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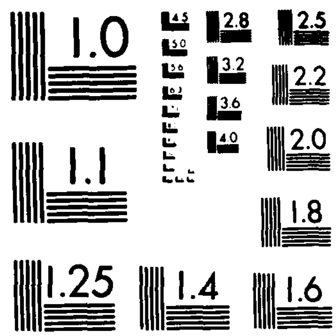

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A COVARIANCE INEQUALITY FOR COHERENT STRUCTURES

by

Kumar Joag-Dev and Frank Proschan

University of Illinois and Florida State University

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A Covariance Inequality for Coherent Structures

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Kumar Joag-Dev and Frank Proschan

ABSTRACT

In this paper, we extend a basic result in reliability theory. ~~We~~ show that the S-shaped property of the reliability function holds when the states of the components are associated; the earlier stronger hypothesis of independence among component states is unnecessarily strong.

0. INTRODUCTION AND NOTATION. An important inequality which is the basis for proving that the reliability function is S-shaped is developed in Barlow and Proschan (1981, Chapte. 2, Section 5). To describe it briefly, let X_i, $i = 1, \ldots, n$, be performance indicating binary (values 0 or 1) random variables. As usual, $X_i = 1$ indicates that the i^{th} component is functioning while $X_i = 0$ indicates that it is failed. Let $\underset{\sim}{X}$ for $(X_1, \ldots, X_n)$, and let the binary function $\phi(\underset{\sim}{X})$ denote the performance indicator of a system with the above n components. The function ϕ is said to be coherent if

(i) ϕ is coordinatewise nondecreasing, that is,

$$\underset{\sim}{x} < \underset{\sim}{y} \Rightarrow \phi(\underset{\sim}{x}) \le \phi(\underset{\sim}{y}) \quad .$$

(ii) All n components are "relevant".

The i^{th} component is said to be "relevant" if there exists at least one configuration of the other (n-1) components such that the system is in the failed state if the i^{th} component is in the failed state and in the functioning state otherwise.

In what follows we assume that $\underset{\sim}{X}$ is "associated"; that is, for every pair of coordinatewise nondecreasing functions (f,g), defined on $R^n \to R$,

$$(1) \quad \mathrm{COV}\,[f(\underset{\sim}{X}),\ g(\underset{\sim}{X})] \ge 0.$$

It is well known that if the X_i are mutually independent then $\underset{\sim}{X}$ is associated.

The main inequality to be considered in this note is

$$(2) \quad \mathrm{COV}\,[\Sigma X_i - \phi(\underset{\sim}{X}), \phi(\underset{\sim}{X})] > 0.$$

Note that the <u>nonstrict version</u> of (2) follows easily from the fact that $\Sigma X_i - \phi(\underset{\sim}{X})$ is coordinatewise nondecreasing in $\underset{\sim}{X}$, and $\underset{\sim}{X}$ being associated implies (1).

The approach in Barlow and Proschan (1981) assumes and uses independence of the components in a crucial manner. As seen from the discussion above, association seems to be the natural condition; we prove inequality (2) in this more general setting.

Our approach is based on a result which states that if a bivariate distribution exhibits "positive quadrant dependence", then uncorrelatedness implies independence. Although the proof of this result needs several steps (see Lehmann (1966)), a very simple proof will be presented for the discrete distribution involved in our case.

In general, the present approach relies directly on the notions of association, independence, and coherence. It is hoped that it will provide a better insight.

1. RESULTS.

LEMMA. Suppose the pair (U,V) satisfies the "positive quadrant dependence" (PQD) condition; that is,

$$(3) \quad P[U \geqq u,\ V \geqq u] \geqq P[U \geqq u]\ P[V \geqq v],$$

for every pair of reals (u,v). Then

$$\mathrm{COV}\ [U,V] = 0 \Rightarrow U,\ V \text{ are independent.}$$

The proof for the special case, relevant for our result, will be given at the end.

REMARK. Suppose (U,V) is associated. Then in view of (1), it is clear that (U,V) satisfies PQD condition (3).

<u>THEOREM</u>. Let ϕ be a system with n ($\geqq 2$) associated components $X_1, \ldots, X_n$. Further, suppose that ϕ is coordinatewise nondecreasing and $0 < P[\phi(\underset{\sim}{X}) = 0] < 1$.

Then

$$\mathrm{COV}\,[\Sigma X_i - \emptyset(\underset{\sim}{X}), \emptyset(\underset{\sim}{X})] = 0 \Rightarrow \quad \text{Only one out of the } n \text{ components is relevant for } \emptyset.$$

PROOF. In view of the Lemma and Remark above, $\Sigma X_i - \emptyset(\underset{\sim}{x})$, $\emptyset(\underset{\sim}{x})$ are independent. Thus

$$(4) \quad P[\emptyset(\underset{\sim}{X}) = 0, \Sigma X_i - \emptyset(\underset{\sim}{X}) = n-1] = P[\emptyset(\underset{\sim}{X}) = 0]\, P[\Sigma X_i - \emptyset(\underset{\sim}{X}) = n-1],$$

$$(5) \quad P[\emptyset(\underset{\sim}{X}) = 1, \Sigma X_i - \emptyset(\underset{\sim}{X}) = 0] = P[\emptyset(\underset{\sim}{X}) = 1]\, P[\Sigma X_i - \emptyset(\underset{\sim}{X}) = 0]$$

Due to the assumptions on $\emptyset$,

$$\Sigma X_i = \{{n \atop 0} \Rightarrow \emptyset(x) = \{{1 \atop 0} \quad ,$$

so that

$$(6) \quad P[\Sigma X_i - \emptyset(\underset{\sim}{X}) = 0] \geqq P[\Sigma X_i = 0] \geqq \prod_1^k P[X_i = 0] > 0 ,$$

where the second inequality in (6) follows from the association of $\underset{\sim}{X}$. Similarly,

$$(7) \quad P[\Sigma X_i - \emptyset(\underset{\sim}{X}) = n-1] \geqq P[\Sigma X_i = n] > 0.$$

From (6) and (7), it follows that the left sides of (4) and (5) are positive. Hence,

$$(8) \quad P[\Sigma X_i = (n-1),\ \emptyset(\underset{\sim}{X}) = 0] > 0,$$

and

$$(9) \quad P[\Sigma X_i = 1,\ \emptyset(\underset{\sim}{x}) = 1] > 0.$$

The inequality (9) tells us that there is a component j such that the system works even if component j is the only working component. On the other hand, (8) implies the existence of a component whose failure causes the system failure, even if all other components are functioning. Due to the nondecreasing character of $\emptyset$, the latter component has to be j. Thus j is the only relevent component as claimed.

PROOF OF LEMMA.

The pair (U,V) relevant for our Theorem is the pair for which the possible values for U are 0, 1, ..., k; while the possible values for V are 0 or 1. Writing $p_{i1} = P(U=i, V=1)$, $p_i = P(U=i)$, where $i=1, \ldots, k$, and $r = P(V=1)$, it follows that,

$$(10) \quad E(U) = \sum_1^k i p_i = \sum_1^k \alpha_i, \; E(V) = r,$$

where $\alpha_i = \sum_{j=i}^k p_j = P(U \geq i)$.

Also,

$$(11) \quad E(UV) = \sum_1^k i p_{i1} = \sum_1^k \beta_i,$$

where

$$\beta_i = \sum_{j=i}^k p_{j1} = P(U \geq i, V = 1).$$

Now the PQD property for U and V above yields

$$P(U \geq i, \; V=1) \geq P(U \geq i) P(V = 1),$$

or equivalently,

$$(12) \quad \beta_i \geq r \alpha_i; \; i=1, \ldots, k.$$

In view of (10) and (11), the uncorrelatedness of U and V implies

$$(13) \quad \sum_{1}^{k} \beta_i = r \sum_{1}^{k} \alpha_i$$

Since α_i and β_i are nonnegative, a strict inequality in (12) for some i, would violate (13). Thus

$$\beta_i = r\alpha_i; \; i=1, \ldots, k.$$

Since V is binary, this implies the independence of U and V.

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