I \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\psi} \end{bmatrix} - \begin{bmatrix} q_x \\ q_z \\ 0 \end{bmatrix} \tag{68}$$

where ∂_x is the partial differential operator with respect to x, and a dot over the variable indicates differentiation with respect to time.

3.3.2. Effective Beam Properties.

For a beam-like truss possessing a plane of symmetry coinciding with its midplane, the coupling coefficients η_{ij} vanish and the three basic beam deformations are uncoupled, leaving three stiffness coefficients, EA, GA and EI to be determined. In this case, these stiffness coefficients can be obtained by isolating a typical substructure and studying its force- deformation behavior in each type of deformation. Trusses of this type were considered by Sun et al [46].

If coupling coefficients η_{ij} are present, it is more convenient to use the inverse relation of (66), i.e.,

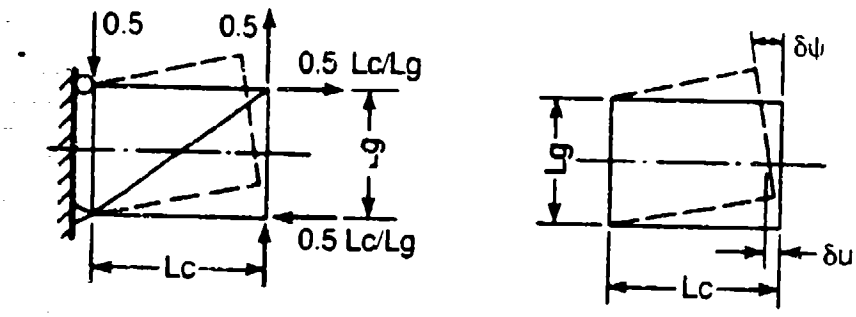
$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial w}{\partial x} + \psi \\ \frac{\partial \psi}{\partial x} \end{bmatrix} = [\eta^*] \begin{bmatrix} N \\ Q \\ M \end{bmatrix} \qquad ((69)$$

where

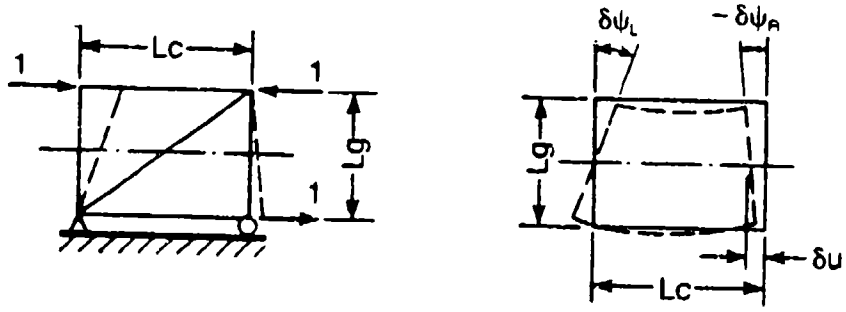
$$[\eta^*] = [\eta]^{-1} \qquad (70)$$

and $[\eta]$ is the 3x3 beam stiffness matrix in (66). The elements η_{ij} can be evaluated column by column by applying unit loads for N, Q and M, respectively, to the typical substructure. For instance, by applying $N = 1$, $Q = M = 0$, the resulting values of $\partial u/\partial x$, $\partial w/\partial x + \psi$, and $\partial \psi/\partial x$ are equal in value to η^*_{11}, η^*_{21} and η^*_{31}, respectively. The other elements η^*_{ij} can be obtained in a similar manner. The elements in $[\eta]$ are then obtained by inverting $[\eta^*]$. Note that if a truss member is shared by two adjacent substructures, the cross-sectional area of this member should be reduced by half.

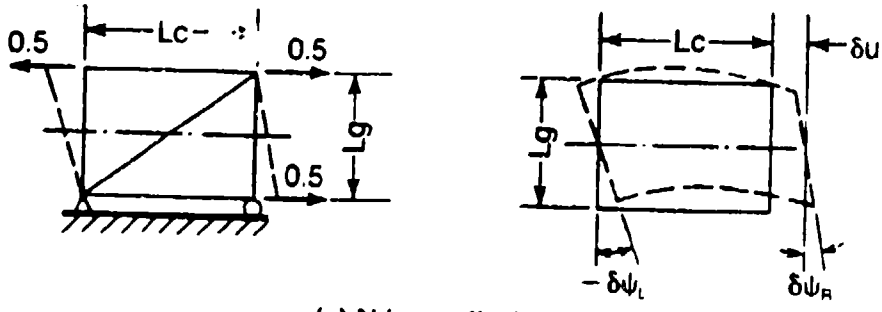
In the above procedure, care must be exercised in interpreting the beam deformations $\partial u/\partial x$, $\partial w/\partial x + \psi$ and $\partial \psi/\partial x$. Figure 4 depicts the deformations of a unit cell of a truss-beam resulting from applications of shear force Q, moment M, and extensional force N.



(a) Q is applied



(b) M is applied



(c) N is applied

Figure 4. Extension-Shear-Bending Coupled Deformation.

Consider the case shown in Figure 4c. Due to a unit extensional force applied to the unit cell, the resulting extension is δu, the rotation at the left end is $\delta\psi_L$ and at the right end is $\delta\psi_R$ and the vertical displacements of the midplane at the left and right ends are δw_L and δw_R (not shown in the figure), respectively. From this deformed geometry, we obtain

$$\eta^*_{11} = \partial u/\partial x \approx \delta u/L_c \tag{71a}$$

$$\eta^*_{12} = \partial w/\partial x + \psi \approx (\delta w_R - \delta w_L)/L_c + (\delta\psi_R + \delta\psi_L)/2 \tag{71b}$$

$$\eta^*_{31} = \partial\psi/\partial x \approx (\delta\psi_R - \delta\psi_L)/L_c. \tag{71c}$$

To obtain and η^*_{21}, η^*_{22}, and η^*_{23}, a unit moment is applied as shown in Figure 4b.

The application of the transverse shear force is more tricky. Due to the finite dimension of the substructure under consideration, the unit shear force applied at the right end would produce a couple at the left end. Thus, a pair of forces of $0.5\,L_c/L_g$ as shown in Figure 4a needs to be added in order to produce a state of shear stress in a continuum. If $L_c \to 0$, then these forces vanish as expected. This pair of horizontal forces correspond to the thickness-shear stress in the continuum theory and should not be confused with the beam bending moment.

It is important to note that the effective stiffnesses EA, GA and EI should be regarded as single entities rather than the product of two constants. The mass inertia terms for the continuum beam, m, R and ρI, are calculated from the typical substructure by calculating the total inertias of the whole substructure first and then distributed uniformly along the beam element. For spatial beam-like trusses, the Timoshenko beam model can be easily extended to include torsion and bi-directional bending.

Example 1. A typical substructure of the example truss shown in Figure 5 exhibits extension-bending coupling. By following the above procedures, the equivalent continuum beam properties are obtained and given as follows.

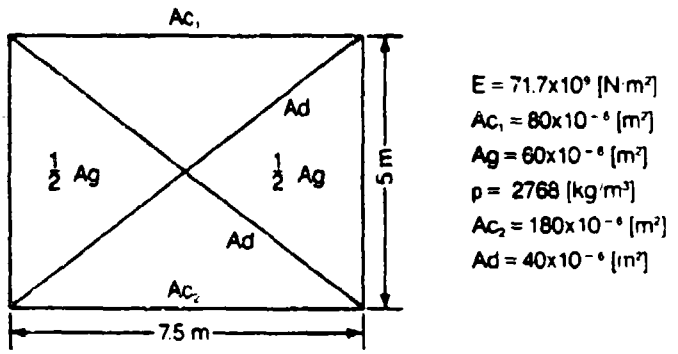


Figure 5. Typical Substructure of a Truss.

$$[\eta] = \begin{bmatrix} 2.13 \times 10^7 & 0 & -1.79 \times 10^7 \\ 0 & 1.47 \times 10^6 & 0 \\ -1.79 \times 10^7 & 0 & 1.17 \times 10^8 \end{bmatrix}$$

$$\rho A = 1.09 kg/m$$

$$R = -0.69 kg$$

$$\rho I = 5.28 kg\text{-}m$$

Equations for the mass and stiffness properties are given in Section 3.3.4. along with equivalent properties.

3.3.3. Loads in Truss Members.

For structural design purposes, loads carried by truss members may be needed. The member load in the actual truss can be obtained from the continuum model solution.

The solution for a Timoshenko beam is given in terms of the longitudinal displacement $u(x)$, the transverse displacement $w(x)$, both in the mid- plane of the beam, and the rotation of the cross-section, $\psi(x)$. The displacement at any joint of the truss can be calculated from the following relations

$$u^*(x,z) = u(x) + z\psi(x) \qquad (72a)$$

$$w^*(x,z) = w(x) \qquad (72b)$$

where the coordinates of the joint (x,z) must be specified.

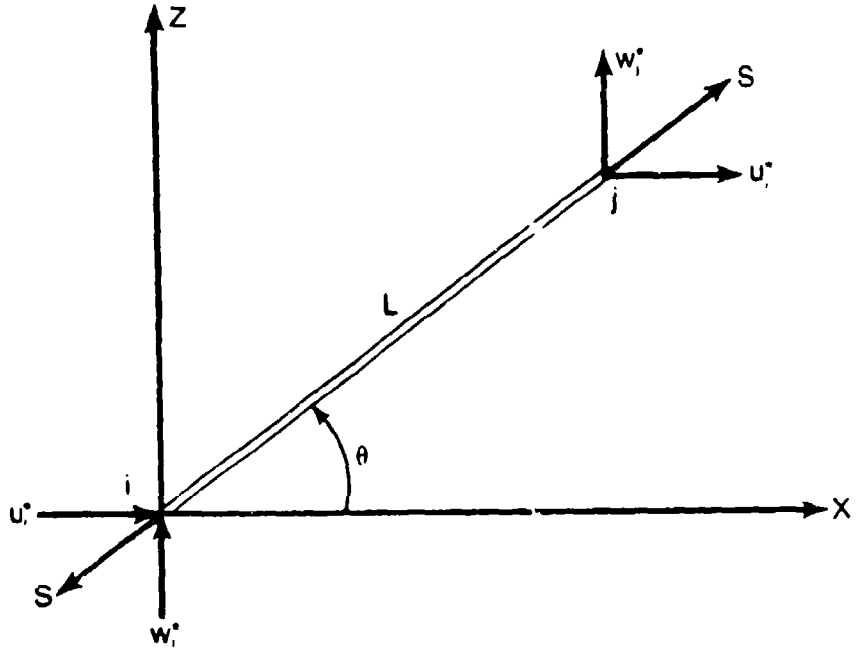


Figure 6. Internal Loads in a Truss Bar.

For a truss bar joining i-joint and j-joint (see Figure 6), the load in the bar is given by

$$S_{ij} = \frac{AE}{L}\{(u_j^* - u_i^*)\cos\theta + (w_j^* - w_i^*)\sin\theta\} \tag{73}$$

where A is the cross-sectional area of the bar, E is Young's modulus, and L is the length. The quantity AE here should not be confused with the extensional stiffness EA of the effective stiffness of the continuum Timoshenko beam model.

3.3.4. Continuum Modeling with Damping.

Damping may add to the dynamic stability of large space structures. In the space environment where air damping is absent, the major source of damping derives from structural deformation, i.e. deformation of structural members and relative movements of joints. If the joints are designed rigid, then material damping may provide the sole source of structural damping in the system. (Fuel sloshing and other "non-structural" forms of damping are neglected here.)

If the stress-strain relation including damping for a truss bar member is known, then the continuum model with a gross damping effect can be derived by the direct structural method as described previously. Abrate and Sun [50] have considered viscous damping in a symmetric truss beam.

For illustrative purposes, consider viscous damping for which the stress- strain relation in a bar is uniaxial and is given by

$$\sigma = E\epsilon + d\dot{\epsilon} \tag{74}$$

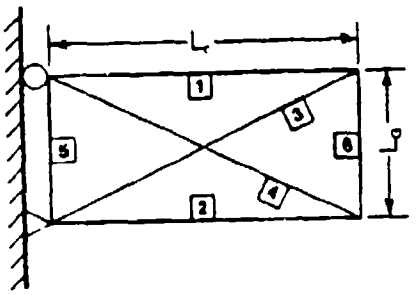


Figure 7. A Typical Truss Cell.

where σ is the uniaxial normal stress, E is the Young's modulus, d is the viscous damping coefficient, and ϵ is the uniaxial strain.

Consider a symmetric truss with a typical substructure as shown in Figure 7. For simplicity assume

$$A_1 = A_2, \qquad A_3 = A_4, \qquad A_5 = A_6 \tag{75a}$$

$$E_1 = E_2, \qquad E_3 = E_4, \qquad E_5 = E_6 \tag{75b}$$

$$d_1 = d_2, \qquad d_3 = d_4, \qquad d_5 = d_6. \tag{75c}$$

Note that in symmetric trusses, the extensional deformation is uncoupled from the flexural deformation. Using the direct structural method, the strains in the members of the substructure can be related to the global bending $\partial\psi/\partial x$, and the transverse shear $\partial w/\partial x + \psi$ by

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_6 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{bmatrix} \frac{\partial \psi}{\partial x} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{bmatrix} \left(\frac{\partial w}{\partial x} + \psi \right) \tag{76}$$

where subscript i indicates member i, and the coefficients a_i and b_i are obtained from the properties of the substructure. If the contraction in the depth direction is neglected, then a_i and b_i can be obtained explicitly as

$$a_1 = -a_2 = L_g/2, \qquad a_3 = a_4 = a_5 = a_6 = 0 \tag{77a}$$

$$b_3 = -b_4 = L_g L_c/(L_g^2 + L_c^2), \qquad b_1 = b_2 = b_5 = b_6 = 0. \tag{77b}$$

The strain rates in the members are given by

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \vdots \\ \dot{\epsilon}_6 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{bmatrix} \frac{\partial \dot{\psi}}{\partial x} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{bmatrix} \left(\frac{\partial \dot{w}}{\partial x} + \dot{\psi}\right). \tag{78}$$

Using (74), (76) and (78), the elastic force and damping force for each member can be calculated, and subsequently the global resultant forces and moments. Then

$$\begin{bmatrix} Q \\ M \end{bmatrix} = \begin{bmatrix} GA & 0 \\ 0 & EI \end{bmatrix} \begin{bmatrix} \partial w/\partial x + \psi \\ \partial \psi/\partial x \end{bmatrix} + \begin{bmatrix} C_{22} & 0 \\ 0 & C_{33} \end{bmatrix} \begin{bmatrix} \partial \dot{w}/\partial x + \dot{\psi} \\ \partial \dot{\psi}/\partial x \end{bmatrix} \tag{79}$$

where

$$GA = \frac{2L_g^2\, L_c\, A_3\, E_3}{(L_g^2 + L_c^2)^{3/2}} \tag{80a}$$

$$EI = \frac{1}{2} L_g^2\, A_1\, E_1 \tag{80b}$$

$$C_{22} = \frac{2\, L_g^2\, A_3\, d_3}{(L_g^2 + L_c^2)^{3/2}} \tag{81a}$$

$$C_{33} = \frac{1}{2} L_g^2 \, A_1 \, d_1. \tag{81b}$$

The equivalent mass properties of the structure are obtained by simply computing the corresponding properties of the substructure and dividing by the length. The equations for the mass and ρI are

$$m = \rho\left[A_5 \, L_G + 2 \, A_1 \, L_c + 2 \, A_3 \, L_3\right]/L_c \tag{82a}$$

$$\rho I = \rho\left[A_5 \, L_G(\frac{L_G^2}{12}) + 2 \, A_1 \, L_c \, (\frac{L_G^2}{2}) + 2A_3 \, L_3 \, (\frac{L_3^2}{12})\right]/L_c. \tag{82b}$$

Differentiation of (79) and substitution into the last equation of (67), gives the following equations of motion:

$$\frac{\partial[GA(\frac{\partial w}{\partial x} + \psi)]}{\partial x} + \frac{\partial[C_{22}(\frac{\partial \dot{w}}{\partial x} + \dot{\psi})]}{\partial x} = m\ddot{w} - q_z. \tag{83a}$$

$$\frac{\partial(EI\frac{\partial \psi}{\partial x})}{\partial x} - GA(\frac{\partial w}{\partial x} + \psi) + \frac{\partial(C_{33}\frac{\partial \dot{\psi}}{\partial x})}{\partial x} - C_{22}(\frac{\partial \dot{w}}{\partial x} + \dot{\psi}) = R\ddot{u} + \rho I\ddot{\psi}. \tag{83b}$$

This continuum model including damping was studied in [47] and [50] for free vibration as well as forced vibration. Excellent results were found.

3.4. Substructuring.

Substructuring is a term associated with the practice of modeling a structure by assembling models of substructures from its component parts. These substructures are analyzed and assembled into the total structure. There are various reasons for substructuring, and various types of substructure models. In the case of large space strucutures, the reasons for substructuring include:

- A multiplicity of particular substructures like solar arrays, habitation modules, etc.
- Model verification advantages – model verification improves as smaller portions of the structure can be tested individually. However, the structure should be designed to minimize the redundancy of load paths at substructure interfaces because these load paths must be verified as well as the substructures themselves.
- Coordinate reduction – although modern digital computers and software have all but eliminated the restrictions on model size which initially gave impetus to substructuring twenty years ago, coordinate reduction is still an important consideration in model verification. When fewer generalized coordinates are required to define the state of a system, fewer structural parameters must be verified experimentally.
- The isolation of nonlinearities – substructuring may be employed to isolate certain types of nonlinearities, such as those due to large rotation, nonlinear hinge mechanisms, and hinge lock- up representing a change in boundary conditions. In this way, linear models may be employed for the substructures themselves, while the nonlinear models are confined to substructure interfaces.
- Contract responsibility – substructuring permits individual contractors to be held responsible not only for the modeling of components they build, but also for their verification.

Various types of substructure models serve different needs. Some of the models appropriate for large space structures include the following:

- Conventional finite element models (FEM) or lumped parameter models (LPM), also referred to as "spatial models" [51], – this is perhaps the simplest type of model, consisting of mass and stiffness matrices defined by physical nodal coordinates. Occasionally the damping matrix may also be included whenever a dominant damping mechanism such as visco-elastic damping can be identified.

- Modal models – these are models whose equations of motion are linear, have constant coefficients, and are expressed in terms of generalized coordinates defined by the normal modes of vibration and/or static shape functions including rigid body modes. (These are the models typically used in "modal synthesis.")
- Equivalent continuum models–as shown in Section 3.3, certain types of substructures lend themselves to equivalent continuum modeling, where the properties of an equivalent continuum are derived from the properties of the actual structure. The continuum model eventually must be transformed to either a finite element-type model or to a modal-type model if it is to be integrated with the discretized models representing other parts of the structure.
- Input/output models, also referred to as "response models" [51] – these models are particularly useful for subsystems which do not lend themselves to conventional analytical modeling, but which can be characterized by experimentally derived input/output relationships.

Modal models are an important class of models for substructuring. They can be derived from each of the other three types of models whenever the systems are linear and time invariant. Modal models are advantageous in system identification because they provide a direct basis of comparison between an analytical model, for which modal characteristics can be derived, and the physical system whose modal characteristics can be observed experimentally. When modal models are used in substructuring, a number of technical issues arise, all related to the fundamental question of how well the substructure modes, with their artificial boundary conditions, represent the behavior of that substructure when it is integrated with the rest of the system. Both deformation and stress must be accurately represented. The key issues are:

1. Selection of boundary conditions - they may be fixed, free or artificially loaded with known mass and/or stiffness elements; these are subsequently removed from the model analytically.
2. Modal truncation - how many subsystem modes must be retained in order to achieve acceptable accuracy in the system modes and how can truncation error be evaluated?
3. Residual mass and stiffness - how can contributions of the residual mass and stiffness of the truncated modes to the frequency response at lower frequencies be recovered?
4. Damping - what substructure damping properties are required to synthesize system damping, and how can they be measured?

System identification adds another layer of complexity and difficulty to the resolution of these issues, because the system model being synthesized from subsystem models is not precisely known. This imposes the additional requirement that subsystem models be capable of adjustment to match physical observations of the actual system's behavior. This may mean, for example, that more subsystem modes must be included in the model. Potential problems of this nature have not yet been addressed; they are beyond the present state of the art.

3.4.1. Mathematical Context.

These issues are better understood within the mathematical context of substructuring. The following development presents the essential features of this context. For simplicity, a system comprised of only two susbsystems, S^a and S^b, is considered.

The equations of motion for a typical subsystem, S^a, may be written as follows:

$$M^a \ddot{x}^a + D^a \dot{x}^a + K^a x^a = f_x^a. \tag{84}$$

In most cases, the damping matrix, D^a, is not known, so the undamped modes are used to effect the transformation

$$x^a = \phi^a q^a. \tag{85}$$

Depending on the boundary conditions selected for the homogeneous form of (84) the mode shapes, ϕ^a, may be free-free, fixed-fixed, or anything in between. In general, ϕ^a may contain three types of modes: rigid body modes (R), static deformation (constraint or attachments) modes (C), and dynamic deformation (normal) modes (N).

$$\phi^a = [\phi^R \; \phi^C \; \phi^N]^a. \tag{86}$$

These modes may be employed in any combination, depending on the model, i.e. they may be all R, all C, all N or any combination of R, C and N - type modes.

Transformation of (84) by (85) leads to equations of the form

$$m^a \ddot{q}^a + d^a \dot{q}^a + k^a q^a = f_q^a \tag{87}$$

where

$$m^a = (\phi^a)^T M^a \phi^a \tag{88a}$$

$$d^a = (\phi^a)^T D^a \phi^a \tag{88b}$$

$$k^a = (\phi^a)^T K^a \phi^a \tag{88c}$$

$$f_q^a = (\phi^a)^T f_x^a. \tag{88d}$$

Substructure equations of motion are coupled by applying displacement compatibility constraints of the form

$$C^a x^a + C^b x^b = 0. \tag{89}$$

This leads to the transformation

$$\begin{bmatrix} q^a \\ q^b \end{bmatrix} = \beta\, q^c \tag{90}$$

which, when applied to the uncoupled equations, leads to

$$\beta^T \begin{bmatrix} m^a & 0 \\ 0 & m^b \end{bmatrix} \beta\, \ddot{q}^c + \beta^T \begin{bmatrix} d^a & 0 \\ 0 & d^b \end{bmatrix} \beta \dot{q}^c + \beta^T \begin{bmatrix} k^a & 0 \\ 0 & k^b \end{bmatrix} \beta q^c = \beta^T \begin{bmatrix} f_q^a \\ f_q^b \end{bmatrix} \tag{91}$$

or

$$m^c \ddot{q}^c + d^c \dot{q}^c + k^c q^c = f_q^c. \tag{92}$$

At this point it is assumed that some estimate of the damping matrix, d^c, will be available, either from analytical considerations which produce D^a and (or) D^b, or from substructure tests which produce d^a and/or d^b. Details of this topic will be taken up later. Knowledge of d^c presents a choice between solving the homogeneous form of (92) for the damped or undamped modes. In either case, the purpose is twofold: (1) to diagonalize or nearly diagonalize the equations of motion so they may be more easily solved for the forced response, and (2) to provide a basis of comparison with experimental data. Both the damped and undamped mode approaches are presented here for reference.

The undamped modes are obtained by solving the eigenproblem

$$(k^c - \lambda_j m^c)\phi_j = 0 \tag{93}$$

where ϕ_j is the jth undamped modal vector (eigenvector) corresponding to the eigenvalue $\lambda_j = \omega_{0_j}^2$. The matrices k^c and m^c are diagonalized by the transformation

$$q^c = \phi q \tag{94}$$

Transformation of (92) thus leads to

$$I\ddot{q} + \xi\dot{q} + \omega_o^2 q = f_q \tag{95}$$

where

$$\phi^T m^c \phi = I \qquad \text{(diagonal)} \tag{96a}$$

$$\phi^T d^c \phi = \xi \tag{96b}$$

$$\phi^T k^c \phi = \omega_o^2 \qquad \text{(diagonal)} \tag{96c}$$

$$\phi^T f_q^c = f_q. \tag{96d}$$

In the case of light damping, i.e. $\xi_{jj}/2\omega_{o_j} = \varsigma_j << 1$, (95) may be solved either in the time domain or the frequency domain by perturbation methods where the modal damping matrix, ξ, is separated into two matrices containing the diagonal elements, ξ_d, and off- diagonal (or nondiagonal) elements, ξ_n, respectively. In the time domain, (95) may be written (for purposes of approximation) as

$$I\ddot{q} + \xi_d\dot{q} + \omega_o^2 q = f_q - \xi_n\dot{q} \tag{97}$$

where the left hand side of the equations has been diagonalized.

The equations of motion are transformed to the frequency domain by taking the Fourier transform of (95). This gives

$$[(\omega_o^2 - \omega^2 I) + i\omega\xi]Q(i\omega) = F_q(i\omega). \tag{98}$$

The matrix in square brackets is known as the complex impedance matrix, $Z(i\omega)$. This matrix may be separated into diagonal and off- diagonal parts, giving

$$[Z_d(i\omega) + Z_n(i\omega)]Q(i\omega) = F_q(i\omega) \tag{99}$$

where

$$Z_d(i\omega) = [(\omega_o^2 - \omega^2 I) + i\omega\xi_d] \tag{100a}$$

$$Z_n(i\omega) = i\omega\xi_n. \tag{100b}$$

Then using $Z_d^{-1/2}(i\omega)$ as a scaling transformation where

$$Q(i\omega) = Z_d^{-1/2}(i\omega)Q'(i\omega) \tag{101}$$

one obtains

$$(I + Z_d^{-1/2} Z_n Z_d^{-1/2})Q' = Z_d^{-1/2} F_q \tag{102}$$

where the argument $(i\omega)$ has been omitted for notational simplicity. In the case of light damping, (98) has the following approximate solution:

$$\begin{aligned} Q &= Z_d^{-1/2}(I + Z_d^{-1/2} Z_n Z_d^{-1/2})^{-1} Z_d^{-1/2} F_q \\ &\approx Z_d^{-1/2}(I - Z_d^{-1/2} Z_n Z_d^{-1/2}) Z_d^{-1/2} F_q \end{aligned} \tag{103}$$

The significance of this result is that the frequency-dependent complex matrix inversion is avoided, leading to an efficient way of solving the equations of motion for complex frequency response, without having to solve the $2n \times 2n$ eigenproblem for complex eigenvalues and eigenvectors.

The frequency response matrix in the generalized q-coordinate system is recognized to be

$$H_q = Z_d^{-1/2}(I - Z_d^{-1/2} Z_n Z_d^{-1/2}) Z_d^{-1/2}. \tag{104}$$

In the original x-coordinate system it is simply

$$H_x = \begin{bmatrix} H_x^a \\ H_x^b \end{bmatrix} = T H_q T^T \tag{105}$$

where

$$T = \begin{bmatrix} \phi^a & 0 \\ 0 & \phi^b \end{bmatrix} \beta\phi = \phi^c \beta\phi. \tag{106}$$

The damped modes of (92) are obtained by placing the second-order equations in first- order form

$$\begin{bmatrix} d^c & m^c \\ m^c & 0 \end{bmatrix} \begin{bmatrix} \dot{q}^c \\ \ddot{q}^c \end{bmatrix} + \begin{bmatrix} k^c & 0 \\ 0 & -m^c \end{bmatrix} \begin{bmatrix} q^c \\ \dot{q}^c \end{bmatrix} = \begin{bmatrix} f_q^c \\ 0 \end{bmatrix}$$

or

$$A^c \dot{r}^c + B^c r^c = f_r^c. \tag{107}$$

Complex eigenvalues, Λ_j, and eigenvectors, Φ_j, are then obtained by solving the eigenproblem

$$(A^c\Lambda_j + B^c)\Phi_j = 0. \tag{108}$$

If d^c is assumed to be symmetric, then both A^c and B^c are real symmetric matrices, and the eigenvectors, Φ, will diagonalize both of them. Normalization of Φ such that

$$\Phi^T A^c \Phi = I \tag{109}$$

leads to a particularly convenient diagonalized form of (107)

$$I\ddot{r} + \Lambda r = f_r \tag{110}$$

where

$$\Phi^T B^c \Phi = \Lambda \tag{111a}$$

$$\Phi^T f_r^c = f_r. \tag{111b}$$

Transformation of (110) to the frequency domain gives

$$(\Lambda + i\omega I)R(i\omega) = F_r(i\omega). \tag{112}$$

The frequency response matrix in the generalized r-coordinate system is therefore

$$H_r(i\omega) = (\Lambda + i\omega I)^{-1}. \tag{113}$$

Transformation back to the x-coordinate system gives

$$H_x(i\omega) = TH_r(i\omega)T^T \tag{114}$$

where

$$T = \begin{bmatrix} \phi^a & 0 \\ 0 & \phi^b \end{bmatrix} \beta \phi^U = \phi^c \beta \Phi^U \tag{115}$$

and where ϕ has been partitioned such that

$$r^c = \begin{bmatrix} q^c \\ \dot{q}^c \end{bmatrix} = \begin{bmatrix} \Phi^U \\ \Phi^L \end{bmatrix} r = \Phi r. \tag{116}$$

It is of interest to note here that

$$\Phi^L = \Phi^U \Lambda. \tag{117}$$

The damped mode approach has been deliberately formulated to reveal the parallel nature of the damped and undamped mode approaches. The trade-offs between the two are then obvious. In summary, the frequency response in both cases is given by

$$H_x(i\omega) = \phi^c \beta H_{q^c}(i\omega) \beta^T (\phi^c)^T. \tag{118}$$

In the undamped mode case, $H_{q^c}(i\omega)$ is given by

$$\begin{aligned} H_{q^c}(i\omega) &= \phi Z_d^{-1/2} (I + Z_d^{-1/2} Z_n Z_d^{-1/2})^{-1} Z_d^{-1/2} \phi^T \\ &\approx \phi (Z_d^{-1} - Z_d^{-1} Z_n Z_d^{-1}) \phi^T \end{aligned} \tag{119}$$

where Z_d and Z_n are given by (100). In the damped mode case, $H_{q^c}(i\omega)$ is given by

$$H_{q^c}(i\omega) = \Phi^U (\Lambda + i\omega I)^{-1} (\Phi^U)^T. \tag{120}$$

Thus it is seen that the benefit of the complex mode formulation is that the approximation in (119) is avoided. Conversely, the benefit of the real mode formulation is that the computational burden of solving the $2n \times 2n$ complex eigenproblem is avoided. It can be

shown mathematically † that for light damping (*e.g.* $\varsigma < 0.1$) the error of approximation in (119) is negligible, even for closely spaced modes. In any case, the error is less than that introduced by the assumption of a diagonal modal damping matrix, which is equivalent to ignoring the term, $Z_d^{-1} Z_n Z_d^{-1}$, in (119).

3.4.2. Boundary Conditions.

Inasmuch as model verification is one of the primary reasons for substructuring, the boundary conditions selected for substructure modes must be compatible with experimental capability. For example, very large reaction masses required to achieve fixed boundary conditions on large structures may be impractical. Mass loading and stiffness loading at interface boundaries will be subject to practical limitations consistent with space, geometry, attachment points, etc. The easiest boundary conditions to implement in the test laboratory are free boundaries. However, unless a particular substructure boundary is also free in the assembled system, it will not serve well in modal synthesis unless the resulting modal model is augmented with residual mass and stiffness as discussed in Section 3.4.4.

The best choice for boundary conditions is that which most nearly represents the actual structural interface in the assembled structure. Residual mass and stiffness contributions to the modal mass and stiffness matrices will be required to the extent that optimum conditions are not achieved. The simulation of this structural interface is an art, and requires considerable understanding and insight.

Historically, the fixed-interface method was first proposed by Hurty [53]. A simplification to Hurty's method was suggested by Craig and Bampton [54]. The methods of

† *Gershgorin's Disk Theorem [52] states that all of the eigenvalues of the complex matrix G lie in at least one of the disks of radius* $r_j = \sum_k |G_{jk}|$ *centered at* G_{jj}.

Gladwell [55] and Benfield and Hruda [56] represent the earliest reports on the use of artificially loaded-interface modes. Procedures reported by Goodman [57] and Hou [58] rely on free-interface boundary conditions. Since the publication of these earlier papers, numerous variations and applications have been reported, e.g. [59]-[65]. Modal synthesis methods have also been described in at least two text-books [66], [67]; Craig [68] and Fleming [69] present recent reviews of the methods, Craig from a theoretical viewpoint, and Fleming from an aerospace applications point of view.

3.4.3. Modal Truncation and Convergence.

Modal truncation introduces modeling error. The object of component mode synthesis is to replace the conventional finite element mass and stiffness model with a modal model which incorporates a limited number of modes. The more representative the isolated component modes are of the component's deformation and stress within the assembled structural system, the fewer the number of component modes required to achieve a given level of accuracy. Studies have been made to assess this truncation error, and to derive various convergence indicators which may be applied to estimate truncation error without having to rerun the problem with a larger number of modes. Hurty presented a convergence indicator for eigenvalues based on perturbation analysis [70]. Hasselman extended this analysis to obtain a convergence indicator for eigenvectors [71], [72]. Hasselman also derived a convergence indicator based on the gradient of the Rayleigh Quotient in a subspace orthogonal to the lower modes [71], [72]. In general, the convergence indicator for the nth system mode is given as the squared magnitude of the gradient, projected onto a subspace orthogonal to the first $n-1$ modes. For small errors where the small perturbation assumption holds, this convergence indicator reduces to Hurty's eigenvalue convergence indicator. However, whereas the perturbation indicators are meaningless when the small perturbation assumption fails, the gradient indicator provides a lower bound on the eigenvalue error even when the error is not small, making it more useful in pratical situations.

3.4.4. Residual Mass and Stiffness.

As might be expected, the convergence of modal synthesis which employs free- interface component modes is generally poor relative to the other methods where the structural interface is loaded in some manner. Free- interface modes should only be employed whenever proper account is taken of the "residual" (leftover) stiffness and mass associated with the truncated (eliminated) modes. McNeal [73] proposed the use of a residual stiffness matrix to augment a modal model based on free-interface modes. Rubin [74] extended this concept to include residual mass (and damping) matrices and showed how the parameters could be extracted from an experimentally derived mobility matrix. Martinez, et al, [75] employed a similar technique in deriving residual stiffness and mass matrices, specifically addressing the problem of rotational compatibility at the substructure interface. Hruda [76] recently compared the use of free-interface/residual flexibility versus fixed-interface methods for Shuttle loads analysis.

3.4.5. Damping.

When substructuring is not being employed, the treatment of damping in the equations of motion usually presents no problem; uncoupled modal damping is almost always assumed, and is often derived from experimental measurements. The term uncoupled modal damping implies that either the modal damping matrix, e.g. d^a in (88b), or ξ in (95) is diagonal, or that the off- diagonal elements may be neglected without introducing significant error in the analysis. In reality, the modal damping matrix is rarely (if ever) diagonal; i.e. "proportional damping" or other special forms of damping which allow the modal damping matrix to be diagonalized by undamped modes is merely a mathematical convenience. However, even when the off-diagonal terms of the modal damping matrix are of the same order as the diagonal terms, they may be neglected as long as the modal frequencies are not closely spaced [77]. This is not true in modal synthesis, i.e. the off-

diagonal terms of the component modal damping matrices, e.g. d^a in (88b), may not be neglected as explained in [79]. Hasselman [79] suggested a method for generating the full modal damping matrix from experimental estimates of the damped (complex) modes.

If the complex conjugate eigenvalues Λ_j^a and eigenvectors Ψ_j^a of subsystem S^a are given by

$$\Lambda_j^a = \sigma_j^a \pm i\omega_{d_j}^a \tag{121a}$$

$$\Phi_j^a = \Phi_{R_j}^a \pm i\Phi_{I_j}^a \tag{121b}$$

where $i = \sqrt{-1}$, then the diagonal and off-diagonal elements of d^a are given by

$$d_{jj}^a = -2\sigma_j^a = 2\varsigma_j^a \omega_{o_j}^a \tag{122a}$$

$$d_{jk}^a = \omega_{d_j}^a (\Phi_{I_j}^a)^T M^a \Phi_{R_k} + \omega_{d_k}^a (\Phi_{R_j}^a)^T M^a \Phi_{I_k}^a \tag{122b}$$

where ς_j^a is the *jth* modal damping ratio and $\omega_{o_j}^a$ is the *jth undamped* modal frequency. The advantage of (122) is that the full modal damping matrix, d^a, can be evaluated without knowledge of the equivalent viscous damping matrix, D^a, in (88b).

Both time-domain [80], [81] and frequency-domain methods [82], [83] for estimating the complex modes have been developed (see Section 5); however, the small imaginary parts of the complex eigenvectors are not normally used, and therefore not usually reported. It is not known to what extent these estimates have been scrutinized to assess their meaningfulness; they could be contaminated by noise.

The diagonal elements of d^a (or ξ) and perhaps those off-diagonal elements *associated with closely spaced modes* can be estimated along with m and k (Equations 61 and

65) when the estimation is based on frequency response measurements. However, the remaining off-diagonal terms will not be detectable [77]. Reference [78] suggests an alternate method for evaluating the complex eigenvectors and d^a (or ξ) from frequency response measurements whenever the modes are not closely spaced.

Several other papers on damping synthesis have appeared in the literature during the past 10-15 years [84]-[91]. The primary difficulties continue to be (1) the accurate estimation of complex eigenvectors, mass distribution, and the off-diagonal elements of the modal damping matrix; (2) the estimation of damping contributed by substructure interfaces, or portions of the substructures near these interfaces; (3) the treatment of damping nonlinearities; (4) the effect of prestress on damping, and (5) the effect of environmental conditions in general (gravity, atmospheric, thermal, etc.) on structural damping. Reviews on the subject of damping synthesis are presented in [92], [93]; however, these reports are at least five years old and do not reflect more recent publications.

3.4.6. Eigenvalue/Eigenvector Derivatives.

The closed-form evaluation of eigenvalue and eigenvector derivatives plays an important role in parameter estimation. These derivatives indicate the sensitivity of structural modes and frequencies to individual parameter variations, and thereby determine how large a variation is required to effect a desired change in an eigenvalue or eigenvector. Eigenvalue and eigenvector derivatives also play an important role in characterizing modeling uncertainty by first-order statistical methods, as discussed in Section 3.6.

The determination of eigenvalue derivatives is shown in References [94], [95], [96] to be a straighforward and simple calculation, involving only the eigenvector of the eigenvalue derivative being calculated. For undamped systems, the derivative of the jth eigenvalue, λ_j, to the parameter, θ_k, is given by

$$\frac{\partial \lambda_j}{\partial \theta_k} = \phi_j^T \left(\frac{\partial K}{\partial \theta_k} - \lambda_j \frac{\partial M}{\partial \phi_k} \right) \phi_j \tag{123}$$

in the case where the eigenvectors are normalized to unit modal mass. A similar expression is obtained for eigenvector derivatives.

$$\frac{\partial \phi_j}{\partial \phi_k} = \phi \gamma_j \tag{124a}$$

where γ_j is a vector whose elements are

$$\gamma_{ij} = \phi_i^T \left[\frac{1 - \delta_{ij}}{\lambda_j - \lambda_i} \left(\frac{\partial K}{\partial \theta_k} - \lambda_j \frac{\partial M}{\partial \theta_k} \right) - \frac{\delta_{ij}}{2} \frac{\partial M}{\partial \theta_k} \right] \phi_j . \tag{124b}$$

In the case of complex eigenvalues and eigenvectors, the corresponding expressions are

$$\frac{\partial \Lambda_j}{\partial \theta_k} = -\Phi_j^T \left[\Lambda_j \frac{\partial A}{\partial \theta_k} + \frac{\partial B}{\partial \theta_k} \right] \Phi_j \tag{125}$$

and

$$\frac{\partial \Phi_j}{\partial \theta_k} = \Phi \Gamma_j \tag{126a}$$

where

$$\Gamma_{ij} = \Phi_i^T \left[\frac{1 - \delta_{ij}}{\Lambda_j - \Lambda_i} \left(\Lambda_j \frac{\partial A}{\partial \theta_k} + \frac{\partial B}{\partial \theta_k} \right) - \frac{\delta_{ij}}{2} \frac{\partial A}{\partial \theta_k} \right] \Phi_j . \tag{126b}$$

It is observed in (124), as well as in (126), that this method for calculating eigenvector derivatives in general requires all of the eigenvectors; however, a good approximation is usually obtained from a truncated set as long as a sufficient number is retained. For reduced models, there is no need to truncate the modal matrix because the cost of computing the modes is small. For large-order models, Nelson [97] has presented a method for calculating eigenvector derivatives which requires only the eigenvector corresponding to the eigenvector

derivative being calculated. However, a set of linear algebraic equations involving K, M (and D in the case of damped modes) must be solved.

When modal synthesis is employed, the question of truncation error arises with regard to how many component modes are required to achieve satisfactory accuracy in the calculation of system eigenvalue and eigenvector derivatives. Hasselman [98] addresses this problem for the case of undamped modes. It is shown that the number of component modes required to achieve a given degree of accuracy increases as one progresses from eigenvalues to eigenvectors to eigenvalue derivatives to eigenvector derivatives, but that a limited number of component modes can yield an accurate approximation.

3.5. Modeling Uncertainty.

Several types of modeling uncertainty are recognized as being important in the identification of large space structures. Ideally, one would like to associate modeling uncertainty with the degree to which a model is able to predict the response of a system to known inputs, assuming that the response can be measured without error. Modeling uncertainty defined in this way results from two types of modeling error – random error which may be attributed to the lack of perfect control on the experiments conducted to measure structural response, and systematic error which may be attributed to an incorrect model. The distinction is an important one to the extent that systematic errors may be eliminated by identifying the sources of error and removing them. Random error can be reduced by exercising greater control over the experiments; in general it cannot be eliminated.

Modeling uncertainty is reduced during model validation and verification. Validation has been identified with the process of ascertaining whether the structure of a model (equations of motion) is correct, and verification has been identified with refining the parameter estimates and verifying by numerical comparison that the model agrees with

experimental data and prior knowledge of the model (Section 1.3). Both entail the removal of systematic error. Random error can be minimized in the process by carefully controlling the experiments. For example, if flight hardware is being tested, the variations in materials and manufacturing are eliminated as sources of random error by testing the particular hardware being modeled. If prototype hardware is being tested, some control over the experiment is lost to the extent that random variations occur from one piece of hardware to another. The testing of scale models introduces additional error, both random and systematic, but is clearly superior to not testing at all, which may be the only other alternative for large structural assemblies.

Even the most carefully controlled experiments cannot preclude the type of error resulting from environmental effects which cannot be duplicated in a test laboratory, or otherwise accurately observed and accounted for in the model. This type of error may contain both random and systematic error. In general, systematic error which cannot be detected must be treated as random, in the sense that it is defined by probability distributions rather than deterministic quantities.

There are basically two ways to quantify modeling uncertainty; they will be referred to as inductive and deductive. The deductive approach would make direct comparisons between observed quantities, e.g. natural frequencies or dynamic response, and predictions of those same quantities based on the model. Given sufficient observations, modeling uncertainty can be quantified statistically. The inductive approach would use statistical estimates of model parameters to generate variations in the behavior of interest (natural frequencies or dynamic response). These variations can be generated by straightforward simulation (Monte Carlo), or approximate means such as first- order statistical modeling based on a linearized Taylor series expansion of the behavior about a nominal set of parameter values treated as random variables.

As an illustration of this latter approach, one may consider the uncertainty in a natural frequency of vibration as follows: A Taylor series expansion of the undamped natural frequency, ω_{o_j}, about the nominal values of the parameter vector, θ_k, would be written as

$$\omega_{o_j} = {}^{o}\omega_{o_j} + \sum_k \frac{\partial \omega_{o_j}}{\partial \theta_k} \Delta\theta_k + \cdots \quad \text{H.O.T..} \tag{127}$$

Then

$$\Delta\omega_{o_j} = \omega_{o_j} - {}^{o}\omega_{o_j} = \{\partial\omega_{o_j}/\partial\theta\}^T \{\Delta\theta\} \tag{128}$$

The variance of ω_{o_j} is given by

$$E[(\Delta\omega_{o_j})^2] = E[\{\partial\omega_{o_j}/\partial\theta\}^T \Delta\theta\Delta\theta^T \{\partial\omega_{o_j}/\partial\theta\}] \tag{129a}$$

$$\sigma^2_{\omega_{o_j}} = \{\partial\omega_{o_j}/\partial\theta\}^T S_{\theta\theta} \{\partial\omega_{o_j}/\partial\theta\} \tag{129b}$$

where $S_{\theta\theta}$ is the covariance matrix of the parameter vector, θ, and $\{\partial\omega_{o_j}/\partial\theta\}^T$ is a row vector of partial derivatives. The frequency derivatives are directly proportional to the eigenvalue derivatives given in (123) and (125).

Many difficulties are encountered with the inductive approach. For example,

- There are probably more sources of uncertainty than can be identified.
- Even for those which can be identified, there are relatively few sources of data upon which to base quantitative estimates, let alone statistical estimates with any confidence.
- And, even if all sources of uncertainty could be identified and appropriately quantified, one would face the immense task of combining them computationally.

An incomplete list of possible sources of modeling uncertainty is shown in Table 2 to emphasize these points.

Table 2. Sources of Modeling Uncertainty.

A. Material Properties

- Elastic constants
- Inelastic constants
- Creep properties
- Thermal properties
- Damping properties
- Density

B. Manufacturing, Fabrication, Construction, Deployment

- Sectional properties (I, A, t, etc.)
- Dimensional properties (assembly tolerances)
- Joints
- Fasteners
- Prestressing
- Deployed geometry

C. Effects of Ambient Loads

- Thermal loads (static, steady-state, transient)
- Microgravity
- Gravity gradient
- Aerodynamic
- Solar particles

D. Modeling Techniques

- Lumped parameter modeling
  - Equivalent Stiffnesses
  - Mass Aggregation
  - Damping assumptions
- Finite element modeling
  - Mesh size and nodal geometry
  - Assumed displacement fields
  - Assumed mass distribution
  - Coordinate reduction
  - Selected dynamic degrees of freedom
  - Selected response coordinates
- Equivalent continuum modeling
  - Assumption of uniformity
  - Other idealizations of geometry
  - Simplification of boundary conditions
  - Discretization (see finite element modeling)
- Modal synthesis
  - Approximate boundary conditions
  - Modal truncation
  - Treatment of residuals
  - Damping synthesis
- I/O Modeling

- Experimental error
- Measurement noise
- Computational error
- Discretization error (time or frequency)

- Fluid-structure interaction
- Damping
  - Assumed viscous damping
  - Assumed complex modulus damping
  - Other idealizations of damping mechanisms
  - Assumed uncoupled modal damping
  - Variability in measured modal damping ratios
- Modeling of composite materials
  - Orientation of plies
  - Bonding
  - Inhomogeneity
- Effects of nonstructural elements
- Modeling of nonlinearities
  - Material (stress-strain, viscoelastic properties)
  - Geometric (large deformation, free play, joint slippage, local buckling)
  - Amplitude dependence (stiffness and damping)
  - Cycle/history dependence (stiffness and damping)

E. Analysis Methods

- Modal Analysis
  - Convergence of numerical computations
  - Orthogonality of eigenvectors
  - Skipped modes
  - Numerical instability
  - Round-off error
- Linear response analysis
  - Coordinate selection
  - Modal truncation
  - Forcing function uncertainty
- Nonlinear response analysis
  - Numerical stability/convergence
  - Resolution
  - Forcing function uncertainty

As an example of the deductive approach, one may consider expressing modeling uncertainty in terms of the modal matrix parameters, Δm and Δk defined in (64). Here, Δm and Δk are defined in terms of the difference between predicted and measured eigenvalues and the cross-orthogonality of predicted and measured eigenvectors expressed by (63). As shown in [99], a statistical analysis may be performed, given sets of predicted and measured eigenvalues and eigenvectors for generically similar structures. First Δk must be

normalized to remove frequency dependency. This is readily accomplished by normalizing the elements of Δk as follows

$$\Delta\tilde{k}_{jj} = \frac{\omega_{o_j}^2 - {}^o\omega_{o_j}^2}{{}^o\omega_{o_j}^2} + 2(1 - \psi_{jj}) \tag{130a}$$

$$\Delta\tilde{k}_{jk} = \frac{{}^o\omega_{o_j}^2}{{}^o\omega_{o_k}{}^2}(1 - \psi_{jk}) + \frac{{}^o\omega_{o_k}^2}{{}^o\omega_{o_j}^2}(1 - \psi_{kj}). \tag{130b}$$

Now if the elements of Δm and Δk are considered to make up a parameter vector, $\Delta\tilde{\theta}$, such that

$$\Delta\tilde{\theta} = \{\Delta m_{11}, \Delta m_{12}, \cdots, \Delta m_{22}, \Delta m_{23}, \cdots, \Delta m_{nn},$$

$$\Delta\tilde{k}_{11}, \Delta\tilde{k}_{12}, \cdots, \Delta\tilde{k}_{22}, \Delta\tilde{k}_{23}, \cdots, \Delta\tilde{k}_{nn}\}^T \tag{131}$$

then the covariance matrix, $S_{\tilde{\theta}\tilde{\theta}}$, is given by

$$S_{\tilde{\theta}\tilde{\theta}} = E[\Delta\tilde{\theta}\Delta\tilde{\theta}^T]. \tag{132}$$

Application of this generic statistical model to a particular structure requires the rescaling of $S_{\tilde{\theta}\tilde{\theta}}$ in accordance with the particular modal frequencies of that structure. This results in a structure-specific statistical model, $S_{\theta\theta}$, as shown in [99]. The correlation structure of $S_{\theta\theta}$ is an important part of this statistical model. The statistics embodied in $S_{\theta\theta}$ can be propagated forward to evaluate response uncertainty, or backward (relative to the inductive modeling process) to identify dominant sources of physical parameter uncertainty.

The type of uncertainty model represented by $S_{\theta\theta}$ is one based on previous experience in modeling and testing generically similar structures. Structure-specific testing and

model verification not only adjusts a model in the sense of removing systematic error, but also reduces modeling uncertainty. One must be careful, however, to distinguish between improved confidence (reduced uncertainty) in the estimation of mean parameter values, and the reduction of modeling uncertainty one would expect to observe in evaluating the covariance matrices, $S_{\hat{\theta}\hat{\theta}}$ and $S_{\theta\theta}$, on the basis of verified model predictions as opposed to unverified model predictions.

3.6. Experimental Considerations.

The design of experiments for system identification is very important. With reference again to Section 1.3.1 and Figure 2, it is seen that the design of experiments must take into account not only the physical hardware, but also the form of the analytical model, the estimation algorithms to be used in verifying the model, and most importantly, the verification procedures which determine how these algorithms will be used to process experimental data to refine and verify the analytical models. Just as it was not deemed within the scope of this report to review structural modeling procedures in depth, so it is in the case of testing. The discussion here will highlight those aspects of testing and experiment design which are especially pertinent to structural system identification.

The primary goal of testing is to simulate the structure, the service environment and the conditions of service the structure will experience so that the behavior of the structure may be observed and recorded. When testing for purposes of model verification, additional objectives are introduced. They follow from the model verification plan which must coordinate the objectives of testing to fully identify the characteristics of the structure. In the case of large space structures, several types of tests are envisioned, including ground testing as well as testing in space.

Ground testing will be used to identify the most detailed characteristics of small components. Once component models have been verified individually, assemblies of two or more components may be tested to verify their interface models. These tests may proceed up to the point where the size of the structure exceeds the limited space of the test laboratory, or it is no longer feasible to test in an earth-gravity environment. Scale modeling may then be employed to permit ground testing of major assemblies and the complete structural system. Suspension design will attempt to simulate weightless conditions in a limited way. Other limitations affect the ability to simulate the atmospheric and thermal environment of space. Thus, while component, assembly and scale model testing may enable the mass and stiffness properties of a large space structure to be verified under the particular conditions of prestress extant in a ground test laboratory, they will not be able to simulate the prestress conditions of a space environment. Nor will they simulate conditions under which structural damping can be properly estimated.

It is anticipated that some testing in space will be required as part of the identification process, before the structure is placed in service. Again, component testing and/or the testing of assembled components would appear to be desirable. Scale model testing in space may also have some benefit. All of these considerations must be taken into account when contemplating the type of tests required for identification of large space structures.

The subsections which follow address five areas of experiment design:

- The Test Environment
- Structural Boundary Conditions
- Excitation
- Measurement
- Data Acquisition and Reduction

Each of these areas should be covered in the preparation of a model verification plan. The purpose of such a plan is twofold: (1) to establish a set of requirements for testing

in the form of a test specification and (2) to establish specific uses for the test data in the identification process. Test specifications will serve as the basis for writing detailed test plans. A verification plan will guide the subsequent utilization of specific data. It also provides input to technical management, which must coordinate various functional activities.

3.6.1. Test Environment.

The test environment will attempt to simulate those aspects of the service environment which affect structural behavior; i.e. in the case of space structures, they include the gravity environment, the thermal environment, and the atmospheric environment. While this will be difficult for ground vibration testing of the larger assemblies, smaller components and smaller scale models may be tested in a thermal vacuum chamber. Specially designed suspension systems could improve the ability to simulate weightlessness during the ground testing of larger assemblies.

3.6.2. Structural Boundary Conditions.

As discussed in Section 3.4.2, the boundary conditions selected for experimental evaluation of structure and substructure characteristics in the test laboratory should attempt to simulate those which will be in effect under actual service conditions. In the case of substructure testing, the simulation of boundary conditions at substructure interfaces is somewhat less demanding. When interface loading (either mass or stiffness) is employed, it need not necessarily be of the same magnitude as it would be under actual conditions, as long as it represents a reasonable approximation. An alternative is free boundary conditions, with mobility measurements at the boundary to identify residual mass and stiffness [74]. At external boundaries, accurate simulation of the boundary conditions is essential.

3.6.3. Excitation.

In order to observe structural behavior under static or dynamic load conditions it is necessary that the structure be excited in a way which reveals the behavior of interest. Excitation may be provided by ambient (environmental) load conditions or by mechanical actuators. Forced excitation is always preferable because it is easier to measure and control. Various types of actuators are used; the selection for a particular application depends not only on the type of excitation required to produce the desired structural behavior, but also on the environmental conditions in which the actuator must function. Electrodynamic, hydraulic, reciprocating mass and rocket-type actuators are among the types commonly used.

In the case of dynamic excitation, the "waveform" of the applied force determines the frequency content of the information learned about the structure. Commonly used waveforms include stationary sine, slow sine sweep, fast sine sweep (sometimes called chirp) damped (exponentially decaying) sine, impulse, stationary random and burst random. The frequency domain analysis described in Section 4.2 may be used with all of these waveforms. Any of these forms of excitation may be applied at either single points or multiple points simultaneously. Multi- point input testing is a farily recent development, except for stationary sine testing which has utilized multiple inputs for decades.

The duration of stationary excitation or the number of repetitions of transient inputs affects the accuracy of experimental data. Greater accuracy is achieved as more repetitions are used in an averaging process. In the case of stationary random excitation, frequency resolution is directly proportional to the duration of the test. The useful frequency range of experimental results depends on the anti- alias filter characteristics, the data duration, the sampling interval and the processing algorithm. Other things being equal, the larger the duration and the shorter the sampling interval, the broader the frequency range.

3.6.4. Measurements.

A model verification plan must specify a list of measurements, and if possible, a corresponding instrumentation list. Testing in space (or testing large flexible structures in a ground test laboratory) may pose instrumentation requirements which cannot be met with existing measurement devices or techniques. In this case the measurement list would specify the location of the desired measurement, the type of measurement (acceleration, displacement, strain, etc.), the range of measurement, the desired accuracy over that range and, in the case of dynamic measurements, the range of frequencies over which the amplitude and accuracy requirements apply. The duration and sampling rate of the measurements might also conceivably affect the selection or design of instrumentation, especially if power requirements are significant.

The number and type of measurements required for model verification will depend strongly on the model and how it is utilized. For example, rotational motion at a point is difficult to measure. If conventional accelerometers are used, they must be spaced close together and their outputs differenced to detect rotation, [29]. This typically involves taking small differences of large numbers which results in highly inaccurate measurements, if they are meaningful at all. An alternative approach is to measure translational accelerations at more widely separated points, and use the model to "curve fit" the measurements, whereupon rotational motion can be derived from the model. This simple example illustrates the importance of specifying the required measurements in a model verification plan. Not only must specific measurements be selected to verify specific model parameters, but the intended use of the measurements must also be specified.

In preparing a model verification plan, it is useful to develop a matrix for correlating specific measurements with specific parameters whose values will be estimated from those measurements. For example, the measurements may be listed down the left side of the

matrix, and the parameters to be estimated along the top, as shown in Table 3. Symbols are then entered in the appropriate matrix locations to indicate the pairing of particular measurements with particular parameters. The symbols may designate frequency bands over which the measurement data will be used to estimate a particular parameter value.

Table 3. Parameter-Measurement Correlation Matrix.

Measurement	Parameter									
	1	2	3	4	5	6	7	8	9	10
1	•									
2		•	•	○○	••	○				
3		○	○	••	••	•				
4		•	○	••	○○	•				
5		○	○			•		•••		○○○
6							••		•••	
7		○	○			•		○○○		•••
8								•••		•••

• 0.05 ⟷ 0.20 Hz •• 0.25 ⟷ 1.00 Hz ••• 1.00 ⟷ 10.00 Hz.

NOTE: Closed symbols indicate measurements expected to contain information about the parameter indicated. Open symbols indicate measurements involved in the estimation of the parameter indicated, but which are not expected to contain information about the parameter.

Table 3 places the model verification task into proper perspective. If the matrix is square, then there are an equal number of measurements and parameters to be estimated. However, if there is a blank column, the parameter corresponding to that column will not be estimated. If there is a blank row, a measurement is not being used. If the matrix is diagonal, then there is a one-to-one correspondence between parameters and measurements (the ideal situation). A fully populated matrix indicates that all of the measurements will

be used to estimate all of the parameters simultaneously (the least desirable situation). The matrix may be rectangular. It may have more rows than columns, suggesting an over- determined system. If it has more columns than rows it may be under- determined, but not necessarily. Different frequency ranges of the same measurement may contain the information to estimate different parameters. For example, one frequency response measurement from a base-excited two degree-of-freedom spring-mass chain is sufficient to identify the two spring stiffnesses if the two masses are known.

In preparing a matrix such as that in Table 3, one begins by developing a list of the parameters to be estimated. The measurement rows are then added, one by one, until all of the parameters are "covered." This task may be repeated several times as an optimum strategy evolves. It is also advisable that the task be repeated for different candidate sets of parameters to be estimated. Otherwise, it may be found after the tests have been completed that the parameters needing adjustment require measurements which had not been anticipated.

The planning matrix shown in Table 3 suggests a strategy for parameter estimation. Recalling the objective of grouping subsets of parameters and measurements for parameter estimation, one might initially attempt to estimate the parameters of Table 3 in the following sequence of separate estimation runs:

1. Parameter 1; Measurement 1; Frequencies 0.05-0.20 Hz
2. Parameters 2, 3, 6; Measurements 2, 3, 4, 5, 7; Frequencies 0.05-0.20 Hz
3. Parameters 4, 5; Measurements 2, 3, 4; Frequencies 0.25-1.00 Hz
4. Parameter 7; Measurement 6; Frequencies 0.25-1.00 Hz
5. Parameter 9; Measurement 6; Frequencies 1.00-10.00 Hz
6. Parameters 8, 10; Measurements 5, 7, 8; Frequencies 1.00-10.00 Hz

It is unlikely that the optimum strategy will be selected on the first attempt. Some learning is usually required. The learning process is facilitated by the availability of computational tools and graphics which provide insight to the decisional process.

3.6.5. Data Acquisition and Reduction.

The preparation of a model verification plan should take into account the methods, software and hardware available for data acquisition and reduction, as well as the capability for storing raw data. The use of antialiasing filters to prevent the "folding" of higher frequency information onto the frequency range of interest introduces roll-off in the higher end of this frequency range, depending on the type of filters used. A phenomenon called "leakage" occurs when stationary data are processed by FFT (Fast Fourier Transform) analysis. The FFT acts as an imperfect filter, allowing signal content (power) from one frequency to leak to other frequencies causing contamination or "noise." To supress the problem, it is common practice to introduce a time "window" that tapers the data so as to avoid the sharp discontinuities which otherwise occur at the beginning and end of the finite-time record. There are numerous such windows in use and the user should be aware of their respective effects on the data.

There are many other detailed operations and procedures commonly employed to "enhance the quality of spectral analysis." The implication is that there are many potential pitfalls to be avoided. For example, "overlap" averaging is used to recover some of the diminished statistical accuracy of "windowed" data. References [29], and [100]-[103] of the following chapter discuss data acquistion and reduction procedures in detail.

SECTION 4. IDENTIFICATION OF INPUT-OUTPUT RELATIONSHIPS.

This section deals with the formulation of the identification problem in the context of input-output relationships. It deals with some of the commonly used identification techniques for both linear and nonlinear systems. Section 4.1 provides a general framework for the identification process of linear systems in the time domain. Section 4.2 deals primarily with identification techniques used for linear systems in the frequency domain. Section 4.3 introduces series expansion methods which find use mainly in the identification of nonlinear systems.

4.1. Time Domain Input-Output Relationships.

An understanding of the input-output relationships for a large structure is necessary for the formulation and development of a parameter estimation algorithm. The discrete spatial model for the dynamics of a large space structure is generally derived by the finite-element method. The model could be derived by other mathematical techniques such as finite- differences, etc., but the finite-element method is the most common approach. The mathematical model employs two vector functions, one for characterizing the displacement of the spatial nodes of the structure, which will be denoted by the vector $x(t)$, and one for describing the measured nodal displacements, denoted as $y(t)$. The number of spatial nodes on the structure will be taken as n and it will be assumed that the number of measured outputs will be l. The vectors $x(t)$ and $y(t)$ will then be $n \times 1$ and $l \times 1$ respectively. The forces on the structure will be denoted by $f(t)$ and it will be assumed that this is an $m \times 1$ vector with the spatial locations for the applied forces known. At this point assume that there are no unknown forces acting on the structure and there is no noise in the measurements. The elimination of the errors due to the influence of these unknowns in the structure identification and modeling is part of the parameter estimation algorithm selection and model development.

The finite-element model spatial node displacements satisfy the second order vector equation

$$M\,\frac{d^2x(t)}{dt^2} + D\,\frac{dx(t)}{dt} + K\,x(t) = B\,f(t) \tag{133}$$

where $M \in R^{n\times n}$, $D \in R^{n\times n}$, $K \in R^{n\times n}$ and $B \in R^{n\times m}$. If the structure is linear and time-invariant, as implied in (133), then the Laplace transform of (133) gives

$$X(s) = [Ms^2 + Ds + K]^{-1}[BF(s) + (Ms + D)x(0) + M\dot{x}(0)] \tag{134}$$

where $x(0)$ and $\dot{x}(0)$ are the initial conditions of the structure. It is not difficult to show that the characteristic equation of (134) is

$$d(s) = \det[Ms^2 + Ds + K] = \sum_{i=1}^{2n} d_i s^{2n-i} = 0. \tag{135}$$

The characteristic equation of (134) will have $2n$ roots for a lightly damped large flexible structure and complex roots will occur in conjugate pairs with the rigid body modes having roots at $s = 0$. The roots other than those belonging to the rigid body modes will be along the $i\omega$ axis of the complex s-domain with a small real part less than zero. The rigid body mode roots correspond to the undeformed structure being displaced (rotation or translation) with the complex roots belonging to the vibrational modes of the structure. There will be n displacements and vibrational modes. Thus $x(t)$ for the free-response case (no forcing terms) will have the form

$$x(t) = \sum_{i=1}^{n} k_i e^{-\sigma_i t} \sin(\omega_i t + \phi_i) \tag{136}$$

where σ_i, ω_i and k_i are the real part of the roots of $d(s)$, the imaginary part of the roots of $d(s)$ and the partial fraction expansion coefficients of (134) respectively. Additional terms will contribute to the response when the forcing function is active. Identification of σ_i, ω_i and k_i for the n modes is adequate to identify the structure.

Not all of the structure nodes will have an applied force nor will each structure node have a displacement sensor to measure the motion of the node. If C denotes the measurement matrix, the location and transfer characteristics of the motion sensors, the output or measured node displacements will satisfy the equation

$$y(t) = Cx(t) \tag{137}$$

where $C \in R^{l \times n}$. It has been assumed that B and C are matrices with constant elements, but these matrices may also be frequency dependent. The elements of the matrices B and C are dependent upon the actuators and displacement (velocity or acceleometers) sensors and are therefore frequency dependent. Equations (136) and (137) can be combined to define the output from the structure, i. e., the measured displacements.

It has been assumed up to this point that the forces acting on the structure are known. This assumption is not valid for the structure on-orbit as unknown forces will be acting on the structure. The exact description of these forces is not known nor is the location of the force concentration available. The forces may be spatially distributed over the structure and may vary with time. Equation (133) must therefore be modified to include the additional forces with

$$M \frac{d^2x(t)}{dt^2} + D \frac{dx(t)}{dt} + Kx(t) = B_1 f_d(t) + B_2(x,t) f_n(t) \tag{138}$$

where $B_1 f_d(t)$ is the known part of the applied force and $B_2(x,t) f_n(t)$ is the effect of the unknown forces. As stated earlier, $B_2(x,t)$ will be spatially and time dependent. In addition, the force $f_n(t)$ may not be a Gaussian process which makes it more difficult to account for its contribution to the node displacements.

The measured output will depend on the unknown part of the applied force as well as measurement errors. The latter contribution to the output can be consider as additive noise thus $y(t)$ has the form

$$y(t) = C\,x(t) + v(t). \tag{139}$$

The measured vector $y(t)$ can then be obtained by solving (138) and substituting the solution into (139). The structure model in (138) and (139) is the one that must be considered in the identification task.

Some parameter estimation algorithms have been developed with the state variable formulation as the basis. It is a simple matter to place equation (133) and (137) in the state variable form where $x_1(t) = x(t)$ and $x_2(t) = \dot{x}(t)$. The state variable formulation [100] for the node displacement is then given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}Bf(t) \end{bmatrix} \tag{140}$$

or in the compact form

$$\dot{X}(t) = \bar{A}\,X(t) + \bar{B}\,f(t) \tag{141}$$

where $\bar{A} \in R^{2n\times 2n}$ and $\bar{B} \in R^{2n\times m}$. It should be noted that $X(t)$ is now a $2n \times 1$ vector with components $x(t)$ and $\dot{x}(t)$. The measurement equation is now given by

$$Y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \bar{C}X(t) \tag{142}$$

where $\bar{C} \in R^{l\times 2n}$. The closed form solution to the linear time-invariant equation of (141) can be found by taking the inverse of (141) and using the convolution integral for the forcing function term. The vector $X(t)$ is given by

$$X(t) = \Phi(t,0)\,X(0) + \int_0^t \Phi(t,\tau)B\,f(\tau)d\tau \tag{143}$$

where

$$\Phi(t,0) = \Phi(t) = e^{At}$$

when the system is linear and

$$\Phi(t,\tau) = \Phi(t-\tau) = e^{A(t-\tau)}.$$

Equations (133)-(143) are for the continuous time formulation. If the applied forces are deterministic and known, the closed form solutions for the node displacements and measured node displacements can be determined.

The estimation problem must deal with discrete values of $y(t)$ as the measurements are taken at the times $t = kT$ where T is the sampling time. The data that is collected, the measured node displacements, will be discrete. Thus, all of the system equations must be defined in the discrete time domain or in terms of the z-transforms. The z-transform of (133) with a zero-order-hold (ZOH) will have the form

$$x(z) = \frac{N_0 z^{2n-1} + N_1 z^{2n-2} + \cdots + N_{2n-2}}{z^{2n} + d_1 z^{2n-1} + \cdots + d_{2n}} f(z) \tag{144}$$

The difference equation for node displacements relates the input to the output, i. e. applied force to node displacement, and is given by

$$\begin{aligned} x(k) = {} & N_0 f(k-1) + N_1 f(k-2) + \cdots + N_{2n-1} f(k-2n) \\ & - d_1 x(k-1) - d_2 x(k-2) + \cdots + d_{2n} x(k-2n) \end{aligned} \tag{145}$$

where the T has been dropped in the arguments for convenience.

Equation (143) has a simpler form in the z-domain than does the second order vector equation of (133). The z-transform of (143) with a zero-order-hold (ZOH) is

$$x(z) = [zI - e^{AT}]^{-1} A^{-1} [e^{AT} - I] B F(z) \tag{146}$$

with the difference equation

$$x(k) = e^{AT}x(k-1) + A^{-1}[e^{AT} - I]BF(k-1). \tag{147}$$

The measured output is obtained by multiplying $x(k)$ by C in (145) and (147).

The task of system identification is to collect the measured data, i.e. the input time sequence $f(kT)$ and the output time sequence $y(kT)$, and to estimate the time-domain input/output time sequence $\Phi(kT,0)B$. An analytical model can then be verified or corrected as necessary to agree $\Phi(kT,0)B$. This aspect of the identification task is the verification and validation of the model phase.

Several aspects of the identification task must be considered in selecting and implementing an identification algorithm. The factors are:

- What are the external forces acting on the structure over which there is no control? Such forces are accounted for in the second term on the right of Equation (138). These forces are generally unknown but are inputs to the structure.
- What are the values of the initial displacements and velocities of the structure at the onset of the experiment to identify the structure? It is obvious from (134) that the initial conditions enter into the equations of motion. In general the structure will not be stationary but will be constantly in motion. The motion of the structure may be small but can the motion be ignored in the identification task?
- What type of forcing function should be selected for optimum result in the identification task? The total force on the structure is the external forces which may be unknown as well as the applied forces which are known. It may be the case that the external forces may counteract the applied forces and thus lead to a nonoptimal total force on the structure.

It is a common assumption in much of the theory of identification algorithm development that an impulse forcing function can be applied to the system. In practice, there is no way to generate an ideal impulse and the best that can be accomplished is a pseudo-impulse, one that is a finite pulse of amplitude f_0 with a short but finite duration τ. For example, if $F(s)$ in (134) is due to an impulse then $F(s) = f_0$ and if the initial conditions are zero, i.e. $x(0^-)$ and $\dot{x}(^-)$ are zero, then it follows that

$$Y(s) = C[Ms^2 + Ds + K]^{-1}Bf_0 = T(s)f_0 = H(s) \tag{148}$$

where $T(s)$ is the input-output transfer function and $H(s)$ is the impulse response. Similarly using the same assumptions of no unknown forces and initial conditions, the input-output relation from (143) is given by

$$Y(t) = C\int_0^t \Phi(t,\tau)BF_0\delta(\tau)d\tau \tag{149}$$

where $\delta(t)$ is the ideal impulse. As before, the output $y(t)$ is the impulse response which may be a poor approximation to the actual response of the structure for the applied pulse of short but finite duration.

4.2. Frequency Domain Methods.

4.2.1. Linear Relationships.

The identification or estimation of linear input-output relationships is treated extensively in a number of excellent textbooks on the general subject of time series analysis, e.g. [101]-[104]. This highly developed and well documented subject requires little elaboration, except to point out some of the alternative analysis procedures appearing in the recent literature. The basic relationships are introduced to facilitate this discussion.

Input and output time histories denoted by $x(t)$ and $y(t)$, respectively, have Fourier transforms denoted by $x(i\omega)$ and $y(i\omega)$, where the Fourier transform of $x(t)$ is given by

$$x(i\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t} dt \qquad -\infty < t < \infty \tag{150a}$$

and the inverse Fourier transform by

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(i\omega)\, e^{i\omega t} d\omega \qquad -\infty < \omega < \infty \tag{150b}$$

For linear systems, input and output are related in the time domain by the Duhamel or convolution integral

$$y(t) = \int_{0}^{t} H(\tau)\, x(t-\tau) d\tau \tag{151a}$$

and in the frequency domain by the complex product

$$y(i\omega) = H(i\omega) x(i\omega) \tag{151b}$$

where $H(t)$ is the unit impulse response function, and $H(i\omega)$ is the complex frequency response function for the unit impulse input. The functions $H(t)$ and $H(i\omega)$ are Fourier transform pairs, as are $x(t)$ and $x(i\omega)$, and $y(t)$ and $y(i\omega)$.

In practice, the Fourier transform and inverse Fourier transform are computed by a recursive algorithm called the Fast Fourier Transform (FFT) [105]. The FFT operates on digitized versions of $N = 2^k$ samples of the time histories $x(t)$ and $y(t)$ of finite duration NT where T is the sampling time. These finite transforms may be denoted by

$$x_k(\omega, T) = \int_{0}^{T} x_k(t)\, e^{-i\omega t} dt \tag{152a}$$

$$y_k(\omega, T) = \int_{0}^{T} y_k(t)\, e^{-i\omega t} dt \tag{152b}$$

where the index k indicates a particular sample function of the process $x(t)$ or $y(t)$. When $x(t)$ and $y(t)$ are stationary ergodic random processes, the one-sided (for non-negative frequencies) crossspectral and autospectral density functions are defined respectively as

$$G_{xy}(i\omega) = \lim_{T\to\infty} \frac{2}{T} E[x_k^*(i\omega,T)\, y_k(i\omega,T)] \tag{153a}$$

$$G_{xx}(i\omega) = \lim_{T\to\infty} \frac{2}{T} E[x_k^*(i\omega,T)\, x_k(i\omega,T)] \tag{153b}$$

where E denotes the expectation operator and the asterisk denotes the complex conjugate. Estimates of $\hat{G}_{xy}(i\omega)$ and $\hat{G}_{xx}(i\omega)$ are obtained by averaging over a number, n, of samples $x_k(t)$ and $y_k(t)$ in the manner

$$\hat{G}_{xy}(i\omega) = \frac{2}{nT} \sum_{k=1}^{n} x_k^*(i\omega,T)\, y_k(i\omega,T) \tag{154a}$$

$$\hat{G}_{xx}(i\omega) = \frac{2}{nT} \sum_{k=1}^{n} x_k^*(i\omega,T)\, x_k(i\omega,T). \tag{154b}$$

An estimate of the frequency response function for a single input/output problem is then given as

$$\hat{H}(i\omega) = \hat{G}_{xy}(i\omega)/\hat{G}_{xx}(i\omega). \tag{155}$$

When $x(t)$ and $y(t)$ are transients, the energy spectral density functions corresponding to (154a,b) are computed as $T\hat{G}_{xy}(i\omega)$ and $T\hat{G}_{xx}(i\omega)$ and used to compute $H(i\omega)$ in a manner analogous to (151).

The estimation of frequency response will generally involve both random and bias errors. As Bendat and Piersol [103] explain, bias errors can result from input noise that does not pass through the system, unmeasured inputs that do pass through the system and are correlated with the measured input, inadequate resolution in the frequency domain,

and nonlinearity. The intent is to eliminate or minimize these sources of error, and treat what remains as random error. Random error is evaluated from the coherence function, $\gamma_{xy}^2(i\omega)$, defined as

$$\gamma_{xy}^2(i\omega) = \frac{|G_{xy}i(\omega)|^2}{G_{xx}(i\omega)\, G_{yy}(i\omega)}. \tag{156}$$

The normalized random errors in the amplitude and phase of $\hat{H}(i\omega)$ are given respectively, in [103] as

$$\mathcal{E}[|\hat{H}(i\omega)|] \approx \frac{[1-\hat{\gamma}_{xy}^2(i\omega)]^{1/2}}{|\hat{\gamma}_{xy}(i\omega)|\sqrt{2n}} \tag{157a}$$

$$\mathcal{E}[\hat{\phi}(i\omega)] \approx \sin^{-1}\{\mathcal{E}[|\hat{H}(i\omega)|]\} \tag{157b}$$

where the normalized error is expressed as a coefficient of variation, i.e. the standard deviation σ divided by the mean μ. For example,

$$\mathcal{E}[\hat{\phi}(i\omega)] = \frac{\sigma[\hat{\phi}(i\omega)]}{\mu[\hat{\phi}(i\omega)]}$$

provided that $\mu[\hat{\phi}(i\omega)]$ is bounded away from zero. For small error, the normalized amplitude and phase errors are approximately equal and Gaussian.

One of the most troublesome bias-type errors is that which arises from "input noise that does not pass through the system," i.e. noise which contaminates the measurement of the input signal. This is a very common condition under random excitation, as pointed out by Mitchell [106]. The input power spectrum drops drastically near resonance, because of the low system input impedance as seen by the driver. The driver is trying to drive the equivalent of a short circuit; it cannot maintain its normal input force level at a resonant frequency with the result that the input power spectrum, $\hat{G}_{xx}(i\omega)$, tends to notch (drops toward zero). When this occurs, noise on the input measurement appears large relative to

the true input; the input signal to noise ratio is no longer large. As a result, the resonant peak of the frequency response function is consistently underestimated, i.e. biased.

The condition is clearly demonstrated in mathematical terms by considering measured input to be contaminated by noise $m(t)$, and the measured output by noise $n(t)$. If the true input and output are denoted by $u(t)$ and $v(t)$, respectively, then

$$x(t) = u(t) + m(t) \tag{158a}$$

$$y(t) = v(t) + n(t). \tag{158b}$$

Provided that $m(t)$ and $n(t)$ are uncorrelated with each other and with $u(t)$ and $v(t)$, it follows that

$$\hat{G}_{xy}(i\omega) = \hat{G}_{uv}(i\omega)$$

$$\hat{G}_{xx}(i\omega) = \hat{G}_{uu}(i\omega) + \hat{G}_{mm}(i\omega)$$

in which case

$$\hat{H}_1(i\omega) = \frac{\hat{G}_{xy}(i\omega)}{\hat{G}_{xx}(i\omega)} = \frac{\hat{G}_{uv}(i\omega)}{\hat{G}_{uu}(i\omega) + \hat{G}_{mm}(\omega)}. \tag{159}$$

If the true frequency response is denoted by

$$H_o(i\omega) = G_{uv}(i\omega)/G_{uu}(i\omega) \tag{160}$$

it is clear that $H_o(i\omega)$ will be underestimated by $\hat{H}_1(i\omega)$ unless the input signal-to-noise ratio, $\hat{G}_{uu}(i\omega)/\hat{G}_{mm}(i\omega)$, is large.

Mitchell [106] suggests an alternate estimator, $\hat{H}_2(i\omega)$, for use near resonance

$$\hat{H}_2(i\omega) = \frac{\hat{G}_{yy}(i\omega)}{\hat{G}_{yx}(i\omega)} = \frac{\hat{G}_{vv}(i\omega) + \hat{G}_{uu}(i\omega)}{\hat{G}_{vu}(i\omega)} = \frac{\hat{H}_1(i\omega)}{\gamma^2_{xy}(i\omega)}. \tag{161}$$

In this case the output noise power spectral density, $G_{uu}(i\omega)$, appears in the numerator. However, at resonance the output (or response) is large so that the output signal-to-noise ratio is large. Then

$$\hat{H}_2(i\omega) \approx \frac{\hat{G}_{vv}(i\omega)}{\hat{G}_{vu}(i\omega)} = \frac{\hat{G}_{vv}(i\omega)}{\hat{G}_{uv}(i\omega)} = \frac{\hat{G}^*_{uv}(i\omega)}{\hat{G}_{uu}(i\omega)}. \tag{162}$$

It follows from this discussion that in practical applications which involve both input and output measurement noise, one should use $\hat{H}_1(i\omega)$ as given by Equation (159) near frequencies where the frequency response is notched (antiresonances), and use $H_2(i\omega)$ as given by Equation (161) near frequencies where the frequency response is peaked (resonances).

It has been stated that input-output relationships serve as a basis for estimating higher forms of structural modeling quantities such as modal characteristics and design-type structural parameters. They are also used directly in some applications particularly where it is too difficult to formulate an analytical model, and the input-output relationship is too complex to perform a modal decomposition. A frequency response function or a matrix of frequency response functions, for example, can be inverted to obtain the corresponding complex impedance. In the case of linear systems, these empirical models may be superimposed on the analytical frequency- domain models of adjacent substructures to form what is called a hybrid model [51], [107] and [108]. Whenever a modal decomposition of the frequency response is not performed, however, the coupling analysis must be performed at each frequency of the discrete spectrum (resulting from the FFT transformation). This can be costly where many coordinates are involved, because it involves the inversion of substructure frequency response matrices, the coupling (or superposition) of these matrices to obtain the system impedance matrix, and the inversion of the system impedance matrix to obtain the frequency response of the system at each frequency. While

theoretically possible, this process tends to yield poor quality results due to numerical ill conditioning arising from the inversion of matrices containing experimental errors.

4.2.2. Nonlinear Relationships.

Unlike the linear case where general relations for arbitrary stationary random signals passing through arbitrary linear systems have been derived, it is usually necessary to treat nonlinear systems individually. Bendat and Piersol [109] have developed a procedure for the identification of a class of nonlinear systems called finite memory square-law systems. Special bispectral density functions are defined and applied that are functions of a single variable. From measurements of input data and output data only, results are obtained to identify the separate frequency response functions for two models of linear systems in parallel with nonlinear square-law systems. Nonlinear coherence functions are defined from these models which determine the proportion of the output spectrum due to the nonlinear operations. Together with ordinary coherence functions, a measured output spectrum for these models can be decomposed into three components representing the linear operations, the nonlinear operations, and the remaining uncorrelated noise effects. The paper indicates also how to analyze other types of nonlinear models by employing similar techniques. In a later paper [110] these authors address the decomposition of wave forces into linear and nonlinear components.

4.3. Series Expansion Methods.

4.3.1. Volterra and Wiener Input-Output Representations.

As pointed out in Section 2, the Volterra series representation provides a general formulation for a memoryless system where the input and output are measured. The set of kernels (see equation 4) h_1, h_2, $\cdots$, h_n completely characterize the dynamic response of such a system. However the calculation of the kernels is, in general, a difficult job.

Wiener [111] has shown that an equivalent series expansion is valid for such systems when the input used is zero mean, Gaussian white noise (GWN). The series then becomes that given by equation (6) where G_n forms a complete set of orthogonal functions with respect to the GWN input, $u(t)$.

Using equations (7) and (8) in (6), multiplying both sides by $u(t-\tau)$, and taking the expectations, yields

$$g_1(\tau) = E[y(t)\, u(t-\tau)]/P \tag{163}$$

where P is the power spectral density of the input white noise. This relation is analogous to (155), but is in general quicker to compute. The effect of unknown (or ignored) inputs that follow paths which are different from the "input" path are particularly difficult to assess by this method. References [112] and [113] provide some results in this regard. However, the effect of output measurement errors can be more easily assessed, and when the output measurement noise is independent of the input the estimate of g_1 is unaffected by the noise [113]. The kernel g_2 can again be obtained through cross-correlation as

$$g_2(\tau_1,\tau_2) = E\{[y(t) - G_1(g_1 u(t))]u(t-\tau_1)u(t-\tau_2)\}/(2P^2) \tag{164}$$

In general,

$$g_n(\tau_1,\tau_2,\tau_3,\cdots,\tau_n) = E\{[y(t) - \sum_{1}^{n-1} G_m]u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_n)\}/(n!\, P^n). \tag{165}$$

Expressions for the higher order kernels and/or their transforms for various feedback systems can be found in reference [113].

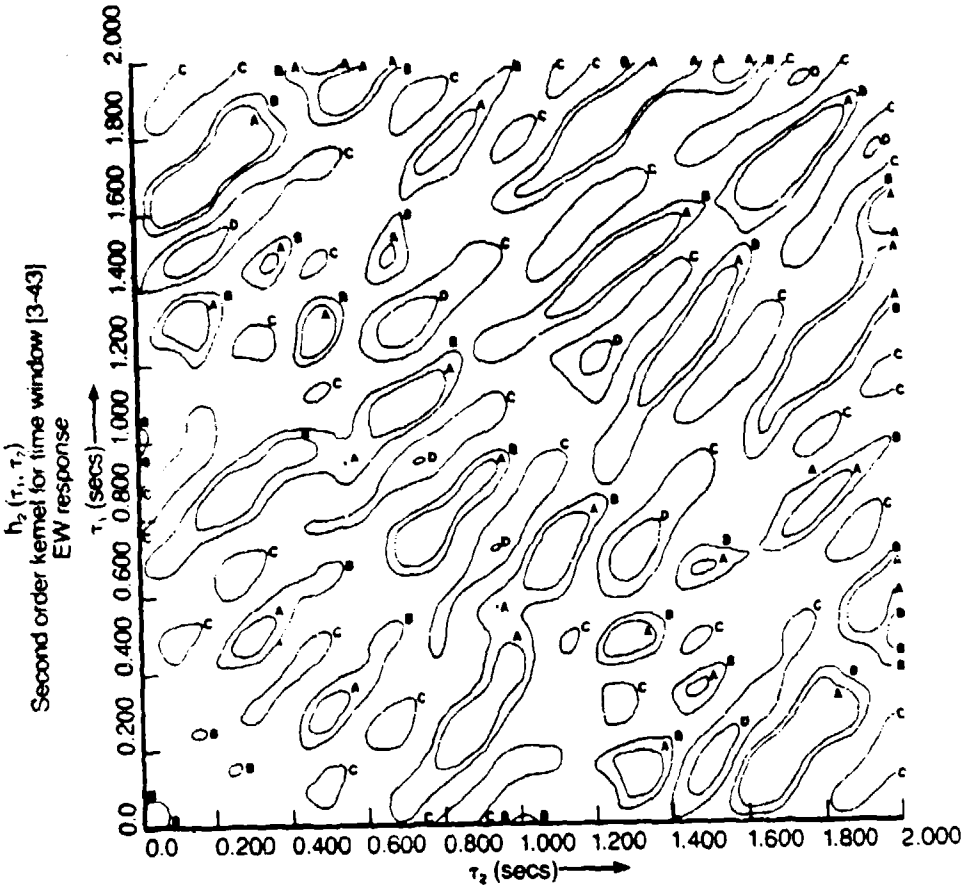


Figure 8. Contour Plot of Kernel $h_2(\tau_1, \tau_2)$
Corresponding to the Time Window {3 – –43} sec.
The Contour Values are A=-125, B=-75, C=100, D=300.

A typical contour plot for the second order kernel obtained from the response of a building structure subjected to a near-white base excitation is shown in Figure 8. Further details may be obtained from reference [113].

The major drawbacks of this techniques are:

1) it requires a Gaussian broad-band input, broad enough to encompass the frequency spectrum of interest for the structure;
2) it is relevant to systems whose input-output relations can be described by such series representations, and hence is not applicable to hysteretic systems; and,
3) it requires fairly large computing times when dealing with higher order kernels.
4) it may not be useful for inputs other than GWN.

The advantages of the method are as follows:

1) GWN being the input signal, one does not need elaborate controllers to track a desired pre-assigned input time history.
2) It provides physical insight into the manner in which the nonlinearities contribute to the system's response. The relative contribution of the nonlinearities to the total response of the system can therefore be evaluated.
3) In feedback systems it provides further insight into the poles (possible 'resonances') of the system caused by the nonlinearities.
45) It makes no assumptions about the model structure.

4.3.2. Expansion of Nonlinearities in Series Expansions.

A technique first developed by Graupe [114] for the identification of nonlinearities in systems, and used in references [115], [116] and [117], is to expand the nonlinearities in terms of series expansions. As opposed to the method of Section 4.3.1 where the input-output response relation was expanded in terms of a series of functionals, here the nonlinear restoring force is expanded in a series of functions. For example, consider a single degree-of-freedom system described by the equation

$$m\ddot{y} + k(y, \dot{y}) = f(t) \tag{166}$$

where the restoring force, k, is a nonlinear function of the displacement y, and the velocity, $\dot{y}$. The forcing function is denoted by $f(t)$ and m is the mass of the system. The restoring force k can be expanded as

$$k(y,\dot{y}) = \sum\sum a_{ij}\phi_i(y)\psi(\dot{y}) \tag{167}$$

where the coefficients a_{ij} are to be identified by minimizing

$$||k(y,\dot{y}) - w(t)||_g$$

where $w(t) = f(t) - m\ddot{y}$ and g is a suitable weighting function. The quantities $w(t)$, $y(t)$, $\dot{y}(t)$ and $\tilde{y}(t)$ are assumed to obtained from measured data. When the functions $\phi_i(y)$ and $\psi_j(\dot{y})$ are chosen so that they are orthonormal with respect to the weighting functions $g_1(y)$ and $g_2(\dot{y})$, the a_{ij} terms may be computed using the least squares approach,

$$a_{ij} = \int\int g_1(y)g_2(\dot{y})w(t)\phi_i(y)\psi_j(\tilde{y})dydd\ddot{y}. \tag{168}$$

The quantities y and $\dot{y}$ are generally replaced by the measurements y and $\dot{y}$.

When the restoring force is separable so that

$$k(y,\dot{y}) = k_1(y) + k_2(\dot{y}) \tag{169}$$

where, $k_1(y) = \sum b_i\phi_i(y)$ and $k_2(\dot{y}) = \sum a_i\psi(\dot{y})$, major computational advantages result. Details on the computational aspects of the problem and results on the error sensitivity of the technique can be found in reference [117].

The major disadvantages of the method are as follows:

1) It is applicable only to nonhysteretic systems. This is because the equation (167) implies a one-to-one correspondence between the restoring force k, the displacement y, and the velocity $\dot{y}$.

2) It determines the best estimates of the coefficients within the range of amplitudes in which the system is exercised by the input. Different inputs will often lead to different characterizations of the system i.e. the parameters a_{ij} will change depending on the range(amplitude) of response exhibited by the system

3) It requires measurements of y, $\dot{y}$ and $\ddot{y}$ along with the force $f(t)$ at each node of the system. This is usually impossible for large, general structural systems. Reference [117] uses the technique for close-coupled dynamic systems. For such special systems, the method gives reasonably good results.

The major advantages of the method are:

1) There is no restriction on the nature of the input signals

2) The identification requires modest computational storage and computing time.

3) The identification results are not very sensitive to measurement noise.

4) The duration of time over which the data is required to be taken is comparatively small compared to the Wiener approach.

SECTION 5. IDENTIFICATION OF MODAL CHARACTERISTICS FOR FLEXIBLE STRUCTURES.

When formulating and solving an identification problem, it is important to have the purpose of the identification clearly in mind. In control problems such as control of large space structures, the final goal is to design control strategies for a particular structural system. On the other hand, there are situations where the primary interest is to analyze the properties of a dynamic system, such as stiffness, damping, frequencies, etc. The control problem might require a fairly accurate model of the system dynamics which will adequately describe the motion of the system. Fundamentally, one seeks to find a set of parameters that builds a mathematical model to best reproduce, according to some criteria, the test data. The mathematical model for a linear finite-dimensional dynamic system typically includes a state matrix, an output influence matrix and an input influence matrix. Among the modal parameters for flexible structures, damping and frequencies constitute the state matrix, modes shapes produce the output influence matrix and modal participation factors yield the input influence matrix. In the field of controls, the process of constructing a model (state space representation) from experimental data is called system realization [29], [124]. The choice of model structure is one of the basic ingredients in the formulation of the identification problem. The choice will influence the character of the realization problem, the computational effort, the possibility to have a minimum order model, etc. The accuracy of identified modal parameters from the system realization are thus affected by the choice of the model.

The purpose of this section is to present methods using experimental data to estimate dynamic properties such as damping, frequencies, mode shapes and modal participation factors which are referred to as modal parameters. The task of modal parameter identification is treated in several ways by different researchers. Many different methods and techniques [29],[124] are analyzed and treated. New methods are suggested en masse and,

as a result, the field appears to look more like a bag of tricks than a unified subject. Therefore, a most difficult assignment is to organize the bewilderingly large number of ideas. Many so-called different methods are in fact quite similar in the sense that they are mathematically equivalent but are presented in different ways. Thus a unified mathematical framework to treat modal parameter identification is needed to achieve some unification of the field. Although it is very difficult to get a complete overview of the field in rapid development, the reader is presented with a basic mathematical foundation which provides insight into the field.

The tuned sine dwell approach is the earliest methods of modal parameter identification. The method requires the employment of special input signals such as sine-wave inputs having variable frequencies for frequency response identification [125]-[138], which makes it possible to determine the frequencies and light damping. On the other hand, the recently developed methods for modal parameter identification are based on parametric models in terms of state equations. Fundamentally, two approaches can be classified with respect to the basic elements for construction of the data matrix. The first approach, which uses the impulse response function to construct the data matrix to realize a model for modal parameter identification, is called the time-domain identification method [139]-[151]. The second approach, which uses the transfer function matrix to realize a model and then identify the modal parameters, is referred to as the frequency-domain method [152]- [154]. The first part of this Section is a brief presentation of the classical method, namely, the forced normal mode excitation approach. The time-domain and frequency-domain identification methods are then addressed using system realization theory. The relation among various existing methods is established and discussed. This Section is a short version of Reference [158].

5.1. Forced Normal Mode Excitation.

Before presenting the general theory of system realization, a brief discussion of the classical tuned sine dwell approach is given in this Section. Significant improvements have been made in this traditional method of modal testing since its introduction more than 35 years ago. Unlike many of the aerospace companies and government agencies in the United States which still use the more traditional and time consuming technique, the Europeans have automated much of the procedure and have been able to reduce test times by an order of magnitude, e.g., from two modes per day to twenty modes per day.

The forced-normal-mode-excitation method of experimental modal analysis is the oldest method for identifying the modal characteristics of complex structures [125]. In more recent times, implementation of this approach, particularly in the aerospace industry, has involved the use of several force transducers (or "shakers"), and many (often hundreds) of response measurements. This method of testing has come to be known as "multipoint sine dwell" testing [126]. In addition to the advantage of being able to simultaneously record the response at hundreds of locations, this method has the advantage of concentrating large amounts of energy in single modes of vibration, and provides the satisfying experience of directly perceiving resonant conditions via the human senses.

The traditional disadvantages of the method include the time required to tune the modes and the associated difficulty of isolating sufficiently "pure" modes, especially when the modes are closely spaced in frequency. Large space structures are expected to exhibit high modal density with many closely spaced modes. With conventional testing techniques, considerable experience and skill are required to achieve satisfactory results. Comparisons between the results of multipoint sine dwell testing and other methods for the experimental identification of structural modes are given in [127],[128]. Several recent survey papers discuss relative advantages and disadvantages [1],[129].

Most of the recent advances in multipoint sine dwell testing have been in the implementation of the force distribution, or "force appropriation" [129]-[138]. One of the most advanced systems is attributed to the German research organization, (DFVLR). Their system involves approximately 500 channels of data and computer-controlled co/quad analysis equipment. The object of force appropriation is, in addition to shaping the forcing function to match the mode being excited, to counterbalance the small damping forces in the structure such that the response throughout the structure is (as nearly as possible) 90 degrees out of phase with the forcing function. This is not always easily accomplished because of limitations in where the force can be applied. The intended result is the excitation of "undamped modes", based on the premise that the undamped modes are more directly comparable to those of analytical models which do not include damping. Whether this is in fact a real advantage is a matter of current debate. Nevertheless, the achievements of DFVLR illustrated in Table 8 from [129],[130] are impressive.

Table 8. Test Duration For Four Aircraft Structures Using the Sine Dwell Method.

Year	Configuration	No. of measured eigenmodes	Test duration
1966	Clean wing	21	6 weeks
1974	5 configurations with 2-6 external stores	110	6 weeks
1978	Clean wing and five configuration with 2-6 external stores	125	10 days
1984	Transport aircraft with 2 engines	460	26 days

5.2. Time-Domain Modal Parameter Identification, [128]- [139].

Many different time-domain methods and techniques in the field of structures were developed, analyzed and tested for modal parameter identification. The question arises whether there exists relationship among these methods. The answer is positive. Indeed, a unified mathematical framework can be developed to present and discuss these methods. The time-domain methods for modal parameter identification in the field of structures start with the transfer function matrix, which yields Markov parameters (generalized impulse response samples). The knowledge of Markov parameters makes it possible to construct a Hankel matrix [118]-[123] as the basis for the realization of a state space discrete-time model. This section will thus start with the derivation of discrete-time models from the continuous-time models which are usually used by structural engineers. It will be followed by the basic concept of minimum realization which was developed by Ho and Kalman [123]. Since the Eigensystem Realization Algorithm (ERA) [139]-[141] for modal parameter identification was developed using the minimum realization theory, it will be presented and discussed first. The Polyreference Technique [144]-[146] and the Least Squares Regression method will then be derived using the mathematical framework developed in the ERA.

5.2.1. State Equations: Continuous-Time and Discrete-Time Models.

The equations of motion for a finite-dimensional linear dynamical system are a set of n_2 second-order differential equations, where n is the number of independent coordinates. Let M, D, and K be the mass, damping and stiffness matrices, respectively. The state equations can be expressed in matrix notation as

$$M\ddot{w} + D\dot{w} + Kw = f(w,t) \tag{170}$$

where $\ddot{w}$, $\dot{w}$ and w are vectors of generalized acceleration, velocity, and displacement respectively and $f(w,t)$ is the forcing function over the period of interest at certain specific

locations. (170) can be rewritten as a first order system of differential equations in a number of ways. Certain reformulations are more suitable in computation than others. One reformulation begins with the following definition:

$$x = \begin{bmatrix} w \\ \dot{w} \end{bmatrix} \tag{171a}$$

$$\bar{A} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \tag{171b}$$

$$\bar{B} = \begin{bmatrix} 0 \\ M^{-1}B_f \end{bmatrix} \tag{171c}$$

where B_f is an $n_2 \times m$, $(n = 2n_2)$, input influence matrix. The integer m is the number of inputs. Equation (170) can thus be written in a more compact form as

$$\dot{x} = \bar{A}x + \bar{B}u. \tag{172}$$

If the dynamic system is excited and measured by the l output quantities in the output vector $y(t)$ using sensors such as strain gages, accelerometers, etc., a matrix output equation can be formulated as

$$y = Cx \tag{173}$$

where C is thus an $l \times n$ output influence matrix.

Equations (172) and (173) constitute a continuous-time model for finite- dimensional dynamic systems. The matrix $\bar{A}$ in (172) is a representation of mass, stiffness, and damping properties. The input matrix $\bar{B}$ characterizes the locations and type of input $u(t)$, whereas the output matrix C describes the relationship between the state vector $x(t)$ and the output measurement vector $y(t)$.

Use of the initial conditions $x(t)$ at time $t = t_0$ gives the solution to the continuous-time equation with an input $u(t)$

$$x(t) = e^{\bar{A}(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{\bar{A}(t-\tau)}\bar{B}u(\tau)d\tau \tag{174}$$

for $t \geq t_0$. This equation describes the change of state variable $x(t)$ with respect to the initial conditions $x(t_0)$ and the input $u(t)$. It will be shown in the following that the evaluation of $x(t)$ at equally spaced intervals of time t can be obtained by a discrete-time representation of (174).

Let the equally spaced times be given by $0, \Delta t, 2\Delta t, \cdots, k\Delta t, \cdots$, where Δt is a constant interval. Substitution of $t = (k+1)\Delta t$ and $t_0 = k\Delta t$ into (174) yields

$$x[(k+1)\Delta t] = e^{\bar{A}\Delta t}x(k\Delta t) + \int_{k\Delta t}^{(k+1)\Delta t} e^{\bar{A}[(k+1)\Delta t-\tau]}\bar{B}u(\tau)d\tau. \tag{175}$$

If $u(t)$ is assumed to be constant over the interval $k\Delta t \leq \tau \leq (k+1)\Delta t$ and has the value $u(k\Delta t)$, (175) with a constant matrix $\bar{B}$ becomes

$$x[(k+1)\Delta t] = e^{\bar{A}\Delta t}x(k\Delta t) + \int_{0}^{\Delta t} e^{\bar{A}(\Delta t+\tau')}d\tau'\bar{B}u(k\Delta t) \tag{176}$$

where the variable τ in (175) has been changed by letting $\tau' = k\Delta t - \tau$. Now, define

$$A = e^{\bar{A}\Delta t} \tag{177a}$$

$$B = \int_{0}^{\Delta t} e^{\bar{A}\tau'}d\tau'\bar{B} \tag{177b}$$

$$x(k+1) = x[(k+1)\Delta t] \tag{177c}$$

and

$$u(k) = u(k\Delta t). \tag{177d}$$

Equation (176) can be then expressed in a compact form

$$x(k+1) = Ax(k) + Bu(k); \qquad k = 0, 1, 2, \cdots \tag{178}$$

and (173) becomes

$$y(k) = Cx(k). \tag{179}$$

Combination of equations (178) and (179) is the discrete-time representation of a dynamical system. This set of equations constitute the basic formulation for system identification of linear, time-invariant dynamical systems, because experimental data are discrete in nature. If experimental data are recorded in a digital computer, continuous physical measurements will be directly sampled and converted into digital words.

What are the response characteristics of the discrete model, i.e., as given in equations (178) and (179)? To observe the response to an impulse in one of the input variables, $u(0) = 1$ and $u(k) = 0 \quad (k = 1, 2, ...)$ are substituted into (178). When the substitution is performed for each input element, the results are combined to obtain the impulse response function matrix Y with dimensions $l \times m$ as follows:

$$Y(0) = CB, \quad Y(1) = CAB, \quad Y(2) = CA^2B, \quad \cdots, Y(k) = CA^kB, \quad \cdots \tag{180}$$

This sequence of constant matrices, known as Markov parameters, can be obtained from the experimental data through the transfer function or impulse responses. The Markov parameters can thus be used as the basis for building mathematical models for dynamical systems.

5.2.2. Basic Concepts of Realization.

The triple of constant matrices $[A,\ B,\ C]$, which represents the system characteristics can be used to determine the system's response at any of the l output points to any input at any of the input points. Such a representation is called a realization of the system. Any system has an infinite number of realizations which will predict the identical response for any particular input.

Let a new vector z be defined such that

$$x = Tz \tag{181}$$

where T is any nonsingular square matrix. Substitution of (181) into (178) and (179) yields

$$z(k+1) = T^{-1}ATz(k) + T^{-1}Bu(k); \qquad k = 0, 1, 2, \cdots \tag{182}$$

$$y(k) = CTz(k). \tag{183}$$

It is obvious that the effect of input $u(k)$ on $y(k)$ will be the same for this new system of equations (182) and (183). Thus, the triple $[T^{-1}AT,\ T^{-1}B,\ CT]$ will also be a realization for the same system. The predicted responses using the realization $[T^{-1}AT,\ T^{-1}B,\ CT]$ will be identical to those predicted using $[A,\ B,\ C]$. Because there exists an infinite number of nonsingular matrices T, there are an infinite number of such realizations.

Minimum realization means a model with the smallest state space dimensions among all realizable systems that has the same input-output relations. All minimum realizations have the same set of eigenvalues, which are parameters of the system itself.

Assume that the state matrix A of order n_0 has a complete set of linearly independent eigenvectors $(\psi_1, \psi_2, \cdots, \psi_{n_0})$ with corresponding eigenvalues $(\lambda_1, \lambda_2, \cdots, \lambda_{n_0})$ which are not necessarily distinct. Define Λ as the diagonal matrix of eigenvalues and ψ the matrix of eigenvectors, i.e.

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n_0}) \tag{184}$$

and

$$\psi = [\psi_1, \psi_2, \cdots, \psi_{n_0}]. \tag{185}$$

The triple $[A,\ B,\ C]$ can then be transformed to the realization $[\Lambda,\ \psi^{-1}B,\ C\psi]$ by choosing $T = \psi$. The diagonal matrix Λ contains the information of modal damping rates and damped natural frequencies. The matrix $\psi^{-1}B$ is defined as the initial modal amplitudes and the matrix $C\psi$ the mode shapes. All the modal parameters of a dynamic system can thus be identified by the triple $[\Lambda,\ \psi^{-1}B,\ C\psi]$. The desired modal damping rates and damped natural frequencies are simply the real and imaginary parts of the eigenvalues $\bar{\Lambda}$, after transformation from the discrete-time domain to the continuous-time domain using the relation $\bar{\Lambda} = ln(\Lambda)/\Delta t$.

In the field of structures, time-domain analysis for identification of modal parameters begins by forming the generalized $(r+1) \times (s+1)$ Hankel matrix, composed of the Markov parameters from (180).

$$H(k) = \begin{bmatrix} Y(k) & Y(k+1) & \cdots & Y(k+s) \\ Y(k+1) & Y(k+2) & \cdots & Y(K+s+1) \\ \vdots & \vdots & \ddots & \vdots \\ Y(k+r) & Y(k+r+1) & \cdots & Y(k+r+s) \end{bmatrix}. \tag{186}$$

If $r+1 \geq n_0$ and $s+1 \geq n_0$ (the order of the system), the matrix $H(k)$ is of rank n_0 . To confirm this point, observe from (180) that

$$H(k) = V_r A^k W_s \tag{187}$$

where

$$V_r = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^r \end{bmatrix} \tag{188a}$$

$$W_s = [\, B \quad AB \quad A^2B \quad \cdots \quad A^sB \,]. \tag{188b}$$

The block matrix V_r is called the observability matrix, whereas the block matrix W_s is called the controllability matrix. If the order of the system is n_0 , then the minimum dimension of the state matrix is $n_0 \leq n_0$. If the system is controllable and observable, the block matrices V_r and W_s are of rank n_0 . Therefore the Hankel matrix is of rank n_0 by (187). Based on the properties of the Hankel matrix composed of the Markov parameters (impulse response function), several methods for modal parameter identification are discussed in the following sections.

5.2.3. The Eigensystem Realization Algorithm (ERA).

The basic development of the time-domain (state-space) concept is attributed to Ho and Kalman [123] who introduced the important principles of minimum realization theory. The Ho-Kalman procedure uses the generalized Hankel matrix (equation 186) to construct a state-space representation of a linear system from noise-free data. The methodology has been recently modified and substantially extended to develop the Eigensystem Realization Algorithm [139]-[143] to identify modal parameters from noisy measurement data.

In contrast to classical system realization methods which use the generalized Hankel matrix given in (186), the ERA algorithm begins by forming a block data matrix which is

obtained by deleting some rows and columns of the generalized Hankel matrix of (186), but maintaining the first block matrix intact. Furthermore, the standard ordering of entries in the generalized Hankel matrix does not need to be maintained.

Let $B = [b_1, b_2, \cdots, b_m]$ and $C^T = [c_1^T, c_2^T, \cdots, c_l^T]$ where the column vector b_i is the control influence vector for the ith control inp t and the row vector c_j^T is the measurement influence vector for the jth measurement sensor. Denote column submatrices of B by B_i, $(i = 0, 1, \cdots, \eta)$ and row submatrices of C by C_j, $(j = 0, 1, \cdots, \xi)$. The matrices B_i and C_j^T are subsets of B and C^T, $[b_1, b_2, \cdots, b_m]$ and $[c_1^T, c_2^T, \cdots, c_l^T]$ respectively. The ERA data block matrix can then be expressed by

$$H(k) = [Y_{ji}(s_i + k + t_i)]; \qquad Y_{ji}(s_i + k + t_i) = C_j A^{s_i+k+t_j} B_i \tag{189}$$

where $s_0 = t_0 = 0$, and s_j and t_i are arbitrary integers. When $i = j = 0$, $Y_{00}(k) = Y(k) = CA^kB$.

The ERA block data matrix (equation 189) allows one to include only good or strongly measured signals without losing any capability. This is useful since some measurement data may be noisier than others or sensors may malfunction during the test. The advantage of this capability is the potential to minimize the distortion of the identified parameters caused by noise. A judicious choice of data and its proper arrangement in the block matrix $H(k)$ can also be used to minimize the computational requirements of the method. For example, the columns of $H(k)$ may be made as independent as possible by properly selecting the data samples to use as entries of the matrix. This effort could substantially reduce the order of the matrix for large problems. For sufficiently low noise data, the order can be the same as that of the true system state matrix A. This fact results from examination of the controllability and observability matrices, to be discussed next.

From (189), it can be shown that

$$H(k) = V_\xi A^k W_\eta \tag{190a}$$

$$V_\xi = \begin{bmatrix} C \\ C_1 A^{s_1} \\ \vdots \\ C_\xi A^{s_\xi} \end{bmatrix} \tag{190b}$$

$$W_\eta = [\, B \quad A^{t_1} B_1 \quad \cdots \quad A^{t_\eta} B_\eta \,] \tag{190c}$$

where V_ξ and W_η are generalized observability and controllability matrices.

Assume that there exists a matrix $H^\dagger$ satisfying the relation

$$W_\eta H^\dagger V_\xi = I_r \tag{191}$$

where I_r is an identity matrix of order r. It will be shown that the matrix $H^\dagger$ plays a major role in deriving the ERA. What is $H^\dagger$? Observe that,

$$H(0) H^\dagger H(0) = V_\xi W_\eta H^\dagger V_\xi W_\eta = V_\xi W_\eta = H(0) \tag{192}$$

The matrix $H^\dagger$ is thus the pseudoinverse of the matrix $H(0)$ in a general sense.

The ERA process starts with the factorization of the block data matrix (equation 189), for $k = 0$, using singular value decomposition of [142]:

$$H(0) = P\, D\, Q^T \tag{193}$$

where the columns of matrices P and Q are orthonormal and D is a rectangular matrix

$$D = \begin{bmatrix} D_r & 0 \\ 0 & 0 \end{bmatrix}$$

with

$$D_r = \text{diag}[d_1,\ d_2,\ \cdots,\ d_n,\ d_{n+1},\ \cdots,\ d_r]$$

and monotonically non-increasing d_i $(i = 1, 2, ..., r)$

$$d_1 \geq d_2 \geq \cdots \geq d_n \geq d_{n+1} \geq \cdots d_r \geq 0.$$

Next, let P_r and Q_r be the matrices formed by the first r columns of P and Q, respectively. Hence the matrix $H(0)$ and its pseudoinverse become

$$H(0) = P_r D_r Q_r^T \qquad \text{where} \quad P_r^T P_r = I_r = Q_r^T Q_r \tag{194}$$

and

$$H^\dagger = Q_r D_r^{-1} P_r^T. \tag{195}$$

Equation (195) can be readily proved by observing (192).

Define O_l as a null matrix of order l, I_l an identity matrix of order l, and $E_l^T = [I_l\ O_l\ \cdots\ O_l]$. Using (189), (190), (191), (194), and (195), a minimum order realization can be obtained as follows:

$$\begin{aligned}
Y(k) &= E_l^T H(k) E_m \\
&= E_l^T V_\xi A^k W_\eta E_m \qquad \text{(use (189) and (190))} \\
&= E_l^T V_\xi [W_\eta H^\dagger V_\xi] A^k [W_\eta H^\dagger V_\xi] W_\eta E_m \qquad \text{(use (191))} \\
&= E_l^T H(0) [Q_r D_r^{-1} P_r^T] V_\xi A^k W_\eta [Q_r D_r^{-1} P_r^T] H(0) E_m \quad \text{(use (190) and (195))} \\
&= E_l H(0) Q_r D_r^{-1/2} [D_r^{-1/2} P_r^T H(1) Q_r D_r^{-1/2}]^k D_r^{-1/2} P_r^T H(0) E_m \quad \text{(use (194))} \\
&= E_l^T P_r D_r^{1/2} [D_r^{-1/2} P_r^T H(1) Q_r D_r^{-1/2}]^k D_r^{1/2} Q_r^T E_m.
\end{aligned} \tag{196}$$

This is the basic formulation of realization for the ERA. The triple

$$\tilde{A} = D_r^{-1/2} P_r^T H(1) Q_r D_r^{-1/2}, \quad \tilde{B} = D_r^{1/2} Q_r^T E_m, \quad \tilde{C} = E_l^T P_r D_r^{1/2} \tag{197}$$

is a minimum realization. The order of the matrix A is r which is equal to n_0 (the order of the system) for sufficiently low noise data. When the matrices P_r and Q_r are obtained through other factorization methods such that $P_r P_r^T \neq I_r$ and $Q_r Q_r^T \neq I_r$, (197) is still valid if the matrices P_r^T and Q_r^T are replaced by $P_r^\dagger$ and $Q_r^\dagger$ respectively.

Due to measurement noise, nonlinearity, and computer round off, the block matrix $H(k)$ will usually be of full rank which does not, in general, equal the true order of the system under test. It should not be the aim to obtain a system realization which exactly reproduces the noisy sequence of data. A realization which produces a smoothed version of the sequence, and which closely represents the underlying linear dynamics of the system, is more desirable. Several accuracy indicators have been investigated for quantitatively partitioning the realized model into pure (principal) and noise (perturbational) portions so that the noise portion can be disregarded. Two principal indicators now available are the singular values of the block data matrix and a parameter referred to as Modal Amplitude Coherence. The number of retained singular values determines the order of the realization, and Modal Amplitude Coherence is used to assess the resulting degree of modal purity.

If (190) and (191) are examined as a whole the equality

$$H(0) = V_\xi W_\eta = [P_r D_r^{1/2}][D_r^{1/2} Q_r^T] \tag{198}$$

defines the controllability and observability Grammians as

$$W_\eta W_\eta^T = D_r \qquad \text{and} \qquad V_\xi^T V_\xi = D_r. \tag{199}$$

The fact that the controllability and observability Grammians are equal and diagonal implies that the realized system $[A, B, C]$ is as controllable as it is observable. This

property is called an internally balanced realization. It means that the signal transfer from the input to the state and then from the state to the output are similar and balanced.

Some singular values, say $d_{n+1}, \cdots, d_r$, may be relatively small and negligible in the sense that they contain more noise information than system information. In other words, the directions determined by the singular values $d_{n+1}, \cdots, d_r$, have less significant degrees of controllability and observability relative to the noise. It would be unwise to require a realization including these directions. The reduced model of order n after deleting singular values $d_{n+1}, \cdots, d_r$ is then considered as the robustly controllable and observable part of the realized system. References [139]-[141] provide the mathematical framework for establishing the relationship between these accuracy indicators and the characteristics of the noise.

5.2.4. The Polyreference Technique (Canonical-Form Realization).

During past the two decades, several algorithms for the construction of canonical-form representation of linear systems have appeared in the controls literature [120],[121]. Researchers in the field of controls are mainly concerned with, for an example, determining a passive or an active network that has a prescribed impedance or transfer function. Although techniques of canonical-form realization are available in the control literature, direct application to modal parameter identification for flexible structures has not been addressed. Among several available methods for canonical-form realization, one based on Hankel matrices will be addressed in this section.

Recently in the field of structures, a method, similar if not identical, to a canonical-form realization, was developed in [130],[131] using frequency- response functions for identification of modal parameters from multi-input and multi-output measurement data. The method is referred to as the Polyreference technique. Mathematical background of the

Polyreference technique will be presented in this section using a new approach which provides insights of correlation between the Polyreference technique and the Canonical-Form Realization.

Form the $(r+1) \times (s+1)$ block Hankel matrix

$$H(0) = \begin{bmatrix} Y(0) & Y(1) & \cdots & Y(s) \\ Y(1) & Y(2) & \cdots & Y(s+1) \\ \vdots & \vdots & \ddots & \vdots \\ Y(r) & Y(r+1) & \cdots & Y(r+s) \end{bmatrix} \tag{200}$$

where r and s are integers which are chosen to be larger than the order of the system. Using the singular value decomposition, find the nonsingular matrices P and Q such that

$$H(0) = PDQ^T = [P_n \quad P_o] \begin{bmatrix} D_n & 0 \\ 0 & 0 \end{bmatrix} [Q_n \quad Q_o]^T \tag{201}$$

where D_n is a diagonal matrix containing monotonically non- increasing nonzero singular values. The integer n is determined by the characteristics of the system noise as discussed in [140]. All singular values numbered after n are considered as zero values. The matrices P_n and P_o denote respectively the first n and the last remaining columns of the orthonormal matrix P. Similarly, Q_n and Q_o denote respectively the first n and the remaining columns of the orthonormal matrix Q.

Now observe, from the definition of Markov parameters, that

$$H(0) = VW; \quad V = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^r \end{bmatrix} \quad \text{and} \quad W = [B \quad AB \quad \cdots \quad A^sB] \tag{202}$$

where V and W are observability and controllability matrices respectively. Since matrices P and Q are orthonormal, i.e., $P^TP = Q^TQ = I$, (201) and (202) lead to

$$P^TH(0)Q = P^TVWQ = \begin{bmatrix} P_n^TVWQ_n & P_n^TVWQ_o \\ P_o^TVWQ_n & P_o^TVWQ_o \end{bmatrix} = \begin{bmatrix} D_n & 0 \\ 0 & 0 \end{bmatrix} \tag{203}$$

which yields

$$P_n^T V W Q_n = D_n, \tag{204}$$

$$P_n^T V W Q_o = 0, \qquad P_o^T V W Q_n = 0, \tag{205}$$

and

$$P_o V W Q_o = 0. \tag{206}$$

Note that the ERA algorithm is developed using the matrices P_n, Q_n and D_n as shown in (204). If the system is assumed controllable and observable, each of the five matrices P_n, V, W, Q_n and D_n are of rank n. By (204), it means that the ranks of matrices $P_n^T V$ and $W Q_n$ are n. Thus (205) and (206) imply

$$P_o^T V = 0 \qquad \text{and} \qquad W Q_o = 0. \tag{207}$$

The matrix P_o provides the left orthonormal basis for the null subspace which is orthogonal to the observability matrix, whereas the matrix Q_o gives the right orthonormal basis for the null subspace which is orthonormal to the controllability matrix. Now partition the matrices P_o and Q_o as

$$P_o^T = [\, P_{o0}^T \quad P_{o1}^T \quad \cdots \quad P_{or}^T \,] \qquad \text{and} \qquad Q_o^T = [\, Q_{o0}^T \quad Q_{o1}^T \quad \cdots \quad Q_{os}^T \,]. \tag{208}$$

Substitution of (202) and (208) into (207) yields

$$\sum_{i=0}^{r} P_{oi}^T C A^i = P_{o0}^T C + P_{o1}^T C A + \cdots + P_{or}^T C A^r = 0 \tag{209a}$$

$$\sum_{i=0}^{s} A^i B Q_{oi} = BQ_{o0} + ABQ_{o1} + \cdots + A^s BQ_{os} = 0 \qquad (209b)$$

Equation (209) is the basic formulation for the Polyreference technique and the Canonical Form Realization. In fact, (209a) is the basis for an observable canonical-form realization whereas (209b) is the basis for a controllable canonical-form realization (see [121], pp. 321, Problem 6-19 and 6-21). Both equations (209) should produce the same results. The question arises as to whether (209a) is more favorable than (209b) or vice versa. The answer is given in the following.

Observe that each submatrix P_{oi} $(i = 0, \cdots, r)$ must have more columns than rows (the number of outputs l). Similarly, each submatrix Q_{oi} $(i = 0, \cdots, s)$ has more columns than rows (the number of inputs m). Choose square matrices $\bar{P}_{oi}$ of order l and $\bar{Q}_{oi}$ of order m respectively from matrices P_{oi} and Q_{oi}, and rewrite (209) such that

$$-\sum_{i=0}^{r-1} [\bar{P}_{oi}^T]^{-1} \tilde{P}_{oi}^T C A^i = CA^r \equiv \sum_{i=0}^{r-1} \tilde{P}_{oi}^T C A^i \qquad (210a)$$

and

$$-\sum_{i=0}^{s-1} A^i B \bar{Q}_{oi} [\bar{Q}_{os}]^{-1} = A^s B \equiv \sum_{i=0}^{s-1} A^i B \tilde{Q}_{oi} \qquad (210b)$$

with

$$\tilde{P}_{oi}^T = [\bar{P}_{or}^T]^{-1} \bar{P}_{oi}^T \qquad \text{and} \qquad \tilde{Q}_{oi} = \bar{Q}_{oi} [\bar{Q}_{os}]^{-1}.$$

Equation (210) can be rearranged into companion matrix form as

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-1} \\ CA^{r-2} \end{bmatrix} A = \begin{bmatrix} 0 & I_l & 0 & \cdots & 0 \\ 0 & 0 & I_l & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1}^T & -\tilde{P}_{02}^T & \cdots & -\tilde{P}_{o(r-1)}^T \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-2} \\ CA^{r-1} \end{bmatrix} \tag{211a}$$

and

$$A[B \quad AB \quad \cdots \quad A^{s-1}B] = [B \quad AB \quad \cdots \quad A^{s-1}B] \begin{bmatrix} 0 & 0 & \cdots & 0 & \tilde{Q}_{o0} \\ I_m & 0 & \cdots & 0 & \tilde{Q}_{o1} \\ 0 & I_m & \cdots & 0 & \tilde{Q}_{o2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_m & \tilde{Q}_{o(s-1)} \end{bmatrix}. \tag{211b}$$

The matrix I_l is an identity matrix of order l and the matrix I_m is an identity matrix of order m. Now it is claimed from (211a) that the triple

$$\hat{A} = \begin{bmatrix} 0 & I_l & 0 & \cdots & 0 \\ 0 & 0 & I_l & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1}^T & -\tilde{P}_{o2}^T & \cdots & -\tilde{P}_{o(r-1)}^T \end{bmatrix} \tag{212a}$$

$$\tilde{B} = \begin{bmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(r-2) \\ Y(r-1) \end{bmatrix} \tag{212b}$$

$$\tilde{C} = [I_l \quad 0 \quad \cdots \quad 0 \quad 0] \tag{212c}$$

is an lr-dimensional realization of the system. Indeed, it can be readily verified that

$$Y(0) = \tilde{C}\tilde{B}, \quad Y(1) = \tilde{C}\tilde{A}\tilde{B}, \quad \cdots, Y(r) = \tilde{C}\tilde{A}^r\tilde{B}.$$

Because of the structure of $\tilde{A}$ and $\tilde{C}$, it is easy to show that the realization is observable. However, it is not in general controllable. This realization is called an observable canonical-form realization. It is not a minimum realization because it is not both observable and controllable.

Similarly, from (211b), it can be verified that the triple

$$\tilde{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 & \tilde{Q}_{o0} \\ I_m & 0 & \cdots & 0 & \tilde{Q}_{o1} \\ 0 & I_m & \cdots & 0 & \tilde{Q}_{o2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_m & \tilde{Q}_{o(s-1)} \end{bmatrix} \tag{212d}$$

$$\tilde{B} = \begin{bmatrix} I_m \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{212e}$$

$$\tilde{C} = [Y(0) \quad Y(1) \quad \cdots \quad Y(s-2) \quad Y(s-1)] \tag{212f}$$

is an ms-dimensional realization of the system. This is a controllable canonical-form realization which is not in general observable. Again this realization is not of minimum order.

Equation (212) can be reduced to a minimum order realization by applying the reduction procedure shown in [121], Chapter 5. However, the canonical form will be destroyed after the application of the reduction procedure.

The order of either observable or controllable canonical-form realization, i.e., lr or ms, is required to be equal to or larger than the order of the system. From a computational point of view, one should choose the one with smaller dimension to work for modal parameter identification. The numerical problem for the eigensolution of the canonical-form realization can be solved in various ways. A technique suitable and efficient for mini-

computer systems has been implemented and shown in [146]. It should be remarked that only a subset of eigenvalues in the realized state matrix $\tilde{A}$ (see (212)) belongs to the actual state matrix A, since the matrix $\tilde{A}$ is generally oversized for multi-input and multi-output measurements.

The method used here to obtain the canonical-form realization is different from that shown in [144]. Orthonormal matrices P_o and Q_o are computed through the application of the singular value decomposition to realize a companion-form state matrix. Since the orthonormal matrices are very close to identity matrices, this thus generates a computationally well-behaved canonical-form realization.

5.2.5. An Alternate Method for the ERA and the Polyreference Techniques.

A minimum order of canonical-form realization is generally impossible for multi-input, multi-output systems due to the constraint that the realized state matrix is a companion form. If the constraint is released, a minimum order realization can be obtained from (211).

In view of (201) and (202), the controllability and observability matrices can be expressed by the following equations

$$V = P_n D_n^{1/2} \qquad \text{and} \qquad W = D_n^{1/2} Q_n^T. \tag{213}$$

Define O as a null matrix, I_{lr} an identity matrix of order lr and $E_{lr} = [I_{lr} \;\; O]$ of dimension $lr \times l(r+1)$.

Hence, equation (211a) can be written with the aid of (212a) as

$$[E_{lr} P_n] D_n^{1/2} A = \tilde{A} [E_{lr} P_n] D_n^{1/2} \quad \rightarrow \quad A = D_n^{-1/2} [E_{lr} P_n]^{\dagger} \tilde{A} [E_{lr} P_n] \tag{214}$$

where $H^{\dagger}$ means the pseudoinverse of H. This is a minimum realization of order n. To compute (214), a simple procedure can be developed as follows. Define σ_l as a shift operator which shifts l columns of a matrix, for an example, $\sigma_l\, E_{lr} = [O \;\; I_{lr}]$. In view of the definition of the shift operator σ_l, the companion matrix $\tilde{A}$ in (212a) and (213), (214) becomes

$$A = D_n^{-1/2}[E_{lr}P_n]^{\dagger}[\sigma_l E_{lr}P_n]D_n^{1/2} \tag{215}$$

Here, $[\sigma_l\; E_{lr}P_n]$ simply means the matrix obtained by deleting the last l rows of the matrix P_n and $[E_{lr}\; P_n]$ represents the matrix obtained by deleting the first l rows of the matrix P_n. This equation was first presented in [147]. Since P_n is an orthonormal matrix, a special and efficient procedure can be developed to compute the pseudoinverse of the matrix $E_{lr}\; P$ using the matrix inversion lemma [147].

Similarly, an equation for the determination of the state transition matrix A can be derived from (211b) as

$$A = D_n^{1/2}[\sigma_m E_{ms}Q_n]^{T}[E_{ms}Q_n]^{T\dagger}D_n^{-1/2} \tag{216}$$

where $[E_{ms}\; Q_n]$ means the matrix obtained by deleting the first m rows of the matrix Q_n and $[\sigma_m\; E_{ms}\; Q_n]$ represents the matrix obtained by deleting the last rows of the matrix Q_n.

Equations (215) and (216) can also be derived by the ERA procedure. Let an oversized Hankel matrix H be formed such that

$$H(0) = E_{lr}\hat{H} \qquad \text{and} \qquad H(1) = \sigma_l E_{lr}\hat{H} \tag{217}$$

The Hankel matrix $H(0)$ is formed by deleting the last l rows of the Hankel matrix $\hat{H}$ whereas the Hankel matrix $H(1)$ is obtained by deleting the first l rows of the matrix $\hat{H}$.

Find the orthonormal matrices P_n and Q_n, and a diagonal matrix D_n such that

$$\hat{H} = P_n D_n Q_n^T. \tag{218}$$

Equation (217) thus becomes

$$H(0) = [E_{lr} P_n] D_n Q_n^T \qquad \text{and} \qquad H(1) = [\sigma_l E_{lr} P_n] D_n Q_n^T. \tag{219}$$

Substituting this equation into the ERA basic formulation (199) and noting that $Q_n^T Q_n = I_n$ yields the triple

$$A = D_n^{-1/2} [E_{lr} P_n]^\dagger [\sigma_l E_{lr} P_n] D_n^{1/2}, \quad B = D_n^{1/2} Q_n^T E_m, \quad C = E_l^T P_n D_n^{1/2} \tag{220}$$

The state transition matrix A is indeed identical to that in (215).

Similarly, it can be written that

$$H(0) = \hat{H} E_{ms}^T = P_n D_n [E_{ms} Q_n]^T \tag{221a}$$

$$H(1) = \hat{H} [\sigma_m E_{ms}]^T = P_n D_n [\sigma_m E_{ms} Q_n]^T \tag{221b}$$

where the Hankel matrix $H(0)$ is obtained by deleting the last m columns of the oversized Hankel matrix $\hat{H}$ and the Hankel matrix $H(1)$ is obtained by deleting the first m columns of the same matrix $\hat{H}$. Substitution of (221) into the ERA basic formulation, with the aid of $P_n^T P_n = I_n$, produces the triple

$$A = D_n^{1/2} [\sigma_m E_{ms} Q_n]^T [E_{ms} Q_n]^{T\dagger} D_n^{-1/2} \tag{222a}$$

$$B = D_n^{1/2} Q_n^T E_m \tag{222b}$$

$$C = E_l^T P_n D_n^{1/2}. \tag{222c}$$

The state transition matrix A is again identical to that in (216).

Realizations (220) and (222) preserve the same features as for the ERA, including a good numerical performance, internal balancedness, and flexibility in determining order-error tradeoff. Based on formulations (220) and (222), a close link between the ERA and the Polyreference techniques is established through the singular value decomposition and the generalized Hankel matrix.

5.2.6 Least Squares Regression Techniques.

The least square regression technique for a discrete-time dynamic model has been derived and used for system identification for more than two decades (see [115], Chapter 5, pp. 97-99). The same technique was rederived and further developed for the use of modal parameter identification in the field of structures, [150],[151]. Here, the least square regression technique will be formulated using system realization theory which provides a good basis for the comparison with other methods.

In view of (190) and (191), the measurement function $Y(k)$ can be obtained through either of two other algorithms as follows:

$$\begin{aligned} Y(k) &= E_l^T H(k) E_m \\ &= E_l^T V_\xi A^k [W_\eta H^\dagger V_\xi] W_\eta E_m \\ &= E_l^T [V_\xi A W_\eta H^\dagger]^k V_\xi W_\eta E_m \\ &= E_l^T [H(1) H^\dagger]^k H(0) E_m \end{aligned} \tag{223}$$

or

$$
\begin{aligned}
Y(k) &= E_l^T H(k) E_m \\
&= E_l^T V_\xi [W_\eta H^\dagger V_\xi] A^k W_\eta E_m \\
&= E_l^T V_\xi W_\eta [H^\dagger V_\xi A W_\eta]^k E_m \\
&= E_l^T H(0) [H^\dagger H(1)]^k E_m.
\end{aligned} \tag{224}
$$

Hence, the triple $[H(1)H^\dagger, H(0)E_m, E_l^T]$ or the triple $[H^\dagger H(1), E_m, E_l^T H(0)]$ is a realization. The matrix $H(1)H^\dagger$ or $H^\dagger H(1)$ constitutes the basis for the least square regression technique (see [115], pp. 97-99).

The matrix $H^\dagger$ is the pseudoinverse of the matrix $H(0)$. For a single input, there exists a case where the rank of $H(0)$ equals the column number of $H(0)$, then

$$H^\dagger = [H(0)^T H(0)]^{-1} H(0)^T$$

On the other hand, there exist a case for a single output where the rank equals the row number, then

$$H^\dagger = H(0)^T [H(0) H(0)^T]^{-1}$$

The matrix $H(1)H^\dagger$ has been used in the structural dynamics field to identify system modes and frequencies (see [150]-[151]). This is a special case representing a single input which cannot realize a system that has repeated eigenvalues or use sufficiently low noise data unless the system order is known *a priori*.

These realizations (equations (223) and (224)) are not of minimum order, since the dimension of $x(t)$ is the number of either columns or rows of the matrix $H(0)$ which is larger than the order of the state matrix A for multi-input and multi-output cases. Examination of (223) and (224) reveals that these two equations are special cases of ERA. Equation (223) is formulated by inserting the identity matrix (equation (191)) on the

right- hand side of the state transition matrix A. On the other hand, (224) is obtained by inserting the identity matrix (equation (191)) on the left-hand side of the state transition matrix A. However, the ERA is formed by inserting the identity matrix (equation (191)) on both sides of the state transition matrix A as shown in (196). Because of the different insertion, the least square regression method does not minimize the order of the system. Mathematically, if the singular-value decomposition technique or another rank detection technique is not included in the computational procedures, the realized triple obtained from (223) or (224) cannot be numerically implemented, unless a certain degree of artificial noise and/or system noise is present. Noise tends to make up the rank deficiency of the data matrix $H(0)$. Since the degree of noise contamination is generally unknown, the least squares regression technique (equation (223) or (224)) is not recommended for modal parameter identification in the structures field.

5.3 Frequency-Domain Modal Parameter Identification.

Time and frequency are two fundamental bases of description for linear dynamic systems [152]. For an example, given a single input and single output linear dynamic system, a three-dimensional space can be constructed for the output response with amplitude as one axis, and time and frequency as the other axes. A sinusoidal time history for each individual frequency (mode) can be treated as a projection on the time plane, existing at some distance from the origin. This distance is measured along the frequency axis. Similarly, the dynamic output response has a projection onto the frequency plane. This projection takes the form of an impulse with an amplitude. The position of the impulse coincides with the corresponding frequency. Summing multiple time plane projections produces the time history of the dynamic response. Similarly, connecting all the frequency components in the frequency plane yields the spectrum diagram. The duality of the time and frequency description of the dynamic response for a linear system becomes evident.

This section exposes the close conceptual connection between time-domain and frequency domain approaches to identification of modal parameters for linear dynamic systems. To identify modal parameters including damping ratios, frequencies, mode shapes, and modal participation factors, many methods have been developed using frequency spectra (transfer function) analysis or time- domain methods. For many years, the multi-shaker sine-dwell approach was the primary experimental method employed in aerospace structural mode surveys. The major advantage is that each mode is experimentally isolated by careful tuning of a series of shakers and verified as part of the experimental process. The disadvantage lies in the long elapsed testing time and the requirement for considerable test engineering expertise.

The current section presents some recently- developed frequency-domain techniques. All of the methods will be correlated using system realization theory. There are a number of important features in the frequency-domain analysis, including overlap averaging and zooming [152]. Overlap averaging is used to smooth the transfer function, while zooming is used to concentrate all the spectral lines into a narrow band in the frequency range of interest.

The transfer functions are basic elements for frequency-domain techniques. This section starts with the discussion of the transfer function characteristics. It then follows by the presentation of Eigensystem Realization Algorithm in frequency-domain and the Polyreference technique in frequency-domain, for the purpose of comparison with time-domain identification techniques. The conceptual connections between time-domain and frequency-domain approaches are exposed and discussed.

5.3.1 Characteristics of the Transfer Function.

As pointed out in the last section, the linear, constant, finite-dimensional dynamical systems can be represented by the continuous-time model

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) \tag{225}$$

$$y(t) = Cx(t). \tag{226}$$

Taking the Laplace transform of (225) and (226) yields

$$y(s) = C[sI - \bar{A}]^{-1}(x(t_0) + \bar{B}u(s)) \tag{227}$$

where I is an identity matrix and s is the Laplace transform parameter. For $x(t_0) = 0$,

$$y(s) = C[sI - \bar{A}]^{-1}\bar{B}u(s). \tag{228}$$

This equation shows the direct relation between the input $u(s)$ and the output $y(s)$. The function defined by

$$Y(s) = C[sI - \bar{A}]^{-1}\bar{B} \tag{229}$$

is called the transfer function matrix of dimension l (the number of outputs) by m (the number of inputs).

Let $Y(t)$ be the impulse response function matrix

$$Y(t) = Ce^{\bar{A}t}\bar{B} \tag{230}$$

which is then related to the transfer function matrix $Y(s)$ by

$$
\begin{aligned}
Y(s) &= \mathcal{L}[Y(t)] \\
&= \int_0^\infty Y(t)e^{-st}dt \\
&= \int_0^\infty Ce^{\bar{A}t}Be^{-st}dt \\
&= C[sI - \bar{A}]^{-1}\bar{B}
\end{aligned}
\tag{231}
$$

or

$$
\begin{aligned}
Y(t) &= \mathcal{L}^{-1}[Y(s)] \\
&= \frac{1}{2\pi i}\int_{\sigma_0-i\infty}^{\sigma_0+i\infty} Y(s)e^{st}ds \\
&= \frac{1}{2\pi i}\int_{\sigma_0-i\infty}^{\sigma_0+i\infty} C[sI - \bar{A}]^{-1}\bar{B}e^{st}ds \\
&= Ce^{\bar{A}t}\bar{B}
\end{aligned}
\tag{232}
$$

where the eigenvalues of A are assumed to have negative real parts and $\mathcal{L}^{-1}[\cdot]$ is the inverse of the Laplace transform operator $\mathcal{L}[\cdot]$. The Laplace transform of the ith derivative of the impulse response function matrix becomes

$$
\begin{aligned}
\mathcal{L}[Y^j(t)] &= \int_0^\infty C\bar{A}^j e^{\bar{A}t}Be^{-st}dt \\
&= C\bar{A}^j[sI - \bar{A}]^{-1}B \\
&= C\bar{A}^j G(s)B \\
&= s^jY(s) - s^{j-1}Y^{(0)}(t=0) - s^{j-2}Y^{(1)}(t=0) - \cdots - Y^{(j-1)}(t=0) \\
&= s^jY(s) - \sum_{k=0}^{j-1} Y^{(k)}(t=0)s^{j-1-k}; \qquad j = 1, 2, \cdots
\end{aligned}
\tag{233}
$$

where $Y^{(j)} = dY^j(t)/dt^j$ and $G(s) = [sI - \bar{A}]^{-1}$.

Let the equally-spaced time be given by $0,\ \Delta t,\ 2\Delta t, \cdots, k\Delta t, \cdots$ where Δt is a constant interval. The numerical Fourier transform of the transfer function $Y(i\omega)$ has the approximation as follows:

$$
\begin{aligned}
Y(i\omega) &= \int_0^\infty Y(t)e^{-i\omega t}dt \\
&\approx \sum_{k=0}^{\infty} Y(k\Delta t)e^{-i\omega\Delta t}\Delta t \\
&= \sum_{k=0}^{\infty} Ce^{\bar{A}k\Delta t}Be^{-i\omega k\Delta t}\Delta t \\
&= \sum_{k=0}^{\infty} CA^kBe^{-i\omega k\Delta t}\Delta t \\
&= \sum_{k=0}^{\infty} Y(k)e^{-i\omega k\Delta t}\Delta t \\
&= CG(i\omega)B
\end{aligned}
\tag{234}
$$

where

$$A = e^{\bar{A}\Delta t} \tag{177}$$

$$G(i\omega) = \sum_{k=0}^{\infty} A^kBe^{-i\omega k\Delta t}\Delta t$$

and

$$Y(k) = CA^kB. \tag{180}$$

The Fourier transform of the jth derivative of the transfer function matrix can correspondingly be approximated by

$$(i\omega)^jY(i\omega) - \sum_{k=0}^{j-1} Y^{(k)}(t=0)(i\omega)^{j-1-k} \approx \sum_{k=0}^{\infty} C\bar{A}^je^{\bar{A}k\Delta t}Be^{-i\omega k\Delta t}\Delta t = C\bar{A}^jG(i\omega)B. \tag{235}$$

Equations (234) and (235) constitute the bases for the frequency-domain parameter identification methods which will be presented in the following.

5.3.2. The Eigensystem Realization Algorithm in Frequency Domain, (ERA-FD).

This subsection presents a brief review of a recently developed technique for modal parameter identification which is referred to as the Eigensystem Realization Algorithm in Frequency-Domain (ERA-FD). In parallel to the Eigensystem Realization Algorithm in time-domain, the ERA-FD starts with a complex data matrix formed by transfer functions and corresponding shifted transfer functions. Using singular value decomposition on the data matrix, a state space triple $[A, B, C]$ is realized to match the transfer functions of the system. Both discrete-time and continuous-time models are considered for comparisons with other methods.

Consider a linear, time- invariant system initially at rest with an m- dimensional input signal $u(t)$ and a l-dimensional output $y(t)$, subject to an additive disturbance $n(t)$,

$$y(t) = \sum_{k=0}^{\infty} Y(k)u(t-k) + n(t) \qquad t = 0, 1, 2, \cdots \tag{236}$$

where $Y(k)$ is the impulse response matrix. This system has the transfer function as shown in (234)

$$Y(i\omega_j) = \sum_{k=0}^{\infty} Y(k)e^{-ik\Delta t\omega_j}\Delta t \qquad j = 0, 1, 2, \cdots \tag{237}$$

The identification problem now is the following. Generate and (or) measure an input signal $u(k), k = 0, 1, 2, \cdots, N$ and measure the corresponding output signal $y(k), k = 0, 1, 2, \cdots, N$. Based on these measurements, form an estimate of the transfer function, subject to the additive noise $n(t)$,

$$Y_o(i\omega_j) = \sum_{k=0}^{N} Y(k)\, e^{-ik\Delta t\omega_j}\Delta t = \sum_{k=0}^{N} CA^k B e^{-ik\Delta t\omega_j}\Delta t = CG(i\omega_j)B; \quad j = 0, 1, \cdots, N \tag{238}$$

where

$$G(i\omega_j) = \sum_{k=0}^{N} A^k e^{-ik\Delta t\omega_j}\Delta t.$$

Given an estimated transfer function $Y_o(i\omega_j)$, the goal is to find a triple $[A, B, C]$ of minimum order such that identities of (238) hold. The triple with minimum order will minimize the effect of noise on the identified modal parameters.

Define a shifted transfer function by

$$\begin{aligned} Y_r(i\omega_j) &\triangleq \sum_{k=0}^{N} Y(k+r)e^{-ik\Delta t\omega_j}\Delta t \\ &= \sum_{k=0}^{N} CA^{k+r}Be^{-ik\Delta t\omega_j}\Delta t \\ &= CA^r G(i\omega_j)B; \quad r = 0, 1, \cdots \end{aligned} \tag{239}$$

A special recursive formula to compute the shifted transfer function has been developed in [153]. The idea of the shifted transfer function is derived from the basic concept of system realization, i.e. the Hankel matrix.

The Eigensystem Realization Algorithm in frequency-domain begins by forming the $r \times N$ complex block matrix

$$H_g(k) = \begin{bmatrix} Y_k(i\omega_0) & Y_k(i\omega_1) & \cdots & Y_k(i\omega_N) \\ Y_{k+t_1}(i\omega_0) & Y_{k+t_1}(i\omega_1) & \cdots & Y_{k+t_1}(i\omega_j) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+t_{r-1}}(i\omega_0) & Y_{k+t_{r-1}}(i\omega_1) & \cdots & Y_{k+t_{r-1}}(i\omega_N) \end{bmatrix} = VA^kW \tag{240}$$

where

$$V = \begin{bmatrix} C \\ CA^{t_1} \\ \vdots \\ CA^{t_{r-1}} \end{bmatrix}$$

$$W = [\, G(i\omega_0)B \quad G(i\omega_1)B \quad \cdots \quad G(i\omega_N)B \,]$$

and $t_j \quad (i = 1, \cdots, r-1)$ are arbitrary integers. The matrix V is the observability matrix and the matrix W is the controllability matrix in frequency domain. Now find the singular value decomposition for the matrix $H_g(0)$

$$H_g(0) = P_n D_n Q_n^*; \qquad {}^*\text{is the complex conjugate transpose} \tag{241}$$

where P_n and Q_n are orthonormal matrices in complex domain, and D_n is a diagonal matrix with positive elements $[d_1,\ d_2\ , \cdots,\ d_n]$ referred to as singular values of $H_g(0)$. The rank of $H_g(0)$ is determined by testing the singular values for zero. The pseudo-inverse of the matrix $H_g(0)$ can then be given by

$$H_g^\dagger = Q_n D_n^{-1} P_n^*. \tag{242}$$

Now observe that, from (240),

$$H_g(0) = H_g(0) H_g^\dagger H_g(0) = VW H_g^\dagger VW \tag{243}$$

which implies

$$W H_g^\dagger V = I_n \tag{244}$$

where I_n is an identity matrix of order n.

Define 0_l as a null matrix of order l, I_l as identity matrix of order l and $E_{li}^T = [0_l, \cdots, I_l, \cdots, 0_l]$ where I_l is located at the ith position. With the aid of (241) - (244), a minimum order- realization can be obtained from

$$\begin{aligned} Y_k(j\omega_i) &= E_{l1}^T H_g(k) E_{mi} \\ &= E_{l1}^T V A^k W E_{mi} \\ &= E_{l1}^T V [W H_g^\dagger V] A^k [W H_g^\dagger V] W E_{mi} \\ &= E_{l1}^T H_g(0) H_g^\dagger V A^k W H_g^\dagger H_g(0) E_{mi} \\ &= E_{l1}^T P_n D_n^{1/2} [D_n^{-1/2} P_n^* V A^k W Q_n D_n^{-1/2}] D_n^{1/2} Q_n^* E_{mi} \\ &= E_{l1}^T P_n D_n^{1/2} [D_n^{-1/2} P_n^* H_g(1) Q_n D_n^{-1/2}]^k D_n^{1/2} Q_n^* E_{mi}. \end{aligned} \tag{245}$$

Examination of (239) and (245) shows that the triple

$$A = D_n^{-1/2} P_n^* H_g(1) Q_N D_n^{-1/2}, \quad B = G_1(i\omega_j) D_n^{1/2} Q_n^* E_{mi}, \quad C = E_{l1}^T P_n D_n^{1/2} \tag{246}$$

is a minimum realization derived from frequency domain analysis. To compute the matrix B, the integer j can be any value from 0 to N. For simplicity, it is set to $i = 0$. Based on the special characteristics of the forms B and C, accuracy indicators were developed in [153] to quantify the system and noise modes. Although the transfer function matrices $Y_k(i\omega_j)$ $(j = 0, \cdots, N)$ are in the complex domain, the block matrix H (k) can be implemented in the real domain by putting the real part of each individual matrix $Y_k(i\omega_j)$ in one block and its imaginary part in consecutive block. In doing this way, all the computations required for the system realization become real arithmetic.

Equation (246) is developed using the d.screte-time dynamic model as the basis so that the realized matrix A represents the state transition matrix (see (178)). The question arises whether a realization can be derived using the continuous-time model (171) to identify a state matrix $\bar{A}$ directly. Recall from (238) that

$$Y_0(i\omega_j) = \sum_{k=0}^{N} Ce^{\bar{A}k\Delta t}Be^{-ik\omega_j\Delta t}\Delta t = CG(i\omega_j)B; \quad i = 0,1,\cdots,N. \tag{247}$$

Redefine the shifted transfer functions $\bar{Y}_k(i\omega_j)$ with the aid of (235) as

$$\bar{Y}_r(i\omega_j) = (i\omega_j)^r Y_0(i\omega_j) - \sum_{k=0}^{r-1} Y^{(k)}(t=0)(i\omega_j)^{j-1-k} = C\bar{A}^r G(i\omega_j)B \tag{248}$$

for $j = 0,1,\cdots,N$. The shifted transfer functions in this case are obtained by differentiating impulse response function matrix r times. For example, if deflection sensors are used for dynamic measurements, $Y_0(i\omega_j)$ represent the transfer function matrix for the deflection responses, $Y_1(i\omega_j)$ Y describe the velocity responses and $Y_2(i\omega_j)$ describe the acceleration responses. It is generally inadvisable to differentiate a measurement signal because noise effects are greatly magnified. On the other hand, if accelerometers are used for the dynamic measurements, $Y_2(i\omega_j)$ represent the corresponding transfer function matrices. Then matrices $Y_1(i\omega_j)$ and $Y_0(i\omega_j)$ can be obtained by integrating the acceleration signals once and twice to respectively describe the velocity and deflection signals. The information of high frequency modes may be lost due to the integration process, however.

Now substituting these shifted transfer functions (248) into the block matrix (240) and performing the same procedures (equations (241) - (246)) such as singular value decomposition, etc., a minimum realization identical to (246) will be obtained except that the state transition matrix A is replaced by the state matrix $\bar{A}$. The detailed description is omitted. Instead, a simple example is discussed.

Let the block matrix $\bar{H}_g(0)$ be formed by

$$\bar{H}_g(0) = \begin{bmatrix} Y_0(i\omega_0) & Y_0(i\omega_1) & \cdots & Y_0(i\omega_N) \\ \bar{Y}_1(i\omega_0) & \bar{Y}_1(i\omega_1) & \cdots & \bar{Y}_1(i\omega_N) \end{bmatrix} = VW \tag{249}$$

where

$$V = \begin{bmatrix} C \\ C\bar{A} \end{bmatrix}$$

and

$$W = [\, G(i\omega_0)B \quad G(i\omega_1)B \quad \cdots \quad G(i\omega_N)B \,].$$

Note that the controllability matrix W is identical to the one shown in (240). Observe that

$$\bar{H}_g(1) = \begin{bmatrix} \bar{Y}_1(i\omega_0) & \bar{Y}_1(i\omega_1) & \cdots & \bar{Y}_1(i\omega_N) \\ \bar{Y}_2(i\omega_0) & \bar{Y}_2(i\omega_1) & \cdots & \bar{Y}_2(i\omega_N) \end{bmatrix} = V\bar{A}W \tag{250}$$

and assume that the number of rows for the matrix $H_g(0)$ is greater than the order of the system. Find the singular value decomposition for the block matrix

$$\bar{H}_g(0) = P_n \bar{D}_n Q_n^* \tag{251}$$

which is similar to (241). The triple

$$\bar{A} = [D_n^{-1/2} P_n^* \bar{H}_g(1) Q_n D_n^{-1/2}]^{-1}, \quad \bar{B} = G^{-1}(i\omega_i) D_n^{1/2} Q_n^* E_{mi}, \quad C = E_{l1}^T P_n D_n^{1/2} \tag{252}$$

is a direct minimum realization for the continuous-time model (equations (120) and (121)).

5.3.3. The Polyreference Technique in the Frequency Domain.

This subsection presents the Polyreference technique in frequency-domain for modal parameter identification using system realization theory. The method makes use of a set of transfer function matrices and shifted transfer function matrices to form a complex Hankel-like data matrix. By employing singular value decomposition of the data matrix, an orthonormal matrix is computed to derive an observable canonical-form realization which

is in parallel to the canonical-form realisation obtained for the Polyreference technique in time-domain.

The first part of this subsection will show the direct canonical-form realization of the discrete-time model. The close relationship of the time- domain and frequency-domain Polyreference techniques is established. The second part of this subsection will show the direct observable-form realization of the continuous-time model. The close relationship between this approach and other existing approaches, [82]-[154], is discussed.

Form the $(r+1) \times (s+1)$ block Hankel-like matrix

$$H_g(0) = \begin{bmatrix} Y_0(i\omega_0) & Y_0(i\omega_1) & \cdots & Y_0(i\omega_N) \\ Y_1(i\omega_0) & Y_1(i\omega_1) & \cdots & Y_1(i\omega_N) \\ \vdots & \vdots & \ddots & \vdots \\ Y_k(i\omega_0) & Y_k(i\omega_1) & \cdots & Y_k(i\omega_N) \end{bmatrix} = VW \tag{253}$$

where

$$V = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^r \end{bmatrix} \tag{254a}$$

with

$$W = [\, G(i\omega_0)B \quad G(i\omega_1)B \quad \cdots \quad G(i\omega_N)B \,] \tag{254b}$$

and r is an integer chosen such that the number lr (l=the number of outputs) is greater than the order of the system. Using the same procedures shown in (202)-(206), first find the singular value decomposition of $H_g(0)$ such that

$$\begin{bmatrix} P_n^T \\ P_o \end{bmatrix} H_g(0) [\, Q_n \quad Q_o \,] = \begin{bmatrix} D_n & 0 \\ 0 & 0 \end{bmatrix} \tag{255}$$

which yields

$$P_o^T V W Q_n = 0 \longrightarrow P_o^T V = 0 \tag{256}$$

where P_n, Q_n^T, P_o, and Q_o^T are orthonormal matrices and D_n is a diagonal matrix. The integer n is determined by the characteristics of the system noise as discussed in [140]. All singular values after n are considered as zero values.

Now partition the matrices P_o such that $P_o^T = [P_{o0}^T \; P_{o1}^T \; \cdots \; P_{or}^T]$ and choose square matrices $\bar{P}_{oi}$ of order l from matrices P_{oi} $(i = 0, 1, \cdots, r)$. Substitution of the observability matrix defined in (254) into (256) with the partitioned matrices P_{0i} produces

$$-\sum_{i=0}^{r-1} [\bar{P}_{or}^T]^{-1} \bar{P}_{oi}^T C A^i = C A^r \triangleq \sum_{i=0}^{r-1} \tilde{P}_{oi}^T C A^i \tag{257}$$

where

$$\tilde{P}_{oi}^T = [\bar{P}_{or}^T]^{-1} \bar{P}_{oi}^T.$$

Equation (257) can be rearranged into companion matrix form as

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-2} \\ CA^{r-1} \end{bmatrix} A = \begin{bmatrix} 0 & I_l & 0 & \cdots & 0 \\ 0 & 0 & L_l & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1}^T & -\tilde{P}_{o2}^T & \cdots & -\tilde{P}_{o(r-1)}^T \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-2} \\ CA^{r-1} \end{bmatrix}. \tag{258}$$

The matrix I_l is an identity matrix of order l. Now it is claimed from (258) that the triple

$$\tilde{A} = \begin{bmatrix} 0 & I_l & 0 & \cdots & 0 \\ 0 & 0 & L_l & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1}^T & -\tilde{P}_{o2}^T & \cdots & -\tilde{P}_{o(r-1)}^T \end{bmatrix} \tag{259a}$$

$$G(i\omega_i)\tilde{B} = \begin{bmatrix} Y_0(i\omega_j) \\ Y_1(i\omega_j) \\ \vdots \\ Y_{r-2}(i\omega_j) \\ Y_{r-1}(i\omega_j) \end{bmatrix} \tag{259b}$$

and

$$\tilde{C} = [\, I_l \quad 0 \cdots 0 \quad 0 \,] \tag{259c}$$

is an lr-dimensional realization of the system. Indeed, it can be readily verified that

$$Y_0(i\omega_j) = \tilde{C}G(i\omega_j)\tilde{B}$$

$$Y_1(i\omega_j) = \tilde{C}\tilde{A}G(i\omega_j)\tilde{B}$$

and

$$Y_r(i\omega_j) = \tilde{C}\tilde{A}^r G(i\omega_j)\tilde{B}.$$

Because of the structure of $\tilde{A}$ and $\tilde{C}$, it is easy to show that the realization is observable. However, it is not in general controllable. This realization is called an observable canonical-form realization. Therefore, it is not a minimum realization. Since the controllability matrix W defined in (254) does not have the form which explicitly involves the state transition matrix as the observability matrix does, there does not exist an explicit controllable-form realization as that for the time-domain Polyreference technique.

Equation (259) is developed using the discrete-time dynamic model as the basis such that the realized matrix $\tilde{A}$ represents the state transition matrix. It is natural to question whether a realization similar to (259) can be derived directly using the continuous-time model. The answer is affirmative. Replacing the shifted transfer function matrices in (253)

by the ones shown in (248) and performing the same procedures (254)- (259), an observable canonical-form realization identical to (259) will be obtained except that the triple $[\tilde{A}, \tilde{B}, \tilde{C}]$ is replaced by $[\bar{A}, \bar{B}, \bar{C}]$ where $\bar{A}$ represents the state matrix for the continuous-time model. The detailed description is omitted. Instead, a simple example is discussed for comparison with other existing methods [82],[154].

Let the Hankel-like matrix $\bar{H}_g(0)$ be formed as

$$\bar{H}_g(0) = \begin{bmatrix} Y_0(i\omega_0) & Y_0(i\omega_1) & \cdots & Y_0(i\omega_N) \\ \bar{Y}_1(i\omega_0) & \bar{Y}_1(i\omega_1) & \cdots & \bar{Y}_1(i\omega_N) \\ \bar{Y}_2(i\omega_0) & \bar{Y}_2(i\omega_1) & \cdots & \bar{Y}_2(i\omega_N) \end{bmatrix} = VW \tag{260}$$

$$C = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \tag{261a}$$

where

$$W = [\, G(i\omega_0)B \quad G(i\omega_1)B \quad \cdots \quad G(i\omega_N)B \,]. \tag{261b}$$

Assume that the number of rows of the matrix $H_g(0)$ is greater than the order of the system. Following the same procedures shown from (255) to (258) and using the same notations produces a polynomial equation

$$C\bar{A}^2 + \tilde{P}_{o1}^T C\bar{A} + \tilde{P}_{o0} C = 0. \tag{262}$$

Equation (257) can be rearranged into companion matrix form as

$$\begin{bmatrix} C \\ C\bar{A} \end{bmatrix} \bar{A} = \begin{bmatrix} 0 & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1} \end{bmatrix} \begin{bmatrix} C \\ C\bar{A} \end{bmatrix}. \tag{263}$$

The matrix I_l is an identity matrix of order l. The triple

$$\bar{A} = \begin{bmatrix} 0 & I_l \\ -\tilde{P}_{o0}^T & -\tilde{P}_{o1} \end{bmatrix} \tag{264a}$$

$$G(i\omega_j)\tilde{B} = \begin{bmatrix} Y_0(i\omega_j) \\ Y_1(i\omega_j) \end{bmatrix} \tag{264b}$$

and

$$\tilde{C} = [\, I_l \quad 0\,] \tag{264c}$$

is a $2l$-dimensional realization of the system. Examination of (171) and (264) reveals that the mass matrix M, the damping matrix D and the stiffness matrix K for a finite-dimensional system may be related by

$$M^{-1}K = \tilde{P}_{o0}^T \quad \text{and} \quad M^{-1}D = \tilde{P}_{o1}^T. \tag{265}$$

Modal parameters of the estimated system can be obtained by solving the eigenvalue problem of (264). If real and undamped modes are sought, just solve the eigenvalue problem with the absence of the matrix $\tilde{P}_{o1}^T$ in the state matrix $\tilde{A}$. The approach used here to obtain the observable canonical-form realization is different from that shown in [82] which first introduced the companion-form state matrix $\tilde{A}$ shown in (264) for modal parameter identification. However, both methods, which use the same transfer function matrices to build the state space model, are conceptually similar.

The orthonormal matrix P_o is computed through the application of the singular value decomposition to realize a companion-form state matrix. Since the orthonormal matrix is very close to the unity matrix, the method presented here thus generates a computationally well-behaved realization.

Now consider a simplest case where the Hankel-like matrix $\bar{H}_g(0)$ is formed by only two series of transfer function matrices $Y_C(i\omega_j)$ and $\bar{Y}_1(i\omega_j)$ $(i = 0, 1, \cdots, N)$ as shown in (250). Assume that the number of rows, $2l$, of the Hankel-like matrix is at least twice

the order of the system. Following the same procedures as shown from (255) to (257) yields a first degree polynomial equation as

$$C\tilde{A} + \tilde{P}_{o0}^{T} C = 0. \tag{266}$$

The triple

$$\tilde{A} = -\tilde{P}_{o0}^{T}, \quad G(i\omega_j)\tilde{B} = Y_0(i\omega_j), \quad \tilde{C} = I_l \tag{267}$$

is a l-dimensional realization of the system. Equation (266) was introduced in [154] for modal parameter identification for flexible structures. In contrast to the minimum realization shown in (252), this realization (267)) is not of minimum order in the sense that the identified state matrix is usually oversized if the order of the system is not known *a priori*. Furthermore, the number of the sensors, l, must be greater than the order of the system.

5.3.4. An Alternate Method for the ERA-FD Technique and the Polyreference Technique in the Frequency Domain.

A minimum-order of canonical-form realization in frequency domain is generally impossible for multi-input and multi-output systems due to the constraint that the realized state matrix is a companion form. If the constraint is released, a minimum order realization in frequency-domain can be obtained from (259). The procedures to derive the minimum order realization are identical to that shown in Section 5.2.5 for the time domain case. Using the block Hankel-like matrix (253) and notations defined in Section 5.2.5. The triple

$$A = D_n^{-1/2}[E_{lr}P_n]^{\dagger}[\sigma_l E_{lr}P_n]D_n^{1/2} \tag{268}$$

$$B = G^{-1}(i\omega_j)D_n^{1/2}Q_n^{*}E_m \tag{269}$$

and

$$C = E_l^T P_n D_n^{1/2} \tag{270}$$

is a minimum realization of order n. Here, $[\sigma_l\ E_{lr}\ P_n]$ simply means the matrix obtained by deleting the last l rows of the matrix P_n and $[E_{lr}\ P_n]$ represents the matrix obtained by deleting the first l rows of the matrix P_n. Since P_n is an orthonormal matrix, a special and efficient procedure can be developed to compute the pseudoinverse of the matrix $E_{lr}P$ using the matrix inversion lemma [193].

Based on (266), a close link between the ERA-FD and the Polyrefernce technique in Frequency domain is established. A minimum realization similar to (266) can also be derived for the direct realization of the continuous- time model.

After several methods in frequency-domain for modal parameter identification are derived using system realization theory, the derivation of the least square regression technique in frequency domain becomes trivial and thus is omitted.

5.4. Concluding Remarks.

In this section, several methods for modal parameter identification have been presented and derived using system realization theory. The relations between different techniques are reasonably well understood and the choice of methods can be made largely on the basis of the final purpose of the identification, tor example control of flexible structures. Most methods are claimed to work well on simulated and test data. In spite of a large literature on identification, there are few papers which compare different techniques using experimental data. Unfortunately, conclusive results have not been obtained. However, for a person engaged in application, it would be highly desirable to have comparisons available. This section illustrates the mathematical relations among several recently developed

methods via system realization theory, which provides a basis and insight for comparison and evaluation.

It is hoped and expected, through the interaction of control and structure fields, that the field of modal parameter identification is moving towards more unification and that there will be more comparisons of different methods. One of the purposes of this section is to contribute to the goal of unification. The contribution may be fairly small but it should serve as a starting point to stimulate more research toward this goal. It is believed that interaction with other fields such as controls, artificial intelligence, etc. is essential for progress in developing identification algorithms for structures.

For a section like this size, it is out of question to strive for completeness. This means that there exists many other techniques available in the fields of modal parameter identification. The reader is directed to the Bibliography for further information.

SECTION 6. IDENTIFICATION OF STRUCTURAL MODEL PARAMETERS.

When discussing "identification of structural model parameters", it is necessary to specify the set of parameters and the mathematical model that are being used to characterize the structure. The model and the particular set of parameters to be identified plays a central role in determining which identification methods are most suitable for use. Therefore, before discussing the identification techniques of current interest for large space structures, it is important to discuss the various parameterization methods available. The presentation in this section is tutorial. Planar structures will be used to provide simple examples of the basic ideas.

6.1. Parameterization Methods: An Overview.

In Section 3, finite element methods are reviewed. This is the most common and arguably the most powerful method of discretizing a general large flexible structure. The global mass and stiffness matrices can be expressed as

$$M = \sum_{i=1}^{N} M_i(\theta), \qquad K = \sum_{i=1}^{N} K_i(\theta) \tag{271}$$

where M, K are the contribution of the ith element (or collection of elements, or substructure) to the assembled global matrices and θ is an $n \times 1$ vector of associated physical modeling parameters (densities, Young's Moduli, cross-section areas, lengths, discrete masses etc.). Thus θ can be conceived of as the collection of all parameters which constitute the "argument list" of the finite element modeling process. Obviously, for a given application, there may be an infinity of interpretations implicit in (271).

Since θ is not always contained linearly in (271), one often makes use of local linearizations and recasts (271) in another form

$$M = M_0 + \sum_{j=1}^{n} \left.\frac{\partial M}{\partial \theta_j}\right|_0 (\theta_j - \theta_{j_0}) \tag{272}$$

$$K = K_0 + \sum_{j=1}^{n} \left.\frac{\partial K}{\partial \theta_j}\right|_0 (\theta_j - \theta_{j_0}) \tag{273}$$

where

$$M_0 = M(\theta_0) \tag{274}$$

$$K_0 = K(\theta_0) \tag{275}$$

and the partial derivative (sensitivity) matrices are assembled as

$$\left.\frac{\partial M}{\partial \theta_j}\right|_0 = \sum_{i=1}^{N} \left.\frac{\partial M_i}{\partial \theta_j}\right|_0 \tag{276}$$

$$\left.\frac{\partial K}{\partial \theta_j}\right|_0 = \sum_{i=1}^{N} \left.\frac{\partial K_i}{\partial \theta_j}\right|_0 . \tag{277}$$

It is evident that (272) and (273) can be used to apply small changes to the parameter vector in order to update $M(\theta)$ and $K(\theta)$ consistent with response measurements. It is of interest to note the close resemblance of (272) and (273) with the submatrix scaling equations (279a) and (279b) where the submatrices have now become the sensitivity matrices and the scale factors are the changes in the physical parameters.

Many space structures, although large in size and complex in detail, behave grossly as a beam, plate, or shell. In view of this fact, an interesting method of parameterization is one in which the structure is treated as an equivalent simple continuum model, leading to a small set of equivalent parameters, as discussed in Section 3.3. This approach is most attractive for beam-like and plate-like lattice structures with repeating lattice patterns

and has proven to be quite accurate in predicting natural frequencies and mode shapes associated with the macroscopic deformations of these structures. This family of methods is suited for identifying the structure such that the macroscopic motions are well modeled.

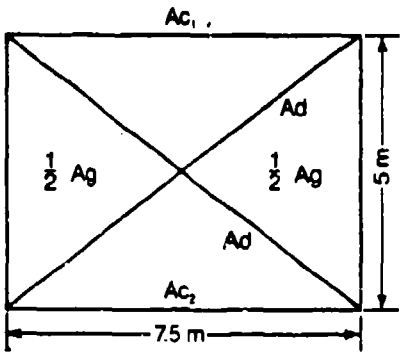


Figure 9. Simple Planar Structure.

EXAMPLE 2: To illustrate the use of simple continuum models for parameterization, the equivalent Timoshenko beam parameters are given for the planar structure of Figure 9. The details for deriving these parameters are presented in Section 3.3.

The horizontal members have cross-sectional area A_c, the vertical members $(1/2)\ A_G$ and the diagonal members A_D. All members have modulus of elasticity E and density ρ.

The equivalent shear and bending rigidities are

$$\overline{GA} = 2\ L_C\ L_G^2\ E\ A_D / L_D^3 \tag{278}$$

$$\overline{EI} = \frac{1}{2} L_G^2\ E\ A_C. \tag{279}$$

The equivalent mass properties are

$$\overline{m} = \rho \left[A_G L_G + 2A_C L_C + 2A_D L_D\right]/L_C \tag{280}$$

$$\overline{\rho I} = \rho \left[A_G L_G \left(\frac{L_G^2}{12}\right) + 2A_D L_D \left(\frac{L_D^2}{12}\right) + 2A_C L_C \left(\frac{L_G^2}{2}\right)\right]/L_C. \tag{281}$$

Upon deriving these equivalent parameters, notice that (278) -(281) provide a reduction in the number of parameters from eight (L_C, L_G, L_D, A_C, A_G, A_D, E, ρ) *to four* ($\overline{GA}$, $\overline{EI}$, $\overline{m}$, $\overline{\rho I}$). *Upon considering a structure composed of many repeating substructures, it is obvious that the reduction in number of model parameters is potentially very significant.*

In some identification techniques, the parameter set does not necessarily correspond to physical properties inherent to the structure. These techniques can be separated into two general classes: those which utilize an initial, approximate model of the structure (through finite-element analysis, for example), and those which are independent of any modeling process. As mentioned in Section 3.2, one method of parameterization, which requires the availability of an initial finite-element model, is to update the global mass and stiffness matrices by adding (assembling) scaled versions of prescribed submatrices in such a way that improves the accuracy of the global model (vis-a-vis matching measured response or eigenvalue/eigenvector data). These submatrices can represent single finite elements or, more commonly, groups of elements having the same geometry, material properties, boundary conditions, and modeling assumptions. White and Maytum [169] show that an array of energy distribution (potential and/or kinetic) among these similar element groups can be used for identification of the parameter set, which now consists of the submatrix scaling factors. A potentially crucial pitfall here is that constraining a linear system to only match measured eigenvalues and eigenvectors does not uniquely identify the system [175]. An infinity of linear systems have a given set of eigenvalues and eigenvectors.

EXAMPLE 3: To illustrate the use of submatrix scaling factors, consider the structure in Example 2. Let the horizontal members, the vertical members, and the diagonal members be arbitrarily defined as three separate element groups so that the global model can now be updated (improved) in the following manner

$$M = M_0 + \alpha_1 M_C + \alpha_2 M_G + \alpha_3 M_D \tag{282}$$

$$K = K_0 + \beta_1 K_C + \beta_2 K_G + \beta_3 K_D \tag{283}$$

where

M, K *are the final estimated mass and stiffness matrices*

M_0, K_0 *are the initial mass and stiffness matrices*

M_C, K_C *are the mass and stiffness contributions from the horizontal members*

M_G, K_G *are the mass and stiffness contributions from the vertical members*

M_D, K_D *are the mass and stiffness contributions from the diagonal members*

α_i, β_i *are the to-be-determined submatrix scaling factors.*

In another approach, which also utilizes an initial model of the structure, it is desired to obtain improved mass and stiffness matrices by adding small modifications to their initial approximations. Berman and Flannely [170] and Berman and Nagy [171] determine minimum changes in the initial mass and stiffness matrices such that the final model satisfies the eigenvalue condition

$$K\Phi = M\Phi\Omega^2 \tag{284}$$

and the orthonormality condition

$$\Phi^T M \Phi = I \tag{285}$$

where M is the unknown mass matrix, K is the unknown stiffness matrix, Φ is a matrix of measured mode shapes, and Ω is a diagonal matrix of measured natural frequencies. Clearly, the dimension of the system is dictated by the number of sensors, their location being consistent with the implicit discretization. This method has been commonly referred to as Analytical Model Improvement (AMI). In a similar approach Chen et al [174] obtained improved mass and stiffness matrices based on the theory of matrix perturbations. Small perturbations are introduced to the initial mass and stiffness matrices and to the analytically predicted eigenvectors and eigenvalues such that (284) and (285) are satisfied. In each of the above methods, the parameter set consists of the mass and stiffness matrix modifications. Once again, the issue of uniqueness is worth emphasizing: it will be shown later that these methods do not usually give the correct linear system, even for a complete set of perfectly measured eigenvalues and eigenvectors.

Since all of the parameterization methods listed above require the availability of an approximate model of the structure, the identification and modeling processes become inherently coupled. However, if there is little or no reliable information available for the structure (which would, of course, be the worst possible scenario), then there is no modeling process to rely upon and it becomes necessary to identify all elements of the mass, damping, and stiffness matrices [175]-[177]. It is anticipated from the onset that this approach is highly redundant for most applications since there are often many more elements in the mass, damping, and stiffness matrices than the needed number of more fundamental physical and (or) geometrical parameters previously described. The attractive feature of this approach is that the parameters (matrix elements) appear linearly in the equations of motion. The order of the system, and therefore the maximum size of the uniquely determined parameter set, is established by the number of measurement locations prescribed during the data gathering step of the identification process. If it is desired to

present the equations of motion in state-space form, it may be preferable to identify the elements of the state matrix [114],[178].

In the foregoing, linear models have been emphasized. One expects that nonlinear behavior of certain structural elements, such as joints, as well as large deformation material and geometrical nonlinearities will complicate at least a significant minority of the applications. It is impossible to generalize methodology to cover all conceivable nonlinear models, but several principles should govern. The most important strategy is to perform component testing of substructures with special emphasis upon components, such as coupling mechanisms (joints), where nonlinear behavior is likely. These could be characterized in the laboratory using, for example, the force-state mapping technique [179] of O'Donnell and Crawley. Once a parametric model is established for the nonlinear elements, it is possible to include them directly in the structural model and use (for example) nonlinear least- squares estimation approaches to recover best estimates for these parameters along with the rest of the structural parameters. Even though an element is nonlinear, one may still benefit from a "best-fitting linear" model. If the residuals are sufficiently small, it may be possible to fit local behavior linearly within a certain operating region of the state space. Virtually all control strategies for high dimensioned systems will rely upon a best-fitting linear model.

6.2. Parametric Identification Methods.

In Section 6.1, various parameterization methods (to be used in conjunction with identification) are reviewed. The identification techniques applicable to large space structures are now discussed in more detail, with examples to illustrate and explore the relative merits of each. In each example, a code is listed which classifies the method and its application according to Table 1.

Consider first those methods which identify the fundamental physical and (or) geometrical properties of the structure. The major advantages of these methods are that the actual physical properties of the structure are identified and the internal consistency of the finite-element model (or any other discretized representation) is maintained. The major disadvantage stems from the fact that the structural responses (natural frequencies, mode shapes, displacements, etc.) are generally nonlinear functions of the parameters, therefore requiring iterative procedures. Convergence is dependent upon the nonlinearity of the particular problem and the accuracy of the starting estimates. There are two methods which are most commonly used: the least- squares (or weighted- least- squares) method and the Bayesian method, which is simply a weighted- least- squares method with the incorporation of *a priori* parameter estimates. The least- squares method and its close cousins, minimal variance and maximum likelihood estimation, usually do not utilize statistical information (confidence levels) of the initial parameter estimates, whereas the Bayesian method allows for rigorous processing of the *a priori* parameter estimates and their associated confidence levels in the form of the parameter covariance matrix. This covariance matrix can be used to qualify the updated parameter estimates relative to their *a priori* values and covariance estimates. Also, the least squares method utilizes all of the measured data in batch form and is therefore applicable to off-line identification only; the Bayesian estimator allows for sequential processing of the measured data and is therefore (likely) applicable to both off-line and on-line identification. However, due to the dependence upon the initial parameter confidence levels, caution must be used when implementing the Bayesian estimator, for many different results can be obtained by manipulation of these confidence levels. A tutorial on the relationship between the least- squares method, the Bayesian method, and other similar methods is provided by Isenberg [156].

A common starting point for each of the methods is a discretized model of the structure, in the form of mass, damping, and stiffness matrices, which contains the values of the

structural properties originally assigned by the engineer. The corresponding eigenvalue problem then becomes

$$[\lambda_j^2 M(\theta) + \lambda_j D(\theta) + K(\theta)]\Phi_j = 0 \tag{286}$$

where M, D, K: $n \times n$ mass, damping, and stiffness matrices

λ_j : jth eigenvalue

Φ_j : $n \times 1$ jth eigenvector

θ : $m \times 1$ vector of structural parameters.

The objective is to improve the initial parameter vector, θ_0, so as to correlate the analytically determined modal characteristics with those measured on the actual structure. Since the modal characteristics are generally nonlinear functions of the parameters, each method requires an iterative scheme to determine the optimal parameter estimates. The measured eigenvalues and eigenvectors can be related to their predicted values in the following manner

$$\begin{bmatrix} \tilde{\lambda} \\ \tilde{\Phi} \end{bmatrix} = \begin{bmatrix} \lambda(\theta_0 + \Delta\theta) \\ \Phi(\theta_0 + \Delta\theta) \end{bmatrix} + \begin{bmatrix} y_\lambda \\ y_\Phi \end{bmatrix} \tag{287}$$

where

λ : $q \times 1$ vector containing the first q eigenvalues

$\tilde{\lambda}$, $\tilde{\Phi}$: measured quantities

y_λ, y_Φ : vectors of residual errors (the sum of measurement and model errors).

If (286) is expanded in a first-order Taylor's series a linear relationship between residuals and parameter corrections can be obtained:

$$Y \equiv \begin{bmatrix} \tilde{\lambda} - \lambda(\theta_0) \\ \tilde{\Phi} - \Phi(\theta_0) \end{bmatrix} = T\Delta\theta. \tag{288}$$

The Jacobian matrix is now defined as

$$T = \begin{bmatrix} \partial\lambda/\partial\theta \\ \partial\Phi/\partial\theta \end{bmatrix}. \tag{289}$$

This matrix is generally rectangular, depending on the number of measured quantities (number of rows of the Jacobian matrix) and the number of parameters to be identified (number of columns of the Jacobian matrix). The differences in the identification methods lie in the determination of $\Delta\theta$ from (187).

The classical least- squares solution to (288) is given as [157]

$$\Delta\theta = (T^T W T)^{-1} T^T W Y \tag{290}$$

where W is a symmetric, positive definite weight matrix. If the weight matrix corresponds to the inverse measurement error covariance matrix then (290) is also interpreted as a maximum likelihood as well as a minimal variance estimator. Hendricks et al [158] used a set of measured eigenvalues to estimate the parameters of a flexible structure using (290). Using Hendricks approach, an iterative solution is required fo. the estimation process, in which predicted eigenvalues are reevaluated at each iteration (from (286), using the currently estimated parameter set). A typical element of the Jacobian matrix, now consisting of eigenvalue sensitivities only, is determined by taking the derivative of (286) with respect to the parameter θ_i to obtain

$$\frac{\partial\lambda_j}{\partial\theta_i} = \frac{-\Phi_j^T[\lambda_j^2 \frac{\partial M}{\partial\theta_i} + \lambda_j \frac{\partial D}{\partial\theta_i} + \frac{\partial K}{\partial\theta_i}]\Phi_j}{\Phi_j^T[D + 2\lambda_j M]\Phi_j}. \tag{291}$$

To compute the Jacobian matrix from (291), the global mass, damping, and stiffness matrix sensitivities are required. These can easily be computed for relatively simple "academic"

problems because the global matrix elements are usually known in terms of the parameters. If a large, complex finite-element model is required to discretize the structure, the local (element level) matrix elements are known in terms of the parameters, so that the global matrix sensitivities can be determined by simple assembly of the local matrix sensitivities (analogous to the assembly of the local mass and stiffness matrices).

EXAMPLE 4: To illustrate the use of least squares estimation, the parameters of the undamped planar structure given below will be identified. Code (FEM-L/RMP/I,D,B,I)

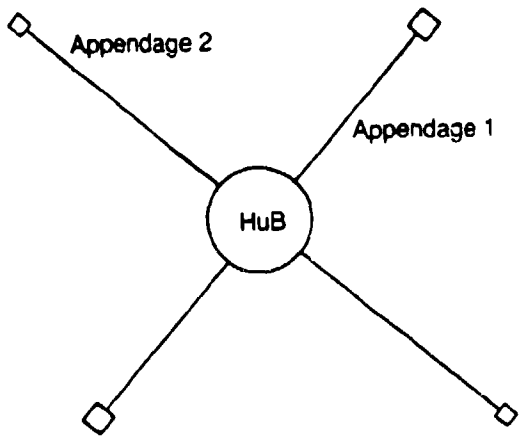


Figure 10. Schematic of Draper Model.

Ten measured frequencies, simulated from a "true" model and corrupted with Gaussian noise (zero mean, standard deviation = 0.05), are used to identify the seven parameters inherent to the structure: the bending rigidity (EI), mass density (ρ), and length (L) of the appendages, the tip mass (M_T) and tip mass moment of inertia (I_T) of the appendages, and the hub mass moment of inertia (I_H) and radius (R).

The procedure works well provided that a time scale, a mass scale, and a length scale are defined. The time scale is fixed from the measured frequencies, but the mass and

length scales must be determined by fixing two of the above parameters at their original values. For example, if the appendage length and mass density are fixed then a length scale (L) and a mass scale ($\rho * L$) are defined. Alternatively, a priori measured values could be introduced and treated as additional measurements in the estimation process.

The results below present the initial and final parameter estimates, and the measured and final predicted natural frequencies for a test case in which the hub inertia and radius are held fixed.

Table 5. Draper Model Parameters.

PARAMETER	INITIAL	FINAL	TRUTH
$EI\ (n \cdot m^2)$	135,350	127,220	128,910
$\rho\ (kg/m)$	0.0023835	0.0022058	0.00227
$L\ (m)$	48.96	48.195	48.0
$M_T\ (kg)$	0.16275	0.15362	0.155
$I_T\ (kg \cdot m^2)$	0.273	0.25328	0.26
$I_H\ (kg \cdot m^2)$	1670	1670	1670
$R\ (m)$	12	12	12

Table 6. Natural Frequencies of Draper Model.

MEASURED FREQUENCIES (rad/sec)	INITIALLY COMPUTED FREQUENCIES (rad/sec)	FINAL ESTIMATED FREQUENCIES (rad/sec)
4.3853	4.2612	4.365
7.0007	7.0261	7.0108
51.624	49.771	51.661
52.631	50.762	52.587
160.05	154.29	160.11
160.70	154.89	160.67
337.50	325.20	337.44
337.81	325.65	337.87
577.15	555.96	577.11
577.49	556.39	577.52

The scaling problem, present in the above example, arises because results from the free-vibration, homogeneous equations of motion are used for identification. Since (286) is homogeneous, it is evident that at least one arbitrary scale factor can be absorbed

into the mass, damping, and stiffness matrices, making their identification non-unique (given only the eigenvalues and eigenvectors). Creamer and Hendricks [159], investigating this problem further, conclude that knowledge (or direct measurement) of a single mass and length scale is a necessary condition, but not always sufficient. They show that the presence of symmetry in the structure can hinder the performance of the identification procedure, requiring knowledge of a mass and length scale for each symmetric portion of the structure. Of course, if all of the symmetry constraints are imposed in the structural model prior to identification, this problem will often be alleviated. Ref. [181] deals with the observability/identifiability problems in a general way and establishes general necessary conditions.

Chen and Garba [160] utilize a different approach to solve (288). They assume that there are only a few very accurate measurements available to identify a larger set of parameters. Therefore, a minimum-norm type solution is introduced to select one of the infinite number of possible solutions. The solution which yields the smallest possible changes in the values of the parameters is chosen. Defining a positive quantity Q as

$$Q = (\Delta\theta_1)^2 + (\Delta\theta_2)^2 + \cdots + (\Delta\theta_m)^2 \tag{292}$$

a solution $\Delta\theta$ is sought such that (288) is satisfied and Q is minimized. If s measurements are available ($s < m$), then the first s parameter changes can be written, from (288), in the form

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_s \end{bmatrix} = f(\Delta\theta_{s+1},\ \Delta\theta_{s+2},\ \cdots,\ \Delta\theta_m). \tag{293}$$

Therefore, Q can be minimized from

$$\frac{\partial Q}{\partial \Delta\theta_{s+1}} = \frac{\partial Q}{\partial \Delta\theta_{s+2}} = \cdots = \frac{\partial Q}{\partial \Delta\theta_m} = 0. \tag{294}$$

Equations (288) and (294) contain m equations to solve for the m unknowns, $\Delta\theta$.

Again, the procedure requires an iterative scheme to identify the parameters. Instead of reevaluating the eigenvalues and eigenvectors from the free-vibration equations, which can be computationally expensive for high-order systems, Chen utilizes matrix perturbation theory to update the predicted eigenvalues and eigenvectors at each iteration. This is more computationally efficient but is only accurate for very small changes in the parameters at each iteration. These approximation errors can accumulate and degrade the convergence of this approach. Claiming that analytical expressions for the Jacobian matrix would be difficult to obtain for complex structures, the authors use a finite-difference scheme, in conjunction with the perturbation method, to compute the eigenvalue and eigenvector sensitivities at each iteration. In applying this method to the Viking Orbiter propulsion system, it is shown that the final identified model can be highly dependent upon the measurements available and the selection of parameters used to update the model. Observe that minimizing the correction norm of (292), while it may appear reasonable, is nonetheless an *ad hoc* decision. Experience suggests that this approach should be applied to adjust a very small subset of the most important parameters whose *a priori* estimates are least certain. As is evident in the example below, obtaining good estimates using this method is not routinely achieved.

EXAMPLE 5: The method of Chen and Garba is used to identify the parameters of the simple spring-mass system below. Code (LPM/RMP/I,D,B,I)

Assume that three parameters are to be identified with two measured frequencies. Equation (286) then becomes

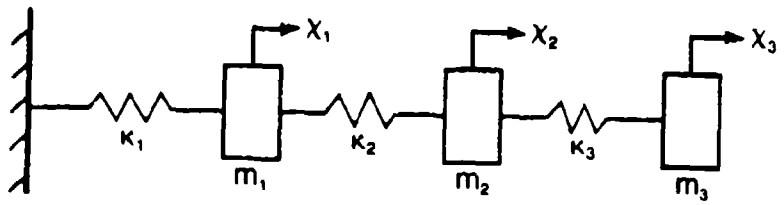


Figure 11. Typical Spring-Mass Configuration.

$$T_{11}\Delta\theta_1 + T_{12}\Delta\theta_2 + T_{13}\Delta\theta_3 = \Delta y_1 \tag{295}$$

$$T_{21}\Delta\theta_1 + T_{22}\Delta\theta_2 + T_{23}\Delta\theta_3 = \Delta y_2 \tag{296}$$

which then can be rewritten as

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{bmatrix} = \begin{bmatrix} \Delta y_1 - T_{13}\Delta\theta_3 \\ \Delta y_2 - T_{23}\Delta\theta_3 \end{bmatrix}. \tag{297}$$

To obtain the final equation, Q is defined as

$$Q = (\Delta\theta_1)^2 + (\Delta\theta_2)^2 + (\Delta\theta_3)^2 \tag{298}$$

which can be rewritten, using (297), as

$$Q = Q(\Delta\theta_3). \tag{299}$$

Therefore, minimizing Q yields

$$\frac{\partial Q}{\partial \Delta\theta_3} = 0. \tag{300}$$

Thus there are three equations, (297) and (300), to determine the three parameters.

Three test cases were performed for comparison of results.

Table 7. Spring-Mass Test Cases.

CASE	MEASUREMENTS	PARAMETERS
I	ω_1, ω_2	K_1, K_2, K_3
II	ω_1, ω_2	K_1, K_2, m_1
III	ω_2, ω_3	K_1, K_2, K_3

The results below show that no unique solution exists even for this simple problem. The results are dependent upon the measurements used and the parameters to be identified.

Table 8. Estimated Parameters: Spring-Mass System.

CASE	K_1 $(1.0N/M)^*$	K_2 $(2.0N/M)^*$	K_3 $(3.0N/M)^*$	m_1 $(1.0kg)^*$
I	0.99087	2.0914	2.8422	—
II	0.99487	2.1128	—	1.0359
III	0.93558	2.1032	2.9237	—

(* values in parentheses are true values)

Table 9. Estimated Frequencies: Spring-Mass System.

CASE	ω_1 $(0.50827)^*$	ω_2 $(1.7321)^*$	ω_3 $(2.7824)^*$
I	0.50827	1.7321	2.7568
II	0.50827	1.7321	2.8026
III	0.497790	1.7321	2.7824

(* values in parentheses are true values)

Notice that the estimated model frequencies match only the frequencies used as measurements.

The Bayesian estimation method [161],[162] is a statistical method for updating a set of parameters by utilizing prescribed confidence levels associated with the initial values of the parameters. Of the various forms used for writing the Bayesian estimate from (288), perhaps the most common is

$$\Delta\theta = \theta_i - \theta_{i-1} = (T^T S_{\varepsilon\varepsilon}^{-1} T + S_{\theta\theta}^{-1})^{-1} \left[T^T S_{\varepsilon\varepsilon}^{-1} Y + S_{\theta\theta}^{-1}(\theta_0 - \theta_{i-1})\right] \qquad (301)$$

where

θ_i : vector of updated parameter values (ith iteration)

θ_{i-1} : vector of current parameter values ($(i-1)th$ iteration)

θ_0 : vector of initial (*a priori*) parameter estimates

Y : vector of measured response minus predicted response at current $(i-1)$ iteration

$S_{\theta\theta}$: initial parameter covariance matrix

$S_{\varepsilon\varepsilon}$: measurement error covariance matrix

The confidence levels of the new estimate, computed after (301) has converged, are obtained from

$$S_{\theta\theta}^{*} = (T^T S_{\varepsilon\varepsilon}^{-1} T + S_{\theta\theta}^{-1})^{-1}. \qquad (302)$$

The diagonals of the covariance matrix represent the variances, σ_θ^2, of each parameter estimate. As is evident from (301), if the *a priori* parameter estimates are very poor ($S_{\theta\theta}^{-1} \rightarrow 0$), then the Bayesian estimator reduces to the minimal variance estimator, as described earlier. It is also of interest to note that the identical estimation algorithm of (301) results if (i) the weighted- least- squares approach is used and the initial estimates of θ_0 are treated as additional measurements, or (ii) the minimal variance approach is used and one seeks the optimum linear combination of the initial estimates θ_0 and the measurements Y. These results are proven in Junkins [157].

Many applications of the Bayesian estimator can be found in the literature. Collins et al [33] use the Bayesian estimator to identify a set of bending and shear rigidity parameters for the Saturn-Apollo launch vehicle. Beliveau [163] utilizes the Bayesian estimator to identify viscous damping properties of a shear building. Dobbs and Nelson [164] use

the estimator to determine some mass and stiffness parameters for an offshore platform. To reduce the numerical effort associated with the estimator, Dobbs incorporates a "parameter linking" constraint procedure in which certain common parameters are held in fixed proportion to one another. Hasselman and Johnson [180] and Fries and Cooperrider [183] use the Bayesian estimator to identify rail vehicle parameters. Fries and Cooperrider utilize frequency response measurements in place of eigenvalue-eigen measurements. Their Jacobian matrix represents the sensitivity of the frequency response function to the parameters of interest. Other uses of frequency response measurements for Bayesian estimation are given in Refs. [32],[184] through [186]. In a recent paper, Martinez [187] reviews and examines the estimation of a set of physical parameters via least- squares and statistical (Bayesian) methods and their relation to a more general Extended Kalman Filter algorithm. In using frequency response data with the EKF algorithm, Martinez marches through the frequency spectrum one frequency at a time, as opposed to the previous applications in which sequential processing of batch data is utilized.

Investigating the Bayesian estimation method more carefully, Hasselman [165] postulates the following bounds for evaluating the estimated parameters in terms of their associated increase in confidence:

(i) For confirmation of the prior estimate

$$|\Delta\theta| \leq \sigma_\theta^* \tag{303}$$

(ii) For a significant change in the prior estimate

$$|\Delta\theta| \geq 2\sigma_\theta^* \tag{304}$$

(iii) For an acceptable new estimate

$$|\Delta\theta| \leq \sigma_\theta + \sigma_\theta^* \tag{305}$$

(iv) For an unacceptable new estimate

$$|\Delta\theta| \geq 2(\sigma_\theta + \sigma_\theta^*) \tag{306}$$

In the above bounds, σ_θ is the standard deviation of the original parameter estimate and σ_θ^* is the standard deviation of the revised parameter estimate. The transition zones between the bounds require the engineer's judgement to draw one conclusion or the other. If any one of the parameter estimates falls in the range of unacceptability, then the model and/or the measurements should be closely scrutinized and, perhaps, rejected. To complete the picture, Hasselman includes the (somewhat arbitrary) bound

$$(\frac{\sigma_\theta}{\sigma_\theta^*} - 1) \geq 0.5$$

to distinguish between those estimates which are statistically significant and those which are not. For visual inspection of the qualification of the estimate, Hasselman presents a "significance" plot in reference [165] which incorporates the above bounds in graphical form. In concluding, Hasselman observes that correct use of the Bayesian estimation method requires (i) realistic estimates of the measurement uncertainty, (ii) realistic estimates of the initial parameter uncertainty, and (iii) use of measurements that are sufficiently sensitive to the parameters being estimated.

EXAMPLE 6: To illustrate the use of the Bayesian estimator, a set of parameters for the planar structure given in Figure 10 will be identified. Code (FEM-L/RMP/I,S,B,I)

Ten measured frequencies (zero mean, standard deviation = 0.05) are used to identify the following six parameters: the bending rigidities of each appendage ($EI1$, $EI2$), the tip masses ($M1_T$, $M2_T$), and the tip inertias ($I1_T$, $I2_T$).

The results below present the initial and final parameter estimates (with respective variances), and the measured and predicted natural frequencies.

Table 10. Estimated Parameters: Draper Model.

PARAMETER	INITIAL	σ	FINAL	σ^*	TRUTH
$EI1\ (n \cdot m^2)$	67676	5000	64372	143.28	64453
$EI2\ (n \cdot m^2)$	122460	10000	128890	208.08	128910
$M1_T\ (kg)$	0.16275	0.015	0.16312	0.0047944	0.155
$M2_T\ (kg)$	0.14725	0.015	0.15602	0.0033790	0.155
$I1_T\ (kg \cdot m^2)$	0.273	0.015	0.25481	0.0069702	0.26
$I2_T\ (kg \cdot m^2)$	0.247	0.015	0.25988	0.0050982	0.26

Table 11. Initial and Final Frequencies: Draper Model.

MEASURED FREQUENCIES (rad/sec)	INITIAL ESTIMATED FREQUENCIES (rad/sec)	FINAL ESTIMATED FREQUENCIES (rad/sec)
3.4700	3.5069	3.4262
6.2445	6.2647	6.2982
36.808	37.699	36.812
52.168	50.986	52.163
113.35	115.94	113.40
160.41	156.73	160.39
238.83	244.06	238.79
337.59	329.88	337.61
408.27	417.49	408.28
577.28	563.80	577.27

To assess the quality of the final estimate (303) can be used along with (306). The following results give the qualification of the estimate.

Table 12. Quality of Estimates:Draper Model.

PARAMETER	σ	σ^*	$\lvert\Delta\theta\rvert$	$\lvert\Delta\theta\rvert - \sigma^*$
$EI1$	5000	143.28	3304	3160.72
$EI2$	10000	208.08	6430	6221.92
$M1_T$	0.015	0.0047944	0.00037	-0.0044244
$M2_T$	0.015	0.0033790	0.00877	0.005391
$I1_T$	0.015	0.0069702	0.01819	0.0112198
$I2_T$	0.015	0.0050982	0.01288	0.0077818

From (305) it is concluded that all estimates are acceptable. Also, from (304) it is concluded that all parameters except $M1_T$ showed statistically significant improvements. From (303) it is concluded that the original estimate of $M1_T$ is statistically confirmed. Therefore, the revised model is considered an acceptable model for the structure.

In the above least- squares and Bayesian estimation examples, only frequency measurements were used. One could, additionally, use time domain response measurements (free or forced), such as displacements, velocities, and accelerations, as long as the response sensitivities could be calculated. The Jacobian matrix would now become augmented with both eigenvalue/eigenvector sensitivities and time domain response sensitivities. This allows for more flexibility in the estimation process, but also requires increased effort in the computational and experimental phases.

The large parameter sets associated with high degree of freedom, complex space structures may require computationally expensive identification efforts, regardless of the method employed. As discussed in Section 3.3, many large space structures can be treated grossly as a beam, plate, or shell, or a simple assembly thereof. These equivalent continuum models contain far fewer parameters than the discrete models and are, therefore, quite attractive for some identification purposes. Noor and Anderson [166], Chen and Sun [167], and Sun et al [168] develop methods for obtaining the equivalent continuum parameters for lattice structures. The partial differential equations governing the motion of these simple continuum models can be solved either analytically or by simple discretization procedures.

EXAMPLE 7: The equivalent continuum parameters are identified for a Timoshenko beam model of the ten-bay planar structure given below. Code (ECM-C/RMP/I,D,B,I)

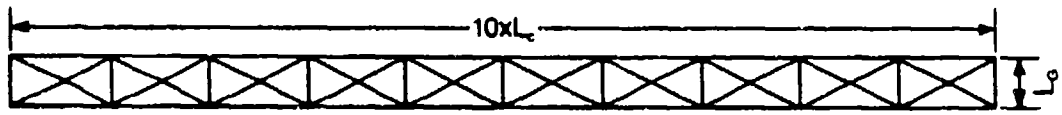


Figure 12. Ten Bay Symmetrical Truss.

The cross-sectional areas of the horizontal, vertical, and diagonal members are A_C, A_G, *and* A_D, *respectively. A representative element of the structure will be used to obtain the parameters.*

The partial differential equations governing the classical equivalent Timoshenko beam are

$$\frac{\partial}{\partial x}(\overline{EI}\frac{\partial \psi}{\partial x}) + \overline{GA}(\frac{\partial w}{\partial x} + \psi) = \overline{\rho I}\ddot{\psi} \tag{307}$$

$$\frac{\partial}{\partial x}[\overline{GA}(\frac{\partial w}{\partial x} + \psi)] = \bar{m}\ddot{w} \tag{308}$$

where w *is the centerline displacement and* ψ *is the rotation of the beam cross-section. From Section 3.3, the equivalent beam parameters can be shown to be*

$$\overline{GA} = 2\, L_C\, L_G^2\, E\, A_D / L_D^3 \tag{309}$$

$$\overline{EI} = \frac{1}{2} L_G^2\, E\, A_C. \tag{310}$$

$$\bar{m} = \rho\,[A_G L_G + 2A_C L_C + 2A_D L_D]/L_C \tag{311}$$

$$\bar{\rho I} = \rho\left[A_G L_G\left(\frac{L_G^2}{12}\right) + 2A_D L_D\left(\frac{L_D^2}{12}\right) + 2A_C L_C\left(\frac{L_G^2}{2}\right)\right]/L_C. \tag{312}$$

The numerical values are given below for the ten-bay truss:

Table 13. Parameters of Ten Bay Truss.

$L_G = 5\ (m)$	$\overline{\rho I} = 5.5199318\ (kg \cdot m^2)$
$E = 71.7 \times 10^9\ (N/m^2)$	$\overline{EI} = 7.17 \times 10^7\ (N \cdot m^2)$
$L_C = 10\ (m)$	$\overline{GA} = 1,026,087\ (N)$
$A_C = 80 \times 10^{-6}\ (m^2)$	$\overline{m} = 0.7734975\ (kg/m)$
$A_G = 60 \times 10^{-6}\ (m^2)$	
$A_D = 40 \times 10^{-6}\ (m^2)$	
$\rho = 2768\ (kg/m^3)$	

Using the least- squares estimation method, the equivalent bending and shear rigidities are identified for the ten-bay structure. The first six bending frequencies (the first three modes are rigid-body modes and the seventh is an axial mode), obtained from a finite element model of the actual structure and corrupted with Gaussian noise (zero mean, standard deviation = 0.20), are used as measurements. To simulate an approximate model, the modulus of elasticity used for computing the equivalent bending and shear rigidities is 10from a finite element model of the equivalent Timoshenko beam, are given below.

Table 14. Frequency Estimates: Timoshenko Beam.

INITIALLY APPROXIMATED FREQUENCIES (rad/sec)	*FINAL ESTIMATED FREQUENCIES (rad/sec)*	*MEASURED FREQUENCIES (rad/sec)*
21.68	*21.29*	*20.51*
55.55	*53.63*	*52.88*
100.19	*95.05*	*95.26*
152.13	*142.12*	*142.58*
209.22	*193.03*	*193.76*
268.72	*245.52*	*244.83*

The equivalent mass parameters $\overline{m}$ and $\overline{\rho I}$ are held equal to their calculated values, whereas $\overline{GA}$ and $\overline{EI}$ are estimated to best fit the six measured frequencies. The initial and final values for the equivalent rigidities are given below.

Table 15. Estimated Parameters of the Timoshenko Beam.

PARAMETER	*INITIAL APPROXIMATION*	*FINAL ESTIMATE*
$\overline{GA}$	1128696 (N)	905280 (N)
$\overline{EI}$	7.887×10^7 $(N \cdot m^2)$	7.7313×10^7 $(N \cdot m^2)$

In the previously discussed identification methods, the parameter set consisted of physical properties of the structure, such as the bending rigidity and the mass density. The remaining identification methods utilize parameter sets consisting of matrix scaling factors or matrix elements to fit the experimental results. The first methods to be discussed are those which require an initial approximate model of the structure. White and Maytum [169] identify a set of scaling factors which, when multiplied by prescribed submatrices, improve the accuracy of the global model. The eigenvalue problem for the true model is given by

$$K\Phi = M\Phi\Omega^2 \tag{313}$$

where

M, K : the true mass and stiffness matrices

Ω^2 : the diagonal matrix of true (measured) eigenvalues

Φ: the true modal matrix

The estimates of the true matrices are assumed to be linear variations of the nominal model as follows

$$M = M_0 + \sum_{r=1}^{Q} \alpha_r M_r \tag{314}$$

$$K = K_0 + \sum_{r=1}^{P} \beta_r K_r \tag{315}$$

where

Q : the total number of mass elements

P : the total number of stiffness elements

M_0, K_0 : the initial global mass and stiffness matrices

M_r, K_r : the rth element of substructure mass and stiffness matrices transformed into the global system

α_r : the rth mass element scale factor

β_r : the rth stiffness element scale factor.

The element mass and stiffness matrices can represent a single finite element or a group of finite elements having the same assumed geometry, material properties, boundary conditions, and modeling assumptions. An important point to note is that the use of the scale factors as described in (314) and (315) conserves the consistency of the original finite element model so that no unmodeled coupling occurs as a result of the identification process. White assumes that the true and approximate eigenvectors are related in the following manner

$$\Phi = \Phi_0 \Psi \tag{316}$$

where Φ_0 is the initial modal matrix and Ψ is a general transformation matrix. Using (314) through (316) in (313) and premultiplying by Φ_0^T gives

$$(I + \sum_{r=1}^{Q} \alpha_r \Phi_0^T M_r \Phi_0)\Psi\Omega^2 = (\Omega_0^2 + \sum_{r=1}^{P} \beta_r \Phi_0^T K_r \Phi_0)\Psi. \tag{317}$$

In (317), Ω_0^2 is the diagonal eigenvalue matrix of the original model. To identify the scale factors, White makes an approximation to (313), and therefore to (317), by using the initial modal matrix in place of the true (measured) modal matrix ($\Psi = I$ in (316)). A typical row of (317) now becomes

$$\Omega_i^2 - \Omega_{0_i}^2 = -\Omega_i^2 \sum_{r=1}^{Q} \alpha_r \Phi_{0_i}^T M_r \Phi_{0_i} + \sum_{r=1}^{P} \beta_r \Phi_{0_i}^T K_r \Phi_{0_i}$$
$$= E_{0_i} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{318}$$

To fit N frequencies $(N > P + Q)$, (318) can be used to solve for the scale factors, in a least- squares sense, in the following manner

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = (E_0^T E_0)^{-1} E_0^T (\Omega^2 - \Omega_0^2). \tag{319}$$

The matrix E_0 represents the energy distribution of each mode of the nominal model. The importance of the energy matrix is that it allows for direct examination of the modal coupling present among the various element groups. This method is often very attractive in practice since one can use the energy distribution of each mode as a very useful intuitive guide in defining a "divide and conquer" strategy for solving reduced dimensioned sub-problems (to improve selected parameters prior to a final global solution). The procedure described above may require a repeated iterative process to achieve acceptable frequency correlation. For the undamped case, it is evident, upon comparing (288), (291), and (318), that White's method is very closely related to the method used by Hendricks [158].

EXAMPLE 8: To illustrate the White/Maytum identification method, and contrast it with the equivalent continuum method, the ten-bay structure of Example 7 will be identified. It is assumed that the mass matrix for the structure is correct and the stiffness matrix must be modified to fit measured frequencies. Code (FEM- S/RMP/I,D,B,I)

To simulate an initial inaccurate stiffness matrix, the following properties are chosen for the modulus of elasticity of the truss elements:

Table 16. Modulus of Elasticity of Truss Elements.

STIFFNESS	TRUE VALUE	APPROXIMATE VALUE
$E_c\ (N/m^2)$	71.7×10^9	81.375×10^9
$E_G\ (N/m^2)$	71.7×10^9	61.2×10^9
$E_D\ (N/m^2)$	71.7×10^9	87.45×10^9

To identify the structure, three element groups are chosen as follows:

Table 17. Truss Element Groups.

GROUP	MEMBERS
A	20 horizontal members
B	11 vertical members
C	20 diagonal members

To contrast with Example 7, the same six frequencies will be treated as measurements for identifying the three stiffness scale factors. The fractional energy distributions for the elastic modes, obtained by dividing the ith row of $[E_0]$ by $\Omega_{0_i}^2$, are given below.

Table 18. Energy Distribution by Groups.

MODE NO.	GROUP ENERGY (per cent)		
	A	B	C
4	92.9	0.0	7.1
5	77.4	0.0	22.6
6	60.6	0.0	39.4
8	45.2	0.0	54.8
9	32.3	0.0	67.7
10	21.9	0.0	78.1

It can be observed that Group B contains no energy in the six modes of interest and, therefore is not required for model improvement to best fit these measured frequencies.

The initial frequencies, final frequencies (after three iterations), and measured frequencies are compared below.

Table 19. Estimated Frequencies: Ten Bay Truss.

Ω_i	*INITIAL VALUE (rad/sec)*	*FINAL VALUE (rad/sec))*	*MEASURED VALUE (rad/sec)*
4	*22.11*	*20.75*	*20.51*
5	*56.87*	*53.05*	*52.88*
6	*102.59*	*95.05*	*95.26*
8	*155.08*	*142.80*	*142.58*
9	*211.40*	*193.69*	*193.76*
10	*268.29*	*244.84*	*244.83*

In another approach, which also utilizes an initial discretized model of the structure, Berman and Flannely [170] and Berman and Nagy [171] improve the initial mass and stiffness matrices by adding (hopefully small) modifications determined from a set of measured modes and frequencies. In updating the model, the eigenvalue condition

$$K\Phi = M\Phi\Omega^2 \tag{320}$$

and the orthonormality condition

$$\Phi^T M\Phi = I \tag{321}$$

are enforced as equality constraints. In these equations, M and K represent the unknown $n \times n$ mass and stiffness matrices (initially approximated by M_A and K_A), Φ is an $n \times m$ matrix of measured eigenvectors, and Ω^2 is an $m \times m$ diagonal matrix of measured eigenvalues. If the modal displacements are measured at a subset of the coordinates of the analytical model, the first step of the identification is to obtain an estimate of the modal displacements at the remaining coordinates. This is determined by ordering M_A and K_A so that the upper coordinates correspond to those at which the measurements were made. Equation (320) can now be written, for the ith mode, as

$$\left(\begin{bmatrix} K_1 & K_2 \\ K_2^T & K_3 \end{bmatrix} - \Omega_i^2 \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix} \right) \begin{bmatrix} \Phi_{1_i} \\ \Phi_{2_i} \end{bmatrix} = 0. \tag{322}$$

The remaining modal displacements are approximated, consistent with (322), as

$$\Phi_{2_i} = -(K_3 - \Omega_i^2 M_3)^{-1}(K_2^T - \Omega_i^2 M_2^T)\Phi_{1_i}. \tag{323}$$

The second step is to derive the expression for the mass matrix improvement [172]. By defining the quantity $\mathcal{E}$ as

$$\mathcal{E} = \|M_A^{-1/2}(M - M_A)M_A^{-1/2}\| \tag{324}$$

the updated mass matrix can be obtained from minimization of $\mathcal{E}$ and use of the orthonormal condition of (321). Notice that $\mathcal{E}$ is simply a weighted sum square change in the mass matrix, with the inverse of the mass matrix Cholesky decomposition used as weights. The resulting updated mass matrix becomes

$$M = M_A + M_A \Phi m_A^{-1}(I - m_A)m_A^{-1}\Phi^T M_A \tag{325}$$

where

$$m_A = \Phi^T M_A \Phi$$

The third step is to derive a similar expression for the stiffness matrix improvement [173]. Analogous to (324), the quantity $\mathcal{E}$ can be defined such that

$$\mathcal{E} = \|M^{-1/2}(K - K_A)M^{-1/2}\|. \tag{326}$$

Minimizing this function subject to the constraint equations

$$\Phi^T K \Phi = \Omega^2 \tag{327}$$

$$K = K^T \tag{328}$$

yields the updated stiffness matrix

$$K = K_A - K_A\Phi\Phi^T M - M\Phi\Phi^T K_A + M\Phi\Omega^2\Phi^T M + M\Phi\Phi^T K_A\Phi\Phi^T M. \tag{329}$$

The advantages of this method are (i) the computational procedure is simple and requires no iterative process, and (ii) the improved model predicts the measured mode shapes and frequencies exactly (to within arithmetic errors). The disadvantages are (i) the physical significance of the updated parameters is not conserved, (ii) the final mass and stiffness matrices often do not resemble the correct mass and stiffness matrices and, in fact, physically unrealistic coupling often occurs, and (iii) both mode shape and frequency measurements are required.

EXAMPLE 9: Berman's method is used to identify the mass and stiffness matrices of the following planar structure. Code (LPM/RMP/D,D,B,I)

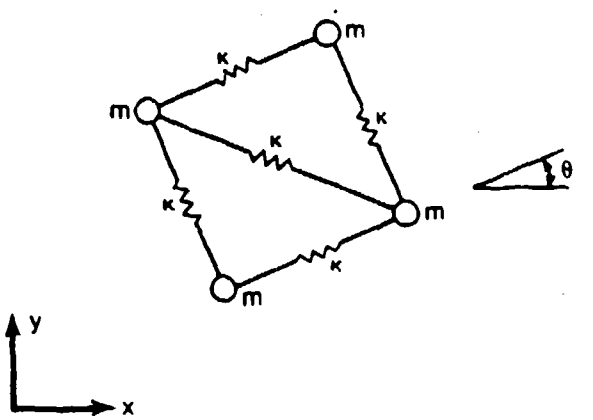


Figure 13. Spring-Mass Model of Truss Cell.

In the structure above, there are two degrees of freedom per mass (x_i, y_i). *The eight total degrees of freedom consist of three rigid body modes* (X, Y, θ) *and five elastic modes. The true mass and stiffness matrices are given below.*

$$M = m\,I$$

$$K = \frac{k}{2}\begin{bmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 \\ -2 & 0 & 3 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 3 & 1 & -1 & 0 & -2 \\ 0 & 0 & -1 & 1 & 3 & -1 & -2 & 0 \\ 0 & -2 & 1 & -1 & -1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 \end{bmatrix}$$

where $m = 100$, *and* $k = 36$.

To simulate an approximate model the mass matrix is increased by 10% and the stiffness matrix is decreased by 10%. The first four elastic modes from the true model are used as measurements for correcting the model. The resulting corrected mass and stiffness matrices and the corresponding frequencies are given below.

$$M = \begin{bmatrix} 104.12 & -0.88 & 2.87 & 2.13 & 2.13 & -2.13 & 0.88 & 0.88 \\ -0.88 & 104.12 & -2.13 & 2.13 & 2.13 & 2.87 & 0.88 & 0.88 \\ 2.87 & -2.13 & 105.88 & -0.88 & -0.88 & 0.88 & 2.13 & 2.13 \\ 2.13 & 2.13 & -0.88 & 105.88 & 0.88 & -0.88 & -2.13 & 2.87 \\ 2.13 & 2.13 & -0.88 & 0.88 & 105.88 & -0.88 & 2.87 & -2.13 \\ -2.13 & 2.87 & 0.88 & -0.88 & -0.88 & 105.88 & 2.13 & 2.13 \\ 0.88 & 0.88 & 2.13 & -2.13 & 2.87 & 2.13 & 104.12 & -0.88 \\ 0.88 & 0.88 & 2.13 & 2.87 & -2.13 & 2.13 & -0.88 & 104.12 \end{bmatrix}$$

$$K = \begin{bmatrix} 35.55 & -0.45 & -34.91 & -1.09 & -1.09 & 1.09 & 0.45 & 0.45 \\ -0.45 & 35.55 & 1.09 & -1.09 & -1.09 & -34.91 & 0.45 & 0.45 \\ -34.91 & 1.09 & 51.38 & -15.38 & -15.38 & 15.38 & -1.09 & -1.09 \\ -1.09 & -1.09 & -15.38 & 51.38 & 15.38 & -15.38 & 1.09 & -34.91 \\ -1.09 & -1.09 & -15.38 & 15.38 & 51.38 & -15.38 & -34.91 & 1.09 \\ 1.09 & -34.91 & 15.38 & -15.38 & -15.38 & 51.38 & -1.09 & -1.09 \\ 0.45 & 0.45 & -1.09 & 1.09 & -34.91 & -1.09 & 35.55 & -0.45 \\ 0.45 & 0.45 & -1.09 & -34.91 & 1.09 & -1.09 & -0.45 & 35.55 \end{bmatrix}$$

Table 20. Estimated Frequencies (I): Spring-Mass System.

Ω_i	*TRUE VALUE (rad/sec)*	*INITIAL VALUE (rad/sec)*	*CORRECTED VALUE (rad/sec)*
4	*0.45922*	*0.41538*	*0.45922*
5	*0.84853*	*0.76752*	*0.84853*
6	*0.84853*	*0.76752*	*0.84853*
7	*0.84853*	*0.76752*	*0.84853*
8	*1.10866*	*1.00282*	*1.00282*

The corrected mass and stiffness matrices, being fully populated, do not resemble the true model even though the first four elastic frequencies agree exactly with the measurements. The last frequency does not agree with the truth because this frequency was not used as a measurement. In fact, the initial value for the frequency was not improved at all. Another point worth mentioning is that a completely different corrected model would be obtained if the initial approximate model was chosen differently even though, again, the frequencies (and mode shapes) from the newly corrected model would exactly match their measured counterparts. In view of these results, it is wise to use great caution when employing this method. Although the corrected model frequencies agree with their measured values, a forced-response analysis of the identified structure would be quite inaccurate.

In an approach similar to Berman's method, Chen et al [174] utilize matrix perturbation theory to improve initial mass and stiffness matrices. In this method the mass and stiffness matrices and the eigenvector and eigenvalue matrices are expressed as follows

$$M = M_A + \epsilon M_B \tag{330}$$

$$K = K_A + \epsilon K_B \tag{331}$$

$$\Phi = \Phi_A + \epsilon \Phi_B \tag{332}$$

$$\Omega = \Omega_A + \epsilon\Omega_B \tag{333}$$

where subscript A refers to the initial values, subscript B refers to perturbed values, and ϵ is a small perturbation. By substituting (330) through (333) into (321) and (327) the perturbed mass and stiffness matrices become

$$\epsilon M_B = M_A\Phi_A(2I - \Phi_A^T M_A\Phi - \Phi_A^T M_A\Phi_A)\Phi_A^T M_A \tag{334}$$

$$\epsilon K_B = M_A\Phi_A(2\Omega_A\Omega - \Phi_A^T K_A\Phi - \Phi^T K_A\Phi_A)\Phi_A^T M_A. \tag{335}$$

Similar to Berman's method, there is no iterative process used (in Chen's implementation) for this method although the improved model does not predict the measured mode shapes and frequencies exactly.

EXAMPLE 10: Using Chen's method, the structure of Example 9 is identified using the same true model, approximate model, and measurements. The corrected mass and stiffness matrices and the corresponding frequencies are given below. Code (LPM/RMP/D,D,B,I)

$$M = \begin{bmatrix} 126.78 & 8.72 & -20.02 & -30.04 & 6.85 & -11.96 & -3.61 & 38.28 \\ 8.72 & 146.61 & -11.96 & -8.60 & -39.42 & -39.85 & 42.66 & 11.84 \\ -20.02 & -11.96 & 128.68 & 36.38 & -5.51 & 10.62 & 6.85 & -35.04 \\ -35.04 & -8.60 & 36.38 & 166.50 & -11.75 & 9.94 & 10.40 & -57.84 \\ 6.85 & -39.42 & -5.51 & -11.75 & 155.42 & 40.76 & -46.76 & 10.40 \\ -11.96 & -39.85 & 10.62 & 9.94 & 40.76 & 148.51 & -39.42 & -8.60 \\ -3.61 & 42.66 & 6.85 & 10.40 & -46.76 & -39.42 & 153.52 & -13.64 \\ 38.28 & 11.84 & -35.04 & -57.84 & 10.40 & -8.60 & -13.64 & 164.60 \end{bmatrix}$$

$$K = \begin{bmatrix} 49.22 & 5.24 & -48.57 & -23.10 & 1.58 & -4.59 & -2.23 & 22.45 \\ 5.24 & 60.90 & -4.59 & -7.53 & -25.68 & -60.25 & 25.03 & 6.88 \\ -48.57 & -4.59 & 65.04 & 6.63 & -18.05 & 21.06 & 1.58 & -23.10 \\ -23.10 & -7.53 & 6.63 & 87.32 & 7.88 & -8.94 & 8.59 & -70.85 \\ 1.58 & -25.68 & -18.05 & 7.88 & 80.80 & 9.21 & -64.33 & 8.59 \\ -4.59 & -60.25 & 21.06 & -8.94 & 9.21 & 76.72 & -25.68 & -7.53 \\ -2.23 & 25.03 & 1.58 & 8.59 & -64.33 & -25.68 & 64.98 & -7.94 \\ 22.45 & 6.88 & -23.10 & -70.85 & 8.59 & -7.53 & -7.94 & 71.50 \end{bmatrix}$$

Table 21. Estimated Frequencies (II): Spring-Mass System.

Ω_i	*TRUE VALUE (rad/sec)*	*INITIAL VALUE (rad/sec)*	*CORRECTED VALUE (rad/sec)*
4	*0.45922*	*0.41538*	*0.46141*
5	*0.84853*	*0.76752*	*0.79915*
6	*0.84853*	*0.76752*	*0.79915*
7	*0.84853*	*0.76752*	*0.85258*
8	*1.10866*	*1.00282*	*1.00282*

The corrected matrices are farther from the truth than the initial approximate model even though the first four elastic frequencies are improved. In a variety of other applications, one finds that neither Berman's method or Chen's method is reliably "better" and it is apparent that valid results using either method typically requires (i) a very good a priori approximation and (ii) use of forced response measurements to confirm and/or further modify the estimated system matrices.

If there is no approximate mathematical model available for the structure, or if the model is deemed unsatisfactory, the elements of the mass, damping, and stiffness matrices (or the state matrix) can be identified if sufficient measured data are available. Rajaram [175], Rajaram and Junkins [176], and Hendricks et al [177] use a set of forced response measurements of displacements, velocities, and accelerations to determine the elements of the mass, damping, and stiffness matrices. For a force vector of dimension less than the order of the system, the equations of motion can be written in the partitioned form

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{12}^T & D_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \end{bmatrix} \tag{336}$$

where the mass, damping, and stiffness matrices are of order $n \times n$ and the configuration vectors x_1 and x_2 are of order $n_1 \times 1$ and $n_2 \times 1$ with $(n_1 + n_2 = n)$, respectively. To obtain the elements of the matrices the first equation of (336) is rewritten as

$$[\ddot{x}_1^T \quad \dot{x}_1^T \quad x_1^T \quad \ddot{x}_2^T \quad \dot{x}_2^T \quad x_2^T] \begin{bmatrix} M_{11} \\ D_{11} \\ K_{11} \\ M_{12}^T \\ D_{12}^T \\ K_{12}^T \end{bmatrix} = [f_1^T]. \tag{337}$$

If (337) is sampled at discrete times $(t_1, t_2, \cdots, t_m \,; m > 3n)$ then an overdetermined set of equations can be obtained. However, these equations cannot be solved because the resulting matrix of measured responses is rank deficient. As can be seen from the second equation of (336), $\ddot{x}_2$ can be written as a linear combination of $\ddot{x}_1$, $\dot{x}_1$, x_1, $\dot{x}_2$, and x_2. To alleviate this problem M_{12} must be known *a priori*. This condition will be satisfied if the mass matrix is confidently known or if the mass matrix is diagonal $(M_{12} = 0)$. Then the equations can be written as

$$AP = U \tag{338}$$

where the jth row of A is

$$[\ddot{x}_1^T(t_j) \quad \dot{x}_1^T(t_j) \quad x_1^T(t_j) \quad \dot{x}_2^T(t_j) \quad x_2^T(t_j)], \tag{339}$$

the jth row of U is

$$[f_1^T(t_j) - \ddot{x}_2^T(t_j) M_{12}^T], \tag{340}$$

and the P matrix is

$$P = \begin{bmatrix} M_{11} \\ D_{11} \\ K_{11} \\ D_{12}^T \\ K_{12}^T \end{bmatrix}. \tag{341}$$

The elements of P can now be determined by the least squares method. The elements of M_{22}, D_{22}, and K_{22} can be obtained by solving the following equations

$$[B] \begin{bmatrix} M_{22} \\ D_{22} \\ K_{22} \end{bmatrix} = -[V] \begin{bmatrix} M_{12} \\ D_{12} \\ K_{12} \end{bmatrix} \tag{342}$$

where the jth row of B is

$$[\ddot{x}_2^T(t_j) \quad \dot{x}_2^T(t_j) \quad x_2^T(t_j)] \tag{343}$$

and the jth row of V is

$$[\ddot{x}_1^T(t_j) \quad \dot{x}_1^T(t_j) \quad x_1^T(t_j)]. \tag{344}$$

Equation (342) can be obtained from the second part of (336). As observed in the above formulation, the order of the system is determined by the number of sensor locations available on the structure.

EXAMPLE 11: Rajaram's method is used to identify the mass and stiffness matrices of the structure in Example 9. Code (LPM/STH/D,D,B,I)

The configuration vector takes the form

$$[x]^T = [x_1 \quad y_1 \quad x_2 \quad y_2 \quad x_3 \quad y_3 \quad x_4 \quad y_4]$$

and the force vector becomes

$$[f]^T = [\, f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \quad f_7 \quad f_8 \,]$$

To excite all of the modes of the structure, an ON-OFF type forcing history, as shown below, is used [177].

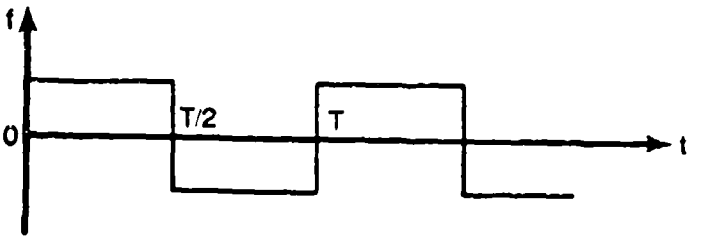


Figure 14. Square-Wave Forcing Function.

The amplitude, f, and the period, T, of each force are given by

$$f_i = 1.0\ N$$

$$T_i = 4\pi/(i+1)\ sec \qquad i = 1, 2, 3, \cdots, 8$$

Using a set of simulated noise-free measurements, taken at one-second intervals for thirty seconds, the following mass and stiffness matrices are identified:

$$M = \begin{bmatrix} 100.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 100.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 100.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 100.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 100.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 100.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 100.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 100.0 \end{bmatrix}$$

$$K = \begin{bmatrix} 36.0 & 0.0 & -36.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 36.0 & 0.0 & 0.0 & 0.0 & -36.0 & 0.0 & 0.0 \\ -36.0 & 0.0 & 54.0 & -18.0 & -18.0 & 18.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -18.0 & 54.0 & 18.0 & -18.0 & 0.0 & -36.0 \\ 0.0 & 0.0 & -18.0 & 18.0 & 54.0 & -18.0 & -36.0 & 0.0 \\ 0.0 & -36.0 & 18.0 & -18.0 & -18.0 & 54.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -36.0 & 0.0 & 36.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -36.0 & 0.0 & 0.0 & 0.0 & 36.0 \end{bmatrix}$$

By comparing these results with the true matrices given in Example 9 it is observed that the mass and stiffness matrices are very accurately identified. It has been found that the numerical conditioning of this method is heavily dependent upon the frequency content and linear independence of the excitation forces. Typically square wave bang-bang excitation is adequate, but broad-band random excitation may be preferred for large complicated structures with a dense natural frequency spectrum. Of course, experience with reductions of synthetic data from a fixed- order discrete system (as in the present example) does not readily extrapolate to applying this approach to fit the response of a high order system by a low- order model.

The method described above does not utilize any knowledge of sparsity which might be present in the matrices (i.e. banded matrices). Also, the symmetry of matrices M_{ii}, D_{ii}, and K_{ii} for $(i = 1,2)$ is not accounted for. The most computationally efficient method would be one which utilizes all *a priori* knowledge of the elements of the matrices to reduce the dimensions of the unknown parameter vector, and to make the estimates consistent with the mathematical model of the system.

An interesting and enlightening theoretical discussion on the identifiability of the elements of the mass and stiffness matrices and the state matrix is presented by Sirlin et al [181]. Necessary and sufficient conditions on the number and placement of sensors

and actuators are determined which insure identifiability of conservative linear mechanical systems.

6.3 Determination of Spatial Models From the Frequency Response Function.

As mentioned in Section 6.2, the use of frequency response measurements for Bayesian estimation of physical parameters of a structure has seen some application (Refs.[32],[183]-[187]). In this section two closely related methods will be examined for identifying the elements of the mass, damping, and stiffness matrices (in the form of submatrix scale factors) using frequency response measurements, eigenvalue/eigenvector measurements, and the Spectral Decomposition Theorem as given by (120). The difference in the two methods lies in the availability of modal measurements and frequency response measurements. In the first method it is assumed that $m < n$ eigenvalues and eigenvectors have been determined (e.g. the methods discussed in Section 5) and one element of the frequency response function has been measured throughout the frequency range of interest. The second method does not require *a priori* eigenvector measurements but does require the measurement (or estimation) of one complete column (or row) of the frequency response function. Before continuing with the details, it is necessary to establish the following notation. The classical (n) second- order equations of motion can be written in the ($2n$) symmetric first-order form

$$A\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} + B\begin{bmatrix}x\\ \dot{x}\end{bmatrix} = \begin{bmatrix}0\\ f\end{bmatrix} \tag{345}$$

where

$$A = \begin{bmatrix}-K & 0\\ 0 & M\end{bmatrix}$$

$$B = \begin{bmatrix}0 & K\\ K & D\end{bmatrix}.$$

The $2n$ eigenvalues and eigenvectors, occurring in complex conjugate pairs, take on the form

$$\lambda_j = \sigma_j + i\omega_j, \qquad \lambda_j^* = \sigma_j - i\omega_j \tag{346}$$

$$\hat{\Phi}_j = \begin{bmatrix} \Phi_j \\ \lambda_j \Phi_j \end{bmatrix} \qquad \hat{\Phi}_j^* = \begin{bmatrix} \Phi_j^* \\ \lambda_j^* \Phi_j^* \end{bmatrix} \quad j = 1, 2, \cdots, n \tag{347}$$

where σ_j is the jth damping factor, ω_j is the jth natural frequency, and Φ_j is the jth displacement mode shape. The $2n \times 2n$ modal matrix is now defined as

$$\Phi = \left[\hat{\Phi}_1 \quad \hat{\Phi}_2 \quad \cdots \quad \hat{\Phi}_n \;\vdots\; \hat{\Phi}_1^* \quad \hat{\Phi}_2^* \quad \cdots \quad \hat{\Phi}_n^* \right]. \tag{348}$$

Investigating the first method more closely, the spectral decomposition of the measured FRF element, $\tilde{H}_{jk}$, can be written (for the general viscous damping case) as

$$\tilde{H}_{jk}(i\omega) = \sum_{r=1}^{m} Y(i\omega) + \sum_{r=m+1}^{n} Y(i\omega) \tag{349}$$

where

$$Y(i\omega) = \frac{\hat{\Phi}_{jr}\hat{\Phi}_{kr}}{i\omega - \tilde{\lambda}_r} + \frac{\hat{\Phi}_{jr}^*\hat{\Phi}_{kr}^*}{i\omega - \tilde{\lambda}_r^*}$$

and

m : the highest frequency mode within the frequency range of interest

$\tilde{\lambda}_r$: the rth measured eigenvalue

$\hat{\Phi}_{jr}$: the jtn component of the rth mode shape.

The matrix equivalent of (349) is given by (120). One problem implicit in (349) is that the mode shape, $\hat{\Phi}_r$, must be normalized with respect to the true (unknown) A matrix (see (345)), whereas the measured mode shapes, defined as $\tilde{\psi}_r$, will not be normalized in

this fashion. The other problem is that eigenvalue/eigenvector measurements will usually be incomplete, measurements being available only for $m < n$ modes. It is therefore necessary to make some approximation to (349). Motivated by Ewins [29],[182], (349) is approximated as follows

$$\tilde{H}_{jk}(i\omega) \approx \frac{a_1}{i\omega} + \sum_{r=1}^{m}\left(\frac{\tilde{\psi}_{jr}\tilde{\psi}_{kr}}{i\omega - \tilde{\lambda}_r}\gamma_r + \frac{\tilde{\psi}^*_{jr}\tilde{\psi}^*_{kr}}{i\omega - \tilde{\lambda}^*_r}\gamma^*_r\right) + a_2 \tag{350}$$

where a_1 and a_2 are complex constants and the γ's are scale factors which normalize the measured mode shapes ($\tilde{\psi}_r\sqrt{\gamma_r} = \hat{\Phi}_r$). (Ewins chooses to combine the terms in the summation and, therefore, obtains a different form of (350)). The first term of (350) represents the contribution from any unmeasured rigid-body modes (obtained by setting $\tilde{\lambda}_r$ to zero in the summation). The last term represents an approximation for the contribution of unmeasured high-frequency modes ($\lambda_r > \omega$). To determine the unknown constants (a_1, a_2, γ's) (350) is "sampled" at different frequencies distributed throughout the frequency range of interest. The resulting set of equations becomes

$$\begin{bmatrix} \tilde{H}_{jk}(i\omega_1) \\ \tilde{H}_{jk}(i\omega_2) \\ \vdots \\ \tilde{H}_{jk}(i\omega_N) \end{bmatrix} = \begin{bmatrix} \frac{1}{i\omega_1} & L_{11} & L_{12} & \cdots & L_{1m} & L^*_{11} & L^*_{12} & \cdots & L^*_{1m} & 1 \\ \frac{1}{i\omega_2} & L_{21} & L_{22} & \cdots & L_{2m} & L^*_{21} & L^*_{22} & \cdots & L^*_{2m} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{i\omega_N} & L_{N1} & L_{N2} & \cdots & L_{Nm} & L^*_{N1} & L^*_{N2} & \cdots & L^*_{Nm} & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ \gamma \\ \gamma^* \\ a_2 \end{bmatrix} \tag{351}$$

where the definition

$$L_{pq} = \frac{\tilde{\psi}_{jq}\tilde{\psi}_{kq}}{i\omega_p - \tilde{\lambda}_q}, \qquad L^*_{pq} = \frac{\tilde{\psi}^*_{jq}\tilde{\psi}^*_{kq}}{i\omega_p - \tilde{\lambda}^*_q}$$

is used. A least squares solution of (351) yields estimates for the unknown constants.

If there are no measurements assumed available for the eigenvectors, then (350) must be modified as follows

$$\tilde{H}_{jk}(i\omega) = \frac{a_1}{i\omega} + \sum_{r=1}^{m}\left(\frac{{}_r\Gamma_{jk}}{i\omega - \tilde{\lambda}_r} + \frac{{}_r\Gamma^*_{jk}}{i\omega - \tilde{\lambda}^*_r}\right) + a_2 \tag{352}$$

where ${}_r\Gamma_{jk}$ are the to-be-determined "modal constants" [182]. A least squares estimate can be obtained for the modal constants as described above. The desired normalized eigenvectors are obtained from the relationship

$${}_r\Gamma_{jk} = \hat{\Phi}_{jr}\hat{\Phi}_{kr}. \tag{353}$$

It now becomes evident that the price which must be paid for not using measured eigenvector data is that at least one complete column (or row) of the frequency response function must be measured in order to obtain the m normalized eigenvectors. The point frequency response measurement, $\tilde{H}_{kk}$, is used to determine the kth component of each eigenvector,

$$\begin{aligned} \tilde{H}_{kk} &\rightarrow [{}_1\Gamma_{kk},\ {}_2\Gamma_{kk},\ \cdots,\ {}_m\Gamma_{kk}] \\ &\rightarrow [\hat{\Phi}^2_{k1},\ \hat{\Phi}^2_{k2},\ \cdots,\ \hat{\Phi}^2_{km}] \end{aligned} \tag{354}.$$

The remaining transfer response measurements, $\tilde{H}_{jk}$, are then utilized along with the results from (354), to determine the remaining components of the eigenvectors,

$$\begin{aligned} \tilde{H}_{jk} &\rightarrow [{}_1\Gamma_{jk},\ {}_2\Gamma_{jk},\ \cdots,\ {}_m\Gamma_{jk}] \\ &\rightarrow [\hat{\Phi}_{j1}\hat{\Phi}_{k1},\ \hat{\Phi}_{j2}\hat{\Phi}_{k2},\ \cdots,\ \hat{\Phi}_{jm}\hat{\Phi}_{km}]. \\ &\rightarrow [\hat{\Phi}_{j1},\ \hat{\Phi}_{j2},\ \cdots,\ \hat{\Phi}_{jm}] \end{aligned} \tag{355}$$

Once the normalized eigenvectors have been obtained, whether from the scaling factors of (350) or the modal constants of (352), the spatial model of the structure (the mass, damping, and stiffness matrices) can be identified. First it is obvious that if the full $2n \times 2n$ modal matrix of (348) has been obtained then the A and B matrices can be identified

(and, therefore, the mass, damping, and stiffness matrices) by using the orthogonality relationships

$$\hat{\Phi}^T A \hat{\Phi} = I \tag{356}$$

$$\hat{\Phi}^T B \hat{\Phi} = -\Lambda = -\mathrm{diag}(\lambda_j) \tag{357}$$

to obtain

$$A = \hat{\Phi}^{-T} \hat{\Phi}^{-1} \tag{358}$$

$$B = -\hat{\Phi}^{-T} \Lambda \hat{\Phi}^{-1}. \tag{359}$$

In the more practical situation of having $m < n$ eigenvalues and normalized eigenvectors the submatrix scale factors can be utilized, as defined in Section 6.2, (314) and (315). In this case, the orthogonality conditions of (356) and (357) can be expanded, using the definitions of A, B, and $\hat{\Phi}_r$, to obtain

$$\begin{bmatrix} \Phi_r^T(-\tilde{\lambda}_r^2 M_0 + K_0)\Phi_r + 1 \\ \Phi_r^T(-\tilde{\lambda}_r D_0 - 2K_0)\Phi_r - 1 \end{bmatrix} = \begin{bmatrix} J_{1r} & \vdots & J_{2r} & \vdots & J_{3r} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \cdots \\ \beta_1 \\ \beta_2 \\ \vdots \\ \cdots \\ \delta_1 \\ \delta_2 \\ \vdots \end{bmatrix} \tag{360}$$

where

$$J_{1r} = \begin{bmatrix} \tilde{\lambda}_r^2 \Phi_r^T M_1 \Phi_r & \tilde{\lambda}_r^2 \Phi_r^T M_2 \Phi_r & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

$$J_{2r} = \begin{bmatrix} -\Phi_r^T K_1 \Phi_r & -\Phi_r^T K_2 \Phi_r & \cdots \\ 2\Phi_r^T K_1 \Phi_r & 2\Phi_r^T K_2 \Phi_r & \cdots \end{bmatrix}$$

$$J_{3r} = \begin{bmatrix} 0 & 0 & \cdots \\ \tilde{\lambda}_r \Phi_r^T D_1 \Phi_r & \tilde{\lambda}_r \Phi_r^T D_2 \Phi_r & \cdots \end{bmatrix}$$

and α_i, β_i, δ_i are the mass, stiffness and damping scale factors, respectively.

Collecting the set of the above equations for each measured eigenvalue provides a system of equations to solve for the scale factors. The advantages of this method are that the parameters are contained linearly in the equations, eliminating the need for a good initial model, and the consistency of the original model is maintained, yielding no unmodeled coupling due to the identification process. Since (350) represents an approximation to the frequency response function an iterative procedure can be used, whereby the unmeasured eigenvalues and eigenvectors are predicted (using the present best estimate of the system parameters) and used in (350) in lieu of measurements. The process converges well if the actual system is represented satisfactorily by the first estimate of the system.

For high-dimensioned and/or poorly conditioned problems, it is anticipated that the replacement of least- squares and matrix inversions by judicious solutions via singular value decomposition will prove constructive.

SECTION 7. SYSTEM CHARACTERIZATION AND NUMERICAL ASPECTS OF IDENTIFICATION.

This section will discuss some of the problems associated with the parameter estimation or system identification task with primary emphasis of identification of on-orbit space structures. The emphasis of this section will be on the continuous-discrete time relationship, the distinction between some continuous and discrete time algorithms, optimum input test signals, optimum actuators and sensor locations, optimum sampling rates and uncertainties in system modelling and system inputs. There will be no attempt to discuss the exact mathematical formulation of the various parameter estimation or identification algorithms. The discussion will be limited to underlying mathematical principles that affect the algorithms in a rather superficial sense.

7.1. Continuous Versus Discrete Time Systems.

The identification of a system generally deals with discrete data rather than continuous data. It is therefore necessary to understand the mathematical basis of discrete systems. There is a limited number of algorithms that will identify a system in the continuous domain as opposed to the discrete domain. One of these algorithms is the quadrature method described in Bellman, Kalaba and Lockett [188] which requires that the data be sampled at specific times which are not equispaced. Since data is usually collected at equispaced times, the general theory of discrete or z- transforms should be utilized in the identification task. Most of the currently used algorithms are of the discrete formulation. Among the widely known algorithms are the Fourier transform spectrum (FFT) [106], the minimal realization (ERA, Kalman-Ho) [189], the maximum likelihood estimator (MLE) [190], the auto-regressive moving average (ARMA) [191], the lattice filter (LF) [192], the Prony method (PA) [193], the instrumental variable method (IV) [194] as well as extensions and variants of these algorithms. Extreme care must be taken to properly formulate the

problem in the discrete domain. The questions of parameter sensitivity in the discrete domain must be addressed with discrete mathematics, z-transforms in the most general case. It is not the intent herein to discuss the various algorithms but to discuss some of the limitations such as computational load, model parameters, computational errors, and other precautions in applying discrete algorithms to obtain continuous model parameters.

One of the major problems encountered with most of the algorithms devoted to parameter estimation is that the time functions between sample time are unknown but the functions are generally assumed to be slowly varying with the discrete data at the sampling time kT as an approximation to the actual functions. This means that frequency contents higher than the sampling frequency are subject to being lost or distorted in the sampling process. This limitation implies that the data is a zero-th order approximation to the actual function with the data assumed to be constant between samples. The data does not take into account first and higher order derivatives of the function. In addition to the above limitation, the A/D sampler used to collect the actual data has a $|sin(x)|/x$ frequency distribution, thus, the actual data will be distorted in the frequency domain and the system parameters for the high frequency modes will not be true parameters.

In the task of structure identification from the continuous domain the spatial discretized model is utilized with the Laplace transform of the displacement equation having the form

$$[\, M\, s^2 + D\, s + K\,]\, X(s) \;=\; B\, F(s) \tag{361}$$

with B the input force matrix and $F(s)$ the Laplace transform of the force acting on the structure. The structure matrices, M, D, and K, are $n \times n$ matrices where n may be as large as 1000. The structure will have at least $2n$ vibrational modes. The bracketed

term on the left side of (361) is a matrix polynomial frequently called a lambda matrix. The transfer function for the displacement can be obtained from the equation

$$X(s) = [M s^2 + D s + K]^{-1} F(s). \tag{362}$$

The measured response is then given by

$$Y(s) = C X(s) \tag{363}$$

where C is the measurement matrix for the structure. The initial conditions on (362) have been set to zero. In addition to the matrix polynomial characterization given above, the system can be given in the state variable form with

$$X(s) = \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} sI & -I \\ M^{-1}K & sI + M^{-1}D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} F(s) \tag{364}$$

where $X(s)$ is a new vector with the Laplace transform of $x(t)$ and the derivative of $x(t)$ as its components. The compact notation

$$X(s) = A(s) F(s) \tag{365}$$

can then be used for (364) without loss of generality. The measurement vector is given by (363) with the new definition of $X(s)$.

The discrete versions of (361), (362), (363) and (364) are obtained by first taking the z-transform of the matrix polynomial of (362) with a zero- order-hold or

$$X(z) = (1 - z^{-1}) Z \left[s^{-1} (Ms^2 + Ds + K)^{-1} \right] F(z) \tag{366}$$

where Z indicates the z-transform of the enclosed quantity. The equation in (366) can be rewritten in compact form as

$$X(z) = N(z)|\, d(z)\,|^{-1}\, F(z) \tag{367}$$

where the left-hand polynomials are the results from the z- transform operation. It then follows that $Y(z)$ is defined as

$$Y(z) = C\, X(z) \tag{368}$$

where $X(z)$ can be written as in (365) but where $X(z)$ is the z-transformed equation of (362). Equations (367) and (368) have M, D, and K imbedded in the equations in some complex form which is not a simple matter to unravel.

The Bellman, Kalaba and Lockett procedure for identification is one of the few methods by which the structure in the s-domain can be identified. The method uses the discrete quadrature summation

$$Y(s) = \sum_{i=1}^{N} w_i\, e^{s-1}\, y(t_i) \tag{369}$$

where $y(t_i)$ is the measured response, w_i is the ith weight function, and t_i is the ith zero of the shifted Legrendre polynomials $P_N(t)$. The algorithm gives the values of $Y(s)$ at finite integer values of s but does not allow for the selection of optimum equispaced sampling times which will probably be necessary for large space structures due to the large dimensions of the problem. The mass, damping, and stiffness matrices must then be obtained from $Y(s)$. Numerical experiments indicate that this algorithm suffers from the failure to use optimal sampling as well as being sensitive to noise and round-off errors due to finite word length. The quadrature method does not give accurate numerical Laplace transforms when the time function is lightly damped. Thus this procedure will not be a valid candidate for structure identification.

There are some researchers who use the Fourier transform to carry out system identification but here again the process is discrete if one uses the fast Fourier transform (FFT) as this task computes the spectrum from discrete data. Although the FFT is quite efficient in analyzing the data to determine frequency contents, it can be shown that the processing of the frequency spectrum data to identify system parameters is computer intensive requiring a considerable number of multiplies and divides. Any efficiencies gained through the FFT processing are lost in the final computation of the system parameters. There is the additional limitation that it is difficult to separate closely spaced modes of the system, particularly when the eigenvalues of the system have small real parts. Such modes of the system are lightly damped and are generally difficult to identify as there is an overlapping of the frequency response curves for the individual modes.

The discrete algorithms that are the most frequently discussed in the literature are the ERA, (Kalman-Ho), the MLE, the ARMA, the lattice, the Prony, the instrumental variable, and the adaptive estimator. Each of these methods utilizes modified forms of the discrete model with the input- output relationship having the form

$$A(z^{-1})\, Y(z) \;=\; B(z^{-1})\, F(z) \tag{370}$$

where $A(z^{-1})$ and $B(z^{-1})$ are the parameters of the system obtained from the z-transform of (364) or directly from (366). One of the difficulties of the identification task is that of unraveling the identified matrix polynomial A and B to obtain the mass, damping and stiffness matrices. Insufficient work has been carried out on discrete z- transforms polynomials of second or higher order to fully understand the make-up of A and B in terms of M, D, and K. Matrix polynomials, or lambda matrices have been studied extensively and the properties of these functions are known but the z-transform properties of matrix polynomials are not well understood.

The ERA model takes the z-transformed discrete state variable form given in (364). This form can be manipulated for the identification task to obtain eigenvalues of the discrete state matrix which are identical to the discrete latent roots of the discrete lambda matrix associated with (366). The system eigenvalues close to unity are difficult to compute accurately since the eigenvalues of the discrete state matrix are of the form $\exp(p_i T)$ where T is the sampling time and p_i is an eigenvalue of the system. If the product $p_i T$ is close to zero, small errors can lead to large errors in the computed values of p_i. The sensitivity of a structure eigenvalue is quite high for discrete eigenvalues close to unity. The computations of the mass, damping and the stiffness matrices can be carried out if the eigenvalues (latent roots) and the eigenvectors (latent vectors) of the state matrix are known, although there are still some difficulties on the interpretation of the structure of the state matrix. An associated problem is that of dimensionality. The "curse of dimensionality" in this case differs from that of dynamic programming where it is associated with memory. In the identification problem it is that of numerical accuracy when a large number of modes are present in the system response. Insufficient data exists to fully analyze the numerical errors encountered when a large structure and a large number of modes are to be identified. Methods to circumvent the dimensionality problem when the ERA method is utilized will most likely be found but the method must mature before the computational problems are solved. Clever ways of combatting numerical errors for large structures must be devised prior to applying most identification algorithms to vibrational problems.

The maximum likelihood estimator (MLE) would appear to be a suitable candidate for the estimation task since the method does not depend on having a full characterization of the noise. The algorithm provides a means of computing the covariance of the noise but the difficulty with the method is the number of computations required. The algorithm is iterative with an update provided on the covariance at each iterative stage. The added covariance computations, along with the identification computational load, places a rather

large computational burden on the digital machine when this algorithm is implemented. As in all discrete- time formulations, the continuous parameters must be computed from the discrete- time data. If the system noise is Gaussian with mean zero value, the MLE and the LS algorithms will be identical.

The ARMA model is quite popular in adaptive estimation as well as for adaptive control. Since it is not necessary to compute exact model parameters but only to provide control signals, the adaptive algorithm has found favor with researchers involved with devising control algorithms. One advantage of the adaptive type algorithms is the ability to carry out the parameter estimation on-line in real time provided that a fast digital machine is available. Data available in the literature suggest that the adaptive algorithms, whether for estimation or control, are limited to systems of small order. Literature on the use of these algorithms for systems of order ten or higher is rare. This type of algorithm will probably be limited to those problems where the number of unknown parameters is small and exact system parameters are not necessary but only approximate selected parameter values sufficient to provide control are needed. Even though the shortcomings of the adaptive type of algorithm may be, the identification of the system parameters still involves transforming the discrete parameters to continuous parameters before the mass, damping and stiffness matrices are identified.

Most of the other algorithms mentioned above suffer from the limitations stated in the previous paragraphs. Exact knowledge for transforming the discrete time parameters obtained from the discrete data to the mass, damping, and stiffness matrices is lacking for large structures. In addition to this problem or limitation, the numerical errors from a finite word length and the dimensionality of the structure may be rather serious. Many of these problems can be circumvented by providing distributed computations either onboard the space structure or in the computational center that processes the data coming from the structure. Numerical algorithms for accurately substructuring the computations do

not exist at the present time other than in a information flow sense. There is also the outstanding question of how well the structure must be identified for control of vibrations. The exact task that the space structure is performing as well as the materials utilized in the structure will no doubt be a major factor in the parameter estimation task.

7.2. Optimal Input and Sampling.

In order to limit the amount of data that must be processed in the identification problem as well as providing some checks on the numerical errors, the identification task should be carried out with an optimal input signal and optimal sampling rates. The input signal should excite all of the vibrational modes of the structure and all of the natural modes of the structure should be observable if the structure is to be identified. This means that the system must be controllable as well as observable. Controllability assures that all of the modes will be excited whereas observability means that the output data contains information on all of the vibrational modes of the structure. For the state variable formulation of a linear time-invariant system the controllability matrix is

$$Q_c = [\, B \; AB \; A^2B \cdots A^{n-1}B \,] \tag{371}$$

whereas the observability matrix is

$$Q_o = [\, C^T \; A^TC^T \; (A^T)^2C^T \cdots (A^T)^{n1}C^T \,] \tag{372}$$

where the system triplet is $[A, B, C]$. The triplet in not known in the identification task as it is the matrix triplet or some other system parameters that are sought.

The most general input signal to assure that all vibrational modes have been excited is that of pseudo-white noise since pure white noise is only a mathematical abstraction. White noise requires infinite power and is impractical. Pseudo-white noise will excite all

of the modes of a vibrating structure provided that the sampling rate is sufficiently high to place the folding frequency f, where f equals 3.14159/sampling time, above the highest mode of the structure. A high sampling rate is necessary if all of the modes are to be identified; thus, the folding frequency restriction does not impose additional restrictions on the sampling rate. The simplest pseudo white noise source to implement is that of the maximum length null sequence (MLNS). This noise source is generated through the use of a M-bit hardware shift register randomly loaded with zeros and ones. The contents of the register are shifted to the right with selected bits of the register used in an exclusive OR operation, denoted by $\oplus$, to compute a new bit for location 1 of the register. The feedback bit locations for shift register lengths of M=18 through M=24 are given in the table below.

Table 22. Feedback Bits for Shift Register.

M	Feedback Bits
18	$7 \oplus 18$
19	$1 \oplus 2 \oplus 5 \oplus 19$
20	$3 \oplus 20$
21	$2 \oplus 21$
22	$1 \oplus 22$
23	$5 \oplus 23$
24	$1 \oplus 2 \oplus 7 \oplus 24$

It has been assumed that the register bits are numbered from left to right with the lowest bit, 1, to the left and bit M to the right. If the shift register is M bits long then the periodicity of the pseudo-white noise generated will be $2^M - 1$ long. That is , the bit pattern of the shift register will repeat itself every $2^M - 1$ shift operations. Since the hardware implemented shift register contains zeros and ones, the software implementation would require replacing the base 2 numbers (0, 1) with -1 and +1. Floating point values are generated by passing the binary bits through an optimally designed all-pass digital filter. The output of the all-pass filter will be pseudo-white Gaussian noise with mean

zero and variance set by the all-pass filter coefficients. The spectrum of the output of the pseudo-white noise generator is given by

$$\Phi_N(\omega) = \frac{\delta(\omega)}{N} + \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{N+1}{N}\left(\frac{sin(\frac{m\pi}{N})}{\frac{m\pi}{N}}\right)^2 \delta(\omega - 2\frac{m\pi}{NT}) \tag{373}$$

where $\delta(\omega - 2\pi/NT)$ is the unit impulse at $\omega = 2m\pi$.

One all-pass digital filter that is suboptimal is given by

$$\begin{aligned} W(k) = a_0\, N(k) + a_1\, N(k-1) + a_2\, N(k-2) + a_3\, N(k-3) \\ + b_1\, W(k-1) + b_2\, W(k-2) + b_3\, W(k-3) \end{aligned} \tag{374}$$

where the numerical values of the coefficients are $a_0 = 0.5828$, $a_1 = 0.8948$, $a_2 = 1.1900$, and $a_3 = 1.0000$ with $b_1 = -a_3$, $b_2 = -a_2$, and $b_3 = -a_1$. $W(k)$ is the generated pseudo-white noise in floating point numbers, $N(k)$ is the contents of one cell of the shift register. The four values $N(k)$, $N(k-1)$, $N(k-2)$ and $N(k-3)$ should be taken from any four fixed adjacent cells of the shift register, i. e. $N(1)$, $N(2)$, $N(3)$, and $N(4)$. The all-pass filter may not be necessary if the binary values are passed directly to the force transducers. The MLNS noise source is described in Graupe [114] as well as in several other references. The weighting coefficients for (373) were found by Rowe and Kerr [195].

The optimal sampling rate is quite important if a minimum amount of data is to be manipulated in the identification process. The selection of the sampling rate is also important in assuring that all modes of the structure are observed. There are two methods of selecting the sampling rate. The first of these is to compute the Fisher information matrix from several choices of the sampling frequency. This method is a not a viable computational means of establishing the optimum sampling and the method must generally be replaced by a simpler method. Let Q_o be the observation matrix and Q_c be the controllability matrix. Let H denote the Hankel matrix with $H = [Q_o^T\ Q_c]$ from which it follows that the optimal sampling rate can be computed by selecting a sampling time

T such as to maximize the determinant of H. Since the observability and controllability matrices cannot be determined reliably *a priori*, the Markov matrices of the output data must be used in place of the Hankel matrix. This matrix is the same one used in the ERA or Ho method. The procedure to determine a near optimum sampling rate is to collect data for several sampling times, form the Hankel matrix for each sampling time and then determine the best sampling time by using a least squares method to determine the sampling time dependency from the maximum determinant from the series of Markov matrices. Although it will not be possible to determine the exact optimal sampling time by this procedure, the determined sampling time should be suboptimal.

7.3. Parameter Sensitivity to Word Length and Bias.

One of the key unanswered questions in parameter estimation is that of parameter sensitivity to changes in the input-output data and numerical accuracies. There has been very little work on this topic and yet the resolution of the question on which algorithm is the most effective in a numerical sense depends on the sensitivity. It would be foolish in system design work to select the design which is most sensitive to component changes or digital word length. To illustrate the sensitivity issue, sensitivity of digital filters to computer word length has received attention and it is widely known that the parallel implementation of a digital filter is less subject to round off errors due to the digital word length than is the cascade design. There is also some evidence to show that a lattice filter is even more insensitive to word length than is the parallel implemented filter. Research on parameter sensitivity has been limited and since this report has to be written from the "state-of-practice" viewpoint, there is insufficient time to address this aspect of identification. The sensitivity question will be placed in the category of future research problems for the identification of large space structures.

It is well known that certain estimation algorithms are biased and the resulting parameters will be dependent on the amount of the bias. It therefore follows that the unbiased estimator would be preferred when all other disadvantages are equal. The least- squares algorithms tend to be biased estimators whereas the instrumental variable algorithm is unbiased. The literature on identification addresses this issue although the bias- sensitivity combination has not been examined. Attention should be given to this aspect of the identification problem.

7.4. Optimal Actuator and Sensor Locations.

The placement of the force transducers on the structure also plays a role in the excitation of the structure modes. The optimal placement of the force transducers would require a minimum number of inputs to the structure as well as a minimum number of transducers on the structure. Optimal placement of the motion sensors would minimize the number of measured values to handle in the parameter estimation algorithm. There is a problem with the placement of a minimum number of transducers and sensors such that each transducer and sensor is optimally placed since this implies that the structure is known *a priori*. Determination of the optimum location of the force actuators and the motion sensors is not a simple task. The problem of locating the motion sensors has been addressed by Udwaida, [196],[28]. These papers utilize the covariance matrix of the parameter estimates to determine the sensor locations.

In order to excite all vibrational modes of the structure the controllability matrix should be of full rank. The problem with this is that the number of modes or the rank of the controllability matrix will not necessarily be available. *A priori* information for the rank of the controllability matrix as well as the parameters of the controllability matrix are not known. Similarly, the observability matrix must be of full rank but here again, sufficient information does not exist on the rank of this matrix. The only information

available *a priori* is the input- output data from which the Markov matrix can be constructed. The rank of the Markov matrix is generally a measure of the number of modes in the system. The rank of the Hankel or Markov matrix can be determined by the singular value algorithm (SVD). The question then arises as to what singular value of the Hankel or Markov matrices implies usable data and what value indicates measurement or system noise. It is probably not possible to determine the exact rank of the system or to determine the optimal sampling time or the force-sensor locations for a structure. The best that can be done is to determine a suitable value for each and then to use these values in the identification algorithm. This means that optimal location of the actuators and sensors from state-of-practice algorithms cannot be determined, all that can be achieved is to determine an approximate location for the components. A structure actuator-sensor location algorithm needs to be devised that will place the actuators and sensors in a near optimal sense.

7.5. Frequency Response of Force Actuators, Sensors and Samplers.

The input-output data which will be utilized in the model identification will be subject to the transfer functions of the forcing and measurement devices on board the structure. None of the devices are ideal with respect to frequency response and therefore impose "distortion" factors in the identification task. The frequency distortion effects of the force actuators, the motion sensors and the A/D samplers must be corrected in the various parameter estimation algorithms.

The transfer functions of the actuators and the sensors will depend upon the design of the particular device. The transfer functions of these devices can be measured in the laboratory by sinusoidal testing and the frequency responses obtained. There is no reason to believe that the frequency response of the devices will change on-orbit other than perhaps a scaling of the frequency response magnitudes.

The frequency response of the sampler, A/D converters, is known, as it is the response of the zero-order-hold (ZOH) circuit. The transfer function for the ZOH is given by

$$T_{ZOH}(s) = \frac{\left[1 - e^{-sT}\right]}{s} \tag{375}$$

where T is the sampling time and s is the Laplace transform variable. Substituting $s = j\omega$ in (374), the magnitude response is found to be

$$|T_{ZOH}(\omega)| = \frac{T\ sin(\omega T/2)}{(\omega T/2)} \tag{376}$$

whereas the phase response is

$$\Phi_{ZOH}(\omega) = \frac{-\omega T}{2}. \tag{377}$$

The magnitude response is periodic with period of π/T and has a value of T at $\omega = 0$ and 0 at $\omega = 2\pi/T$. The folding frequency, that frequency about which the magnitude response squared is periodic and symmetric, is $\omega_T = \pi/T$.

The frequency and phase response distortion due to the sampling can be removed by applying a numerical inverse filter whose frequency and phase response is the inverse of the A/D sampler. Extreme care must be taken in the implementation of the inverse filter as it may be numerically unstable. Since the sampling frequency should be twice as high as the frequency contents of the measured data, the inverse filter can be cutoff at approximately three-fourths of the sampling frequency.

SECTION 8. COMPUTATIONAL AND DATA ACQUISITION ISSUES.

The parameter estimation problem to be implemented depends on the selection of good software codes which are efficient and accurate. The selection of the code to be utilized will partially be determined by the existing hardware available for running the code. In addition, the collection of data for the parameter estimation task will be an integral part of the requirements on the software code. The hardware may be located on the ground, partially on the space structure, or totally on the structure. There will be different requirements for each of these hardware configurations. If the hardware is on the ground, the force and displacement data must be collected on the structure and then sent to the ground hardware site. In contrast to off-structure hardware is the case where all of the hardware is on the structure with all processing on the structure. These issues, as well as some of the computational requirements, will be discussed in this section. Since the hardware, computers as well as actuators and sensors, is constantly changing, the issues addressed here are in line with presently available hardware and predictions that seem to hold for the future. The views expressed here may change significantly with hardware developments over the next year or two. It should also be recognized that available funding for hardware development can change in a drastic sense over a few years, thus enforcing or voiding some of the comments in this section.

8.1. On-Orbit Versus Ground-Based Computations.

This section will address those issues of the identification task by considering some of the hardware requirements for identification and the division of the hardware-software directed aspects for realizing the goal of structure identification and control. Since the structure is on-orbit, there is not a clear division of what aspects of the task will be carried out on the structure and those that must be completed at the ground base control point. These two subtasks of the identification phase will be denoted as either on-structure or

off-structure. Although it may not be possible to place a large computational facility on the structure for every mission, there will be missions for an on-orbit structure which may require that the mission requirements be carried out completely independent of ground control. It is obvious that certain aspects of the identification process must be on-structure whereas some other procedures may be on- structure or off-structure, depending on the hardware requirements. For example, the force actuators and displacement sensors must be on-structure. Some data transmission facilities must be available on-structure to preprocess the acquired data for transmission to the off-structure facilities.

The generated data, force actuator signals, and the collected data from the motion sensors, may be processed in real time or collected and then processed as batch data. The computations in real time will be considered as on-line processing whereas the batch type of processing will be classified in this report as off-line. Some of the issues for the on-line vs. off-line computations will be considered in the following material.

8.1.1. On-Orbit Data Processing.

The computational task associated with on-orbit identification of structures can be broken down into three hardware requirements: the on-structure excitation, measurements and preprocessing, bidirectional data communication from on-structure to off-structure , and data processing in the identification algorithm. The on-structure hardware consist of force sensors and motion sensors that have analog-digital conversion capabilities if the data link is digital, as well as some computational facilities for preprocessing the data to minimize the data communication requirements. In addition, it may be desirable to generate on-structure the signal for the force actuators which excites the modes of the structure. The data communication link will be necessary if there are insufficient computational facilities on-structure. The data link would send the on-structure information to the off-structure processors for the identification algorithm. The off-structure computers

would then process the down-link data to arrive at an acceptable model. It is assumed that the control strategy and the associated algorithms are not to be included in the computations but this assumption may not be valid in a totally integrated on-structure computer facility.

Although the data handling capabilities could be categorized as on-line vs. off-line, it would seem more appropriate to use the on-structure and off- structure classification. Rather than use the above structure descriptors, the terms will be used interchangeably with on-orbit and off-orbit facilities where on-orbit means on-structure and off-orbit means off- structure. The computational task may be off-line, meaning not in real time, or on-line, which would be real time with the on-structure and off- structure computations belonging to both types of computations. The on-line and off-line computational task management will be discussed as a separate section. The reason for the structure division of tasks is in keeping with distributed types of computers which are now possible with fast microprocessors. It is now possiole to think of parallel computations in terms of distributed special purpose computers at less cost than large general purpose computers. The on-orbit structures could then have their own integrated identification-control algorithms on-board with sufficient capabilities for control and identification of the structure for multipurpose use. Present state-of-practice does not provide the ability to implement the identification and control strategy described above but there do not appear to be limitations to this state-of-art approach.

8.1.2. On-Structure Data Acquisition and Computer Requirements.

The on-orbit structure will require some computationai facilities even though the task to be performed may be rather simple. For example, assume that all force commands are transmitted to the structure hardware through the data link with the commands in digital form. The incoming data must be translated for use by the force actuators to properly

excite the structure. The resulting motion from the forces is then measured by the motion sensors and the measured data is then sent back to the off-orbit facility through the data link. This hardware configuration: force actuators, displacement sensors, analog to digital converters, and data link interface, is the minimum set required for parameter estimation. Timing information for coordinating the sampling can either be provided from the on-orbit or off- orbit hardware but it should be recognized that a time delay will be placed in the data loop if the timing pulses are generated off-orbit. It would therefore seem appropriate to generate the timing on-orbit with a submaster clock synchronized to the off-orbit master clock.

The next level of hardware implementation would require the same hardware as described but with several other simple tasks. The force excitation signal could be generated on-structure and then sent to the force actuators. In addition the sensor outputs could be collected at a central point for some preprocessing prior to placing the data on the communication link. It may be necessary to prefilter the sensor outputs prior to sending the data to the off-orbit facility. Suitable filtering algorithms can be implemented in microprocessors on the structure.

There is no reason to restrict the preprocessing computations to prefiltering when some of the computations assigned to the parameter estimation algorithm can be carried out on-structure with microprocessors. The concept of decentralized control has been studied for several years and applied to large systems such as power control in electrical power networks. There does not seem to be any major mathematical limitations which would prohibit the extension of the decentralized concept to parameter estimation. The decentralization concept is to perform some of the computations at localized computer nodes in the network with minimum, but sufficient, information for system identification flowing between the computer nodes. In the case of large structures on-orbit, local microprocessors could be implemented to collect data from a mathematically partitioned structure and

processed for a subtask of the overall identification algorithm. The locally processed information would be relayed to an on-structure or off- structure centralized computer facility for final processing. The force actuators and motion sensors network would communicate only with the subtask processors with the decentralized information flowing only between the subtask processors and the final processor. Although the decentralized concept does not fit into state-of-practice algorithms, it is strongly recommended that this concept be studied, as it may be the only way to identify large space structures. The decentralized concept is closely related to substructure modeling, partitioning of systems, or in a mathematical sense to parallel processing in state-of-art array processors. The identification computer network could also be utilized for the control of the structure once the identification task has been completed and would be well suited for adaptive identification and control of multipurpose space structures.

Much of the hardware for the on-structure facility exists although there has not been a major effort to catalog hardware components for on-structure applications. The only cataloging effort known to the ASCE task force is the report by Charles Draper Laboratories supported by NASA Johnson Space Center with the results published in a report [197]. The data published in that report are now out of date due to recent developments. Force actuators using magnetic forces such as the ones developed for NASA Langley Research Center by the University of Virginia [198] are probably state-of-art, although there have been no published results on application to space structures. Motion sensors are available for the on-structure sensing of the motion of the structure. The force balanced integrated accelerometer developed by the Sandia Laboratories [199] would appear to be the best candidate for the vibrational sensor. This accelerometer has a five-six decade linear response range and can be constructed to have a very low threshold, on the order of 0.001g force. The accelerometer is a solid state device in an integrated chip package which occupies a very small space and is lightweight. The accelerometer has three degrees of

freedom output, thus one chip will measure acceleration along in the three reference axes. Since the accelerometer is constructed on a solid state chip, there is no reason to believe that analog to digital conversions could not be included in the design if it is not included now.

Commercially available microprocessors with clock rates of 20 megahertz are now available in sample lots. The clock rate of 20 megahertz will most likely be increased to 30 megahertz within the next two years [200]. Higher speed chips are also being developed under the DOD program for very high speed integrated circuits (VHSIC) [201]. Special purpose microprocessors with clock rates of perhaps 100 megahertz will probably be available in the near future under this program. These clock rates, along with high density memory chips, would seem to indicate that on-structure 32-48 bits computers which are lightweight and compact with throughput of 5-20 million floating point operations per second (Mflops) and 20 megabytes of associated memory should be available within the next 1-3 years. Hypercube computers with performance specifications of as high as 500 megaflops are presently in the design stage. Computers with these performance specifications configured in a distributed network should have sufficient computer power to perform most of the identification task on-structure with a small demand on the communication link and the off-structure computer facility. The major drawback to these exotic machines is the lack of software which will probably require several years to develop. Certification of the high-speed microprocessors and associated chips for military space applications are currently underway [201].

8.1.3. Data Link Hardware.

The required communication link for transferring the data from on-structure to off-structure can be implemented with existing hardware. Communication hardware with

transfer rates in excess of 10 megabit/second is presently available as well as special equipment with higher transfer rates. This aspect of the identification problem should be implementable with existing hardware and is of minor significance in the on-orbit parameter estimation task. The requirement for data transfer should ease as more of the identification hardware and software is placed on-structure. It may be necessary to transfer a small amount of data to the off-structure facility for monitoring purposes but this should be possible with narrow bandwidth channels.

8.1.4. Off-Structure Data Acquisition and Computer Requirements.

It is difficult to make a division between the on-structure and off- structure data acquisition and computer facilities until the identification and control algorithms are selected. Since the actuators and sensors are placed on the structure, this part of the data acquisition need not be considered off-structure. The off-structure facility should have a communication interface to the data link and these data should be transferred to the off-structure computer. There is no limitation placed on the off-structure computer facility, particularly if the off-structure site is the ground. The selection of the ground for the off-stucture will be assumed in which case the computer can be sized according to the computational load. Machines in the class of the CRAY-2, with throughput of 1 billion flops, should have sufficient throughput to handle the most demanding identification task. The off-structure computer can be sized downward to meet the demands of the overall identification and control task as the computer load is shifted from off to on structure as the algorithms mature.

8.2. On-Line Versus Off-Line Computation.

For the sake of clarity, the on-line computations will be considered as those calculations done in real time. Similarly, off-line calculations will be those that do not have to be carried

out in real time. Identification or estimation algorithms can be divided into two classes, those of real time and those not requiring real time with perhaps a mix in some cases. The adaptive estimation algorithm would require that all of the computations be in real time, whereas the Ho or ERA algorithm can be done in either mode. If the on-orbit structure is in a fixed orientation and orbit and is stable with respect to vibrations, there is no reason to do the identification in real time. As an example of the mixed mode of calculations, consider the case in which off-orbit substructure data is available to be utilized in the identification algorithm. The substructure data would be collected off- orbit with some secondary computations required to store the information in the most usable form. These data would then be used in real time to supplement the on-orbit collected data.

It is difficult to address the on-line versus off-line computational requirements. The only possible analysis that can be done is to consider all of the various mathematical operations that are utilized in one operation of an algorithm, such as an FFT computation, a matrix inverse, etc. Rather than consider each algorithm in its totality, perhaps it is more logical to look at the some of the mathematical algorithms that are utilized in a parameter estimation algorithm. In general, the number of computational operations that require the major amount of time in computations are the multiply and divide operations. These two operations require approximately equal time, thus there will be no distinction between the two in counting mathematical operations. Add-subtract operations require very little time and should not be included in the operations count. The table below tabulates some of the mathematical operations used in some of the identification algorithms. The number of operations is approximate with each given only as the order of counts.

The mathematical operations in Table 23 are as follows: A^{-1} is a matrix inverse, SVD is the singular value decomposition, λ, L is the eigenproblem (right or left) , $A^{\dagger}$ is the generalized inverse, AB is a matrix multiply, Ax is a matrix-vector multiply, and $Ax = b$ is a linear equation solver. If it is assumed that N=4096 and n=200, the number of operations

Table 23. Multiply-Divide Counts for Mathematical Operations.

Data Points	Algorithm	Operations
N	FFT	N log(N)
$n \times n$	A^{-1}	n^3
$n \times n$	SVD	n^3
$n \times n$	λ, L-General	n^3
$n \times n$	λ, L-Symmetric	n^3
$m \times n$, $m > n$	$A^{\dagger}$	mn^2
$(n \times n)(n \times n)$	AB	n^3
$(n \times n)(n \times 1)$	Ax	n^2
$(n \times n)(n \times 1)$	Ax=b	n^3

is approximately 24000 for the FFT and approximately 8×10^6 for SVD. It is clear that efficient algorithms of the N log(N) class need to be developed to reduce computation time for processing large amounts of numerical data for identification algorithms.

Although it may be without mathematical rigor, some estimate of multiply- divide operations should be made to determine an upper limit to the computational load. Suppose that the structure has 100 vibrational modes that must be considered in the identification task with 30 force actuators and 30 displacement sensors. It will also be assumed that the spectral density of the structure is such that no vibrational mode in excess of 50 hertz needs to be considered. Using this upper frequency, a sampling frequency of 100 hertz is suggested by the Nyquist sampling theorem. This means that the sensors must be sampled 100 times a second and with the 30 sensors, a total of 3000 data words must be collected each second. The sampling rate does not place an undue restriction on the samplers as analog- digital (A/D) convertors are available with specifications far in excess of these requirements. Since there are 100 modes to be considered, the Markov matrix in the ERA algorithm should have a minimum of approximately 300 rows and 200 columns if the least- squares algorithm or the SVD algorithm is to be used. Approximately 60000 data points will then be needed to populate the Markov matrix. The resulting matrix is

300 by 200 and using $(row)^2$ by (col) as an estimate of the number of multiply-divide operations for the least squares or SVD algorithm, the throughput for these operations requires 18 Mflops. The least- squares or SVD algorithm is not the only operation required for the parameter estimation, others are necessary. Using the 30 inputs and outputs, the number of operations will probably be multiplied by at least 5, thus approximately 90 Mflops will be required for determining the solution. Even if this number is reduced by one-half, present sequential digital machines, except for those with array processors or those in the supercomputer class, cannot achieve this throughput. The above numbers are in the "worst case" range.

If the above analysis is correct then the computations cannot be carried out in real-time if a few seconds of data is to be processed all at once for the model identification with standard designed sequential computers. The only hope for real time identification is to use recursive algorithms where incoming data is processed on-line with an iterative algorithms which computes a new update after each new set of data points. Even this computational approach would probably require an array processor. An on- structure distributed computer facility based on "state-of-art" array processors would require a concerted design effort but should be able to process the data in real time. Array processors with throughput of 10-40 Mflops are available, thus 5-20 array processors would suffice for on-line operation. Parameter estimation algorithms would have to be modified to run on distributed array processors as little work has been done to implement the algorithms for such a computer environment. The above number of array processors will probably be reduced by a factor of 2 if an extrapolation is used to best guess what will be available in 3-5 years.

The number of data points collected each second to be utilized in the parameter estimation algorithm could easily be transmitted to the off- structure computer facility for off-line computations. The sampling frequency of f=100 used in the above example

requires that only 3000 (32 bits) words be transmitted to the off-structure facility. This value is well within a data link bandwidth of 1 megabits/second transmission rate. The data link with a 1 megabit bandwidth does not impose a severe limitation in implementing any of the parameter estimation algorithms in an off-line procedure. There does not seem to be a limitation on any aspect of an off- line (or off-structure) implementation if a super computer is available for data processing. However, the off-line implementation with a supercomputer is not recommended as this places a rather severe restriction to later modification of the algorithm to on-structure computations. It is recommended that an off-structure distributed computer facility be established so that at a later date the machines can be placed on the structure. This would permit the on-orbit structures to eventually have adaptive control strategies for multi-missiosn without dependence on off-orbit computer facilities. Although the initial hardware cost may be higher, the total cost will be lower for the family of future structures to be placed on-orbit.

The parameter estimation algorithm selected for the task of identification and control of large space structures will not be finalized for several years. An intermediate period during which tests of various algorithms will be conducted is the most probable scenario and provisions for a broad coverage of computer needs should be established. It is also recommended that a research and development program be established for a structure identification facility at the same site of the distributed computer to develop computer hardware for a broad range of structures. The program goal would be to produce hardware and software packages for the purpose of adaptive identification and control of orbiting structures and certification of the hardware for space applications. Hopefully, such a program would produce hardware and software applicable to all structures on-orbit regardless of their mission.

SECTION 9. CONCLUSIONS AND RECOMMENDATIONS.

The ASCE Committee on Methods for Identification of Large Space Structures has reviewed and evaluated the "state-of-the-practice" in system identification for application to large space structures. A distinction is made between state-of-the-practice and state-of-the-art because much of the published literature on structural system identification relates to theoretical methods and numerical demonstration of the methods using analytically simulated "test data"; the theory has not been applied to real problems involving real hardware and real test data, and yet is considered by many to be state-of-the-art.

The report emphasizes identification as an integrated process involving the analytical abstraction of a physical system for purposes of achieving specific modeling objectives. This process involves conventional analytical modeling whenever possible, and the experimental verification of those models using parameter estimation. Nonparametric estimation for input- output mapping may be employed in situations where analytical models are difficult to formulate. Thus the identification process is presented as a broad range of activities possibly affecting the design of hardware itself.

One of the primary modeling objectives for large space structures is to produce reliable structural models for use with control systems which will maneuver, damp, shape and point these structures, or portions of them in accordance with mission objectives. Since the models will be "flown" with the spacecraft, they will have to be flight certified to confirm their predictive accuracy and reliability. The ability to certify these models will depend strongly on the identification process, and the procedures used for model verification. To the best of the committee's knowledge, no effort of this magnitude has ever been undertaken. It is anticipated that new techniques for modeling, testing and estimation, as well as new actuators and sensors for identification and control, new computer hardware

and software, for ground testing as well as on-orbit testing, will ultimately be required to meet the challenge of identifying large space structures.

The subsections which follow draw attention to a number of issues that need to be resolved in developing a capability for identification of large space structures. Some of the issues have already been discussed. Other issues or parts of issues have been identified in the process of writing the report but have not been substantively addressed. Finally, an attempt has been made to incorporate issues raised by the reviewers of this report.

9.1. Current Issues and Needs Relative to Large Space Structures.

During the past decade, the U.S. has developed a space transportation system capable of sending people and materials into low earth orbit and bringing them back safely. The Space Shuttle can support its crew in space for only a few weeks – too short a period to really exploit many of the practical benefits of space operation. Consequently, the U.S. has decided to take the next logical step, to build a space station capable not only of supporting humans for extended periods of time, but also enabling them to work productively in this new frontier.

From its user studies, NASA developed a set of requirements for the proposed space station, [202]. It should be capable of servicing satellites, tending free-flying platforms, and serving as a base for the construction of other large space structures. It should provide facilities for research and development. The basic structure should be expandable to accommodate additional laboratories and living quarters, solar arrays (up to 300 kilowatts of power), and radiators to dispose of the waste heat.

Building the space station in a low gravity field presents NASA with major challenges. Major space station components will be constructed on earth, but final assembly will occur in space. Several shuttle flights will be necessary to carry the components into orbit. The

station's truss structures will probably be made of composites which have less weight and lower coefficients of thermal expansion than metal. To assemble the space station in space, two construction approaches are under consideration: "deployable" structures that automatically erect themselves by simply unfolding, or "erectable" structures that would be constructed by remote manipulator arms and (or) astronauts. In those areas where it is a straightforward truss structure, a deployable design may be better used. In other cases where complicated geometry or devices are attached, erectable designs will probably be used.

To maintain a space station in an orbit subject to disturbances such as a tenuous but measurable atmosphere, thermal gradients etc., some means of stabilization is required. Conventional control moment gyros may be used to fine-tune the station's attitude. Control rockets may be required to periodically reboost the station when aerodynamic drag has slowed it enough to lose attitude. In addition, other control devices may be needed to passively or actively damp out the vibrational motion due to the structural flexibility of the space station. However, most existing control and filtering schemes assume that a dynamic model is available for the system. A common approach is to develop a theoretical dynamic model from physical laws and then perform experimental verification. On the other hand, if an experimental identification of the structure is performed, an empirical dynamic model can (theoretically) be obtained by measuring input and output signals.

Some of the most technically challenging large space systems have been proposed by the Department of Defense. For example, Space Based Lasers (SBL) have been proposed in some SDI architecture to attack enemy ballistic missiles. These 10-50 meter structures are small and stiff compared to the Space Station, but their need to maintain optical telescope tolerances while subjected to severe on-board vibration disturbances and rapid retargeting maneuvers has driven new requirements for actively controlled optical structures.

The identification of a model for an on-orbit structure is a particularly complex problem which will require a rather large amount of data collection and computer operations to process the data [25]. The major problems associated with identifying a model of an on-orbit structure are the choice of a mathematical model for the structure, selection of a parameter estimation algorithm (recognizing it limitations), handling of noise and uncertainties, and verification of the model from the parameter estimation. In addition to these problems, it may be difficult to relate the prelaunch structure model data to the on-orbit model and on-orbit data due to gravitational and other environmental effects. The on-orbit structure will have insufficient structural integrity to be assembled on earth for testing. One key question arises: will it be possible to construct substructures for testing, with these data used later for the on-orbit phase of identification?

The on-orbit structure will be exposed to a harsh environment consisting of particle radiation, solar effects, gravitational anomalies, extreme temperatures and of course, near vacuum. In addition to these effects, the structure may undergo physical parameter changes due to preloaded stress and fatigue as well as the deterioration of composite materials due to outgasing and other aging effects. Also entering the identification task and creating additional complexities is the accuracy of the computations.

The following specific issues are presented to highlight system identification goals for large space structures, and indicate directions for pursuing those goals.

9.1.1. Choice of Experimental Conditions.

Before a space system is put in orbit, laboratory experiments are normally carried out to characterize the system. The design of an identification experiment involves a number of choices such as the choice of actuators and actuator locations, input signals, choice of response measurements, sensors and sensor locations, antialiasing filters, sample rates, etc.

The experimental conditions affect the covariance matrix of the estimates. With a bad choice of input, some parameters of interest may not be identifiable. The objective of experiment design is to choose an input that enhances the estimates of the parameters of interest. Theoretically, an impulse or white noise random excitation applied at appropriate locations will excite all modes of the system. Significant participation of all important modes is a necessary condition for identification. This is not always achieved in practice because of bandwidth limitations on the input and output devices. It is desirable to select an input which maximizes the sensitivity of the system output to unknown parameters. To date, the concept of optimal input has not been satisfactorily addressed in experimental design for structural identification. In general, it is good practice to choose an input for an identification experiment that is, as far as possible, similar to the inputs the system will experience during operation.

Sampling can be taken over equally spaced intervals, logarithmically spaced intervals, or by any other spacing schemes. The use of equally spaced samples is more widespread than any other technique. For a linear system, the sampling rate should be at least twice the highest frequency of interest. In practice, it is not wise to press the two-sample-per-cycle rule too closely, particularly when the noise level is high or significant damping is present in the system. To provide a margin of safety, the sampling rate should be slightly greater than twice the highest significant frequency being sampled. For FFT data processing, the sampling rate is limited by the data block size of the FFT processor (typically a power of the integer 2 such as 512, 1024, 2048, etc.). The sampling rate, r_s, is determined by the data block size, N, the minimum frequency resolution bandwidth, f, and the highest frequency of interest (i.e. Nyquist cutoff frequency), f_c. The relationship is $r_s = N(\delta f/f_c) = N/(Tf_c)$ where T is the duration of the record consisting of N samples.

9.1.2. Modeling Accuracy.

The prospect of certifying structural dynamic models to be "flown" with the control systems of large space structures raises fundamental questions having to do with the interface between structural dynamics and controls technologies. Given that control systems can be designed with robustness to accommodate some degree of modeling inaccuracy, one might first question the cost-benefit tradeoffs between effort spent in accurate identification vs. effort spent in designing a robust control system. The question appears simple on the surface, but quickly leads to more difficult questions, such as how modeling accuracy, robustness and controller performance should be measured. All three involve multi-dimensioned qualities, all of which are likely to be mission-dependent.

The reduced structural dynamic models used in control systems are particularly susceptible to modeling error because of the need to compress large volumes of structural information into the framework of a few generalized coordinates. These generalized coordinates usually correspond to normal modes of the structure, selected on the basis of frequency and participation in critical response variables. These response variables may be associated with actuator and sensor locations, large structural (or nonstructural) masses, or large structural motion. If shape control of an antenna (for example) is sought, then those modes which determine the shape of the antenna must be included in the model.

The accuracy of a model therefore depends on how many modes are included in the model and how accurately those modes can be identified. When light damping is present, the latent roots of the lambda matrix

$$Ms^2 + Ds + K = \Lambda(s) \tag{378}$$

occur in complex conjugate pairs. The measurement sequence $y(kT)$ will be obtained from the measurement equation

$$Y(kT) = Cx(kT) \tag{379}$$

where $x(kT)$ represents the actual motion of the structure measured at the sensor locations. The measurement matrix C will in general contain differential and integral operators and thus be frequency dependent when transformed to the frequency domain.

In addition to the errors due to structural idealization and modal truncation, inaccuracies will arise from measurement errors, as indicated above, and computational errors, with the two types of errors difficult to separate. Even if the measurements are error free, the word length of the computer used for data processing will introduce errors in the parameter estimation. These errors will be particularly significant for short word lengths and a model of high order. To illustrate, the method of the least- squares algorithm for solving the overdetermined linear equation

$$Ax = b \tag{380}$$

by the equation

$$x = (A^T A)^{-1} A^T b \tag{381}$$

might be utilized in the identification process when observation errors and noise are present. If $(A^T A)$ is ill-conditioned the above algorithm will produce a solution with much larger errors than will the singular value decomposition algorithm applied to the same data. The larger the number of modes to be identified, the more nearly singular the $(A^T A)$ matrix will be. For a problem with the same data to be processed, the SVD algorithm will produce a more accurate solution to equation (380) than will (381) in terms of the least square error. An excellent comparison of these two algorithms is given in Noble and Daniel [203].

9.1.3. Model Optimization and Uniqueness.

The concepts of model optimization and uniqueness were introduced in Section 2.1. Uniqueness was described in terms of "identifiability", the ability to "uniquely tie down the parameters of a given model." As explained there, one might intuitively expect a model with more parameters to match observations of a system's behavior better than a model with fewer parameters. However, one might expect the model with more parameters to experience "greater problems with nonuniqueness".

To illustrate the uniqueness problem, one may recall from Section 5.2.2 that the Ho algorithm does not identify a unique triplet $[A, B, C]$ for the transfer function of a multivariable system defined by

$$T(s) = C[sI - A]^{-1}B. \tag{382}$$

The transfer function will be unique but not the triplet. The same problem exists in identifying the mass, damping and stiffness matrices of the transfer function defined by

$$T(s) = C[Ms^2 + Ds + K]^{-1}B. \tag{383}$$

The transfer function might be found to be unique, but this does not imply that the mass, damping and stiffness matrices can be uniquely determined. As an alternative to the question of uniqueness, another question that might be asked in the identification task is: what are the important parameters for the model and how are these parameters used in the control of a large space structure [204]. Is it important in the control of the structure to identify all of the parameters of a model uniquely?

This line of reasoning leads to the notion of a model which in some sense is "optimum". The qualities of an optimum model may be considered apart from the question of

uniqueness, momentarily. The following questions suggest the qualities one might associate with an optimum model:

"What is the optimal set of actuator/sensor locations, input time- histories and system (structural) characterizations required to meet control performance goals?"

"What is the optimal set of actuator/sensor locations, input time- histories and system (structural) characterizations required for a particular structure so that errors in system modeling (e.g. ignored nonlinearties) would have minimal effect on the identified results?"

Imbedded in these questions are questions such as the following:

"Given a structural design, how does one find the set of response variables which, when sensed, will best provide an adequate measure of how well the desired control performance is met?" (related to "observability")

"How does one establish the best set of actuator forces which will best enable strong and complete influence on the chosen set of response variables?" (related to "controllability")

All of these questions have to do with formulating a model for purposes of identification. Inasmuch as uniqueness is related to identifiability, one might surmise from the foregoing qualities of an optimum model that the ability to identify a model uniquely depends to some extent on the formulation of an optimum model. From a practical standpoint, one might pose one final question as a corollary to the others :

"What degree of nonuniqueness in identification can be tolerated without affecting controller performance?"

The questions are easier to ask than answer. Their importance, however, lies in the directions they suggest for future research. The questions are really intended to stimulate

further questions, rather than solicit specific answers at this time. The state-of-the-art will advance as the questions are refined, expanded, and eventually answered.

As pointed out in Section 2.1, *a priori* knowledge will play a crucial role in structural system identification. Since it is possible to ground test substructures and interconnected substructures prior to assembling them on-orbit, knowledge can be gained by making measurements in laboratories. Substructure testing programs can be used to help define an optimal model for on-orbit identification. Although the laboratory environment will not be the same as the on-orbit environment, there is no reason to believe that a model which is optimal in the laboratory will not be suitable for on-orbit identification. The success of on-orbit identification will probably depend on substructure testing; a research and development program with sufficient support should be established to carry out this work. There are research efforts underway at several laboratories [205] but available computer programs and structural modeling efforts are too limited to assure success in determining optimal structure models in the near future. General purpose structural modeling and multi-body dynamics computer programs such as NASTRAN and TREETOPS [206], respectively, can be used to define and investigate complex models including large space structures. There is no assurance, however, that optimum structural models can be generated with these programs above.

9.1.4. System Uncertainties.

The identification of a system or structure from input-output data is further complicated by uncertainties in system modeling. As discussed in Section 3.5, uncertainties are due to system parameters which are not fully understood, such as nonlinearities, deadband effects, hysteresis, model dimensions, elastic deformations, truncation errors, and a general lack of full characterization of the structure's materials. In addition to system uncertainties, forces may be acting on the system during the period of parameter identification

which are not taken into account as input data. It is generally assumed that the system is linear and time invariant, i.e. the system input/output behavior is the same from one test period to another. If there are unaccounted outside forces acting on the structure, or if the structural properties change due to the preloaded stress, the identified system parameters will vary from one test period to another. It is impossible to predict how the structure may change on- orbit without knowledge of the environment and its effect on the structure. If the effect is small, it may be treated as model uncertainty; if not, it will have to be modeled.

Although there may be some information on the changes of physical parameters of the structural components due to the environment of space, there is insufficient test data on composite materials in the extreme space environment to fully characterize these materials. The effects of solar radiation, electrons, gamma rays, and other forms of radiation for long term exposure of the structure and the resulting change in the dynamic properties of the structure are unknown. Of course, the cycling of temperature and vibration deformations of the structure accumulate microscopic damage to the material and makes it difficult-to-model time- variation of the material properties. These effects should be included in the uncertainties until a better understanding of these effects is established.

The uncertainties in the structure's physical components and the forces acting on the structure and its vibrational behavior can probably best be handled by some form of adaptive estimation and control algorithm. As stated above, the adaptive strategy has been applied only to systems with a limited number of modes or system parameters. References to the application of the adaptive scheme to systems with 50-100 parameters have not been found in the open literature. This does not necessarily mean that such applications do not exist, it can only be concluded that the literature search did not uncover any references on the subject matter. The paper by Nurre, et al [205] tends to support the above statement

as their paper does not provide definite procedures on how to include the uncertainties in the identification process.

9.1.5. Damping Estimation.

The term damping as used defines the energy dissipation properties of a material or system. In most cases, damping is broken down into two major headings identified as material damping and system damping [207]. Material damping, sometimes called internal damping, internal friction, or hysteresis damping, is related to energy dissipation in a volume of macrocontinuous media. Viscoelastic damping is an idealized form of material damping. System damping involves configurations of distinguishable parts or interaction among various phenomena. Among the types of system damping are joints, interfaces, and dashpots. Aerospace engineers have become interested in measuring damping more accurately because damping for flexible space structures is a major factor in designing a controller to supress vibrational motion. A wide variety of experimental techniques have been used to estimate damping. It is difficult to compare results found by different investigators. When it is possible, the reported values often differ by substantial amounts, e.g. by ratios of five to one or more. Part of these differences can be attributed to material and system uncertainties, invalid assumptions and inadequate identification methods. It is believed that any test ought to bear a close resemblance to service conditions in order to reduce uncertainties. Work done to date has not been completely effective in modeling damping mechanisms. Looking to the future, the ability to successfully design and measure structural damping will be important for both active and passive control. The damping properties of specific materials and configurations under specific test conditions do, of course, provide valuable data. However, to properly interpret and effectively use such data in engineering applications requires more general theories and computational procedures.

9.1.6. Approximate Linear Models for Nonlinear Systems.

All structures encountered in practice are nonlinear to some extent. The inherent complexity of nonlinear vibrational systems makes a purely theoretical approach difficult in deriving analytic models. Therefore, models based on (or improved by use of) measured quantities are needed. Nonproportionality of restoring forces in response to certain input forces is a very common phenomenon in structures. For example, joint-dominated structures may exhibit nonlinear behavior depending upon forcing conditions. Identification of specific nonlinear structures using estimation methods has been studied. The choice among various approaches for nonlinear system identification is dictated by the process and purpose of identification. Unfortunately, most algorithms are very difficult to implement, or apply only to a narrow class of systems. Since identification methods for linear systems are well established and have been widely applied, the question arises as to whether a nonlinear system can be represented by a linear model which approximates the dynamic behavior of the system. For moderate system nonlinearities, an approximate model may be adequate for response prediction. In fact, many nonlinear functions can be approximated in terms of a series expansion of a finite number of sine and cosine functions (as in the use of describing functions, for example). A system response representation in terms of a sine and cosine series is equivalent to a linear model in modal space which is characterized by the frequencies and coefficients of the series. Further research in this direction should be performed.

9.1.7. Decision to Perform an Identification in Orbit.

With detailed analysis and ground testing, the dynamic characteristics of a space system may be estimated. However, once the system is in orbit, its characteristics may be quite different because of the environment, construction anomalies, or other unforeseen factors. To overcome this problem there should be predetermined times during the course

of the mission for calibration and parameter updates. Although there are potentially a great number of parameters to be identified, there is certainly no need for continuous identification of all parameters. Questions arise as to how often and how many parameters require updating. The decision for making parameter updates should depend on some significant inference made from observed data. Any catalogued parameter of mission interest will probably have associated with it both a nominal value and some acceptable tolerance around the nominal. If the parameter is observable through the measurement data, then it is indeed a good candidate for a detection scheme to statistically test whether its value has remained in the neighborhood of the nominal. With this background, there exist various criteria for designing detectors to decide when and how many parameters require updates. Criteria such as the Neyman-Pearson optimal detector which maximizes the probability of detection for a given false alarm rate, or the Bayes optimal detector which minimizes expected operating costs, may be tailored for specific application.

9.2. RECOMMENDATIONS.

It is evident that the state-of-the-practice in system identification is far from being adequate to meet the enormous challenges posed by the deployment or erection of large space structures. This report is concluded by making the following recommendations:

1. *Experimental Design: New, innovative experimental techniques and input schemes must be evolved that will permit excitation of a large number of global and no local modes of the structure, its components or its scaled model during ground vibration testing or vibration testing in space.*

2. *Type of Identification: We anticipate extremely large flexible space structures being placed into orbit within the next 20 to 30 years. Parametric methods involving the estimation of thousands of parameters (model or modal) are out of the question; methods*

of parameterization which lead to the definition of a relatively few distributed parameters should be sought instead. Modal parameterization methods may be especially viable for LSS because reasonable estimates of highly sparse modal mass, stiffness and damping matrices of the structure can be obtained through an appropriate updating scheme applied to modal matrices which are initially diagonal. Research along this direction may yield fruitful results. Research should also be directed toward the development of highly efficient non-parametric methods that recover an input-output map of critical structural loadpaths.

3. *Benchmark on Estimation Algorithms: It is recommended that a comparative study of estimation algorithms be undertaken with a view to evolving some highly efficient and practical algorithms most appropriate for identification of LSS. The research community should also be seeking algorithms that are essentially linear, that is to say those that operate on the input-output data in a linear way much like the FFT procedure. We should be looking for algorithms that are more appropriate for parallel computation, those that do not require operations on large matrices, those that can handle rank deficient matrices through Singular Value Decomposition and those that are direct rather than iterative with no guarantee of convergence.*

4. *Practical Considerations: It is difficult to assess the adequacy of the state of the art until it has been reduced to practice. One of the biggest deficiencies and most urgent needs is the practical implementation of existing theory. Some of the practical problems are:*

   - *Diagnosing the cause(s) of unsuccessful parameter estimation. Possible causes include:*

     - *Experimental errors*

- *Modeling errors, including the improper assessment of initial parameter uncertainty*

- *Inappropriate definition of parameters*

- *Insufficient data*

- *Inappropriate selection of data*

- *Inappropriate sequencing of data when sequential processing is used.*

• *Determining what parts of a model can reliably be identified from ground vibration tests and what parts must be identified or confirmed by testing in space.*

• *Determining how to verify a reduced-order plant model in space, including the selection of parameters to be identified, the selection of inputs and the selection of measurement quantities.*

5. *On-Orbit, Off-Orbit Computations: Hardware and software aspects of identification to handle large amounts of data collected on board must be addressed and resolved.*

6. *Statistical Estimation: Statistical methods for both parametric and nonparametric estimation are recommended over nonstatistical methods because they provide a means of evaluating the reliability of the estimates. Statistical estimation is considered to be necessary although not always sufficient for this purpose. Experimental errors and modeling errors can invalidate statistical measures of reliability; however even invalid measures of statistical reliability often provide a correct indication that an estimate is in error.*

7. *Uncertainties: A great deal of research is needed to provide a better characterization of modeling uncertainty, including the identification and removal of systematic errors. Systematic errors include joint nonlinearity and other types of nonlinearities, material*

damping, etc. The remaining model uncertainty can be treated as random but should be realistically and reliably quantified.

REFERENCES.

1. Allemang, R.J., and Brown, D.L., "Multiple-Input Experimental Modal Analysis - A Survey," The International Journal of Analytical and Experimental Modal Analysis, Vol. 1, No. 1, January 1986.

2. Hsieh, G.J., Kot, C.A., and Srinivasan, M.G., "Evaluation of System Identification Methodology and Application," NUREG/CR-3388, for the U.S. Nuclear Regulatory Commission, May 1983.

3. Martinez, D.R., "Parameter Estimation in Structural Dynamics Models," Report No. SAND 80-0135, Sandia National Laboratories, March 1981.

4. Ibanez, Paul, "Review of Analytical and Experimental Techniques for Improving Structural Dynamic Models," Welding Research Council Bulletin No. 249, June 1979.

5. Hart, G.C., and Yao, J.T.P., "System Identification in Structural Dynamics," ASCE Journal of Engineering Mechanics Division, Vol. 103 No. EM6, December 1977.

6. Berman, J.G., "Determining Structural Parameters from Dynamic Testing," Shock and Vibration Digest, Vol. 7, January 1975.

7. Collins, J.D., Young, J.P., and Kiefling, L.A., "Methods and Applications of System Identification in Shock and Vibration," System Identification of Vibrating Structures - Mathematical Models from Test Data, ed. Pilkey, W. and Cohen, R., Annual Meeting of the ASME, 1972.

8. Gersch, W., "Parameter Identification Stochastic Process Techniques," The Shock and Vibration Digest, Vol. 1, November 1975.

9. Pilkey, W. and Cohen, R. (eds.) "System Identification of Vibrating Structures - Mathematical Models from Test Data," Annual Meeting of the ASME, 1972.

10. Rodeman, R., and Yao, J., "Structural Identification - Literature Review," Report CE-STR-73-3, School of Civil Engineering, Purdue University, Dec. 1973.

11. Schiff, A.J., "Identification of Large Structures Using Data from Ambient and Low Level Excitations," System Identification of Vibrating Structures-Mathematical Models from Test Data, ed. Pilkey, W. and Cohen, R., Annual Meeting of the ASME, 1972.

12. Young, J.P. and On, F.J., "Mathematical Modeling via Direct Use of Vibration Data," SAE Paper 790715, SAE National Aeronautic and Space Engineering and Manufacturing Meeting, Los Angeles, CA, October 1969.

13. Natke, H.G., "Survey of European Ground and Flight Vibration Test Methods," SAE paper 760878, Aerospace Engineering and Manufacturing Meeting, San Diego, November 1976.

14. Eykhoff, P., **System Identification**, Wiley, London, 1975.

15. Denham, M. J., "Canonical Forms for the Identification of Multivariate Systems," IEEE Trans. AC, AC-19, No. 6, pp. 646-656.

16. Udwadia, F. E., Shah, P. and Sharma, D., "Uniqueness of Damping and Stiffness Distributions in the Identification of Soil and Structural Systems," Journ. of Applied Mechanics, ASME, Vol. 45, Mar. 1978.

17. Udwadia, F. E. and Sharma, D., "Some Uniqueness Results in the Identification of Structural Systems," SIAM Journal of Applied Mathematics, Vol 43, 1978.

18. Gabor, D., Wilby, W. and Woodcock, R., "A Universal Nonlinear Filter, Predictor and Simulator Which Optimizes Itself by a Learning Process," IEEE, 108B, pp. 422.438.

19. George, D. A., "Continuous Nonlinear Systems," Tech. Report 355., Research Lab. of Electron.,M.I.T.,Cambridge, Mass. (1959).

20. Udwadia, F. E. and Marmarelis, P., "Identification of Building Structural Systems: Part I and Part II," Bulletin. Seismological Society of America, Vol. 66, no. 1, Feb. 1976.

21. Wolovich, W., **Linear Multivariate Systems**, Berlin: Springer, Vol. 11, 1974.

22. Sage, A. and Melsa, J., **System Identification**, Vol. 80, Academic Press, 1971.

23. Box, M., Davies, J. and Swann, W., **Nonlinear Optimization Techniques**, Oliver and Boyd, Edinburgh.

24. Astrom, K. J., and Bohlin, T., **Numerical Identification of Linear Dynamic Systems from Normal Operating Records**, Theory of Self- Adaptive Systems, Plenum Press, 1965, N. Y.

25. Ljung, L. and Soderstrom, T., **Theory and Practice of Recursive Identification**, MIT Press, 1982.

26. Udwadia, F. E. and Shah, P. C., "A Methodology for Optimal Sensor Locations for Identification of Dynamic Systems," Journal of Applied Mechanics, Vol.ume 45, March 1978.

27. Udwadia, F. E., "Optimal Sensor Locations for the Identification of Structural Systems," Proceedings of the NASA Workshop on Control and Identification of Space Structures, NASA, San Diego, July 1984.

28. Udwadia, F. E., "Some Results on the Optimal Spacing of Measurements in the Identification of Structural Systems," Quarterly of Applied Mathematics, Vol. 43, November, 1985.

29. Ewins, D. J., **Modal Testing, Theory and Practice** Research Studies Press Ltd., England, 1984.

30. Guyan, Robert J., "Reduction of Stiffness and Mass Matrices," AIAA Journal, Vol. 3, No. 2, February 1965.

31. Archer, J.S., "Consistent Mass Matrix for Distributed Mass Systems," Journal of the Structural Division, ASCE, Vol. 89, ST4, August, 1963.

32. Hasselman, T.K., and Chrostowski, J.D., "Dynamic Model Verification of a Multi-Component System," SAE Technical Paper No. 841585, Aerospace Congress and Exposition, Long Beach, California, October 15-18, 1984.

33. Collins, J.D., Hart, G.C., Hasselman, T.K., and Kennedy, B., "Statistical Identification of Structures," AIAA Journal, Vol. 12, No. 2, February 1974.

34. Hasselman, T.K., "A Perspective on Dynamic Model Verification," Modal Testing and Model Refinement, AMD-Vol. 59, American Society of Mechanical Engineers, New York, NY, 1983.

35. Dobbs, M.W., and Nelson, R.B., "Parameter Identification of Large Models - Concept and Reality," Modal Testing and Model Refinement, AMD-Vol. 59, American Society of Mechanical Engineers, New York, NY, 1983.

36. Lee, L. T., and Hasselman, T.K., "Dynamic Model Verification of Large Structural Systems," SAE Technical Paper No. 781047, Aerospace Meeting, San Diego, November 27-30, 1978.

37. Garba, J.A., and Wada, B.K.,"Application of Perturbation Methods to Improve Analytical Model Correlation with Test Data," 1977 SAE Aerospace Meeting, Sept. 1977.

38. Heki, K. and Saka, T., "Stress Analysis of Lattice Plates as Anisotropic Continuum Plates," Proceedings of the 1971 IASS Pacific Symposiu, Part II on Tension Structures and Space Frames, Tokyo and Kyoto, Japan, 1972, pp. 663-674.

39. Nayfeh, A.H. and Hefzy, M.S., "Continuum Modeling of Three- Dimensional Truss-like Space Structures," AIAA J., vol. 16, No. 8, 1978, pp. 779-787.

40. Nayfeh, A.H. and Hefzy, M.S., "Effective Constitutive Relations for Large Repetitive Frame-like Structures," Int. J. Solids and Structures, vol. 18, No. 11 1982, pp. 975-987.

41. Sun, C.T. and Yang, T.Y., "A Continuum Approach Toward Dynamics of Gridworks," J. Appl. Mech., vol. 40, No. 1 1973, pp. 186-192.

42. Noor, A.K., Anderson, M.S. and Greene, W.H.,"Continuum Models for Beam and Plate-like Lattice Structures," AIAA J., vol. 16, No. 12, 1978, pp. 1219-1228.

43. Noor, A.K. and Nemeth, M.P., "Micropolar Beam Models for Lattice Grids with Rigid Joints," Comp. Meths. Appl. Mech. Eng., 21, 1980, pp. 249-263.

44. Noor, A.K. and Nemeth, M.P., "Analysis of Spatial Beam-like Lattices with Rigid Joints," Comp. Meths. Appl. Mech. Eng., 24, 1980, pp. 35-59.

45. Sun, C.T., Kim, B.J. and Bogdanoff, J.L., "On the Derivation of Equivalent Simple Models for Beam- and Plate-like Structures in Dynamic Analysis," Proceedings AIAA 22nd Structures, Structural Dynamics and Materials Conference (April 6-9) and AIAA Dynamics Specialists Conference (April 9-10), Atlanta, Georgia, 1981, pp. 523-531.

46. Sun, C.T. and Kim, B.J., "Continuum Modeling of Periodic Truss Structures," in Damage Mechanics and Continuum Modeling (edited by N. Stubbs and D. Krajcinovic), ASCE 1985, pp. 57-71.

47. Sun, C.T. and Juang, "Modeling Global Structural Damping in Trusses Using Simple Continuum Models," Proceedings AIAA 24th Structues, Structural Dynamics, and Materials Conference, Lake Tahoe, Nevada, May 2-4, 1983, pp. 722-729. Also to appear in AIAA J.

48. Chen, C.C. and Sun, C. T., "Transient Analysis of Large Frame Structures by Simple Models," The Journal of the Astronautical Sciences, vol. 31, No. 3, 1983, pp. 359-379.

49. Abrate, S. and Sun, C.T., "Dynamic Analysis of Geometrically Nonlinear Truss Structures," Computers and Structures, vol. 17, No. 4, 1983, pp. 491-497.

50. Abrate, S. and Sun, C.T., "Continuum Modeling of Damping in Large Space Structures," to be published.

51. Ewins, D.J., "Modal Test Requirements for Coupled Structures Analysis Using Experimentally-Derived Component Models," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, Joint ASCE/ASME Mechanics Conference, Albuquerque, NM, June 24-26, 1985.

52. Gantmacher, G.R., The Theory of Matrices, Vol. I, Chelsea Publishing Co., New York, NY, 1959.

53. Hurty, W.C., "Dynamic Analysis of Structural Systems Using Component Modes," AIAA Journal, Vol. 3, No. 4, pp. 678-585, 1965.

54. Craig, R.R., and Bampton, M.C.C., "Coupling of Substructures for Dynamic Analysis," AIAA Journal, Vol. 6, No. 7, July 1968.

55. Gladwell, G.M.L., "Branch Mode Analysis of Vibrating Systems," Journal of Sound and Vibration, Vol. 1, pp. 41-59, 1964.

56. Benfield, W.A., and Hruda, R.F., "Vibration Analysis of Structures by Component Mode Substitution," AIAA/ASME 11th Structures, Structural Dynamics, and Materials Conference, Denver, Colorado, April 1970.

57. Goldman, R.L., "Vibration Analysis by Dynamic Partitioning," AIAA Journal, Vol. 7, No. 6, pp. 1152-1154, 1969.

58. Hou, S.N., "Review of Modal Synthesis Techniques and a New Approach," Technical Memorandum 69-2031-5, Bellcomm., Inc., September 26, 1969.

59. Hintz, R.M., "Analytical Methods in Component Modal Synthesis," AIAA Journal, Vol. 13, No. 8, pp. 1007-1016, August 1975.

60. Craig, R.R. Jr., and Chang, C-J., "A Review of Substructure Coupling Methods for Dynamic Analysis," 13th Annual Meeting, Society for Engineering Science, Advances in Engineering Science, Vol. 2, NASA CP-2001, pp. 393.408, November 1976.

61. Herting, D.N., and Hoesly, R.L., "Development of an Automated Multistage Modal Synthesis System for NASTRAN," NASA-CP-2018, pp. 435-448, 1977.

62. Geering, H.P., "New Methods in Substructuring," Proceedings AIAA/ASME/ASCE-/AHS 21st Structures, Structural Dynamics and Materials Conference, pp. 801-808, May 1980.

63. Bertram, A., "Using Modal Substructuring Techniques in Modeling Large Flexible Spacecraft," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, AMD-Vol. 67, The American Society of Mechanical Engineers, New York, NY, 1985.

64. Hasselman, T.K., and Kaplan, A., "Dynamic Analysis of Large Systems by Complex Mode Synthesis," Journal Dynamic Systems, Measurement and Control, ASME, Vol. 96, Ser. G, pp. 327-333, September 1974.

65. Craig, R.R. Jr., and Chung, "Generalized Substructure Coupling Procedure for Damped Systems," AIAA Journal, Vol. 20, No. 3, pp. 442-444, March 1982.

66. Craig, R.R. Jr., **Structural Dynamics - An Introduction to Computer Methods.** J. Wiley and Sons, Inc., New York, 1981.

67. Meirovitch, L., **Computational Methods in Structural Dynamics**, Sijthoff and Noordhoff, Alphen dan den Rign, Netherlands, 1980.

68. Craig, R.R. Jr., "A Review of Time-Domain and Frequency- Domain Component Mode Synthesis Methods," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, AMD-Vol. 67, presented at the Joint ASCE/ASME Mechanics Conference, Albuquerque, NM, June 24- 26, 1985.

69. Fleming, E.R., "The Use of Component Mode Synthesis in the Dynamic Analysis of Aerospace Systems," Combined Experimental/Analytical Modeling of Dynamic Struc-

tural Systems, AMD-Vol. 67, presented at the Joint ASCE/ASME Mechanics Conference, Albuquerque, NM, June 24- 26, 1985.

70. Hurty, W.C., "Criterion for Selecting Realistic Natural Modes of a Structure," TM 33.36, Jet Propulsion Lab., Pasadena, CA, Nov 1967.

71. Hasselman, T.K., "Solution of the Structural Dynamics Eigenproblem by Modal Synthesis: Sensitivity to Coordinate Selection and Parameter Variation," Doctoral Dissertation, UCLA; UCLA Tech. Report, UCLA- ENG-7239, June 1972.

72. Hasselman, T.K., and Hart, G.C., "A Minimization Method for Treating Convergence in Modal Synthesis," AIAA Journal, Vol. 12, No. 3, March 1974.

73. MacNeal, R.H., "A Hybrid Method of Component Mode Synthesis," Computers and Structures, Vol. 1, 1971, pp. 581-601.

74. Rubin, S., "An Improved Component Mode Representation," AIAA Journal, Vol. 13, 1975, pp. 995-1006.

75. Martinez, D.R., Miller, A.K., and Carne, T.G., "Combined Experimental/Analytical Modeling of Shell/Payload Structures," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, ASME, AMD- Vol. 67, June 1985.

76. Hruda, R.F., "A Recommendation on the Use of Free- Free/Residual Flexibility Versus Fixed-Interface Dynamic Models for Shuttle Loads Analyses," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, ASME, AMD-Vol. 67, June 1985.

77. Hasselman, T.K., "Modal Coupling in Lightly Damped Structures," AIAA Journal, Vol. 14, No. 11, November 1976.

78. Hasselman, T.K., "Damping Synthesis from Substructure Tests," AIAA Journal, Vol. 14, No. 10, November 1976.

79. Hasselman, T.K., "Method for Constructing a Full Modal Damping Matrix from Experimental Measurements," AIAA Journal, Vol. 10, No. 4, April 1972, pp. 526-527.

80. Ibrahim, S.R., and Mikulcik, E.C., "The Experimental Determination of Vibration Parameters from Time Responses," Shock and Vibration Bulletin, No. 46, August 1976.

81. Juang, Jer-Nan, and Papa, Richard S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," Journal of Guidance, Control, and Dynamics, AIAA, Vol. 8, No. 5, September-October 1985, pp. 621-627.

82. Coppolino, R.N., "A Simultaneous Frequency Domain Technique for Estimation of Modal Parameters from Measured Data," SAE Technical Paper No. 811046, Aerospace Congress and Exposition, Anaheim, CA, October 5-8, 1981.

83. Richardson, M., and Potter, R., "Identification of the Modal Properties of an Elastic Structure from Measured Transfer Function Data," Instrument Society of America, 20th International Symposium, Albuquerque, NM, May 21-23, 1974.

84. Kana, D.D. and Huzar, S., "Synthesis of Shuttle Vehicle Damping Using Substructure Test Results," Southwest Research Institute Interim Report No. 02-3131, Prepared for NASA, June 1972.

85. Kana, D.D., and Unruh, J.F., "Substructure Energy Methods for Prediction of Space Shuttle Modal Damping," Journal Spacecraft, 12, pp. 294-301, 1975.

86. Soni, M.L., Kluesener, M.F., and Drake, M.L., "Damping Synthesis and Damped Design for Flexible Spacecraft Structures," Computer and Structures, Vol. 20, No. 1-3, pp. 563.574, 1985.

87. Soni, M.L., "Prediction of Damping for Flexible Spacecraft Appendages," Proceedings of the 2nd International Modal Analysis Conference, Orlando, Florida, 1984.

88. Beliveau, J.G., and Soucy, Y., "Damping Synthesis Using Complex Substructure Modes and a Hermitian System Representation," AIAA Journal, Vol. 23, No. 12, December 1985.

89. Lips, K.W., and Vigneron, F.R., "Damping Synthesis for a Spacecraft Using Substructure and Component Data," Journal of Spacecraft and Rockets, Vol. 23, No. 2, March-April 1986.

90. Lips, K.W., and Vigneron, F.R., "Damping Synthesis for a Spacecraft Using Substructure and Component Data," Dept. of Communications, Ottawa, Canada, CRC Report 1365, August 1984.

91. Jezequel, L., "A Method of Damping Synthesis From Substructure Tests," Journal Mechanical Design, ASME 102, pp. 286-294, 1980.

92. Maddah, E., and Imbert, J.F., "A Comparison of Damping Synthesis Methods for Space Vehicle Dynamic Analysis," Centre National d'Etudes Spatiales, Report No. CNES-80/CT/PRT/SST/SM/316, Toulouse, France, 1980.

93. Soni, M.L., Kluesener, M.F., Gogner, F.K., and Drake, M.L., "Damping of Fexible Spacecraft Appendages," University of Dayton Research Institute, Report UDR-TR-81-28, Dayton, Ohio, 1981.

94. Taylor, D.L., and Kane, T.R., "Multiparameter Quadratic Eigenvalue Problems," Journal of Applied Mechanics, Vol. 42, Ser. E, pp. 478- 483, June 1975.

95. Fox, R.L., and Kapoor, M.P., "Rates of Change of Eigenvalues and Eigenvectors," AIAA Journal, Vol. 6, pp. 2426-2429, December 1968.

96. Rogers, L.C., "Derivatives of Eigenvalues and Eigenvectors," AIAA Journal, Vol. 8, pp. 943-944, May 1970.

97. Nelson, R.B., "Simplified Calculation of Eigenvector Derivatives," AIAA Journal, Vol. 14, No. 9, pp. 1201-1205, September 1976.

98. Hasselman, T.K., and Hart, G.C., "Modal Analysis of Random Structural Systems," ASCE Journal of the Engineering Mechanics Division, Vol. 98, No. EM3, pp. 562-579, June 1972.

99. Hasselman, T.K., "Structural Uncertainty in Dynamic Analysis," SAE Technical Paper No. 811049, SAE Aerospace Congress and Exposition, Anaheim, California, also published in SAE Transactions, 1981.

100. Zadeh, L. A., and Desoer, C. A., **Linear System Theory, The State Space Approach**, First Edition, McGraw-Hill Book Company, New York, N. Y., 1963.

101. Otnes, R.K., and Enochson, L., "Digital Time Series Analysis", Wiley Interscience, New York, 1972.

102. Bendat, J.S., and Piersol, A.G., **Random Data: Analysis and Measurement Procedures**, Wiley Interscience, New York, 1971.

103. Bendat, J.S., and Piersol, A.G., **Engineering Applications of Correlation and Spectral Analysis**, Wiley Interscience, New York, 1980.

104. Bendat, J.S., and Piersol, A.G., **Random Data: Analysis and Measurement Procedures**, 2nd Edition, Revised and Expanded, John Wiley and Sons, New York, 1986

105. Cooley, J.W. and Tukey, J.W., "An Algorithm for the Machine Calculation of Complex Fourier Series," Mathematics of Computation, Vol. 19, No. 90, p. 297, April 1965.

106. Mitchell, L.D., "Improved Methods for the Fast Fourier Transform (FFT) Calculation of the Frequency Response Function," ASME Journal of Mechanical Design, Vol. 104, pp. 277-279, April 1982.

107. Ewins, D.J. and Sidhu, J., "Correlation of Finite Elements and Modal Test Studies of a Practical Structure," Proc. Int. Modal Anal. Conf. 2, 1984.

108. Coppolino, R.N., "Hybrid Experimental/Analytical Dynamic Models of Aerospace Structures," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, Joint ASCE/ASME Mechanics Conference, Albuquerque, NM, June 24-26, 1985.

109. Bendat, J.S., and Piersol, A.G.,"Spectral Analysis of Non- Linear Systems Involving Square-Law Operations," Journal of Sound and Vibration, Vol. 81, pp.199-213, 1982.

110. Bendat, J.S., and Piersol, A.G., "Decomposition of Wave Forces into Linear and Nonlinear Components," to be published in the Journal of Sound and Vibration, May 1986.

111. Wiener N., **Nonlinear Problems in Random Theory**, MIT Press, Cambridge, 1958.

112. Udwadia F. and Marmarelis P., "Identification of Building Structural Systems: I the Linear Case", Bulletin of the Seismological Society, Vol. 66, No. 1, pp. 153-171, February 1976.

113. Marmarelis P. and Udwadia F. , "Identification of Building Structural Systems: The Nonlinear Case", Bulletin of the Seismological Society, Vol. 66, No. 1, pp. 125-151, February 1976.

114. Graupe D., **Identification of Systems**, New York, pp. 106-115, 1975.

115. Masri S. and Caughey T, "A Non-Parametric Identification Technique for Nonlinear Dynamic Problems", J. of Appl. Mech., 46, pp. 433- 447, 1979.

116. Udwadia F. "Nonparametric Identification of a Class of Close-Coupled Dynamic Systems", Earthquake Engineering and Structural Dynamics, Vol. 9, pp. 385-409, 1981

117. Udwadia F., "Nonparametric Identification for Nonlinear Dynamic Systems", NASA Report, Jet Propulsion Laboratory, Report No. 81-48, 1980

118. Kailath, T., **Linear Systems**, Prentice-Hall, Inc., Englewood Cliff, New Jersey, 1980

119. Wiberg, D. M., **State Space and Linear Systems**, McCraw-Hill Book Co., New York, New York, 1971.

120. Chen, C. T., **Introduction to Linear System Theory**, Holt, Rienhart, and Winston, Inc., New York, New York, 1970.

121. Chen, C. T., **Linear System Theory and Design**, Holt, Rienhart and Winston, Inc., New York, New York, 1984.

122. Seinfeld, J. H. and Lapidus, L., **Mathematical Methods in Chemical Engineering**, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974.

123. Ho, B. L. and Kalman, R. E., "Effective Construction of Linear State-Variable Models From Input/Output Data," Proc. 3rd Annual Allerton Conference on Circuit and Systems Theory, 1965, pp. 449-459; also Regelungstechnik, Vol.14, 1966, pp. 545.548.

124. Goodwin, G. C. and Payne, R. L., **Dynamic System Identification: Experiment Design and Data Analysis**, Academic Press, New York, 1977.

125. Walker, P.B. and A.W. Clegg, "The Technique of Wing Resonance Testing", Report No. A.D. 3105/M.T. 11767 of the Royal Aircraft Establishment, South Farnborough, England, (February 1939).

126. Lewis, R.C. and Wrisley, D.L., "A System for the Excitation of Pure Natural Modes of Complex Structures," J. Aero. Sci., 17, 705.722 (1950).

127. Leppert, E.L., Lee, S.H., Day, F.O., Chapman, P.C. and Wada, B.K., "Comparison of Modal Test Results: Multipoint Sine Versus Single- Point Random," SAE Paper No. 760879, San Diego, CA (November 29-December 2, 1976).

128. Chen, J.C., "Evaluation of Modal Testing Methods," AIAA Paper No. 84-1071, Palm Springs, CA (1984).

129. Stroud, R.C., "Excitation, Measurement, and Analysis Methods for Modal Testing," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, - ASME Publication AMD-Vol 67, Joint ASCE/ASME Mechanics Conference, Albuquerque, NM, (June 24-26, 1985).

130. Niedbal, N., "Experimental System Identification for Experimental/Analytical Correlation and Modeling," Combined Experimental/Analytical Modeling of Dynamic Structural Systems, ASME Publication AMD-Vol 67, Joint ASCE/ASME Mechanics Conference, Albuquerque, NM (June 24-26, 1985).

131. Hawkins, F.J., "An Automatic Resonance Testing Technique for Exciting Normal Modes of Vibration of Complex Structures," Symposium IUTAM, "Progres Recents de la Mecanique des Vibrations Lineaires," 37-41 (1965).

132. Hawkins, F.J., "GRAMPA - An Automatic Technique for Exciting the Principal Modes of Vibration of Complex Structures," Roy. Aircraft Establishment, RAE-TR-67-211 (1967).

133. Taylor, G.A., Gaukroger, D.R., and Skingle, C.W., "MAMA - A Semi-Automatic Technique for Exciting the Principal Modes of Vibration of Complex Structures," Aero. Res. Council, ARC-R/M-3590 (1967).

134. Craig, R.R., Jr. and Su, Y.W.T., "On Multiple-Shaker Resonance Testing," AIAA J., 12, 924-931 (1974).

135. Feix, M., "An Iterative Self-Organizing Method for the Determination of Structural Dynamic Characteristics," European Space Agency, ESA-TT-232 (N76-331183), 36-60 (1975).

136. Ibanez, P., "Force Appropriation by Extended Asher's Method," SAE Paper No. 760873 (1976).

137. Morosow, G., "Exciter Force Apportioning for Modal Vibration Testing Using Incomplete Excitation," PhD Dissertation, Univ. of Colorado at Boulder (1977).

138. Morosow, G. and Ayre, R.S., "Force Appropriation for Modal Vibration Testing Using Incomplete Excitation," Shock and Vib. Bul., 48, Part 1, 39-48 (1978).

139. Juang, J. N. and Pappa, R. S., "An Eigensystem Realization Algorithm (ERA) for Modal Parameter Identification and Model Reduction," Presented at NASA/JPL Workshop on Identification and Control of Flexible Space Structures, San Diego, Ca., June 4-6, 1984. Journal of Guidance, Control and Dynamics, Vol. 8, No. 5, Sept.-Oct. 1985, pp. 620-627.

140. Juang, J. N., and Pappa, R. S., "Effects of Noise on Modal Parameters Identified by the Eigensystem Realization Algorithm," To appear in the Journal of Guidance, Control and Dynamics.

141. Pappa, R. S. and Juang, J. N., "Galileo Spacecraft Modal Identification Using an Eigensystem Realization Algorithm," Journal of the Astronautical Sciences, Vol. 33, No. 1, Jan.- Mar. 1985, pp. 15.33.

142. Klema, V. C. and Laub, A. J., "The Singular Value Decomposition: Its Computation and Some Applications," IEEE Transactions on Automatic Control, Vol. AC-25, No. 2, April 1980, pp. 164-176.

143. Pappa, R. S. and Juang, J. N., "The ERA User Manual," NASA Langley Research Center, Hampton, Virginia.

144. Vold, H., Kundrat, J., Rocklin, G. T., and Russell, R., "A Multi-Input Modal Estimation Algorithm for Mini-Computers," SAE Paper 820194, Feb. 1982.

145. Vold, H. and Russell, R., "Advanced Analysis Methods Improve Modal Test Results," Sound and Vibration, March 1983, pp. 36-40.

146. **User Manual for Modal Analysis 9.0**, SDRC/CAE International Inc., 1985.

147. Kung, S., "A New Identification and Model Reduction Algorithm via Singular Value Decomposition, 12th Asilomar Conference on Circuits, Systems and Computers, Nov. 1978, pp. 705.714.

148. Clarke, D. W., "Generalized Least-Squares Estimation of the Parameters of a Dynamic Model," Paper 3.17, Proc. I.F.A.C. Symp. on Identification, Praque, 1967.

149. Graupe, D., Swanick, B. H., and Cassir, G. R., "Application of Regression Analysis to Reduction of Multivariable Control Problems and to Process Identification," Proc. National Electronics Conf., Vol. 23, pp. 20-25, Chicago, 1967.

150. Ibrahim, S. R., and Mikulcik, E. C., "A Method for the Direct Identification of Vibration Parameters from the Free Response," Shock and Vibration Bulletin, No. 47, Pt. 4, Sept. 1977, pp. 183-198.

151. Ibrahim, S. R., "Application of Random Time Domain Analysis to Dynamic Flight Measurements," Shock and Vibration Bulletin, No. 49, Part 2, Sept. 1979, pp. 165.170.

152. Ramirez, R. W., **The FFT Fundamentals and Concepts**, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1985

153. Juang, J. N. and Suzuki, H, "An Eigensystem Realization in Frequency Domain for Modal Parameter Identification," Presented at the AIAA Guidance, Navigation and Control Conference, Williamsburg, Virginia, August 18-20, 1986.

154. Zhang, L., Kanda, H., Brown, D. L., and Allemang, R. J., "A Polyreference Frequency Domain Method for Modal Parameter Identification," ASME Paper 85.DET-106,

Presented at the ASME Design Engineering Division Conference and Exhibit on Mechanical Vibration and Noise, Cincinnati, Ohio, Sept. 10-13 1985.

155. Juang, J. N., "Mathematical Correlation of Modal Parameter Identification Methods for Flexible Structures," NASA Technical Memorandum, April 1986.

156. Isenberg, J., "Progressing from Least Squares to Bayesian Estimation," ASME Winter Annual Meeting, New York City, New York, December, 1979.

157. Junkins, J.L., **Optimal Estimation of Dynamical Systems**, Sijthoff and Noordhoff Alpen aan den Rijn, The Netherlands/Rockville, Maryland, U.S.A., 1978.

158. Hendricks, S.L., Hayes, S.M., and J.L. Junkins, "Structural Parameter Identification for Flexible Spacecraft," AIAA Paper No. 84-0060, Presented at the AIAA 22nd Aerospace Sciences Meeting, Jan. 9-12, 1984.

159. Creamer, N.G., and S.L. Hendricks, "Structural Parameter Identification Using Modal Response Data," Proceedings of the Fifth VPI & SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, VPI & SU, Blacksburg, VA, L. Meirovitch, ed., June, 1985.

160. Chen, J.C. and J.A. Garba, "Analytical Model Improvement Using Modal Test Results," AIAA Journal, Vol. 18, No. 6, June, 1980, pp. 684-690.

161. Broemeling, L.D., **Bayesian Analysis of Linear Models**, Marcel Dekker, Inc., New York, 1985.

162. Deutsch, R., **Estimation Theory**, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965.

163. Beliveau, J., "Identification of Viscous Damping in Structures From Modal Information," J. Applied Mech., Vol. 43, pp. 335-339, June, 1976.

164. Dobbs, M.W. and R.B. Nelson, "Parameter Identification of Large Structural Models - Concept and Reality," ASME Winter Annual Meeting, Boston, Massachusetts, November, 1983.

165. Hasselman, T.K., "A Perspective on Dynamic Model Verification," ASME Winter Annual Meeting, Boston, Massachusetts, November, 1983.

166. Noor, A.K. and Anderson, C.M., "Analysis of Beam-Like Lattice Trusses," Comp. Meth. Appl. Mech. Engrg. Vol. 20, 1979, pp. 53-70.

167. Chen, C.C., and Sun, C.T., "Transient Analysis of Large Frame Structures by Simple Models," Proceedings of Symposium on Engineering Science and Mechanics, National Cheng Kung University/American Astronautical Society, Dec. 28-31, 1981, also to appear in J. Astronautical Sciences.

168. Sun, C.T., Kim, B.J., and Bogdanoff, J.L., "On the Derivation of Equivalent Simple Models for Beam- and Plate-Like Structures in Dynamic Analysis," Proceedings AIAA Specialists Conference, Atlanta, Georgia, April 6.8, 1981, pp. 523-532.

169. White, C.W. and Maytum, B.D., "Eigensolution Sensitivity to Parametric Model Perturbations," Shock and Vibration Bulletin, Bulletin 46, part 5, Aug., 1976, pp. 123-133.

170. Berman, A. and Flannely, W.G., "Theory of Incomplete Models of Dynamic Structures," AIAA Journal, Vol. 9, No. 8, August, 1971, pp. 1481-1487.

171. Berman, A. and Nagy, E.J., "Improvement of Large Analytical Model Using Test Data," AIAA Journal, Vol. 21, No. 8, August, 1983, pp. 1168-1173.

172. Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," AIAA Journal, Vol. 17, No. 10, October, 1979, pp. 1147- 1148.

173. Wei, F.S., "Stiffness Matrix Corrections from Incomplete Test Data," AIAA Journal, Vol. 18, Oct. 1980, p. 1274.

174. Chen, J.C., Kuo, C.P. and Garba, J.A., "Direct Structural Parameter Identification by Modal Test Results," 24th Structures Structural Dynamics and Materials Conference AIAA/ASME/ASCE/AAS, Part 2 of Proceedings, May, 2-4, 1983.

175. Rajaram, S., "Identification of Vibration Parameters of Flexible Structures", Doctoral Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA., May, 1984.

176. Rajaram, S. and J.L. Junkins, "Identification of Vibrating Flexible Structures," AIAA Journal of Guidance, Control, and Dynamics, Vol. 8, No. 4, pp. 463-470, July-August, 1985.

177. Hendricks, S.L., Rajaram, S., Kamat, M.P., and J.L. Junkins, "Identification of Large Flexible Structures Mass, Stiffness and Damping From On-Orbit Experiments," AIAA Journal of Guidance, Control, and Dynamics, Vol. 7, No. 2, pp. 244-245, March-April, 1984.

178. Ibrahim, S.R. and Mikulcik, E.C., "A Time Domain Modal Vibration Test Technique," Shock and Vibration Bulletin, Bulletin 43, Part 4, 1973, pp. 21-37.

179. O'Donnell, K.J., and Crawley, E.F., "Identification of Nonlinear System Parameters in Space Structure Joints Using the Force-State Mapping Technique," MIT Space Systems Laboratory Report No. SSL# 16.85, July, 1985.

180. Hasselman, T.K. and L. Johnson, "Validation and Verifi cation of Rail Vehicle Models," ASME Paper No. 79-WA/ DSC-8, Presented at the ASME Winter Annual Meeting, New York, New York, Dec. 2-7, 1979.

181. Sirlin, S.W., Longman, R.W., and J.N. Juang, "Identifi ability of Conservative Linear Mechanical Systems," Journal of the Astronautical Sciences, Vol. 33, No. 1, Jan-March, 1985, pp. 95-118.

182. Ewins, D.J. and P.T. Gleeson, "A Method for Modal Identification of Lightly Damped Structures," Journal of Sound and Vibration 84(1), 1982, pp. 57-79.

183. Fries, R.H. and N.K. Cooperrider, "Bayesian Estimation of Transit Rail Vehicle Parameters," Journal of Dynamic Systems, Measurements, and Control, June 1985, Vol. 107, pp. 151-158.

184. Lee, L.T. and T.K. Hasselman, "Dynamic Model Verification of Large Structural Systems," Paper No. 781047 presented at the SAE Aerospace Meeting, San Diego, CA, Nov. 27-30, 1978.

185. Chrostowski, J.D., Evensen, D.A., and Hasselman, T.K., "Model Verification of Mixed Dynamic Systems," ASME Journal of Mechanical Design, Vol. 100, April 1978.

186. Hasselman, T.K. and J.D. Chrostowski, "Parameter Estimation For Large Space Structures," Proceedings of the Session Sponsored by the ST Div./ASCE/Spring Convention, Denver, CO, May 1, 1985.

187. Martinez, D.R., "Estimation Theory Applied to Improving Dynamic Structural Models," Sandia Report No. SAND82-0572, Sandia National Laboratories, Albuquerque, NM, Nov. 1984.

188. Bellman R. E., R. E. Kalaba, and J. A. Lockett, **Numerical Inversion of the Laplace Transform**, American Elsevier, 1967.

189. Ho, B. L., "On Effective Construction of Realizations from Input-Output Descriptions", Ph. D. Dissertation, Stanford U., 1966.

190. Kashyap, R. L., "Maximum Likelihood Identification of Stochastic Linear Systems", IEEE Trans. Auto. Cont., Vol. AC-15, pp. 25- 34, 1970.

191. Talmon, J. L., and A. J. W. van den Boom,"On the Estimation of Transfer Function Parameters of Process and Noise Using a Single-Stage Estimator", Proc. 3 IFAC Symp. Identification and System Parameter Estimation, Hague, Holland, 1973.

192. Friedlander, B., "Lattice Methods for Spectral Estimation", Proc. IEEE, Vol. 70, pp. 990-1017, 1982.

193. Crittenden, J. L., R. J. Mulholland, J. Hill IV, and E. F. Martinez,"Sampling and Data Analysis Properties of Prony's Method Applied to Model Identification", Int. J. System Sci., Vol. 145, pp. 571- 581, 1983.

194. Soderstrom, T., and P. G. Stoica, **Instrumental Variable Methods for System Identification**, Springer-Verlag, 1983.

195. Rowe, I. H., and I. M. Kerr, "A Broad-Spectrum Pseudorandom Gaussian Noise Generator", IEEE Trans. Auto. Cont., Vol. AC- 15, pp. 529-535, 1970.

196. Shah, P. C., and F. E. Udwadia, "A Methodology for Optimal Sensor Locations for Identification of Dynamic Systems", Trans. ASME, Vol. 45, pp. 188-195, 1977.

197. Fogel, E., "Preliminary Flight Control Avionic Requirements for Orbiter-Attached Large Structures", Charles Draper Laboratories, Inc., Cambridge, Ma., Rpt. CSDL-R-1531, 1982.

198. Private Communication, NASA, Langley Research Center Contract to University of Virginia.

199. Private Communication, Sandia Laboratories, Albuquerque, New, Mexico

200. Voelcker, J. (Assoc. Editor), "Microprocessors", IEEE Spectrum, Vol. 23, pp. 46-48, January 1986.

201. Adams, J. A. (Assoc. Editor), "Aerospace and Military", IEEE Spectrum, Vol. 23, pp. 76-81, 1986.

202. Nichols, R. G., "Space Station: Government and Industry Launch Joint Venture, High Technology, Vol. 5, No. 4, April 1985, pp. 18-23.

203. Horta, L. G. and Juang, J.-N., "Identifying Approximate Linear Models for Simple Nonlinear Systems," AIAA Paper No. 85-0686-CP, To appear in the Journal of Guidance, Control and Dynamics.

204. Noble, B. and J. W. Daniel, **Applied Linear Algebra**, Prentice-Hall, Inc., 1977.

205. Nurre, G. S., R. S. Ryan, H. N. Scofield, and J. L. Sims, "Dynamics and Control of Large Space Structures", J. Guid. and Cont., Vol. 7, pp. 514-526, 1983.

206. TREETOPS was developed by the Honeywell Research Laboratories under contract to NASA Marshall Flight Center, Huntsville, Al.

207. Ruzicka, J. E., (Editor), **Structural Damping**, The American Society of Mechanical Engineers, New York, New York, 1959.

BIBLIOGRAPHY.

PARAMETER ESTIMATION AND IDENTIFICATION OF SYSTEMS.

Ackerman, J. E., and R. S. Bucy, "Canonical Minimal Realization of a Matrix of Impulse Response Sequences", J. Inform. and Cont., Vol. 19, pp. 224-231, 1971.

Ackerman, J. E., "On Partial Realizations", IEEE Trans. Auto. Cont., Vol. AC-17, Pp. 625-627, 1972.

Ahmed, M. S., "Structure Determination and Parameter Estimation of Multivariable Systems by Instrumental Variables", 6th Proc. Symp. Identification and System Parameter Estimation, Vol. 2, Pp. 1079- 1084, 1983.

Akaike, H., "Modern Development of Statistical Methods", in Trends and Progress in System Identification, Edited by P. Eykhoff, Pergamon Press, 1981.

Akiake, H., "A New Look at the Statistical Model Identification", IEEE Trans. Auto. Cont. AC-19, Pp. 716-723, 1974.

Akiake, H., "Information Theory and An Extension of the Maximum likelihood Principle", Proc. 2nd Int. Symp. Information Theory, Supp. to Problems of Control and Inform. Theory, Pp. 267-281, 1972.

Akiake, H., "On a Decision Procedure for System Identification", Proc. IFAC Symp. System Eng. Approach Computer Control, Kyoto, Japan, paper 30.1, 1970.

Akiake, H., "Some Problems in the Application of the Cross-spectral Method", in Spectral Analysis of Time Series, Edited by B. Harris, John Wiley and Sons, New York, Pp. 81-107, 1967.

Akiake, H., "Stochastic Theory of Minimal Realization", IEEE Trans. Auto. Cont., Vol. AC-17, Pp. 667-674, 1972.

Alam, M. A., "Orthonormal Lattice Filters: a Multi-Stage Multi- channel Estimation Technique", Geophysics, Vol. 43, No. 7,pp. 1368-1383, Dec. 1983.

Alam, M. A., "Adaptive Spectral Estimation", Proc. 1977 Joint Auto. Cont. Cont., San Francisco, Ca. Vol. 1, Pp. 105-112.

Alberston, H. D., and B. F. Womack, "An Algorithm for Construction of Irreducible Realization of Time-invariant Systems", J. Franklin Inst., Vol. 285, Pp. 110-124, 1968.

Allemang, R. J., "Experimental Modal Analysis Bibliography", Proc. IMAC2, Feb. 1984.

Anderson, B. D. O., "An Algebraic Solution to the Spectral Factorization Problem". IEEE Trans. Auto. Cont. AC-12, Pp. 410-414, 1967.

Anderson, B. M., F. M. Barash and P. V. Lopresti, "The Sequential Construction of Minimal Partial Realizations From Finite Input- Output Data", SIAM J. Cont., Vol. 13, Pp. 552-570, 1975.

Anderson, J. E., "Another Look at Sine-Dwell Mode Testing", J. of Guidance and Control, Vol. 5, No. 4, Pp. 358-365, 1982.

Anderson, J. H., "Geometrical Approach to Reduction of Dynamical Systems", Proc. IEE, Vol. 114, Pp. 1014-1018, 1967.

Aplevich, J. D., "Approximation of Discrete Linear Systems", Int. J. Cont., Vol. 17, Pp. 565-575, 1973.

Arimoto, S. and H. Kimura, Optimal Input Test Signals for System Identification-an Information Theoretic Approach", Int. Jour. Syst. Sci., Vol. 1, Pp. 279-290, 1971.

Arthurs, T. D., "Structural Dynamics", Aeronautics and Astronautics, Vol. 17, Pp. 95-97, 1979.

Ashimov, A. A. and D. Zh. Sydikov, "Identification of High Dimensional Systems by the General Parameter Method", Proc. of the Eighth Triennial World Cong., Pergamon Press, Kyoto, Japan, Vol. 1, Pp. 637-642, 1981.

Astrom, K. J. and B. Wittenmark, "Problems of Identification and Control", J. of Math. Anl. and Appl., Vol. 34, Pp. 90-113, 1971.

Astrom, K. J. and P. Eykhoff, "System Identification-A Survey", Automatica, Vol. 7, Pp. 123-162, 1971.

Astrom, K. J., "A New Algorithm for Recursive Estimation of Controlled ARMA Processes", Pro. 6th Symp. Identification and System Parameter Estimation, Vol. 1, Pp. 175-179, 1983.

Astrom, K. J., "Lectures On the Identification Problem-The Least Squares Method", Report 6806, Div. of Auto. Cont., Lund Inst. of Tech., Lund, Sweden, 1968.

Astrom, K. J., "On the Achieveable Accuracy in Identification Problems", 1st IFAC Symp. On Identification in Auto. Cont. Systems, Prague, Czech., 1967.

Astrom, K. J., and T. Bohlin, "Numerical Identification of Linear Dynamic Systems From Normal Operating Records", Theory of Self Adaptive Cont. Syst., Edited by P. Hammond, Plenum Press, N. Y., 1965.

Audley, D. R., and W. J. Rugh, "On the H-Matrix System Representaion", IEEE Trans. Auto. Cont., Vol. AC-18, Pp. 235-242, 1973.

Audley, D. R., S. L. Baumgartne and H. J. Rugh, "Linear System Realization Based On Data Set Representaion", IEEE Trans. Auto. Cont., Vol. AC-20, Pp. 432-433, 1975.

Balakrishnan, A. V., "Identification and Adaptive Control: An Application to Flight Control Systems", J. of Optimization Theory and Applications, Vol. 9, Pp. 187-213, 1972.

Balas, M. J. and J. H. Lilly, "Adaptive Parameter Estimation of Large Scale Systems by Reduced Ordering Modeling", Proc. 20th Conf. Decision and Controls, Symp. On Adaptive Proc., Pp. 233-239, Dec. 1981.

Banks, H. T., "Algorithm for Estimation in Distributed Models With Application to Large Space Structures", Workshop On Applications of Distributed System Theory to the Control of Large Space Structures, JPL Publication 83-46, July 1983.

Banks, H. T., P. L. Daniels and E. S. Armstrong, "A Spline Based Parameter and State Estimation Technique for Static Models of Elastic Surfaces", 4th VPI&SU AIAA Symp. On Dynamics of Large Struct., Blacksburg, Va., 1983.

Banon, G. and J. Aguilar-Martin, "Estimation Lineaire Recurrente de Parametres des Processus Dynamiques Soumis a Des Perturbation Aleatoires", Revue Du Cethes, Vol. 9, Pp. 39-86, 1972.

Baram, Y, and N. R. Sandell, Jr., "An Information Theoretic Approach to Dynamical System Modeling and Identification", IEEE Trans. Auto. Cont., Vol. AC-23, Pp. 61-66, 1978.

Baram, Y., "Information, Consistent Estimation and Dynamic System Identification",

Report ESL-R-718, Electronics Systems Lab., Dept. of Elec. Eng., MIT, Cambridge, Mass. 1976.

Bard, Y. **Nonlinear Parameter Estimation**, Academic Press, 1974.

Barraud, A., and P. DeLarminat, "Minimal Realization and Approximation of Linear Systems From Normal Operating Records", IFAC Symp. Identification and System Parameter Estimation, 1973.

Bar-Shalom, Y., "Optimal Simultaneous State Estimation and Parametric Identification in Linear Discrete-Time Systems", IEEE Trans. Auto. Cont. Vol. AC-17, Pp. 308-319, 1972.

Baruch, M., "Proportional Optimal Orthogonalisation of Measured Modes", AIAA Jour., Vol. 18, 1980.

Baruh, H., "Parameter Estimation and Control of Distributed Parameter Systems", Ph. D. Disseration, VPI&SU, Aug. 1981.

Bauer, R. D. and F. C. Tung, "Application of Maximum Likelihood Estimation of Modal Parameters", AIAA-AAS Astrodynamics Conference, Seattle, Wash., August 1984.

Beauchamp, K. G., **Walsh Functions and Their Applications**, Academic Press, 1975.

Bekey, G. A. and G. Sardis, (Ed.), "Identification and System Parameter Estimation", Proc. of 6th IFAC Symp., Washington, D. C., Pergamon Press, 1982.

Bekey, G., "System Identification: An Introduction and a Survey", Simulation, Vol. 15, Pp. 151-166, 1970.

Bellman, R. E. and K. J. Astrom, "On Structural Indentifiability", Math. Biosci., Vol. 7, Pp. 329-339, 1970.

Bellman, R. E., R. E. Kalaba and J. A. Lockett, **Numerical Inversion of the Laplace Transform**, American Elsevier, 1966.

Belorusets, V. B., "Methods of Auxiliary Systems for the Identification of Dynamic Systems with Unknown Inputs", Avtomatika I Telemekhanika, Pp. 76-82, 1981, Russian. (Automation and Remote Control, Vol. 42, Pp. 1056-1061, 1981, Translation)

Bendat, , J. S., and A. G. Piersol, **Measurement and Analysis of Random Data**, John Wiley and Son, 1966.

Bendat, J. S., and A. G. Piersol, **Random Data: Analysis and Measurement Procedures**, Wiley-Interscience, 1975.

Benhabib R. J., and F. C. Tung, "Large Space Structures Control: System Identification Versus Direct Adaptive Control", JACC, 1980.

Benhabib, R. J. and F. C. Tung, "Large Space Structure Control System Identification Versus Direct Adaptive Control", Proc. JACC, Vol. 2, San Francisco, Ca., 1980.

Berman, A. and W. G. Flannely, "Theory of Incomplete Models of Dynamic Structure", AIAA Journal, Vol. 9, Pp. 1481-1487, 1971.

Berman, A, "Mass Matrix Correction Using An Incomplete Set of Measured Modes", AIAA Journal, Vol. 17, Pp. 1147-1148, 1979.

Berman, A. and E. J. Nagy, "Improvement of Large Analytical Model Using Test Data", AIAA Journal, Vol. 21, Pp. 1168-1173, 1983.

Berman, A., and F. S. Wei, "Automated Dynamic Analytical Model Improvement", NASA Contract Report 3452, 1981.

Beshelli, S. and R. P. Guidorzi. "Transformation Between Input- output Multistructural Models: Properties and Application", Int. J. Cont., Vol. 37, Pp. 1385-1400, 1983.

Best, J. F., "Parameter Identification of Linear Systems Via the Maximum Likelihood Method", Tech. Paper TR-78-2, Dept. of Elec, Eng. and Comp. Sci., U. of Conn., 1978.

Bierman, G. J., **Factorization Methods for Discrete Sequential Estimation**, Academic Press, 1977.

Bierman, G. J., "Summary of Numerical and Computational Aspects of the Parameter and State Estimation Problem", 6th Proc. Symp. Identification and System Parameter Estimation, Vol. 1, Pp. 93-95, 1983.

Bishop, R. E. D. and J. W. Pendered, "Note On Resonance Testing", Jour. of Mech. Eng. Sci., Vol. 5, Pp. 343-344, 1963.

Bishop, R. E. D., and G. M. L. Gladwell, "An Investigation Into the Theory of Resonance Testing", Proc. Roy. Soc. Phil. Trans., Vol. 255, 1963.

Bodnya, V. G., A. A. Rurua and V. M. Chadeev, "Two-Step Identification Algorithm for Linear Plants", Avtomatika I Telemekhanika, Russian, (Translation; Automation and Remote Control, Vol. 42, Pp. 1041- 1047, 1982).

Bohlin, T., "Using Statistical Identification Methods", Technical Paper TP-18.206, IBM Nordic Lab., Lidingo, Norway.

Bohlin, T., "Four Cases of Identification of Changing Systems", in System Identification-

Advances and Case Studies, Edited by R. Mehra and D. G. Lainiotis, Academic Press, N.Y.

Bohlin, T. "On the Problem of Ambiguites in Maximum Likelihood Identification", Automatica, Vol. 7, Pp. 199-210, 1971.

Bokor, J., and J. Peto, "Minimal Realization Procedure From Partial Fraction Representation of a Rational Transfer Matrix", Problems of Control and Information Theory, Vol. 10, 1981.

Bonivento, C., and R. Guidorzi, "Canonical Input-output Description of Linear Multivariable Systems", Ricerche Di Automatica, Vol. 2, Pp. 72-83, 1971.

Bonivento, C., R. Guidori and C. Marro, "Irreducible Canonical Realization From External Data Sequences", Int. J. Cont., Vol. 18, Pp. 553-563, 1973.

Bosgra, O. H., and A. J. J. VanderWeiden, "Input-output Invariants for Linear Multivariable Systems", IEEE Trans. Auto. Cont., Vol. AC-25, Pp. 20-36, 1980.

Bosgra, O. H., "Realization and the Pole/zero Structure at Infinity of Non-proper and Interconnected Linear Multivariable Systems", JACC , 1981.

Boudon, P., H. Dan Van Mien, M. Fliess and D. Normand-Cyrot, "Application to Nuclear Power Plants of Nonlinear Identification and Realization Techniques", Proc. 6th IFAC Symp. On Identification and System Parameter Estimation, Washington D. C., 1982.

Box, G. E. P. and G. M. Jenkins, **Time Series Analysis: Forecasting and Control**, Holden-Day, San Francisco, Ca., 1970.

Bretthauer, G. "New Results for the Joint Input-Output Identification Method", Proc.

Eighth Triennial World Cong., Pergamon Press, Kyoto, Japan, Vol. 1, Pp. 693-697, 1981.

Brewer, M. J., "Estimation of the State Vector of a Dynamic System Using Limited Memory Least Squares", M. Sc. Thesis, Dept. Computing Cont., Imperial College, London, 1971.

Briggs, F. A. N., P. H. Hammond, M. T. G. Hughes and G. O. Plumb, "Correlation Analysis of Process Dynamics Using Pseudo-random binary Test Perturbations", Proc. Inst. Mech. Eng., Vol. 179, Part 3H, pp. 37- 51, 1965.

Brigham, E. O., **Fast Fourier Transforms**, Prentice-Hall, 1974.

Brockett, R. W., "Finite and Infinite Dimensional Bilinear Realizations", J. Franklin Inst., Vol. 301, Pp. 509-526, 1976.

Brown, D. L., R. J. Allemang, R. Zimmerman, and M. Mergeay, "Parameter Estimation Technique for Modal Analysis", SAE Congress and Exposition, Detroit, Mich., 1979.

Brown, R. F., "Role of Identifiability in Validation Models", Math. Comput. Simul., Vol. 24, Pp. 483-486, 1982.

Bruni, C., A. Isidori and A. Ruberti, "A Method of Realization Based On the Moments of the Impulse-response Matrix", IEEE Trans. Auto. Cont., P 203, 1969.

Budin, M. A., "Minimal Realization of Discrete Linear Systems From Input-output Observations", IEEE Trans. Auto. Cont., Vol. AC-16, Pp. 395-401, 1971.

Budin, M. A., "Minimal Realization of Continous Linear Systems From Input-output Observations", IEEE Trans. Auto. Cont., Vol. AC-17, Pp. 252-253, 1972.

Budin, M. A., "Canonical Forms for Multi-input/multi-output Minimal Systems", IEEE Trans. Auto. Cont., Vol. AC-17, P 554, 1972.

Burke, M. E. and Z. W. Rekasius, "On Optimal and Suboptimal Input Sequences for the Identification of Linear Systems", 1980 Joint Auto. Control Conf., San Francisco, Ca.

Caines, P. E., "Minimal Realization of Transfer Function Matrices", Int. J. Cont., Vol. 13, Pp. 529-547, 1971.

Caines, P. E., "Stationary Linear and Nonlinear System Identification and Predictor Set Completeness", IEEE Trans. Auto. Cont., pp. 583-594, 1978.

Caines, P. E. and C. W. Chan, "Estimation, Identification and Feedback", **System Identification: Advances and Case Studies**, Edited by R. Mehra and D. G. Lainiotis, Academic Press, N. Y., Pp. 349-405, 1976.

Caines, P. E., "The Parameter Estimation of State Variable Models and Multivartiable Linear Systems", Proc. UKAC Conf. On Multivariable Systems, Manchaster, England, 1971.

Caines, P. E., and S. Lafortune, "Adaptive Control with Recursive Identification for Stochastic Linear Systems- Univariate Case", 6th Proc. Symp. Identification and System Parameter Estimation , Vol. 1, pp. 653-657, 1983.

Candy, J. V., "Realization of Invariant System Descriptions From Markov Sequences", Ph. D. Disseration, Univ. of Florida, 1976.

Candy, J. V., M. E. Warren and T. E. Bullock, "Realization of Invariant System Descriptions From Infinite Markov Sequences", IEEE Trans. Auto. Cont., Vol. AC-23 , Pp. 93-96, 1978.

Caravani, P, M. L. Watson and W. T. Thomson, "Recursive Least Squares Time Domain Identification of Structural Parameters", Jour. Appl. Mech., Vol. 44, Pp. 135-140, 1977.

Carmichael, N. and M. D. Quinn, "Joint State and Parameter Estimation", Workshop On Application of Distributed System Theory to the Control of Large Space Structures, JPL Publication 83-46, July 1983.

Carne, T. G. Amd S. R. Ibrahim, "Modal Identification of a Rotating Blade System", AIAA Paper 83-0815.

Chan, Y. T., "Parameter Estimation of Linear Multivariable Plants Using the Instrumental Variable Approach", Proc. 3rd IFAC Symp. Identification and System Parameter Estimation, the Netherlands, Paper PV-6, 1973.

Chen, C. T., "Comments On "Realization of Invariant System Descriptions From Infinite Markov Sequences"", IEEE Trans. Auto. Cont., Vol. AC-25 , Pp. 1013-1014, 1980.

Chen, C. T., "Identification of Linear Time-Invariant Multivariable Systems", 6th Proc. Symp. Identification and System Parameter Estimation, Vol. 2, Pp. 1039-1042, 1983.

Chen, H. F., "Strong Consistency in System Identification Under Correlated Noise", 6th Proc. Symp. Identification and System Parameter Estimation, Vol. 2, Pp. 1091-1096, 1983.

Chen, J. C. and B. K. Wada, "Matrix Perturbations for Structural Dynamic Analysis", AIAA Journal, Vol. 15, Pp. 1095-1100, 1977.

Chen, J. C., C. P. Kuo and J. A. Garba, "Direct Structural Parameter Identification by Modal Test Results", 24th Structures, Structural Dynamics, and Material Conf., AIAA/ASME/ASCE/AAS, Proc. Part 2, May, 1983.

Chen, J. C., and D. P. Mittal, "A Simplified Irreducible Realization Algorithm", IEEE Trans. Auto. Cont., Vol. AC-17, Pp. 535-537, 1972.

Chintapalli, P., D. E. Seborg and D. G. Fisher, "Modal Identification of Linear State Space Models", 6th IFAC Triennial World Congress, Boston, Ma., 1975.

Chow, J. H. and P. V. Kokotovic, "A Decomposition of Near- Optimum Regulators for Systems with Slow and Fast Modes", IEEE Trans. Auto. Cont., Vol. AC-21, Pp. 701-705, 1976.

Christiansen, R. G. and W. P. Wallace, "An Experimental Technique for Determining Vibration Modes of Structures with Quasi- Stationary Random Forcing Functions", Shock and Vibration Jour., Bulletin 42, Part 4, Jan. 1972.

Christopher, P. A. T., Y. N. Cheng, K. F. Man and E. W. Osbourn, "A Practical Method for Identification of Dynamic Processes", IFAC Workshop On Application of Nonlinear Programming to Optimization and Control, San Francisco, Ca. 1983

Chu, D. F. H., (Editor), "Modal Testing and Model Refinement", Proc. Symp., Boston, Ma., ASME , Applied Mechanics Symposia Series, AMD Volume 59, 1983.

Ciancy, S. J., and W. J. Rugh, "On the Realization Problem for Stationary Homogeneous Discrete-Time Systems", Automatica, 1978.

Clarke, D. W., "Generalized Least-Squares Estimation of the Parameters of a Dynamic Model", Proc. IFAC Symp. Identification Auto. Cont. Systems, Paper 3.17, 1967.

Cobelli, A., A. Lepschy and G. Romanin-Jacur, "Comments On "On the Relationship Between Structural Identifiability and the Controllability, Observability Properties"", Trans. IEEE Auto. Cont., Vol. AC-23, Pp. 965- 966, 1978.

Collins, J. D. and G. C. Hart, "Statistical Identification of Structures", AIAA Journal, Vol. 12, Pp. 185-190, 1974.

Collins, J. D., J. P. Young and L. Keifling, "Methods and Application of System Identification in Shock and Vibration", in Identification of Vibrating Structures, Edited by W. Pilkey and R. Cohen, ASME, Pp. 42-71, 1972.

Cooley, J. W., and J. W. Tukey, "An Algorithm for the Machine Calculation of Complex Fourier Series", Math. of Comp., Vol. 19, Pp. 297- 301, 1965.

Copley, J. C., "Numerical Analysis of Vector Responses", Roy. Aircraft Est. Tech Report 80135, 1980.

Correa, G. D., and K. Glover, "Multivariable Idntification Using Pseudo-canonical Forms", Proc. 6th Symp. Identification and System Parameter Estimation, Vol. 2, Pp. 1241-1245, 1983.

Crittenden, J. L., R. J. Mulholland, J. Hill IV, and E. F. Martinez, "Sampling and Data Analysis Properties of Prony's Method Applied to Model Identification", Int. J. System Sci., Vol. 14, Pp. 571-581, 1983.

Curry, R. **Estimation and Control with Quantized Measurements**, MIT Press, 1970.

D'Alessandro, P., A. Isidori, and A. Ruberti, "Realization and Structure Theory of Bilinear Systems", SIAM J. Cont., Vol. 12, Pp. 517-535, 1974.

Damen, A. A. H., and A. K. Hajdasinski, "Practical Tests with Different Approximate Realizations Based On the Singular Value Decomposition of the Hankel Matrix", Proc. 6th IFAC Symp. On Identification and Parameter Estimation , Washington, D. C., 1982.

Davies, W. D. T., **System Identification for Self-Adaptive Control**, Wiley-Interscience, London, 1970.

Davies, P. and J. K. Hammond, "A Comparsion of Fourier and Parametric Methods for Structural System Identification", ASME Trans., J. Vibrations, Acoustics, Stress, and Reliability in Design, Vol. 106, Pp. 40- 48, 1984.

Davison, E. J., "A Method for Simplifying Linear Dynamic Systems", IEEE Trans. Auto. Cont., Vol. 11, Pp. 93-101, 1966.

Davison, E. J., "A New Method for Simplifying Large Linear Dynamic Systems", IEEE Trans. Auto. Cont., Vol. AC-13, Pp. 214-215, 1968.

Davison, E. J., W. Gesing, and S. H. Wang, "An Algorithm for Obtaining the Minimal Realization of a Linear Time-invariant System and Determining If a System Is Stabilizable-detectable", IEEE Trans. Auto. Cont., Vol. AC- , Pp. , 1978.

De La Sen, M., "Two New Algorithms for Parameter Estimation in Adaptive Multivariable Systems", Proc. 6th Symp. Identification and System Parameter Estimation", Vol. 1, Pp. 457-462, 1983.

Delosme, J. M., "Algorithms and Implementation for Linear Least Squares Estimation", Ph. D. Dissertation, Stanford Univ,. Stanford, Ca., 1982.

Denham, M. J., "Canonical Forms for the Identification of Multivariable Linear Systems", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 646-655, 1974.

Denham, M. J., "Recent Results in Realisation Theory of Systems With Discrete-Time Bilinear and Multilinear Response Maps", Proc. IEE, 1981.

Denman, E. D. and J. Leyva-Ramos, "Algorithm for Identification and Analysis of Large Space Structures", J. of Guidance and Cont., J. of Guidance and Control, Vol. 5, No. 4, Pp. 414-416, 1982.

Dobson, B. J., "Modification of Finite Element Models Using Experimental Modal Analysis", Proc. IMAC2, Feb. 1984.

Dejong, L. S., "Numerical Aspects of Realization Algorithms in Linear System Theory", Ph. D. Disseration, Univ. Eindhoven, Holland, 1975.

Dejong, L. S., "Numerical Aspects of Recursive Realization Algorithms", SIAM J. Guid. and Opt., 1978.

Desoer, C. A., and P. Variya, "The Minimal Realization of a Nonanticipative Impulse Response Matrix", SIAM Jour. Appl. Math., Vol. 15, Pp. 754-764., 1967.

Deutsch, R. Estimation Theory, Prentice-Hall, 1965.

Dewey, A. G., "A Computational Procedure for the Minimal Realisation of Transfer Function Matrices", Centre for Computing and Automation, Imperial College of Science and Technology, 1967.

Dickenson, B. W., M. Morf, and T. Kailath, "A Minimal Realization Algorithm for Matrix Sequences", IEEE Trans. Auto. Cont., Vol. AC-19 , Pp. 31-38, 1974.

Dickenson, B. W., "Uniqueness of Minimal Partial Realization", IEEE Trans. Auto. Cont., Vol. AC-20 , Pp. 271 , 1975.

DiPillo, G., L. Grippo and Lucertini, Proc. 3rd IFAC Symp. Identification and System Parameter Estimation, the Netherlands, 1973.

Distefano III, J. J. "On Tbe Relationship Between Structural Identifiability and the Controllability, Observabiltiy Properties", IEEE Trans. Auto. Cont., Vol. AC-22, Pp. 652, 1977.

Dodds, M. W., "Parameter Identification of Large Structure Models: Concepts and Reality", in Proc. Symp. Modal Testing and Model Refinement, Boston, Ma., Pp. 77-84, 1983.

Dunyak, J. P., J. L. Junkins and L. T. Watson, "Robust Nonlinear Least Squares Estimation Using the Chow-Yorke Homotopy Method", (Submitted to Jour. of Guidance, Control and Dynamics)

Durbin, J., "The Fitting of Time Series Models", Rev. L'Institut Int. De Statique, Vol. 28, Pp. 233-243, 1960.

Dwolatzky, B., "A General Solution to the System Identification Problem Using Intermediate Domain Processing", Int. J. of Cont., Vol. 36, pp. 493-510, 1982.

Emre, E., L. M. Silverman, and K. Glover "Generalized Dynamic Covers for Linear Systems with Application to Deterministic Identification and Realization Problems", IEEE Trans. Auto. Cont. Vol. AC-22 , Pp. 26-35, 1977.

Ewins, D. J., and P. T. Gleeson, "A Method for Modal Identification of Lightly Damped Structures", J. Sound and Vibrations, Vol. 84, Pp. 57-79, 1982.

Ewins, D. J., **Modal Testing,** Research Studies Press Ltd., John Wiley and Sons, 1984.

Ewins, D. J., and J. Griffin, "A State of the Art Assessment of Mobility Measurement Techniques - Results for the Mid-range Structures", J. Sound and Vibration, Pp. 197-222, 1981.

Ewins, D. J., and J. Sidhu, "Correlation of Finite Elements and Modal Test Studies of a Practical Structure", Proc. IMAC2, Feb. 1984.

Ewins, D. J., J. M. M. Silva, and G. Maleci, "Vibration Analysis of a Helicopter with An Externally-attached Carrier Structure", Shock and Vibration Bullletin, Bulletin 50, Pp. 155- 171, 1980.

Eykhoff, P., (Editor) **Trends and Progress in System Identification**, Pergamon Press, 1981.

Eykhoff, P., **System Identification**, John Wiley and Sons, 1974.

Fevour, J. D. and J. M. Lebrun, "Transient Synthesis for Mechanical Testing", Instruments and Control Systems, Vol. 43, Pp. 125-127, 1970.

Fevour, J. D., M. C. Mitchell and N. L. Olson, "Transient Test Techniques for Mechanical Impedance and Modal Survey Testing", Shock and Vibration Bulletin, Bulletin 42, Part 1, Pp. 71-82, 1972.

Finigan, B., "Strongly Consistent Parameter Estimation Using the Instrumental Variable Method", Ph. D. Thesis, U. of Toronto, Canada, 1975.

Finigan, B. M., and I. H. Rowe, "Strongly Consistent Parameter Estimation by the Introduction of Strong Instrumental Variables", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 825-830, 1974.

Fisher, E. E., "Identification of Linear Systems", Preprint JACC, Troy, N. Y., 1965.

Fisher, F. **The Identification Problem in Econometrics**, McGraw- Hill, 1966.

Flannely, W. G. and A. Berman, "The State of the Art of System Identification of Vibrating Structures", in System Identification of Vibrating Structures, Edited by W. Pilkey and R. Cohen, ASME, Pp. 121-131, 1972.

Fogel, E., "Reduced Order Modeling of Large Space Structures Via Least Squares Estimation", Proc. 20th Conf. On Dec. and Cont., and Symp. On Adaptive Processes, Pp. 943-948, 1981.

Fox, L. and I. B. Parker, **Chebyshev Polynomials in Numerical Analysis**, Oxford Press, 1968.

Friedlander, B., "Lattice Implementation of Some Recursive Parameter-estimation Algorithms", Int. J. Cont., Vol. 37, Pp. 661-684

Friedlander, B., "A Modified Pre-filter for Some Recursive Parameter Estimation Algorithms, IEEE Trans. Auto. Cont., Vol. AC-27, Pp. 232-235, 1982.

Friedlander, B., "System Identification Techniques for Adaptive Noise Cancelling", Trans. IEEE Acoustics, Speech and Signal Processing, Vol. ASSP-30, Pp. 699-708, 1982.

Friedlander, B., L. Ljung and M. Morf, "Lattice Implementation of the Recursive Maximum Likelihood Algorithm", Proc. 20th IEEE Conf. Dec. and Cont., Pp. 1083-1084, 1981.

Friedlander, B., T. Kailath, M. Morf and L. Ljung, "Extended Levinson and Chandrasekhar Equations for General Discrete-Time Linear Estimation Problems", Trans. IEEE Auto. Cont., Vol. AC-23, Pp. 653-659, 1978.

Froisy, J. B., C. L. Smith, A. B. Corripie and P. W. Morrilli, "Closed-Loop Identification of System Dynamics in the Presence of Noise and Unmeasured Disturbances", Proc. 1974

JACC Conf., Austin, Tx., 1974.

Fuhrt, B. P., "New Estimator for the Identification of Dynamic Processes", IBK Report, Institut Boris Kidric Vinca, Belgrade, Yugoslavia.

Gagliardt, R. M., "Input Selection for Parameter Identification in Discrete Systems", IEEE Trans. Auto. Cont., Vol. AC-12, Pp. 597-599, 1967.

Gaukroger, D. R., C. W. Skingle, and K. H. Heron, "Numerical Analysis of Vector Response Loci", J. Sound and Vibration, Vol. 29, Pp. 341-353, 1973.

Gauthier, A. and I. D. Landau, "On the Recursive Identification of Multi-input, Multi-output Systems", Automatica, Vol. 14, Pp. 609-614, 1978.

Gerardys, R., "Probabilistic Finite State System Identification", Int. J. Gen. Sys., Vol. 8, Pp. 229-242, 1982.

Gerth, W., "On the Construction of Low Order Linear State- variable Models From Measured Data", Proc. 2nd Symp. Multivariable Cont. Syst., Vol. 1, Paper 1.2.1, 1971.

Gertler, J. and Cs. Banyasz, "A Recursive (On-Line) Maximum Likelihood Identification Method", IEEE Trans. Auto. Cont., Vol AC-19, Pp. 816-820, 1974.G

Ghlaim, K. H., and K. F. Martin, "Reduced Component Modes in a Damped System", Proc. IMAC2, Feb. 1984.

Gilbert, E. G., "Controllability and Observability in Multivariable Control Systems", SIAM J. Cont., Vol. 1, Pp. 128-151, 1963.

Godbole, S. S., "Comments On "Construction of Minimal Linear State-variable Models

From Finite Input-output Data"", IEEE Trans. Auto. Cont., Vol. AC-17, Pp. 173-175, 1972.,

Godfrey, K. R., "The Theory of the Correlation Method of Dynamic Analysis and Its Application to Industrial Processes and Nuclear Power Plant", Measurement and Control, Vol. 2, Pp. T65-T72, 1969.

Gopinath, B., "On the Identification and Control of Linear Systems", Ph. D. Disseration, Stanford U., 1968.

Glover, K. and J. C. Willems, "Parameterization of Linear Dynamical Systems: Canonical Forms and Identifiability", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 640-646, 1974.

Glover, K. and J. C. Willems, "On the Identifiability of Linear Dynamic Systems", Proc. 3rd IFAC Symp. Identification and System Parameter Estimation, the Netherlands, 1973.

Glover, K., "Structural Aspects of System Identification", Report ESL-R-516, Electronic System Lab., Decision and Control Sciences Group, Dept. of Elec. Eng., MIT, Cambridge, Mass., 1973.

Glover, K., "An Approximate Realization Algorithm Which Directly Identifies the System Poles", IFAC, 1978.

Goodrich, G. C. and P. E. Caines, "Time Invariant Linear Systems Identification From Nonstationary Cross-Sectional Data", IEEE Trans. Auto. Cont., Vol. AC-24, Pp. 403-411, 1979.

Goodwin, G. C. and R. L. Payne, **Dynamic System Identification: Experimental Design and Data Analysis**, Academic Press, 1977.

Goodwin, G. C., R. L. Payne and P. V. Kabaila, "On Canonical Forms and Algorithms for Multivariable Time Series Analysis", Proc. 4th IFAC Sym. Ident. and Sys. Param. Estimation, Tbilisi USSR, 1976.

Goodwin, G. C., M. B. Zarrop and R. L. Payne, "Coupled Design of Test Signal, Sampling Interval and Filters for System Identification", IEEE Trans. Auto. Cont., Vol. AC-19,pp 748-753, 1974.

Goodwin, G. C., "Application of Curvature Methods to Parameter and State Estimation", Proc. IEE, Vol. 116, Pp. 1107-1110, 1969.

Gopinath, B. "On the Identification of Linear Time-invariant Systems From the Input-output Data", Bell Tech. J., Vol. 48, 1969.

Goyder, H. G. D., "Methods and Application of Structural Modelling From Measured Structural Frequency Response Data", J. Sound and Vibration, Vol. 68, Pp. 209-230, 1980.

Graupe, E., **Identification of Systems**, 2nd Edition, Krieger, New York, 1976.

Gravitz, S. I., "An Analytical Procedure for Orthogonalization of Experimentally Measured Mode", Jour. of Aerospace Sci., Vol. 25, Pp. 721-722, 1958.

Gray, A. H. Jr., and J. D. Markel, "Digital Lattice and Ladder Filter Synthesis", IEEE Trans. Audio and Electroacoustics, Vol. AU-21, Pp. 491-500, 1973

Grewal, M. S., and K. Glover, "Identifiability of Linear and Nonlinear Dynamical Systems", IEEE Trans. Auto. Cont., Vol. AC- 21, Pp. 833-837, 1976.

Guidorizi, R. P., M. P. Losito and T. Muratori, "The Range Error Test in the Structural

Identification of Linear Multivariable Systems", IEEE Trans. Auto. Cont., Vol AC-27, Pp. 1044-1054, 1982.

Guidorzi, R. P., "Canonical Structures in the Identification of Multivariable Structures", Automatica, Vol. 11, Pp. 361-374, 1975.

Gupta, N. K., M. G. Lyons, J. N. Aubrun and G. Margulies, "Modeling, Control, and System Identification for Flexible Structures", Spacecraft Pointing and Positioning Control, AGARD-AG 260, 1981.

Gupta, N. K., "Efficient Computation of Gradient and Hessian of Likelihood Function in Linear Dynamic Systems", IEEE Trans. Auto. Cont., Vol. AC-21, Pp. 781-783, 1976.

Gupta, N. K. and R. K. Mehra, "Computational Aspects of Maximum Likelihood Estimation and Reduction in Sensitivity Function Calculation", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 774-783, 1974.

Gupta, R. D. and F. W. Fairman, "An Explicit Approach to Minimal Realization From System Markov Parameters", Trans. IEEE Auto. Cont., Vol. AC-18, Pp. 52-54, 1973.

Gupta, R. D., and F. W. Fairman, "Derivation of the Ho Algorithm", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 270-271, 1974.

Gupta, R. D., and F. W. Fairman, "System Realization From Inexact Markov Parameters", IEEE Trans. Auto. Cont., Vol. AC-19 , Pp. 606-608, 1974.

Gvers, M. and V. Werts, "A Recursive Least Squares D-Step-Ahead Predictor in Lattice and Ladder Form", Trans. IEEE Auto. Cont., Vol. AC-28, 1983.

Hahagud, S., M. Meyyappa, Y. P. Cheng, and J. I. Craig, "Identification of Structural Dy-

namic Systems with Nonproportional Damping", Proc. 25th Conf. Structures, Structural Dynamics, and Materials, Palm Spring, Ca., Part 2, Pp. 283-291, 1984.

Hajdasinski, A. K., and A. A. H. Damen, "Realization of the Markov Parameter Sequence Using the Singular Value Decomposition of the Hankel Matrix", TH-Report 79-E-95, Eindhoven Univ. Techn., Dept. Elec. Eng., 1979.

Halvorsen, W. G., and D. L. Brown, "Impulse Technique for Structure Frequency Response Testing", J. Sound and Vibration, Vol. 8, 1977.

Hanagud, S.V., M. Meyappa and J. I. Craig, "Method of Multiple Scales and Identification of Nonlinear Structural Dynamical Systems", Proc. 24th Sructural Dynamics and Materials Conf., Lake Tahoe, Nev., Part 2, Pp. 50-57, 1983.

Hang, C. C. and E. L. Yap, "Unbiased Identification of State Space Structures", Proc. Eighth Triennail World Cong., Koyoto, Japan, Vol. 1, Pp. 649-654, 1981.

Hart, G. C., and J. T. P. Yao, "System Identification in Structural Mechanics", Jour. Aerospace Sci., 1977.

Hassan, M. F., "A Development of a Decomposed Estimation Algorithm for Large Scale Systems", Int. J. Sys. Sci., Vol. 14, Pp. 269-279, 1983.

Hasting-James, R. and M. W. Sage, "Recursive Generalized Least Squares Procedure for On-Line Identification of Process Parameters", Proc. IEE, Vol. 116, Pp. 2057-2062, 1969.

Hazewinkel, M, and J. C. Willems, **Stochastic Systems: the Mathematics of Filtering and Identification and Applications**, Reidel Publishing Co., 1981.

Hendricks, S. L., S. M. Hayes, and J. L. Junkins, "Structural Parameter Identification for

Flexible Spacecraft", AIAA 22th Aerospace Mtg., Reno, Nev., AIAA Paper 84-0060, 1984.

Hendricks, S. L., S. Rajaram, M. P. Kamat, and J. L. Junkins, "Identification of Large Flexible Structure Mass/Stiffness and Damping From On-orbit Experiments", J. Guid. Cont., and Dym., Vol. 7, Pp. 224-245, 1984.

Heylen, W. and F. Van Honacker, "An Automated Technique for Improving Modal Matrices by Means of Experimentally Obtained Dynamic Data", 24th Structures, Structural Dynamics and Materials Conf., AIAA/ASME/ASCE/AAS,Proc. Part 2, 1983.

Hicken, J., "Reduction of Linear Systems by Canonical Forms", Electronic Letters, 1976.

Hijab, O., "Asymptotic System Identification and Nonlinear Filtering", 21th Conf. Dec. Cont., Orlando, Fla., Vol. 3, Pp. 1178-1182, 1982.

Hillary, B., and D. J. Ewins, "The Use of Strain Gauges in Force Determination and Frequency Response Function Measurements", Proc. IMAC2, Feb. 1984.

Ho, B. L., and R. E. Kalman, "Effective Construction of Linear State-variable Models From Input/Output Data", Proc. 3rd Allerton Conf. Circuit and System Theory, Pp. 449-459, 1965. (also Regungstech, Vol. 14, Pp. 545-548, 1966)

Ho, B. L., "On Effective Construction of Realizations From Input-Output Descriptions", Ph. D. Disseration, Stanford U., 1966.

Holst, J., "Adaptive Prediction and Recursive Estimation", Report TFRT-1013, Dept. of Automatic Cont., Lund Inst., Lund, Sweden, 1977.

Honig, M. L. and D. G. Messerschmitt, "Convergence Models for Adaptive Gradient and Least-Squares Algorithms", Proc. Int. Conf. Acoust. Speech Signal Process, Atlanta,

Ga., Pp. 267-270, 1981.

Honig, M. L., "Recursive Fixed Order Convariance Least-Squares Algorithms", Bell Tech J., Vol. 62, Pp. 2961-2992, 1983.

Hsia, T. C., "A Study of An Output-error Recursive Identificaion Algorithm", Proc. JACC, Charlottesville, Va., Vol. 2, 1981.

Hsia, T. C., " a Multistage Least Squares Technique for System Identification", Proc. 13th Asilomar Conf. On Circuits, Systems, and Computers, Pp. 206-208, 1979.

Hsia, T. C., **Identification: Least Squares Methods**, Lexington Books, 1977.

Hsia, T. C., " a New Iterative Least Squares Algorithm for System Parameter Identification", JACC, Paper No. C1-1, 1974.

Huang, H. L., "A Generalized-Jordan Form Approach One-Step Irreducible Realization of Matrices", IEEE Trans. Auto. Cont., Vol. AC-19, Pp. 271-272, 1974.

Huan, S. L., B. C. McInnis, and E. D. Denman, "Identification of Structural Systems Using Naturally Induced Vibration Data in the Presence of Measurement", Comp. Methods in Appl. Mech. and Eng., Vol. 41, Pp. 123-128, 1983.

Huan, S. L., B. McInnis, and E. D. Denman, "Analysis of the Random Decrement Method", Int. J. System Sci., Vol. 14, Pp. 417- 423, 1983.

Hurtz, W. C., "Vibrations of Structural Systems by Component Mode Synthesis", Proc. ASCE, Vol 86, Pp. 51-69, 1960.

Hwang, Chyi and Tons-Yi Guo, "Transfer-function Matrix Identification in MIMO Systems

Via Shifted Legendre Polynomials", Int. J. Cont., Vol. 39, Pp. 807-814, 1984.

Hwang, Chyi and Tons-Yi Guo, "Parameter Identification Via Laguerre Polynomials", Int. J. Sys. Sci., Vol. 13, Pp. 209-217, 1982.

Ibrahim, S. R., "Time Domain Parameter Identification and Modeling of Structures", 1983 American Control Conf., San Francisco, Ca., 1983.

Ibrahim, S. R. and E. C. Mikulcik, " a Method for Direct Identification of Vibration Parameters From the Free Response", Schock and Vibration Bulletin, No. 43, Pp. 21-37, 1973.

Ibrahim, S. R., and R. S. Pappas, "Large Modal Survey Testing Using the Ibrahim Time Domain Identification Technique", J. of Spacecraft and Rockets, Vol. 19, No. 5, Pp. 459-465, 1982.

Ibrahim, S. R. and E. C. Mikulcik, "A Time Domain Modal Vibration Test Technique", Shock and Vibration Bulletin, Bulletin 43, Part 4, Pp. 21-37, 1973.

Ibrahim, S. R. and Mikulcik, "The Expermential Determination of Vibration Parameters From Time Response", Shock and Vibration Bulletin, Bulletin 46, Part 5, Pp. 187-196, 1976.

Ibrahim, S. R. and E. C. Mikulcik, "Limitations of Random Input Forces in Random Decrement Computations for Modal Identification", Shock and Vibration Bulletin, Part 3, Pp. 99-112, 1980.

Isermann, R. and U. Baur, "Two-Step Process Identification with Correlation Analysis and Least Squares Parameter Estimation", Trans. ASME J. of Dynamic Systems, Measurement, and Control (ser. G), Vol. 96, Pp. 425-432, 1974.

Isermann, R., U. Baur, W. Bamberger, P. Knepo and H. Siebert, "Comparsion of Six On-Line Identification and Parameter Estimation Methods", Automatica, Vol. 10, Pp. 81-103, 1974.

Isermann, R., "New Results On the Identification of Processes", Automatica, Vol. 7, Pp. 191-197, 1971.

Iwen, R. P., R. J. Benhabib and R. L. Jackson, "A Unified Approach to the Design of Large Space Structure Control Systems", 1980 JACC, 1980.

Isidori, A. "Direct Construction of Minimal Bilinear Realizations From Nonlinear Input-output Maps", IEEE Trans. Auto. Cont., Vol. AC-18, Pp. 626-631, 1973.

Jaaksoo, U., and U. Nurges, "Comments On "Approximate Linear Realization of Given Dimension Via Ho's Algorithm", IEEE Trans. Auto. Cont., Vol. AC-20, Pp. 302-303, 1975.

Jamshidi, M. "An Overview On Reduction of Large-Scale Systems Models", Submitted to IFAC Automatica, 1983.

Jategaonkar, R. V. and J. R. Raol, "Determination of Model Order for Dynamical Systems", IEEE Trans. Systems, Man and Cybernetics, Pp. 56-62, 1982.

Jazinski, A. H., **Stochastic Process and Filtering Theory**, Academic Press, 1970.

Johnson, W., "Developement of a Transfer Function Method for Dynamic Stability Measurement", NASA TN D-8522, July 1977.

Johnson, W. and N. K. Gupta, "Instrumental Variable Algorithm for Modal Parameter Identification in Flutter Theory", AIAA Jour., Vol. 16, No. 8, Pp. 800-806, 1978.

Joshi, S. M. and G. L. Goglia, "On Parameter Identification for Large Space Structures", NASA-CR-170306 Rept., Available NTIS., 1983.

Juang, J. N. and E. C. Wong, "System Identification of Large Space Structures", AIAA 18th Aerospace Sci. Mtg., Pasadena, Ca., 1980.

Juang, J. N. and C. T. Sun, "Parameter Identification of Civil Engineering Structures", 19th Conf. On Decision and Control and Symp. On Adaptive Processes, Albuquerque, NM, 1980.

Juang, J. N. and C. T. Sun, "System Identification of Large Flexible Structures by Using Simple Continuum Models", J. of Astronautical Sci., Vol. 31, Pp. 77-98, 1983.

Juang, J. N., and R. S. Pappa, "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction", NASA/JPL Workshop On Identification and Control of Flexible Structures, June 1984. (Also See Jour. Guid. Cont, and Dynam., 1985.)

Juang, J. N., and R. S. Pappa, "Effects of Noise On ERA- Identified Modal Parameters", AAS Paper 85-422, Aug. 1985.

Junkins, J. L., **Optimal Estimation of Dynamical Systems**, Sijthoiff and Noordhoff Alpen Aan Den Rijn, the Netherland/Maryland, 1978.

Kaczorek, T., "Polynomial Equation Approach to Exact Model Matching Problem in Multivariable Linear Systems", Int. J. Cont., Pp. 531-539, 1982.

Kagiwada, H. H., **System Identification**, Addison-Wesley, 1974.

Kailath, T., **Lectures On Linear Least-Squares Estimation**, Springer-Verlag, 1976.

Kalaba, R. and K. Spingarn, **Control, Identification, and Input Optimization**, Plenum Press, New York, 1982.

Kalaba, R. and K. Spingarn, "Optimal Inputs and Sensitivities for Parameter Estimation", JOTA, Vol. 11, Pp. 56-67, 1973.

Kalman, R. E. and R. S. Englar, "Computations of a Minimal Realization, a User Manual for ASPC", NASA Rept., 1966.

Kalman, R., "A New Approach to Linear Filtering and Prediction Problems", Trans. ASME, Jour. Basic Eng., Vol. 82, Pp. 35-45, 1960.

Kalman, R. E., "Mathematical Description of Linear Dynamical Systems", SIAM Jour. On Control, Vol. 1, Pp. 152-192, 1963.

Kalman, R. E., "Irreducible Realizations and the Degree of a Rational Matrix", SIAM Jour., Vol. 13 , Pp. 520-544,1963.

Kalman, R. E., "On Canonical Realizations", Proc. 3rd Allerton Conf., Pp. 449-459, 1964.

Kalman, R. E., P. L. Falb and M. A. Arib, **Topics in Mathematical System Theory**, McGraw-Hill, 1969.

Kalman, R. E., "On Minimal Partial Realization of a Linear Input/output Map", in **Aspects of Network and System Theory**, Edited by R. E. Kalman and N. DeClaris, Holt, Reinhard, and Winston, Pp. 385-407, 1971.

Kapitonenko, V. V., "Minmax Identification in the Case of Uncertain Original Information", Avtomatika I Telemekhanika, Russian, (Translation: Automation and Remote Control, Vol. 43, Pp. 166-169, 1982).

Kashyap, R. L., "Maximum Likelihood Identification of Stochastic Linear Systems", IEEE Trans. Auto. Cont., Vol AC-15, Pp. 25-34, 1970.

Kellermayr, K. H., "A Walsh Function Approach to the Identification of Multivariable Systems", Auto. Cont. Theory and Appl., Vol. 8, Pp. 67-71, 1980.

Kel'mans, G. K., "Active Identification of Dynamic Systems in Face of "Colored" Disturbances", Int. J. Cont., Vol. 13, Pp. 1201-1221, 1982.

Kennedy, C. C. and C. D. F. Pancu, "Use of Vectors in Vibration Measurement and Analysis", Jour. Aeronaut. Sci., Vol. 14, Pp. 603-625, 1947.

Kheir, N. A. and D. Sutherland, "On-Line Identification Methods", Comp. and Electr. Eng., Vol. 7, pp. 35-43, 1980.

Kibardin, V. M. and T. I. Tovstukha, "Modified Quasioptimal Identification Algorithm", Avtomatika I Telemekhanika, Russian, (Translation: Automation and Remote Control, Vol. 44, Pt. 2, pp. 769-777, 1983).

Kim, C. E., and J. T. Cain, "An Instrumental Variable Method for Stochastic Subsystem Identification", 6th Symp. Indentification and System Parameter Estimation", Vol. 2, pp. 1085- 1090, 1982.

Kirszenblatt, A. and M. Chetrit, "Least Squares Nonlinear Parameter Estimation by the Iterative Continuation Methods", AIAA Jour., Vol. 12, pp. 1751-1752, 1974.

Kirshenboim, J., and D. J. Ewins, "A Method for Recognizing Structural Nonlinearities in Steady State Harmonic Testing", ASME Trans., J. Vibration, Acoustics, Stress, and Reliability in Design, Vol. 106., pp. 49-52, 1984.

Koch, G., "A Realization Theorem for Infinite Dimensional Bilinear Systems", Ricerche Di Automatica, Vol. 3, 1972.

Kondrashova, N. V., "Accuracy and Noise Stability of Multipoint Discrete Transformation", Sov. Autom. Cont., Vol. 16, pp. 46-53, 1983.

Kokotvic, P. V., "Overview of Multimodeling by Singular Perturbation", in System Engineering for Power:Organizational Form for Large-Scale Systems, pp. 13-14, US DOE, Washington, D. C., 1979.

Kopp, R. E. and R. J. Orford, "Linear Regression Applied To System Identification for Adaptive Control Systems", AIAA Jour., Vol. 1, pp. 2300-2306, 1963.

Kovacevic, B., and S Stankovic, "Robust Real-Time Identification of Linear Multivariable Systems", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 1039-1043, 1982.

Krasnov, A. M., and A. G. Moiseev, "Quasioptimal Algorithm for Estimating Process With Measurable Parameters", (Avtomatika Telemekhanika, Apr. 1983, Russian), Translated in Automation and Remote Control, Vol. 44, Part 2, pp. 506-510, 1983.

Kung, S., "A New Identification and Model Reduction Algorithm Via Singular Value Decomposition", 12th Asilomar Conf., Circuits, Systems and Computers, 1978.

Kung, S. Y., and Y. H. Hu,"Improved Pisarenko's Sinsuoidal Spectrum Estimate Via SVD Subspace Approximation Methods", 21st Conf. Dec. Cont., Orlando, Fla., Vol. 3, pp. 1312-1314, 1982.

Kuo, Y. L., "On the Irreducible Jordan Form Realization and the Degree of a Rational Matrix", IEEE Trans. Auto. Cont., Vol. AC- 17, pp. 322-332, 1970.

Lainiotis, D. G., "Optimal Adaptive Estimation: Structure and Parameter Estimation", IEEE Trans. Auto. Cont., Vol. AC-16, pp. 160-169, 1971.

Lal, M. and R. Mitra, "Simplification of Large Scale Dynamics Using Moment Evaluation Algorithm" IEEE Trans. Auto. Cont., Vol. AC-9, pp. 602-603, 1974.

Lal, M., and H. Singh, "On Minimal Realization From Symmetric Transfer-Function Matrix", Proc. IEEE, 1972.

Lal, M., S. C. Puri and H. Singh, "On the Realization of Linear Time-Invariant Dynamical Systems", IEEE Trans. Auto. Cont., Vol. AC-17, pp. 251-252, 1972.

Lal, M., H. Singh, and K. A. Khan, "A Simplified Minimal Realization Algorithm for Symmetric Impulse Response Using Moments", IEEE Trans. Auto. Cont., Vol. AC-18, pp. 683-684., 1973.

Landau, I. D., "Unbiased Recursive Identification Using Model Reference Techniques", IEEE Trans. Auto. Cont., Vol. AC-21, pp. 194-202, 1976. (Also See I. D. Landau, "An Addendum To "Unbaised Recursive ....."", IEEE Trans. Auto. Cont. Vol. AC- 23, pp. 97-99, 1978)

Landau, I. D., "Model Reference Adaptive Systems–A Survey (MRAS) – What Is Possible and Why", Jour. Dynamical Systems, Measurement and Control, Vol. 94G, pp. 119-132, 1972.

Leuridan, J. M., D. L. Brown, and R. J. Allemang, "Direct System Parameter Identification of Mechanical Structures With Application To Modal Analysis", in 23rd AIAA Conf. Structures, Structural Dynamics, and Materials, New Orleans, La., pp. 548-556, 1982.

Levadi, V. S., "Design of Input Signals for Parameter Estimation", IEEE Trans. Auto.

Cont., Vol. AC-11, pp. 205-211, 1966.

Levy, B., S. Y. Kung, and M. Morf, "New Results in 2-D System Theory, 2-D State-Space Model Realization and the Notion of Controllability, Observability, and Minimality", AFOSR-TR-77-0370, 1976.

Lewis, T. O. and P. L. Odell, **Estimation and Linear Models**, Prentice-Hall, Engelwood Cliffs, N. J., 1971.

Liebelt, P. B., **An Introduction To Optimal Estimation**, Addison-Wesley Publishing Co., 1967.

Litman, S. and W. H. Huggins, "Growing Exponentials As a Probing Signal for System Identification", Proc. IEEE, Vol. 51, pp. 917-923, 1963.

Liu, R., and L. C. Suen, "Minimal Dimension Realization and Identifiability of Input-Output Sequences", IEEE Trans. Auto. Cont., Vol. AC-22, pp. 99-100 , 1977.

Ljung, L., "Asymptotic Gain and Search Direction for Recursive Identification Algorithms", Proc. IEEE Conf. Dec. and Cont., Albuquerque, N. M., 1980.

Ljung, L., "Convergence Analysis of Parametric Identification Methods", IEEE Trans. Auto. Cont., Vol. AC-23, pp. 770-783, 1978.

Ljung, L., M. Morf and D. Falconer, "Fast Calculations of Gain Matrices for Recursive Estimation Schemes", In. J. Cont., Vol. 27, pp. 1-19, 1978.

Ljung, L. "Consistency of the Least-Squares Identification Method", IEEE Trans. Auto. Cont., Vol. AC-21, pp. 779-781, 1976.

Ljung, L., "Identification Methods", 6th Symp. Identification and System Parameter Estimation, Vol. 12, pp. 57-64 , 1983.

Ljung, L. and T. Soderstrom, **Theory and Practice of Recursive Identification**, MIT Press, 1983.

Ljung, L., "On the Consistency of Prediction Error Identification Methods", in **System Identification, Advances and Caes Studies**, Edited by R. K. Mehra and D. G. Lainiotis, Academic PRess, 1976.

Ljung, L., T. Soderstrom and Gustavsson, "Counterexamples To General Convergence of a Commonly Used Recursive Identification Method", IEEE Trans. Auto. Cont., Vol. AC-20, pp. 643-652, 1975.

Ljung, L., I. Gustavsson and T. Soderstrom, "Identification of Linear, Multivariable System Operating Under Linear Feedback Control", IEEE Trans. Auto. Cont., Vol AC-19, pp. 836-840, 1974.

Ljung, S., "Fast Algorithms for Integral Equations and Recursive Identification", Ph. D. Disseration No. 93, Dept. Elec. Eng., Linkoping Univ., Linkoping, Sweden, 1983.

Lu, G. Z., X. F. Wang, Z. M. Wang, and E. C. Liu, "Canonical Structure and Identification Algorithm of Multivariable Systems", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 857-862, 1983.

Luders, G. and K. S. Narendra, "Stable Adaptive Schemes for State Estimation and Identification of Linear Systems", IEEE Trans. Auto. Cont., Vol. AC-19, pp. 290-293, 1967.

Luk, Y. W., and L. D. Mitchell, "System Identification Via Modal Analysis", Modal Testing and Model Refinement, Proc. Symp., Boston, Ma., pp. 31-49, 1983.

Lynch, H. M. and V. D. Vande Linde, "Multiple Equilbria of Systems Identifiers Using Lypaunov-Designed Model Builders", Proc. 13th JACC Conf., pp. 821-825, 1972.

Mahmoud, M. S., and M. G. Singh, **Large-Scale System Modelling**, pp. 156-161, Pergamon Press, Oxford, England.

Makhoul, J., "Stable and Efficient Lattice Methods for Linear Prediction", IEEE Trans. Acoustics, Speech and Signal Processing., Vol. ASSP-25, Pp423-428, 1977.

Makhoul, J., "Linear Prediction: A Tutorial Review", Proc. IEEE, Vol. 63, No. 4, pp. 561-580, 1975.

Malowski, A., "Optimal Estimation of Linear Space-Time Dynamic Systems: I. Theory, II. Applications", Academie Polonaise Der Sciences, Bulletin, Serie Des Science Techniques, Vol. 21, pp. 47- 57, 1973. (Available From NTIS, Document 74A11862)

Malowski, A., "Optima Estimation of Nonlinear Space-Time Dynamical Systems Using Multilevel Optimization", Zagadnienia Drgan Nieliniowych, Vo. 17, 1976. (Available From NTIS, Document 76A43407)

Markl, J., "Simple Recursive Estimator With On-Line Orthogonalization of the Input Sequence", Kybernetika, Vol. 19, pp. 345-356, 1983. (English)

Marple, S. L., and L. R. Rabiner, "Performance of a Fast Algorithm for FIR System Identification Using Least-Squares Analysis", Bell System Tech. J., Vol. 62, pp. 717-742, 1983.

Marple, S., "Efficient Least-Squares FIR System Identification", IEEE Trans. Acoustics, Speech, and Signal Proc., Vol. ASSP-29, pp. 62-73, 1981.

Maslowski, A., "A Method of Optimal Estimation of Spatial- Temporal Physical Processes", Archiwum Budomy Maszyn, Vol. 20, pp. 415-426, 1974. (In Polish, Available From NTIS , Document 74A10072).

Maslowski, A., "Optimal Estimation of Linear Space-Time Dynamic Systems: I. Theory, II. Applications", Academie Polonaise Der Sciences, Bulletin, Serie Des Science Techniques, Vol. 21, pp. 47- 57, 1973. (Available From NTIS, Document 74A11862)

Maslowski, A., "Optima Estimation of Nonlinear Space-Time Dynamical Systems Using Multilevel Optimization", Zagadnienia Drgan Nieliniowych, Vo. 17, 1976. (Available From NTIS, Document 76A43407)

Maybeck, P. S., "Combined State and Parameter Estimation for On- Line Applications", Ph. D. Disseration, MIT, Cambridge, Ma., 1972.

Mayne, D. Q., "Canonical Forms for Identification", Publication 74/49, Imperial College, England, 1974.

Mayne, D. Q., "Parameterization and Identification of Linear Multivariable Systems", Lectures Notes in Mathematics, Springer- Verlag, Berlin, Vol. 294, pp. 56-61, 1972.

Mayne, D. Q., "A Canonical Form for Identification of Multivariable Linear Systems", IEEE Trans. Auto. Cont., Vol. AC- 17, pp. 726-729, 1972.

Mayne, D. Q., "Optimal Non-Stationary Estimation of the Parameters of a Linear System With Gaussian Noise", Jour. Elect. Cont., Vol. 14, pp. 101-112, 1963.

Mayne, D. Q., "Computational Procedure for the Minimal Realisation of Transfer-function Matrices", Proc. IEE, Vol. 115, pp. 1363-1368, 1968.

Mayne, D. Q., "An Elementary Derication of Rosenbrock's Minimal Realization Algorithm", IEEE Trans. Auto. Cont., pp. 306-307, 1973.

McGrew, J., "Orthogonalization of Measured Modes and Calculation of Influence Coefficients", AIAA Jour., Vol. 7, pp. 774-776, 1969.

Meditch, J. S., **Stochastic Optimal Linear Estimation and Control**, McGraw-Hill, 1969.

Mehra, R. K., and D. G. Lainiotis (Editors), **System Identification: Advances and Case Studies**, Academic Press, 1976.

Mehra, R. K., "Choice of Input Signals", in P. Eykoff, Trends and Progress in System Identification, Pergamon Press, Oxford, England, 1981.

Mehra, R. K., "Synthesis of Optimal Inputs for Multiinput- multioutput Systems With Process Noise", in **System Identification-Advances and Case Studies**, Edited by R. K. Mehra and D. G. Lainiotis, Academic Press, 1976.

Mehra, R. K., "Optimal Input for System Identification", IEEE Trans. Auto. Cont., Vol. AC-19, pp. 192-200, 1974.

Mehra, R. K., "Synthesis of Optimal Inputs for Multiinput- Multioutput (MIMO) Systems With Noise", "Tech. Rept. 649, Div. Eng. Appl. Physics, Harvard U., 1974.

Mehra, R. K., "Optimal Input Signals for Parameter Estimation in Dynamic Systems-Survey and Results", Trans. IEEE Auto. Cont., Vol. AC-19, pp. 753-768, 1974.

Mehra, R. K. and J. S. Tyler, "Case Studies in Aircraft Parameter Identification", Proc. 3rd IFAC Sym. On Identification and System Parameter Estimation, Netherlands, North-

Holland, 1973.

Mehra, R. K., "Identification of Stochastic Linear Dynamic Systems Using Kalman Filter Representation", AIAA Jour., Vol. 9, pp. 28-31, 1971.

Mehra, R. K., "On-Line Identification of Linear Dynamic Systems With Applications To Kalman Filtering", IEEE Trans. Auto. Cont., Vol. AC-16, pp. 12-21, 1971

Meirovitch, L., **Computational Methods in Structural Mechanics**, Sijthoff-Noordhoff, the Netherlands, 1980.

Meirovitch, L., **Elements of Vibration Analysis**, McGraw-Hill, 1975.

Meirovitch, L., and H. Baruh, "Identification of Distributed- Parameter Vibrating Systems", 6th Symp. Identification and System Parameter Estimation, Washington, D. C., Vol. 2, pp. 1283-1288., 1983.

Mendel, J. M., "Discrete Techniques of Parameter Estimation: the Equation Error Formulation", Dekker, New, York, 1973.

Mendel, J. M., and S. V. Kung, "State Space Modelling With Geophysical Applications", Geophysics, Vol. 5, pp. , 1980.

Miramand, N., J. F. Billard, F. Le Leux and J. P. Kernevez, "Identification of Structural Parameters by Dynamic Tests At a Single Point", Shock and Vibration Bulletin, Bulletin 46, Part 5, pp. 197-212, 1976.

Mishra, R. N. and D. A. Wilson, "A New Algorithm for Optimal Reduction of Multivariable Systems", Int. J. Cont., Vol. 31, pp. 443-466, 1980.

Mital, D. P., and C. T. Chen, "Irreducible Canonical Form Realization of a Rational Matrix", IEEE Trans. Auto. Cont., pp. , 1973.

Mitchell, L. D. and L. D. Mitchell, "Modal Analysis Bibliography - an Update - 1980-1983", Proc. IMAC2, Feb. 1984.

Montgomery, R. C., and N. Sundararajan, "Identification of the Dynamics of a Two-dimensional Grid Structure Using Least-Squares Lattice Filters", American Cont. Conf., 1984.

Moore, B. C., "Singular Value Analysis of Linear Systems", Proc. IEEE Conf. Dec. and Cont., pp. 66-73, 1979.

Moore, B. C., "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Resuction", IEEE Trans. Auto. Cont., Vol. AC-26, pp. 17-32, 1981.

Moore, J. B., "On Strong Consistency of Least-Square Identification Algorithms", Automatica, Vol. 14, pp. 505-509, 1978.

Morf, M., and J. M. Delosme, "Matrix Decomposition and Inversions Via Elementary Signature-orthogonal Transformation", Int. Sym. Mini-and Micro-Computer in Cont. and Measurements, San Francisco, CA., 1981.

Morf, M., "Ladder Forms in Estimation and System Identification", Proc. 11th Asilomar Conf. On Circuits, Systems and Computers, pp. 424-429, 1977.

Morf, M., A. Vieira and D. Lee, "Ladder Forms for Identification and Speech Processing", Proc. IEEE Conf. Dec. and Cont., New Orleans, La., pp. 1074-1078, 1977.

Morf, M., T. Kailath and L. Ljung, "Fast Algorithm for Recursive Identification" in Proc.

IEEE Conf. Dec. and Cont., New Orleans, La., pp. 916-921, 1977.

Morf, M., " Fast Algorithms for Multivariable Systems", Ph. D. Dissertation, Stanford U., Stanford, Ca., 1974.

Mullis, C. T. and R. A. Roberts, "The Use of Second-order Information in the Approximation of Discrete-Time Linear Systems, IEEE Trans. Acoustics, Speech and Signal Processing, Vol. ASSP- 24, No. 3, pp. 226-238, 1976.

Munro, N., "Minimal Realisation of Transfer-function Matrices Using the System Matrix", Proc. IEE Vol. 11, pp. 1298-1301, 1971.

Murthy, D. N. P. and McGriffin, "Parameter Estimation for Auto- Regressive Models With Uncertain Observation", 1980 Joint Automatic Control Conf., San Francisco, Ca.

Nahi, N. E. and G. A. Napjus, "Design of Optimal Probing Signals for Vector Parameter Estimation", Preprint IEEE Conf. Dec. and Cont., Miami, Fla., 1971.

Nahi, N. E. and D. R. Wallis, Jr., "Optimal Inputs for Parameter Estimation in Dynamic Systems With White Observation Noise", Paper IV-A5, Proc. JACC, Boulder, Colo., pp. 506-512, 1969.

Nahi, N. E., **Estimation Theory and Applications** John Wiley & Sons, New York, 1969.

Narendra, K. S., "Stable Identification Schemes" in **System Identification: Advances and Case Studies**, Academic Press, pp. 165-209, 1976.

Newland, D. S. **Random Vibrations and Spectral Analysis**, Longman Press, 1975.

Ng, T. S., G. C. Goodwin and B. D. O. and Erson, "Identifiability of MIMO Linear Dynamic Systems Operating in Closed Loop", Automatica, Vol. 13, pp. 477-485, 1977.

Nieman, R. E., D. G. Fisher and D. E. Saborg, "A Review of Process Identification and Parameter Estimation Techniques", Int. J. Cont., Vol. 13, P 209, 1971.

Otnes, R. K., and L. Enochson, **Applied Time Series Analysis: Vol. 1. Basic Techniques**, John Wiley and Sons, 1978.

Ovcharenko, V. N., "Active Identification of Stochastic Discrete Dynamic Systems", (Avtomatika I Telemekhanika, pp. 84-93, Apr. 1983, in Russian), Tranlation in Automatic and Remote Control, Vol. 44, Pt. 2, pp. 475-483, 1983. (Available From NTIS A84-13848)

Paass, G., "Consistency Results for Bayesian Estimators of Multivariate ARMA Processes", 6th Symp. Identification and System Parameter Estimation, Vol. 1, pp. 187-192 , 1983.

Pack, J. D. and E. H. Satorius, "Least Squares, Adaptive Algorithms", NOSC Tech. Rept. TR 423, 1979.

Panda, S. P., and C. T. Chen, "Irreducible Jordan Form Realization of a Rational Matrix", IEEE Trans. Auto. Cont., Vol. AC-14, pp. 66-69, 1969.

Pandya, R. N., and B. Pagurek, "Two Stage Least-Squares Estimation and Their Recursive Approximation", 3rd IFAC Symp. On Identification and System Parameter Estimation, Netherlands, June 1973.

Panuska, V., "A Stochastic Approximation Method for Identification of Linear Systems Using Adaptive Filtering", JACC Ann Arbor, Mich., 1968.

Pappa, R. S., "Close Mode Identification Performance of the ITD Algorithm", 24th Conf. Structures, Structural Dynamics and Materials, Lake Tahoe, Nv., pp. 193-205, 1983.

Pappa, R. S. and S. R. Ibrahim, "A Parametric Study of the Ibrahim Time Domain Identification Algorithm", Shock and Vibration Bulletin, Bulletin 51, 1981.

Pappa, R. S., and J. N. Juang, "Galileo Spacecraft Modal Identification Using an Eigensystem Realization Algorithm", AIAA Paper 84-1070, May, 1984. (also J. of the Astronautical Sci., March-April, 1985)

Parzen, E. "Some Recent Advances in Time Series Modeling", IEEE Trans. Auto. Cont., Vol. AC-19, pp. 723-729, 1974.

Payne, R. L., G. C. Goodwin and M. B. Zarrob, "Frequency Domain Approach for Designing Sampling Rates for System Identification", Automatica, Vol. 11, pp. 189-193, 1975.

Payne, R. L. and G. C. Goodwin, "A Bayesian Approach To Experiment Design With Applications To Linear Multivariable Dynamic Systems", IMA Conf. Comp. Problems Stat., Univ. of Essex, 1973.

Pazdera, J. S., and H. J. Pottinger, "Linear System Identification Via Lyapunov Design Technique", Proc. JACC, pp. 795-801, 1969.

Pendered, J. W., "Theoretical Investigation Into the Effects of Close Natural Frequencies Resonance Testing", Jour. Mech. End.. Sci., Vol. 7, pp. 372-379, 1965.

Pernebo, L, and L. M. Silverman, "Model Reduction Via Balanced State Space Representation", IEEE Trans. Auto. Cont., Vol. AC- 27, pp. 382-387, 1982.

Peterka, V., "Bayesian Approach To System Identification", in P. Eyhoff, Trends and

Progress in System Identification, Pergamon Press, Oxford, England, 1979.

Peterka, V., "Bayesian System Identification", Proc. 5th IFAC Sym. On Identification and System Parameter Estimation", Darmstadt, FRG, Pergamon Press, 1979.

Peterka, V., "A Square Root Filter for Real Time Multivariate Regression", Kybernetika, Vol. 11, pp. 53-67, 1975.

Polyak, B. T. and Ya. Z. Tsypkin, "Robust Identification", Automatica, Vol. 16, P 53-63, 1980.

Polyak, B. T. and Ya. Z. Tsypkin, "Adaptive Estimation Algorithms: Convergence, Optimality, Robustness", Avomatika I Telemekhanika, Russian. (Translation, Automation and Remote Control, Vol. 3, pp. 71-84, 1979)

Porat, B. "Contributions To the Theory and Application of Lattice Forms", Stanford Univ., Stanford, Ca., 1982.

Prado, G. and R. K. Pearson, "Efficient Techniques for System Identification of Large Space Structures—Prediction Error Approach With Applications To Vibration Control", Proc. JACC, San Francisco, Ca., 1980.

Puri, S., and H. Takeda, "Minimal Realization of a Symmetric Transfer-function Matrix Using Moments", IEEE Trans. Auto. Cont., Vol. AC-18 , pp. 305-306, 1973.

Radford, D. G., "The Engineering Aspects of Hankel Matrix Realization Algorithm", M. S. Thesis, Univ. Toronto, Cand., 1978.

Rafajlowicz, E., "Decomposition of the Least-Squares Identification Algorithm On the Basis of a Two-Stage Experiment", Syst. Sci., Vol. 7, pp. 105-109, 1981.

Rajaram, S., "Identification of Vibration Parameters of Flexible Structures", D. Sc. Disseration, VPI&SU, Blacksburg, Va., 1984.

Rajaram, S., and J. L. Junkins, "Identification of Vibrating Flexible Structures", 1984 AIAA-AAS Astrodynamics Conf., Seattle, Wash., 1984.

Rajbman, N. S., **What Is System Identification?**, Russian, Nauka, 1970.

Rajbman, N. S., (Editor), **Identification and System Parameter Estimation**, North-Holland, 1978.

Raney, J. P., "Identification of Complex Structures Using Near Resonance Testing", Shock and Vibration Bulletin, Bulletin 38, Part 2, 1968.

Richalet, J. A. and R. Pouliquen, **Process Identification by the Model Method**, French, Gordon and Breach, 1970.

Rissanen, J. and L. Ljung, "Estimation of Optimum Structures and Parameters for Linear Systems", Conf. Alg. Syst. Theory, Udine, Italy, 1975.

Rissanen, J. and P. E. Caines, "Consistency of Maximum Likelihood Estimates for ARMA Processes", Rept. 7424, Dept. Elec. Eng., Univ. Toronto, Cont. Sys. Rept., 1974.

Rissanen, J., "Recursive Identification of Linear Systems", SIAM J. Cont., Vol. 9, pp. 420-430, 1971.

Rissanen, J., "Realization of Matrix Sequences", IBM Research Report RJ-1032, IBM, San Jose, Ca., 1972.

Rissanen, J., and T. Kailath, "Partial Realization of Random Systems", Automatica, Vol.

8, pp. 389-396, 1972.

Roberts, D. M., "A Computer Program for the Minimal Realization of Transfer Function Matrices", IEEE Trans. Auto. Cont., Vol. AC- 14, pp. 412-413, 1969.

Robins, A. J., and P. E. Wellstead, "Recursive System Identification Using Fast Algorithms", Int. J. Cont., Vol. 33, pp. 455-480, 1981.

Rodden, W. P., "A Method for Deriving Structural Influence Coefficients From Ground Vibration Tests", AIAA Jour., Vol. 5, pp. 991-1000, 1967.

Rodriquez, G., "A Function Space Approach To Maximum Likelihood Identification of Parameters in Elliptic Systems", 1980 Joint Automatic Control Conf., San Francisco, Ca.

Rodriquez, G., and R. E. Scheid Jr., "State and Model Errors Estimation for Elliptic Systems", Workshop On Application of Distributed System Theory To the Control of Large Space Structures, JPL Publication 83-46, July 1983.

Rodriquez, G., M. Hamidi and D. Eldred, "Maximum Likelihood Identification of Parameters in Static and Dynamic Shape Estimation Models", 4th VPI&SU Symp. On Dynamics and Cont. of Large Structures, Blacksburg, Va., 1983.

Roman, J., "Low Order Observer Design Via Realization Theory", Ph. D. Disseration, Univ. of Florida, 1975.

Roman, J., and T. E. Bullock, "Minimal Partial Realization in a Canonical Form", IEEE Trans. Auto. Cont., Vol. AC-20, pp. 529- 533, 1975.

Roman, J., and T. E. Bullock, "Design of Minimal Order Stable Observers To Estimate Linear Functions of the State Via Realization Theory", IEEE Trans. Auto. Cont., Vol.

AC-20, pp. 613-623, 1975.

Rosenbrock, H. H., "Computation of Minimal Representation of a Rational Transfer-Function Matrix", Proc. IEE, Vol. 115, pp. 325- 327, 1968.

Rossen, R. H., and L. Lapidus, "Minimum Realization and System Modeling: I. Fundemental Theory and Algorithm", Alche Jour., Vol. 18, pp. 673-684, 1972.

Rossen, R. H. and L. Lapidus, "Minimum Realization and System Modeling: II Theoretical and Numerical Extensions", AIChe Jour., Vol. 18, pp. 881-892, 1972.

Rossen, R. H., "Minimum Partial Realization of Chemical Engineering Systems", Ph. D. Disseration, Princeton U., 1972.

Roveda, C. A., and R. M. Schmid, "Upper Bound On the Dimension of Minimal Realizations of Linear Time-invariant Dynamical Systems", IEEE Trans. Auto. Cont., Vol. AC-15, pp. 639-644, 1970.

Rozsa, P. and N. K. Sinha, "Efficient Algorithm for Irreducible Realization of a Rational Matrix", Int. J. Cont, Vol. 20, pp. 739-751, 1974.

Rozsa, P. and N. K. Sinha, "Minimal Realization of a Transfer Function Matrix in Canonical Forms", Int. J. Cont., Vol. 21, pp. 273-284, 1975.

Ruan, R. Y., "Software of Identification and Modeling for Linear Multivariable Stochastic System", 6th Symp. Identification and System Parameter Estimation, Vol. 1, pp. 337-342 , 1983.

Ryaciotaki-Boussalis, H. A., and D. Boussalia, "Modeling and Stability Analysis of a Class of Flexible Space Structures", 19th Asilomar Conf. Cir., Syst., and Comp., 1985.

Sage, A. P. and J. L. Melsa, **System Identification**, Academic Press, 1971.

Sage, A. P., and J. L. Melsa, **Estimation Theory With Applications To Communication and Control**, McGraw-Hill, 1971.

Sage, A. P., **Methodologies for Large-scale Systems**, McGraw- Hill, New York, 1977.

Sakrison, D. J., "The Use of Stochastic Approximations To Solve the System Identification Problem", IEEE Trans. Auto. Cont., Vol. AC-12, pp. 563-567, 1967.

Salehi, S. V., "On the State-Space Realization of Matrix Fraction Descriptions for Multivariable Systems", IEEE Trans. Auto. Cont., Vol. AC-24, pp. 1054-1058, 1978.

Salter, J. P., **Steady State Vibration**, Kenneth Mason Press, 1969.

Sandell, N. R. and K. Yared, "Maximum Likelihood Identification of State Space Models for Linear Dynamic Systems", Rept. ESL-R- 814, Electronic System Lab., MIT, Cambridge, Ma., 1978.

Saridis, G. N., "Stochastic Approximation Methods for Identification and Control-A Survey", IEEE Trans. Auto. Cont., Vol. AC-19, pp. 798-809, 1974.

Saridis, G. N., "Comparison of Six On-Line Identification Algorithms", Automatica Vol. 10, pp. 69-79, 1974.

Sarkar, T. K., V. K. Jain and J. Nebat, "A Comparison of the Pencil-of-function Method With Prony's Method, Wiener Filters and Other Identification Techniques", Rept. AD-A119250, Ref. 83N70875, Avail. NTIS, Springfield, Va., 1983.

Schweppe, F. C., "System Identification of Linear Time Invariant Systems", Elec. Power

Syst. Eng. Lab., MIT, 1976.

Sehitoglu, H., "Real-Time Parameter Identification in a Class of Distributed Systems Using Lyapunov Design Method: Part I. Theory", Int. J. Cont. Vol. 38, pp. 747-756, 1983.

Seinfeld, J. H., and L. Lapidus, "Process .dentification: Linear Systems", Chap. 9 in Mathematical Methods in Chemical Engineering. Vol. 3: Process Modeling, Estimation, and Identification, 1974.

Sen, A. and N. K. Sinha, "On-Line Estimation of the Parameters of a Multivariable System Using Matrix Pseudoinverse", Int. J. Systm. Sci., 1976.

Sen, A. and N. K., "On-Line System Identification Algorithm Combining Stochastic Approximation and Pseudoinverse", Automatica, Vol. 11, pp. 425-429, 1975.

Shamash, Y., "Linear System Reduction Using Pade Approximation To Allow Retention of Dominant Modes", Int. J. Cont., Vol 21, pp. 257-272, 1975.

Shamash, Y., "Multivariable System Reduction Via Modal Methods and Pade Approximations", IEEE Trans. Auto. Cont., Vol. AC-20, pp. 815-817, 1975.

Shrikhande, V. L., D. P. Mital, Amd L. M. Ray, "On Minimal Canonical Realization From Input-output Data Sequences", IEEE Trans. Auto. Cont., Vol. AC-25, pp. 309-312, 1980.

Silverman, L. M., "Realization of Linear Dynamical Systems", IEEE Trans. Auto. Cont., Vol. AC-16, P 554, 1971.

Siferd, R. E., and P. S. Maybeck, "Identifiability of Nonlinear Dynamical Systems", 21th Conf. Dec. Cont., Orlando, Fla., Vol. 3, pp. 1167-1171, 1982.

Sinha, N. K., "Identification of Multivariable Continous Time Systems From Samples of Input-output Data", 21th Conf. Dec. Cont., Orlando, Fla., Vol. 3, pp. 1189-1194, 1982.

Silverman, L. M., and H. E. Meadows, "Equivalence and Synthesis of Time-Variable Linear Systems", Proc. 4th Allerton Conf., pp. 776-784, 1966.

Silverman, L. M., "Representation and Realization of Time- Variable Systems", Ph. D. Disseration, Columbia U., 1966.

Silverman, L. M., "Synthesis of Impulse Response Matrices by Internally Stable and Passive Realization", IEEE Trans. Auto. Cont., Vol. AC-14, P203, 1969.

Silverman, L. M., "Balanced Realization and Approximation of Time-Variable Systems", 4th Int. Symp. Mat. Theory of Networks and Systems, 1979.

Simon, M. and G. R. Tomlinson, "Application of Hilbert Transforms in the Modal Analysis of Nonlinear Systems", Jour. Sound and Vibration, Vol. 96, 1984.

Sinha, N. K. and B. Kuszta, **Modeling and Identification of Dynamic Systems**, Van Nostrand 1983.

Sinha, N. K. and G. J. Lastman "Identification of Continuous- Time Multivariable Systems From Sampled Data", Int. J. Cont., Vol. 35, pp. 117-126, 1982.

Sinha, N. K. and A. Sen, "Critical Evaluation of On-Line Identification Methods", Proc. IEE, Vol. 121, pp. 1153-1158., 1975.

Sinha, N. K. and W. Pille, "On-Line Parameter Estimation Using the M. trix Pseudoinverse", Proc. IEE, Vol. 118, pp. 1041-1046, 1971.

Soderstrom, T. and P. Stoica, **the Instrumental Variable Method Approach To System Identification**, Springer-Verlag, Berlin, FDR, 1983.

Soderstrom, T., L. Ljung and I. Gustavsson, "A Theoretical Analysis of Recursive Identification Methods", Automatica, Vol. 14, pp. 231-244, 1978.

Soderstrom, T., "On Model Structure Testing in System Identification" Int. J. Cont., Vol. 26, pp. 1-18, 1977.

Soderstrom, T., "On the Uniqueness of Maximum Likelihood Identification", Vol. 11, pp. 193-197, 1975.

Soderstrom, T., I. Gustavsson and L. Ljung, "Identifiability Conditions for Linear System Operating in Closed-Loop", Int. J. Cont., Vol. 21, pp. 243-255, 1975.

Soderstrom, T., "Notes On the Design of Optimal Identification Experiments", Rept. UPTEC 75-63-R, Uppsala Univ., Sweden, 1975.

Soderstrom, T., "On the Uniqueness of Maximum Likelihood Identification", Automatica, Vol. 11, pp. 193-197, 1975.

Soderstrom, T., L. Ljung and I. Gustavsson, "On the Accuracy Problem in Identification", Proc. 6th IFAC Cong., Boston, Ma., 1975.

Soderstrom, T., L. Ljung and I. Gustavsson, "On the Accuracy of Identification and the Design of Identification Experiments", Rept. 7428, Dept. Auto. Cont., Lund Inst., Lund, Sweden, 1974.

Soderstrom, T., "A Comparative Study of Recursive Identification Methods", Rept. 7427, Dept. Auto. Cont., Lund Inst., Sund, Sweden, 1974.

Soderstrom, T., "An On-Line Algorithm for Approximate Maximum Likelihood Identification of Linear Dynamic Systems", Rept. 7308, Div. Auto. Cont., Lund Inst., Lund, Sweden, 1973.

Soderstrom, T., "On the Convergence Properties of the Generalized Least-Squares Identification Method", Rept. 7228, Div. Auto. Cont., Lund Inst., Lund, Sweden, 1972.

Sontag, E. D., "Realization Theory of Discrete-Time Nonlinear Systems:Part I-The Bounded Case", IEEE Trans. Auto. Cont., Vol. AC-25, pp. , 1979.

Soong, F., "Fast Least-Squares Estimation and Its Application", Ph. D. Dissertation, Stanford Univ., Stanford, Ca., 1981.

Sorenson, H. W., "Parameter and State Estimation - Introduction and Interrelation", 6th Symp. Identification and System Parameter Estimation, Vol. 12, pp. 85-89 , 1983.

Sorenson, H. W., **Parameter Estimation: Principles and Problems**, Marcel Dekker, 1980.

Spalding, G. R., "Modeling and Identification of Distributed Parameter Systems Using Jacobi Vectors", 6th Symp. Identification and System Parameter Estimation, Vol. 1, pp. 145-150 , 1983.

Speedy, C. B., R. F. Brown and G. C. Goodwin, **Control Theory: Identification and Optimal Control**, Oliver and Boyd, Edinburgh, England, 1970.

Sridhar, B., "Maximum Likelihood Identification Using Array Processors", 19th Asilomar Conf. Cir., Syst., and Comp., 1985.

Staar, J., "A Study of MBH-Type Algorithms", Automatica, 1981.

Staar, J., M. Wemans, and J. Vandewalle, "Comparsion of Multivariable MBH Realization Algorithms in the Presence of Multiple Poles, and Noise Disturbing the Markov Sequence", 4th Int. Conf. On Analysis and Optimization of Systems, 1980.

Staar, J., J. Vandewalle, and M. Wemans, "Realization of Truncated Impulse Response Sequences With Prescribed Uncertainty", 8th IFAC World Congress, 1981.

Staar, J., J. Vandewalle, and M. Wemans, "A General Class of Numerically Reliable Algorithms for the Realization of Truncated Impulse Responses", 8th IFAC World Congress, 1981.

Staar, J., M. Wemans, and J. Vandewalle, "Reliable Multivariable Modeling Algorithms for Adaptive Controllers", 13th CIRP Int. Seminar On Microprocessors in Manufacturing Systems, 1981.

Staar, J., "Concepts for Reliable Modelling of Linear Systems With Application To On-Line Identification of Multivariable State Space Descriptions", Ph. D. Disseration, Katholieke U., Leuven, 1982.

Staar, J., J. Vandewalle, F. Claeys, and H. DeBock, "On the Recursive Identification of Multivariable State Space Models", in Application of Multivariable System Theory, 1982.

Stoica, P. and T. Soderstrom, "Optimal Instrumental Variable Methods for the Identification of Multivariable Linear Systems", Automatica, Vol. 19, 1983.

Staley, R. M. and P. C. Yue, "On System Parameter Identifiability", Inf. Sci., Vol. 2, pp. 127-138, 1970.

Steiglitz, K. and L. E. McBride, "A Technique for the Identification of Linear Systems", IEEE Trans. Auto. Cont., Vol. AC-10, pp. 461-464, 1965.

Stengel, D. N., D. G. Luenberger, R. E. Larson and T. S. Cline, "A Descriptor Variable Approach To Modeling and Optimization of Large-Scale Systems", Rept, CONS-2858-T1, US DOE, Oak Ridge, Tenn., 1979.

Stepinski, T., "Comparative Study of Robust Methods of Vehicle State Estimation", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 829-834, 1983.

Stoica, P. and T. Soderstrom, "Identification of Multivariable Systems Using Instrumental Variable Methods", Proc. 6th IFAC Symp. On Identification and System Parameter Estimation, Washington, D. C., Pergamon Press, 1982.

Stoica, P., and T. Soderstrom, "Identification of Multivariable Systems Using Instrumental Variable Methods", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 1073- 1078 , 1983.

Stoica, P., T. Soderstrom, A. Ahlen and G. Solbrand, "On the Aymptotic Accuracy of Pseudo-Linear Regression Algorithms", Int. J. Cont., Vol. 39, pp. 115-126, 1984.

Suen, L. C., "Minimal Dimension Realization Versus Minimal Realization", IEEE Trans. Auto. Cont., Vol. AC-24, Pp.99-100, 1978.

Sun C. T., and J. N. Juang, "Paramter Estimation in Truss Beams Using Timoshenko Beam Model With Damping", Workshop On Application of Distributed System Theory To the Control of Large Space Structures, JPL Publication 83-46, July 1983.

Sunahara, Y., Sh. Aihara and F. Kojima, "A Method for Parameter Estimation of a Class On Nonlinear Distributed Systems Under Noisy Observations", 6th Symp. Identifiaction and System Parameter Estimation, Vol. 2, pp. 1277-1282, 1983.

Sundararajan, N. and R. C. Montgomery, "Experiments Using Least-Squares Lattice Fil-

ters for the Identification of Structural Dynamics", 24th Conf. Structures, Structural Dynamics and Materials, Lake Tahoe, Nov., pp. 206-210, 1983.

Sundararajan, N. and R. C. Montgomery, "Decoupling the Structural Modes Estimated Using Recursive Lattice Filters", 21th IEEE Conf. Dec. and Cont., Orlando, Fl., 1982.

Sundararajan, N. and R. C. Montgomery, "Adaptive Identification for the Dynamics of Large Space Structures", in Guidance and Control Conf. Proc., San Diego, Ca., pp. 379-387, 1982.

Sundarajan, N. and R. C. Montgomery, "Adaptive Control of a Flexible Beam Using Least-Squares Lattice Filters", 4th VPI&SU Symp. On Dynamics and Control of Large Structures, Blacksburg, Va. ,1983.

Sundarajan, N., J. P. Williams, and R. C. Montgomery, "Parameter Testing for Lattice Filter Based Adaptive Modal Control Systems", AIAA Guid. Cont. Conf., Gatlinburg, Tenn., Paper 83-0880- CP, 1983.

Sussman, H. J., "Existence and Uniqueness of Minimal Realization of Nonlinear Systems", Math. Syst. Theory, 1976.

Targoff, W. P., "Orthogonal Check and Correction of Measured Modes", AIAA Jour., Vol. 14, 1976.

Teles, M. L. S., and P. B. L. Castrucci, "On the Minmax Approach for Dynamic System Identification", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 1235- 1240., 1983.

Tether, A. "Construction of Minimum Linear State Variable Models From Input/output Data", IEEE Trans. Auto. Cont., Vol. AC-15, pp. 427-436, 1970.

Thau, F. E., and R. C. Montgomery, "Adaptive/Learning Control of Large Space Structures-System Identification Techniques for Multi-configuration Flexible Spacecraft", JACC, San Francisco, Ca., 1980.

Thau, F. E., R. C. Montgomery and G. C. Horner, "On-Line Structural Parameter Identification", Proc. AIAA Guid. and Cont. Conf., 1981.

Thau, F. E., T. Eliazov, and R. C. Montgomery, "Least-squares Sequential Parameter and State Estimator for Large Space Structures", Proc. American Cont. Conf., pp. 1-6, 1982.

Thoren, R. P., "Derivation of Mass and Stiffness Matrices From Dynamic Test Data", 13th AIAA/ASME/SAE Conference, San Antonio, Tx., AIAA Paper 72-346, 1972.

Tse, E., and H. Weinert, "Structure Determination and Parameter Identification for Multivariable Stochastic Linear Systems", IEEE Trans. Auto. Cont., Vol. AC-20, pp. 603-612, 1975.

Tse, E., "Information Matrix and Local Identifiability of Parameters", Paper 20-3, JACC, Columbus, Oh., 1973.

Tse, E., and J. J. Anton, "On the Identifiability of Parameters", IEEE Trans. Auto. Cont., Vol. AC-17, pp. 637-645, 1972.

Tsypkin, Ya. Z., "Optimal Parameter Estimation Algorithms in Identification Problems", Avtomatika I Telemekhanika, Russian (Translation: Automation and Remote Control, Vol. 43, pp. 1505, 1517, 1982).

Tsypkin, Ya. Z., "Optimality in Identification of Linear Plants", Int. J. Syst. Sci., Vol. 14, pp. 59-74, 1983.

Tung, F. C., "Application of System Identification Technique To the Modeling of Large Space Structures", IEEE Guidance and Control Conf., Albuquerque, N. M., 1981.

Turner, J. M., "The Lattice Structure and Its Use in Estimation and Filtering", Proc. Conf. Info. Sci. and Syst., Princeton, N. J., 1980.

Turtle, D. P. and P. H. Phillipsen, "Simultaneous Identification and Control", Automatica, Vol. 7, pp. 445-452, 1971.

Vandewalle, J., and J. Staar, "On the Recursive Identification of Multivariable State Space Models", Int. J. on Circuit Theory and Application, Vol. 9, pp. 488-492., 1982.

Vandewalle, J., and J. Staar, "Modelling of Linear Systems: Critical Examples and Numerically Reliable Approaches",Proc. ISCAS, Rome, Italy, pp. 915-918, 1982.

Vandewalle, J., J. Staar, B. DeMoor, and J. Lauwers, "An Adaptive Singular Value Decomposition Algorithm and Its Application To Adaptive Realization", Proc. 6th Int. Conf. on Analysis and Optimization of Systems", June, 1984.

Van Mien, H. D., "Multivariable Identification and Control of Electric Power Plants", the Institute of Measurement and Control, 1982.

Van Overbeek, A. J. and L. Ljung, "On-Line Structure Selection for Multivariable State-Space Models", Automatica, Vol. 18, pp. 529-543, 1982.

Van Trees, H. L., **Detection, Estimation, and Modulation Theory**, John Wiley and Sons, 1968.

Van Zee, G. A., and O. H. Bosgra, "The Use of Realization Theory in the Robust Identification of Multivariable Systems", Proc. 5th IFAC Symp. on Identification and System

Parameter Estimation, Darmstadt, Germany, 1979.

Varlaki, P., and B. Szucs, "Nonparametric Identification of Continous Nonlinear Systems Using Rajman's Dispersional Method", 6th Symp. Identification and System Parameter Estimation, Vol. 1, pp. 97-101 , 1983.

Verriest, E. I., "On Balanced Realization for Time Variant Linear Systems", Ph. D. Dissertation, 1980.

Verrist, E. I., and T. Kailath, "On Generalized Balanced Realization", 19th Cont. on Dec. and Cont., 1981.

Vogt, W. G., and R. W. Gunderson, "Canonical Realization", 1st Asilomar Conf., pp. 782-789, 1967.

Vold, H., J. Kundrat, G. T. Rocklin and R. Russell, "A Multi- Input Modal Estimation Algorithm for Mini-Computers", SAE Int. Congress and Exposition, Detroit, Mich., 1982.

Vorchik, V. G., "Plant Identification in a Stochastic Closed- Loop System", Avtomatika I Telemekhanika, Russian, (Translation: Automation and Remote Control, Vol. 36, 1975.)

Walter, E., "Identifiability Testing for Linear and Nonlinear State Space Models", Proc. 10th IMACS World Cong. Syst. Simulation and Sci. Comput., Montreal, Can., pp. 220-222, 1982.

Wang, B. P., and F. H. Chu, "Structure Model Refinement Using Reanalysis Techniques", Modal Testing and Model Refinement, Proc. Symp., Boston, Ma. ASME, pp. 135-141, 1983.

Wang, J., "Identification of Multivariable Linear Systems", 18th Allerton Conf. on Comm.,

Cont. and Computing, Monticello, Ill., pp. 275-284, 1980.

Wang, J. C., "On-Line Identification of Markov Parameters and Discrete State Variable Model From Finite Input-Output Data", ASME Paper 72-WA/AUT-14, 1972.

Warren, A. W., "Local Maximum Likelihood State Estimation for Discrete Systems With State Dependent Noise", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 1117- 1122 , 1983.

Wasan, M. T., **Parametric Estimation**, McGraw-Hill, 1970.

Weeks, C., "The Control and Estimation of Large Space Structures- -Optimal Shape Variations With Respect To Space and Time", JACC, San Francisco, Ca., 1980.

Wei, F. S., "Stiffness Matrix Correction From Incomplete Data", AIAA Jour., Vol. 18, P 1274, 1980.

Wells, W. R., "Application of Modern Parameter Estimation Methods To Vibrating Structures", Shock and Vibration Bulletin, Bulletin 47, Part 4, pp. 155-159, 1977.

Wells, W., K. Rattan, and S. Banda, "Frequency Domain Paramter Estimation Based on Sampling and Packing Theorms", 6th Symp. Identification and System Parameter Estimation, Vol. 2, pp. 1145- 1148, 1983.

Wellstead, P. E. and J. M. Edmunds, "Least-Squares Identification of Closed-Loop Systems", Int. J. Cont., Vol. 21, pp. 698-699, 1975.

Wemans, M., J. Staar, and J. Vandewalle, "ADAPT: An Interactive Program for the Analysis and Design of Adaptive Controllers", 13th CIRP Int. Seminar on Microprocessors in Manufacturing Systems, 1981.

White, C. W. and B. D. Maytum, "Eigensolutioin Senstivity To Parametric Model Perturbations", Shock and Vibration Bulletin, Bulletin 46, Part 5, pp. 123-133, 1976.

White, R. G. and R. J. Pinnington, "Practical Application of the Rapid Frequency Sweep Technique for Structural Frequency Response Measurements", Jour. Roy. Aero. Soc., 1982.

Whittle, P., "On the Fitting of Multivariable Autoregression and Approximate Canonical Factorization of a Spectral Density Matrix", Biometrika, Vol. 50, pp. 129-134, 1963.

Widrow, B. and S. D. Stearns, **Adaptive Signal Processing**, Prentice-Hall, 1985.

Wieslander, J., "Real Time Identification-Part I", Rept. 6908, Div. Auto. Cont., Lund Inst. Lund, Sweden, 1969.

Willsky, A. S., "Digital Signal Processing and Control and Estimation Theory: Points of Tangency, Areas of Intersection, and Parell Directions", MIT Press, 1979.

Wilson, R. G., "Complementary Sequences for Multivariable System Identification", 19th Conf. Dec. and Cont., Albuquerque, N. M., pp. 693-694, 1980.

Wolovich, W. A., "The Determination of State Space Representation for Linear Multivariable Systems", Automatica, Vol. 9, pp. 97-106, 1973.

Wolovich, W. A., "The Determination of State-Space Representation for Linear Multivariable Systems", IFAC Conf., Dusseldorf, Germany, 1971.

Wolovich, W. A., "The Determination of State-Space Representation for Linear Multivariable Systems", 2nd IFAC Symp. on Multivariable Technical Control Systems, 1971.

Wong, K. Y. and E. Polak, "Identification of Linear Discrete Time Systems Using the Instrumental Variable Method", IEEE Trans. Auto Cont., Vol. AC-12, pp. 707-712, 1967.

Wu, Y. W. A., "Guaranteed Error Estimation/identification and Its Application To Large Space Structures", JACC, 1980.

Xia, G. H., "Transfer Function Identification for Multivariable Linear Systems", 18th Allerton Conf. Comm., Cont. and Computing, Monticello, Ill., pp. 295-302, 1980.

Yashkir, D. V., "Quasigradient Method for Identification of Physical Parameters Using Linear Integral Equations of the First Kind", Sov. Auto. Cont., Vol. 15, pp. 13-17, 1982.

Yedavalli, R. K., "Critical Parameter Selection in the Vibration Supression Problem of Large Space Structures", Guidance and Control Conf., San Diego, Ca., pp. 495-499, 1982.

Youla, D. C., "The Synthesis of Linear Dynamical Systems From Prescribed Weighting Patterns", SIAM J. Appl. Math., Vol. 14, pp. 527-549, 1966.

Young, P. C., "The Use of Linear Regression and Related Procedures for the Identification of Dynamic Processes", Proc. 7th IEEE Sym. on Adaptive Processes, UCLA, 1968.

Young, P. C., "An Instrumental Variable Method for Real-Time Identification of a Noisy Process", Automatica, Vol. 6, pp. 271- 287, 1970

Zaveri, K., "Modal Analysis of Large Structures-Multiple Exciter Systems", Bruel and Kjaer Publications.

Zarrop, M. B., "Optimal Experiment Design for Dynamic System Identification", Springer-Verlag, Berlin, 1979.

Zarrop, M. B., "Experiment Design for System Identification", M. Sc. Dissertation, Imperial College, 1973.

Zasworsky, R. M., D. Jensen, and W. P. Baker, "The Set of All Minimal Partial Realizations", IEEE Trans. Auto. Cont., Vol. AC- 25. pp. 966-970, 1979.

Zehna, P. W., "Invariance of Maximum Likelihood Estimation", Ann. Math. Stat., Vol. 37, P 755, 1966.

Zeiger, H. P. and A. J. McEwen, "Approximate Linear Realization of Given Dimension Via Ho's Algorithm", IEEE Trans. Auto. Cont., Vol. AC-19, P 153, 1974.

Zeiger, H. P., "Ho's Algorithm, Commutative Diagrams, and the Uniqueness of a Minimal Linear System", J. Inform. and Cont., Vol. 11, pp. 71-79, 1967.

Zwang, H. Y., L. S. Shieh and R. E. Yates, "Identification and Model Reduction of Multivarible Continuous Systems Via a Block- Pulse Function Scheme", Applied Math. Modelling, Vol. 6, pp. 369- 373, 1982.