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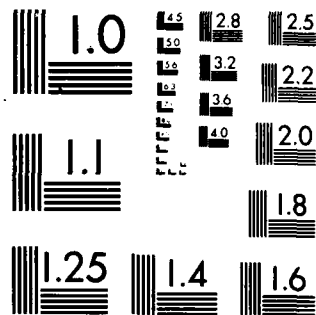
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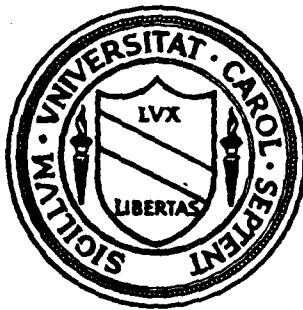
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CENTER FOR STOCHASTIC PROCESSES

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by

Olav Kallenberg

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ON THE THEORY OF CONDITIONING IN POINT PROCESSES

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Abstract: This paper, written for the Proceedings of the First World Congress of the Bernoulli Society of Mathematical Statistics and Probability in Tashkent, 1986, summarizes the most fundamental concepts and relationships in the theory of conditioning in point processes. Though most results are taken from the author's book on Random Measures or from an earlier survey paper, the present emphasis is somewhat different, and there are even a couple of new results, mentioned here without proof.

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ON THE THEORY OF CONDITIONING IN POINT PROCESSES

by Olav Kallenberg, Göteborg, Chapel Hill, Auburn

1. Introduction

By a point process we mean a random configuration of isolated points in some topological space. Point processes arise naturally in a great variety of contexts, both theoretical and applied.

In a physical or biological application, the 'points' may e.g. be the positions of particles or organisms, whereas in queueing theory they may be the arrival times of customers. However, the scope of applications is not restricted to situations describable in terms of discrete points, since other types of geometrical objects, such as planes or circles, may be identified with points in suitable parameter spaces. In theoretical applications, the 'points' may be the level crossings of a random process, or the times and sizes of its jumps. Following Itô, we may further consider the point process of excursions from a fixed state of a Markov process, marked in a diagram versus local time at that state. In general, it seems that a point process description may be fruitful whenever there are random objects which may be counted.

Mathematically, point processes may be described as locally finite random sets. Since the counting properties are fundamental, it is often convenient to identify the collection of points with the associated counting random measure. Conversely, every integer valued random measure on a sufficiently nice space is the counting measure of some locally finite random set, provided the 'points' are allowed to have multiplicities.

In the sequel, we shall only consider simple point processes ξ , where no multiplicities occur. For convenience, the topological space S where ξ is defined will be taken to be a locally compact

second countable Hausdorff space. The number of points ξ_B in a bounded Borel set $B \subset S$ will be assumed to be finite and measurable, hence a random variable.

Given a point process ξ , it is natural to look at the conditional distribution of ξ (or of some other random object defined on the same probability space), given some partial information about ξ , such as the configuration in some subset. In statistical mechanics, for instance, one is interested in descriptions in terms of the interaction potential between the particles of the conditional distribution in a bounded set B , given the configuration outside.

Another type of conditioning is to specify a number of points of ξ , and to ask for the distributional properties of the remainder of the point process. This kind of conditioning arises naturally in the context of queueing systems, when one wants to describe what happens to a 'typical' customer.

A third type of conditioning arises if one wants to describe a point process of events on the time axis in a dynamical way, and look at the conditional probability of having an event in a small time interval dt , when the history up to time t is known. This leads to the familiar and powerful martingale approach to point processes. If the time axis is replaced by some more general space, the possibility of a dynamical description is lost, but it still makes sense to consider the conditional contribution to a space element dt , given the configuration outside t . Such ideas have turned out to be useful in stochastic geometry.

Historically, theories of conditioning have been developed independently in the different situations described above. It was only recently that the various aspects of conditioning could be merged into an allembicing and systematic theory of conditioning

in point processes. Here we shall briefly outline the contours of this theory. For further information, as well as for historical remarks and complete bibliography, the reader is referred to Kallenberg (1983/86) and Kallenberg (1984).

2. Palm measures

Univariate Palm probabilities were introduced in 1943 by the Swedish mathematician and engineer C. Palm in an extensive but non-rigorous study of telephone congestion. Hinchin later adopted some of Palm's ideas for his 1955 monograph on queuing theory, hence the name Palm-Hinchin theory for the early developments in this area. However, the modern theory of Palm distributions and Palm measures originated with Ryll-Nardzewski's approach of 1961.

Here the reduced Palm distribution Q_s of a point process ξ at the point $s \in S$ is defined a.e. as the Radon-Nikodym derivative

$$Q_s(A) = \frac{E[\xi(ds); \xi - \delta_s \in A]}{E\xi(ds)}, \quad s \in S. \quad (1)$$

Note that this approach requires the intensity measure $E\xi$ to be σ -finite. Similarly, the multivariate (reduced) Palm distributions are given by

$$Q_{s_1, \dots, s_n}(A) = \frac{E[\xi^{(n)}(ds_1 \dots ds_n); \xi - \delta_{s_1} - \dots - \delta_{s_n} \in A]}{E\xi^{(n)}(ds_1 \dots ds_n)}, \quad (2)$$

where $\xi^{(n)}$ denotes the restriction of the product measure $\xi^n = \xi \times \dots \times \xi$ to the set where all coordinates are distinct. It is understood that regular (i.e. measure valued) versions should be chosen in (1) and (2). Thus these formulas define disintegrations of the so called Campbell measures C_1 and C_n on the right. When $E\xi$ and $E\xi^{(n)}$ are infinite, it is still possible to obtain disintegrations

$$C_n(B \times A) = \int_B q_{s_1 \dots s_n}(A) \nu_n(ds_1 \dots ds_n), \quad (3)$$

for some suitable supporting measures ν_n and Palm measures $q_{s_1 \dots s_n}$,

where the latter may now be unbounded and are unique only up to an arbitrary normalization. The final step is to put $Q_\mu = Q_{s_1 \dots s_n}$ or $q_\mu = q_{s_1 \dots s_n}$ for a finite counting measure μ with atoms at $s_1, \dots, s_n$.

The conditioning interpretation of the kernels Q and q may be justified in various ways. One is to show that Q_μ can be approximated by the conditional distribution of ξ outside the support of μ , given that ξ has 'points' within small neighbourhoods of $s_1, \dots, s_n$. Thus Q_μ may indeed be thought of as the conditional distribution of $\xi - \mu$, given that $\xi \geq \mu$. The interpretation of q_μ is similar.

Another relation to ordinary conditioning which justifies the above definitions is the fact that, for bounded Borel sets $B \subset S$,

$$P[B^C \xi \in A | B\xi] = \frac{q_{B\xi} \{u: B^C u \in A, \mu B = 0\}}{q_{B\xi} \{u: \mu B = 0\}}. \quad (4)$$

Here $B\xi$ and $B^C\xi$ denote the restrictions of ξ to B and its complement B^C . Implicit in (4) is the fact that the denominator on the right is a.s. finite and positive. When the Palm distributions Q_μ exist, (4) expresses the intuitively obvious fact that conditioning on $\{B\xi = \mu\}$ is equivalent to conditioning first on $\{\xi \geq \mu\}$, and then, in the resulting conditional distribution, on the event $\{(\xi - \mu)B = 0\}$.

3. Gibbs and Papangelou kernels

The so called first order Papangelou kernel $\eta = \eta_1$ of a point process ξ was introduced by the present author in 1978 through the formula

$$\eta B = \frac{P[\xi B = 1 | B^C \xi]}{P[\xi B = 0 | B^C \xi]} \quad \text{a.s. on } \{\xi B = 0\}, \quad (5)$$

plus the requirement that $\eta(\text{supp } \xi) = 0$. It may be shown that this a.s. uniquely defines a locally finite random measure η on S .

Similarly, the n -th order Papangelou kernel is a random measure on S^n given by

$$\eta_n(B_1, \dots, B_n) = \frac{P[\xi B_1 = \dots = \xi B_n = 1, \xi B = n | B^C \xi]}{P[\xi B = 0 | B^C \xi]} \quad \text{a.s. on } \{\xi B = 0\},$$

for disjoint subsets $B_1, \dots, B_n$ of B , plus the requirements that the

one-dimensional projections of η_n should not charge $\text{supp } \xi$, and further that all diagonal planes in S^n should have measure zero. The final step is to define the Gibbs kernel Γ as the random measure on the space of finite counting measures on S , given by

$$\Gamma(A \cap \{\mu B^C = 0\}) = \frac{P[\xi B \in A | B^C \xi]}{P[\xi B = 0 | B^C \xi]} \quad \text{a.s. on } \{\xi B = 0\}, \quad (7)$$

again with an extra requirement that Γ should a.s. be restricted to measures μ with $\mu(\text{supp } \xi) = 0$. It is easy to check that Γ and the η_n are related through the formula

$$\Gamma(A) = \sum_{n=0}^{\infty} \frac{1}{n!} \eta_n \{\delta_{s_1} + \dots + \delta_{s_n} \in A\} \quad \text{a.s.}, \quad (8)$$

provided we put $\eta_0 = \delta_0$. Other interesting facts are the supermultiplicativity

$$\Gamma_{m+n}(ds_1 \dots ds_m dt_1 \dots dt_n) \geq \eta_m(ds_1 \dots ds_m) \eta'_n(dt_1 \dots dt_n), \quad (9)$$

where η'_n denotes the Papangelou kernel of the point process $\xi + \xi_{s_1} + \dots + \xi_{s_m}$, and further the inequality

$$E f(\xi) \geq E \left[\int f(\xi + \mu) 1_{\{\mu B^C = 0\}} \Gamma(d\mu); \xi B = 0 \right]. \quad (10)$$

It may be shown that equality holds in (9) and (10) whenever

$$(\Sigma): P[\xi B = 0 | B^C \xi] > 0 \quad \text{a.s. for bounded } B \subset S. \quad (11)$$

In that case, the random measures η_n may be obtained from the corresponding Campbell measures C_n by disintegration in the second variable:

$$C_n(B \times A) = E[\eta_n B; \xi \in A]. \quad (12)$$

In this sense, the Papangelou kernels may be regarded as 'duals' to the reduced Palm measures. Under (Σ) , even the Gibbs kernel may be obtained through disintegration of a suitable measure, namely the so called compound Campbell measure C , given by

$$C(M \times A) = \sum_{n=0}^{\infty} \frac{1}{n!} C_n(\{\delta_{s_1} + \dots + \delta_{s_n} \in M\} \times A), \quad (13)$$

where M is a set of finite counting measures. Here the term with

index 0 should be interpreted as $1\{0 \in M\}P\{\xi \in A\}$, and the disintegration of C is given by

$$C(M \times A) = E[\Gamma(M); \xi \in A]. \quad (14)$$

The measure C may also be defined directly through the formula

$$C(M \times A) = E \sum_{u \leq \xi} 1\{u \in M, \xi - u \in A\}, \quad (15)$$

where the summation extends over all finite counting measures $u \leq \xi$.

This shows in particular that C is symmetric whenever ξ is a.s. finite. Since the Palm measures and the Gibbs kernel were both obtained by disintegration of C , but in different directions, the two kernels must be essentially equal, apart from a normalization.

In fact, it may be shown that, for finite ξ ,

$$\Gamma(M) = \frac{q_{\xi}(M)}{q_{\xi}\{0\}} \quad \text{a.s.} \quad (16)$$

In this sense, the sets of Palm measures and Papangelou kernels may thus be considered as 'self-dual'. It is interesting to compare the above fact with formula (4).

We conclude this section by indicating the role of the Gibbs and Papangelou kernels in statistical mechanics. Here it is assumed that each η_n is absolutely continuous with respect to Lebesgue measure, and admits a strictly positive density Y_n . The process $-\log Y_n(s_1, \dots, s_n)$ is then interpreted as the local energy required to add new particles at $s_1, \dots, s_n$. Under the physically reasonable assumption (Σ) in (11), formula (9) turns into an equality, expressing simply the conservation law for energy. Needless to say, the Gibbs kernel is named after J.W. Gibbs, the founder of statistical mechanics, in whose writings it occurs implicitly.

4. Local conditioning

A point process ξ on R_+ may be regarded as a (local) submartingale with respect to its natural filtration $(\mathcal{F}_t)$, and therefore has a Doob-Meyer decomposition into a predictable and non-decreasing component $\hat{\xi}$, and a local martingale $\xi - \hat{\xi}$. It is natural to identify ξ and its compensator $\hat{\xi}$ with the associated random measures, and to characterize $\hat{\xi}$ through the equation

$$E \int V d\xi = E \int V d\hat{\xi}, \quad (17)$$

valid for arbitrary predictable and non-negative processes V . It is well-known that $\hat{\xi}$ can also be constructed explicitly through discrete approximations of the type

$$\hat{\xi}_t = \lim_{n \rightarrow \infty} \sum_{j=1}^n P[\xi_{t_j} > \xi_{t_{j-1}} | \mathcal{F}_{t_{j-1}}], \quad (18)$$

where $0 = t_0 < t_1 < \dots < t_n = t$, and it is assumed that the mesh size of the n -th partition tends to zero.

In general spaces, it is natural to imitate the above procedure by forming the sums

$$\sum_j P[\xi I_{nj} = 1 | I_{nj}^C \xi] \leq \sum_j P[\xi I_{nj} > 0 | I_{nj}^C \xi] \leq \sum_j E[\xi I_{nj} | I_{nj}^C \xi], \quad (19)$$

where the I_{nj} for fixed n form a partition of S , and where the summation extends over all I_{nj} in a fixed set B . Assuming that $(I_{n+1,j})$ is a refinement of (I_{nj}) , that the diameters of I_{nj} tend uniformly to zero, and that B is a finite union of sets I_{nj} , it may be shown that the first two sums in (19) (and even the third when $E\xi B < \infty$) converge a.s. to some random variable ζB , where ζ is a locally finite random measure on S , which is independent of (I_{nj}) and related to the Papangelou kernel η via the formulas

$$\zeta(ds) = \frac{\eta(ds)}{1 + \eta\{s\}}, \quad \eta(ds) = \frac{\zeta(ds)}{1 - \zeta\{s\}}, \quad s \notin \text{supp } \xi. \quad (20)$$

The random measure ζ is known as the conditional intensity of ξ .

The name Papangelou kernel for η derives from the fact that

F. Papangelou was the first, in 1974, to prove convergence of one of the sums in (19), in a special case when $\zeta = \eta$. He also established the important fact that, for S of the form $R \times S'$, ξ is a mixture of stationary Poisson processes iff ζ is a.s. shift invariant. This fact has some rather remarkable consequences in stochastic geometry.

By the similarity of (18) and (19), one may expect that even the conditional intensity ζ may be characterized, like $\hat{\xi}$ in (17), as a dual projection with respect to some suitable σ -field $\mathcal{Z}$ in the product space $S \times \Omega$, where Ω denotes the underlying probability space. In fact, van der Hooven showed in 1982 that $\mathcal{Z}$ can be chosen as the exvisible σ -field generated by all sets of the form $B \times F$, where B is a bounded Borel set in S , while F lies in the completion of $\sigma(B^C \xi)$. Thus, in analogy with (17), ζ can be characterized as the a.s. unique 'exvisible' random measure such that

$$E \int Z d\xi = E \int Z d\zeta \quad (21)$$

holds for all 'exvisible' and non-negative processes Z on S .

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