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I. OVERVIEW

↘ The first quarter of the OCCAM contract effort was especially fruitful. Several pure results were obtained in optical conceptual computing and associative memories. These results include the formal definitions of and theorems on bidirectional associative memories and fuzzy associative memories, the first-principles proof that differential Hebbian learning subsumes standard Hebbian learning, the construction of a new optically computable fuzzy integral, a quantitative theory of fuzzy cognitive map combination and inferencing, the design and preliminary simulation of a novel all-optical dynamical associative memory, the first optical design for implementing the fundamental fuzzy set/logic operations of pairwise minimum and maximum, the design and preliminary simulation of a translation, rotation, and scale invariant optical preprocessor suitable for pattern recognition by associative memory, and the design and construction of an associative memory demonstration computer board. Several of these results are currently in preparation as technical papers. Some have been presented at professional speaking engagements. ←

The VERAC and UCSD research teams worked closely together on essentially a daily basis during the the 1st Quarter. Many insights and improvements were jointly obtained. Nevertheless, there was sufficient

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division of research labor to warrant a discussion of 1st Quarter OCCAM activities in separate VERAC and UCSD sections, bearing in mind, again, the close interaction of the researchers.

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II. OCCAM AT VERAC

Bart Kosko directs the VERAC OCCAM effort. Robert Sasseen provides simulation and analysis support. Robert and all the OCCAM research team have successfully completed Bart's Fuzzy Theory course at UCSD. The key research results and activities of VERAC's OCCAM effort are listed below.

1. Bidirectional Associative Memory. Bart defined bidirectional stability and proved that every matrix is a bidirectional associative memory (BAM). With Clark Guest he devised a basic phase-conjugate resonator implementation of a BAM. The paper "Bidirectional Associative Memories" is in preparation.

Let M be an arbitrary n -by- p real matrix. Let A be an n -dimensional binary vector and B be a p -dimensional vector. Let $M(\dots)$ and $M^T(\dots)$ be nonlinear operators that depend on M and M^T , where M^T is the matrix transpose of M . Suppose $B = M(A)$, $A' = M^T(B)$, $B' = M(A')$, $A'' = M^T(B')$, and so on. Then M is bidirectionally stable if for every initial pair (A, B) there exists a fixed pair (A_f, B_f) such that $B = M(A)$, $A' = M^T(B)$, . . . , $B_f = M(A_f)$, $A_f = M^T(B_f)$, $B_f = M(A_f)$, Hence if M is bidirectionally stable, M behaves a heteroassociative content addressable memory (CAM). We know of no other heteroassociative CAM in the literature.

Which matrices are bidirectionally stable? That depends on how the nonlinear operators $M(\dots)$ and $M^T(\dots)$ are specified. Surely the simplest operators are threshold linear:

$$a_i = \begin{cases} 1 & \text{if } B M_i^T > 0 \\ 0 & \text{if } B M_i^T \leq 0 \end{cases},$$

$$b_j = \begin{cases} 1 & \text{if } A M^j > 0 \\ 0 & \text{if } A M^j \leq 0 \end{cases},$$

where M_i is the i th row (column) of M (M^T) and M^j is the j th column of M . I.e., vector multiply M by A then hardclip to produce a binary B , vector multiply M^T by B then hardclip to produce a binary A' , and so on. This process can be interpreted as the synchronous interaction of two Grossberg fields of McCulloch-Pitts neurons $F_A = \{a_1, \dots, a_n\}$ and $F_B = \{b_1, \dots, b_p\}$ symmetrically interconnected via the synaptic weights $\{m_{ij}\}$.

Asynchronous neuronal state changes are also permitted.

If $M(\dots)$ and $M^T(\dots)$ are interpreted as threshold-linear adjoint operators, then our question has been answered with a decisive theorem: Every matrix is bidirectionally stable. The theorem is proven by identifying a Lyapunov or energy with the operation of the bidirectional threshold-linear operator. The correct energy potential turns out to be

$$E(A, B) = -1/2 A M B^T - 1/2 B M^T A^T.$$

Observe that $B M^T A^T = B (A M)^T = (A M B^T)^T = A M B^T$, where the last inequality follows since the transpose of a scalar equals the scalar. Hence E is equivalent to

$$E(A, B) = -A M B^T.$$

We can then show that the energy change $E_2 - E_1$ due to a state change in neuron a_k , or the entire neuron vector A , is negative. Since E is bounded below by the negative of the sum of the absolute values of the entries of M , E converges to a local energy minimum. Since M was arbitrary, the theorem follows for both synchronous and asynchronous state changes. Hence every matrix M can be decoded as an associative memory.

This result subsumes the result of Hopfield et al that a square symmetric zero-diagonal matrix is unidirectionally stable (an autoassociative CAM). The Hopfield case follows if M is square symmetric with zero (or more generally nonnegative) diagonal elements and $A = B$. Moreover, in general the Hopfield associative memory is stable only for asynchronous (serial) recall, a serious restriction that does not apply to a BAM. For instance, one of the simplest Hopfield associative memories stores the vector (1 0) as

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Now multiply this M by the bipolar vector $X = (1 \ 1)$. This gives $X^C = (-1 \ -1)$. But $X^C M = (1 \ 1)$. Hence the iterative recall procedure forever oscillates or blinks back and forth between X and X^C . In other words, in synchronous operation (vector multiplication), M is unidirectionally unstable but bidirectionally stable!

The BAM storage algorithm allows m -many heteroassociative pairs (A_i, B_i) to be encoded in M by suitably sculpting the energy surface defined by D . Transform the binary pair (A_i, B_i) into the bipolar pair (X_i, Y_i) . Memorize the vector pair by forming the correlation matrix $X_i^T Y_i$. (Since $X_i^C = -X_i$, this encoding technique also memorizes the complement association (X_i^C, Y_i^C) since $(X_i^C)^T Y_i^C = X_i^T Y_i$.) Form M now by superimposition: simply pointwise add the m correlation matrices $X_i^T Y_i$. We observe, as most have overlooked when using correlation memorization techniques, that the BAM encoding algorithm employs Grossberg reciprocal outstar coding. Indeed the BAM operation corresponds to a simple form of Grossberg adaptive resonance between fields F_A and F_B .

It follows from BAM analysis that the storage capacity of M is given by a generalization of the familiar bound for autoassociators:

$$m < \min(n, p)$$

for reliable coding.

Finally, we comment that in the OCCAM 2nd Quarter research effort a continuous/differentiable version of the BAM theorem has been proved. The

difference-equation version of this theorem allows fuzzy unit, or fit, vectors (with element values in $[0, 1]$) to be used in the BAM recall and storage procedure. With Clark Guest a preliminary phase-conjugate resonator BAM implementation has been constructed.

2. Fuzzy Associative Memory. Fuzzy associative memory (FAM) is term occasionally used (for instance in TRW's so called FAM (WAM) VHSIC processor) but seldom defined. Bart defined a FAM as a fuzzy relation M that maps input fuzzy sets A to output fuzzy sets B , and proved that every fuzzy heteroassociative pair (A, B) can be stored and recalled with perfect reliability in an easily constructed relation M . M is realized by an n -by- p matrix of elements in $[0, 1]$, i.e., a point in the space $[0, 1]^{n \times p}$. A is an n -fit vector and B is a p -fit vector, i.e., A and B are respectively points in the unit hypercubes $[0, 1]^n$ and $[0, 1]^p$. These results are included in "Fuzzy Associative Memories," in preparation, invited to appear in a special Addison-Wesley edition on fuzzy expert systems.

The key insight is that association generalizes the familiar logical operation of modus ponens--if A and $A \rightarrow B$, then B . The (fuzzy) logical conditional $A \rightarrow B$ stores the pair (A, B) . When a key C is applied to the memory, B is recalled if $C = A$. More generally if C is approximately A , then B' is recalled where B' is approximately B . Storing the pair (A, B) corresponds to heteroassociative memory. As a special case, storing the redundant pair (A, A) corresponds to autoassociative memory.

The fundamental fuzzy operation of set-relation composition (analogous to vector-matrix multiplication) is max-min composition. $A \circ M$ denotes max-

min composition. The operation $A \circ M$ is performed by intersection the fuzzy set A with the fuzzy sets of M represented as columns. This is directly analogous to the vector operation AM where the vector A multiplies the columns of M. In particular if $A \circ M = B$, the j th element of fuzzy set B is found by taking the maximum of the pairwise minima of A's fit values with the fit values of the j th column of M:

$$b_j = \max\{ \min(a_1, m_{1j}), \dots, \min(a_n, m_{nj}) \}.$$

This directly analogous to the vector-multiply operation of taking the global sum of pairwise products. One difference, however, that min and max do not disturb the data on which they act. They only effect order. Hence if $B = A \circ M$, then every element of B is some element of A or M. We note that M is in fact the conditional possibility distribution of B given A.

We now briefly state our results. Suppose we wish to memorize the fuzzy set $A = (.3 \ 1 \ .4 \ .7)$. Hence we wish to store the autoassociative pair (A, A) in M. The Compositional Rule of Inference, propounded by Zadeh, says that we form the relation (conditional possibility distribution) M by identifying m_{ij} with the pointwise fuzzy logical implication or truth value t_{ij} . With this we agree. However, Zadeh et al suggest the Lukasiewicz implication value $t_{ij} = \min(1, 1 - a_i + b_j)$ (where in the autoassociative case $b_j = a_j$). This gives the fuzzy relation M:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ .3 & 1 & .4 & .7 \\ .3 & 1 & 1 & 1 \\ .3 & 1 & .4 & 1 \end{pmatrix}.$$

and hence in fact $A \circ M = A$. However, this is only true because this technique always produces 1s along the main diagonal. Moreover, $t_{ij} = 1$ if (and only if) $a_i \leq b_j$, which tends to occur at least as often as $a_i > b_j$ occurs. Hence M tends to consist of 1s. This precludes heteroassociative recall reliability, tending to produce $B = (1, 1, \dots, 1)$ for any input A .

We have proven that the correct implication operator is $t_{ij} = \min(a_i, b_j)$, which is essentially a fuzzy Hebb law. This is equivalent to representing M as the fuzzy cartesian product $M = A \times B = A^T \circ B$. This produces the memory M :

$$\begin{pmatrix} .3 & .3 & .3 & .3 \\ .3 & 1 & .4 & .7 \\ .3 & .4 & .4 & .4 \\ .3 & .7 & .4 & .7 \end{pmatrix},$$

and hence again $A \circ M = A$. There are two key properties at work in this selection of M . First, $A = \text{Diagonal}(M)$ since $t_{ii} = \min(a_i, a_i) = a_i$. Second, the diagonal entries always dominant the column entries: $m_{ii} \geq m_{ji}$ for all j , for each i , which again follows from the nature of the minimum operation. Our autoassociative theorem states that A can be perfectly memorized by an M such $A = \text{Diagonal}(M)$ and M is diagonal dominant. Hence $A^T \circ A$ works, as well as the simpler choice of M that lists A down the main diagonal and puts 0s elsewhere. Another theorem says that for all other A' , $A' \circ M \subset A$, i.e., the elements of $A' \circ M$ are pairwise dominated by the

elements of A. Hence M is a subset classifier as opposed to the more specialized (and more popular) metric classifiers. The further A' is A, the more A' o M approaches the empty set (0, 0, . . . , 0).

Our heteroassociative theorem says that $m_{ij} = \min(a_i, b_j)$ perfectly memorizes (A, B) subject to one condition: for every element b_j of B, there exists some element a_i of A such that $b_j \leq a_i$. Note that if A and B are binary, this condition is always satisfied since only $A = (0, 0, . . . , 0)$ could violate it, but then $A^T B$ produces the zero matrix! Moreover, M is a subset classifier since for all A', $A' o M \subset B$. We comment that M^T also produces the bidirectional memory relation subject to the same dominance condition. We also comment that the recent AT&T Bell Labs fuzzy logic VLSI chip implements our fuzzy associative memory without realizing it! Through personal communication with its developer, Masaki Togai (now at Rockwell), we learned that the chip designers selected the min operation after exhaustive simulations because only it worked as an implication operator.

Suppose we wish to store the pair (A, B) where $A = (.3 \ 1 \ .4 \ .7)$ as before and $B = (.5 \ .2)$. The key condition of the theorem is satisfied since a 1 occurs in A. Hence $M = A^T B$,

$$\begin{pmatrix} .3 & .2 \\ .5 & .2 \\ .4 & .2 \\ .5 & .2 \end{pmatrix}$$

perfectly learns the association since $A \circ M = B$. Suppose we present the partial pattern $A' = (.3 \ 0 \ .4 \ 0)$. Then $A' \circ M = (.3 \ .2) \subset B$. The bidirectional FAM M^T will be a good but suboptimal memory since the key condition is not satisfied: no element in B is at least as large as $a_2 = 1$. Hence $B \circ M^T = A'' = (.3 \ .5 \ .4 \ .5) \subset A$.

The FAM theorems can be viewed as new theorems in the new field of fuzzy eigensets. Our construction technique shows how to find the fuzzy relation M that has a given fuzzy set A as an eigenset in the sense of max-min composition: $A \circ M = A$. It turns out, however, that the m -many FAMs M_i storing the pairs (A_i, B_i) cannot simply added or maxed together to store the pairs. The eigenset property is too pervasive. For instance, suppose M is formed by a pointwise maximum operation: $M = \max(M_1, \dots, M_m)$. Then

$$M = (\max(A_1, \dots, A_m))^T \circ \max(B_1, \dots, B_m),$$

in other words, the eigenset pair of M is $(\max A_i, \max B_i)$. Hence all patterns (A, B) get mapped into a subset of the max pair. If pairwise max is replaced with normalized addition, then all pairs get mapped into subsets of the normalized sum pair, and so forth. So although these FAMs permit parallel distributed storage of patterns, these storage media cannot be naively superimposed. For many applications, including the AT&T Bell Labs fuzzy logic chip, such superimposition is not necessary.

3. Differential Hebbian Subsumption of Hebbian Learning. We report the first derivation of Hebbian learning phenomena that we know of in the literature. Bart derived this simple but powerful result in his work on kinetic-energy Lyapunov functions for neural networks.

Hebb postulated that synaptic change is driven by the correlation of pre-synaptic and post-synaptic activity. If we denote the nonnegative activation of neuron i by $X_i(t)$ at time t and the directed edge or synapse from X_i to X_j by the real-valued function $e_{ij}(t)$ at time t , then the Hebbian learning law takes the form

$$\dot{e}_{ij} = -e_{ij} + X_i X_j, \quad (1)$$

where the "forget function" $-e_{ij}$ has been appended to represent the passive decay of neurotransmitter release when X_i and X_j are inactive. Equation (1) takes many forms in the literature--other terms are added, functions of X_i and X_j are correlated, etc.--but the recurring structure is that concurrent (or lagged) activation drives learning. Unfortunately the activation product in (1) grows synaptic connections between neurons at an exponential clip. If a forget term is not added, or if it has, as experimentally it seems to have, a small weight, then e_{ij} rapidly saturates at its maximum positive strength. Total connectivity results and no learning occurs. This is, we add, an abundant simulation phenomenon among neural net researchers.

The differential Hebbian hypothesis is that concurrent (or lagged) change, or concomitant variation, drives learning:

$$\dot{e}_{ij} = -e_{ij} + \dot{x}_i \dot{x}_j . \quad (2)$$

We stress, however, that the forget term need not occur in (2). It is added for sake of comparison. For instance, in causal reasoning, causal connections do not passively decay away! So for sake of contrast let us replace (1) and (2) with (3) and (4):

$$\dot{e}_{ij} = x_i x_j , \quad (3)$$

$$\dot{e}_{ij} = \dot{x}_i \dot{x}_j . \quad (4)$$

Hence in (4) learning is governed by a natural correlation sign law: the edge strengthens if the activations agree in sign and weakens or tends to be inhibitory if they disagree in sign.

Which learning law is more accurate? Through personal communication, we have found that many researchers pursuing this question answer it with a natural, yet diplomatic, compromise:

$$\dot{e}_{ij} = x_i x_j + \dot{x}_i \dot{x}_j . \quad (5)$$

This model seems natural enough--just add first, and perhaps second, order variables. Indeed it is natural, but we can now prove it is an unavoidable restatement of (4).

Our argument focuses on the relatively noncontroversial model of shortterm memory or x_i activation:

$$\begin{aligned}\dot{x}_i &= -a x_i + \sum_p e_{pi} S(x_p) + \dots \\ &= -a x_i + O_i ,\end{aligned}\tag{6}$$

where $a > 0$ is the shortterm memory decay constant (function), S is some nondecreasing, usually sigmoid, signal function, and O_i represents other terms. Then upon substitution of (6) into (4), we obtain the unambiguous prediction that some Hebbian learning behavior occurs:

$$\begin{aligned}\dot{e}_{ij} &= \dot{x}_i \dot{x}_j \\ &= a^2 x_i x_j + O_{ij} ,\end{aligned}\tag{7}$$

where O_{ij} denotes other terms in the learning equation (that may or may not be positive). Upon rearrangement and rescaling as necessary, we see that (7) is in fact equivalent to (5).

Equation (7) summarizes a new synaptic theory. It predicts where and to what extent Hebbian learning occurs and quantitatively suggests why a strict Hebb law conflicts with neurophysiological data. For example, if a regression analysis is performed on synaptic behavior, we expect the explanatory contribution of the Hebb component to be negligible when the behavior does not involve (much) passive decay. Since activation decay is fundamental to both neural and causal processes, the Hebbian prediction of (7) is quite robust. It even suggests where to search for an electro-

chemical mechanism for Hebbian behavior, namely in the interaction of two resting-potential media along a conducting medium.

4. A New Fuzzy Integral and Expectation Operator. How can a function f be integrated over a fuzzy set A ? Traditional fuzzy answers to this question have produced noncomputable sup-min structures where the supremum is taken over an uncountably infinite interval, usually $[0, 1]$. We find this approach unfruitful for a variety reasons. Bart has developed an alternative theory of integration.

We propose an abstract fuzzy integral defined in terms of the positive measure Sigma-Count. The sigma count of a fuzzy set is simply the sum of the fit values. For instance, $\text{Sigma-Count}(.2 \ .3 \ 1 \ .8) = 2.3$; hence fuzzy cardinality can be a real number, not just an integer. Though it is beyond the scope of this R&D Status Report, we define the fuzzy integral of f over A to be a sum of products--the product of f_i with the Sigma-Count of A intersected with all the points x such that $f(x) = f_i$.

We define the fuzzy expectation of f with respect to A as simply the sum of the products $f(x) m_A(x)$, where $m_A(x)$ is the degree of membership, or fit value, of x in the subset A . The point is that, unlike the probabilistic expectation, we do not require that $\text{Sigma-Count}(A) = 1$.

Our fundamental theorem is that this Sigma-Count fuzzy integral equals this intuitive fuzzy expectation operator! Besides the many theoretical

problem this solves, it makes a fuzzy integral easy to compute, often by hand. For instance, suppose the domain $X = \{1, 2, 3, 4\}$ and A is given as before by $A = (.2 \ .3 \ 1 \ .8)$. Then if f is the squaring function, $f(x) = x^2$, then $E_A(f) = .2 \times 1 + .3 \times 4 + 1 \times 9 + .8 \times 16 = 13.6$. This theory is presented in the paper, "Fuzzy Expectations," also in preparation. E_A , and hence the fuzzy integral equivalent to it, are obviously trivial to optically implement.

5. Fuzzy Knowledge Combination Theory for Arbitrary Many Fuzzy Cognitive Maps--Hidden Patterns. Bart developed a new theory for combining arbitrary many fuzzy cognitive maps (FCMs) obtained from arbitrary many experts of arbitrary credibility. These results are in the paper "Adaptive Cognitive Processing," also in preparation

We limit the discussion to simple FCMs. These are fuzzy signed digraphs. An edge e_{ij} from concept variable C_i to concept variable C_j has a weight or degree of causal strength in the fuzzy causal interval $[-1, 1]$. $e_{ij} = 0$ indicates no causal connection. $e_{ij} > 0$ indicates that C_i causally increases C_j ; the larger e_{ij} , the more C_i increases C_j . $e_{ij} < 0$ indicates that C_i causally decreases C_j -- C_i up implies C_j down, C_i down implies C_j up. A FCM can be represented by the fuzzy relation or square matrix F , where $f_{ij} = e_{ij}$.

Suppose k -many experts represent their knowledge of some complex situation in k -many FCMs F_i of different square dimensions. What have the experts given us? How can we combine their knowledge? What can we do with

it when we have combined it? Nontrivial answers to these questions follow from a simple examination of fundamentals.

We observe that the complex situation represented can contain a mix of factual and conceptual variables deeply interconnected through partial causality. The factual variables might include agricultural exports or enemy mortality. The conceptual variables might include stress, utility, or love. What sorts of inferences do people draw from such entangled fuzzy concepts? How do they conceptually compute? Surely they associate input patterns with predicted or output patterns. Although some people can articulate some of the serial causal paths in their inferences about complex phenomena, most do not. Indeed, from an evolutionary point of view, inference articulation is far less important than inference accuracy.

This suggests that we can perform simple yet interesting associative inferences on FCMs. Indeed we observe that each F_i matrix has the bipolar form that is so important for threshold-linear dynamics. So let us proceed as follows. As a first approximation, let each concept node C_i be either on or off (1 or 0) at any given time. The simplest rule for deciding whether C_i fires is the threshold-linear rule: if the gated summed inputs to C_i exceed 0, then C_i turns on; else off. (Incidentally this threshold law, unlike the BAM and Hopfield threshold laws, approximates the passive exponential decay of causal activation.) In synchronous operation we therefore have reduced FCM operations to vector operations:

$$S_{i+1} = F(S_i) .$$

where S_i is the binary FCM state vector at iteration i . Since each FCM F_i is nonsymmetric, we expect to observe rich dynamical behavior in terms of stable limit cycles. In this setting, however, limit cycles (generalized fixed points) are quite welcome. They are temporal predictions, forecasts of sequences of events. Since all the weights in each F_i are in $[-1, 1]$, we do not expect that any given F_i will possess many stable limit cycles relative to the number of nodes. In other words, the perceived regularity of responses of the experts to what-if questions (state vectors) corresponds to mapping the 2^n possible questions to no more than n answers.

The next problem is how to deal with the different nodes, causal concepts, the experts discuss. The i th expert includes n_i nodes in his FCM F_i . In general the node sets N_i and N_j overlap, i.e., the experts tend to discuss many of the same concepts in their causal explanations. We view this in a simple way. We assume every expert discusses every node (all the nodes in the union of $N_1, \dots, N_k$). However, many of these nodes are effectively undiscussed because the expert believes they are not causally connected to any other nodes.

In summary, we augment the FCM matrices F_i to include all the nodes discussed by all the experts, nodes $C_1, \dots, C_n$. The rows and columns of each augmented connection matrix M_i are suitably permuted to bring them into mutual coincidence.

The knowledge combination procedure is now clear. The simplest way to superimpose the fuzzy bipolar matrix memories M_i is to add them together pointwise:

$$M = \sum_i M_i .$$

This combination technique amounts to an intuitive voting scheme. If 50 experts say C_i causes $(+1) C_j$ and 50 say C_i causally decreases $(-1) C_j$, then the synthesized $e_{ij} = m_{ij} = 0$. In general m_{ij} reflects the preponderance of excitatory over inhibitory connections, or inhibitory over excitatory connections.

We then conjecture that M embodies certain hidden patterns. A hidden pattern is a resonant or equilibrium state of the activated FCM M : $P = M(P)$, where P is a finite limit cycle. An intuitive interpretation of a hidden pattern is that it is the consensus eventually reached by a round-table discussion among experts. A topic or situation (state vector) is proposed, then fairly soon a rough agreement is reached. The point is that the final agreement may differ from the complete position of each expert. It emerges from associative group interaction. Nor need the final consensus be a unanimous opinion (fixed point). It can be agreed upon sequence or set of conditions, or a clear-cut disagreement, all of which intuitively correspond to a stable limit-cycle. The task is to decode the hidden patterns in M 's edges.

The basic quantitative relationship that governs the dynamical shape of the hidden patterns of M is the inverse relationship between symmetry of M and occurrence of limit cycles. The more symmetric M --the closer M to M^T --the fewer and the shorter the limit cycles among the hidden patterns.

Symmetry up, limit cycles down. For instance, if $M = M^T$, a simple unidirectional Lyapunov argument (discrete version of the Cohen-Grossberg Theorem) shows that all hidden patterns are fixed vectors. The less symmetric M is, the more complicated the feedback loops in M , and thus the greater change of oscillation.

We now derive a rough estimate of the frequency and length of limit cycles "hidden" in M . Fix the total number of nodes discussed by the k -many experts at n but let k vary. The fewer experts there are discussing the same concepts, the sparser each augmented FCM M_i tends to be, and hence the sparser M tends to be. But the sparser M is, the more M approximates a symmetric matrix, since the more often $m_{ij} = m_{ji} = 0$ tends to occur. Similarly, the more experts there are relative to the number of concepts discussed, the less sparse M_i tends to be and thus the less symmetric M tends to be. A similar conclusion follows when k is fixed and n is varied. Hence the dynamics of M are driven by its symmetry; in turn the symmetry of M is driven by the ratio k/n . Therefore if L denotes the expected frequency/length of limit cycles, L can be approximated by

$$L \approx k/n.$$

Suppose now each expert i has a credibility weight w_i in $[0, 1]$. How do we form the weighted augment FCM matrices $M_i^{w_i}$? Since M is formed by summing the M_i matrices, the natural operation for "gating" the knowledge of i by w_i is simply to multiply the connections in M_i by w_i :

$$M^W = \sum_i M_i^W = \sum_i w_i M_i.$$

Hence if i is highly credible (w_i is near 1), M_i will be relatively well represented in M . If i is incredible (w_i is near 0), M_i will make little contribution to M .

How do the weights $w = (w_1, \dots, w_k)$ affect L^W , the limit cycle behavior of M^W ? Note that if all $w_i = 1$, then $k = w_1 + \dots + w_k = W$, where W denotes the sum of weights. Otherwise, $k > W$. On average, the smaller W is relative to k , the smaller each w_i tends to be; hence, the smaller the edges in M^W tend to be; hence, the closer M^W is to M ; hence, the more symmetric M^W tends to be. So, generalizing the above estimate for L , we can approximate L^W by

$$L^W \approx W/n.$$

6. BAM Simulation and Demonstration. Robert Sasseen successfully demonstrated both unidirectional and bidirectional bivalent associative memories on VERAC's Texas Instruments Explorer AI Workstation. This graphics intensive software is written in the object-oriented programming language FLAVORS, which seems especially appropriate, as well commodious, for representing network behavior. We mention that Robert has developed several other Explorer simulations of much more complex network behavior. Currently the utility of using FLAVORS to model networks on the Explorer is heavily constrained by the Explorer's arithmetic processing capabilities. To

relieve these constraints, VERAC's Adaptive Systems Group has entered an agreement with TI to beta-test their new Odyssey Board (containing four TMS-32020 DSP processors) on the Explorer. We have agreed to receive the Odyssey Board in November 1986. We expect that this neural network accelerator will greatly increase our ability to test new network theories and hypotheses.

At the American Association for Artificial Intelligence (AAAI) Conference in Philadelphia in early August 1986, Robert successfully demonstrated some of these simulations at the TI exhibit booth. The responses were quite positive.

While at the AAAI conference in Philadelphia, Bart and Robert were given a tour of Nabil Farhat's optics lab at the University of Pennsylvania. The tour was thorough and courteous, and we have invited Nabil Farhat to tour the VERAC-UCSD OCCAM facilities.

III. OCCAM AT UCSD

This section summarizes the results of the 1st Quarter OCCAM research effort at UCSD's Optics Lab. The principle points are six: (1) recruitment of project personnel, (2) design and simulation of a new form of all-optical dynamical associative memory, (3) design and simulation of a translation, rotation, and scale invariant optical preprocessor suitable for pattern

recognition by associative memory, (4) design and systems for performing optical minimum and maximum operations that are fundamental to implementations of fuzzy logic/sets, (5) design and construction of an associative memory demonstration computer board, and presentation of a tutorial titled "Holographic Approach to Associative Memory."

1. Recruitment of OCCAM personnel. Assistant Professor Clark Guest directs the OCCAM effort at UCSD. When OCCAM commenced, Clark had recruited three Ph.D. candidate graduate students in the department of Electrical Engineering and Computer Science (EECS) to participate in the project. Those students are Myung Soo Kim, Robert Te Kolste, and Hedong Yang. Clark personally grounded all three in associative memory and neural net methodology through independent study projects; they also successfully completed Bart's Fuzzy Theory course.

2. New Optical Dynamical Neural Network. A new optical implementation of a crossbar associative network with feedback, the type studied by Kohonen, Amari, Grossberg, et al and made popular by Hopfield, has been designed and simulated. The design uses the sigmoidal response of the Hughes Liquid Crystal Light Valve (LCLV) device to implement the threshold neuron processing elements.

The LCLV modulates light through a birefringent polarization conversion that nominally yields a sine squared output intensity response to an increasing input intensity. Proper electrical biasing ensures that the

output saturates at the first peak of the sine squared curve, thereby yielding the approximately sigmoidal response necessary for noise-quenching/signal-enhancing processing element behavior.

The crossbar interconnection of nodes is achieved with an optical matrix-vector multiplier designed around the LCLV, as shown in Figure 1 attached. The matrix of connection strengths is imaged onto the output side of the LCLV with linearly polarized light. A uniform beam with the orthogonal polarization is also shown onto the side of the LCLV. Bipolar connection weights are achieved through a comparison of the two beams. Where the matrix image intensity exceeds the uniform beam intensity a positive weight is coded; otherwise, a negative weight is coded.

The current system has several advantages over other electrooptical implementation of feedback associative memories. Connection matrix weights are entered as light intensity, an image on the face of a CRT will suffice. Thus connection strengths can be readily changed according to any desired adaptation algorithm. Bipolar connection weights are achieved without requiring separate display elements of the positive and negative values. The LCLV intrinsically provides the nonlinear response of the neurons, thereby eliminating the need for optical-to-electronic and electronic-to-optical conversions on every iteration. The LCLV is a high resolution device, and should eventually be able to support 500 or more neuron elements.

A computer simulation of the system was coded in the programming language C. The simulation incorporated not only the general feedback

associative memory architecture, but the sine squared characteristic of the LCLV, the dynamical time response of the LCLV, and the polarization encoding of bipolar weights as well. Simulation results show that the LCLV feedback associative memory dependably converges to correct recall within a few response times of the LCLV. Since in the 2nd Quarter of the OCCAM program convergence was proved for the continuous version of the BAM, an LCLV implementation seems promising.

Based on these results, Hughes was approached and subsequently consented to donate a LCLV to the OCCAM project. Characterization measurements are currently being conducted on the device, and experimental implementation of the feedback associative memory will begin soon.

3. Translation, Scale, and Rotation Invariant Optical Preprocessor for Associative Memory. One task specified at the beginning of the OCCAM program was the determination of operating systems for optical associative memories. An important part of this task is the characterization of the boundary between the associative memory and the support environment that makes up its operating system. Specifically, we are currently studying the interface of preprocessing and associative processing in the field of image pattern recognition.

The first phase will be to use a traditional optical preprocessing system to achieve translation, scale, and rotation invariant feature extraction. The feature values will serve as inputs to an associative memory that will perform image classification. In the second phase,

invariance operations will be incorporated into the associative part of the system, simplifying and eventually eliminating the optical preprocessor. A comparison of the approaches will be made to determine which tasks are most efficiently carried out through preprocessing and which tasks are appropriate for associative systems.

Pursuant to the first phase objective, an optical preprocessor capable of translation, scale, and rotation invariant feature extraction has been designed. This preprocessor is represented in Figure 2 attached. Translation invariance is achieved in the first stage by taking the magnitude (with the LCLV) of the two-dimensional optical Fourier transform of the object. The processor second stage is a phase-coded matched filter that uses circular harmonics as rotationally invariant features. Radial moments r^k , $k = 1, 2, \dots$, of the circular harmonics are taken. When the scale of the input object is changed, all moments are scaled in a predictable way. Use of an on-center off-surround competitive feature detector will compensate for this feature scaling, thus permitting invariant recognition.

The use of circular moments for scale and rotation invariant feature extraction has been simulated with a computer model. The four alphabetic letters A, E, F, and R were used as input images. Twenty-five circular moments have been calculated for each letter in a variety of scales and rotations. The results demonstrate good intraclass recognition and interclass discrimination. Currently, the method of mutual information is being used to select a smaller set of moments that will be used in the

planned optical implementation. Simulation of associative memory classification of detected features is also proceeding.

4. Optical Min and Max Fuzzy Operators. Optical implementations of fuzzy logic and fuzzy cognitive maps (FCMs) is a key objective of the OCCAM project. The operations of maximum and minimum play roles in fuzzy logic computations, as discussed above in the section on FAMS, that are parallel to the roles of addition and multiplication in matrix algebra. Many fuzzy operations, e.g., the compositional rule of inference, can be cast as matrix vector products with max and min substituting for sum and product. Optical implementations of matrix vector multipliers are well known. Recent work in OCCAM has identified optical implementations of the max and min operations that can be incorporated into fuzzy logic processors.

There are two basic approaches to optical implementation of min and max. The first is an indirect approach. An optical implementation of the Boolean test ($A > B$) is performed bitwise parallel on two data pages. The binary mask obtained from this operation is applied to data page A, the complement of the mask is applied to page B. The two resulting images are then combined, yielding an image with the bitwise maximum of data pages A and B. If the mask and its complement are interchanged, the bitwise minimum is formed.

The second approach yields directly a bitwise max or min data page, with the intermediate masking steps. It is based on the identifies

$$\max(a, b) = (a + b + |a - b|) / 2 ,$$

$$\min(a, b) = (a + b - |a - b|) / 2 ,$$

which follow by checking the three cases $a = b$, $a < b$, and $a > b$. An optical implementation of this approach is shown in Figure 3. The system uses coherent subtraction, and implements the absolute value operation with an LCLV, which must be operated in the linear range of its response curve.

5. Associative Memory Demonstration Board. For educational and demonstration purposes, a neural net demonstrator device has been designed and built. The device consists of a single board microcontroller and a custom designed display board. The LED display can indicate pairs of input and output vectors that have been associated in memory. A trial input vector may be supplied by the user, and an output vector pattern is generated. The microcontroller is fully programmable and many memorization and recall algorithms can be implemented, included BAM and optimal linear heteroassociators. The board is currently being programmed, and the hardware is being tested.

6. SPIE Tutorial. At the invitation of SPIE, Clark presented a half day tutorial entitled "Holographic Approach to Associative Memory" on 17 August 1986. The tutorial was attended by ten people and well received. A copy of the notes is attached to this R&D Status Report.

Finally, an optical volume holographic associative memory has been designed, and analysis of its characteristics has begun. Other implementations of associative memories in photorefractive crystals are under investigation by other groups, but they make no use of phase encoding of the associated beams. This is an important aspect of any system that will fully use the storage capacity of volume phase holograms, and is the current center of our attention. Experiments with association of phase encoded beams in photorefractive crystals are planned.

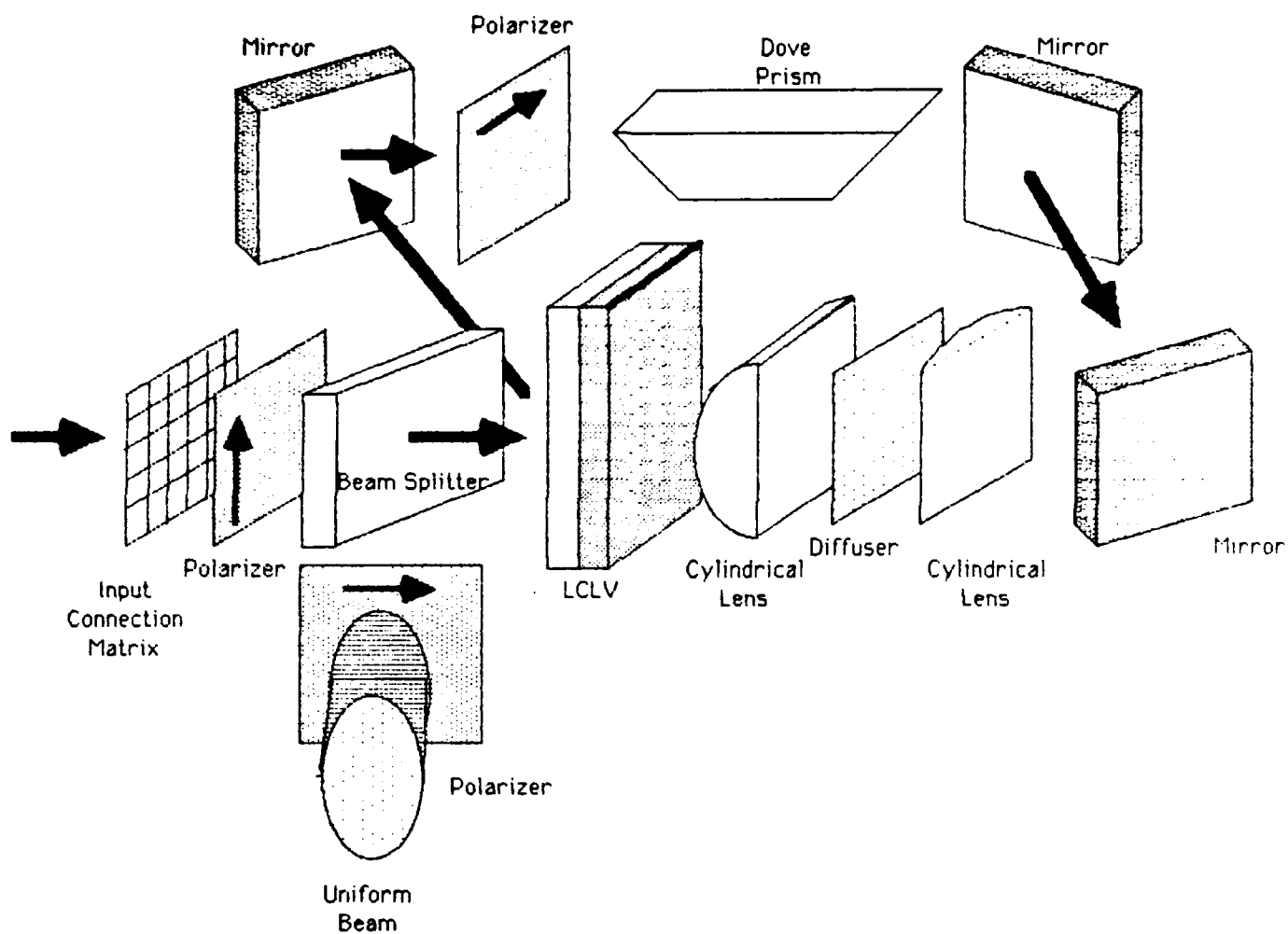


Figure 1) LCLV Feedback Associative Memory

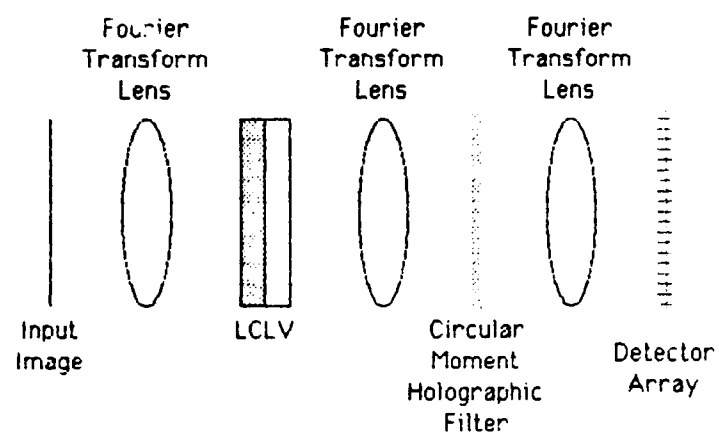


Figure 2) Invariant feature optical preprocessor

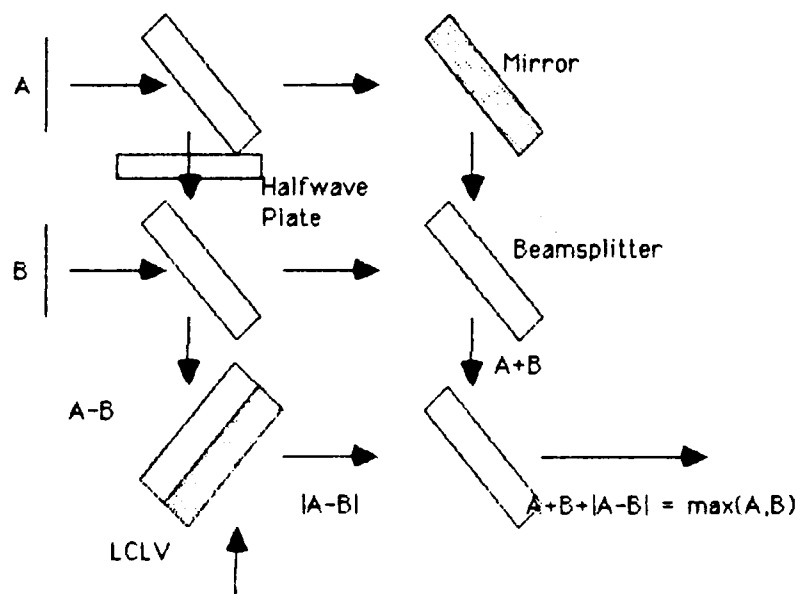


Figure 3) Optical implementation of max operation