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TWO NEW SERIES OF SEARCH DESIGNS FOR 3(N) FACTORIAL
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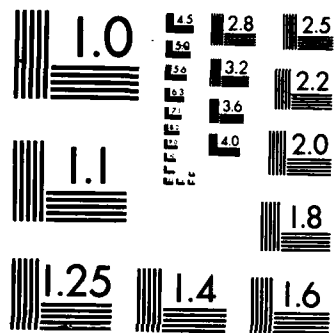
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Technical Report No. 144

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TWO NEW SERIES OF SEARCH DESIGNS FOR

3^m FACTORIAL EXPERIMENTS

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0. Summary

In this paper two new series of search designs with very small number of treatments are presented for 3^m factorial experiments. The first series of designs can search one nonzero two factor interaction and estimate it along with the general mean and the main effects. The second series of designs can search one nonzero three factor interaction and estimate it along with the two factor and lower order interactions.

Short Running Title: Search Designs for 3^m Factorials

AMS 1970 Subject Classifications: Primary and Secondary: 62K15

Keywords and Phrases: Estimability, Interactions, Main Effects, Resolution III and V Plans, Search Designs.

*The work of the first author is sponsored by the Air Force Office of Scientific Research under Grant AFOSR-86-0048.

**The work of the second author is supported by a fellowship from the University of California, Riverside.



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1. Introduction

Consider the linear model

$$(1) \quad E(\underline{y}) = X_1 \underline{\beta}_1 + X_2 \underline{\beta}_2,$$

$$(2) \quad V(\underline{y}) = \sigma^2 I,$$

where $\underline{y}$ ($N \times 1$) is a vector of observations and for $i = 1, 2$, X_i ($N \times v_i$) are known matrices, $\underline{\beta}_i$ ($v_i \times 1$) are vectors of fixed parameters and σ^2 is a constant which may or may not be known. Moreover, $\underline{\beta}_1$ is completely unknown, but we have partial information about $\underline{\beta}_2$. We know that at most p elements of $\underline{\beta}_2$ are nonzero and the remaining elements are negligible, where p is a nonnegative integer which may or may not be known. In this paper we assume p is known to be 1. However, we do not know exactly which element of $\underline{\beta}_2$ is nonzero. The problem is to search the nonzero element of $\underline{\beta}_2$ and draw inference on it in addition to the elements of $\underline{\beta}_1$. Such models are called search linear models and were introduced in Srivastava (1975). We want X_1 and X_2 to be such that the above problem can be resolved; the underlying design corresponding to X_1 and X_2 is called a search design.

In a 3^m factorial experiment the treatments are denoted by $(a_1, \dots, a_m)$, $a_i = 0, 1, 2$; the factorial effects are denoted by $F_1^{c_1} \dots F_m^{c_m}$, $c_i = 0, 1, 2$; the observation corresponding to the treatment $(a_1, \dots, a_m)$ is denoted by $y(a_1, \dots, a_m)$. The expectation form of the model is

$$(3) \quad E(y(a_1, \dots, a_m)) = \sum b_1 \dots b_m F_1^{c_1} \dots F_m^{c_m},$$

where the values of b_1 depend on a_1 and c_1 and are given in Table 1.

Table 1
The values of b_1

$a_1 \backslash c_1$	0	1	2
0	1	-1	1
1	1	0	-2
2	1	1	1

We now consider the following two situations:

S1: The vector β_1 consists of the general mean and the main effects and the vector β_2 consists of the 2-factor interactions. The 3-factor and higher order interactions are assumed to be zero. We assume $m \geq 3$.

S2: The vector β_1 consists of the general mean, the main effects and the 2-factor interactions and the vector β_2 consists of the 3-factor interactions. The 4-factor and higher order interactions are assumed to be zero. We assume $m \geq 4$.

For given N treatments, we can write the equation (3) in the form of the equation (1) for both S1 and S2. Let D_1 be a design with $(1+2m)$ treatments as the treatment with all factors at level 2, treatments with the i th ($i = 1, \dots, m$) factor at levels of 0 and 1 and the other factors at level 2. We know that under D_1 we can estimate the

elements of β_1 in S1 with the assumption that $\beta_2 = 0$ (i.e., a Resolution III plan). Let D_2 be a design with $(1+2m+4\binom{m}{2})$ treatments as $(1+2m)$ treatments in D_1 , treatments with the (i,j) th factors $(i,j=1,\dots,m, i < j)$ at levels $(0,0), (0,1), (1,0), (1,1)$ and the other factors at level 2. We know that under D_2 we can estimate the elements of β_1 in S2 with the assumption that $\beta_2 = 0$ (i.e., a Resolution V plan). Let D_3 be a design with m treatments as treatments with the i th $(i=1,\dots,m)$ factor at level 0 and the other factors at level 1. We prove that the design $D^{(1)}$ consisting of treatments in D_1 and D_3 and the design $D^{(2)}$ consisting of the treatments in D_2 and D_3 are in fact search designs in S1 and S2, respectively.

In all Taguchi design methods, See Taguchi and Wu (1985), popular in statistical quality control experimentations, the higher order interactions (2-factor and higher order in most plans) are assumed to be zero. A few of those higher order interactions may have significant effect on the optimal experimental condition. The use of search designs may be a potential tool in improving upon the Taguchi design methods.

There is a vast literature available in the construction of search designs for 2^m factorial experiments. Near minimal resolution IV plan which permit search and estimation of three or fewer nonzero two factor interactions for p^m factorial experiments are available in Anderson and Thomas (1980) under the assumption that 3-factor and higher order interactions are all zero. Chatterjee and Mukherjee

(1986) presented search design for $s^r \times w^{(m-r)}$ where s and w are any positive integers, under the assumption that 3-factor and higher order interactions are all zero. Our $D^{(2)}$ in S2 is therefore totally new and there is no other competitor in literature. Our design $D^{(1)}$ in S1 is although new but has competitors in Chatterjee and Mukherjee (1986) and in Anderson and Thomas (1980). However, the design $D^{(1)}$ has an edge over designs in those papers in terms of the smaller number of treatments. This can be seen by taking examples 4.5 and 4.6 in Chatterjee and Mukherjee with $r = m$ and $s = 3$ and comparing with the number of treatments $(1+3m)$ in $D^{(1)}$. In S1, the design in the example 4.5 in Chatterjee and Mukherjee has $(1+6m)$ treatments with $3m$ more treatments than in $D^{(1)}$. In example 4.6 ($m=3$), Chatterjee and Mukherjee has 8 more treatments than in $D^{(1)}$. Indeed, both Chatterjee and Mukherjee, Anderson and Thomas designs can be used in factorial experiments other than 3^m .

2. Preliminary Results

We first introduce the following notations.

$y_{\alpha ij}^{\beta \gamma}$ = The observation corresponding to the treatment with levels of all factors except i and j are α , the level of the i th factor is β and the level of the j th factor is γ .

$y_{\alpha i}^{\beta}$ = The observation corresponding to the treatment with levels of all factors except i are α and the level of the i th factor is β .

y_α = The observation corresponding to the treatment with levels of all factors are α .

$S(c_1=u)$ = Sum of factorial effects $F_1^{c_1} \dots F_m^{c_m}$ with $c_1=u$.

$S(c_1=u, c_j=v)$ = Sum of factorial effects $F_1^{c_1} \dots F_m^{c_m}$ with $c_1=u$ and $c_j=v$.

We now present the minimum variance unbiased estimators (MVUE) of

$S(c_1=u)$ and $S(c_1=u, c_j=v)$ under (1) and (2) with $\beta_2=0$ for both designs D_1 and D_2 .

Table 2
MVUE's of $S(c_1=u)$ and $S(c_1=u, c_j=v)$ for D_1 and D_2

Design	Parameter	MVUE
D_1	2 $S(c_1=1)$	$y_2 - y_{21}^0$
	6 $S(c_1=2)$	$y_2 - 2y_{21}^1 + y_{21}^0$
D_2	4 $S(c_1=1, c_j=1)$	$y_2 - y_{21}^0 - y_{2j}^0 + y_{21j}^{00}$
	12 $S(c_1=1, c_j=2)$	$(y_2 - y_{21}^0) + (y_{2j}^0 - y_{21j}^{00}) - 2(y_{2j}^1 - 2y_{21j}^{01})$
	12 $S(c_1=2, c_j=1)$	$(y_2 - y_{2j}^0) + (y_{21}^0 - y_{21j}^{00}) - 2(y_{21}^1 - y_{21j}^{10})$
	36 $S(c_1=2, c_j=2)$	$(y_2 + y_{21}^0 - 2y_{21}^1) + (y_{2j}^0 - 2y_{21j}^{10} + y_{21j}^{00}) - 2(y_{2j}^1 - 2y_{21j}^{11} + y_{21j}^{01})$

The requirement on X_1 and X_2 for a design to be a search design (see, Srivastava (1975)) is that for $\binom{v_2}{2}$ models

$$(4) \quad E(\underline{y}) = X_1 \underline{\beta}_1 + X_2^{(1)} \underline{\beta}_2^{(1)}, \quad i=1, \dots, \binom{v_2}{2},$$

where $X_2^{(1)}$ ($N \times 2$) is a submatrix of X_2 and $\underline{\beta}_2^{(1)}$ is a (2×1) subvector of $\underline{\beta}_2$, the parameters $\underline{\beta}_1$ and $\underline{\beta}_2^{(1)}$ are unbiasedly estimable. Both of $D^{(1)}$ and $D^{(2)}$ consist of two component designs namely (D_1, D_3) and (D_2, D_3) . For $u=1, 2, 3$, we denote the observations corresponding to D_u by $\underline{y}_u$ and write (4) as

$$(5) \quad E(\underline{y}_u) = X_{u1} \underline{\beta}_1 + X_{u2}^{(1)} \underline{\beta}_2^{(1)}, \quad i=1, \dots, \binom{v_2}{2}.$$

For $u=1, 2$, we have

$$(6) \quad \text{Rank } X_{u1} = v_1,$$

$$(7) \quad E[-X_{31} X_{u1}^{-1} \underline{y}_u + \underline{y}_3] = [X_{32}^{(1)} - X_{31} X_{u1}^{-1} X_{u2}^{(1)}] \underline{\beta}_2^{(1)}.$$

We denote

$$W_u^{(1)} = [X_{32}^{(1)} - X_{31} X_{u1}^{-1} X_{u2}^{(1)}], \quad u=1, 2, i=1, \dots, \binom{v_2}{2}.$$

For $u=1, 2$, the requirement for the design $D^{(u)}$ to be a search design is that

$$\text{Rank } W_u^{(1)} = \text{Rank } W_u^{(1)'} W_u^{(1)} = 2, \quad i=1, \dots, \binom{v_2}{2}.$$

It is to be noted that columns of $W_u^{(1)}$ (rows and columns of $W_u^{(1)'} W_u^{(1)}$)

correspond to a pair of elements of β_2 . Since both designs $D^{(1)}$ and $D^{(2)}$ are balanced arrays of full strength, it can be seen that for two pairs of elements of β_2 isomorphic under the symmetric group of permutations of degree m on the set $\{1, 2, \dots, m\}$, the $W_u^{(1)}$ matrices are identical except for permutations of rows and columns. We now present in Table 3 and 4 the nonisomorphic pairs of elements in β_2 for S1 and S2. We note that 2-factor interactions can be any of 3 types $F_i F_j$, $F_i^2 F_j$, $F_i F_j^2$ for different values of i, j without assuming the restriction $i < j$. Similarly 3-factor interactions can be any of 4 types $F_i F_j F_k$, $F_i^2 F_j F_k$, $F_i F_j^2 F_k$, $F_i^2 F_j^2 F_k$ for different values of i, j, k without assuming the restriction $i < j < k$.

Table 3
The nonisomorphic pairs of elements in β_2 for S1

$(F_i F_j, F_i F_u)$	$(F_i F_j, F_i^2 F_u^2)$	$(F_i^2 F_j, F_u^2 F_v)$
$(F_i F_j, F_u F_v)$	$(F_i F_j, F_u^2 F_v^2)$	$(F_i^2 F_j, F_i^2 F_j^2)$
$(F_i F_j, F_i^2 F_j)$	$(F_i^2 F_j, F_i^2 F_u)$	$(F_i^2 F_j, F_i^2 F_u^2)$
$(F_i F_j, F_i^2 F_u)$	$(F_i^2 F_j, F_j^2 F_i)$	$(F_i^2 F_j, F_j^2 F_u^2)$
$(F_i F_j, F_u^2 F_i)$	$(F_i^2 F_j, F_j^2 F_u)$	$(F_i^2 F_j, F_u^2 F_v^2)$
$(F_i F_j, F_u^2 F_v)$	$(F_i^2 F_j, F_u^2 F_i)$	$(F_i^2 F_j^2, F_i^2 F_u^2)$
$(F_i F_j, F_i^2 F_j^2)$	$(F_i^2 F_j, F_u^2 F_j)$	$(F_i^2 F_j^2, F_u^2 F_v^2)$

Table 4

The nonisomorphic pairs of elements in β_2 for S2

$(F_1 F_j F_k, F_1 F_j F_u)$	$(F_1 F_j F_k, F_1^2 F_u^2 F_v^2)$	$(F_1^2 F_j F_k, F_j^2 F_k^2 F_1)$	$(F_1^2 F_j^2 F_k, F_1^2 F_u^2 F_j)$
$(F_1 F_j F_k, F_1 F_u F_v)$	$(F_1 F_j F_k, F_u^2 F_v^2 F_w^2)$	$(F_1^2 F_j F_k, F_j^2 F_k^2 F_u)$	$(F_1^2 F_j^2 F_k, F_1^2 F_u^2 F_k)$
$(F_1 F_j F_k, F_u F_v F_w)$	$(F_1^2 F_j F_k, F_1^2 F_j F_u)$	$(F_1^2 F_j F_k, F_j^2 F_u^2 F_k)$	$(F_1^2 F_j^2 F_k, F_1^2 F_u^2 F_v)$
$(F_1 F_j F_k, F_1^2 F_j F_k)$	$(F_1^2 F_j F_k, F_1^2 F_u F_v)$	$(F_1^2 F_j F_k, F_j^2 F_u^2 F_1)$	$(F_1^2 F_j^2 F_k, F_u^2 F_k^2 F_j)$
$(F_1 F_j F_k, F_1^2 F_j F_u)$	$(F_1^2 F_j F_k, F_j^2 F_1 F_k)$	$(F_1^2 F_j F_k, F_j^2 F_u^2 F_v)$	$(F_1^2 F_j^2 F_k, F_u^2 F_k^2 F_v)$
$(F_1 F_j F_k, F_1^2 F_u F_v)$	$(F_1^2 F_j F_k, F_j^2 F_1 F_u)$	$(F_1^2 F_j F_k, F_u^2 F_v^2 F_k)$	$(F_1^2 F_j^2 F_k, F_u^2 F_v^2 F_j)$
$(F_1 F_j F_k, F_u^2 F_j F_k)$	$(F_1^2 F_j F_k, F_j^2 F_u F_k)$	$(F_1^2 F_j F_k, F_u^2 F_v^2 F_1)$	$(F_1^2 F_j^2 F_k, F_u^2 F_v^2 F_k)$
$(F_1 F_j F_k, F_u^2 F_j F_v)$	$(F_1^2 F_j F_k, F_j^2 F_u F_v)$	$(F_1^2 F_j F_k, F_u^2 F_v^2 F_w)$	$(F_1^2 F_j^2 F_k, F_u^2 F_v^2 F_w)$
$(F_1 F_j F_k, F_u^2 F_v F_w)$	$(F_1^2 F_j F_k, F_u^2 F_j F_k)$	$(F_1^2 F_j F_k, F_1^2 F_j^2 F_k^2)$	$(F_1^2 F_j^2 F_k, F_1^2 F_j^2 F_k^2)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_k)$	$(F_1^2 F_j F_k, F_u^2 F_1 F_k)$	$(F_1^2 F_j F_k, F_1^2 F_j^2 F_u)$	$(F_1^2 F_j^2 F_k, F_1^2 F_j^2 F_u)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_u)$	$(F_1^2 F_j F_k, F_u^2 F_1 F_v)$	$(F_1^2 F_j F_k, F_1^2 F_u^2 F_v^2)$	$(F_1^2 F_j^2 F_k, F_1^2 F_k^2 F_u^2)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_k)$	$(F_1^2 F_j F_k, F_u^2 F_v F_k)$	$(F_1^2 F_j F_k, F_u^2 F_1^2 F_k^2)$	$(F_1^2 F_j^2 F_k, F_1^2 F_u^2 F_v^2)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_v)$	$(F_1^2 F_j F_k, F_u^2 F_v F_w)$	$(F_1^2 F_j F_k, F_u^2 F_v^2 F_k^2)$	$(F_1^2 F_j^2 F_k, F_u^2 F_k^2 F_v^2)$
$(F_1 F_j F_k, F_u^2 F_j^2 F_k)$	$(F_1^2 F_j F_k, F_1^2 F_j^2 F_k)$	$(F_1^2 F_j F_k, F_u^2 F_v^2 F_w)$	$(F_1^2 F_j^2 F_k, F_u^2 F_v^2 F_w)$
$(F_1 F_j F_k, F_u^2 F_j^2 F_v)$	$(F_1^2 F_j F_k, F_1^2 F_j^2 F_u)$	$(F_1^2 F_j^2 F_k, F_1^2 F_j^2 F_u)$	$(F_1^2 F_j^2 F_k, F_1^2 F_j^2 F_u)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_k^2)$	$(F_1^2 F_j F_k, F_1^2 F_u^2 F_k)$	$(F_1^2 F_j^2 F_k, F_1^2 F_k^2 F_j)$	$(F_1^2 F_j^2 F_k, F_1^2 F_u^2 F_v^2)$
$(F_1 F_j F_k, F_1^2 F_j^2 F_u^2)$	$(F_1^2 F_j F_k, F_1^2 F_u^2 F_v)$	$(F_1^2 F_j^2 F_k, F_1^2 F_k^2 F_u)$	$(F_1^2 F_j^2 F_k, F_u^2 F_v^2 F_w^2)$

It follows from (7) that $W_u^{(1)'} W_u^{(1)} \beta_2^{(1)}$ is a set of two equations in two elements of $\beta_2^{(1)}$ and the checking of $\text{Rank } W_u^{(1)'} W_u^{(1)} = 2$ can be done by showing two independent unbiasedly estimable equations in elements of $\beta_2^{(1)}$. We approach this problem by considering Table 2, the equation (5) for $u = 3$ and the equation (5) for the treatment with all factors at level 2.

3. Main Results

We now present our two main results and their proofs.

Theorem 1. The design $D^{(1)}$ is a search design for S_1 .

Proof. The proof consists of showing two independent unbiasedly estimable parametric functions of the elements in every pair in Table 3. We explain the nature of the proof by considering only one out of 21 pairs in Table 3 for the lack of space. We consider the model (4) with the elements of $\beta_2^{(1)}$ as $(F_1 F_j, F_1^2 F_j^2)$. From the design D_1 and the parametric function $S(c_u=1)$ in Table 2, it follows that the parametric functions (i) $F_1 + F_1 F_j$, (ii) $F_j + F_1 F_j$ and (iii) F_u , $u \neq 1, j$, are unbiasedly estimable. Again, from the design D_1 and $S(c_u=2)$ in Table 2, it can be seen that (iv) $F_1^2 + F_1^2 F_j^2$, (v) $F_j^2 + F_1^2 F_j^2$ and (vi) F_u^2 , $u \neq 1, j$, are unbiasedly estimable. For the treatments $1, j$ and the other treatments in D_3 , we find from the equation (5) that (vii) $\mu - F_1 + F_1^2 - 2F_j^2 - 2F_1^2 F_j^2$, (viii) $\mu - F_j + F_j^2 - 2F_1^2 - 2F_1^2 F_j^2$ and (ix) $\mu - 2(F_1^2 + F_j^2)$ are unbiasedly estimable. For the

treatment with all factors at level 2, we find from (5) that (x) $\mu + F_1 + F_j + F_1^2 + F_j^2 + F_1 F_j + F_1^2 F_j^2$ is unbiasedly estimable. From (ix)-(vii)+3(iv)-(i), it follows that $-F_1 F_j + 5F_1^2 F_j^2$ is unbiasedly estimable. From (vii)+(viii)+(x)-3(ix)-6((iv)+(v)), we find that $F_1 F_j - 15F_1^2 F_j^2$ is unbiasedly estimable. We thus have displayed two independent unbiasedly estimable parametric functions in $F_1 F_j$ and $F_1^2 F_j^2$. The checkings for other pairs in Table 3 are similar to the above. This completes the proof of the theorem.

Theorem 2. The design $D^{(2)}$ is a search design for S_2 .

Proof. The proof consists of showing two independent unbiasedly estimable parametric functions of the elements in each of 68 pairs in Table 4 and we explain the nature of the proof by considering the only pair $(F_1^2 F_j F_k, F_1^2 F_j^2 F_u)$. From the design D_2 and the parametric functions $S(c_v=1, c_w=1)$ it follows that (i) $F_v F_w$ with $(v,w) \neq (j,k)$ and (ii) $F_1 F_k + F_1^2 F_j F_k$ are unbiasedly estimable. From the parametric functions $S(c_v=1, c_w=2)$, $S(c_v=2, c_w=1)$ and $S(c_v=2, c_w=2)$, it follows that (iii) $F_v^2 F_w$ with $(v,w) \neq (1,j), (1,k), (1,u), (j,u)$, (iv) $F_1^2 F_j + F_1^2 F_j F_k$, (v) $F_1^2 F_k + F_1^2 F_j F_k$, (vi) $F_1^2 F_u + F_1^2 F_j^2 F_u$, (vii) $F_j^2 F_u + F_1^2 F_j^2 F_u$, (ix) $F_v^2 F_w^2$ with $(v,w) \neq (1,j)$ and (x) $F_1^2 F_j^2 + F_1^2 F_j^2 F_k$ are all unbiasedly estimable. It follows from $S(c_v=1)$ and $S(c_v=2)$ that (xi) F_v , $v \neq j,k,u$, (xii) $F_j + F_1^2 F_j + F_1 F_j + F_1^2 F_j F_k$, (xiii) $F_k + F_1^2 F_k + F_1 F_k + F_1^2 F_j F_k$, (xiv) $F_u + F_1^2 F_u + F_1^2 F_u + F_1^2 F_j^2 F_u$, (xv) F_v^2 , $v \neq 1,j$, (xvi) $F_1^2 + F_1^2 F_j + F_1^2 F_k + F_1^2 F_u + F_1^2 F_j^2 + F_1^2 F_j F_k + F_1^2 F_j^2 F_u$, (xvii) $F_j^2 + F_1^2 F_j + F_1^2 F_j^2 + F_1^2 F_j^2 F_u$ are unbiasedly estimable. For

the treatments i, j, k, u and the other (for $m > 4$) treatments in D_3 , we get from (5), (ix), (xi) and (xv) that (xviii) $\mu + F_1^2 - 2F_j^2 - 2F_1^2 F_j^2$, (xix) $\mu - F_j^2 + F_j^2 - 2F_1^2 + 2F_1^2 F_j^2 - 2F_1^2 F_j^2$, (xx) $\mu - F_k^2 - 2F_1^2 - 2F_j^2 + 2F_1^2 F_k^2 + 4F_1^2 F_j^2$, (xxi) $\mu - F_u^2 - 2F_1^2 - 2F_j^2 + 2F_1^2 F_u^2 + 2F_j^2 F_u^2 + 4F_1^2 F_j^2 - 4F_1^2 F_j^2 F_u^2$ and (xxii) $\mu - 2F_1^2 - 2F_j^2 + 4F_1^2 F_j^2$ are all unbiasedly estimable. (We in fact will not use the equation (xxii).) From (xix) - (xviii) + 3 (vii) - 3 (vi) - 3 (v) - 6 (iv) - (ii) - 3 (xvii) + 3 (xvi) + (xi) and (xx) - (xviii) - 9 (x) - 3 (vi) - 6 (v) - 3 (iv) - (ii) + 3 (xvi) + (xi) we find that $-6F_1^2 F_j F_k$ and $-6F_1^2 F_j F_k - 9F_1^2 F_j^2 F_u$ are unbiasedly estimable. This displays two independent linear functions of $F_1^2 F_j^2 F_k$ and $F_1^2 F_j^2 F_u$ which are unbiasedly estimable. The checkings for the other pairs in Table 4 can be done similarly. This completes the proof of the theorem.

4. Concluding Remarks.

- a. The design $D^{(1)}$ has $(1+3m)$ treatments and the design $D^{(2)}$ has $(1+m+2m^2)$ treatments. Minimal resolutions III, V and VII plans require $(1+2m)$, $(1+2m^2)$ and $(1+2m^2+8\binom{m}{3})$ treatments.
- b. The design $D^{(1)}$ can search one nonzero two factor interaction. A natural question comes up, "can $D^{(1)}$ search one nonzero 3-factor or higher order interaction?" The answer is "NO". For example, in case $m=5$ if we consider the model (4) with the elements of $\beta_2^{(1)}$ as $(F_1 F_2, F_3^2 F_4^2 F_5)$, we can not find two independent unbiasedly estimable linear functions of $F_1 F_2$ and $F_3^2 F_4^2 F_5$. There are in fact many such pairs.

- c. By calling the level 2 as the level 0 (the method of collapsing levels), and omitting the replicated treatments, we get essentially the two series of search designs obtained in Srivastava and Ghosh (1976), Srivastava and Gupta (1979) for 2^m factorial experiments. However, the designs thus obtained have more strength in the sense that they can search any nonzero 1-factor or higher order interactions, where $i = 2$ and 3.
- d. This research started from an unpublished technical report of Ghosh (1985) and the motivation was to reduce the number of treatments. The designs $D^{(1)}$ and $D^{(2)}$ show the success in our effort.

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY NA		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE NA			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report No. 144		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 86 - 2008	
6a. NAME OF PERFORMING ORGANIZATION University of California, Riverside	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM	
6c. ADDRESS (City, State and ZIP Code) Department of Statistics, University of California, Riverside, CA 92521		7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (If applicable) nm	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-86-0048	
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448		10. SOURCE OF FUNDING NOS.	
		PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304
11. TITLE (Include Security Classification) TWO NEW SERIES OF SEARCH DESIGNS FOR 3 ^m FACTORIAL EXPERIMENTS		TASK NO. A5	WORK UNIT NO.
12. PERSONAL AUTHOR(S) Subir Ghosh and Xiao Di Zhang			
13a. TYPE OF REPORT Interim	13b. TIME COVERED FROM Dec '85 to June '86	14. DATE OF REPORT (Yr., Mo., Day) 11 July 1986	15. PAGE COUNT 15
16. SUPPLEMENTARY NOTATION Submitted to Utilitas Mathematica			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB. GR.	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) In this paper two new series of search designs with very small number of treatments are presented for 3 ^m factorial experiments. The first series of designs can search one nonzero two factor interaction and estimate it along with the general mean and the main effects. The second series can search one nonzero three factor interaction and estimate it along with the two factor and lower order interactions.			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL Major Brian W. Woodruff		22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5027	22c. OFFICE SYMBOL AFOSR/NM

END

12-86

DTIC