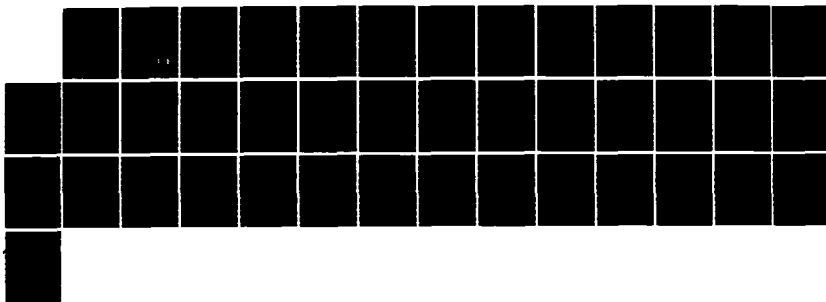


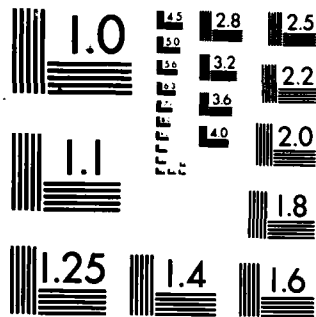
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FUNDAMENTAL QUANTUM 1/F NOISE IN ULTRASMALL SEMICONDUCTOR DEVICES AND THEIR OPTIMAL DESIGN PRINCIPLES

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FIRST ANNUAL REPORT

AFOSR Grant No. 85 - 0130

Starting Date: May 1, 1985; Date of this Report: May 30, 1986

Abstract

During this period a second - quantized derivation of the quantum $1/f$ effect was developed by the principal investigator. This derivation is based on the pair correlation function and automatically includes the right form of exchange between fermions and between bosons.

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As a first step of a more general study of $1/f$ noise in semiconductor devices n^+-p diodes have been investigated with emphasis on HgCdTe photodetectors. Quantum $1/f$ noise has been calculated in the surface and bulk recombination currents, in the diffusion and field currents, and in the tunneling currents. Due to the large localized electric field at the surface, a larger fractional quantum $1/f$ noise power is obtained for surface recombination currents than for similar bulk recombination currents. All quantum $1/f$ noise calculations are first principles calculations with no free parameters, based on the quantum $1/f$ effect in scattering and recombination cross sections, as well as in tunneling rates.

Together, the quantum $1/f$ mobility fluctuations, bulk and surface recombination speed fluctuations and tunneling rate fluctuations can account for the observed $1/f$ noise and can be used for optimizing small devices, as indicated by experiments at SBRC, Univ. of Minnesota and Florida. Some suggestions are given at the end of Sec. IV. For devices larger than 10 - 100 microns coherent state quantum $1/f$ noise becomes important, according to a new interpolation formula developed by the author in June - July 1985. The new interpolation formula which bridges the gap between conventional (incoherent) quantum $1/f$ noise and coherent state quantum $1/f$ noise is in general agreement with measurements in p-n diodes and transistors, and will be tested in more detail in the near future. The theory has also been successfully applied to SQUIDS.



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I. INTRODUCTION

Progress has been achieved during the first year of this grant in three directions: 1) Performance of a second - quantized derivation of the conventional quantum 1/f effect, which allows for a better physical understanding of this phenomenon as we shall see in Sec.II; 2) Calculation of the effect of a finite mean free path of the current carriers on the conventional quantum 1/f effect in condensed matter as presented in Sec. III; 3) Detailed calculation of 1/f noise in $n^+ - p$ HgCdTe photodetectors which will be reviewed in Sec. IV; 4) Generation of a heuristic practical formula connecting "conventional" and "coherent state" quantum 1/f noise, which will be reviewed in Sec. V; and 5) Performance of a simple calculation of quantum 1/f noise in SQUIDs which will be considered in Sec. IV.

The results of 1/f noise calculations in $n^+ - p$ HgCdTe diodes performed on the basis of the quantum 1/f noise theory are presented and compared with measurements at temperatures of 77K, 193K and 300K, as well as with other experimental data. Although the calculations do not contain adjustable parameters, a very encouraging correspondence between theory and experiment is found, and a unified first principles description of 1/f noise contributions from

scattering, tunneling, or surface and bulk recombination cross sections is obtained.

Experiments on n^+-p HgCdTe diodes performed at temperatures between 77K and 300K indicate a linear dependence of the $1/f$ current noise power spectrum on dark current at sufficiently large bias and a quadratic dependence at very low bias. The observed values of the Hooge parameter are compared with the predictions of the quantum $1/f$ theory in the diffusion model and found to be between coherent and incoherent quantum $1/f$ values, closer to the latter. The inclusion of bulk and surface quantum $1/f$ recombination rate fluctuations improves the agreement with the quantum $1/f$ theory for n^+-p diodes.

An electrically charged particle includes the bare particle and its field. The field has been shown in the last two decades to be in a coherent state, which is not an eigenstate of the Hamiltonian. Consequently, the physical particle is not described by an energy eigenstate, and is therefore not in a stationary state. In Sec. V of this report we show that the fluctuations arising from this non-stationarity have a $1/f$ spectral density and affect the ordered, collective, or translational motion of the current carriers. This "coherent" quantum $1/f$ noise should be present along with the familiar quantum $1/f$ effect of elementary cross sections and process rates introduced ten years ago, just as the magnetic energy of a biased semiconductor sample coexists with the kinetic energy of the individual, randomly moving, current carriers. The amplitude of the quantum $1/f$ effect is always the difference of the coherent quantum $1/f$ noise amplitudes in the "out" and "in" states of the process under consideration and dominates in small samples, while large samples should exhibit the larger coherent quantum $1/f$ noise.

In the last section of this report, Sec. VI, the fundamental quantum $1/f$ fluctuations of the cross sections and transition rates which determine the

normal resistance are evaluated for the case of a Josephson junction.

Considering the velocity change in the quantum $1/f$ formula equal to twice the

Fermi velocity and the concentration of carriers in the barrier 10^{19}cm^{-3} , a

spectral density of fractional fluctuations in the normal resistance of the barrier

of $4 \cdot 10^{-14}/f$ is obtained for a junction with a volume of the barrier of 10^{-12}cm^3 .

These fluctuations are inversely proportional to the barrier volume and result in

voltage fluctuations both directly and through the dependence of the critical

current on the normal resistance, in good agreement with the experimental data.

II. SECOND - QUANTIZED FORMULATION OF THE QUANTUM $1/f$ EFFECT

In this Section we focus on the particles emerging from an interaction of any

kind, e.g. scattering, tunneling, emission processes, etc.. Our goal is to calculate

the pair - correlation function both for the case that the outgoing particles,

which are always considered identical noninteracting particles, are fermions, and

in the case that they are bosons.

2.1. THE CASE OF FERMIONS

The state of two fermions, both outgoing from the same interaction, but with statistically independent bremsstrahlung energy losses accompanying the process for them, is

$$\frac{1}{\sqrt{2}} \int d\xi \int d\eta \left(e^{ik\xi} + \sum_{\alpha} b(\alpha) e^{i(k-\alpha)\xi} \right) \times \\ \left(e^{ik'\eta} + \sum_{\alpha'} \beta(\alpha') e^{i(k'-\alpha')\eta} \right) \psi_s^\dagger(\xi) \psi_{s'}^\dagger(\eta) |0\rangle; \quad (2.1)$$

where s and s' are the spins, while $b(\alpha)$ and $\beta(\alpha')$ are the spontaneous bremsstrahlung energy loss amplitudes of the two particles. For any α they differ only by their independent random phases. Here ψ designates the field operators of argument x, y , etc, $= u - vt$, and u is in general different for each of

the arguments mentioned, while t is considered the same for all. u is the coordinate along the scattered beam, and $v = du/dt$ is the velocity of the particles in the scattered beam.

The operator of the pair - correlation is

$$\Theta = \sum_{s,s'} \psi_s^+(x_1) \psi_{s'}^+(x_2) \psi_{s'}(x_2) \psi_s(x_1). \quad (2.2)$$

This corresponds to a density autocorrelation function. Using the well known anticommutation relations

$$\psi_s^+(x) \psi_{s'}(y) + \psi_{s'}(y) \psi_s^+(x) = \delta(x-y) \delta_{ss'}, \quad (2.3)$$

as well as the corresponding homogeneous relations for operators of the same kind, we obtain:

$$\begin{aligned} \langle S_{\uparrow\uparrow}^z | \Theta_{\uparrow\uparrow} | S_{\uparrow\uparrow}^z \rangle &= \langle 0 | \psi_{\uparrow}(\eta) \psi_{\uparrow}(\xi) \psi_{\uparrow}^+(x_1) \psi_{\uparrow}^+(x_2) \psi_{\uparrow}(x_2) \psi_{\uparrow}(x_1) \psi_{\uparrow}^+(\xi) \psi_{\uparrow}^+(\eta) | 0 \rangle \\ &= [-\delta(\eta'-x_1)\delta(\xi'-x_2) + \delta(\xi'-x_1)\delta(\eta'-x_2)] [\delta(\eta-x_2)\delta(\xi-x_1) - \delta(\xi-x_2)\delta(\eta-x_1)], \end{aligned} \quad (2.4)$$

$$\begin{aligned} \langle S_{\uparrow\uparrow}^z | \Theta_{\uparrow\downarrow} | S_{\uparrow\downarrow}^z \rangle &= \langle 0 | \psi_{\uparrow}(\eta) \psi_{\uparrow}(\xi) \psi_{\uparrow}^+(x_1) \psi_{\downarrow}^+(x_2) \psi_{\downarrow}(x_2) \psi_{\uparrow}(x_1) \psi_{\downarrow}^+(\xi) \psi_{\uparrow}^+(\eta) | 0 \rangle \\ &= \delta(\eta'-x_2)\delta(\xi'-x_1)\delta(\eta-x_2)\delta(\xi-x_1), \end{aligned} \quad (2.5)$$

$$\begin{aligned} \langle S_{\uparrow\uparrow}^z | \Theta | S_{\uparrow\uparrow}^z \rangle &= \langle 0 | \psi_{\uparrow}(\eta) \psi_{\downarrow}(\xi) \psi_{\uparrow}^+(x_1) \psi_{\downarrow}^+(x_2) \psi_{\downarrow}(x_1) \psi_{\uparrow}^+(\xi) \psi_{\downarrow}^+(\eta) | 0 \rangle \\ &= \delta(\xi'-x_2)\delta(\eta'-x_1)\delta(\xi-x_2)\delta(\eta-x_1), \end{aligned} \quad (2.6)$$

We also obtain three similar expectation values with all spins reversed.

The spin - averaged pair correlation function is then

$$A = (1/4) \sum_{ss'} \langle S_{ss'}^z | \Theta_{\uparrow\uparrow} + \Theta_{\uparrow\downarrow} + \Theta_{\downarrow\uparrow} + \Theta_{\downarrow\downarrow} | S_{ss'}^z \rangle. \quad (2.7)$$

Substituting the calculated expectation values, we obtain

$$A(x_1, x_2) = 1/2 + \sum_{\mathbf{k}} |b(\mathbf{k})|^2 [2 - \cos \mathbf{k}(x_1 - x_2)] + \sum_{\mathbf{k}, \mathbf{k}'} |b(\mathbf{k})|^2 |b(\mathbf{k}')|^2 [1 - (1/2) \cos(\mathbf{k} - \mathbf{k}')(x_1 - x_2)], \quad (2.8)$$

which yields the fractional spectral density

$$\frac{S_n}{(n)^2} = \frac{S_j}{(j)^2} = \frac{2 |b(\mathbf{k})|^2}{1 + 4 \sum |b(\mathbf{k})|^2 + 2 \sum |b(\mathbf{k})|^2 |b(\mathbf{k}')|^2} \cong 2 |b(\mathbf{k})|^2 = \frac{2 \alpha A}{\omega^{1-\alpha A}} = \frac{2 \alpha A}{f^{1-\alpha A}} \quad (2.9)$$

which is in agreement with our previous results, and also includes a 180° phase shift due to the exclusion principle which is important only at short distances between the particles. In the final form we have transformed to the frequency f .

2.2. CASE OF THE BOSONS

In this case we replace all anticommutators with commutators and obtain

$$A(x_1, x_2) = 2 + 2 \sum |b(\mathbf{k})|^2 [1 + \cos \mathbf{k}(x_1 - x_2)] + \sum \sum |b(\mathbf{k})|^2 |b(\mathbf{k}')|^2 [1 + \cos(\mathbf{k} - \mathbf{k}')(x_1 - x_2)],$$

$$\frac{S_n}{(n)^2} = \frac{S_j}{(j)^2} = |b(\mathbf{k})|^2 = \frac{\alpha A}{\omega^2}. \quad (2.10)$$

Here at short distances we notice an increase of A .

Both results generalize our previous results to the case of short distances.

III. EFFECT OF A FINITE MEAN FREE PATH

Here we calculate the radiation from a large number N of very frequent scattering events and compare with what we would expect from a single isolated scattering of the current carrier. First we review the derivation of the power radiated in a collision with acceleration $\vec{v}$ of the carrier of charge e :

$$\square \vec{A} = -\frac{4\pi}{c} \vec{j}$$

$$\vec{A}(x) = (1/c) \int \frac{\vec{j}(x', t - \frac{|x-x'|}{c})}{|x-x'|} d^3 x' = (1/c\eta) \int \vec{j}(x) - \frac{|x'-x|}{c} d^3 x'$$

$$\vec{E} = -\frac{1}{c} \dot{\vec{A}} = \frac{e \vec{v}}{c r} \quad \vec{B} = \frac{\vec{k}}{|\mathbf{k}|} \times \vec{E}$$

$$\vec{Y} = \frac{c}{4\pi} \vec{E} \times \vec{H} \quad P = \int \vec{Y} \cdot d\vec{S} = \frac{2e^2}{3c^2} \vec{v}^2$$

For one collision we separate a random part of zero average $\vec{\Delta v}$, and an average part proportional with the previous velocity, in the total velocity change:

$$\vec{v} = \delta(t) (\vec{\Delta v})_{\text{tot}} \quad (\vec{\Delta v})_{\text{tot}} = \underbrace{\vec{\Delta v}}_{\text{totally random}} - \gamma \vec{v} \quad (\text{Def of } \gamma) \quad 0 < \gamma < 1.$$

The total energy emitted by one particle is

$$W = \int_{-\infty}^{\infty} P dt = \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} dt \left| \left[(\vec{\Delta v})_{\text{tot}} \int_{-\infty}^{\infty} e^{2\pi i f t} dt + (\vec{\Delta v})_{\text{tot}} \int_{-\infty}^{\infty} e^{2\pi i f (1-t)} dt + \dots \right] \right|^2.$$

Neglecting the short times between collisions, we obtain in the low f limit

$$W = \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} df e^{2\pi i f (t-t')} \times \left| \vec{\Delta v} - \gamma \vec{v} + \Delta_2 \vec{v} - \gamma [\Delta_1 \vec{v} + (1-\gamma) \vec{v}] + \Delta_3 \vec{v} - \gamma \{ \Delta_2 \vec{v} + (1-\gamma) [\Delta_1 \vec{v} + (1-\gamma) \vec{v}] \} \right. \\ \left. + \Delta_4 \vec{v} - \gamma \{ \Delta_3 \vec{v} + (1-\gamma) [\Delta_2 \vec{v} + (1-\gamma) \vec{v}] \} + \dots \right|^2 \quad \text{for } f \rightarrow 0.$$

Summing the geometric series repeatedly,

$$W = \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} df \left| -\gamma \vec{v} \sum_{n=0}^{N-1} (1-\gamma)^n + \Delta_1 \vec{v} \left[1 - \gamma \sum_{n=0}^{N-2} (1-\gamma)^n \right] + \Delta_2 \vec{v} \left[1 - \gamma \sum_{n=0}^{N-3} (1-\gamma)^n \right] + \dots \right|^2 \\ = \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} df \left| \vec{v} \left[1 - (1-\gamma)^N \right] + \Delta_1 \vec{v} (1-\gamma)^{N-1} + \Delta_2 \vec{v} (1-\gamma)^{N-2} + \dots + \Delta_N \vec{v} \right|^2$$

Finally, we average over the independent and random $\vec{\Delta v}$ values. Summing up geometric series again, we obtain for N going to infinity in the limit

$$\overline{W} = \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} df \left\{ v^2 \left[1 - (1-\gamma)^N \right]^2 + (\vec{\Delta v})^2 \frac{1 - (1-\gamma)^{2N}}{1 - (1-\gamma)^2} \right\} \\ \xrightarrow{N \rightarrow \infty} \frac{4e^2}{3c^3} \int_{-\infty}^{\infty} df \left\{ v^2 + \frac{(\Delta v)^2}{\gamma(2-\gamma)} \right\}$$

The e.l.f. emission of radiation from a large number of statistically similar random collisions of a current carrier is thus close to the emission from just one average collision or scattering process, as I had assumed before.

The corresponding quantum $1/f$ effect in the current carried just by this carrier will therefore be

$$\frac{S_{\delta j}(f)}{(j^2)} = 2 \cdot \frac{4e^2}{3c^3 hf} \left\{ v^2 + \frac{(\overline{\Delta v})^2}{\gamma(2-\gamma)} \right\} = \frac{4\alpha}{3\pi f} \left\{ \frac{v^2}{c^2} + \frac{(\overline{\Delta v})^2}{c^2 \gamma(2-\gamma)} \right\}$$

The two terms in curly brackets should be roughly equal in average. Note that the corrective factor including the memory parameter γ is of the order of the unity, and usually close to unity.

IV. 1/f NOISE STUDY OF n^+p HgCdTe PHOTODETECTORS

Infrared detector applications have led to increased interest in understanding and controlling 1/f noise in n^+p junctions, as reflected in several recent studies¹⁻⁶. These studies have associated 1/f noise with surface and bulk leakage currents. The leakage currents causing 1/f noise have been identified as generation - recombination (GR) currents when the gate voltage was optimized ($V_G=0$), and tunneling currents otherwise. The study by Radford and Jones has also noticed a lower fractional 1/f noise power in double - epilayer junctions ($4 \cdot 10^{-8}$) compared to ion - implanted diodes (10^{-6}), and by one or two orders of magnitude lower noise in SiO_2 passivated diodes compared with ZnS passivated junctions.

The present report presents the results of our experiments on 1/f noise in n^+p $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ photodiodes and in other junction devices together with the application of the quantum 1/f noise theory⁷⁻¹⁵, in order to gain both a better understanding of 1/f noise and a tool for better controlling it. We will emphasize the comparison of theory and experiment at every step.

We first address the problem of 1/f noise phenomenologically in Sec. 4.1 which also explains our method of comparing experiment and theory. Sec. 4.2 presents some predictions of the quantum 1/f noise theory, including both conventional quantum 1/f noise and coherent states quantum 1/f noise. Section 4.3 brings a comparison with the experiment, and Sec. 4.4 presents some quantum 1/f noise contributions which are important for n^+p diodes. Sec. 4.5 contains a concluding discussion of the 1/f noise problem in n^+p $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ photodiodes.

4.1. PHENOMENOLOGY OF 1/F NOISE

There are many forms of low frequency current noise observed in diodes and other junction devices. As forms which are not of 1/f type we mention here: 1) Shot noise with a

white $2eI$ spectrum for any current I ; 2) GR noise with Lorentzian spectral density components $4eI^2/(1 + 4\pi^2 f^2 \tau^2)$ for each current I of carriers of lifetime τ ; 3) Burst noise with a spectrum and current dependence similar to GR noise with the time constant of the controlling trap(s) substituted for τ ; 4) Thermal relaxation fluctuations, also characterized by a Lorentzian spectrum, with the thermal relaxation time substituted for τ ; 5) Nonstationarity due to the relaxation of the material defects in time will yield a f^{-2} spectral component which may give a power law spectrum with an exponent below -2 in combination with other noise processes, over a limited spectral interval. The first three of these also have an associated quantum $1/f$ noise contribution, as we shall see.

If these well known forms of low frequency noise are brought under control by straightforward technological improvements in the process of fabrication, the device will remain limited by $1/f$ noise which is also observed in homogeneous samples.

In agreement with earlier observations, Hooge found in 1969 that in semiconductor resistors the fractional $1/f$ noise power density $S_I(f)/(I)^2$ was inversely proportional to the number N of carriers in the sample. He could therefore write

$$S_I(f)/(I)^2 = \alpha_H / fN, \quad (4.1)$$

where α_H is an empirical factor, the Hooge parameter, that is defined by Eq. (4.1). For long samples Hooge found $\alpha_H = 2 \times 10^{-3}$, for short devices we found that α_H can be much smaller. We come back to this in a moment.

If V is the applied voltage, g the conductance of the sample and L the device length, we have

$$I = gV = q\mu NV/L^2 \quad (4.2)$$

so that I can fluctuate if μ , or N , or both fluctuate. Hence

$$S_I(f)/(I)^2 = S_{\mu N}(f)/(\mu N)^2, \text{ where } d(\mu N) = N d\mu + \mu dN. \quad (4.3)$$

If we now write

$$\mu N = \sum_{i=1}^N \mu_i \quad \text{so that } \bar{\mu} = \mu \quad (4.4)$$

where $\mu_i(t)$ is the mobility of an individual carrier, we find

$$S_{\mu N}(f)/(\mu N)^2 = S_N(f)/(N)^2 + S_{\mu}(f)/N(\mu)^2. \quad (4.5)$$

Since $S_{\mu_i}(f)/(\mu_i)^2$ is independent of i and N , and varies as $1/f$, we equate it to α_H/f . Hence mobility fluctuation $1/f$ noise always yields the Hooge relation, whereas number fluctuation noise yields a Hooge type form $\alpha_H/(Nf)$ if and only if $S_N(f)$ is proportional to N . This is the case for MOSFETs but may not be true for other devices.

In view of the above one would expect Eq. (4.1) to be true for semiconductor resistors, MOSFETs, JFETs with very low g-r noise and in junction devices in which the current flow is governed by diffusion. It is not valid for GaAs MESFETs and MODFETs because the noise is often governed by fluctuations of the trap occupancy in the space charge region and is not of the $1/f$ type.

In most cases the carrier distribution is non-uniform. Obviously one must then replace Eq. (4.1) by

$$S_I(x, f)/I^2(x) = \alpha_H/fN(x)\Delta x, \quad (4.6)$$

for any section Δx ; here $N(x)$ is the carrier density per unit length, and α_H is independent of x . Calculating the current noise $S_I(f)$ contributed to the external circuit by a section Δx , summing over all sections Δx and expressing it in terms of the external current I one may obtain

$$S_I(f) = I^2 \alpha_H / f N_{eff} \quad (4.6a)$$

in many cases; here N_{eff} is an effective number of carriers. If N_{eff} is independent of the current I , $S_I(f)$ varies as I^2 ; if N_{eff} is proportional to I as is, e.g., the case for injection

processes, $S_I(f)$ varies as I . Equation (4.6a) is not valid for MOSFETs and JFETs, because of the nonlinearity of the device, but in many junction $1/f$ noise mechanisms (4.6a) holds and N_{eff} is proportional to I .

In general we use Eq. (4.6) to calculate the expected noise power spectrum as the unknown Hooge parameter times a known function of current, frequency, temperature and other device parameters. Measuring $S_I(f)$ we then determine an experimental value of the Hooge parameter α_H . Finally we compare this result with the theoretical value of the Hooge parameter derived from the theory of the quantum $1/f$ effect. This method is preferable because it allows us to recognize cases in which the dominant quantum $1/f$ noise contribution does not come from mobility fluctuations, but from fluctuations of the surface or bulk recombination rates, or of the injection or trapping rates. Such cases are treated by us with a similar method as the one based on Eq. (4.6), with, e.g., the fluctuations of the recombination cross sections replacing the scattering cross sections reflected in the mobility fluctuations.

We must now start from Eq. (4.6) and calculate $S_I(f)$. This yields for MOSFETs and JFETs

$$S_{I_D}(f) = \alpha_H q \mu I_D V_D / f L^2 \quad (4.7)$$

below saturation. Here V_D is the drain voltage, I_D the drain current, μ the carrier mobility in the channel and L the channel length. Since all parameters are easily measurable, the observed values of $S_{I_D}(f)$ yield α_H with an accuracy of 20%.

For n^+p-n transistors the collector $1/f$ noise is found to be

$$S_{I_C}(f) = \alpha_{Hn} I_C (q D_n / f W_b^2) \ln [N(0)/P(W_b)] \quad (4.8)$$

due to electron injection into the base region, and

$$S_{I_B}(f) = \alpha_{Hp} I_B (q D_p / f W_e^2) \ln [P(0)/P(W_e)] \quad (4.9)$$

for the base $1/f$ noise due to hole injection into the emitter. Here W_b and W_e are the base and emitter lengths, respectively, $N(0)$ and $N(W_b)$ the electron concentrations per unit length at the end points of the base, etc. Since $\ln[N(0)/N(W_b)]$ and $\ln[P(0)/P(W_e)]$ are slow functions of bias, $S_{Ic}(f)$ and $S_{Ib}(f)$ should vary approximately as I_c and I_b . Since the emitter is very heavily doped, D_p is inaccurately known and $\ln[N(0)/N(W_b)]$ and $\ln[P(0)/P(W_e)]$ are inaccurately known, so that the values of α_H obtained from the data may be off by a factor 3-5 on either side especially for $S_{Ib}(f)$.

Equations (4.8) and (4.9) also hold for short diodes [$W \ll (D\tau)^{1/2}$], but W_b and W_e must be replaced by the length W of the diffusion region. For long diodes [$W \gg (D\tau)^{1/2}$] we obtain¹⁶

$$S_I(f) = \alpha_H I I q F(\delta) / \tau = \alpha_H I I q / 3 f \tau, \quad (4.10)$$

with

$$F(\delta) = 1/3 - 1/2\delta + 1/3\delta^2 - (qV/KT)^3; \quad \delta = \exp(qV/KT) - 1, \quad (4.10a)$$

where τ is the minority carrier lifetime in the diffusion region and V the applied bias. The last form of Eq. (4.10) is a useful approximation valid for the frequently encountered case $qV/KT \gg 1$. Often τ is inaccurately known, so that the value of α_H , evaluated from the data with the help of (4.10) may be off by a factor 5 either way. More accurate values of τ are urgently needed.

4.2. QUANTUM 1/F NOISE THEORY

For large devices ($L > 1000 \mu\text{meter}$) I have introduced the concept of coherent state quantum $1/f$ noise¹¹⁻¹². In that case the Hooge parameter α_H may be written

$$H = \alpha_H^{coh} = 2\alpha/\pi = 4.6 \times 10^{-3}, \quad (4.11)$$

where $\alpha = 1/(137)$ is the fine structure constant. This is of the same order of magnitude as the empirical value $\alpha_H = 2 \times 10^{-3}$ that Hooge found for long devices. It is therefore proposed that Hooge's empirical value for α_H is due to coherent state quantum $1/f$ noise, so that it has a very fundamental origin. Coherent quantum $1/f$ noise will be discussed in detail in Sec. V.

For small devices ($L < 100 \mu\text{meter}$) I have proposed incoherent^{7-10, 13-15} quantum $1/f$ noise. In that case α_H may be written

$$\alpha_H = \alpha_H^{\text{incoh}} = (4\alpha/3\pi) [(\Delta v)^2 / (c^2)] \quad (4.12)$$

where Δv is the change in the velocity of the carriers in the interaction process considered. This expression holds for any $1/f$ noise source describable by fluctuating cross-sections. Since usually $(\Delta v^2 / c^2) \ll 1$, except for carriers with a very small effective mass, we now have $\alpha_H \ll 3.1 \times 10^{-3}$. This may explain the low values of α_H (in the range of $\alpha_H = 10^{-5} - 10^{-9}$) for very small devices. In between one can introduce a parameter $S = f(L/L_0)$ where L_0 is a characteristic length and write¹²

$$\alpha_H = \alpha_H^{\text{incoh}} [1/(1+S)] + \alpha_H^{\text{coh}} [S/(1+S)] \quad (4.13)$$

with $S \rightarrow \infty$ for $L/L_0 \gg 1$ and $S \rightarrow 0$ for $L \ll L_0$. According to this rough approximation¹², $L_0 \cong 100 \mu\text{meter}$ for samples with a concentration c of carriers of 10^{16}cm^{-3} and varies proportional to $c^{-1/2}$. This describes the transition from Eq. (4.11) to Eq. (4.12) when one goes to devices with smaller and smaller lengths.

For carriers with a Maxwellian velocity distribution we have in the elastic collision approximation

$$\overline{\Delta v^2} = 2\overline{v^2} = 6kT/m^* \quad (4.14)$$

Substituting into Eq. (4.12) one obtains values for α_H that are a factor 30 - 1000 smaller than what is found experimentally. A more accurate calculation¹⁴ of the normal phonon

interaction processes gives values that are only a factor 3 larger than Eq. (4.12). Normal scattering, or non - Umklapp electron - phonon collision processes can therefore not explain these data.

In the Umklapp quantum 1/f noise process¹⁷ the carrier can transfer a momentum h/a to or from the lattice, where a is the lattice constant and h is Planck's constant. If the corresponding velocity change of the carriers is $\Delta \vec{v}$, we have

$$m^* \Delta v = h/a; \quad (\Delta v/c)^2 = (h/m^* a c)^2, \quad (4.15)$$

so that

$$\alpha_H = \alpha_{Hu} = (4\alpha/3\pi) [h/m^* a c]^2. \quad (4.15a)$$

When this is applied to low-noise Silicon n-channel JFETs and p-channel MOSFETs, one obtains values that are about a factor 3 too large, whereas the accuracy of the measurements is 20-30%. The reason is that we have forgotten to multiply (4.15a) by the probability $\exp(-\Theta_D/2T)$, where Θ_D is the Debye temperature, that the interaction process is of the Umklapp type. Hence a better approximation should be

$$(\alpha_H) = \alpha_{Hu} = (4\alpha/3\pi) [h/m^* a c]^2 \exp(-\Theta_D/2T) \quad (4.15b)$$

This agreed with measurements¹⁸ on n-channel JFETs and p-channel MOSFETs within the experimental accuracy of 20-30%. In the case of n-channel JFETS the measurements were done in the 250-400 K temperature range and the temperature dependence $\exp(-\Theta_D/2T)$ of α_H was verified¹⁹ within 10%. We see from Eq. (4.15b) that α_H decreases rapidly for $\Theta_D/2T \gg 1$.

At this point we provide a brief physical explanation of the quantum 1/f effect. Consider, for example, Coulomb scattering of charged particles by a fixed charge. The outgoing (scattered) Schroedinger field monitored by a detector at an angle φ from the direction of the incoming beam contains a main non - bremsstrahlung part and various

contributions which lost small amounts of energy $E = hf$ due to the emission of a bremsstrahlung photon of arbitrarily low frequency f , and therefore have a DeBroglie frequency lowered exactly by f . The expression of the outgoing scattered current density is quadratic in the outgoing Schrodinger field and will contain a major non-bremsstrahlung part, a small bremsstrahlung part, and two cross terms proportional to both the non-bremsstrahlung and the bremsstrahlung parts of the scattered charged particles wave function. These cross terms oscillate with the beat frequency f . Photons are emitted at any frequency, and therefore the cross terms will contain any frequency f with an amplitude proportional to the bremsstrahlung scattering amplitude. The fluctuating cross terms will be registered at the detector as $1/f$ noise in the scattering cross section.

For an elementary derivation of the quantum $1/f$ effect we start with the classical (Larmor) formula for the power $2q^2\dot{v}^2/3c^2$ radiated by an accelerated charge. The sudden acceleration $\dot{v} = \Delta v \delta(t)$ suffered by the charged particle during scattering has a constant Fourier transform $\vec{v}_f = \Delta \vec{v}$, where $\Delta \vec{v}$ is the velocity change during the scattering process and $\delta(t)$ the delta function of Dirac. Therefore the spectral density $4q^2|\dot{v}_f|^2/3c^3$ of the radiated energy can be written in the form $4q^2(\Delta \vec{v})^2/3c^3$ and does not depend on f . Dividing by the energy hf of a photon, we get the number spectrum $4q^2(\Delta \vec{v})^2/3c^3hf$ of the radiated photons, where h is Planck's constant. Since the amplitude of each of the two cross terms is proportional to the amplitude of the bremsstrahlung part of the scattered particles wave function, the fractional spectrum of the observed $1/f$ noise is twice the number spectrum of the emitted photons per scattered charge carrier:

$$I^{-2}S_I(f) = 8q^2(\Delta v)^2/3c^3hf = \alpha_H/f \quad (4.16)$$

With the definition $\alpha = 2\pi q^2/hc$ of the fine structure constant this is identical with Eq. (4.12), and this completes our elementary derivation of the conventional (incoherent) quantum $1/f$ effect.

All scattering cross sections and process rates defined for the current carriers must fluctuate with a fractional spectral density given by Eq. (4.16). Applied to scattering cross sections, this means that the collision frequency, the mean time between collisions, and the mobility of each carrier independently, must all fluctuate with the same fractional spectrum. This allows us to apply the derivation of the $1/N$ factor for mobility fluctuations, presented in Eqs. (4.2)-(4.5). The Hooge formula²⁰ is thus derived from first principles as a quantum $1/f$ result with α_H given by Eq. (4.12). All $1/f$ noise formulae derived on the basis of the Hooge formula can therefore be taken over as quantum $1/f$ results with the appropriate quantum $1/f$ Hooge parameter, but they will provide only the quantum $1/f$ contributions from the scattering cross sections. Therefore they will not describe the experimental results properly in general, until we add the complementary contributions from quantum $1/f$ fluctuations of the surface and bulk recombination cross sections, from quantum $1/f$ fluctuations in tunneling rates, or possible injection - extraction contributions from the velocity changes Δv in transitions of the carriers in junctions and at contacts. Some of these complementary contributions turn out to be similar to results of earlier calculations based on the McWhorter²¹ and North-Fonger²² models, in which the correct quantum $1/f$ expression of recombination rate fluctuations are replacing the rate fluctuations postulated by McWhorter and North. McWhorter had considered transitions to and from traps in the surface oxide layer and North thermal fluctuations of the surface potential as the final cause of $1/f$ noise. Other quantum $1/f$ contributions, finally do not even bear a formal resemblance to earlier calculations. We conclude that the quantum $1/f$ approach provides both a foundation and a properly weighted synthesis of earlier calculations, as well as additional contributions. In the same time the quantum $1/f$ approach eliminates all free parameters or fudge factors, leaving only the fine structure constant as a common factor of all electromagnetic quantum $1/f$ contributions. Unfortunately not all quantum $1/f$ noise contributions have been worked out

in sufficient detail so far, as will be seen in Sec. 4.4, due to limitations in time and secretarial services.

4.3. EXPERIMENTAL VERIFICATION

For the best n-channel MOSFETS $(\alpha_H)_{\text{exp}}$ was a factor 50 larger than $(\alpha_H)_{\text{theory}}$, indicating that the quantum $1/f$ noise is here masked by another noise source, which is probably surface $1/f$ noise caused by traps in the oxide. Apparently the p-channel MOSFETs have lower oxide trap densities, whereas α_{HU} is about one order of magnitude larger; as a consequence the quantum limit (2.15b) can be reached in the best units.

For a device length L comparable to or smaller than the free path length²² for Umklapp processes, α_{HU} must be multiplied by a factor $g(L/\lambda)$, where $g(L/\lambda) \approx 1$ for $L/\lambda \gg 1$ and $g(L/\lambda) \approx 0$ for $L/\lambda \ll 1$ (ballistic limit). The same is true for normal collisions, but with a different λ . The function $g(L/\lambda)$ has still to be evaluated.

In the $1/f$ noise of the base current I_b of transistors Eq. (4.9) indicates that S_{I_b} should vary practically linearly with I_b . This was verified²³ for GE82-185 silicon p⁺-n-p transistors, and the value of α_H agreed roughly with Eq. (2.15b). In NEC 57807 Silicon n⁺-p-n microwave transistors $S_{I(f)}$ varied as I_b^n , with $n = 1.5$ at larger currents and somewhat smaller at lower currents. Hence the parameter α_H could not be properly defined. The quantum $1/f$ noise effect in the mobility seems here to be masked by (most likely also quantum $1/f$ type) fluctuations in the surface recombination velocity s at the surface of the emitter-base space charge region. Zhu²³ gave $(\alpha_H)_{\text{exp}} < 1.1 \times 10^{-7}$, but this estimated value of α_H could be 10 times larger due to an inaccurate estimate of D_p , so that $(\alpha_H)_{\text{exp}}$ and $(\alpha_H)_{\text{theory}} = 4 \times 10^{-7}$ are now comparable.

The $1/f$ noise in the collector current I_c of silicon transistors, both of the n⁺-p-n variety and of the p⁺-n-p type, is extremely small. It is estimated that α_H is at least a factor 100 smaller than the value deduced from Eq. (4.15b). Apparently the Umklapp $1/f$ noise is here absent. The NEC 57807 n⁺-p-n microwave transistor had a base width of

0.35 μ meter so that it could be operating near the ballistic limit. For the GE82-185 the base width was 1.65 μ meter and hence it should be operating in the collision mode. This absence of Umklapp 1/f noise is caused by the manufacturing of the devices on 100 crystal surfaces which avoids the Umklapp processes through the Umklapp - unfavorable position of the energy minima of silicon for this particular orientation. *Some collector 1/f was found recently by Pavlikievich and van der Ziel.*

We have obtained some preliminary data²⁴ on n+-p Hg_{1-x}Cd_xTe diodes with x = 0.30.

The diodes were made on an epitaxial layer 10 μ meter thick, the emitter contact was 150x150 μ meter, the length of the p-region was about 2 mm and the geometry was very far from rectangular. For a rectangular approximation the effective volume V_{eff} of the p-region would be $10^{-3} \times 1.5 \times 10^{-2} \times 0.2 \text{ cm}^3 = 3 \times 10^{-6} \text{ cm}^3$, but for the actual geometry V_{eff} could be larger or smaller. The error could be a factor 5 in any direction.

At 300 K and forward bias it was found that $S_I(f)/I^2$ was constant, as expected for the 1/f noise of the series resistance R_s of the p-region. Since for x = 0.30 the material is nearly extrinsic and $\alpha_n \gg \alpha_p$, this noise comes from electrons; hence Eq. (4.1) may be written

$$S_I(f)/I^2 = \alpha_H/fN = \alpha_H/fn_0 V_{eff}. \quad (4.16)$$

where n_0 is the electron concentration in the p-region. Evaluating n_0 and estimating V_{eff} as shown, Wu and van der Ziel²⁴ found $\alpha_H = 1.7 \times 10^{-2}$. This is a factor 3.5 larger than Handel's value of 4.6×10^{-3} found for coherent state quantum 1/f noise, but the possible error may be a factor 5 in any direction.

At 300 K and lower temperatures it was found that $S_I(f)/I$ was about constant for not too small back bias. This meant that Eq. (4.10) might be applicable. The difficulty was that the life time τ_n of the carriers was poorly known. The relative shape of the $\tau_n(T)$ versus T curves is known; τ_n should have a maximum at intermediate temperatures and drop off appreciably at lower and at higher temperatures²⁴. Wu and van der Ziel²⁴ therefore took $\tau_n(300) = 1.0 \times 10^{-8}$ sec, $\tau_n(193) = 5 \times 10^{-8}$ sec and $\tau_n(77) = 0.5 \times 10^{-8}$ sec. This

yielded $(\alpha_H)_{\text{exp}} = 2 \times 10^{-4}$, whereas $(\alpha_H)_{\text{theory}} = 4 \times 10^{-5}$, according to Eq. (4.15b) and $(\alpha_H)_{\text{coh}} = 4.6 \times 10^{-3}$. This means that $(\alpha_H)_{\text{exp}}$ lies between $(\alpha_H)_{\text{theory}}$ and $(\alpha_H)_{\text{coh}}$, closer to the former. *Recently improved τ values from h.f. impedance measurements give better agreement with the theory (Wu and van der Ziel)*

How should this be interpreted? In view of the fact that Umklapp 1/f noise seemed to be absent in the collector 1/f noise of bipolar transistors, and that this noise, if present, would have been generated in the base region, we cannot be sure that Umklapp 1/f noise is generated in the p-region of long $n^+ - p$ diodes. The safest conclusion that can be taken from the fact that $(\alpha_H)_{\text{exp}} = 10(\alpha_H)_{\text{theory}}$ is that the presence of Umklapp 1/f noise in $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ has not been demonstrated, nor has the absence of Umklapp 1/f noise been proved.

A somewhat more optimistic conclusion cannot be completely ruled out, however. In long $n^+ - p$ diodes one has to split the p-region into a diode part and a series resistance part. The boundary lies about 1-2 diffusion lengths from the junction, so that the diode part of the p-region may be a few hundred meter long. This might bring the generated noise about halfway between coherent state 1/f noise and the Umklapp 1/f noise.

1/f noise in $n^+ - p$ $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ occurs in many forms and each form should be tested. If a Hooge parameter α_H can be defined from the measured data, one can investigate whether or not the measured value of α_H agrees with the theoretical value, lies above it, or lies below it. If a Hooge parameter cannot be defined one can still measure spectra at various currents, compare with theoretical spectra and see whether the quantum 1/f noise is masked or is absent.

4.4. REVIEW OF QUANTUM 1/f NOISE SOURCES APPLICABLE TO $n^+ - p$ DIODES

The various quantum 1/f noise sources suggested by the author, which we will consider here, have in common that they can be described by Eq. (4.12) but with a $\vec{A}v^2$ value described by an "energy" $E = qV_{\text{eff}}$

$$(\vec{A}v^2)/c^2 = 2e/m \cdot c^2 = 2qV_{\text{eff}}/m \cdot c^2 \quad (4.17)$$

Examples:

(a) Recombination quantum 1/f noise in the bulk space charge region⁶:

$$S_I(f) = \alpha_H q I_{gr} [\tanh(qV/2kT)] / f(\tau_{no} + \tau_{po}); \quad (4.18)$$

$$\alpha_H = (4\alpha/3\pi) [2q(V_{dif} - V) + 3kT] (m_n^*)^{1/2} + (m_p^*)^{1/2} - 2c^{-2},$$

where $I_{gr} = qA W n_i (e^{qV/2kT} - 1) / (\tau_{no} + \tau_{po})$ is the recombination current, τ_{no} and τ_{po} the Shockley-Hall-Read lifetimes, V the applied voltage and V_{dif} the diffusion potential of the junction. Introducing an "effective carrier number" $N_{eff} =$

$I_{gr}(\tau_{no} + \tau_{po}) / q [\tanh(qV/2kT)]$, Eq. (4.18) may be written in the form of Eq. (4.6a)

$$S_I(f) = \alpha_H I_{gr}^2 / f N_{eff}. \quad (4.18a)$$

(b) Quantum 1/f noise in the surface recombination current of $n^+ - p$ diodes. This effect is caused by quantum 1/f fluctuations of the surface recombination cross sections. The calculation is similar to the previous (bulk) case, but the GR process is localized at the surface, and the additional electric field arising from the potential jump $2U$ at the interface between the bulk and the oxide and passivation layers will lead to increased velocity changes of the carriers in the recombination process and to larger α_H values. Including also the quantum 1/f mobility fluctuation noise of the spreading resistance caused by the concentration of generation and recombination currents at the intersection of the depletion region with the surface of the diode, we obtain a current noise contribution

$$S_I(f) = \alpha'_H q I'_{gr} [\tanh(qV/2kT)] / f(\tau'_{no} + \tau'_{po}) + \frac{\alpha (I'_{gr})^2}{4\pi^2 f W^2} \ln(A/W^2), \quad (4.19)$$

$$\alpha'_H = (4\alpha/3\pi) [2q(V_{dif} + U - V) + 3kT] (m_n^*)^{1/2} + (m_p^*)^{1/2} - 2c^{-2}.$$

Here the primed quantities refer to the surface, i.e. we have introduced the surface GR current I'_{gr} , the lifetimes in the vicinity of the surface and the α'_H parameter for surface recombination. P is the perimeter, A the area and W the width of the junction. The quantum $1/f$ mobility fluctuation part is expressed in terms of the global α parameter which includes all types of scattering weighed with the appropriate mobility ratio factors. Introducing again an "effective carrier number" $N'_{eff} = |I'_{gr}|(\tau'_{no} + \tau'_{po})/q[\tanh(qV/2kT)]$, the first term of Eq. (4.19) may be also be written in the form of Eq. (4.6a)

$$S_I(f) = \alpha'_H I'^2_{gr} / f N'_{eff}. \quad (4.19a)$$

Due both to the surface potential jump $2U$ and the $1/f$ noise of the spreading resistance, the surface recombination current will be noisier than an equal bulk recombination current, and this is in agreement with the experimental data.

(c) Injection - extraction quantum $1/f$ noise⁶, due to injection or extraction of carriers across barriers. In this case, for not too small currents

$$S_I(f) = \alpha_H |I| q / f \tau; \\ \alpha_H = (4\alpha/3\eta)[2q(V_{dif} - V) + 3kT]/m_n^* c^2, \quad (4.20)$$

where I is the injected current, and τ is the time of passage of a carrier through the barrier region. Introducing again an effective carrier number $N_{eff} = I\tau/q$, Eq. (4.20) may be written in the more general form (4.6a), valid also for very small I

$$S_I(f) = \alpha_H I^2 / f N_{eff}. \quad (4.20a)$$

Note that in each case N_{eff} , as long as it is larger than 1, is proportional to I (otherwise $N_{eff} = 1$, see below), and that α_H depends on bias.

(d) Recombination in the p region of a long n^+p diode of length $w \gg (D_n \tau_n)^{1/2}$. Here, for not too small bias

$$S_I(f) = \alpha_H |I| q / 3f \tau_n = \alpha_H I^2 / f N_{eff}; \quad (4.21)$$

$$N_{eff} = 3II\tau_n/q, \quad \alpha_H = (4\alpha/3\pi)[3kT/m_n^*c^2], \quad (4.21a)$$

since $(\Delta v)^2 = 2E/m$ and $E = 3kT/2$. In this case α_H is very small. At small bias ($qV < kT$) the factor $q/3$ must be replaced by $qF(\delta)$ with F given by Eq. (4.10a).

(e) Quantum $1/f$ noise in the tunneling rate. Tunneling is observed in n^+-p diodes with sufficient gate bias². If we assume that the momentum change of the carriers in the tunneling process is of the order of the thermal r.m.s. momentum, we obtain a minimal quantum $1/f$ noise power spectrum

$$S_I(f) = \alpha_H I^2 / f N_{eff}; \quad N_{eff} = II\tau/q, \quad (4.22)$$

$$\alpha_H = (4\alpha/3\pi)[3kT/m_n^*c^2],$$

where τ is the time of passage through the barrier, or tunneling, i.e. the time during which each carrier contributes to the current through the barrier. Since the width of the barrier crossed by tunneling is small, this time is very short, of the order of 10^{-14} s. N_{eff} will then become larger than 1 at currents exceeding 10^{-5} A, leading to a linear current dependence of the noise power. At lower bias N_{eff} must be set equal to unity in Eq. (4.22). and this gives a quadratic current dependence.

(f) Umklapp $1/f$ noise in a long n^+-p diode. This can be put in the same form as in the previous case

$$S_I(f) = \alpha_H IIqF(\delta)/f\tau_n = \alpha_H I^2 / f N_{eff}; \quad N_{eff} = II\tau/qF(\delta), \quad (4.23)$$

but now α_H is given by (4.15b) and is much larger than in the previous case. The function $F(\delta)$ is given by Eq. (4.10a). This contribution, together with the normal (non - Umklapp) phonon scattering, intervalley scattering, impurity scattering and optical phonon scattering contributions, determines the quantum $1/f$ mobility fluctuation Hooge parameter α_H . However, not all of these contributions are important, in general, in a given semiconductor.

The following are two cases in which a satisfactory quantum $1/f$ calculation has not been performed yet. Both are rather noisy processes.

(a') Recombination at the surface of the base region in a Ge transistor. This process is responsible for the base current I_b and the base noise $S_{Ib}(f)$. Since I_b is proportional to the surface recombination velocity s , where s does not depend on position

$$\delta I_b / I_b = \delta s / s; \quad S_{Ib}(f) / I_b^2 = S_s(f) / s^2, \quad (4.24)$$

where $S_s(f)/s^2$ should be independent of bias. $S_{Ib}(f)$ should thus be proportional to I_b^2 , and this agrees with the experiment.

(b') Recombination at the surface of the emitter space charge region. This can occur in silicon transistors, even if the base current is mainly due to injection of carriers into the emitter region, because the recombination process is a noisy process. Let the recombination occur mainly in a narrow area around x_1 and let the potential at x_1 change by an amount V_1 when bias is applied. Then, if p_1 is the hole concentration at x_1 for zero bias, the hole concentration at x_1 with bias is $p_1 \exp(qV_1/KT)$. Hence the recombination current I_r may be written

$$I_r = q p_1 \exp(qV_1/KT) A_{eff} s; \quad \delta I_r = q p_1 \exp(qV_1/KT) A_{eff} \delta s$$

or

$$S_{Ir}(f) = (q p_1 A_{eff})^2 \exp(2qV_1/KT) S_s(f), \quad (4.25)$$

where $S_s(f)$ is independent of bias and equal to (constant) $/f$. Since $I = I_0 \exp(qV/KT)$,

$$S_{Ir}(f) = \text{const. } I^{2V_1/V} / f. \quad (4.26)$$

For $V_1 = V/2$, $S_I(f)$ is proportional to I , for $V_1 = 0.75$, as expected for p+-n-p transistors, $S_I(f)$ is proportional to $I^{1.5}$.

These are "classical" theories; they still have to be translated into the quantum picture.

What happens to the equation

$$S_I(f) = \alpha_H I^2 / f N_{eff} \quad (4.27)$$

if $N_{eff} < 1$? This might be the case for small devices at very small currents. The factor N_{eff} in the denominator is caused by the incoherence of the single - carrier contributions when more than one carrier are testing a certain cross section or process rate simultaneously. This, however, is not the case for $N_{eff} < 1$. Therefore N_{eff} should be replaced by unity in that case in all formulae, so that

$$S_I(f) = \alpha_H I^2 / f \quad (4.28)$$

in this limit. In cases in which N_{eff} is proportional to I , $S_I(f)$ would be proportional to I at high currents and to I^2 at very low currents in junctions. This is in agreement with experimental observations by Radford and Jones², and by DeWames et al., but should be further investigated in each case in relation to the quantum $1/f$ noise coherence length and the junction area dependence of the noise power in this limit. Indeed, the proportionality to I^2 is usually predicted by the theory even for $N_{eff} > 1$ because N_{eff} stops being proportional to I at low currents, and becomes constant. In such devices the limit $N_{eff} < 1$ can not be reached unless the size of the device (e.g. the junction area) is further reduced.

4.5. DISCUSSION

The quantum $1/f$ noise formulae presented above have been applied by Radford and Jones² to $1/f$ noise in GR, diffusion and tunneling currents in both double epitaxial layer and ion implanted n^+-p HgCdTe diodes. They obtained good agreement with the experimental data in general, but were a factor 20 below the measured values at positive gate bias, when an inversion layer formed at the surface. This discrepancy may be due both to the

presence at positive gate bias of a noisy surface GR contribution (Eqs. 4.19-19a), and to kinetic energies of the tunneling carriers above the thermal level in the vicinity of the inversion layer.

Another previously unexplained fact noted was the difference in the fractional noise level of surface and bulk recombination currents. This is caused in Eqs. (4.19-19a) both by the surface potential jump $2U$ of the order of 1 Volt present at the interface between the bulk and the oxide and passivation layers, and by the quantum $1/f$ mobility fluctuation ($\sim \alpha_\mu$) noise in the spreading resistance which affects the passage of carriers to and from the perimeter of the junction. Furthermore, the higher noise level of ZnS - passivated diodes may be caused by a larger surface recombination speed associated with these coatings compared to SiO_2 passivations, and by a larger effective value of U . The larger surface recombination speed pulls more of the recombination current from the bulk to the surface where it has higher fractional noise. The larger potential jump U increases the applicable Hooge parameter according to Eq. (4.19). Finally, the larger fractional $1/f$ noise levels of ion implanted junctions is mainly caused by the 1-2 orders of magnitude lower carrier lifetimes in Eqs. (4.18-23), which yield 1-2 orders of magnitude smaller N_{eff} values and larger fractional noise power values by the same factor.

In order to reduce the fractional noise level, the theory suggests the use of a surface passivation which lowers the surface recombination speed and the surface potential jump. The ideal "surface" would be a gradual increase of the gapwidth starting from the bulk through compositional changes leading to a completely insulating stable surface outwards, without the generation of surface recombination centers. In addition, the life time of the carriers should be kept high, and abrupt or pinched regions in the junction should be avoided. The reasonable choice of other junction parameters, including the steepness of the junction and the geometry, should yield lower injection - extraction and bulk recombination noises by emphasizing the presence of the larger hole masses in the

denominators of the above expressions. Finally, coherent state $1/f$ noise should be avoided in any case by optimizing the dimensions.

V. COHERENT STATES QUANTUM $1/f$ NOISE AND THE QUANTUM $1/f$ EFFECT

A physical, electrically charged, particle should be described in terms of coherent states of the electromagnetic field, rather than in terms of an eigenstate of the Hamiltonian. This is the conclusion obtained from calculations²⁵ (of the infrared radiative corrections to any process) performed both in Fock space (where the energy eigenstates are taken as the basis, and the particle is considered to have a well defined energy) and in the basis of coherent states. Indeed, all infrared divergences drop out already in the calculation of the matrix element of the process considered, as it should be according to the postulates of quantum mechanics, whereas in the Fock space calculation they drop out only a posteriori, in the calculation of the corresponding cross section, or process rate. From a more fundamental mathematical point of view, both the description of charged particles in terms of coherent states of the field, and the undetermined energy, are the consequence of the infinite range of the Coulomb potential²⁶. Both the amplitude and the phase of the physical particle's electromagnetic field are well defined, but the energy, i.e. the number of photons associated with this field, is not well defined. The indefinite energy is required by Heisenberg's uncertainty relations, because the coherent states are eigenstates of the annihilation operators, and these do not commute with the Hamiltonian.

A state which is not an eigenstate of the Hamiltonian is nonstationary. This means that we should expect fluctuations in addition to the (Poissonian) shot noise to be present. What kind of fluctuations are these? This question was answered in a previous paper²⁷. The additional fluctuations were identified there as $1/f$ noise with a spectral density of $24/\pi f$ arising from each electron independently, where $\alpha = 1/137$ is the fine structure constant.

We will briefly derive this result again here, but we will stress the connection between the coherent quantum $1/f$ noise and the usual quantum $1/f$ effect.

5.1. COHERENT QUANTUM $1/f$ NOISE

The coherent quantum $1/f$ noise will be derived again in three steps: first we consider just a single mode of the electromagnetic field in a coherent state and calculate the autocorrelation function of the fluctuations which arise from its nonstationarity. Then we calculate the amplitude with which this mode is represented in the field of an electron. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector q , the angular frequency $\omega = cq$ and the polarization ϵ . Denoting the variables q and ϵ simply by q in the labels of the states, we write the coherent state²⁵⁻²⁷ of amplitude $|z_q|$ and phase $\arg z_q$ in the form

$$\begin{aligned} |z_q\rangle &= \exp[-(1/2)|z_q|^2] \exp[z_q a_q^\dagger] |0\rangle \\ &= \exp[-(1/2)|z_q|^2] \sum_{n=0}^{\infty} (z_q^n / n!) |n\rangle. \end{aligned} \quad (5.1)$$

Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n! \sqrt{\pi})^{-1/2} \exp[-x^2/2] H_n(x) e^{i n \omega t} \quad (5.2)$$

This yields for the coherent state $|z_q\rangle$ the representation

$$\begin{aligned} \Psi(x) &= \exp[-(1/2)|z_q|^2] \exp[-x^2/2] \sum_{n=0}^{\infty} \{ [z_q e^{i\omega t}]^n / [n! (2^n \sqrt{\pi})]^{1/2} \} H_n(x) \\ &= \exp[-(1/2)|z_q|^2] \exp[-x^2/2] \exp[-z^2 e^{-2i\omega t} + 2xz e^{i\omega t}] \end{aligned} \quad (5.3)$$

In the last form the generating function of the Hermite polynomials was used²⁷. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q , is, for $|z_q| \ll 1$,

$$\begin{aligned} P_q(t, x) &= \langle |\psi_c|^2 |\psi_c|^2 \rangle_{t, t} \\ &= (1 + 8x^2 |z_q|^2 [1 + \cos \omega t] - 2|z_q|^2) \exp[-x^2/2]. \end{aligned} \quad (5.4)$$

Integrating over x from $-\infty$ to ∞ , we find the autocorrelation function

$$A^1(t) = (2\pi)^{-1/2} (1 + 2|z_q|^2 \cos \omega t) \quad (5.5)$$

This result shows that the probability contains a constant background with small superposed oscillations of frequency ω . Physically, the small oscillations in the total probability describe a particle which has been emitted, or created, with a slightly oscillating rate, and which is more likely to be found in a measurement at a certain time than at other times in the same place. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations.

We now determine the amplitude z_q with which the field mode q is represented in the physical electron. One way to do this²⁷ is to let a bare particle dress itself through its interaction with the electromagnetic field, i.e. by performing first order perturbation theory with the interaction Hamiltonian

$$H' = \vec{A} \cdot \vec{j} = -(e/c) \vec{V} \cdot \vec{A} + e\phi, \quad (5.6)$$

where $\vec{A}$ is the vector potential and ϕ the scalar electric potential. Another way is to Fourier expand the electric potential $e/4\pi r$ of a charged particle in a box of volume V . In both ways we obtain²⁷

$$|z_q|^2 = (e/q)^2 (hcqV)^{-1}. \quad (5.7)$$

Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (5.5)

$$\begin{aligned} A(\tau) &= C \prod_q \{1 + 2|z_q|^2 \cos \omega_q \tau\} = C \{1 + 2 \sum_q |z_q|^2 \cos \omega_q \tau\} \\ &= C \{1 + 2(V/2^3 \pi^3) \int d^3 q |z_q|^2 \cos \omega_q \tau\} \end{aligned} \quad (5.8)$$

Here we have again used the smallness of z_q and we have introduced a constant C. Using Eq. (5.7) we obtain

$$\begin{aligned} A(\tau) &= C \{1 + 2(V/2^3 \pi^3) (4\pi^2/V) (e^2/2hc) \int (dq/q) \cos \omega_q \tau\} \\ &= C \{1 + 2(\alpha/\pi) \int \cos \omega \tau d\omega/\omega\}. \end{aligned} \quad (5.9)$$

Here $\alpha = e^2/4\pi\hbar c$ is the fine structure constant $1/137$. The first term in curly brackets is unity and represents the constant background, or the d.c. part. The autocorrelation function for the relative, or fractional density fluctuations, or for current density fluctuations in the beam of charged particles, is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khinchine theorem, the coefficient of $\cos \omega \tau$ is the spectral density of the fluctuations, $S_{\Psi\Psi}$, or S_j for the current density

$$\vec{j} = e(\hbar/m)\Psi^2$$

$$\frac{S_{\Psi\Psi}(\omega)}{\langle \Psi^2 \rangle} = \frac{S_j(\omega)}{\langle j^2 \rangle} = 2 \frac{\alpha}{\pi} \frac{1}{f} \approx \frac{4.6 \cdot 10^{-3}}{f} \quad (5.10)$$

Here we have included the total number N of charged particles which are observed simultaneously in the denominator, because the noise contributions from each particle are independent. This result is related to the well known quantum $1/f$ effect²⁸⁻³³. If a beam of charged particles is scattered, passes from one medium into another medium (e.g. at contacts), is emitted, or is involved in any kind of transitions, the amplitudes z_q which describe its field will change. Then, even if the initial state was prepared to have a well-determined energy, the final state will have an indefinite energy, with an uncertainty

determined by the difference between the new and old z_q amplitudes, Δz_q . This, however, is just the bremsstrahlung amplitude Δz_q . We thus regain the familiar quantum $1/f$ effect, according to which the small energy losses from bremsstrahlung of infraquanta yield a final state of indefinite energy, and therefore lead to fluctuations of the process rate, or cross section, of the process in which the electrons have participated, and which has occasioned the bremsstrahlung in the first place. The calculation of piezoelectric $1/f$ noise³⁴ which deals with phonons as infraquanta, was phrased in terms of the coherent field amplitudes z_q for the first time, although it is concerned only with the usual quantum $1/f$ effect. It has α substituted by the piezoelectric coupling constant g .

3.2. CONNECTION WITH THE USUAL QUANTUM $1/f$ EFFECT

The assumptions included in the derivation of the above coherent quantum $1/f$ noise result are :

1 - The "bare particle" does not have compensating energy fluctuations which could cancel the fluctuations present in the field. The latter are due to the interaction with distant charges, and have nothing to do with the bare particle. Therefore, this assumption is quite reasonable.

2 - The experimental conditions do not alter the physical definition of the charged particle as a bare particle dressed by a coherent state field. This second assumption depends on the experimental conditions.

One way to understand this second assumption is based on the spatial extent of the beam of particles or of the physical sample containing charged particles, and is specifically based on the number of particles per unit length of the sample. According to this model, the coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m , per unit length, of

the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int (B^2/8\pi) d^3x = [nevS/c]^2 \ln(R/r) \quad (5.11)$$

is much smaller than the total kinetic energy E_K of the drift motion of the individual carriers

$$E_K = \sum mv^2/2 = nSmv^2/2 = E_m/s. \quad (5.12)$$

Here we have introduced the magnetic field B , the carrier concentration n , the cross sectional area S and radius r of the sample, the radius R of the electric circuit, and the "coherent ratio"

$$s = E_m/E_K = 2ne^2S/mc^2 \ln(R/r) = 2e^2N'/mc^2, \quad (5.13)$$

where $N' = nS$ is the number of carriers per unit length of the sample and the natural logarithm $\ln(R/r)$ has been approximated by one in the last form. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2) S_{\mu}(f) = (1/(1+s)) [2\alpha A/fN] + (s/(1+s)) [2\alpha/\pi fN] \quad (5.14)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. Eq. (5.14) needs to be tested experimentally. In this equation $A = 2(\alpha v/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the quantum $1/f$ effect²⁸⁻³³. This equation does not include the quantum $1/f$ noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio s introduced here equals the unity for the critical value $N' = N'' = 2 \cdot 10^{12}/\text{cm.}$, e.g. for a cross section $S = 2 \cdot 10^{-4} \text{cm}^2$ of the sample when $n = 10^{16}$.

For small samples with $N' \ll N$ only the first term survives, and for $N' \gg N$ only the second term remains in Eq. (5.14). We hope that an expression similar to Eq. (5.14) will allow us to extend the present good agreement between theory and experiment to the case of large semiconductor samples³⁵⁻³⁸.

VI. QUANTUM $1/f$ NOISE IN SQUIDS

Any cross section or process rate defined for electrically charged particles must fluctuate in time with a $1/f$ spectral density according to quantum electrodynamics, as a consequence of infrared - divergent coupling to low - frequency photons³⁹. This fundamental effect discovered 10 years ago, leads to quantum $1/f$ noise observed in many systems with a small number of carriers, and is also present in the cross sections and process rates which determine the resistance and tunneling rate in Josephson junctions, providing a lower limit of the observed $1/f$ noise.

This Section gives a brief and physical explanation and derivation of the quantum $1/f$ effect, followed by a discussion of the application to Josephson junctions and SQUIDS.

Consider a scattering experiment, e.g. Coulomb scattering of electrons on a fixed charge and focus on the scattered current which reaches the detector. Part of the Schrodinger field of this outgoing beam has lost energy hf due to bremsstrahlung in the scattering process. This part interferes with the main, nonbremsstrahlung part yielding beats at any frequency f in the outgoing DeBroglie waves. These beats are observed as fluctuations of the scattered current and interpreted as cross section fluctuations.

The bremsstrahlung amplitude is known to be $(2\alpha/3\pi f)^{1/2} v/c$, where v is the change in the velocity vector of the particles during the scattering process, and α is the fine structure constant $e^2/\hbar c = 1/137$. The beat current is proportional to this amplitude, and the spectral density of the fractional current j (or cross section σ) fluctuations is therefore

$$J^{-2}S_J(f) = G^{-2}S_G(f) = 4\alpha v^2 / 3\pi N f c^2 \quad (6.1)$$

which duplicates the number spectrum of the emitted photons. Here N is the number of the particles which are simultaneously measuring the cross section.

In a Josephson junction the normal resistance R_n of the barrier is proportional to a scattering cross section or transition rate experienced by the electron in quasiparticle tunnelling and by the Cooper pairs below the critical current I_c . Therefore

$$R_n^{-2}S_{Rn}(f) = (4\alpha/3\pi) (v^2/c^2 N f) = (8\alpha/3\pi) (v_F^2/c^2 N f) = 4 \cdot 10^{-14} / f \Omega \quad (6.2)$$

where we have approximated v^2 with $2v_F^2$, v_F being the Fermi velocity, and the number of carriers N simultaneously present in the barrier of volume Ω (μ^3) by 10^7 , for barriers wider than 10^{-7} cm.

Assuming a linear relationship between the critical current I_c and $G_n = R_n^{-1}$, we obtain similar to Rogers and Buhrman⁴⁰

$$S_V(f) = (4/f) 10^{-12} (T/3K) [R_S g(V) / (R_S + R_J)]^2 \\ [I_c R_n (I_c^2 / I_c^2 - 1)^{-1/2} + \overline{g(V) V_J^2}]^2 \quad (6.3)$$

where $R_J(V)$ is the junction resistance, R_S the shunt resistance, and $g(V) = R_n / R_J$.

The noise caused in a SQUID by the source considered above can be obtained as the sum of the noise contributions from the two junctions.

In addition to this noise present in each junction, SQUID'S may also allow us to see coherent quantum $1/f$ noise

$$I^{-2}S_I(f) = 2 \frac{\alpha}{\pi} \frac{1}{f} = 4.6 \cdot 10^{-3} / f N_{eff} \quad (6.4)$$

where N_{eff} is an effective number of carriers which define the coherent current state in the vicinity of the two Josephson junctions. This noise is caused by the coherent character of the field of each current carrier, which leads to uncertainty in its energy and thereby generates an additional form of quantum $1/f$ noise in the current. In practice the

current will fluctuate less, depending on the electric circuit which feeds the SQUID, but the flux through the junction will fluctuate, thereby exhibiting a departure from a perfectly coherent field state, i.e. $1/f$ amplitude and phase fluctuations.

The above quantum $1/f$ results of Eqs.(6.2) and (6.3) are in good quantitative agreement with the experimental data⁴⁰, but the application to SQUIDs of the coherent quantum $1/f$ result (6.4) needs further investigation before a meaningful comparison with the experiment can be performed.

REFERENCES

1. S.P.Tobin, S. Iwasa and T.J. Tredwell, IEEE-ED 27, 43 (1980).
2. W.A. Radford and C.E. Jones, J. Vac. Sci. Technol. A 3, 183 (1985).
3. J. Bajaj, G.M. Williams, M.H. Sheng, M. Hinnrichs, D.T. Cheung, J.P. Rode and W.E. Tennant, J. Vac. Sci. Technol. A 3, 192 (1985).
4. T.G.M. Kleinpenning, J. Vac. Sci. Technol. A 3, 176 (1985).
5. H.K. Chung, M.A. Rosenberg and P.H. Zimmermann, J. Vac. Sci. Technol. A 3, 189 (1985).
6. A. van der Ziel and P.H. Handel, IEEE-ED 32, 1802 (1985).
7. P.H. Handel, Phys. Rev. Lett. 34, 1492, 1495 (1975).
8. P.H. Handel, Phys. Rev. A 22 745 (1980).
9. T.S. Sherif and P.H. Handel, Phys. Rev. A 26, 596 (1982).
10. P.H. Handel and D. Wolf, Phys. Rev. A 26, 3727 (1982).
11. P.H. Handel, Noise in Physical Systems and $1/f$ Noise edited by M. Savelli, G. Lecoy and J.P. Nougier (North Holland, 1983), p. 97.
12. P.H. Handel, Proc. VIII. Int. Conf on Noise in Physical Systems and $1/f$ Noise, Rome, 1985, to be published.
13. A. van der Ziel, Proc. VIII. Int. Conf on Noise in Physical Systems and $1/f$ Noise, Rome, 1985, to be published.

14. C.M. van Vliet and G.S. Kousik, Proc. VIII. Int. Conf on Noise in Physical Systems and 1/f Noise, Rome, 1985, to be published.
15. A. van der Ziel, P.H. Handel, X.C. Zhu and K.H. Duh, IEEE-ED 32, 667 (1985).
16. A. van der Ziel and J.B. Anderson, Proc. VIII. Int. Conf on Noise in Physical Systems and 1/f Noise, Rome, 1985, to be published.
17. P.H. Handel, Very low Frequency Quantum Electronics, 1/f Noise and Infra Quantum Physics ONR final Technical Report N00014-79-0405 (1982).
18. K.H. Duh and A. van der Ziel, IEEE-ED 32, 662 (1985).
19. A. Pavlikiewicz and A. van der Ziel, IEEE Electron Devices Letters, October 1985 issue.
20. F.N. Hooge, Phys. Lett. A 29, 139 (1969).
21. A.L. McWhorter, MIT Lincoln Lab. Rept. No.80 (1955).
22. W.H. Fonger in Transistors I, RCA labs, Princeton, NJ, (1956).
23. X.C. Zhu and A. van der Ziel, IEEE-ED 32, 658 (1985).
24. X.L. Wu and A. van der Ziel, Proc. VIII. Int. Conf on Noise in Physical Systems and 1/f Noise, Rome, 1985, to be published.
25. V. Chung, Infrared divergence in quantum electrodynamics, Phys. Rev. 140B (1965) 1110-1122.
26. J.M. Jauch, and F. Rohrlich, The Theory of Photons and Electrons (Springer, Heidelberg, 1976).
27. P.H. Handel, Any particle represented by a coherent state exhibits 1/f noise in Noise in Physical Systems and 1/f Noise, M. Savelli, G. Lecoy, and J.P. Nougier (Eds.), Elsevier Science Publ. B.V., 1983, pp. 97-100.
28. P.H. Handel, 1/f Noise - an "infrared" phenomenon, Phys. Rev. Lett. 34 (1975).
29. P.H. Handel, Quantum approach to 1/f noise, Phys. Rev. A22 (1980) 745-757.
30. T.S. Sherif and P.H. Handel, Unified treatment of diffraction and 1/f noise, Phys. Rev. A26 (1982) 596-602.

31. P.H. Handel, and T.S. Sheriff, Direct calculation of the Schroedinger field which generates quantum $1/f$ noise in Noise in Physical Systems and $1/f$ Noise, M. Savelli, G. Lecoy, and J.P. Nougier (Eds.), Elsevier Science Publ. B.V., 1983.
32. P.H. Handel, and D. Wolf, Characteristic functional of quantum $1/f$ noise, Phys. Rev. A26 (1982) 3727-3730.
33. K.M. Van Vliet, P.H. Handel, and A. van der Ziel, Superstatistical emission noise, Physica 108A (1981) 511-513.
34. P.H. Handel, and T. Musha, Quantum $1/f$ noise from piezoelectric coupling, in Noise in Physical Systems and $1/f$ Noise, M. Savelli, G. Lecoy, and J.P. Nougier (Eds.), Elsevier Science Publ. B.V., 1983.
35. A. van der Ziel, P.H. Handel, X.C. Zhu, and K.H. Duh, A theory of the Hooge parameters of solid state devices, IEEE Transactions ED-32, 667-671 (1985).
36. J. Kilmer, C.M. van Vliet, G. Bosman, A. van der Ziel and P.H. Handel, Evidence of electromagnetic quantum $1/f$ noise found in gold films, Phys. Status Solidi 121B, 429-432 (1984).
37. A. van der Ziel, and P.H. Handel, $1/f$ Noise in n^+p diodes, IEEE Transactions, ED-September 1985.
38. A. van der Ziel, Theory of and experiments on quantum $1/f$ noise, Proc. VIII Int. Conf on Noise in Physical Systems and IV. Int. Conf. on $1/f$ Noise, Rome, Italy, Sept. 1985; Invited paper.
39. P.H. Handel, Phys. Rev. Lett. 34, 1492, 1495 (1975); Phys. Rev. A22, 745 (1980); T.S. Sherif and P.H. Handel, A26, 596 (1982); P.H. Handel and D. Wolf, A26, 3727 (1982); K.M. Van Vliet, P.H. Handel and A. van der Ziel, Physica 108A, 511 (1981); A. Widom et al, Phys. Rev. B26, 1475 (1982); B27, 3412 (1983); A. van der Ziel, P.H. Handel, and K.M. van Vliet, Physica 124B, 299 (1984).
40. C.T. Rogers and R.A. Buhrman, IEEE Trans MAG-19, 453 (1983).

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