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ROBUST PREALLOCATED PREFERENTIAL DEFENSE MODEL

Jerome Bracken

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(With a contribution by Richard M. Soland)

September 1986

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INSTITUTE FOR DEFENSE ANALYSES

IDA Independent Research Program

PREFACE

This study was conducted as part of the Independent Research Program of the Institute for Defense Analyses, under which significant issues of general interest to the defense research community are investigated.

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I. INTRODUCTION

RPPDM is a computer model designed to solve an extended version of the preallocated preferential defense game as presented by Bracken, Brooks and Falk in [1]. The problem revolves around a defender's attempt to protect T targets against A reentry vehicles. The targets may possess different relative values. (In [1] all the targets were assumed to be identical.) We will assume that these valuations are objective and that the defender is unable to hide the values of particular targets from the attacker. The defender has at his disposal D interceptors with which to destroy the RV's. Each interceptor and RV may be assigned to one and only one target. We assume that the attacker knows the total number of interceptors but is unaware of the specific allocation to defend each target.

In [5] and [6] Matheson solved the case, known as the Basic Game, where the attack size A is known to the defender. When we loosen this restriction, however, the solution of the Basic Game no longer suffices, for the defender must now rely on a single strategy, which must be useful against more than one attack size. In [1] a criterion is proposed for a robust defense which most closely approximates the expected outcome of the Basic Game. Robustness problems are formulated as linear programs for four alternate scenarios depicting different combinations of behavioral assumptions regarding the attacker and the defender. The present model, RPPDM, solves the most interesting of these scenarios: the defender believes that the attacker can discover and thus optimize whatever defense he chooses to employ, and the defender is correct (case II,II of [1]).

As it is currently designed, RPPDM may be used in either batch or interactive mode. In interactive mode, RPPDM prompts the user for every piece of information necessary to execute the program. In batch mode a separate program is run to create an input file for use with RPPDM. The user has a choice of sending the report to a file, the terminal, or both. The user may choose to solve any number of robust defenses for a given set of Basic Game parameters. (This will be made clear in the examples.) The output report presents a summary of the game parameters, the optimal strategies for the Basic Game, the expected outcome of the Basic Game, the robust defense, the optimal attacks against the robust defense, and the resulting expected target-survival rates.

The code is organized around a main program, MAIN, whose major function is to query the user (or read from a file) for specific parameters needed to set up the problem, such as the number of interceptors, the number of targets, etc. MAIN then uses that information to call on a series of subroutines to conduct the appropriate numerical calculations and to generate the solution reports.

The subroutines are organized into three categories:

- 1) PIJ subroutines
- 2) LP subroutines
- 3) REPORT subroutines

The PIJ subroutines include SIMAT1, SIMAT2, SEQAT1, and SEQAT2. They are used to generate the PIJ's associated with particular combinations of attack and defense methodologies. Only one of these subroutines is used in any one run of RPPD.

The LP subroutines include BG, YROUBUST and XROBUST. These subroutines are the heart of the model. They set up the linear programming equivalents of the problem in a format that can be accepted by XMP, a linear programming system. BG is responsible for returning the optimal attacker and defender strategies the expected outcome (game value) of the Basic Game. YROBUST calculates the robust defense. XROBUST solves for the optimal attacks against the robust defense and the resulting expected survival rates.

The REPORT subroutines include SUMMARY, STPRINT, YPRINT, VPRINT, ALPRINT, ALYRPRINT, ALVPRINT, and RVINTCOUNT. They are used to produce the output reports.

The code is written in FORTRAN-77 to run on a VAX 8600 computer.

Note that computer programs which solve the Basic Game are documented in [3], [4], and [7].

II. MAIN PROGRAM

Program MAIN acts as a coordinator between the user and the subroutines where the actual "work" takes place. MAIN first asks the user whether interactive or batch mode is to be used. If batch was chosen it asks for the name of the input file. The flow of MAIN may then be broken roughly into five stages:

- 1) Generate the PIJ's
- 2) Solve the Basic Game

- 3) Produce the solution report for the Basic Game
- 4) Solve the robust game
- 5) Produce the solution report for the robust game

During each stage, MAIN will prompt the user (or read from file in batch mode) for the additional information needed to process that stage. We will discuss each of these stages separately, focusing on the needed inputs and the subroutines called.

Stage 1: Generate the PIJ's

RPPDM is equipped to handle five combinations of attack and defense methodologies:

- A) Simultaneous Attack with One Shot
- B) Simultaneous Attack with Shoot-Look-Shoot
- C) Sequential Attack of Unknown Size with One Shot
- D) Sequential Attack of Unknown Size with Shoot-Look-Shoot
- E) Sequential Attack of Known Size with Shoot-Look-Shoot

Depending upon the answers to two questions (three if the attack is sequential), MAIN will call on one of four subroutines, SIMAT1 for A, SIMAT2 for B, SEQAT1 for C and D, and SEQAT2 for E, to generate the appropriate set of PIJ's. For each of the PIJ subroutines the following inputs are necessary: the maximum number of RVs that may defend a single target (S), the failure probability of the RV's (PFA) and that of the interceptors (PFD). For cases involving shoot look shoot defenses, two failure probabilities for the interceptors are necessary (PFD1 and PFD2 rather than PFD).

Stage 2: Solve the Basic Game

Main prompts for the minimum attack size (MINRV), the maximum attack size (MAXRV), and the attack size increment (INCRV). These values are then used to calculate the actual set of attack sizes (RV) and the number of attack sizes (A). The number of interceptors (INT) and the total number of targets (TARGETS) are also needed. In addition, if there are more than one type of target, the relative value VTYPE) and the number of targets (NTAR) belonging to each type must also be entered. The value and the size of each target type is then calculated as the fraction of all the targets. Subroutine BG is called to solve the Basic Game. BG returns the attacker's minimax strategy in XBG, the defender's minimax strategies in YBG, and the game values in VBG.

Stage 3: Produce the solution report for the Basic Game

MAIN calls on the REPORT subroutines to print out the results of the Basic Game on the selected output device(s). SUMMARY provides a listing of the problem's parameters as entered. STPRINT prints out XBG and YBG, VPRINT prints out VBG, ALPRINT prints out allocation tables based on XBG and YBG, ALVPRINT prints out the expected number of targets that will survive for each target type, and RVINTCOUNT prints out the numbers RV's and interceptors assigned to each target type. (See Section VII, "Notes on Output" for more details.)

Stage 4: Solve the Robust Game

MAIN prompts for the number of robust strategies desired. Stages 4 and 5 are repeated until all of them have been solved.

MAIN prompts for a lower and an upper bound on the attack sizes for which a robust defense is to be found. Both bounds must lie in the set of attack sizes defined for the Basic Game during stage 2. Subroutine YROBUST is then called to solve for the robust defense (YII). With the robust defense defined, MAIN calls XROBUST to find the optimal attacks (XII) against YII and the resulting expected survival rates (VII).

Stage 5: Produce the solution report for the Robust Game

MAIN calls on YPRINT to print out YII, STPRINT to print out XII, VPRINT to print out VII, ALYPRINT to print out the allocation table for YII, ALPRINT to print out the allocation table for XII, ALVPRINT to print out the expected number of targets that will survive for each target type, and RVINTCOUNT to print out the numbers of RV's and interceptors assigned to each target type.

III. PIJ SUBROUTINES

The PIJ subroutines SIMAT, SIMAT2, SEQAT1, and SEQAT2 are called on to generate the appropriate set of PIJ's associated with particular attack and defense methodologies.

SIMAT1 is designed to generate the PIJ's when the attack is simultaneous and the defense has only one opportunity to intercept the RV's.

SIMAT2 applies when the attack is still simultaneous, but the defense has two opportunities to intercept the RV.

SEQAT1 applies when the attack is sequential of unknown size and the defender has one or two chances to intercept the RV's. The first case is accomplished by passing 1.0 for the failure rate of the first interceptor salvo (PFD1) and assigning PFD to PFD2. In both cases the defender does not know the number of RV's that will arrive at a particular target.

SEQAT2 applies when the attack is sequential, the defender has shoot-look-shoot, and the defender knows (after the attack begins) how many RV's will engage him at a given target.

The resulting PIJ's are returned to MAIN in the two dimensional array P. The first index of P is the number of RV's that attacks a target, and the second index is the number of interceptors that defends a target. Thus $P(I,J)$ is the probability that a target will survive when attacked by I RV's and defended by J interceptors. See Appendix H for discussions of the concepts on which these PIJ subroutines are based.

IV. LP SUBROUTINES

The purpose of BG, YROBUST, and XROBUST is to set up the appropriate linear programs for processing by XMP. In the following subsections we will examine each individually in terms of the linear programs they solve. Please consult program listing in the Appendix and XMP documentation for details regarding the mechanics of the interactions between BG, etc., and XMP. In the linear programs that follow, all variables are, or can be assumed to be, non-negative.

BG

BG is designed to solve the Basic Game. A linear program of the Basic Game may be set up as follows (see Appendix G for the derivation):

$$\begin{aligned}
\text{VBG} &= \max_{Y(k,j), s(k), t} \left[\sum_{k=1}^{\text{NTYPE}} s(k) \cdot (\text{RV}/\text{TARGETS}) \cdot t \right] \\
\text{subject to} & \left. \begin{aligned}
& s(k) - \text{VF}(k) \sum_{j=0}^S \left[P(0,j) \cdot Y(k,j) \right] \leq 0 \\
& s(k) - \text{NF}(k) \cdot t - \text{VF}(k) \sum_{j=0}^S \left[P(1,j) \cdot Y(k,j) \right] \leq 0 \\
& \vdots \\
& s(k) - \text{NF}(k) \cdot R \cdot t - \text{VF}(k) \sum_{j=0}^S \left[P(R,j) \cdot Y(k,j) \right] \leq 0
\end{aligned} \right\} 1 \leq k \leq \text{NTYPE} \\
& \sum_{j=0}^S Y(k,j) = 1 \quad 1 \leq k \leq \text{NTYPE} \\
& \sum_{k=1}^{\text{NTYPE}} \sum_{j=0}^S \left[j \cdot \text{NF}(k) \cdot Y(k,j) \right] = \text{INT}/\text{TARGETS}
\end{aligned}$$

The problem is solved first for RV(1), the smallest attack size, and the solutions stored in VBG, XBG (the dual variables of the inequality constraints) and YBG. Then RV(1) is substituted by RV(2) and the problem reoptimized. BG keeps substituting RV's until all of the attack sizes are solved.

XBG and YBG are passed back to MAIN as three dimensional arrays. The first index designates the attack size, the second index is the target type, and the third index is the number of RV's (for XBG) or interceptors (for YBG). XBG(A,K,I) and YBG(A,K,J), are, respectively, the fraction of type K targets attacked by I RV's and the

fraction of type K targets defended by J interceptors for the basic game with the A'th attack size (RV(A)).

VBG is passed back to MAIN as a one dimensional array with the index designating the attack size. VBG(A) is the game value for the A'th attack size.

YROBUST

YROBUST is designed to solve for the robust defense. A linear program of the robust game may be set up as follows (see Appendix G for the derivation):

$$\begin{aligned}
 & \max_{Y(k,j), s(A,k), t(A)} \quad \rho \\
 & \text{subject to} \\
 & \left. \begin{aligned}
 & \text{VF}(k) \sum_{j=0}^S \left[P(0,j) \cdot Y(k,j) \right] - Z(k,0) = 0 \\
 & \text{VF}(k) \sum_{j=0}^S \left[P(R,j) \cdot Y(k,j) \right] - Z(k,R) = 0
 \end{aligned} \right\} \quad 1 \leq k \leq \text{NTYPE} \\
 & \rho \cdot \text{VBG}(A) - \sum_{k=1}^{\text{NTYPE}} s(A,k) + \left[\text{RV}(A)/\text{TARGETS} \right] \cdot t(A) \leq 0 \quad 1 \leq A \leq \bar{A} \\
 & \left. \begin{aligned}
 & s(A,k) - Z(k,0) \leq 0 \\
 & s(A,k) - Z(k,1) - \text{NF}(k) \cdot t(A) \leq 0 \\
 & \vdots \\
 & s(A,k) - Z(k,R) - \text{NF}(k) \cdot R \cdot t(A) \leq 0
 \end{aligned} \right\} \quad \begin{aligned} & 1 \leq A \leq \bar{A} \\ & 1 \leq k \leq \text{NTYPE} \end{aligned} \\
 & \sum_{j=0}^S Y(k,j) = 1 \quad 1 \leq k \leq \text{NTYPE} \\
 & \sum_{k=1}^{\text{NTYPE}} \sum_{j=0}^S \left[j \cdot \text{NF}(k) \cdot Y(k,j) \right] = \text{INT}/\text{TARGETS}
 \end{aligned}$$

The Y that solves this program is stored in YII and passed back to MAIN as a two dimensional variable. The first index is target type, and the second index is the number of interceptors assigned to a target. YII(K,J) is the fraction of type K targets defended by J interceptors under the robust defense.

XROBUST

Given the robust defense, YII, XROBUST finds the optimal attack, XII, and the resulting expected survival rate using the following LP:

$$\begin{aligned}
 & \text{VII} = \min_{X(k,i)} \sum_{k=1}^{\text{NTYPE}} \text{VF}(k) \sum_{i=0}^R \left\{ \left[\sum_{j=0}^S P(i,j) \text{YII}(k,j) \right] X(k,i) \right\} \\
 & \text{subject to} \\
 & \sum_{i=0}^R X(k,i) = 1 \quad 1 \leq k \leq \text{NTYPE} \\
 & \sum_{k=1}^{\text{NTYPE}} \sum_{i=0}^R \left[i \cdot \text{NF}(k) \cdot X(k,i) \right] = \text{RV/TARGETS}
 \end{aligned}$$

After the LP is solved, and the X that yields VII is stored in XII for every attack size.

XII and VII are passed back to MAIN as, respectively, two and one dimensional variables. The indices are the same as their counterparts in the Basic Game, XBG and VBG.

V. REPORT SUBROUTINES

The report subroutines generate formatted output for RPPDM. The SUMMARY subroutine creates a copy of the parameters of the game as specified and send it to the output device selected. The STPRINT subroutine prints out attacker and defender strategies that are indexed by attack sizes (i. e., XBG, YBG, and XII). The YPRINT

subroutine prints out the robust defense. The VPRINT subroutine prints out the expected survival rate for the chosen range of attack sizes. ALPRINT prints out target allocation tables for attacker and defender strategies indexed by attack sizes. ALYPRINT prints out target allocation tables for the robust defense. ALVPRINT prints out the expected numbers of targets that will survive for each target type. RVINTCOUNT prints out the missile (RV or interceptor) allocation by target types.

VI. NOTES ON INPUT

A separate program, BATCH, is provided to facilitate the use of batch mode. It is an interactive program that creates an input file that may be read by RPPDM. It will query the user for every piece of information that is needed to run RPPDM (in the same order as the interactive mode), and write the information to a sequential file named by the user. Thus when a user seeks to run a series of problems with changes in only a small number of the parameters, he or she could create the first problem using BATCH, then use an editor to modify the input file for the first problem to fit the data for the other problems. We demonstrate this procedure in the second example of Section VIII.

VII. NOTES ON OUTPUT

In any given run of RPPDM a minimum of 9 tables are printed, and for each robust defense desired, an additional 8 tables are added. (For runs involving targets of equal value, the numbers are 7 and 6, since the RV and interceptor allocation tables are eliminated.)

The first table is a table of the basic parameters that define the Basic and the Robust Games.

The next eight tables list the results of the Basic Game. The first two of these tables print out, respectively, the attacker and defender strategies indexed by target types and then by attack sizes. For each attack size A, there are N entries which define the minimax strategy for a given target type K. The i'th entry corresponds to the fraction of type K targets that will be assigned i-1 RV's or interceptors. Each row contains a maximum of 10 entries. The first row will include the first 10 entries, the second row the next 10 entries, etc. Therefore, an entry in the third row and sixth column is the fraction of targets (of that target type) to receive 25 RV's or interceptors.

The third table of the Basic Game is the game value, or the expected survival rate associated with the minimax strategies of the Basic Game. Each entry is the fraction of the "total value" that is expected to survive. Thus .3567 means that the expected value of all the targets that survive the attack is 35.7% of the original value before the attack.

The fourth and the fifth tables are just a variation on the strategy tables discussed earlier. These target allocation tables, instead of using fractions, give the actual numbers that correspond to the minimax strategies. The allocation tables are first subdivided by attack sizes and then by target types. For each attack size the strategy for all the different target types are listed together. The Kth entry now corresponds to the number of targets (of the target type under consideration) that should be assigned K-1 missiles.

The seventh table lists the number of targets of each target type that is expected to survive in the Basic Game.

The eighth and ninth tables lists the RV and interceptor allocations by target type. Each entry corresponds to the total number of missiles assigned to all the targets of that target type. (As noted above, these tables are not created when there is only one type of target.)

The next eight tables correspond to the first Robust Game (if any). The tables are printed out and the entries defined in an identical way to those in the Basic Game except that the defender's strategy and target allocation tables are not indexed by attack sizes. The first table lists the robust defender's strategy, the second the optimal attacker strategies against the robust defense, the third the expected value of the robust game, the fourth and fifth the target allocation tables for the defender and the attacker (respectively), the sixth the expected number of targets surviving, by target type, and the seventh and eighth the missile allocation tables for the attacker and the defender in this, the first Robust Game.

If another robust defense is called for, then eight more tables will be printed in the same order as the eight associated with the first Robust Game.

VIII. HOW TO USE RPPDM

The following is a list of the FORTRAN files in RPPDM and their contents:

<u>File</u>	<u>Contents</u>
RPPDM.FOR	MAIN

PIJ1.FOR
PIJ2.FOR
PIJ3.FOR
PIJ4.FOR
LP1.FOR
LP2.FOR
LP3.FOR
REPORT1.FOR

REPORT2.FOR

REOIRT3.FOR
BATCH.FOR

SIMAT1
SIMAT2
SEQAT1
SEQAT2
BG
YROBUST
XROBUST
SUMMARY,STPRINT
YPRINT,VPRINT
ALYPRINT
ALPRINT,ALVPRINT
RVINTCOUNT
BATCH

In addition, the XMP linear programming package is necessary for the use of RPPDM.

Together with the XMP subroutines, all the files except BATH.FOR must be compiled and then linked into an executable image. BATCH.FOR should be compiled and linked separately.

The following examples illustrate typical runs of the model. In the first example the user wishes to examine the robust defense under simultaneous attack and one shot defense with only one type of target. In the second example the user wants to find two different robust defenses under a more complicated combination of attack and defense methodologies with 3 different types of targets. In the first case he or she interactively provides the model with all the specifications needed. In the second case he or she will create an input file with BATCH and then use batch mode with RPPDM.

In order to solve the Basic Game, RPPDM needs the attack and defense methodologies, the RV and interceptor failure rates, the minimum and maximum attack sizes, the attack size increment, the number of interceptors, the total number of targets, the relative value of each target type, and the number of targets in each target type. For the Robust Game, RPPDM needs to know the minimum and maximum attack sizes in the range of attack sizes for which the robust defense is to be found; both must be in the set of attack sizes for the Basic Game. The same increment from the Basic Game is used here.

These sample runs are shown exactly as they would have appeared on a terminal screen except for two items: the '*****USER SAYS >' segment that preceed every user entry and the comments outlined by slashes. They are provided to highlight and clarify certain situations.

A. EXAMPLE 1

```
$ LINK RPPDM, PIJ/LIB, LP/LIB, XMP/LIB, REP/LIB
$ RUN RPPDM
```

```
//////////
// On the VAX/VMS operating system there is a library function
// which allows object files to be placed together in a library
// file. Because the PIJ, LP, REPORT, and XMP subroutines were
// all stored in several different files, library files were
// created to avoid confusion.
//
//////////
```

THERE ARE TWO INPUT OPTIONS:

- 1) TERMINAL (INTERACTIVE)
- 2) FILE (BATCH)

PLEASE ENTER THE NUMBER OF THE DESIRED OPTION
*****USER SAYS> 1

You have three options for the output of the results :

- 1) TERMINAL only
- 2) FILE only
- 3) TERMINAL and FILE

Please enter the number for the desired option ?
.....USER SAYS> 3

Please type in the desired file name (of less than 10 characters including the extension) ?
.....USER SAYS> TEST1.OUT

////////////////////////////////////
// A file named TEST1.OUT will be created to hold a copy of the
// output report. If an extension had not been specified, .DAT
// would be added by default.
////////////////////////////////////

The MAXIMUM number of RV's (up to 30) at a single target ?
.....USER SAYS> 10

The MAXIMUM number of INTERCEPTORS (up to 30) at a single target ?
.....USER SAYS> 10

Select one of the following attack methodologies:

- 1) SIMULTANEOUS ATTACK
- 2) SEQUENTIAL ATTACK

Please input the number of the desired attack ?
.....USER SAYS> 1

The FAILURE rate of the RV's ?
.....USER SAYS> .3

Select one of the following defense methodologies:

- 1) ONE SHOT
- 2) SHOOT LOOK SHOOT

Please input the number for the desired option ?
.....USER SAYS> 1

The FAILURE rate of the interceptors ?
.....USER SAYS> .3

////////////////////////////////////
// All the information needed to generate the Pij's are passed
// to SIMAT1, since SIMAT1 corresponds with the attack-defense
// combination the USER has selected. If another combination
// had been selected, MAIN may ask for additional information.
// The second example illustrates this case.
////////////////////////////////////

The MINIMUM attack size ?
.....USER SAYS> 1000

The MAXIMUM attack size ?
.....USER SAYS> 10000

The attack size INCREMENT ?
.....USER SAYS> 1000

The NUMBER of interceptors ?
.....USER SAYS> 6000

The TOTAL NUMBER of targets ?
.....USER SAYS> 1000

The number of TYPES of targets ?
.....USER SAYS> 1

////////////////////////////////////
/ All the information needed to solve the Basic Game is passed /
/ to BG, which will call on the XMP subroutines to solve the /
/ linear program. /
////////////////////////////////////

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 10000 REAL: 10000

XMAPS...YOU HAVE ROOM FOR 2987 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 10000 TO 6509

////////////////////////////////////
/ The above messages are sent by XMP. Ignore them. If you /
/ selected an output device other than the terminal, the /
/ messages will not appear there. /
////////////////////////////////////

////////////////////////////////////
 / SOLUTION REPORT FOR THE BASIC GAME /
 //////////////////////////////////////

THE PARAMETERS OF THIS PREALLOCATED PREFERENTIAL DEFENSE GAME

.....

THE ATTACK METHODOLOGY	SIMULTANEOUS
THE DEFENSE METHODOLOGY	ONE SHOT
THE FAILURE RATE OF RV'S	0.300
THE FAILURE RATE OF THE INTERCEPTORS	0.300
MAXIMUM NUMBER OF RV'S ATTACKING A SINGLE TARGET	10
MAXIMUM NUMBER OF INTERCEPTORS DEFENDING A SINGLE TARGET	10
THE MINIMUM NUMBER OF RV'S	1000
THE MAXIMUM NUMBER OF RV'S	10000
THE ATTACK SIZE INCREMENT	1000
THE NUMBER OF INTERCEPTORS	6000
THE TOTAL NUMBER OF TARGETS	1000
THE NUMBER OF TARGET TYPES	1
TARGET TYPE 1:	
NUMBER OF TARGETS	1000
RELATIVE VALUE	1.000

.....

THE ATTACKER'S BASIC GAME MINIMAX STRATEGIES

.....									
ATTACK SIZE	TARGET TYPE 0	1	2	3	4	5	6	7	8 9
1000	: 0.8138 ; : 0.0000 ;	: 0.0169 ;	: 0.0081 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.1611 ;	: 0.0000 ;	: 0.0000 ;
2000	: 0.6277 ; : 0.0000 ;	: 0.0338 ;	: 0.0162 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3223 ;	: 0.0000 ;	: 0.0000 ;
3000	: 0.4415 ; : 0.0000 ;	: 0.0506 ;	: 0.0244 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.4834 ;	: 0.0000 ;	: 0.0000 ;
4000	: 0.2554 ; : 0.0000 ;	: 0.0675 ;	: 0.0325 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.6446 ;	: 0.0000 ;	: 0.0000 ;
5000	: 0.0592 ; : 0.0000 ;	: 0.0844 ;	: 0.0406 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.8057 ;	: 0.0000 ;	: 0.0000 ;
6000	: 0.0000 ; : 0.0000 ;	: 0.0641 ;	: 0.0736 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.2475 ;	: 0.6148 ;	: 0.0000 ;
7000	: 0.0000 ; : 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 1.0000 ;	: 0.0000 ;
8000	: 0.0000 ; : 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 1.0000 ;
9000	: 0.0000 ; : 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 1.0000 ;
10000	: 0.0000 ; : 1.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;
.....									

THE DEFENDER'S BASIC GAME MINIMAX STRATEGIES

ATTACK SIZE	TARGET TYPE		1: 100.00% OF TOTAL TARGETS, WITH 100.00% OF TOTAL VALUE						
	0	1	2	3	4	5	6	7	8
1000	: 0.1306 ; : 0.3786 ;	: 0.1462 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3446 ;	: 0.0000 ;	: 0.0000 ;
2000	: 0.1306 ; : 0.3786 ;	: 0.1462 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3446 ;	: 0.0000 ;	: 0.0000 ;
3000	: 0.1306 ; : 0.3786 ;	: 0.1462 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3446 ;	: 0.0000 ;	: 0.0000 ;
4000	: 0.1306 ; : 0.3786 ;	: 0.1462 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3446 ;	: 0.0000 ;	: 0.0000 ;
5000	: 0.1306 ; : 0.3786 ;	: 0.1462 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.3446 ;	: 0.0000 ;	: 0.0000 ;
6000	: 0.3398 ; : 0.5280 ;	: 0.0343 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;
7000	: 0.4000 ; : 0.6000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;
8000	: 0.4000 ; : 0.6000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;
9000	: 0.4000 ; : 0.6000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;
10000	: 0.4000 ; : 0.6000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;	: 0.0000 ;

THE GAME VALUES

.....

ATTACK SIZE

1000	:	0.8777
2000	:	0.7554
3000	:	0.6331
4000	:	0.5109
5000	:	0.3886
6000	:	0.2826
7000	:	0.1923
8000	:	0.1281
9000	:	0.0853
10000	:	0.0568

.....

THE ATTACKER'S BASIC GAME TARGET ALLOCATION

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	813.8	16.9	8.1	0.0	0.0	0.0	161.1	0.0	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	627.7	33.8	16.2	0.0	0.0	0.0	322.3	0.0	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	441.5	50.6	24.4	0.0	0.0	0.0	483.4	0.0	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	255.4	67.5	32.5	0.0	0.0	0.0	644.6	0.0	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	69.2	84.4	40.6	0.0	0.0	0.0	805.7	0.0	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	64.1	73.6	0.0	0.0	0.0	247.5	614.8	0.0	0.0
	0.0									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1000.0	0.0	0.0
	0.0									

ATTACK SIZE = 8000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	1000.0 :	0.0 :
	: 0.0 :								

ATTACK SIZE = 9000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	1000.0 :
	: 0.0 :								

ATTACK SIZE = 10000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 1000.0 :								

.....

THE DEFENDER'S BASIC GAME TARGET ALLOCATION

.....

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 130.6 :	146.2 :	0.0 :	0.0 :	0.0 :	0.0 :	344.6 :	0.0 :	0.0 :	0.0 :
	: 378.6 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 130.6 :	146.2 :	0.0 :	0.0 :	0.0 :	0.0 :	344.6 :	0.0 :	0.0 :	0.0 :
	: 378.6 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 130.6 :	146.2 :	0.0 :	0.0 :	0.0 :	0.0 :	344.6 :	0.0 :	0.0 :	0.0 :
	: 378.6 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 130.6 :	146.2 :	0.0 :	0.0 :	0.0 :	0.0 :	344.6 :	0.0 :	0.0 :	0.0 :
	: 378.6 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 130.6 :	146.2 :	0.0 :	0.0 :	0.0 :	0.0 :	344.6 :	0.0 :	0.0 :	0.0 :
	: 378.6 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 339.8 :	34.3 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	97.9 :	0.0 :	0.0 :
	: 528.0 :									

TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	: 400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 600.0 :									

ATTACK SIZE = 8000									
0	1	2	3	4	5	6	7	8	9
1	: 400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 600.0 :								

ATTACK SIZE = 9000									
0	1	2	3	4	5	6	7	8	9
1	: 400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 600.0 :								

ATTACK SIZE = 10000									
0	1	2	3	4	5	6	7	8	9
1	: 400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 600.0 :								

.....

THE EXPECTED NUMBER OF TARGETS SURVIVING

.....

ATTACK SIZE :		TARGET TYPE
		1
1000	:	877.71
2000	:	755.43
3000	:	633.14
4000	:	510.86
5000	:	388.57
6000	:	282.57
7000	:	192.34
8000	:	128.08
9000	:	85.30
10000	:	56.81

.....

Please enter the number of different ranges of RV's for which robust solutions are to be found ?

..... Enter 0 if no robust solution is desired.....

```

////////////////////////////////////
//                               //
// If '0' were entered, the program would //
// terminate immediately //
//                               //
////////////////////////////////////

```

.....USER SAYS> 1

The lower and upper bounds for the RV ranges must be between 1000 and 10000

The lower bound :

.....USER SAYS> 1000

The upper bound :

.....USER SAYS> 10000

```

////////////////////////////////////
// Let SIZE be an attack size in the RV range for which a robust //
// solution is to be found. The LP for the robust defense //
// requires that the game values for SIZE be known. Thus the //
// set of attack sizes over which a robust defense is desired //
// must be a subset of the attack sizes for the Basic Game. //
//                               //
////////////////////////////////////

```

```

////////////////////////////////////
//                               //
// YROBUST is called to solve the robust defense. Then XROBUST //
// is called to find the optimal attacks and the expected //
// survival rates. //
//                               //
////////////////////////////////////

```

XMAPS...WORDS OF MEMORY AVAILABLE
 INTEGER: 100000 REAL: 50000

XMAPS...YOU HAVE ROOM FOR 27998 NON-ZEROS IN THE BASIS FACTORS
 YOU COULD REDUCE REAL MEMORY FROM 100000 TO 89901

XMAPS...WORDS OF MEMORY AVAILABLE
 INTEGER: 1000 REAL: 1000

XMAPS...YOU HAVE ROOM FOR 2987 NON-ZEROS IN THE BASIS FACTORS
 YOU COULD REDUCE REAL MEMORY FROM 1000 TO 591

```

////////////////////////////////////
//                               //
// More messages from XMP //
//                               //
////////////////////////////////////

```

//////////////////////////////////////
 /
 SOLUTION REPORT FOR THE ROBUST GAME
 /
 //////////////////////////////////////

THE ROBUST DEFENSE STRATEGY FOR RV RANGE 1000 TO 10000 :

0	1	2	3	TARGET TYPE					8	9
				4	5	6	7			
: 0.2468 : 0.0066 : 0.1489 : 0.0402 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :										
: 0.5575 :										

THE OPTIMAL ATTACK STRATEGIES AGAINST THE ROBUST DEFENSE

ATTACK SIZE	TARGET TYPE 1: 100.00% OF TOTAL TARGETS, WITH 100.00% OF TOTAL VALUE								
	0	1	2	3	4	5	6	7	8
1000	: 0.0000 ;	1.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
2000	: 0.0000 ;	0.0000 ;	1.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
3000	: 0.0000 ;	0.0000 ;	0.0000 ;	1.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
4000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.6667 ;	0.0000 ;	0.0000 ;	0.3333 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
5000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.3333 ;	0.0000 ;	0.0000 ;	0.6667 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
6000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	1.0000 ;	0.0000 ;	0.0000 ;
	: 0.0000 ;								
7000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	1.0000 ;	0.0000 ;
	: 0.0000 ;								
8000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	1.0000 ;
	: 0.0000 ;								
9000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	1.0000 ;
	: 0.0000 ;								
10000	: 0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;	0.0000 ;
	: 1.0000 ;								

THE EXPECTED TARGET SURVIVAL RATE WITH THE ROBUST DEFENSE

.....

ATTACK SIZE

1000	:	0.8157
2000	:	0.7021
3000	:	0.5884
4000	:	0.4822
5000	:	0.3759
6000	:	0.2697
7000	:	0.1791
8000	:	0.1191
9000	:	0.0793
10000	:	0.0528

.....

THE ROBUST DEFENSE ALLOCATION FOR RV RANGE 1000 TO 10000 :

0	1	2	3	TARGET TYPE 1					8	9
				4	5	6	7			
: 246.8 :	6.6 :	148.9 :	40.2 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
: 557.5 :										

THE OPTIMAL ATTACK ALLOCATION AGAINST THE ROBUST DEFENSE

		ATTACK SIZE = 1000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	1000.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0									

		ATTACK SIZE = 2000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	1000.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0									

		ATTACK SIZE = 3000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	1000.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0									

		ATTACK SIZE = 4000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	0.0	666.7	0.0	0.0	333.3	0.0	0.0
	0.0									

		ATTACK SIZE = 5000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	0.0	333.3	0.0	0.0	666.7	0.0	0.0
	0.0									

		ATTACK SIZE = 6000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1000.0	0.0	0.0
	0.0									

		ATTACK SIZE = 7000								
TARGET TYPE	0	1	2	3	4	5	6	7	8	9
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1000.0	0.0	0.0
	0.0									

ATTACK SIZE = 8000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	1000.0 :	0.0 :
	: 0.0 :								

ATTACK SIZE = 9000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	1000.0 :
	: 0.0 :								

ATTACK SIZE = 10000									
0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 1000.0 :								

.....

THE EXPECTED NUMBER OF TARGETS (SURVIVING) WITH THE ROBUST DEFENSE

.....

ATTACK SIZE :	TARGET TYPE
1000	1
2000	815.70
3000	702.06
4000	588.41
5000	482.17
6000	375.93
7000	269.68
8000	179.08
9000	119.12
10000	79.29
	52.80

.....

FORTRAN STOP

\$

TEST1.OUT contains a complete solution report

B. EXAMPLE 2

\$ LINK BATCH
\$ RUN BATCH

////////////////////////////////////
/ BATCH will be used to generate an input file for use /
/ with RPPDM /
////////////////////////////////////

Please type in the desired file name (of less than 10 characters
including the extension) for storage of the parameters ?
.....USER SAYS> TEST2.IN

You have three options for the output of the results :

- 1) TERMINAL only
- 2) FILE only
- 3) TERMINAL and FILE

Please enter the number for the desired option ?
.....USER SAYS> 2

Please type in the desired file name (of less than 10 characters
including the extension) ?
.....USER SAYS> TEST2.OUT

The MAXIMUM number of RV's (up to 30) at a single target ?
.....USER SAYS> 15

The MAXIMUM number of INTERCEPTORS (up to 30) at a single target ?
.....USER SAYS> 19

Select one of the following attack methodologies:

- 1) SIMULTANEOUS ATTACK
- 2) SEQUENTIAL ATTACK

Please input the number of the desired attack ?
.....USER SAYS> 2

The FAILURE rate of the RV's ?
.....USER SAYS> .2

Select one of the following defense methodologies:

- 1) ONE SHOT
- 2) SHOOT LOOK SHOOT

Please input the number for the desired option ?
.....USER SAYS> 2

Is the defender aware, after the attack begins, of the number
of RV's slated for each target (Y or N) ?
.....USER SAYS> N

The FAILURE rate for the first salvo interceptors ?

.....USER SAYS> .2

The FAILURE rate for the second salvo interceptors ?

.....USER SAYS> .3

The MINIMUM attack size ?

.....USER SAYS> 1000

The MAXIMUM attack size ?

.....USER SAYS> 8000

The attack size INCREMENT ?

.....USER SAYS> 1000

The NUMBER of interceptors ?

.....USER SAYS> 4000

The TOTAL NUMBER of targets ?

.....USER SAYS> 1000

The number of TYPES of targets ?

.....USER SAYS> 3

Enter first the RELATIVE VALUE and then the NUMBER of targets for each type. Separate the two entries for each target type with a comma and hit <CR> following the entries for each target type:

.....USER SAYS> 1, 300

.....USER SAYS> 2, 400

.....USER SAYS> 3.5, 300

Both BATCH and RPPDM will check to make sure that the sum of numbers of targets for the target types equal the total number entered earlier. The programs will terminate if the values are inconsistent.

Please enter the number of different ranges of RV's for which robust solutions are to be found ?

..... Enter 0 if no robust solution is desired.....
.....USER SAYS> 2

The lower and upper bounds for the RV ranges must be between 1000 and 8000

The lower bound :
.....USER SAYS> 1000

The upper bound :
.....USER SAYS> 8000

NEXT

The lower bound :
.....USER SAYS> 4000

The upper bound :
.....USER SAYS> 8000

FORTRAN STOP

\$ TYPE TEST2.IN

////////////////////////////////////
//
// PRINT OUT THE INPUT FILE
//
////////////////////////////////////

TEST2. OUT 2
 15
 19

 2
0.20000000 2

N

0.20000000
0.30000000

1000

8000

1000

4000

1000

3

1.00000000

2.00000000

3.50000000

2

1000

8000

4000

8000

300
400
300

\$ RUN RPPDM

THERE ARE TWO INPUT OPTIONS:

- 1) TERMINAL (INTERACTIVE)
- 2) FILE (BATCH)

PLEASE ENTER THE NUMBER OF THE DESIRED OPTION ?

*****USER SAYS> 2

PLEASE ENTER THE NAME OF THE INPUT FILE (OF LESS THAN 12
CHARACTERS INCLUDING THE EXTENSION) ?

*****USER SAYS> TEST2.IN

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 60000 REAL: 60000

XMAPS...YOU HAVE ROOM FOR 27998 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 60000 TO 43291

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 100000 REAL: 50000

XMAPS...YOU HAVE ROOM FOR 26798 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 100000 TO 95151

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 1000 REAL: 1000

XMAPS...YOU HAVE ROOM FOR 2987 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 1000 TO 661

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 100000 REAL: 50000

XMAPS...YOU HAVE ROOM FOR 26798 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 100000 TO 95151

XMAPS...WORDS OF MEMORY AVAILABLE
INTEGER: 1000 REAL: 1000

XMAPS...YOU HAVE ROOM FOR 2987 NON-ZEROS IN THE BASIS FACTORS
YOU COULD REDUCE REAL MEMORY FROM 1000 TO 661

FORTTRAN STOP

\$ TYPE TEST2.OUT

//////
// The file TEST2.OUT is displayed on the terminal
//
//////

THE PARAMETERS OF THIS PREALLOCATED PREFERENTIAL DEFENSE GAME

.....

THE ATTACK METHODOLOGY	SEQUENTIAL WITH ATTACK SIZE AT A TARGET UNKNOWN TO THE DEFENDER SHOOT LOOK SHOOT
THE DEFENSE METHODOLOGY	
THE FAILURE RATE OF RV'S	0.200
THE FAILURE RATE OF THE FIRST SALVO INTERCEPTORS	0.200
THE FAILURE RATE OF THE SECOND SALVO INTERCEPTORS	0.300
MAXIMUM NUMBER OF RV'S ATTACKING A SINGLE TARGET	15
MAXIMUM NUMBER OF INTERCEPTORS DEFENDING A SINGLE TARGET	19
THE MINIMUM NUMBER OF RV'S	1000
THE MAXIMUM NUMBER OF RV'S	8000
THE ATTACK SIZE INCREMENT	1000
THE NUMBER OF INTERCEPTORS	4000
THE TOTAL NUMBER OF TARGETS	1000
THE NUMBER OF TARGET TYPES	3
TARGET TYPE 1:	
NUMBER OF TARGETS	300
RELATIVE VALUE	1.000
TARGET TYPE 2:	
NUMBER OF TARGETS	400
RELATIVE VALUE	2.000
TARGET TYPE 3:	
NUMBER OF TARGETS	300
RELATIVE VALUE	3.500

.....

THE ATTACKER'S BASIC GAME MINIMAX STRATEGIES

ATTACK SIZE	TARGET TYPE 1: 30.00% OF TOTAL TARGETS, WITH 13.95% OF TOTAL VALUE								
	0	1	2	3	4	5	6	7	8
1000	: 0.7619	: 0.0700	: 0.0717	: 0.0964	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
2000	: 0.5237	: 0.1401	: 0.1435	: 0.1927	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
3000	: 0.2856	: 0.2101	: 0.2152	: 0.2891	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
4000	: 0.0844	: 0.2693	: 0.2758	: 0.3705	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
5000	: 0.0000	: 0.2174	: 0.2226	: 0.2365	: 0.3100	: 0.0136	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
6000	: 0.0000	: 0.0000	: 0.3964	: 0.0893	: 0.2531	: 0.2569	: 0.0043	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
7000	: 0.0000	: 0.0000	: 0.3268	: 0.1037	: 0.0000	: 0.4887	: 0.0789	: 0.0019	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
8000	: 0.0000	: 0.0000	: 0.2705	: 0.0610	: 0.1920	: 0.0073	: 0.3340	: 0.1353	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

ATTACK SIZE	TARGET TYPE 2: 40.00% OF TOTAL TARGETS, WITH 37.21% OF TOTAL VALUE								
	0	1	2	3	4	5	6	7	8
1000	: 0.7530	: 0.0350	: 0.0359	: 0.0381	: 0.0390	: 0.0466	: 0.0524	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
2000	: 0.5060	: 0.0700	: 0.0717	: 0.0762	: 0.0780	: 0.0931	: 0.1049	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
3000	: 0.2591	: 0.1051	: 0.1076	: 0.1143	: 0.1169	: 0.1397	: 0.1573	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
4000	: 0.0000	: 0.1347	: 0.1379	: 0.1681	: 0.0000	: 0.4075	: 0.0969	: 0.0549	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
5000	: 0.0000	: 0.0000	: 0.2459	: 0.0579	: 0.1653	: 0.0000	: 0.2293	: 0.2965	: 0.0051
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
6000	: 0.0000	: 0.0000	: 0.1982	: 0.0447	: 0.1265	: 0.1034	: 0.1110	: 0.0354	: 0.1887
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
7000	: 0.0000	: 0.0000	: 0.1634	: 0.0368	: 0.1043	: 0.0853	: 0.0826	: 0.0882	: 0.0974
	: 0.1415	: 0.1472	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
8000	: 0.0000	: 0.0000	: 0.1353	: 0.0305	: 0.0864	: 0.0802	: 0.0016	: 0.1676	: 0.0872
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

: 0.0000 : 0.1230 : 0.2883 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :

		TARGET TYPE 3: 30.00% OF TOTAL TARGETS, WITH 48.84% OF TOTAL VALUE								
		0	1	2	3	4	5	6	7	8 9
1000		: 0.7406	: 0.0200	: 0.0205	: 0.0218	: 0.0223	: 0.0266	: 0.0241	: 0.0249	: 0.0116 : 0.0384 :
		: 0.0493	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000			
2000		: 0.4812	: 0.0400	: 0.0410	: 0.0435	: 0.0445	: 0.0532	: 0.0483	: 0.0497	: 0.0231 : 0.0768 :
		: 0.0985	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000			
3000		: 0.2218	: 0.0600	: 0.0615	: 0.0653	: 0.0668	: 0.0798	: 0.0724	: 0.0746	: 0.0347 : 0.1152 :
		: 0.1478	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000			
4000		: 0.0000	: 0.0769	: 0.0788	: 0.0837	: 0.0856	: 0.1023	: 0.0928	: 0.0953	: 0.0466 : 0.1444 :
		: 0.1882	: 0.0054	: 0.0000	: 0.0000	: 0.0000	: 0.0000			
5000		: 0.0000	: 0.0000	: 0.1405	: 0.0317	: 0.0897	: 0.0831	: 0.0035	: 0.1718	: 0.0898 : 0.0000 :
		: 0.0000	: 0.2214	: 0.1569	: 0.0117	: 0.0000	: 0.0000			
6000		: 0.0000	: 0.0000	: 0.1133	: 0.0255	: 0.0723	: 0.0591	: 0.0573	: 0.0649	: 0.0395 : 0.0856 :
		: 0.1052	: 0.0206	: 0.0000	: 0.0593	: 0.2740	: 0.0235			
7000		: 0.0000	: 0.0000	: 0.0000	: 0.1531	: 0.0000	: 0.0837	: 0.0290	: 0.0481	: 0.0391 : 0.0846 :
		: 0.0750	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.4874			
8000		: 0.0000	: 0.0000	: 0.0000	: 0.1290	: 0.0000	: 0.0485	: 0.0240	: 0.0927	: 0.0319 : 0.0000 :
		: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.6739			

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THE DEFENDER'S BASIC GAME MINIMAX STRATEGIES

ATTACK SIZE	1: 30.00% OF TOTAL TARGETS, WITH 13.95% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.3016	: 0.2058	: 0.4300	: 0.0626	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
2000	: 0.3016	: 0.2058	: 0.4300	: 0.0626	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
3000	: 0.3016	: 0.2058	: 0.4300	: 0.0626	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
4000	: 0.2611	: 0.2105	: 0.2758	: 0.2527	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
5000	: 0.2446	: 0.1010	: 0.2001	: 0.0196	: 0.4347	: 0.0000	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
6000	: 0.4638	: 0.0000	: 0.0845	: 0.0785	: 0.0161	: 0.3572	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
7000	: 0.4952	: 0.0000	: 0.0624	: 0.0753	: 0.0000	: 0.0151	: 0.3521	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
8000	: 0.5939	: 0.0000	: 0.0381	: 0.0660	: 0.0070	: 0.0000	: 0.2495	: 0.0456	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

ATTACK SIZE	2: 40.00% OF TOTAL TARGETS, WITH 37.21% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.1408	: 0.1111	: 0.1071	: 0.1140	: 0.0077	: 0.2549	: 0.2643	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
2000	: 0.1408	: 0.1111	: 0.1071	: 0.1140	: 0.0077	: 0.2549	: 0.2643	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
3000	: 0.1408	: 0.1111	: 0.1071	: 0.1140	: 0.0077	: 0.2549	: 0.2643	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
4000	: 0.1621	: 0.0966	: 0.0991	: 0.1093	: 0.0000	: 0.0219	: 0.5110	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
5000	: 0.4048	: 0.0000	: 0.0504	: 0.1246	: 0.0000	: 0.0000	: 0.0000	: 0.3665	: 0.0312
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
6000	: 0.5184	: 0.0000	: 0.0272	: 0.0392	: 0.0316	: 0.0613	: 0.0608	: 0.0000	: 0.0705
	: 0.1912	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
7000	: 0.5426	: 0.0000	: 0.0120	: 0.0359	: 0.0225	: 0.0138	: 0.0593	: 0.0675	: 0.0000
	: 0.0000	: 0.0472	: 0.1961	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
8000	: 0.5584	: 0.0000	: 0.0032	: 0.0194	: 0.0308	: 0.0392	: 0.0000	: 0.0660	: 0.0642

: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.1260 : 0.0928 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :

TARGET TYPE 3: 30.00% OF TOTAL TARGETS, WITH 48.84% OF TOTAL VALUE

0	1	2	3	4	5	6	7	8	9
1000	: 0.0718	: 0.0595	: 0.0611	: 0.0652	: 0.0412	: 0.0310	: 0.1114	: 0.1144	: 0.0000
	: 0.0199	: 0.1244	: 0.3002	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
2000	: 0.0718	: 0.0595	: 0.0611	: 0.0652	: 0.0412	: 0.0310	: 0.1114	: 0.1144	: 0.0000
	: 0.0199	: 0.1244	: 0.3002	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
3000	: 0.0718	: 0.0595	: 0.0611	: 0.0652	: 0.0412	: 0.0310	: 0.1114	: 0.1144	: 0.0000
	: 0.0199	: 0.1244	: 0.3002	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
4000	: 0.1813	: 0.0257	: 0.0546	: 0.0598	: 0.0364	: 0.0226	: 0.1013	: 0.1124	: 0.0000
	: 0.0053	: 0.0779	: 0.3227	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
5000	: 0.3756	: 0.0000	: 0.0263	: 0.0314	: 0.0443	: 0.0265	: 0.0000	: 0.0581	: 0.0442
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.2510	: 0.0645	: 0.0000	: 0.0000	: 0.0000
6000	: 0.5088	: 0.0000	: 0.0018	: 0.0237	: 0.0162	: 0.0132	: 0.0358	: 0.0375	: 0.0000
	: 0.0370	: 0.0546	: 0.0410	: 0.0000	: 0.0000	: 0.0000	: 0.0101	: 0.0000	: 0.1207
7000	: 0.6058	: 0.0000	: 0.0000	: 0.0000	: 0.0273	: 0.0114	: 0.0267	: 0.0231	: 0.0000
	: 0.0347	: 0.0678	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.2031
8000	: 0.6656	: 0.0000	: 0.0000	: 0.0000	: 0.0275	: 0.0000	: 0.0000	: 0.0395	: 0.0250
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.2246

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THE GAME VALUES

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ATTACK SIZE

1000	:	0.8615
2000	:	0.7229
3000	:	0.5844
4000	:	0.4473
5000	:	0.3406
6000	:	0.2707
7000	:	0.2218
8000	:	0.1836

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THE ATTACKER'S BASIC GAME TARGET ALLOCATION

TARGET TYPE	ATTACK SIZE = 1000									
	0	1	2	3	4	5	6	7	8	9
1	228.6 :	21.0 :	21.5 :	28.9 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	301.2 :	14.0 :	14.3 :	15.2 :	15.6 :	18.6 :	21.0 :	0.0 :	0.0 :	0.0 :
	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
3	222.2 :	6.0 :	6.1 :	6.5 :	6.7 :	8.0 :	7.2 :	7.5 :	3.5 :	11.5 :
	14.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

	ATTACK SIZE = 2000									
	0	1	2	3	4	5	6	7	8	9
1	157.1 :	42.0 :	43.0 :	57.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	202.4 :	28.0 :	28.7 :	30.5 :	31.2 :	37.3 :	41.9 :	0.0 :	0.0 :	0.0 :
	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
3	144.4 :	12.0 :	12.3 :	13.1 :	13.4 :	16.0 :	14.5 :	14.9 :	6.9 :	23.0 :
	29.6 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

	ATTACK SIZE = 3000									
	0	1	2	3	4	5	6	7	8	9
1	: 85.7 :	63.0 :	64.6 :	86.7 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	: 103.6 :	42.0 :	43.0 :	45.7 :	46.8 :	55.9 :	62.9 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
3	: 66.5 :	18.0 :	18.4 :	19.6 :	20.0 :	24.0 :	21.7 :	22.4 :	10.4 :	34.6 :
	: 44.3 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

	ATTACK SIZE = 4000									
	0	1	2	3	4	5	6	7	8	9
1	: 25.3 :	: 80.8 :	: 82.7 :	: 111.1 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
2	: 0.0 :	: 53.9 :	: 55.2 :	: 67.2 :	: 0.0 :	: 163.0 :	: 38.8 :	: 22.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
3	: 0.0 :	: 23.1 :	: 23.6 :	: 25.1 :	: 25.7 :	: 30.7 :	: 27.8 :	: 28.6 :	: 14.0 :	: 43.3 :
	: 56.5 :	: 1.6 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				

ATTACK SIZE = 5000									
0	1	2	3	4	5	6	7	8	9
1	0.0 : 65.2 : 0.0	66.8 : 70.9 : 0.0	93.0 : 4.1 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
2	0.0 : 0.0 : 0.0	98.4 : 23.1 : 0.0	66.1 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	91.7 : 118.6 : 0.0	2.1 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
3	0.0 : 0.0 : 0.0	42.2 : 9.5 : 26.9	24.9 : 1.1 : 51.5	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0

ATTACK SIZE = 6000									
0	1	2	3	4	5	6	7	8	9
1	0.0 : 0.0 : 0.0	118.9 : 26.8 : 75.9	77.1 : 1.3 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
2	0.0 : 0.0 : 0.0	79.3 : 17.9 : 50.6	41.4 : 44.4 : 14.2	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
3	0.0 : 0.0 : 0.0	34.0 : 7.7 : 21.7	17.7 : 17.2 : 19.5	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0

ATTACK SIZE = 7000									
0	1	2	3	4	5	6	7	8	9
1	0.0 : 0.0 : 0.0	98.0 : 31.1 : 0.0	146.6 : 23.7 : 0.6	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
2	0.0 : 0.0 : 0.0	65.4 : 14.7 : 41.7	34.1 : 33.1 : 35.3	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
3	0.0 : 0.0 : 0.0	0.0 : 0.0 : 45.9	25.1 : 8.7 : 14.4	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0

ATTACK SIZE = 8000									
0	1	2	3	4	5	6	7	8	9
1	0.0 : 0.0 : 0.0	81.2 : 18.3 : 57.6	2.2 : 100.2 : 40.6	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
2	0.0 : 0.0 : 0.0	54.1 : 12.2 : 34.5	32.1 : 0.6 : 67.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0
3	0.0 : 0.0 : 0.0	0.0 : 0.0 : 38.7	14.5 : 7.2 : 27.8	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0	0.0 : 0.0 : 0.0

THE DEFENDER'S BASIC GAME TARGET ALLOCATION

TARGET TYPE	ATTACK SIZE = 1000									
	0	1	2	3	4	5	6	7	8	9
1	: 90.5 :	61.8 :	129.0 :	18.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	: 56.3 :	44.5 :	42.8 :	45.6 :	3.1 :	102.0 :	105.7 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	: 21.5 :	17.9 :	18.3 :	19.6 :	12.3 :	9.3 :	33.4 :	34.3 :	0.0 :	0.0 :
	: 6.0 :	37.3 :	90.1 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 2000										
	0	1	2	3	4	5	6	7	8	9
1	: 90.5 :	61.8 :	129.0 :	18.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	: 56.3 :	44.5 :	42.8 :	45.6 :	3.1 :	102.0 :	105.7 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	: 21.5 :	17.9 :	18.3 :	19.6 :	12.3 :	9.3 :	33.4 :	34.3 :	0.0 :	0.0 :
	: 6.0 :	37.3 :	90.1 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

	ATTACK SIZE = 3000									
	0	1	2	3	4	5	6	7	8	9
1	: 90.5 :	61.8 :	129.0 :	18.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	: 56.3 :	44.5 :	42.8 :	45.6 :	3.1 :	102.0 :	105.7 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	: 21.5 :	17.9 :	18.3 :	19.6 :	12.3 :	9.3 :	33.4 :	34.3 :	0.0 :	0.0 :
	: 6.0 :	37.3 :	90.1 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

	ATTACK SIZE = 4000									
	0	1	2	3	4	5	6	7	8	9
1	: 78.3 :	63.1 :	82.7 :	75.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	: 64.9 :	38.7 :	39.6 :	43.7 :	0.0 :	8.7 :	204.4 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	: 54.4 :	7.7 :	16.4 :	17.9 :	10.9 :	6.8 :	30.4 :	33.7 :	0.0 :	0.0 :
	: 1.6 :	23.4 :	96.8 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 5000

	0	1	2	3	4	5	6	7	8	9
1	: 73.4 :	: 30.3 :	: 60.0 :	: 5.9 :	: 130.4 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 161.9 :	: 0.0 :	: 20.2 :	: 49.8 :	: 0.0 :	: 0.0 :	: 0.0 :	: 146.6 :	: 9.0 :	: 12.5 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 112.7 :	: 0.0 :	: 7.9 :	: 9.4 :	: 13.3 :	: 7.9 :	: 0.0 :	: 17.4 :	: 23.4 :	: 13.3 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 75.3 :	: 19.3 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :

ATTACK SIZE = 6000

	0	1	2	3	4	5	6	7	8	9
1	: 139.1 :	: 0.0 :	: 25.4 :	: 23.5 :	: 4.8 :	: 107.2 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 207.3 :	: 0.0 :	: 10.9 :	: 15.7 :	: 12.6 :	: 24.5 :	: 24.3 :	: 0.0 :	: 0.0 :	: 28.2 :
	: 76.5 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 152.6 :	: 0.0 :	: 0.5 :	: 7.1 :	: 4.9 :	: 4.0 :	: 10.7 :	: 11.3 :	: 0.0 :	: 0.0 :
	: 11.1 :	: 16.4 :	: 12.3 :	: 0.0 :	: 0.0 :	: 0.0 :	: 3.0 :	: 0.0 :	: 29.8 :	: 36.2 :

ATTACK SIZE = 7000

	0	1	2	3	4	5	6	7	8	9
1	: 148.6 :	: 0.0 :	: 18.7 :	: 22.6 :	: 0.0 :	: 4.5 :	: 105.6 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 217.1 :	: 0.0 :	: 4.8 :	: 14.4 :	: 9.0 :	: 5.5 :	: 23.7 :	: 27.0 :	: 1.2 :	: 0.0 :
	: 0.0 :	: 18.9 :	: 78.4 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 181.7 :	: 0.0 :	: 0.0 :	: 0.0 :	: 8.2 :	: 3.4 :	: 8.0 :	: 6.9 :	: 0.0 :	: 0.0 :
	: 10.4 :	: 20.3 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 60.9 :

ATTACK SIZE = 8000

	0	1	2	3	4	5	6	7	8	9
1	: 178.2 :	: 0.0 :	: 11.4 :	: 19.8 :	: 2.1 :	: 0.0 :	: 74.8 :	: 13.7 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 223.4 :	: 0.0 :	: 1.3 :	: 7.8 :	: 12.3 :	: 15.7 :	: 0.0 :	: 0.0 :	: 26.4 :	: 25.7 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 50.4 :	: 37.1 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 199.7 :	: 0.0 :	: 0.0 :	: 0.0 :	: 8.2 :	: 0.0 :	: 0.0 :	: 11.8 :	: 5.3 :	: 7.5 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 67.4 :

THE EXPECTED NUMBER OF TARGETS SURVIVING

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
1000	255.09	344.93	259.20
2000	210.18	289.87	218.41
3000	165.28	234.80	177.61
4000	144.70	178.50	131.44
5000	137.28	126.95	97.45
6000	100.65	102.71	78.84
7000	92.90	96.33	54.68
8000	64.73	91.88	41.80

.....

THE ATTACKER'S BASIC GAME RV ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE	TARGET TYPE		
	1	2	3
1000	151	370	479
2000	302	740	959
3000	452	1109	1438
4000	580	1567	1853
5000	804	1928	2268
6000	1015	2282	2703
7000	1168	2675	3157
8000	1344	3120	3536

.....

THE DEFENDER'S BASIC GAME INTERCEPTOR ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE	TARGET TYPE		
	1	2	3
1000	376	1423	2201
2000	376	1423	2201
3000	376	1423	2201
4000	456	1519	2025
5000	690	1400	1910
6000	676	1406	1918
7000	762	1606	1632
8000	635	1858	1506

.....

THE ROBUST DEFENSE STRATEGY FOR RV RANGE 1000 TO 8000 :

0	1	2	3	TARGET TYPE 1					8	9
				4	5	6	7			
: 0.3130	: 0.1817	: 0.1818	: 0.0257	: 0.0128	: 0.2850	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000
: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

0	1	2	3	TARGET TYPE 2					8	9
				4	5	6	7			
: 0.3777	: 0.0333	: 0.0631	: 0.1802	: 0.0000	: 0.0451	: 0.0602	: 0.0000	: 0.0000	: 0.0000	: 0.0416
: 0.0461	: 0.1527	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

0	1	2	3	TARGET TYPE 3					8	9
				4	5	6	7			
: 0.3852	: 0.0000	: 0.0407	: 0.0527	: 0.0068	: 0.0242	: 0.2018	: 0.0000	: 0.0000	: 0.0000	: 0.0000
: 0.0260	: 0.0414	: 0.0343	: 0.0057	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0776	: 0.1037

THE OPTIMAL ATTACK STRATEGIES AGAINST THE ROBUST DEFENSE

ATTACK SIZE	1: 30.00% OF TOTAL TARGETS, WITH 13.95% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
2000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
3000	: 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
4000	: 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
5000	: 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
6000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
7000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 :								
8000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 :								

ATTACK SIZE	2: 40.00% OF TOTAL TARGETS, WITH 37.21% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
2000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
3000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
4000	: 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
5000	: 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
6000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
7000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
8000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 :								

TARGET TYPE 3: 30.00% OF TOTAL TARGETS, WITH 48.84% OF TOTAL VALUE									
0	1	2	3	4	5	6	7	8	9
1000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
2000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
3000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
4000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
5000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
6000	: 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
7000	: 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
8000	: 0.0000 : 0.0000 : 0.5667 : 0.0000 : 0.0000 : 0.0000 : 0.3333 : 0.0000 : 0.0000 :								

.....

THE EXPECTED TARGET SURVIVAL RATE WITH THE ROBUST DEFENSE

.....

ATTACK SIZE	
1000	: 0.6828
2000	: 0.5661
3000	: 0.4577
4000	: 0.3503
5000	: 0.2752
6000	: 0.2307
7000	: 0.1872
8000	: 0.1438

.....

THE ROBUST DEFENSE ALLOCATION FOR RV RANGE 1000 TO 8000 :

0	TARGET TYPE 1							
	1	2	3	4	5	6	7	8
:	93.9 :	54.5 :	7.7 :	3.8 :	85.5 :	0.0 :	0.0 :	0.0 :
:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

0	TARGET TYPE 2							
	1	2	3	4	5	6	7	8
:	151.1 :	13.3 :	25.3 :	72.1 :	0.0 :	18.0 :	24.1 :	0.0 :
:	18.4 :	61.1 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	16.6 :

0	TARGET TYPE 3							
	1	2	3	4	5	6	7	8
:	115.6 :	0.0 :	12.2 :	15.8 :	2.0 :	7.3 :	60.5 :	0.0 :
:	7.8 :	12.4 :	10.3 :	1.7 :	0.0 :	0.0 :	0.0 :	23.3 :

THE OPTIMAL ATTACK ALLOCATION AGAINST THE ROBUST DEFENSE

TARGET TYPE	ATTACK SIZE = 1000									
	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	300.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 2000										
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	300.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	400.0 :				
3	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

	ATTACK SIZE = 3000									
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	0.0 :	300.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
3	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

ATTACK SIZE = 4000										
	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	0.0 :	300.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 5000

	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 0.0 :	: 400.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :

ATTACK SIZE = 6000

	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 0.0 :	: 50.0 :	: 0.0 :	: 350.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :

ATTACK SIZE = 7000

	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 0.0 :	: 0.0 :	: 0.0 :	: 400.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :

ATTACK SIZE = 8000

	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :
2	: 0.0 :	: 0.0 :	: 0.0 :	: 400.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
3	: 0.0 :	: 0.0 :	: 200.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 100.0 :	: 0.0 :	: 0.0 :	: 0.0 :

THE EXPECTED NUMBER OF TARGETS (SURVIVING) WITH THE ROBUST DEFENSE

.....

ATTACK SIZE	TARGET TYPE		
	1	2	3
1000	211.75	0.30	0.00
2000	211.75	0.06	0.00
3000	141.79	0.00	0.00
4000	141.79	400.00	0.00
5000	91.06	270.36	0.00
6000	91.06	188.73	300.00
7000	63.04	177.06	179.43
8000	7.00	177.06	152.87

.....

THE DEFENDER'S ROBUST GAME INTERCEPTOR ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
N/A :	630	1521	1850

.....

THE ATTACKER'S ROBUST GAME RV ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
1000 :	300	5200	0
2000 :	300	5600	0
3000 :	600	0	0
4000 :	600	0	0
5000 :	900	400	0
6000 :	900	1100	0
7000 :	1200	1200	600
8000 :	1800	1200	1000

.....

THE ROBUST DEFENSE STRATEGY FOR RV RANGE 4000 TO 8000 :

0	TARGET TYPE 1								
	1	2	3	4	5	6	7	8	9
	: 0.5145	: 0.0000	: 0.1156	: 0.0389	: 0.1918	: 0.1391	: 0.0000	: 0.0000	: 0.0000
	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

0	TARGET TYPE 2								
	1	2	3	4	5	6	7	8	9
	: 0.4972	: 0.0000	: 0.0306	: 0.0407	: 0.0328	: 0.0637	: 0.0632	: 0.0000	: 0.0732
	: 0.1986	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000	: 0.0000

0	TARGET TYPE 3								
	1	2	3	4	5	6	7	8	9
	: 0.4873	: 0.0000	: 0.0042	: 0.0247	: 0.0168	: 0.0137	: 0.0372	: 0.0390	: 0.0000
	: 0.0384	: 0.0567	: 0.0426	: 0.0000	: 0.0000	: 0.0000	: 0.0105	: 0.0000	: 0.1254

THE OPTIMAL ATTACK STRATEGIES AGAINST THE ROBUST DEFENSE

ATTACK SIZE	1: 30.00% OF TOTAL TARGETS, WITH 13.95% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
2000	: 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
3000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
4000	: 0.0000 : 0.0000 : 0.3333 : 0.0000 : 0.0000 : 0.0000 : 0.6667 : 0.0000 : 0.0000 :								
5000	: 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
6000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
7000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 :								
8000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 :								

ATTACK SIZE	2: 40.00% OF TOTAL TARGETS, WITH 37.21% OF TOTAL VALUE								
	TARGET TYPE 0	1	2	3	4	5	6	7	8
1000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
2000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
3000	: 0.5000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
4000	: 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
5000	: 0.4167 : 1.0000 : 0.0000 : 0.5833 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
6000	: 0.0000 : 0.1250 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
7000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 1.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :								
8000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.5000 : 0.0000 : 0.0000 : 0.0000 :								

TARGET TYPE 3: 30.00% OF TOTAL TARGETS, WITH 48.84% OF TOTAL VALUE

0 1 2 3 4 5 6 7 8 9

1000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
2000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
3000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
4000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
5000	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
6000	: 1.0000 : 0.0000 : 0.0000 : 0.2381 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
7000	: 0.0000 : 0.0000 : 1.0000 : 0.7143 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :
8000	: 0.0000 : 0.0000 : 0.6667 : 1.0000 : 0.0000 : 0.0000 : 0.3333 : 0.0000 : 0.0000 :
	: 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 : 0.0000 :

.....

THE EXPECTED TARGET SURVIVAL RATE WITH THE ROBUST DEFENSE

.....

ATTACK SIZE

1000	:	0.5944
2000	:	0.4969
3000	:	0.4404
4000	:	0.3838
5000	:	0.3272
6000	:	0.2707
7000	:	0.2141
8000	:	0.1575

.....

THE ROBUST DEFENSE ALLOCATION FOR RV RANGE 4000 TO 8000 :

0	1	2	3	TARGET TYPE 1				
				4	5	6	7	8
: 154.3 :	0.0 :	34.7 :	11.7 :	57.6 :	41.7 :	0.0 :	0.0 :	0.0 :
: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

0	1	2	3	TARGET TYPE 2				
				4	5	6	7	8
: 198.9 :	0.0 :	12.2 :	16.3 :	13.1 :	25.5 :	25.3 :	0.0 :	0.0 :
: 79.4 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

0	1	2	3	TARGET TYPE 3				
				4	5	6	7	8
: 146.2 :	0.0 :	1.3 :	7.4 :	5.1 :	4.1 :	11.2 :	11.7 :	0.0 :
: 11.5 :	17.0 :	12.8 :	0.0 :	0.0 :	0.0 :	3.2 :	0.0 :	37.6 :

THE OPTIMAL ATTACK ALLOCATION AGAINST THE ROBUST DEFENSE

TARGET TYPE	ATTACK SIZE = 1000									
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	300.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				
2	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	400.0 :	0.0 :	0.0 :				
3	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	: 0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :				

	ATTACK SIZE = 2000									
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
2	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 400.0 :	: 0.0 :				
3	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				

ATTACK SIZE = 3000										
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 300.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
2	: 200.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
3	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				

ATTACK SIZE = 4000										
	0	1	2	3	4	5	6	7	8	9
1	: 0.0 :	: 0.0 :	: 100.0 :	: 0.0 :	: 0.0 :	: 200.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
2	: 400.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				
3	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :
	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :	: 0.0 :				

ATTACK SIZE = 5000

	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	0.0 :	0.0 :	300.0 :	0.0 :	300.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	166.7 :	400.0 :	0.0 :	233.3 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 6000

	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	0.0 :	0.0 :	300.0 :	0.0 :	300.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	0.0 :	50.0 :	0.0 :	400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	300.0 :	0.0 :	0.0 :	71.4 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 7000

	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	0.0 :	0.0 :	0.0 :	300.0 :	300.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	0.0 :	0.0 :	0.0 :	400.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	0.0 :	0.0 :	300.0 :	214.3 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

ATTACK SIZE = 8000

	0	1	2	3	4	5	6	7	8	9
1	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	300.0 :	300.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
2	:	0.0 :	0.0 :	0.0 :	200.0 :	0.0 :	200.0 :	0.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :
3	:	0.0 :	0.0 :	200.0 :	300.0 :	0.0 :	0.0 :	100.0 :	0.0 :	0.0 :
	:	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :	0.0 :

.....

THE EXPECTED NUMBER OF TARGETS (SURVIVING) WITH THE ROBUST DEFENSE

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
1000	170.93	0.10	0.00
2000	132.24	0.02	0.00
3000	132.24	200.00	0.00
4000	59.28	400.00	0.00
5000	118.56	503.91	0.00
6000	118.56	202.10	334.42
7000	82.08	172.52	258.24
8000	27.36	148.20	285.64

.....

THE DEFENDER'S ROBUST GAME INTERCEPTOR ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
N/A	543	1463.	1994

.....

THE ATTACKER'S ROBUST GAME RV ALLOCATION BY TARGET TYPE

.....

ATTACK SIZE :	TARGET TYPE		
	1	2	3
1000	300	5200	0
2000	600	5600	0
3000	600	0	0
4000	1200	0	0
5000	2400	1100	0
6000	2400	1250	214
7000	2700	1200	1243
8000	3300	1600	1900

.....

\$

// If desired, the user may now edit TEST2.IN to change some of
 // the parameters and rerun RPPDM
 //

REFERENCES

1. Bracken, Jerome, Brooks, Peter S. and Falk, James E., Robust Pre-allocated Preferential Defense, P-1816, Institute for Defense Analyses, August 1985.
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4. Key, John C., MVADEM User's Guide and Reference Manual: Revision 1, URH2S-01, Sparta, Inc., April 1984.
5. Matheson, John D., Preferential Strategies, AR 66-2, Analytic Services, Inc., May 1966.
6. Matheson, John D., Preferential Strategies with Imperfect Information, AR 67-1, Analytic Services, Inc., April 1967.
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APPENDIX A
PROGRAM MAIN

PROGRAM MAIN

MAIN will solve for the basic game and the Case II,II as outlined in Bracken, Brooks, and Falk [1]. Eith batch or interactive modes may be employed. XMP is the linear programming package used.

Parameter statements for establishing limits on the problem's size

PARAMETER (MAXNAT=25, MAXR=30, MAXS=30, MAXNTYPE=7)

.....
 MAXNAT THE MAXIMUM ALLOWABLE NUMBER OF ATTACK SIZES
 MAXR THE MAXIMUM ALLOWABLE 'R'
 MAXS THE MAXIMUM ALLOWABLE 'S'
 MAXNTYPE THE MAXIMUM ALLOWABLE TYPES OF TARGETS

Variable type declarations for the basic game

REAL YBG(MAXNAT, MAXNTYPE, 0:MAXR)
 REAL YBG(MAXNAT, MAXNTYPE, 0:MAXS), VBG(MAXNAT)
 REAL VTYPE(MAXNTYPE), VFRAC(MAXNTYPE), NFRAC(MAXNTYPE)
 REAL P(0:MAXR, 0:MAXS), TS(MAXNAT, MAXNTYPE)
 REAL R, S, MAXRV, MINRV, INCRV, INT, TARGETS
 INTEGER RV(MAXNAT), A, NTAR(MAXNTYPE)
 INTEGER TMXBG(MAXNAT, MAXNTYPE), TMYBG(MAXNAT, MAXNTYPE)

.....
 YBG THE ATTACKER'S OPTIMAL BASIC GAME STRATEGY
 YBG THE DEFENDER'S OPTIMAL BASIC GAME STRATEGY
 VBG THE GAME VALUE
 TS THE EXPECTED NUMBER OF TARGETS (SURVIVING)
 P THE PIJ MATRIX
 R THE MAXIMUM NUMBER OF RV'S AT A SINGLE TARGET
 S THE MAXIMUM NUMBER OF INTERCEPTORS AT A SINGLE
 TARGET
 MAXRV THE MAXIMUM ATTACK SIZE
 MINRV THE MINIMUM ATTACK SIZE
 INCRV THE ATTACK SIZE INCREMENT
 INT THE NUMBER OF INTERCEPTORS
 TARGETS THE NUMBER OF TARGETS
 RV THE ATTACK SIZES
 A THE NUMBER OF ATTACK SIZES
 VTYPE THE RELATIVE VALUES OF THE TARGET TYPES
 NTAR THE NUMBER OF TARGETS FOR EACH TYPE
 VFRAC THE FRACTION OF TOTAL VALUE FOR EACH TYPE
 NFRAC THE FRACTION OF TOTAL TARGETS FOR EACH TYPE
 TS TARGET EQUIVALENTS OF EXPECTED 'VBG' AND 'VII'
 TMXBG RV ALLOC. FOR BASIC GAME BY TARGET TYPES
 TMYBG INTERCEPTOR ALLOC. FOR BASIC GAME BY TARGET TYPES


```

C
C
C      Variable type declarations for Case II.II
C
      REAL YII(MAXNTYPE, 0:MAIS)
      REAL XII(MAXNAT,MAXNTYPE,0:MAXR), VII(MAXNAT)
      REAL VBGR(MAXNAT)
      INTEGER MAXRRV, MINRRV
      INTEGER RRV(MAXNAT), AR
      INTEGER TMXII(MAXNAT,MAXNTYPE), TMYII(MAXNAT,MAXNTYPE)
C
C      .....
C      YII      THE ROBUST DEFENDER'S STRATEGY
C      XII      THE OPTIMAL ATTACKER'S STRATEGY AGAINST YII
C      VII      THE EXPECTED SURVIVAL RATE GIVEN YII AND XII
C      VBGR     THE GAME VALUE FOR ROBUST ATTACK SIZE RANGE
C      MAXRRV   THE MAXIMUM ATTACK SIZE IN THE ROBUST RANGE
C      MINRRV   THE MINIMUM ATTACK SIZE IN THE ROBUST RANGE
C      RRV      THE ATTACK SIZES IN THE ROBUST RANGE
C      AR       THE NUMBER OF ATTACK SIZES IN THE ROBUST RANGE
C      TMXII    RV ALLOC. FOR BASIC GAME BY TARGET TYPES
C      TMYII    INTERCEPTOR ALLOC. FOR BASIC GAME BY TARGET TYPES
C      .....
C
      INTEGER TER,IN,FILE,OUT,OUT1,OUT2,QPR
      CHARACTER*1 ANSWER, NAME*80, FILEOUT*12, FILEIN*12
C
C      .....
C      TER      THE I/O UNIT NUMBER FOR THE TERMINAL
C      IN       THE I/O UNIT NUMBER FOR THE INPUT DEVICE
C      FILE     THE I/O UNIT NUMBER FOR AN OUTPUT FILE
C      OUT      THE I/O UNIT NUMBER FOR THE OUTPUT DEVICE
C      QPR      THE I/O UNIT NUMBER FOR DISPLAY OF PROMPTS
C      ANSWER   'Y' OR 'N' ANSWER TO SELECTED QUESTIONS
C      NAME     HEADER FOR OUTPUT
C      FILEOUT  NAME OF AN OUTPUT FILE
C      .....
C
      Set up common statement for input, IOIN, error output, IOERR,
      results output, IOLOG (IMP only)
C
      COMMON/IO_UNIT/IOIN, IOERR, IOLOG
      IOIN=5
      IOERR=5
      IOLOG=5
C
      Set TER to default I/O unit number for terminal
C
      TER=5
C
      Set FILE to TER+1
C
      FILE=TER+1
C

```

```

c      Set IN to the I/O unit number for the input file/device
c
      WRITE(TER, '(A)') 'OTHER ARE TWO INPUT OPTIONS:'
      WRITE(TER, '(A)') 'O      1) TERMINAL (INTERACTIVE)'
      WRITE(TER, '(A)') '      2) FILE      (BATCH)'
      WRITE(TER, '(A)') 'PLEASE ENTER THE NUMBER OF THE DESIRED OPTI
1ON ?'
      READ(TER, *) NIN
      IF (NIN .NE. 1) THEN
        WRITE(TER, '(A)') 'PLEASE ENTER THE NAME OF THE INPUT FILE
1(OF LESS THAN 12'
        WRITE(TER, '(A)') ' CHARACTERS INCLUDING THE EXTENSION) ?'
        READ(TER, '(A)') FILEIN
c
c      Open file for file input
c
        OPEN(UNIT=TER+2, FILE=FILEIN, STATUS='OLD')
        IN=TER+2
c
c      Open junk file for unnecessary output
c
        OPEN(UNIT=TER+3, FILE='JUNK.DAT', STATUS='SCRATCH')
        QPR=TER+3
      ELSE
        QPR=TER
        IN=TER
      ENDIF
c
c      Select output option
c
      WRITE(QPR, '(A)') 'You have three options for the output of th
le results : '
      WRITE(QPR, '(A)') 'O      1) TERMINAL only'
      WRITE(QPR, '(A)') '      2) FILE only'
      WRITE(QPR, '(A)') '      3) TERMINAL and FILE'
      WRITE(QPR, '(A)') 'Please enter the number for the desired opt
1ion ?'
      READ(IN, FMT=*) NOUT
c
c      Ask for file name, if necessary
c
      IF (NOUT .NE. 1) THEN
        WRITE(QPR, '(A)') 'Please type in the desired file name (o
if less than 10 characters'
        WRITE(QPR, '(A)') ' including the extension) ?'
        READ(IN, '(A)') FILEOUT
        OPEN(UNIT=FILE, FILE=FILEOUT, STATUS='NEW')
      ENDIF
c
c      Input R and S
c
      WRITE(UNIT=QPR, FMT='(A,I2,A)') 'The MAXIMUM number of RV's
1(up to 'MAIR.') at a single target ? '
      READ(UNIT=IN, FMT=*) R

```

```

WRITE(UNIT-QPR, FMT='(A,I2,A)') 'The MAXIMUM number of INTERC
ceptors (up to 'MAXS,') at a single target ? '
READ(UNIT-IN, FMT=*) S

```

```

Input failure rates for

```

```

Select attack methodology

```

```

WRITE(UNIT-QPR, FMT='(A)') 'Select one of the following attac
k methodologies: '
WRITE(UNIT-QPR, FMT='(A)') '0      1) SIMULTANEOUS ATTACK'
WRITE(UNIT-QPR, FMT='(A)') '      2) SEQUENTIAL ATTACK'
WRITE(UNIT-QPR, FMT='(A)') 'Please input the number of the de
sired attack ? '
READ(UNIT-IN, FMT=*) NATTYPE

```

```

Input the failure rate for the RV's

```

```

WRITE(UNIT-QPR, FMT='(A)') 'The FAILURE rate of the RV's ? '
READ(UNIT-IN, FMT=*) PFA

```

```

Select defense methodology

```

```

WRITE(UNIT-QPR, FMT='(A)') 'Select one of the following defen
se methodologies: '
WRITE(UNIT-QPR, FMT='(A)') '0      1) ONE SHOT'
WRITE(UNIT-QPR, FMT='(A)') '      2) SHOOT LOOK SHOOT'
WRITE(UNIT-QPR, FMT='(A)') 'Please input the number for the d
esired option ? '
READ(UNIT-IN, FMT=*) NDFTYPE

```

```

If the the attack is sequential, find out whether the defender knows,
after the attack begins, the number of RV's slated for each target.

```

```

IF (NATTYPE .EQ. 2) THEN
  WRITE(UNIT-QPR, FMT='(A)') 'Is the defender aware, after
the attack begins, of the number
of RV's slated for each targ
et (Y or N) ? '
  READ(UNIT-IN, FMT='(A)') ANSWER
  IF (ANSWER .EQ. 'Y') THEN
    IF (NDFTYPE .EQ. 1) THEN
      WRITE(UNIT-QPR, FMT='(A)') 'NOTE: This scenario is equ
livalent to one with a simultaneous attack.'
      NATTYPE=1
    ENDIF
  ENDIF
ENDIF

```

```

Find the failure rates for the interceptors

```

```

IF (NDFTYPE .EQ. 1) THEN
  WRITE(UNIT-QPR, FMT='(A)') 'The FAILURE rate of the inter
ceptors ? '

```

```

      READ(UNIT-IN, FMT=*) PFD
    ELSE
      WRITE(UNIT-QPR, FMT='(A)') 'OThe FAILURE rate for the firs
1t salvo interceptors ? '
      READ(UNIT-IN, FMT=*) PFD1
      WRITE(UNIT-QPR, FMT='(A)') 'OThe FAILURE rate for the seco
1nd salvo interceptors ? '
      READ(UNIT-IN, FMT=*) PFD2
    ENDIF

```

```

Generate the appropriate pij's

```

```

IF (NATTYPE .EQ. 1 .AND. NDFTYPE .EQ. 1) THEN

```

```

  Simultaneous attack without SLS

```

```

  CALL SIMAT1(MAXR, MAXS, PFA, PFD, R, S, P)

```

```

ELSE IF (NATTYPE .EQ. 1 .AND. NDFTYPE .EQ. 2) THEN

```

```

  Simultaneous attack with SLS

```

```

  CALL SIMAT2(MAXR, MAXS, PFA, PFD1, PFD2, R, S, P)

```

```

ELSE IF (NATTYPE .EQ. 2 .AND. NDFTYPE .EQ. 1 .AND.
1ANSWER .EQ. 'N') THEN

```

```

  Sequential attack without SLS and attack at target unknown

```

```

  CALL SEQAT1(MAXR, MAXS, PFA, 1.0, PFD, R, S, P)

```

```

ELSE IF (NATTYPE .EQ. 2 .AND. NDFTYPE .EQ. 2 .AND.
1ANSWER .EQ. 'N') THEN

```

```

  Sequential attack with SLS and attack at target unknown

```

```

  CALL SEQAT1(MAXR, MAXS, PFA, PFD1, PFD2, R, S, P)

```

```

ELSE IF (NATTYPE .EQ. 2 .AND. NDFTYPE .EQ. 2 .AND.
1ANSWER .EQ. 'Y') THEN

```

```

  Sequential attack with SLS and attack at target known

```

```

  CALL SEQAT2(MAXR, MAXS, PFA, PFD1, PFD2, R, S, P)

```

```

ENDIF

```

```

Specific parameters of the game

```

```

WRITE(QPR, '(A)') 'OThe MINIMUM attack size ? '
READ(IN, FMT=*) MINRV
WRITE(QPR, '(A)') 'OThe MAXIMUM attack size ? '
READ(IN, FMT=*) MAXRV
WRITE(QPR, '(A)') 'OThe attack size INCREMENT ? '

```

```

READ(IN, FMT=*) INCRV
WRITE(QPR, '(A)') 'OThe NUMBER of interceptors ?'
READ(IN, FMT=*) INT
WRITE(QPR, '(A)') 'OThe TOTAL NUMBER of targets ?'
READ(IN, FMT=*) TARGETS
WRITE(QPR, '(A)') 'OThe number of TYPES of targets ?'
READ(IN, FMT=*) NTYPE

C
C   Only one target type
C
      IF (NTYPE .EQ. 1) THEN
        VTYPE(1)=1
        NTAR(1)=TARGETS
        TVT=TARGETS

C
C   More than one target type
C
      ELSE
        WRITE(QPR, '(A)') 'OEnter first the RELATIVE VALUE and the
in the NUMBER of targets'
        WRITE(QPR, '(A)') ' for each type. Separate the two entrie
is for each target type with'
        WRITE(QPR, '(A)') ' a comma and hit <CR> following the ent
tries for each target type: '

C
C   Loop through each target type
C
        DO 100 I = 1, NTYPE
          READ(IN, FMT=*) VTYPE(I), NTAR(I)
          TVT=TVT+VTYPE(I)*NTAR(I)
          TNT=TNT+NTAR(I)
100      CONTINUE
        IF (TNT .NE. TARGETS) THEN
          WRITE(TER, '(A)') 'OThe sum of the targets in the indi
lvidual target types does not'
          WRITE(TER, '(A)') ' equal the total number of targets'
          STOP
        ENDIF
      ENDIF

C
C   Calculate the fractional values for VTYPE and NTAR
C
        DO 110 I = 1, NTYPE
          VFRAC(I)=VTYPE(I)*NTAR(I)/TVT
          NFRAC(I)=REAL(NTAR(I))/TARGETS
110      CONTINUE

C
C   Place the attack sizes in an array
C
        A=1+(MAXRV-MINRV)/INCRV
        DO 120 I= 1, A
          RV(I)=MINRV+(I-1)*INCRV
120      CONTINUE
C

```

```

c      Call Subroutine BG to solve for the basic game
c
      CALL BG(XBG, YBG, VBG, R, S, P,
1RV, A, INT, TARGETS,
1MAXNAT, MAXR, MAXS, MAXNTYPE, NTYPE,
1VFRAC, NFRAC)
c
c      Print out results of the basic game
c
      IF (NOUT .EQ. 1) THEN
        OUT1-TER
        OUT2-TER
      ELSE IF (NOUT .EQ. 2) THEN
        OUT1-FILE
        OUT2-FILE
      ELSE
        OUT1-TER
        OUT2-FILE
      ENDIF
c
c      Calculate some statistics for output
c
      DO 130 K=1, A
        DO 131 NT=1, NTYPE
          TS(K,NT)=0
          TMX=0
          TMY=0
          DO 132 I=0, R
            DO 133 J=0, S
              TS(K,NT)=XBG(K,NT,I)*P(I,J)*YBG(K,NT,J)+
1TS(K,NT)
              IF (I .EQ. 0) THEN
                TMY=TMY+J*YBG(K,NT,J)
              ENDIF
133          CONTINUE
              TMX=TMX+I*XBG(K,NT,I)
132          CONTINUE
              TS(K,NT)=TS(K,NT)*NTAR(NT)
              TMXBG(K,NT)=NINT(TM*NTAR(NT))
              TMYBG(K,NT)=NINT(TMY*NTAR(NT))
131          CONTINUE
130          CONTINUE
c
c      DO 140 OUT=OUT1, OUT2
c
c      CALL SUMMARY(NATTYPE, NDFTYPE, ANSWER, MINRV, MAXRV, INCRV,
1      INT, TARGETS, R, S, PFA, PFD, PFD1, PFD2, OUT,
1      MAXNTYPE, NTYPE, VTYPE, NTAR)
c
c      NAME='1THE ATTACKER'S BASIC GAME MINIMAX STRATEGIES
1      CALL STPRINT(NAME, XBG, RV, A, R, OUT, MAXNAT, MAXR, MAXNTYPE,
1      NTYPE, VFRAC, NFRAC)
c

```

```

NAME='1THE DEFENDER'S BASIC GAME MINIMAX STRATEGIES
CALL STPRINT(NAME,YBG,RV,A,S,OUT,MAXNAT,MAXS,MAXNTYPE,
1          NTYPE,VFRAC,NFRAC)
C
NAME='1THE GAME VALUES
CALL VPRINT(NAME,VBG,RV,A,OUT,MAXNAT)
C
NAME='1THE ATTACKER'S BASIC GAME TARGET ALLOCATION
CALL ALPRINT(NAME,YBG,RV,A,R,OUT,MAXNAT,MAXR,MAXNTYPE,
1          NTYPE,VFRAC,NFRAC,TARGETS)
C
NAME='1THE DEFENDER'S BASIC GAME TARGET ALLOCATION
CALL ALPRINT(NAME,YBG,RV,A,S,OUT,MAXNAT,MAXS,MAXNTYPE,
1          NTYPE,VFRAC,NFRAC,TARGETS)
C
NAME='1THE EXPECTED NUMBER OF TARGETS SURVIVING
CALL ALVPRINT(NAME,VBG,RV,A,OUT,MAXNAT,
1          MAXNTYPE,NTYPE,TS)
C
IF (NTYPE .NE. 1) THEN
    ROB=.FALSE.
    NAME='1THE ATTACKER'S BASIC GAME RV ALLOCATION BY TAR
1GET TYPE'
    CALL RVINTCOUNT(NAME,TMXBG,RV,A,OUT,
1          MAXNAT,MAXNTYPE,NTYPE,ROB)
C
    WRITE(OUT, '(/)')
    NAME='0THE DEFENDER'S BASIC GAME INTERCEPTOR ALLOCATI
1ON BY TARGET TYPE'
    CALL RVINTCOUNT(NAME,TMYBG,RV,A,OUT,
1          MAXNAT,MAXNTYPE,NTYPE,ROB)
    ENDIF
140    CONTINUE
C
C    Ask for the number of robust solutions desired
C
    WRITE(QPR,'(A)') '1Please enter the number of different ra
ranges of RV's for which robust
    WRITE(QPR,'(A)') '    solutions are to be found ?'
    WRITE(QPR,'(A)') '    ***** Enter 0 if no robust solu
tion is desired*****
    READ(IN,*) J
    IF (J .EQ. 0) THEN
        STOP
    ENDIF
    WRITE(QPR,'(A)') '0The lower and upper bounds for the RV range
ls must be between'
    WRITE(QPR,'(I5,' and ',I5)') RV(1), RV(A)
C
    DO 200 I1=1, J
        WRITE (QPR,'(A)') '0    The lower bound : '
        READ (IN,*) MINRRV
        WRITE (QPR,'(A)') '0    The upper bound : '

```

```

      READ (IN,*) MAXRRV
      AR=1+(MAXRRV-MINRRV)/INCRV
      DO 210 I=1, AR
210         RRV(I)=MINRRV+(I-1)*INCRV
      DO 220 I=1, A
          IF (MINRRV .EQ. RV(I)) THEN
              DO 221 I2=1, AR
221                 VBGR(I2)=VBG(I+I2-1)
              ENDIF
220         CONTINUE
      C
      C      Find the robust defender's strategy, YII
      C
          CALL YROBUST(YII, VBGR, R, S, P,
1RRV, AR, INT, TARGETS,
1MAXNAT, MAXR, MAXS, MAXNTYPE, NTYPE, NFRAC, VFRAC)
      C
      C      Find the XII's and the VII's
      C
          CALL XROBUST(XII, YII, VII, R, S, P,
1RV, A, INT, TARGETS,
1MAXNAT, MAXR, MAXS, MAXNTYPE, NTYPE, NFRAC, VFRAC)
      C
          DO 230 K3=1, A
              DO 231 NT3=1, NTYPE
                  TS(K3,NT3)=0
                  TMX=0
                  DO 232 I3=0, R
                      DO 233 J3=0, S
                          TS(K3,NT3)=XII(K3,NT3,I3)*P(I3,J3)*
1YII(NT3,J3)+TS(K3,NT3)
233                     CONTINUE
                          TMX=TMX+I3*XII(K3,NT3,I3)
232                     CONTINUE
                          TS(K3,NT3)=TS(K3,NT3)*NTAR(NT3)
                          TMXII(K3,NT3)=NINT(TM*NTAR(NT3))
231                     CONTINUE
230                     CONTINUE
                  DO 240 NT3=1, NTYPE
                      TMY=0
                      DO 241 J3=0, S
                          TMY=TMX+J3*YII(NT3,J3)
241                     CONTINUE
                          TMYII(1,NT3)=NINT(TMY*NTAR(NT3))
240                     CONTINUE
      C
      C      Print out the robust solution
      C
          DO 250 OUT=OUT1, OUT2, FILE-TER
      C
          CALL YRPRINT(YII, MINRRV, MAXRRV, MAXS, S, OUT,
1MAXNTYPE, NTYPE)
      C

```

```

NAME-'1THE OPTIMAL ATTACK STRATEGIES AGAINST THE ROBUS
1T DEFENSE'
1 CALL STPRINT(NAME,XII,RV,A,R,OUT,MAXNAT,MAXR,MAXNTYPE,
NTYPE,VFRAC,NFRAC)
C
NAME-'1THE EXPECTED TARGET SURVIVAL RATE WITH THE ROBU
1ST DEFENSE'
1 CALL VPRINT(NAME,VII,RV,A,OUT,MAXNAT)
C
1 CALL ALYRPRINT(YII,MINRRV,MAXRRV,MAIS,S,OUT,
MAXNTYPE,NTYPE,NFRAC,TARGETS)
C
NAME-'1THE OPTIMAL ATTACK ALLOCATION AGAINST THE ROBUS
1T DEFENSE'
1 CALL ALPRINT(NAME,XII,RV,A,R,OUT,MAXNAT,MAXR,MAXNTYPE,
NTYPE,VFRAC,NFRAC,TARGETS)
C
NAME-'1THE EXPECTED NUMBER OF TARGETS (SURVIVING) WITH
1 THE ROBUST DEFENSE'
1 CALL ALVPRINT(NAME,VII,RV,A,OUT,MAXNAT,MAXNTYPE,NTYPE,
TS)
C
IF (NTYPE.NE.1) THEN
ROB-.TRUE.
NAME-'1THE DEFENDER'S ROBUST GAME INTERCEPTOR ALL
LOCATION BY TARGET TYPE'
1 CALL RVINTCOUNT(NAME,TMYII,RV,A,OUT,
MAXNAT,MAXNTYPE,NTYPE,ROB)
WRITE(OUT,'( / )')
C
ROB-.FALSE.
NAME-'0THE ATTACKER'S ROBUST GAME RV ALLOCATION B
1Y TARGET TYPE'
1 CALL RVINTCOUNT(NAME,TMXII,RV,A,OUT,
MAXNAT,MAXNTYPE,NTYPE,ROB)
ENDIF
C
250 CONTINUE
200 CONTINUE

STOP
END

```

APPENDIX B
SUBROUTINES SIMAT1, SIMAT2, SEQAT1, AND SEQAT2

SUBROUTINE SIMAT1(MAXR,MAXS,PFA,PFDR,S,P)

Subroutine SIMAT1 solves for the probability of a target surviving under simultaneous attack and one shot defense

INPUT variables:

MAXR	INTEGER	The maximum allowable 'R'
MAXS	INTEGER	The maximum allowable 'S'
PFA	REAL	The probability that a missile will fail to destroy a target.
PFDR	REAL	The probability that an interceptor will fail to destroy a missile.
R	INTEGER	The maximum number of missiles that can attack a target.
S	INTEGER	The maximum number of interceptors that can attack a target.

OUTPUT variable:

P(I,J)	REAL	The probability that a target attacked by I missiles and protected by J interceptors will survive.
--------	------	--

INTEGER R, S
REAL P(0:MAXR,0:MAXS), MNFRA

A=1-PFA
D=1-PFDR

Loop through number of attacking missiles

DO 10 I = 0, R

Loop through number of interceptors

DO 20 J = 0, S

Assign value to P(I,J)

```

IF (I .EQ. 0) THEN
  P(I,J) = 1
ELSE IF (A .EQ. 1.0 .AND. D .EQ. 1.0) THEN
  IF (I .GT. J) THEN
    P(I,J) = 0
  ELSE
    P(I,J) = 1
  ENDIF
ELSE
  MNINT = INT(J/I)

```

```

MNFRA = REAL(J)/REAL(I)-MNINT
P1 = (1-A*(1.0-D)**(MNINT+1))**(I*MNFRA)
P2 = (1-A*(1.0-D)**MNINT)**(I*(1-MNFRA))
P(I,J) = P1*P2
ENDIF
C
20      CONTINUE
10      CONTINUE
C
      RETURN
END

```

```

1      SUBROUTINE SIMAT2(MAXR,MAXS,PFA,PFD1,PFD2,
          R,S,P)
C
C      Subroutine SIMAT2 calculates for the probability of a target
C      surviving under simultaneous attack and shoot, look, shoot
C      defense.
C
C      INPUT variables:
C
C      MAXR      INTEGER      The maximum allowable 'R'
C      MAXS      INTEGER      The maximum allowable 'S'
C      PFA       REAL         The probability that a
C                             missile will fail to
C                             destroy a target.
C      PFD1      REAL         The probability that a first
C                             salvo interceptor will fail
C                             to destroy a missile
C      PFD2      REAL         The probability that a second
C                             salvo interceptor will fail
C                             to destroy a missile
C      R         INTEGER      The maximum number of
C                             missiles that can attack a
C                             target
C      S         INTEGER      The maximum number of
C                             interceptors that can
C                             defend a target
C
C      OUTPUT variables:
C
C      P(I,J)    REAL         The probability that a target
C                             attacked by I missiles and
C                             protected by J interceptors
C                             will survive
C
C      INTEGER R,S
C      REAL P(0:MAXR,0:MAXS)
C
C      DEFINITION OF LOCAL VARIABLES:
C
C      INPUTS ARE (1) MAX NUMBER OF
C      ATTACKERS N, (2) MAX NO. OF INTERCEPTORS M, (3) MAX NO.
C      OF SALVOS SAL, (4) SINGLE-SHOT INTERCEPTOR KILL PROBABILITY
C      ON EACH SALVO PKILL(1),...,PKILL(SAL), AND (5) ATTACKER
C      RELIABILITY RHO. OUTPUTS ARE (1) MINIMUM PROBABILITY OF
C      TARGET DESTRUCTION WHEN K SALVOS REMAIN, S2(I,J),
C      I=1,...,M; J=1,...,N; K=1,...,SAL, (2) OPTIMAL NUMBER
C      OF INTERCEPTORS TO USE UNDER EACH CONDITION D(I,J), AND
C      (3) EXPECTED NUMBER OF INTERCEPTORS REMAINING AT THE
C      END OF THE ATTACK W2(I,J). IN CASE OF TIES, THE
C      CHOICE IS MADE TO MAXIMIZE W2(I,J).
C
C      ARRAYS NEEDED ARE S1(M+1,N+1), S2(M+1,N+1), D(M+1,N+1),
C      PA(M+1,N+1), RR(N+1,N), TO(N), T1(N), PKILL(R).

```

```

C      W1(M+1,N+1), AND W2(M+1,N+1).
C
      DIMENSION S1(51,51),S2(51,51),D(51,51),W1(51,51),W2(51,51),
1      PA(51,51),RR(51,50),TO(50),T1(50),PKILL(5)
      INTEGER D,SAL
C
C      SET LOCAL INPUT VARIABLES TO THEIR CORRESPONDING 'SHARED' INPUT
C      VARIABLES
C
      N=R
      M=S
      SAL=2
      RHO=1.-PFA
      PKILL(1)=1.-PFD1
      PKILL(2)=1.-PFD2
C
C      PKILL(1) IS THE S-S KILL PROB. ON THE FIRST SALVO,....
C      PKILL(2) IS THE S-S KILL PROB. ON THE SECOND SALVO.
C
C      INITIALIZE; DO CASE OF 1 SALVO LEFT.
C
      RHOBAR = 1. - RHO
      M1=M+1
      N1=N+1
      DO 200 I=1,M1
      W2(I,1) = I-1
200      S2(I,1) = 0.
      S2(1,2)=RHO
      DO 210 J=3,N1
      J1=J-1
210      S2(1,J)=1.-RHOBAR*(1.-S2(1,J1))
      DO 211 I=1,M1
      DO 211 J=2,N1
      W2(I,J) = 0.0
211      CONTINUE
      Q=1.-PKILL(SAL)
      DO 220 J=2,N1
      IF(J .GE. 2) GO TO 1211
      DO 1200 I=2,M1
      D(I,J) = I-1
1200      S2(I,2) = Q**(I-1)
      GO TO 220
1211      J1=J-1
      ZJ1=J1
      DO 220 I=2,M1
      I1=I-1
      ZI1=I1
      ZIJ=ZI1/ZJ1
      INTIJ=I1/J1
      ZINTIJ=INTIJ+1.E-2
      IF (ZINTIJ .LT. ZIJ) GO TO 212
C
C      ZIJ IS AN INTEGER
C

```

```

      S2(I,J)=1.-(1.-RHO*(Q**INTIJ))**J1
      GO TO 214
212 CONTINUE
C
C      ZIJ IS NOT AN INTEGER
C
      INTIJ1=INTIJ+1
      IEXP1=I1-J1*INTIJ
      IEXP2=J1-IEXP1
      F1=(1.-RHO*(Q**INTIJ1))**IEXP1
      F2=(1.-RHO*(Q**INTIJ))**IEXP2
      S2(I,J)=1.-F1*F2
214 CONTINUE
      D(I,J)=I1
220 CONTINUE
      K=1
C
C      NOW WRITE RESULTS FOR K=1
C
230 CONTINUE
      IF (SAL.EQ. 1) GO TO 900
C
C      MAIN LOOP FOR K SALVOES, K>1
C
      DO 420 K=2,SAL
      KK = SAL-K+1
      Q=1.-PKILL(KK)
C
      DO 300 I=1,M1
      DO 300 J=1,N1
      W1(I,J) = W2(I,J)
300 S1(I,J) = S2(I,J)
C
C      FIRST FIX J
C
      DO 400 J=2,N1
      J1=J-1
      ZJ1=J1
C
C      CALCULATE ALL PA(-,-) VALUES
C
      PA(1,J)=1.
      DO 310 JJ=1,J1
310 PA(1,JJ)=0.
C
C      FIRST CALCULATE TO AND T1 VECTORS
C
      DO 380 I=2,M1
      I1=I-1
      ZI1=I1
      ZIJ=ZI1/ZJ1
      INTIJ=I1/J1
      ZINTIJ=INTIJ+1.E-5
      IF (ZINTIJ.LT. ZIJ) GO TO 330

```

```

C
C   ZIJ IS AN INTEGER
C
    FAC=Q**INTIJ
    DO 320 L=1,J1
    T1(L)=FAC
320  TO(L)=1.-FAC
    GO TO 335
330  CONTINUE
C
C   ZIJ IS NOT AN INTEGER
C
    LIM=J1-I1+J1*INTIJ
    LIM1=LIM+1
    FAC=Q**INTIJ
    DO 332 L=1,LIM
    T1(L)=FAC
332  TO(L)=1.-FAC
    FAC=Q**FAC
    DO 333 L=LIM1,J1
    T1(L)=FAC
333  TO(L)=1.-FAC
335  CONTINUE
C
C   NOW FINISHED WITH TO AND T1
C
    RR(1,1)=TO(1)
    RR(2,1)=T1(1)
    IF (J1.EQ. 1) GO TO 360
    DO 340 L=2,J1
    L1=L+1
    LM1=L-1
    RR(1,L)=RR(1,LM1)*TO(L)
340  RR(L1,L)=RR(L,LM1)*T1(L)
    DO 350 L=2,J1
    LM1=L-1
    DO 350 J2=2,L
    JM1=J2-1
    RR(J2,L)=RR(J2,LM1)*TO(L)+RR(JM1,LM1)*T1(L)
350  CONTINUE
360  CONTINUE
    DO 370 JJ=1,J
    PA(I,JJ)=RR(JJ,J1)
370  CONTINUE
380  CONTINUE
C
C   ALL PA(-,-) VALUES ARE NOW READY
C
C   TEST IS TENTATIVE VALUE OF S(I,J)
C   ITEST IS TENTATIVE VALUE OF D(I,J)
C   WTEST IS TENTATIVE VALUE OF W(I,J)
C
382  CONTINUE
    DO 395 I=2,M1
    TEST=1.

```

```

DO 390 II=1,I
MI-I-II+1
FAC = 0.0
DO 385 JJ=1,J
385 FAC=FAC+PA(II,JJ)*S1(MI,JJ)
TEST-AMIN1(FAC.TEST)
C
C UPDATE TENTATIVE VALUES
C
390 CONTINUE
S2(I,J)=TEST
395 CONTINUE
400 CONTINUE
C
C NOW WRITE RESULTS
C
420 CONTINUE
C
500 CONTINUE
C
C STORE PIJ'S
C
DO 154 I=0,N
DO 155 J=0,M
P(I,J)=1-S2(J+1,I+1)
155 CONTINUE
154 CONTINUE
RETURN
900 END

```

SUBROUTINE SEQAT1(MAXR,MAIS,PFA,PFD1,PFD2,R,S,P)

Subroutine SEQAT1 solves for the probability of a target surviving under unknown sequential attack and one shot or shoot, look, shoot defense

INPUT variables:

MAXR	INTEGER	The maximum allowable 'R'
MAIS	INTEGER	The maximum allowable 'S'
PFA	REAL	The probability that a missile will fail to destroy a target
PFD1	REAL	The probability that a first salvo interceptor will fail to destroy a missile
PFD2	REAL	The probability that a second salvo interceptor will fail to destroy a missile
R	INTEGER	The maximum number of missiles that can attack a target
S	INTEGER	The maximum number of interceptors that may attack a target

OUTPUT variable:

P(I,J)	REAL	The probability that a target attacked by I missiles and protected by J interceptors will survive
--------	------	---

INTEGER R,S
REAL P(0:MAXR, 0:MAIS)

LOCAL VARIABLE DEFINITIONS

RR(.,D) IS THE EXPECTED DAMAGE CURVE WITH ONE SHOT LEFT
Q(.,D) IS THE EXPECTED DAMAGE CURVE WITH TWO SHOTS LEFT
SLOPE(D) YIELDS PRIM READ SLOPE
NQ(D) IS THE NUMBER OF INT TO SHOOT AT 1ST RV, 1ST VOLLEY
NR(D) IS THE NUMBER OF INT TO SHOOT AT NEXT RV, 2ND VOLLEY
NRR(D) IS THE NUMBER REMAINING IF MISS (D-NQ(D))

DIMENSION Q(0:100,0:100),RR(0:100,0:100),SLOPE(0:100),
1 NR(0:100),NQ(0:100),RTRY(0:100),QTRY(0:100),
1 NRR(0:100)

CAPESS-1.-PFA

```

C
C SET Q(A,0)-RR(A,0)-1-PFA**A
C
DO IA=0,R
  IF(PFA.LT.0.000001) THEN
    IF(IA.EQ.0) Q(IA,0)=0.
    IF(IA.GT.0) Q(IA,0)=1.
  ELSE
    Q(IA,0)=1.-(PFA**IA)
  END IF
  RR(IA,0)=Q(IA,0)
END DO
NQ(0)=0
NR(0)=0
C
C SET SLOPE(D)=1.-PFA (UNPROTECTED VALUE)
C AND SET RR(0,D)=Q(0,D)=0
C
DO ID=0,S
  SLOPE(ID)=CAPESS
  Q(0,ID)=0.
  RR(0,ID)=0
END DO
C
C GENERATE RR(..D) AND Q(..D) FOR D=1,S
C
DO ID=1,S
  SLPMAX=0.
  PROD=PFD2*CAPESS
  DO IA=1,R
    RR(IA,ID)=PROD+(1.-PROD)*Q(IA-1,ID-1)
    SLPTRY=RR(IA,ID)/IA
    IF(SLPTRY.GT.SLPMAX) SLPMAX=SLPTRY
  END DO
  SLOPE(ID)=SLPMAX
  NR(ID)=1
  DO IK=2,ID
    SLPMAX=0.
    PROD=(PFD2**IK)*CAPESS
    DO IA=1,R
      RTRY(IA)=PROD+(1.-PROD)*Q(IA-1,ID-1)
      SLPTRY=RTRY(IA)/IA
      IF(SLPTRY.GT.SLPMAX) SLPMAX=SLPTRY
    END DO
    IF(SLPMAX.LT.SLOPE(ID)) THEN
      DO L=1,R
        RR(L,ID)=RTRY(L)
      END DO
      SLOPE(ID)=SLPMAX
      NR(ID)=IK
    END IF
  END DO
  SLPMAX=0.
  DO IA=1,R

```

```

      Q(IA,ID)=RR(IA,ID)
      SLPTRY=Q(IA,ID)/IA
      IF(SLPTRY.GT.SLPMAX) SLPMAX=SLPTRY
    END DO
    NQ(ID)=0
    DO IK=1,ID
      SLPMAX=0.
      POW=PPD1**IK
      DO IA=1,R
        QTRY(IA)=POW*RR(IA,ID-IK)+(1.-POW)*Q(IA-1,ID-IK)
        SLPTRY=QTRY(IA)/IA
        IF(SLPTRY.GT.SLPMAX) SLPMAX=SLPTRY
      END DO
      IF(SLPMAX.LT.SLOPE(ID)) THEN
        DO L=1,R
          Q(L,ID)=QTRY(L)
        END DO
        SLOPE(ID)=SLPMAX
        NQ(ID)=IK
      END IF
    END DO
  END DO
  RETURN THE SURVIVAL PROBABILITIES
  DO I=0,R
    DO J=0,S
      P(I,J)=1.-Q(I,J)
    END DO
  END DO
  DO ID=1,S
    NRR(ID)=ID-NQ(ID)
  END DO
  RETURN
END

```

```

SUBROUTINE SEQAT2(MAXR,MAIS,PFA,PPD1,PPD2,R,S,P)

Subroutine SEQAT2 solves for the probability of a target
surviving under known sequential attack and shoot, look, shoot,
defense

INPUT variables:

MAXR      INTEGER      The maximum allowable 'R'
MAIS      INTEGER      The maximum allowable 'S'
PFA       REAL         The probability that a
                        missile will fail to
                        destroy a target
PPD1      REAL         The probability that a
                        first salvo interceptor
                        will fail to destroy a
                        missile
PPD2      REAL         The probability that a
                        second salvo interceptor
                        will fail to destroy a
                        missile
R         INTEGER      The maximum number of
                        missiles that can attack
                        a target
S         INTEGER      The maximum number of
                        interceptors that can
                        defend a target

OUTPUT variable:

P(I,J)    REAL         The probability that a
                        target attacked by I
                        missiles and protected
                        by J interceptors will
                        survive

INTEGER R,S
REAL P(0:MAXR,0:MAIS)

LOCAL PROGRAM NOTES:

COMPUTES AN OPTIMAL PRIM READ DEFENSE WITH A
SLS CAPABILITY UNDER THE ASSUMPTION THAT THE
DEFENSE CAN DETERMINE THE SIZE OF THE ATTACK
AT THE MOMENT THAT THE ATTACK BEGINS.

DES(...) IS THE EXPECTED DESTRUCTION

REAL SUR(100,100),DES(100,100),FRSLPE(100)
INTEGER DEE(100,100),EE(100,100)

PARAMETER EPSI IS A TOLERANCE

```

```

C      EPSI=0.000001
C
C      SLOPE=1./R
C
C      IIPRIM WILL REMAIN 1 WHEN DES=-JA*SLOPE FOR ALL JA
C
      IIPRIM=1
      DO ID=1,S+1
        SUR(1,ID)=1.
        DES(1,ID)=0.
1010    DO JA=2,R+1
          SUR(JA,ID)=PFA*SUR(JA-1,ID)
          DES(JA,ID)=1.-SUR(JA,ID)
C
C      IPRIM WILL BE SET TO 1 IF PRIMREAD OK AT JA.ID
C
      IPRIM=0
      DEE(JA,ID)=0
      EEE(JA,ID)=0
      SURTRY=SUR(JA,ID)
      DO LODEE=1,ID
        DO LOREE=1,ID-LODEE+1
          INDTEM=ID-LODEE-LOREE+2
          TRYSUR=(1.-PFD1**((LOREE-1))*SUR(JA-1,ID-LODEE+1)+
1              (PFD1**((LOREE-1))*
1              (1.-(PFD2**((LOREE-1)))*(1.-PFA))*
1              SUR(JA-1,INDTEM)
C
C      TEST FOR UPDATE:  "THRIFTY ALLOCATION" BREAKS TIES
C                        IN FAVOR OF THE SMALLEST POTENTIAL
C                        ALLOCATIONS.  IF STILL TIED, IT PICKS
C                        THE SMALLEST INITIAL VOLLEY.
C
      SRTRYN=SURTRY-EPSI
      IF(TRYSUR.LT.SRTRYN) GO TO 1000
      SRTRYP=SURTRY+EPSI
      IF(TRYSUR.LE.SRTRYP) THEN
        ISMINC=DEE(JA,ID)+EEE(JA,ID)+2
        ISMTRY=LODEE+LOREE
        IF(ISMINC.LT.ISMTRY) GO TO 1000
        IF(ISMINC.EQ.ISMTRY) THEN
          IF(DEE(JA,ID).LE.LODEE-1) GO TO 1000
        END IF
      END IF
C      UPDATE IS NECESSARY
      SUR(JA,ID)=TRYSUR
      DES(JA,ID)=1.-SUR(JA,ID)
      DEE(JA,ID)=LODEE-1
      EEE(JA,ID)=LOREE-1
      SURTRY=TRYSUR
C      UPDATE UNNECESSARY
1000    CONTINUE
      END DO

```

```

        END DO
      END DO
    END DO
  C
  C DETERMINE WORST SLOPE FOR EACH DEFENSE SIZE
  C
    DO ID=1,S+1
      PRSLPE(ID)=0.
      DO IA=2,R+1
        SLPTRY=DES(IA,ID)/(IA
        IF(SLPTRY.GT.PRSLPE(ID)) PRSLPE(ID)=SLPTRY
      END DO
    END DO
  C
  C RETURN PIJ'S
  C
    DO I=0,R
      DO J=0,S
        P(I,J)=1-DES(I+1,J+1)
      END DO
    END DO
  C
  C RETURN
  C END

```

APPENDIX C
SUBROUTINES BG, YROBUST, AND XROBUST

```

C
SUBROUTINE BG(XBG, YBG, VBG, R, S, P,
IRV, A, INT, TARGETS,
1MAT, MAXR, MAXS, MAXNTYPE, NTYPE,
1VFRAC, NFRAC)
C
C Subroutine BG finds and returns the solution(s) for the basic game
C
C .....
C INPUT variables:
C
C      R              INTEGER      Maximum RV's at a single
C                               target
C      S              INTEGER      Maximum interceptors at a
C                               single target
C      P(I,J)         REAL          Probability of survival for
C                               a target attacked by I
C                               RV's and protected by J
C                               interceptors
C      RV             INTEGER      The attack sizes
C      A              INTEGER      The number of attack sizes
C      INT            INTEGER      Number of interceptors
C      TARGETS        INTEGER      Number of targets
C      MAT            INTEGER      Maximum permissible A
C      MAXR           INTEGER      Maximum permissible R
C      MAXS           INTEGER      Maximum permissible S
C      MAXNTYPE       INTEGER      Maximum permissible NTYPE
C      NTYPE          INTEGER      The number of target types
C      VFRAC(K)       REAL          Fractional value of all
C                               type K targets
C      NFRAC(K)       REAL          Fractional number of all
C                               type K targets
C
C .....
C Input variable type declaration
C
C      INTEGER R, S, A, INT, TARGETS
C      INTEGER MAT, MAXR, MAXS
C      INTEGER RV(MAT), MAXNTYPE, NTYPE
C      REAL P(0:MAXR,0:MAXS)
C      REAL VFRAC(MAXNTYPE), NFRAC(MAXNTYPE)
C
C .....
C OUTPUT variables:
C
C      XBK(A, K, I)    REAL          Min-max strategy for the
C                               attacker for attack size
C                               A: fraction of K'th type
C                               targets with I RV's
C      YBK(A, K, J)    REAL          Min-max strategy for the
C                               defender for attack size

```



```

C
C
C      Body of Program (SUBROUTINE BG)
C.....
C
C      LENI=LENI1
C      LENMI=LENMI1
C      LENR=LENR1
C      LENMR=LENMR1
C      LOOK=200
C      FACTOR=50
C      PRINT=0
C      BNDTYP=1
C
C      CALL XMAPS(BNDTYP,IOERR,MAPI,MAPR,MAXA,MAXM,MAXN,MEMI,MEMR)
C
C      Prepare XMP to solve for the first attack size RV(1)
C      Set row type and right hand side for the inequality constraints
C
C      N1=NTYPE*(R+1)
C      DO 100 I=1, N1
C          B(I)=0
C          ROWTYP(I)=+1
100  CONTINUE
C
C      Set row type and right hand side for the equality constraints
C
C      N2=NTYPE*(R+2)
C      DO 110 I=N1+1, N2
C          B(I)=1
C          ROWTYP(I)=0
110  CONTINUE
C      B(N2+1)=REAL(INT)/REAL(TARGETS)
C      ROWTYP(N2+1)=0
C
C      Let N be the current number of variables; incremented by XADDAJ
C
C      N=0
C
C      Construct the first NTYPE*(S+1) structural variables associated
C      with the defender's strategy
C
C      DO 120 NT=1, NTYPE
C          DO 121 J=0, S
C              COLLEN=0
C              DO 122 I=0, R
C                  IF (P(I,J) .NE. 0) THEN
C                      COLLEN=COLLEN+1
C                      COLA(COLLEN)=P(I,J)*VFRAC(NT)
C                      COLI(COLLEN)=I+1+(R+1)*(NT-1)
C                  ENDIF
122  CONTINUE
C          COLLEN=COLLEN+1

```

```

C
C      Set the number of constraints
C
C      M=N2+1
C
C      Construct the slack, surplus and artificial variables with XSLACK
C
C      CALL XSLACK(B,BASCB,BASIS,BASLB,BASUB,BLOW,
1          BNDTYP,BOUND,
1          COLA,COLI,COLMAX,IOERR,
1          M,MAPI,MAPR,MAXM,MAXN,MEMI,MEMR,
1          N,ROWTYP,STATUS,UZERO,XBZERO,Z)
C
C      Solve the LP with the primal simplex method
C
C      CALL XPRIML(B,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
1          CAND,CANDA,CANDCJ,CANDI,CANDL,
1          COLA,COLI,COLMAX,
1          FACTOR,IOERR,EOLG,ITER1,ITER2,
1          LOOK,M,MAPI,MAPR,MAXM,MAXN,MEMI,MEMR,
1          N,NTYPE2,PICK,PRINT,STATUS,TERMIN,UNBDDQ,
1          UZERO,XBZERO,YQ,Z)
C
C      Check if solution has been found
C
C      IF (TERMIN .NE. 1) THEN
1'      WRITE (IOERR,'(A)') '1THERE IS AN ERROR IN THE FORMULATION
      WRITE (IOERR,'(A,I4)') ' TERMINATION CODE - ', TERMIN
      STOP
      ENDIF
C
C      Store in XBG, YBG, and VBG the solutions to the LP for RV(1)
C
C      DO 200 NT-1, NTYPE
C          DO 201 I= 0, R
C              XBG(1,NT,I)-UZERO(I+1+(NT-1)*(R+1))
201      CONTINUE
200      CONTINUE
C      DO 210 I= 1, N2+1
C          IF (BASIS(I) .LE. (S+1)*NTYPE) THEN
1          YBG(1,(BASIS(I)-1)/(S+1)+1,
      MOD(BASIS(I)-1,(S+1)))-XBZERO(I)
      ENDIF
210      CONTINUE
      VBG(1)-Z
C
C      Solve for the remaining RV's
C
C      IF (A .EQ. 1) THEN
      RETURN
      ENDIF
C
C      DO 300 I = 2, A

```

```

        COLA(COLLEN)=1.0
        COLI(COLLEN)=N1+NT
        IF (J .NE. 0) THEN
            COLLEN=COLLEN+1
            COLA(COLLEN)=J*NFRAC(NT)
            COLI(COLLEN)=N2+1
        ENDIF

C
C      Call XADDAJ to enter the column for Yj (or variable J+1)
C
        OBJ=0
        CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,IOERR,
1          K,MAPI,MAPR,MEMI,MEMR,N)
121      CONTINUE
120      CONTINUE
C
C      Construct the columns for the 's' variables
C
        DO 130 NT=1, NTYPE
            DO 131 I=1,R+1
                COLA(I)=1
                COLI(I)=I+(NT-1)*(R+1)
131      CONTINUE
C
C      Call XADDAJ to enter the column for s (or variable S+2)
C
        COLLEN=R+1
        OBJ=1
        CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,IOERR,
1          K,MAPI,MAPR,MEMI,MEMR,N)
130      CONTINUE
C
C      Construct the column for the 't' variable
C
        COLLEN=0
        DO 140 NT=1, NTYPE
            DO 141 I=1,R
                COLLEN=COLLEN+1
                COLA(COLLEN)=I*NFRAC(NT)
                COLI(COLLEN)=I+1+(NT-1)*(R+1)
141      CONTINUE
140      CONTINUE
C
C      Call XADDAJ to enter the column for t (or variable S+3)
C
        OBJ=REAL(-RV(1))/REAL(TARGETS)
        CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,IOERR,
1          K,MAPI,MAPR,MEMI,MEMR,N)
C
C      Start all the structural variables at their lower bounds
C
        DO 150 J=1, N
150      STATUS(J)=0
C

```

```

J=NTYPE*(S+2)+1
CJ=REAL(-RV(I))/REAL(TARGETS)
CALL XCHGCJ(CJ,J,MAPI,MAPR,MEMI,MEMR)
C
DO 310 I1=1,M
  J=BASIS(I1)
  CALL XGETAJ(CJ,COLA,COLI,COLLEN,COLMAX,
1 IOERR,J,MAPI,MAPR,MEMI,MEMR)
  BASCB(I1)=CJ
310 CONTINUE
C
CALL XPRIML(B,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
1 CAND,CANDA,CANDCJ,CANDI,CANDL,
1 COLA,COLI,COLMAX,
1 FACTOR,IOERR,EOLG,ITER1,ITER2,
1 LOOK,M,MAPI,MAPR,MAXM,MAXN,MEMI,MEMR,
1 N,NTYPE2,PICK,PRINT,STATUS,TERMIN,UNBDDQ,
1 UZERO,XBZERO,YQ,Z)
C
C Store in XBG, YBG, and VBG the solutions to the LP for RV(I)
C
DO 320 NT=1, NTYPE
  DO 321 I2= 0, R
    XBG(I,NT,I2)=UZERO(I2+1+(NT-1)*(R+1))
321 CONTINUE
320 CONTINUE
DO 330 I2= 1, N2+1
  IF (BASIS(I2) .LE. (S+1)*NTYPE) THEN
    YBG(I, (BASIS(I2)-1)/(S+1)+1,
1MOD(BASIS(I2)-1,(S+1)))-XBZERO(I2)
  ENDIF
330 CONTINUE
  VBG(I)=Z
300 CONTINUE
C
C RETURN
C
END

C
C Subroutines for changing an objective row coefficient
C

SUBROUTINE XCHGCJ(CJ,J,MAPI,MAPR,MEMI,MEMR)
COMMON/XMPLEN/LENI,LENMI,LENMR,LENR
DOUBLE PRECISION CJ,MEMR(LENR)
INTEGER MAPI(LENMI), MAPR(LENMR)
INTEGER MEMI(LENI)

```

```
CALL XDATA5(CJ,J,MEMR(MAPR(3)),MEMI(MAPI(5)))  
RETURN  
END
```

```
SUBROUTINE XDATA5(CJ,J,PROFIT,MAXN)  
DOUBLE PRECISION CJ,PROFIT(MAXN)  
PROFIT(J)=CJ  
RETURN  
END
```

```

C
C      SUBROUTINE YROBUST(YII, VBG, R, S, P,
C      1RV, A, INT, TARGETS,
C      1MAT, MAXR, MAXS, MAXNTYPE, NTYPE, NFRAC, VFRAC)
C
C      Subroutine ROBUST finds and returns the robust defender's strategy
C      over a range of attack sizes
C
C***** PRIMAL VERSION *****
C*****
C
C      INPUT variables:
C
C      VBG(A)          REAL          Game value associated with
C                                   XBG and YBG
C      R              INTEGER        Maximum RV's at a single
C                                   target
C      S              INTEGER        Maximum interceptors at a
C                                   single target
C      P(I,J)         REAL          Probability of survival for
C                                   a target attacked by I RV's
C                                   and protected by J
C                                   interceptors
C      RV             INTEGER        The attack sizes over which
C                                   the robust defense will be
C                                   defined
C      A              INTEGER        The number of attack sizes
C      INT            INTEGER        Number of interceptors
C      TARGETS        INTEGER        Number of targets
C      MAT            INTEGER        Maximum permissible A
C      MAXR           INTEGER        Maximum permissible R
C      MAXS           INTEGER        Maximum permissible S
C      MAXNTYPE       INTEGER        Maximum permissible NTYPE
C      NTYPE          INTEGER        Number of target types
C      NFRAC(K)       REAL          The fraction of targets of
C                                   type K
C      VFRAC(K)       REAL          The fraction of the total
C                                   value for type K targets
C
C*****
C
C      Input variable type declaration
C
C      INTEGER R, S, A, INT, TARGETS
C      INTEGER MAT, MAXR, MAXS, MAXNTYPE, NTYPE
C      INTEGER RV(MAT)
C      REAL P(0:MAXR,0:MAXS), VBG(MAT), NFRAC(MAXNTYPE)
C      REAL VFRAC(MAXNTYPE)
C
C*****
C
C      OUTPUT variables:

```

```

o
o      YII(K,J)          REAL          Robust strategy for the
o                                     defender over attack sizes
o                                     RV: fraction of type K
o                                     targets with J
o                                     interceptors
o
o.....
o
o      Output variable type declaration
o
o      REAL YII(MAXNTYPE,0:MAXS)
o
o.....
o
o      Local variables needed for XMP.  See XMP Dictionary for
o      definitions.
o
o.....
o
o      Integer parameters
o
o      INTEGER MAXM,MAXN,MAXA,COLMAX,PICK
o      PARAMETER (MAXM=1500, MAXN=1700, MAXA=20000)
o      PARAMETER (COLMAX=100, PICK=7)
o      INTEGER LENI1, LENMI1, LENMR1, LENR1
o      PARAMETER (LENI1=100000,LENMI1=9,LENMR1=8,LENR1=50000)
o
o      Double Precision arrays and variables
o
o      DOUBLE PRECISION B(MAXM),BASCB(MAXM),BASLB(MAXM),BASUB(MAXM)
o      DOUBLE PRECISION BLOW(1),BOUND,CANDA(COLMAX,PICK),CANDCJ(PICK)
o      DOUBLE PRECISION CJ,COLA(COLMAX),LJ,MEMR(LENR1),UJ,UZERO(MAXM)
o      DOUBLE PRECISION XBZERO(MAXM),YQ(MAXM),Z
o
o      Integer arrays and variables
o
o      INTEGER BNDTYP,COLLEN,DFEASQ,DTERM,DUMBR,ERROR,FACTOR
o      INTEGER IOERR,IOLOG,ITER,ITER1,ITER2,LOOK,M,MAPI(LENMI1)
o      INTEGER MAPR(LENMR1),N,NTYPE2,PRINT,TERMIN
o      INTEGER UNBDDQ,MEMI(LENI1),BASIS(MAXM),CAND(PICK)
o      INTEGER CANDI(COLMAX,PICK),CANDL(PICK),COLI(COLMAX)
o      INTEGER ROWTYP(MAXM), STATUS(MAXN)
o
o      I/O numbers for in put, error output, and results output
o
o      COMMON/IO_UNIT/IOIN,IOERR,IOLOG
o
o      Other common variables
o
o      COMMON /XMPLEN/ LENI, LENMI, LENMR, LENR
o      INTEGER LENI, LENMI, LENMR, LENR
o
o.....

```

```

C
C
C      Body of Program (SUBROUTINE YROBUST)
C.....
C
C
C      LENI=LENI1
C      LENMI=LENMI1
C      LENR=LENR1
C      LENMR=LENMR1
C      LOOK=200
C      FACTOR=50
C      PRINT=0
C      BNDTYP=1
C
C      CALL XMAPS(BNDTYP,IOERR,MAPI,MAPR,MAXA,MAXM,MAXN,MEMI,MEMR)
C
C      Set row type and right hand sides for the Z equality
C      constraints
C
C      N1=(R+1)*NTYPE
C      DO 100 I=1, N1
C          B(I)=0
C          ROWTYP(I)=0
100      CONTINUE
C
C      Set row type and right hand sides for the RO and ZST
C      inequality constraints
C
C      N2=N1+A+A*NTYPE*(R+1)
C      DO 110 I=N1+1, N2
C          B(I)=0
C          ROWTYP(I)=+1
110      CONTINUE
C
C      Set row type and right hand side for the Y equality constraints
C
C      N3=N2+NTYPE
C      DO 120 I=N2+1, N3
C          B(I)=1
C          ROWTYP(I)=0
120      CONTINUE
C      B(N3+1)=REAL(INT)/TARGETS
C      ROWTYP(N3+1)=0
C
C      Let N be the current number of variables; incremented by XADDAJ
C
C      N=0
C
C      Construct the first NTYPE*S+1 structural variables associated
C      with the defender's strategy
C

```

```

DO 130 NT=1, NTYPE
DO 131 J=0, S
COLLEN=0
DO 132 I=0, R
  IF (P(I,J) .NE. 0) THEN
    COLLEN=COLLEN+1
    COLA(COLLEN)=P(I,J)*VFRAC(NT)
    COLI(COLLEN)=I+1+(R+1)*(NT-1)
  ENDIF
132 CONTINUE
COLLEN=COLLEN+1
COLA(COLLEN)=1.0
COLI(COLLEN)=N2+NT
IF (J .NE. 0) THEN
  COLLEN=COLLEN+1
  COLA(COLLEN)=J*NFRAC(NT)
  COLI(COLLEN)=N3+1
ENDIF
C
C      Call XADDAJ to enter the column for Yj (or variable J+1)
C
OBJ=0
CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,IOERR,
1      K,MAPI,MAPR,MEMI,MEMR,N)
131 CONTINUE
130 CONTINUE
C
C      Construct the columns for the z variables
C
DO 140 NT=1, NTYPE
DO 141 I=1, R+1
  COLA(I)=-1.0
  COLI(I)=I+(NT-1)*(R+1)
DO 142 I1=1, A
  COLA(I1+1)=-1.0
  COLI(I1+1)=N1+A*(R+1)*(NT-1)+(I1-1)*(R+1)+I
142 CONTINUE
C
C      Call XADDAJ to enter the column for z's
C
COLLEN=A+1
OBJ=0
CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,IOERR,
1      K,MAPI,MAPR,MEMI,MEMR,N)
141 CONTINUE
140 CONTINUE
C
C      Construct the column for RO
C
DO 150 I=1, A
  COLA(I)=VBG(I)
  COLI(I)=N1+I
150 CONTINUE

```

```

COLLEN=A
OBJ=1
1 CALL XADDAJ(OBJ, COLA, COLI, COLLEN, COLMAX, IOERR,
             K, MAPI, MAPR, MEMI, MEMR, N)

o
o Construct the column for the 's' variables
o
DO 160 NT=1, NTYPE
DO 161 I=1, A
COLA(1)=-1
COLI(1)=N1+I
DO 162 I1=1, R+1
COLA(I1+1)=1
COLI(I1+1)=N1+A*(R+1)*(NT-1)+(I-1)*(R+1)+I1
162 CONTINUE
COLLEN=R+2
OBJ=0
CALL XADDAJ(OBJ, COLA, COLI, COLLEN, COLMAX, IOERR,
             K, MAPI, MAPR, MEMI, MEMR, N)
1
161 CONTINUE
160 CONTINUE
o
o Construct the column for the 't' variables
o
DO 170 I=1, A
COLA(1)=REAL(RV(I))/REAL(TARGETS)
COLI(1)=N1+I
COLLEN=1
DO 171 NT=1, NTYPE
DO 172 I1=1, R
COLLEN=COLLEN+1
COLA(COLLEN)=-I1*NFRAC(NT)
COLI(COLLEN)=N1+A*(NT-1)*A*(R+1)+(I-1)*(R+1)+I1+1
172 CONTINUE
171 CONTINUE
OBJ=0
o
o Call XADDAJ to enter the column for t (or variable S+3)
o
CALL XADDAJ(OBJ, COLA, COLI, COLLEN, COLMAX, IOERR,
             K, MAPI, MAPR, MEMI, MEMR, N)
1
170 CONTINUE
o
o Start all the structural variables at their lower bounds
o
DO 180 J=1, N
180 STATUS(J)=0
o
o Set the number of constraints
o
M=N3+1
o
o Construct the slack, surplus and artificial variables with XSLACK

```

```

C      CALL XSLACK(B,BASCB,BASIS,BASLB,BASUB,BLOW,
1          BNDTYP,BOUND,
1          COLA,COLI,COLMAX,IOERR,
1          M,MAPI,MAPR,MAXM,MAXN,MEMI,MEMR,
1          N,ROWTYP,STATUS,UZERO,XBZERO,Z)
C
C      Solve the LP with the primal simplex method
C
C      CALL XPRIML(B,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
1          CAND,CANDA,CANDCJ,CANDI,CANDL,
1          COLA,COLI,COLMAX,
1          FACTOR,IOERR,ILOG,ITER1,ITER2,
1          LOOK,
1          M,MAPI,MAPR,MAXM,MAXN,MEMI,MEMR,
1          N,NTYPE2,PICK,PRINT,STATUS,
1          TERMIN,UNBDDQ,
1          UZERO,XBZERO,YQ,Z)
C
C      Check if solution has been found
C
C      IF (TERMIN.NE.1) THEN
1'      WRITE (IOERR,'(A)') '1THERE IS AN ERROR IN THE FORMULATION
      WRITE (IOERR,'(A,I4)') ' TERMINATION CODE = ', TERMIN
      STOP
      ENDIF
C
C      Store in YII the solution to the LP
C
      DO 200 I=1,NTYPE
      DO 201 J=0,S
201      YII(I,J)=0
200      CONTINUE
      DO 210 I=1,M
      IF (BASIS(I).LE.(S+1)*NTYPE) THEN
          YII((BASIS(I)-1)/(S+1)+1,MOD(BASIS(I)-1,(S+1)))
1-XBZERO(I)
      ENDIF
210      CONTINUE
C
      RETURN
C
      END

```

```

O      SUBROUTINE XROBUST(XII, YII, VII, R, S, P,
O      1RV, A, INT, TARGETS,
O      1MAT, MAXR, MAXS, MAXNTYPE, NTYPE, NFRAC, VFRAC)
O
O      Subroutine XROBUST uses the robust defense to find the optimal
O      attack against it and the resultant expected target survival
O      rate for each specified attack size.
O      .....
O      INPUT variables:
O
O      YII(K,J)      REAL      The robust defense found by
O                               YROBUST: fraction of type
O                               K targets protected by J
O                               interceptors
O      R              INTEGER    Maximum RV's at a single
O                               target
O      S              INTEGER    Maximum interceptors at a
O                               single target
O      P(I,J)        REAL      Probability of survival for a
O                               target attacked by I RV's
O                               and protected by J
O                               interceptors
O      RV             INTEGER    The attack sizes
O      A              INTEGER    The number of attack sizes
O      INT            INTEGER    Number of interceptors
O      TARGETS        INTEGER    Number of targets
O      MAT            INTEGER    Maximum permissible A
O      MAXR            INTEGER    Maximum permissible R
O      MAXS            INTEGER    Maximum permissible S
O      MAXNTYPE        INTEGER    Maximum permissible NTYPE
O      NTYPE          INTEGER    The number of target types
O      NFRAC(K)       REAL      The fraction of all targets
O                               that are of type K
O      VFRAC(K)       REAL      The fraction of total target
O                               value that are of type K
O      .....
O      Input variable type declaration
O
O      INTEGER R, S, A, INT, TARGETS
O      INTEGER MAT, MAXR, MAXS, MAXNTYPE, NTYPE
O      INTEGER RV(MAT)
O      REAL P(0:MAXR,0:MAXS), YII(MAXNTYPE, 0:MAXS), NFRAC(MAXNTYPE)
O      REAL VFRAC(MAXNTYPE)
O      .....
O      OUTPUT variables:
O
O      XII(A, K, I)    REAL      The attacker's optimal

```



```

C
C
C      Body of Program (SUBROUTINE XROBUST)
C.....
C
C
C      LENI-LENI1
C      LENMI-LENMI1
C      LENR-LENR1
C      LENMR-LENMR1
C      LOOK-50
C      FACTOR-50
C      PRINT-0
C      BNDTYP-1
C
C      CALL XMAPS(BNDTYP,IOERR,MAPI,MAPR,MAXA,MAXM,MAXN,MEMI,MEMR)
C
C      Set row type and right hand side for the equality constraints
C
C      DO 100 NT=1, NTYPE
C          B(NT)=1
C          ROWTYP(NT)=0
100  CONTINUE
C      B(NTYPE+1)=REAL(RV(1))/REAL(TARGETS)
C      ROWTYP(NTYPE+1)=0
C
C      Let N be the current number of variables; incremented by XADDAJ
C
C      N=0
C
C      Construct the NTYPE*(R+1) structural variables associated with
C      the attacker's strategy
C
C      DO 110 NT=1, NTYPE
C          DO 111 I=0, R
C              COLLEN=1
C              COLA(1)=1
C              COLI(1)=NT
C              IF (I .NE. 0) THEN
C                  COLLEN=2
C                  COLA(2)=I*NFRAC(NT)
C                  COLI(2)=NTYPE+1
C              ENDIF
C              OBJ=0
C              DO 112 J=0, S
112  OBJ=-P(I,J)*VFRAC(NT)*YII(NT,J)+OBJ
C              CALL XADDAJ(OBJ,COLA,COLI,COLLEN,COLMAX,
C                  IOERR,K,MAPI,MAPR,MEMI,MEMR,N)
111  CONTINUE
110  CONTINUE
C
C      Start all the structural variables at their lower bounds
C

```

```

120      DO 120 J=1, N
          STATUS(J)=0
C
C
C      Set the number of constraints
C
          M=NTYPE+1
C
C      Construct the slack surplus and artificial variables with XSLACK
C
          CALL XSLACK(B, BASCB, BASIS, ASLB, BASUB, BLOW,
1             BNDTYP, BOUND,
1             COLA, COLI, COLMAX, IOERR,
1             M, MAPI, MAPR, MAXM, MAIN, MEMI, MEMR,
1             N, ROWTYP, STATUS, UZERO, XBZERO, Z)
C
C      Solve the LP with the primal simplex method
C
          CALL XPRIML(B, BASCB, BASIS, ASLB, BASUB, BNDTYP, BOUND,
1             CAND, CANDI, CANDCJ, CANDL, CANDL,
1             COLA, COLI, COLMAX,
1             FACTOR, IOERR, EOLOG, ITER1, ITER2,
1             LOOK, M, MAPI, MAPR, MAXM, MAIN, MEMI, MEMR,
1             N, NTYPE2, PICK, PRINT, STATUS, TERMIN, UNBDDQ,
1             UZERO, XBZERO, YQ, Z)
C
C      Check if solution has been found
C
          IF (TERMIN .NE. 1) THEN
1             WRITE (IOERR, '(A)') 'THERE IS AN ERROR IN THE FORMULATION
                WRITE (IOERR, '(A,I4)') 'TERMINATION CODE = ', TERMIN
                STOP
            ENDIF
C
C      Store in XII and VII the solutions to the LP for RV(1)
C
          DO 200 I= 1, NTYPE+1
              IF (BASIS(I) .LE. NTYPE*(R+1)) THEN
                  XII(1, (BASIS(I)-1)/(S+1)+1,
1                     MOD(BASIS(I)-1, (S+1)))-XBZERO(I)
              ENDIF
200          CONTINUE
              VII(1)--Z
C
C      Solve for the remaining RV's
C
          IF (A .EQ. 1) THEN
              RETURN
          ENDIF
C
          DO 300 I = 2, A
C
              B(NTYPE+1)-REAL(RV(I))/REAL(TARGETS)

```

```

      CALL XBCOMP(B,BASCB,BNDTYP,BOUND,
1          COLA,COLI,COLMAX,IOERR,
1          M,MAPI,MAPR,MAXM,MAIN,MEMI,MEMR,N,
1          STATUS,XBZERO,Z)
C
      CALL XPRIML(B,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
1          CAND,CANDA,CANDCJ,CANDI,CANDL,
1          COLA,COLI,COLMAX,
1          FACTOR,IOERR,EOLG,ITER1,ITER2,
1          LOOK,M,MAPI,MAPR,MAXM,MAIN,MEMI,MEMR,
1          N,NTYPE2,PICK,PRINT,STATUS,TERMIN,UNBDDQ,
1          UZERO,XBZERO,YQ,Z)
C
C      Store in XII and VII the solutions to the LP for RV(I)
C
      DO 301 I1 = 1, M
      IF (BASIS(I1) .LE. NTYPE*(R+1)) THEN
1          XII(I, (BASIS(I1)-1)/(S+1)+1,
      MOD(BASIS(I1)-1,(S+1)))-XBZERO(I1)
      ENDIF
301      CONTINUE
      VII(I)--Z
300      CONTINUE
C
      RETURN
C
      END

```

APPENDIX D
SUBROUTINES SUMMARY , YRPRINT, STPRINT, AND VPRINT

```

1      SUBROUTINE SUMMARY(NATTYPE,NDFTYPE,ANSWER,MINRV,MAXRV,
1      INCRV,INT,TARGETS,R,S,PFA,PFD,PFD1,
1      PFD2,OUT,MAXNTYPE,NTYPE,VTYPE,NTAR)
C
      INTEGER NATTYPE,NDFTYPE,MINRV,MAXRV,INCRV,INT,TARGETS,R,S
      INTEGER OUT,MAXNTYPE,NTYPE
      INTEGER NTAR(MAXNTYPE)
      REAL PFA,PFD,PFD1,PFD2,VTYPE(MAXNTYPE)
      CHARACTER*1 ANSWER,TITLE*50
C
      WRITE(OUT,'(A)') 'OTHE PARAMETERS OF THIS PREALLOCATED PREFERE
INITIAL DEFENSE GAME'
      WRITE(OUT,1000) 'O'
      TITLE='O THE ATTACK METHODOLOGY'
      IF (NATTYPE .EQ. 1) THEN
        WRITE(OUT,1001) TITLE, 'SIMULTANEOUS'
      ELSE
        WRITE(OUT,1001) TITLE, 'SEQUENTIAL'
        IF (ANSWER .EQ. 'Y') THEN
          WRITE(OUT,1001) ' ' ' WITH ATTACK SIZE AT'
          WRITE(OUT,1001) ' ' ' A TARGET KNOWN TO'
          WRITE(OUT,1001) ' ' ' THE DEFENDER'
        ELSE
          WRITE(OUT,1001) ' ' ' WITH ATTACK SIZE AT'
          WRITE(OUT,1001) ' ' ' A TARGET UNKNOWN TO'
          WRITE(OUT,1001) ' ' ' THE DEFENDER'
        ENDIF
      ENDIF
      TITLE=' THE DEFENSE METHODOLOGY'
      IF (NDFTYPE .EQ. 1) THEN
        WRITE(OUT,1001) TITLE, 'ONE SHOT'
      ELSE
        WRITE(OUT,1001) TITLE, 'SHOOT LOOK SHOOT'
      ENDIF
C
      TITLE='O THE FAILURE RATE OF RV'S'
      WRITE(OUT,1002) TITLE, PFA
      IF (NDFTYPE .EQ. 1) THEN
        TITLE=' THE FAILURE RATE OF THE INTERCEPTORS'
        WRITE(OUT,1002) TITLE, PFD
      ELSE
        TITLE=' THE FAILURE RATE OF THE FIRST SALVO'
        WRITE(OUT,1001) TITLE, ' '
        TITLE=' INTERCEPTORS'
        WRITE(OUT,1002) TITLE, PFD1
        TITLE=' THE FAILURE RATE OF THE SECOND SALVO'
        WRITE(OUT,1001) TITLE, ' '
        TITLE=' INTERCEPTORS'
        WRITE(OUT,1002) TITLE, PFD2
      ENDIF
C
      TITLE='O MAXIMUM NUMBER OF RV'S ATTACKING A '
      WRITE(OUT,1001) TITLE, ' '

```

```

TITLE- '          SINGLE TARGET'
WRITE(OUT,1003) TITLE, R
TITLE- '    MAXIMUM NUMBER OF INTERCEPTORS DEFENDING A '
WRITE(OUT,1001) TITLE, ' '
TITLE- '          SINGLE TARGET'
WRITE(OUT,1003) TITLE, S

c
TITLE- 'O THE MINIMUM NUMBER OF RV'S'
WRITE(OUT,1003) TITLE, MINRV
TITLE- '    THE MAXIMUM NUMBER OF RV'S'
WRITE(OUT,1003) TITLE, MAXRV
TITLE- '    THE ATTACK SIZE INCREMENT'
WRITE(OUT,1003) TITLE, INCRV
TITLE- '    THE NUMBER OF INTERCEPTORS'
WRITE(OUT,1003) TITLE, INT
TITLE- '    THE TOTAL NUMBER OF TARGETS'
WRITE(OUT,1003) TITLE, TARGETS
TITLE- 'O THE NUMBER OF TARGET TYPES'
WRITE(OUT,1003) TITLE, NTYPE
DO 100 I=1, NTYPE

        WRITE(OUT,1004) '          TARGET TYPE ', I, ' '
        TITLE- '          NUMBER OF TARGETS'
        WRITE(OUT,1003) TITLE, NTAR(I)
        TITLE- '          RELATIVE VALUE'
        WRITE(OUT,1002) TITLE, VTYPE(I)
100    CONTINUE
        WRITE(OUT,1000) 'O'

c
1000    FORMAT(A,72(' '))
1001    FORMAT(A50, A)
1002    FORMAT(A50, F5.3)
1003    FORMAT(A50, I5)
1004    FORMAT(A, I2, A)
c

RETURN
END

        SUBROUTINE YRPRINT(YR, MINRRV, MAXRRV, MAXS, N, OUT,
1          MAXNTYPE, NTYPE)
c
        INTEGER MINRRV, MAXRRV, MAXS, MAXNTYPE, NTYPE
        REAL YR(MAXNTYPE,0:MAXS)
        INTEGER OUT, N1, N
c
        WRITE(OUT, 1000) '1THE ROBUST DEFENSE STRATEGY FOR RV RANGE '
1, MINRRV, ' TO ', MAXRRV, ' '
        WRITE(OUT,1001) 'O'
        DO 100 NT=1,NTYPE
            WRITE(OUT,1002) 'O', 'TARGET TYPE ',NT
            WRITE(OUT,1005) (I,I=0,9)
            WRITE(OUT,1003) ' '

```

```

      N1=N/10
      IF (N1 .EQ. 0) THEN
        WRITE(OUT,1004) (YR(NT,I), I=0,N)
      ELSE
        WRITE(OUT,1004) (YR(NT,I), I=0.9)
        DO 101 I1=1, N1
          IF (I1 .EQ. N1) THEN
            WRITE(OUT,1004) (YR(NT,I), I=I1*10, N)
          ELSE
            WRITE(OUT,1004) (YR(NT,I), I=I1*10,I1*10+9)
          ENDIF
        CONTINUE
      ENDIF
101   CONTINUE
      ENDIF
      WRITE(OUT,1003) ' '
100   CONTINUE
      WRITE(OUT,1001) '0'
C
1000  FORMAT(A,I5,A,I5,A)
1001  FORMAT(A,108(' '))
1002  FORMAT(A,47X,A,I2)
1003  FORMAT(A,7X,93(' '))
1004  FORMAT(8X,' ',10(F7.4,' '))
1005  FORMAT(13X,10(I1,8X))
C
      RETURN
      END

      SUBROUTINE STPRINT(STNAME,STBG,RV,A,N,OUT,MAXNAT,MAXN,
1      MAXNTYPE,NTYPE,VFRAC,NFRAC)
C
      INTEGER MAXNAT,MAXN,MAXNTYPE,NTYPE
      CHARACTER*70 STNAME
      REAL STBG(MAXNAT,MAXNTYPE,0:MAXN), NFRAC(MAXNTYPE)
      REAL VFRAC(MAXNTYPE)
      INTEGER RV(MAXNAT), A, N, OUT
C
      WRITE(OUT,1000) STNAME
      WRITE(OUT,1001) '0'
      DO 100 NT=1, NTYPE
        WRITE(OUT,1000) ' '
        WRITE(OUT,1002) '0', 'TARGET TYPE', NT,
1' : ' , 100*NFRAC(NT), '% OF TOTAL TARGETS, WITH ' , VFRAC(NT)*100,
1' % OF TOTAL VALUE'
        IF (NT .EQ. 1) THEN
          WRITE(OUT,1007) ' ATTACK SIZE',(I, I=0.9)
        ELSE
          WRITE(OUT,1007) ' ',(I, I=0.9)
        ENDIF
        WRITE(OUT,1003) ' '
        DO 101 K = 1, A
          N1=N/10
          IF (N1 .EQ. 0) THEN

```

```

        WRITE(OUT,1004) RV(K), (STBG(K,NT,I), I=0,N)
    ELSE
        WRITE(OUT,1004) RV(K), (STBG(K,NT,I), I=0,9)
        DO 102 I1=1, N1
            IF (I1.EQ. N1) THEN
                WRITE(OUT,1005) (STBG(K,NT,I), I=I1*10, N)
            ELSE
                WRITE(OUT,1005) (STBG(K,NT,I), I=I1*10,
1 I1*10+9)
            ENDIF
102      CONTINUE
        ENDIF
        IF (K.NE. A) THEN
            WRITE(OUT,1006)
        ELSE
            WRITE(OUT,1003) ' '
        ENDIF
101      CONTINUE
100      CONTINUE
        WRITE(OUT,1000) ' '
        WRITE(OUT,1001) '0'
C
1000     FORMAT(A)
1001     FORMAT(A,106(' '))
1002     FORMAT(A,17X,A12,2X,I3,A,F6.2,A,F6.2,A)
1003     FORMAT(A,106(' '))
1004     FORMAT(5X, I5, 4X, ' ',10(P7.4, ' '))
1005     FORMAT(14X, ' ',10(P7.4, ' '))
1006     FORMAT(14X,93(' '))
1007     FORMAT(A,7X,10(I1,8X))
C
        RETURN
    END

```

```

        SUBROUTINE VPRINT(VNAME, VG, RV, A, OUT, MAXNAT)
C
C
        INTEGER MAXNAT
        CHARACTER*70 VNAME
        REAL VG(MAXNAT)
        INTEGER RV(MAXNAT), A, OUT
C
        WRITE(OUT,1000) VNAME
        WRITE(OUT,1001) '0'
        WRITE(OUT,1000) '0 ATTACK SIZE'
        WRITE(OUT,1002) ' '
        DO 100 K= 1, A
            WRITE(OUT,1003) RV(K), VG(K)
100      CONTINUE
        WRITE(OUT,1002) ' '

```

```
WRITE(OUT,1001) '0'
C
1000 FORMAT(A)
1001 FORMAT(A,29(' '))
1002 FORMAT(A,29('-'))
1003 FORMAT(5X, I5, 5X, ':', F11.4)
C
RETURN
END
```

APPENDIX E

**SUBROUTINES ALYRPRINT, ALPRINT, ALVPRINT, AND
RVINTCOUNT**

```

1          SUBROUTINE ALYRPRINT(YR, MINRRV, MAXRRV, MAXS, N, OUT,
c          MAXNTYPE, NTYPE, NFRAC, TARGETS)

          INTEGER MINRRV, MAXRRV, MAXS, MAXNTYPE, NTYPE
          REAL YR(MAXNTYPE, 0:MAXS), NFRAC(MAXNTYPE)
          INTEGER OUT, N1, N, TARGETS

          WRITE(OUT, 1000) '1THE ROBUST DEFENSE ALLOCATION FOR RV RANGE '
1, MINRRV, ' TO ', MAXRRV, ' : '
          WRITE(OUT, 1001) '0'
          DO 100 NT=1, NTYPE
              WRITE(OUT, 1002) '0', 'TARGET TYPE ', NT
              WRITE(OUT, 1005) (I, I=0, 9)
              WRITE(OUT, 1003) ' '
              N1=N/10
              IF (N1 .EQ. 0) THEN
                  WRITE(OUT, 1004) (YR(NT, I)*NFRAC(NT)*TARGETS, I=0, N)
              ELSE
                  WRITE(OUT, 1004) (YR(NT, I)*NFRAC(NT)*TARGETS, I=0, 9)
                  DO 101 I1=1, N1
                      IF (I1 .EQ. N1) THEN
                          WRITE(OUT, 1004) (YR(NT, I)*NFRAC(NT)*TARGETS,
1 I-I1*10, N)
                      ELSE
                          WRITE(OUT, 1004) (YR(NT, I)*NFRAC(NT)*TARGETS,
1 I-I1*10, I1*10+9)
                      ENDIF
101          CONTINUE
              ENDIF
              WRITE(OUT, 1003) ' '
100          CONTINUE
          WRITE(OUT, 1001) '0'
c
1000         FORMAT(A, I5, A, I5, A)
1001         FORMAT(A, 106(' '))
1002         FORMAT(A, 46X, A, I2)
1003         FORMAT(A, 7X, 93(' '))
1004         FORMAT(8X, ' : ', 10(F7.1, ' '))
1005         FORMAT(13X, 10(I1, 8X))
c
          RETURN
          END

```

```

1          SUBROUTINE ALPRINT(STNAME, STBG, RV, A, N, OUT, MAXNAT, MAXN,
c          MAXNTYPE, NTYPE, VFRAC, NFRAC, TARGETS)

          INTEGER MAXNAT, MAXN, MAXNTYPE, NTYPE
          CHARACTER*70 STNAME
          REAL STBG(MAXNAT, MAXNTYPE, 0:MAXN), NFRAC(MAXNTYPE)
          REAL VFRAC(MAXNTYPE)

```

```

c      INTEGER RV(MAXNAT), A, N, OUT, TARGETS

      WRITE(OUT,1000) STNAME
      WRITE(OUT,1001) '0'
      DO 100 K=1,A
        WRITE(OUT,1000) ' '
        WRITE(OUT,1002) '0', 'ATTACK SIZE -', RV(K)
        IF (K .EQ. 1) THEN
          WRITE(OUT,1007) ' TARGET TYPE', (I, I=0,9)
        ELSE
          WRITE(OUT,1007) ' ', (I, I=0,9)
        ENDIF
        WRITE(OUT,1003) ' '
        DO 101 NT=1,NTYPE
          N1=N/10
          IF (N1 .EQ. 0) THEN
            WRITE(OUT,1004) NT,
1 (STBG(K,NT,I)*NFRAC(NT)*TARGETS, I=0,N)
          ELSE
            WRITE(OUT,1004) NT,
1 (STBG(K,NT,I)*NFRAC(NT)*TARGETS, I=0,9)
            DO 102 I1=1, N1
              IF (I1 .EQ. N1) THEN
                WRITE(OUT,1005)
1 (STBG(K,NT,I)*NFRAC(NT)*TARGETS, I=I1*10, N)
              ELSE
                WRITE(OUT,1005)
1 (STBG(K,NT,I)*NFRAC(NT)*TARGETS, I=I1*10,I1*10+9)
              ENDIF
            102 CONTINUE
          ENDIF
          IF (NT .NE. NTYPE) THEN
            WRITE(OUT,1006)
          ELSE
            WRITE(OUT,1003) ' '
          ENDIF
        101 CONTINUE
      100 CONTINUE
      WRITE(OUT,1000) ' '
      WRITE(OUT,1001) '0'

c      1000 FORMAT(A)
      1001 FORMAT(A,106(' '))
      1002 FORMAT(A,50X,A13,I6)
      1003 FORMAT(A,106('-'))
      1004 FORMAT(5X, I3, 6X, ' ',10(F7.1, ' '))
      1005 FORMAT(14X, ' ',10(F7.1, ' '))
      1006 FORMAT(14X,93('-'))
      1007 FORMAT(A,7X,10(I1,8X))

c      RETURN
      END

```

```

1          SUBROUTINE ALVPRINT(VNAME, VG, RV, A, OUT, MAXNAT,
c                                     MAXNTYPE, NTYPE, TS)
c
c          INTEGER MAXNAT, MAXNTYPE, NTYPE
c          CHARACTER*70 VNAME, CNAME, BLANK
c          REAL VG(MAXNAT), TS(MAXNAT, MAXNTYPE)
c          INTEGER RV(MAXNAT), A, OUT
c          CHARACTER*10 DASH(20), STAR(20)
c
c          BLANK='
c          WRITE(OUT,1000) VNAME
c
c          DO 200 NT=1, NTYPE
c              STAR(NT)='*****'
c              DASH(NT)='-----'
200          CONTINUE
c
c          CNAME=BLANK(1:NTYPE*5)//'TARGET TYPE'
c          WRITE(OUT,1001) 'O',(STAR(NT), NT-1, NTYPE)
c          WRITE(OUT,1002) 'O', CNAME
c          WRITE(OUT,1003) 'ATTACK SIZE', (I, I-1, NTYPE)
c          WRITE(OUT,1004) ' ', (DASH(NT), NT-1, NTYPE)
c          DO 100 K= 1, A
c              WRITE(OUT,1005) RV(K), (TS(K,NT), NT-1, NTYPE)
100          CONTINUE
c          WRITE(OUT,1004) ' ', (DASH(NT), NT-1, NTYPE)
c          WRITE(OUT,1001) 'O',(STAR(NT), NT-1, NTYPE)
c
c          1000  FORMAT(A)
c          1001  FORMAT(A,16(' '),10A)
c          1002  FORMAT(A,10X,A)
c          1003  FORMAT(2X,A11,2X,' ',10(I4,6X))
c          1004  FORMAT(A,16(' '),10A)
c          1005  FORMAT(5X,I5,5X,' ',10(F7.2,3X))
c
c          RETURN
c          END

```

```

1          SUBROUTINE RVINTCOUNT(VNAME, TM, RV, A, OUT,
                                MAXNAT, MAXNTYPE, NTYPE, ROB)
C
C
      INTEGER MAXNAT, MAXNTYPE, NTYPE
      CHARACTER*70 VNAME, CNAME, BLANK
      INTEGER RV(MAXNAT), A, OUT, TM(MAXNAT, MAXNTYPE), N
      CHARACTER*10 DASH(20), STAR(20)
      LOGICAL ROB

C
      BLANK='
      WRITE(OUT,1000) VNAME
      DO 100 NT=1, NTYPE
          STAR(NT)='*****'
          DASH(NT)='-----'
100      CONTINUE
      CNAME=BLANK(1:NTYPE*5)//'TARGET TYPE'
      WRITE(OUT,1001) 'O',(STAR(NT), NT-1, NTYPE)
      WRITE(OUT,1002) 'O', CNAME
      WRITE(OUT,1003) 'ATTACK SIZE', (I, I=1, NTYPE)
      WRITE(OUT,1004) ' ', (DASH(NT), NT-1, NTYPE)
      IF (ROB .EQ. .FALSE.) THEN
          DO 50 K= 1, A
              WRITE(OUT,1005) RV(K), (TM(K, NT), NT-1, NTYPE)
50          CONTINUE
      ELSE
          WRITE(OUT,1006) 'N/A', (TM(1, NT), NT-1, NTYPE)

      ENDIF
      WRITE(OUT,1004) ' ', (DASH(NT), NT-1, NTYPE)
      WRITE(OUT,1001) 'O',(STAR(NT), NT-1, NTYPE)

C
1000     FORMAT(A)
1001     FORMAT(A,16(' '),10A)
1002     FORMAT(A,10X,A)
1003     FORMAT(2X,A11,2X,' ',10(I4,6X))
1004     FORMAT(A,16(' '),10A)
1005     FORMAT(5X, I5, 5X,' ',10(I6,4X))
1006     FORMAT(6X, A3, 6X,' ',10(I6,4X))

C
      RETURN
      END

```

APPENDIX F
PROGRAM BATCH

PROGRAM BATCH

BATCH creates an input file that will be accepted by RPPDM

PARAMETER (MAXNTYPE-7, MAXR-30, MAXS-30)
 INTEGER R, S, MAXRV, MINRV, INCRV, INT, TARGETS
 INTEGER NTAR(MAXNTYPE)
 REAL VTYPE(MAXNTYPE)

.....
 R THE MAXIMUM NUMBER OF RV'S AT A SINGLE TARGET
 S THE MAXIMUM NUMBER OF INTERCEPTORS AT A SINGLE
 TARGET
 MAXRV THE MAXIMUM ATTACK SIZE
 MINRV THE MINIMUM ATTACK SIZE
 INCRV THE ATTACK SIZE INCREMENT
 INT THE NUMBER OF INTERCEPTORS
 TARGETS THE NUMBER OF TARGETS
 VTYPE THE RELATIVE VALUES OF THE TARGET TYPES
 NTAR THE NUMBER OF TARGETS FOR EACH TYPE

INTEGER NROBUST, MAXRRV, MINRRV

.....
 NROBUST THE NUMBER OF ROBUST DEFENSES DESIRED
 MAXRRV THE MAXIMUM ATTACK SIZE IN THE ROBUST RANGE
 MINRRV THE MINIMUM ATTACK SIZE IN THE ROBUST RANGE

INTEGER TER, OUT
 CHARACTER*12 FILEOUT, NAME, ANSWER*1

.....
 TER THE I/O UNIT NUMBER FOR THE TERMINAL
 OUT THE I/O UNIT NUMBER FOR THE OUTPUT DEVICE
 NAME A CHARACTER VARIABLE SENT BY THE USER
 ANSWER A CHARACTER VARIABLE SENT BY THE USER
 FILEOUT NAME OF THE OUTPUT DEVICE

Set TER to default I/O unit number for terminal

TER=5

Set OUT to TER+1

OUT=TER+1

WRITE(TER, '(A)') 'Please type in the desired file name (of 1e
 less than 10 characters'
 WRITE(TER, '(A)') ' including the extension) for storage of t
 he parameters ?'

```

READ(TER, '(A)') FILEOUT
OPEN(UNIT-OUT, FILE=FILEOUT, STATUS='NEW')

o
o
o
o
Select output option

WRITE(TER, '(A)') 'You have three options for the output of the
le results (with RPPDM): '
WRITE(TER, '(A)') '0      1) TERMINAL only'
WRITE(TER, '(A)') '      2) FILE only'
WRITE(TER, '(A)') '      3) TERMINAL and FILE'
WRITE(TER, '(A)') 'Please enter the number for the desired option ?'
READ(TER, FMT=*) NOUT
WRITE(OUT, FMT=*) NOUT

o
o
o
Ask for file name, if necessary

IF (NOUT .NE. 1) THEN
WRITE(TER, '(A)') 'Please type in the desired file name (only
if less than 10 characters'
WRITE(TER, '(A)') 'including the extension) ?'
READ(TER, '(A)') NAME
WRITE(OUT, FMT='(A12)') NAME
ENDIF

o
o
o
Input R and S

WRITE(UNIT-TER, FMT='(A, I2, A)') 'The MAXIMUM number of RV's
1 (up to 'MAXR') at a single target ? '
READ(UNIT-TER, FMT=*) R
WRITE(UNIT-OUT, FMT=*) R
WRITE(UNIT-TER, FMT='(A, I2, A)') 'The MAXIMUM number of INTERC
1EPTORS (up to 'MAXS') at a single target ? '
READ(UNIT-TER, FMT=*) S
WRITE(UNIT-OUT, FMT=*) S

o
o
o
Select attack methodology

WRITE(UNIT-TER, FMT='(A)') 'Select one of the following attack
lk methodologies: '
WRITE(UNIT-TER, FMT='(A)') '0      1) SIMULTANEOUS ATTACK'
WRITE(UNIT-TER, FMT='(A)') '      2) SEQUENTIAL ATTACK'
WRITE(UNIT-TER, FMT='(A)') 'Please input the number of the de
lsired attack ? '
READ(UNIT-TER, FMT=*) NATTYPE
WRITE(UNIT-OUT, FMT=*) NATTYPE

o
o
o
Input the failure rate for the RV's

WRITE(UNIT-TER, FMT='(A)') 'The FAILURE rate of the RV's ? '
READ(UNIT-TER, FMT=*) PPA
WRITE(UNIT-OUT, FMT=*) PPA

o

```

```

o   Select defense methodology
o
    WRITE(UNIT-TER, FMT-'(A)') 'OSelect one of the following defen
lse methodologies:
    WRITE(UNIT-TER, FMT-'(A)') 'O      1) ONE SHOT'
    WRITE(UNIT-TER, FMT-'(A)') '      2) SHOOT LOOK SHOOT'
    WRITE(UNIT-TER, FMT-'(A)') 'OPlease input the number for the d
lesired option ?
    READ(UNIT-TER, FMT-*) NDFTYPE
    WRITE(UNIT-OUT, FMT-*) NDFTYPE

o
o   If the the attack is sequential, find out whether the defender knows.
o   after the attack begins, the number of RV's slated for each target.
o
    IF (NATTTYPE .EQ. 2) THEN
        WRITE(UNIT-TER, FMT-'(A)') 'OIs the defender aware, after
lthe attack begins, of the number'
        WRITE(UNIT-TER, FMT-'(A)') ' of RV''s slated for each targ
let (Y or N) ?
        READ(UNIT-TER, FMT-'(A)') ANSWER
        WRITE(UNIT-OUT, FMT-'(A1)') ANSWER
        IF (ANSWER .EQ. 'Y') THEN
            IF (NDFTYPE .EQ. 1) THEN
                WRITE(UNIT-TER, FMT-'(A)') 'ONOTE: This scenario is
l equivalent to one with a simultaneous attack.'
            ENDIF
        ENDIF
    ENDIF

o
o   Find the failure rates for the interceptors
o
    IF (NDFTYPE .EQ. 1) THEN
        WRITE(UNIT-TER, FMT-'(A)') 'OThe FAILURE rate of the inter
lceptors ?
        READ(UNIT-TER, FMT-*) PFD
        WRITE(UNIT-OUT, FMT-*) PFD
    ELSE
        WRITE(UNIT-TER, FMT-'(A)') 'OThe FAILURE rate for the firs
lt salvo interceptors ?
        READ(UNIT-TER, FMT-*) PFD1
        WRITE(UNIT-OUT, FMT-*) PFD1
        WRITE(UNIT-TER, FMT-'(A)') 'OThe FAILURE rate for the seco
lnd salvo interceptors ?
        READ(UNIT-TER, FMT-*) PFD2
        WRITE(UNIT-OUT, FMT-*) PFD2
    ENDIF

o
o   Specific parameters of the game
o
    WRITE(TER, '(A)') 'Othe MINIMUM attack size ?'
    READ(TER, FMT-*) MINRV
    WRITE(UNIT-OUT, FMT-*) MINRV
    WRITE(TER, '(A)') 'Othe MAXIMUM attack size ?'
    READ(TER, FMT-*) MAXRV

```

```

WRITE(UNIT-OUT, FMT=*) MAXRV
WRITE(TER, '(A)') 'Othe attack size INCREMENT ?'
READ(TER, FMT=*) INCRV
WRITE(UNIT-OUT, FMT=*) INCRV
WRITE(TER, '(A)') 'Othe NUMBER of interceptors ?'
READ(TER, FMT=*) INT
WRITE(UNIT-OUT, FMT=*) INT
WRITE(TER, '(A)') 'Othe TOTAL NUMBER of targets ?'
READ(TER, FMT=*) TARGETS
WRITE(UNIT-OUT, FMT=*) TARGETS
WRITE(TER, '(A)') 'Othe number of TYPES of targets ?'
READ(TER, FMT=*) NTYPE
WRITE(UNIT-OUT, FMT=*) NTYPE

c
IF (NTYPE .NE. 1) THEN
    WRITE(TER, '(A)') 'OEnter first the RELATIVE VALUE and the
in the NUMBER of targets'
    WRITE(TER, '(A)') ' for each type. Separate the two entrie
ls for each target type with'
    WRITE(TER, '(A)') ' a comma and hit <CR> following the ent
tries for each target type: '
c
c
c
    Loop through each target type
    DO 100 I = 1, NTYPE
        READ(TER, FMT=*) VTYPE(I), NTAR(I)
        WRITE(UNIT-OUT, FMT=*) VTYPE(I), ', ', NTAR(I)
        TNT=TNT+NTAR(I)
100    CONTINUE
        IF (TNT .NE. TARGETS) THEN
            WRITE(TER, '(A)') 'Othe sum of the targets in the indi
lvidual target types does not'
            WRITE(TER, '(A)') ' equal the total number of targets'
            STOP
        ENDIF
    ENDIF
c
c
    WRITE(TER, '(A)') 'lPlease enter the number of different ranges
1 of RV's for which robust
    WRITE(TER, '(A)') ' solutions are to be found ?'
    WRITE(TER, '(A)') ' ***** Enter 0 if no robust solution
1 is desired*****
    READ(TER, *) NROBUST
    WRITE(UNIT-OUT, FMT=*) NROBUST
    IF (NROBUST .EQ. 0) THEN
        STOP
    ENDIF
    WRITE(TER, '(A)') 'Othe lower and upper bounds for the RV range
ls must be between'
    WRITE(TER, '(I5, ' and ', I5)') MINRV, MAXRV
c
    DO 200 I1=1, NROBUST
        WRITE (TER, '(A)') '0 The lower bound : '

```

```

      READ (TER,*) MINRRV
      WRITE(UNIT-OUT, FMT=*) MINRRV
      WRITE (TER, '(A)') '0'      The upper bound :
      READ (TER,*) MAXRRV
      WRITE(UNIT-OUT, FMT=*) MAXRRV
      WRITE(TER, '(//)')
      IF (I1 .NE. 1) THEN
        WRITE(TER, '(A)') ' NEXT'
      ENDIF
      WRITE(TER, '(//)')
200  CONTINUE
c
      STOP
      END

```

APPENDIX G

DERIVATION OF THE LP'S USED IN BG AND YROBUST

1. Basic Game

Let $\bar{K}$ be the set of all target types,
 T be the total number of targets,
 RV be the number of reentry vehicles attacking the targets, and
 INT be the defending interceptors.

Let R be the maximum number of RVs allowed to attack a single target and
 S be the maximum number of interceptors allowed to defend a single target.

Let $VF_k = V_k \cdot N_k / \sum_{k \in \bar{K}} (V_k \cdot N_k)$

and $NF_k = N_k / \sum_{k \in \bar{K}} N_k$

where V_k = the relative value of a type k target,

N_k = the number of targets of type k ,

and k is a target type in $\bar{K}$.

Define a strategy X (or Y) as the set of X^k (or Y^k) for all $k \in \bar{K}$ where

$$X^k = (X_0^k, \dots, X_R^k)$$

$$Y^k = (Y_0^k, \dots, Y_S^k)$$

and

X_i^k = fraction of type k targets assigned i RVs

Y_j^k = fraction of type k targets assigned j interceptors.

Let P be the matrix $[P_{ij}]$

and P_i the i th row of P ,

where an element P_{ij} is the probability that a target under attack by i RVs and defended by j interceptors will survive.

The formation of the minimax problem is as follows.

Let VBG = expected fraction of the total value $\sum_{k \in \bar{K}} (N_k \cdot V_k)$ that will survive.

$$VBG = \max_{Y^k \geq 0} \min_{X^k \geq 0} \sum_{k \in \bar{K}} (VF_k \cdot X^{k''} \cdot P \cdot Y^k)$$

$$\sum_{j=0}^S Y_j^k = 1, k \in \bar{K} \quad \sum_{i=0}^R X_i^k = 1, k \in \bar{K}$$

$$\sum_{k \in \bar{K}} \sum_{j=0}^S (j \cdot Y_j^k \cdot NF_k) = \frac{INT}{T} \quad \sum_{k \in \bar{K}} \sum_{i=0}^R (i \cdot X_i^k \cdot NF_k) = \frac{RV}{T},$$

where $X^{k''}$ is X^k transposed.

Taking the dual of the inside problem yields the following linear program.

LP1 is:

$$\begin{aligned}
 \text{VBG} = & \max_{Y_j^k \geq 0, s_k, t} \left[\sum_{k \in \bar{K}} s_k - \frac{RV}{T} \cdot t \right] \\
 \text{subject to} & \left. \begin{aligned}
 s_k & \leq VF_k \cdot P_0 \cdot Y^k \\
 s_k - NF_k \cdot t & \leq VF_k \cdot P_1 \cdot Y^k \\
 & \vdots \\
 & \vdots \\
 s_k - R \cdot NF_k \cdot t & \leq VF_k \cdot P_R \cdot Y^k \\
 \sum_{j=0}^S Y_j^k & = 1 \\
 \sum_{k \in \bar{K}} \sum_{j=0}^S j \cdot Y_j^k \cdot NF_k & = INT/T, \\
 Y_j^k & \geq 0, \text{ and } s_k \text{ and } t \text{ unrestricted.}
 \end{aligned} \right\} \text{for all } k \in \bar{K}
 \end{aligned}$$

The Y which yields VBG is the optimal minimax defender's strategy, Y*.

X*, the attacker's minimax strategy, is equivalent to the dual variables of the inequality constraints.

In addition s_k and t may be assumed to be positive. (See [1] on Page R-1.)

2. Robust Game

Assume that the attacker can always discover and thus optimize against Y. The defender wishes to find the robust strategy Y_{II} which will solve the following problem:

$$\max_{Y \in Y_p} \min_{A \in \bar{A}} \{R_A(X, Y)\}$$

where:

(1) Y_p is the set of all Y's such that

$$\begin{aligned}
 \sum_{j=0}^S Y_j^k & = 1 \text{ for all } k \in \bar{K} \\
 \sum_{k \in \bar{K}} \sum_{j=0}^S (j \cdot NF_k \cdot Y_j^k) & = \frac{INT}{T},
 \end{aligned}$$

(2) $\bar{A}$ is the set of possible attack sizes, and

$$(3) R_A(X, Y) = \frac{1}{VBG(A)} \min_{\substack{\sum_{k \in \bar{K}} (VF_k \cdot X^k \cdot P \cdot Y^k) \\ \text{the set of all } X^k = (X_0^k, \dots, X_R^k)}}$$

where

$$\sum_{i=0}^R X_i^k = 1 \text{ for all } k \in \bar{K} \text{ and}$$

$$\sum_{k \in \bar{K}} \sum_{i=0}^R (i \cdot NF_k \cdot X_i^k) = \frac{RV(A)}{T}.$$

Taking the dual,

$$R_A(X, Y) = \frac{1}{VBG(A)} \max_{Y_j^k \geq 0, s_k(A), t(A)} \left[\sum_{k \in \bar{K}} s_k(A) - \frac{RV(A)}{T} t(A) \right]$$

subject to

$$\left. \begin{aligned} s_k(A) &\leq VF_k \cdot P_0 \cdot Y_k \\ s_k(A) - R \cdot NF_k \cdot t(A) &\leq VF_k \cdot P_1 \cdot Y^k \\ &\vdots \\ s_k(A) - R \cdot NF_k \cdot t(A) &\leq VF_k \cdot P_R \cdot Y^k \end{aligned} \right\} \text{ for all } k \in \bar{K}.$$

Inserting this back into the original problem we have:

$$\max_{Y \in Y_P} \min_{A \in \bar{A}} \left[\frac{1}{VBG(A)} \max_{s_k(A), t(A)} \sum_{k \in \bar{K}} s_k(A) - \frac{RV(A)}{T} t(A) \right]$$

which yields the following linear program.

LP2 is:

$$\begin{array}{ll}
 \max & \rho \\
 Y_j^k \geq 0, s_k(A), t(A) & \\
 \text{subject to} & \\
 \text{VBG}(A) \quad \rho \leq \sum_{k \in \bar{K}} s_k(A) - \frac{RV(A)}{T} t(A) & \\
 s_k(A) \leq VF_k \cdot P_0 \cdot Y^k & \\
 s_k(A) - NF \cdot t(A) \leq VF_k \cdot P_1 \cdot Y^k & \\
 \vdots & \\
 s_k(A) - R \cdot NF_k \cdot t(A) \leq VF_k \cdot P_R \cdot Y^k &
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} k \in \bar{K} \left. \vphantom{\begin{array}{l} \\ \\ \\ \\ \\ \end{array}} \right\} A \in \bar{A}.$$

Once more we can assume $s_k(A)$ and $t(A)$ to be nonnegative (see [1] on Page R-1).

(Substituting $Z_k(i)$ for $VF_k \cdot P_i \cdot Y^k$ yields the LP used in YROBUST.)

By assumption, the attacker "knows" Y_{II} , the Y which solves LP2. Thus we can find X_{II} , the optimal attack against Y_{II} , and V_{II} , the expected survival rate for any attack size A , by solving the following linear program:

LP3 is:

$$\begin{array}{ll}
 V_{II}(A) = \min_{X^k \geq 0} & \sum_{k \in \bar{K}} (VF_k \cdot X^k \cdot P \cdot Y_{II}^k) \\
 \text{subject to} & \sum_{i=0}^R X_i^k = 1 \text{ for all } k \in \bar{K}, \\
 \sum_{k \in \bar{K}} \sum_{i=0}^R i \cdot X_i^k \cdot NF_k = \frac{RV(A)}{T}. &
 \end{array}$$

APPENDIX H
EQUATIONS USED IN SIMAT1, SIMAT2, SEQAT1, AND SEQAT2

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1. Equation for SIMAT1--Simultaneous Attack with One Opportunity to Shoot¹

We assume that a single target is under a simultaneous attack by A identical missiles, and is being defended by D identical interceptors. Let

d = probability that a defending interceptor will destroy the attacking at which it is directed,

a = probability that an attacking missile will destroy the target, given that it evades all defending interceptors.

We assume that the defense can see the entire attack, and must decide on the number of interceptors that it assigns to each of the attacking missiles.

Given suitable independence assumptions, the probability of the target surviving an attack of n_j attacking missiles, each of which are being attacked by j defending interceptors, is

$$(1 - a(1 - d)^j)^{n_j}. \quad (1)$$

Thus, if the target is attacked by A missiles and, for each j between 0 and D, the defender assigns a total of $j n_j$ interceptors against n_j missiles, such that j interceptors are assigned against each of these n_j missiles, where

$$\sum_{j=0}^D n_j = A$$

and

$$\sum_{j=0}^D j n_j = D,$$

then the probability that the target survives is

$$\prod_{j=0}^D (1 - a(1 - d)^j)^{n_j}. \quad (2)$$

¹Source: Appendix A of Reference [1] on page R-1.

The defender wishes to select the n_j in order to maximize this probability.

We wish to show that the "uniform defense" obtained by spreading the D interceptors as equally as possible over the A attackers is optimal.

Consider an allocation of interceptors to attackers which is not uniform. Then there is a pair $i < j$ with $n_i, n_j > 0$ where $i + 2 \leq j$. Consider a new (and more uniform) allocation obtained by allocating $i + 1$ interceptors to one of the n_i attackers, and $j - 1$ interceptors to one of the n_j attackers. The probability that the target now survives is

$$\frac{(1 - a(1 - d)^{i+1})(1 - a(1 - d)^{j-1})}{(1 - a(1 - d)^i)(1 - a(1 - d)^j)}$$

times the old probability, and this is easily shown to be greater than 1.

The most uniform of defenses assigns

$$i = [D/A] \text{ defenders to } n_i = A(1 - \langle D/A \rangle) \text{ attackers}$$

and

$$j = [D/A] + 1 \text{ defenders to } n_j = A \langle D/A \rangle \text{ attackers}$$

(where $[x]$ and $\langle x \rangle$ denote the integer and fractional parts of x). Thus, the optimal defense is:

$$n_j = \begin{cases} A(1 - \langle D/A \rangle) & \text{for } j = [D/A] \\ A \langle D/A \rangle & \text{for } j = [D/A] + 1 \\ 0 & \text{otherwise.} \end{cases}$$

Substituting these values for n_j into (2) yields

$$P(A, D) = (1 - a(1 - d)^{[D/A]+1})^A \langle D/A \rangle (1 - a(1 - d)^{[D/A]})^A (1 - \langle D/A \rangle),$$

where $P(A, D)$ is the probability the target survives given that it is attacked by A missiles and defended by D interceptors that are allocated against these attacking missiles according to this uniform defense. Table 1 gives the numerical values of $P(A, D)$ for four examples.

Table 1. VALUES OF P(A,D) (SIMULTANEOUS ATTACK)

a = 7 d = 7

A \ D	0	1	2	3	4	5	6	7	8	9	10
0	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
1	3000	7500	9370	9811	9943	9983	9995	9998	1 0000	1 0000	1 0000
2	0900	2370	6241	7402	8780	9193	9626	9755	9887	9926	9966
3	0270	0711	1872	4930	5848	6936	8227	8614	9019	9444	9571
4	0081	0213	0562	1479	3895	4620	5479	6499	7708	8071	8451
5	0024	0064	0169	0444	1169	3077	3650	4329	5134	6090	7223
6	0007	0019	0051	0133	0351	0923	2431	2883	3420	4056	4811
7	0002	0006	0015	0040	0105	0277	0729	1920	2278	2702	3204
8	0001	0002	0005	0012	0032	0083	0219	0576	1517	1799	2134
9	0000	0001	0001	0004	0009	0025	0066	0173	0455	1199	1422
10	0000	0000	0000	0001	0003	0007	0020	0052	0137	0360	0947

a = 7, d = 9

A \ D	0	1	2	3	4	5	6	7	8	9	10
0	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
1	3000	9300	9930	9993	9999	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
2	0900	2790	8649	9235	9860	9923	9986	9992	9999	9999	1 0000
3	0270	0837	2595	8044	8588	9170	9791	9854	9916	9979	9985
4	0081	0251	0778	2413	7481	7987	8528	9106	9723	9785	9847
5	0024	0075	0234	0724	2244	6957	7428	7931	8469	9042	9655
6	0007	0023	0070	0217	0673	2087	6470	6908	7376	7876	8409
7	0002	0007	0021	0065	0202	0626	1941	6017	6425	6860	7325
8	0001	0002	0006	0020	0061	0188	0582	1805	5596	5975	6380
9	0000	0001	0002	0006	0018	0056	0175	0542	1679	5204	5557
10	0000	0000	0001	0002	0005	0017	0052	0162	0504	1561	4840

a = 9, d = 7

A \ D	0	1	2	3	4	5	6	7	8	9	10
0	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
1	1000	7300	9190	9757	9927	9978	9993	9998	9999	1 0000	1 0000
2	0100	0730	5329	6709	8446	8967	9520	9686	9855	9905	9956
3	0010	0073	0533	3890	4897	6185	7762	8240	8749	9289	9451
4	0001	0007	0053	0389	2840	3575	4501	5666	7133	7573	8040
5	0000	0001	0005	0039	0284	2073	2610	3285	4136	5207	6555
6	0000	0000	0001	0004	0028	0207	1513	1905	2398	3019	3801
7	0000	0000	0000	0000	0003	0021	0151	105	1391	1751	2204
8	0000	0000	0000	0000	0000	0002	0015	0110	0806	1015	1278
9	0000	0000	0000	0000	0000	0000	0002	0011	0081	0589	0741
10	0000	0000	0000	0000	0000	0000	0000	0001	0008	0059	0430

a = 9, d = 9

A \ D	0	1	2	3	4	5	6	7	8	9	10
0	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
1	1000	9100	9910	9991	9999	1 0000	1 0000	1 0000	1 0000	1 0000	1 0000
2	0100	0910	8281	9018	9821	9901	9982	9990	9998	9999	1 0000
3	0010	0091	0828	7536	8206	8937	9732	9812	9892	9973	9981
4	0001	0009	0083	0754	6857	7488	8133	8857	9645	9724	9803
5	0000	0001	0008	0075	0686	6240	6796	7401	8059	8777	9558
6	0000	0000	0001	0008	0069	0624	5679	6184	6735	7334	7987
7	0000	0000	0000	0001	0007	0062	0568	5168	5628	6129	6674
8	0000	0000	0000	0000	0001	0008	0057	0517	4703	5121	5577
9	0000	0000	0000	0000	0000	0001	0006	0052	0470	4279	4660
10	0000	0000	0000	0000	0000	0000	0001	0005	0047	0428	3894

2. Equation for SIMAT2--Simultaneous Attack with Two Opportunities to Shoot

The following material is extracted from Richard M. Soland, "Optimal Terminal Defense Tactics Against Simultaneously Arriving RVs When Several Sequential Engagements are Possible," TR-85/12, Institute for Reliability and Risk Analysis, School of Engineering and Applied Science, The George Washington University, 18 October 1985. References for this section are given on page H-18.

a. INTRODUCTION

An important and frequent element of ballistic missile defense models is the presence and role of a terminal defense that defends a single target, either an area target such as a city or a point target such as an ICBM silo. Given that such a terminal defense consists of a given number of interceptors, the question immediately arises as to the tactics to be employed by the defense in using those interceptors and how the expected damage to the target varies as a function of the number of attacking reentry vehicles (RVs).

The RVs may be assumed to arrive sequentially, so that the defense never knows how many are coming, or else simultaneously, so that full knowledge of the attack size is obtained. Intermediate cases may be more realistic of course. The defense may have more than one opportunity to engage some or all of the RVs, and it may receive information on the results of some engagements before undertaking subsequent engagements. Here we treat the case in which RVs are assumed to arrive simultaneously and a fixed number of engagements of each RV are possible, with shoot-look-shoot capability between them.

The scenario we consider is as follows: a single target is

attacked simultaneously by A reentry vehicles; it is defended by a terminal defense consisting of D interceptors. The RVs act independently, and the expected fraction of the target destroyed by each unintercepted RV is ρ , where $0 < \rho \leq 1$. The defense may engage each RV up to K times, with one or more interceptors used at each engagement, and it has shoot-look-shoot capability between successive engagements. That is, the defense observes whether or not a particular RV survives a particular engagement before deciding how many interceptors, if any, to use against it at the next engagement. All interceptors are assumed to operate independently, even when used simultaneously against the same RV. The single-shot kill probability of one interceptor against one RV may vary with the number of the engagement in the sequence of K engagements.

The objective of the defense is to minimize the expected fraction of the target destroyed. A policy for the defense is a rule that specifies for each engagement, how many interceptors to use against each of the RVs that has not been destroyed at previous engagements. We desire to find an optimal policy as a function of A, D, and K.

Chapter 3 of Eckler and Burr (1972) also deals with the tactics to be employed by a terminal defense and presents several different models, each based on specific assumptions. One model, that of section 3.5.1, is a special case of ours; it corresponds to $K = 2$ and $\rho = 1$. Some typical results are presented, but details of the methodology are omitted.

Burr et al. (1985) and Falk (1985b) treat the case in which RVs arrive sequentially but only one engagement of each RV is possible, and Falk (1985a) extends such analyses to the case of several engagements

and shoot-look-shoot capability between them. In these three papers the optimality criterion is minimization of the number of interceptors D required to guarantee that the expected fraction of the target destroyed, as a function of A , lies below a given bounding function. Our criterion is minimization of the expected fraction of the target destroyed; as we show, however, with a little additional work we can also treat this other criterion.

A brief outline of the paper is as follows. In the next section we define notation and point out the intuitive result (which is proved in the Appendix) that at each engagement the interceptors used should be spread as uniformly as possible among the attacking RVs. Using this result, we present a dynamic programming algorithm that determines optimal policies. The following section presents several extensions of the basic model; two of them deal, respectively, with the expected number of interceptors remaining after the attack and determination of the minimum number of interceptors needed to provide a desired level of protection of the target. The final section contains illustrative numerical examples.

b. ANALYSIS

It is convenient to number the engagements in reverse chronological order, so we shall refer generally to there being k engagements remaining before the end of the attack. Here $k = 0, 1, \dots, K$, where $k = 0$ indicates that no further engagements are possible. For $k = 1, \dots, K$, let p_k be the single-shot kill probability of one interceptor against one RV on the k th

engagement before the end, and let $q_k = 1 - p_k$. We assume that each q_k satisfies $0 < q_k < 1$; the contrary cases are not of interest.

We define

$S(a,d,k)$ = the expected fraction of the target destroyed
if k engagements remain, the defense has d
interceptors left, there are a RVs left
undestroyed, and the defense follows an
optimal policy for the remaining k engagements.

$S(a,d,k)$ is defined for $a=0,1,\dots,A$; $d=0,1,\dots,D$; $k=0,1,\dots,K$. Now
define, for $j=0,1,\dots,a$; $a=0,1,\dots,A$; $i=0,1,\dots,d$; $d=0,1,\dots,D$; $k=1,\dots,K$,

$P(j|a,i,d,k)$ = the probability that j RVs survive the
 k th engagement from the end when there are
 a RVs left before that engagement and i
of the d remaining interceptors are used
in an optimal manner at that engagement.

The principle of optimality of dynamic programming now allows us to
write the following recursive equation which is valid for $a=1,\dots,A$;
 $d=0,1,\dots,D$; $k=1,\dots,K$:

$$S(a,d,k) = \text{Min}_{i=0,\dots,d} \left\{ \sum_{j=0}^a P(j|a,i,d,k) S(j,d-i,k-1) \right\}. \quad (1)$$

In order to use this recursion to calculate $S(a,d,k)$, we need appropriate
boundary conditions in the form of $S(0,d,k)$ and $S(a,d,0)$. These are
clearly as follows:

$$S(0,d,k) = 0 \quad \text{for } d=0,1,\dots,D; k=1,\dots,K,$$

$$S(a,d,0) = 1 - (1-p)^a \quad \text{for } a=0,1,\dots,A; d=0,1,\dots,D.$$

It remains to provide the $P(j|a,i,d,k)$ before the above recursion can be used. This is rendered relatively straightforward by the following result, whose proof is provided in the Appendix: an optimal manner in which to use i of d interceptors against a RVs is to spread them as uniformly as possible among the a RVs. Thus if i/a is an integer, say I , each of the a RVs is assigned I interceptors and has survival probability q_k^I . The number of RVs that survive the engagement thus has a binomial distribution, so

$$P(j|a,i,d,k) = \binom{a}{j} q_k^{jI} (1-q_k^I)^{a-j}.$$

More generally, i/a is not an integer. Then it is easily shown that $a + a[i/a] - i$ of the a RVs are assigned $[i/a]$ interceptors each and the remaining $i - a[i/a]$ RVs are assigned $\lceil i/a \rceil$ interceptors each (here $[x]$ is the greatest integer less than or equal to x and $\lceil x \rceil$ is the smallest integer greater than or equal to x). From this it follows that the number of RVs that survive the engagement is the sum of two independent binomial random variables with slightly different success probabilities. The needed probabilities $P(j|a,i,d,k)$ may then be obtained by numerically convoluting the two binomial distributions in question.

An alternative way to obtain the needed probabilities, the one actually used for our computational results, is as follows. Let J be the random variable whose probability distribution is $P(j|a,i,d,k)$.

Then J is the sum of a independent Bernoulli random variables, each being equal to one if a specific one of the a RVs survives the engagement, and equal to zero otherwise. We may thus write $J = \sum_{\ell=1}^a X_{\ell}$, where $P(X_{\ell} = 1) = r_{\ell}$ and $P(X_{\ell} = 0) = 1 - r_{\ell}$. The r_{ℓ} are given by $r_{\ell} = q_k^{\alpha}$ for $\ell=1, \dots, a + a\lfloor i/a \rfloor - i$ and $r_{\ell} = q_k^{\beta}$ for $\ell = a + a\lfloor i/a \rfloor - i + 1, \dots, a$, where $\alpha = \lfloor i/a \rfloor$ and $\beta = \lceil i/a \rceil$. By defining the random variables $J_s \equiv \sum_{\ell=1}^s X_{\ell}$ for $s = 1, \dots, a$, so that $J = J_a$, we can calculate the probability distributions of the J_s successively from the recursion (which is valid for $j=1, \dots, s$ and $s=2, \dots, a$)

$$P(J_s = j) = (1-r_s) P(J_{s-1}=j) + r_s P(J_{s-1}=j-1)$$

along with the boundary conditions

$$P(J_s = 0) = \prod_{\ell=1}^s (1-r_{\ell}), \quad P(J_1=1) = r_1.$$

A closed-form expression for $S(a,d,1)$ is sometimes useful, and it is easily obtained from the uniform-defense property and fairly simple probabilistic analysis; the result is

$$S(a,d,1) = 1 - (1 - \rho q_1^{\alpha})^{a+a\alpha-d} (1-\rho q_1^{\beta})^{d-a\alpha},$$

where $\alpha = \lfloor d/a \rfloor$ and $\beta = \lceil d/a \rceil$.

C. EXTENSIONS OF THE BASIC MODEL

In anticipation of the possibility of another attack after the present one is over, the terminal defense may be interested in, as a secondary criterion, the expected number of interceptors remaining to

it at the end of the current attack. We thus define

$T(a,d,k)$ = the expected number of interceptors left after the attack if k engagements remain, the defense presently has d interceptors left, there are a RVs left undestroyed, and the defense follows an optimal policy for the remaining k engagements.

Like $S(a,d,k)$, $T(a,d,k)$ is defined for $a=0,1,\dots,A$; $d=0,1,\dots,D$; $k=0,1,\dots,K$. We can calculate $T(a,d,k)$ along with $S(a,d,k)$ as follows:

Let i^* be a minimizing value of i in the recursion (1) used to determine $S(a,d,k)$. Then

$$T(a,d,k) = \sum_{j=0}^a P(j|a,i^*,d,k) T(j,d-i^*,k-1), \quad (2)$$

for $a=1,\dots,A$; $d=0,1,\dots,D$; $k=1,\dots,K$. The necessary boundary conditions are

$$T(0,d,k) = d \quad \text{for } d=0,1,\dots,D; k=0,1,\dots,K,$$

$$T(a,d,0) = d \quad \text{for } a=1,\dots,A; d=0,1,\dots,D.$$

The recursion (2) does not serve to uniquely determine the $T(a,d,k)$ unless the minimizing i^* in (1) are unique. But the index i^* is not necessarily unique, so we adopt the following convention, which then serves to determine a unique value for $T(a,d,k)$. If i^* in (1) is not unique, use the smallest value of i^* that maximizes $T(a,d,k)$ as computed by (2).

The boundary condition $S(a,d,0) = 1 - (1-\rho)^a$ given above was based on the assumption of RVs that act independently and, if unintercepted,

each destroy an expected fraction ρ of the target. The dynamic programming scheme given works equally well with the more general boundary condition $S(a,d,0) = g(a)$ for $a=0,1,\dots,A$; $d=0,1,\dots,D$, where g is nondecreasing, $g(0) = 0$, and $g(A) \leq 1$. This allows use of an arbitrary relationship between the number of unintercepted RVs and the expected fraction of the target destroyed.

In damage-limitation studies it is sometimes of interest to determine the smallest number of interceptors needed by the defense to provide a desired level of protection of the target. One way to interpret the phrase "desired level of protection" is by specifying a nondecreasing maximum damage function f and requiring that the expected fraction of the target destroyed by a RVs not exceed $f(a)$ for $a=1,\dots,A$. For example, see Burr et al. (1985) and Falk (1985b) for analyses of this problem when the RVs are assumed to arrive sequentially and the defense has no shoot-look-shoot capability, and see Falk (1985a) for an extension to the case in which the defense does have shoot-look-shoot capability. In the present context, and for fixed A and K , we may phrase the problem as

$$\begin{aligned} &\text{Minimize } D \\ &\text{subject to } S(a,D,K) \leq f(a), \quad a=1,\dots,A. \end{aligned}$$

We can solve this problem easily by continuing the computations of the dynamic programming scheme for successively larger values of D until one is found that satisfies all the indicated constraints.

d. EXAMPLES

For the case $\rho = 0.7$, $p_1 = 0.8$ and $p_2 = 0.9$, Table 1 gives $S(A,D,2)$, i^* , and $T(A,D,2)$ for $A,D = 1(1)10$; the three quantities appear in respective rows. For example, with $A = 7$ RVs, $D = 9$ interceptors and 2 engagements left, the defense should use $i^* = 7$ interceptors and hold 2 in reserve for possible use at the last engagement. The expected fraction of the target destroyed is $S(7,9,2) = 0.0631$ and the expected number of interceptors left after the attack is $T(7,9,2) = 0.957$.

Figure 1 shows the expected fractional damage $S(A,D,2)$ as a function of A for $D = 2(2)10$.

Figure 2 shows solutions of the damage limitation problem minimize D subject to $S(a,D,K) \leq f(a)$, $a=1,\dots,10$, for the linear function $f(a) = sa$. Solutions are given for a range of values of the slope s and for both $K = 1$ and $K = 2$; the other parameters of the problem are $\rho = 0.9$ and $p_1 = p_2 = 0.7$. For example, if it is desired to have a defense whose expected fractional damage per attacking RV is limited to $s = 0.125$, then 9 interceptors are needed if only one engagement is possible ($K = 1$), but only 8 interceptors are required if two engagements are possible ($K = 2$).

TABLE 1

Results for 2 Engagements when
 $p_1 = 0.8$, $p_2 = 0.9$ and $\rho = 0.7$

TABLE OF EXPECTED FRACTION OF TARGET DESTROYED, OPTIMAL NUMBER OF
 INTERCEPTORS TO USE, AND EXPECTED NUMBER OF INTERCEPTORS LEFT OVER

	1=A	2=A	3=A	4=A	5=A	6=A	7=A	8=A	9=A	10=A
D= 1	0.0700	0.7210	0.9163	0.9749	0.9925	0.9977	0.9993	0.9998	0.9999	1.0000
D= 1	1	1	1	1	1	1	1	1	1	1
D= 1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D= 2	0.0070	0.1351	0.7405	0.9222	0.9766	0.9930	0.9979	0.9994	0.9998	0.9999
D= 2	2	2	2	2	2	2	2	2	2	2
D= 2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D= 3	0.0007	0.0326	0.1956	0.7587	0.9276	0.9783	0.9935	0.9960	0.9994	0.9998
D= 3	3	2	3	3	3	3	3	3	3	3
D= 3	0.000	0.810	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D= 4	0.0001	0.0076	0.0550	0.2519	0.7756	0.9327	0.9798	0.9939	0.9982	0.9995
D= 4	4	2	3	4	4	4	4	4	4	4
D= 4	0.000	1.620	0.729	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D= 5	0.0000	0.0026	0.0146	0.0803	0.3043	0.7913	0.9374	0.9812	0.9944	0.9983
D= 5	5	2	3	4	5	5	5	5	5	5
D= 5	0.000	2.430	1.458	0.656	0.000	0.000	0.000	0.000	0.000	0.000
D= 6	0.0000	0.0006	0.0062	0.0237	0.1079	0.3530	0.8059	0.9418	0.9825	0.9948
D= 6	6	4	3	4	5	6	6	6	6	6
D= 6	0.000	1.960	2.197	1.312	0.590	0.000	0.000	0.000	0.000	0.000
D= 7	0.0000	0.0001	0.0020	0.0110	0.0349	0.1373	0.3983	0.8195	0.9458	0.9838
D= 7	7	4	3	4	5	6	7	7	7	7
D= 7	0.000	2.940	2.916	1.968	1.181	0.531	0.000	0.000	0.000	0.000
D= 8	0.0000	0.0000	0.0009	0.0041	0.0171	0.0481	0.1679	0.4404	0.8321	0.9496
D= 8	8	4	4	4	5	6	7	8	8	8
D= 8	0.000	3.920	3.208	2.624	1.771	1.063	0.478	0.000	0.000	0.000
D= 9	0.0000	0.0000	0.0002	0.0024	0.0069	0.0245	0.0631	0.1994	0.4796	0.8439
D= 9	9	4	6	4	5	6	7	8	9	9
D= 9	0.000	4.901	2.911	3.280	2.362	1.594	0.957	0.430	0.000	0.000
D=10	0.0000	0.0000	0.0000	0.0009	0.0042	0.0105	0.0330	0.0799	0.2313	0.5160
D=10	10	4	6	4	5	6	7	8	9	10
D=10	0.000	5.881	3.881	3.937	2.952	2.126	1.435	0.861	0.387	0.000

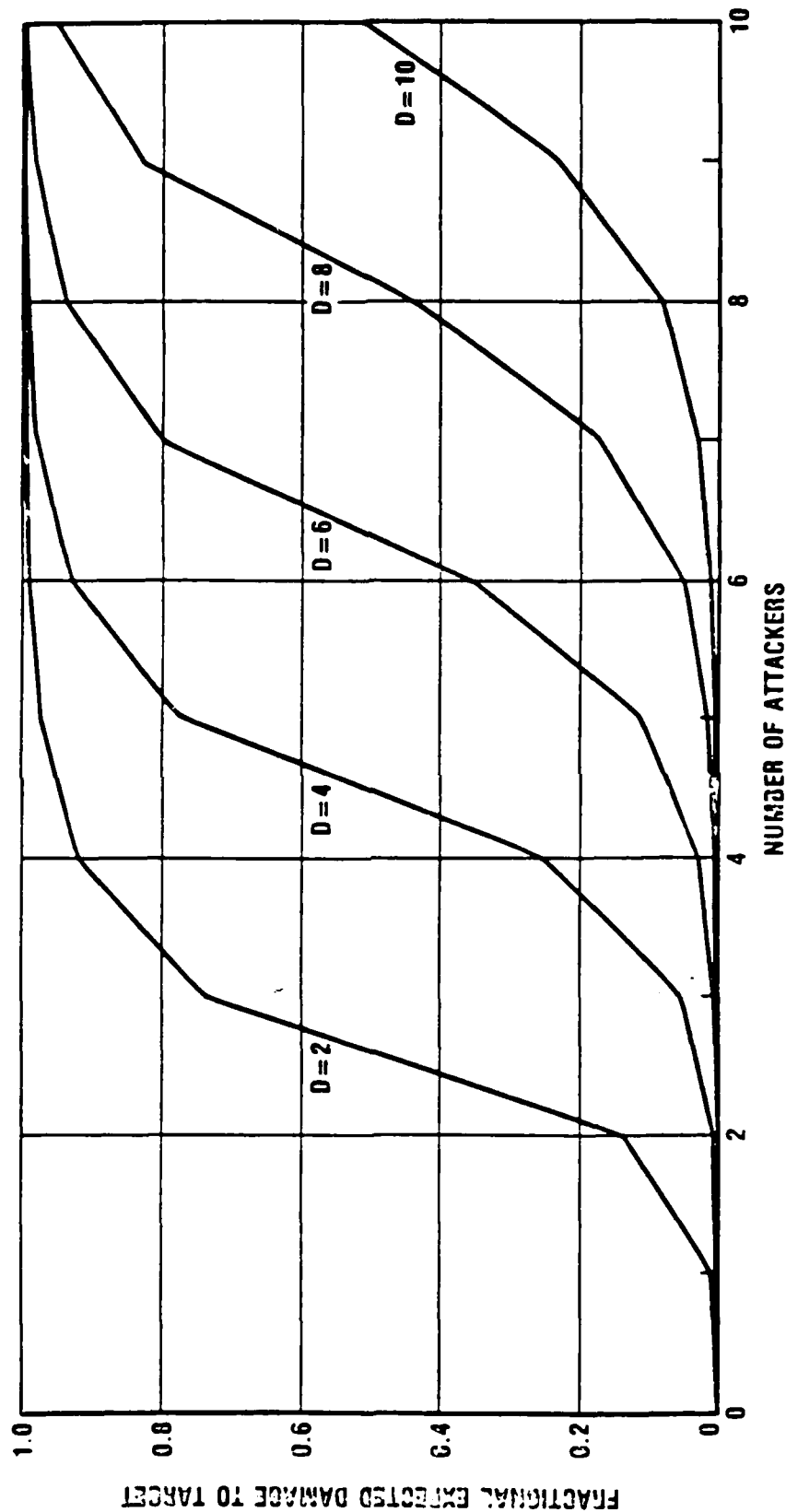


Figure 1. Expected Damage Curves for Various Defense Levels D

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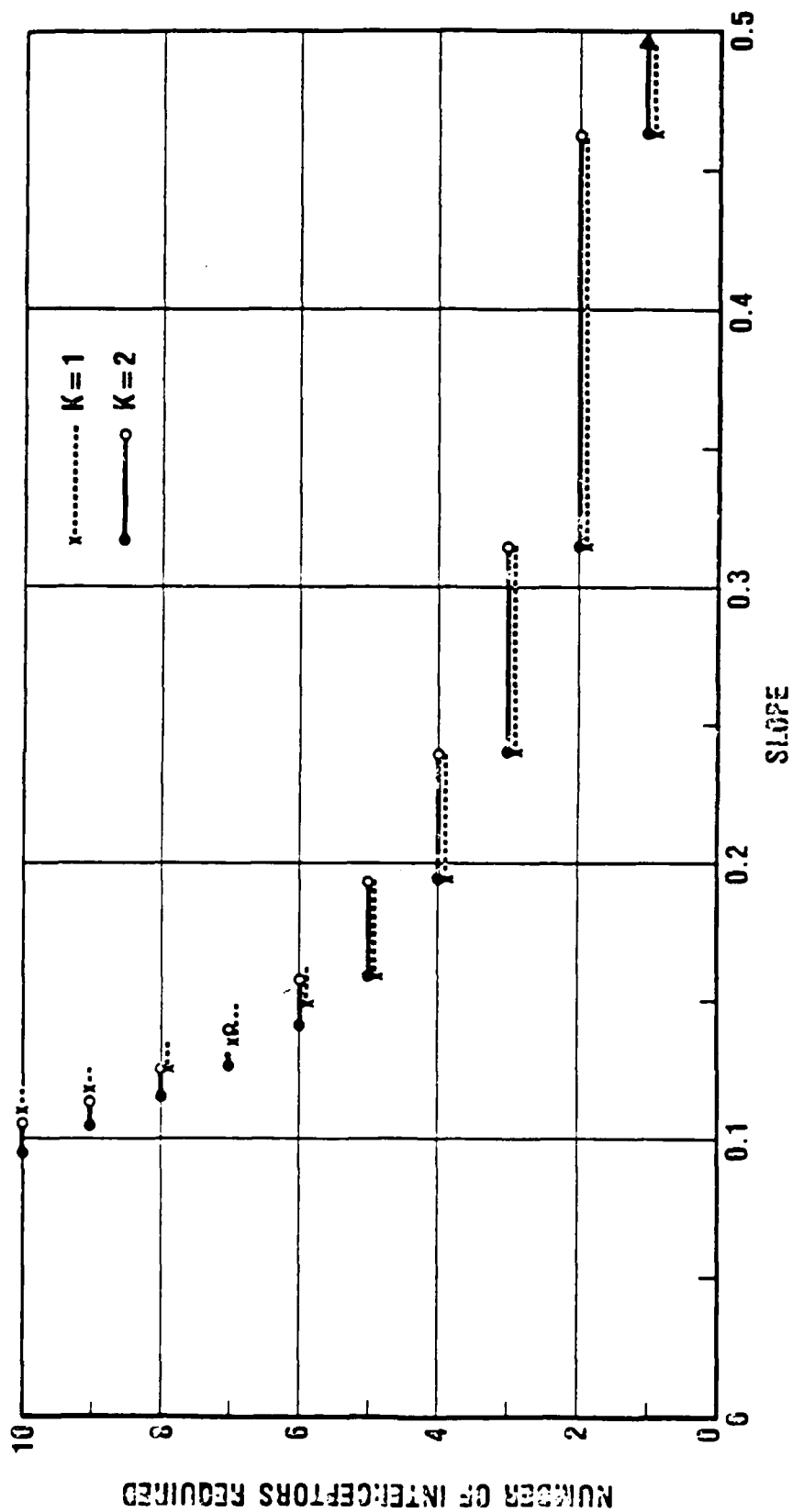


Figure 2. Number of Interceptors Required for a Given Slope

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ANNEX

Here we provide a verification of the intuitive result that, regardless of the number of engagements remaining, an optimal manner in which to use i of d interceptors against a RVs is to spread them as uniformly as possible among the a RVs, i.e., use a uniform defense.

Suppose there are $k \geq 1$ engagements remaining and suppose there is an optimal way to use i of d interceptors against the a RVs that is not uniform, i.e., there are two RVs such that one of them is assigned at least 2 more interceptors than the other. We will show that the expected damage done to the target is not increased if one interceptor is switched from the heavily attacked RV to the lightly attacked RV. By repetition of this step a finite number of times it then follows that a uniform defense is optimal.

Let $u(\ell)$ be the number of interceptors assigned to RV ℓ ($\ell=1, \dots, a$), so that $\sum_{\ell} u(\ell) = i$. Without loss of generality, we may assume $u(1) \geq u(2) + 2$. Define random variables $X_1, \dots, X_a$ as $X_{\ell} = 1$ if RV ℓ survives its attack by $u(\ell)$ interceptors and $X_{\ell} = 0$ otherwise.

Let $X_{12} = X_1 + X_2$. Then $J \equiv X_{12} + \sum_{\ell=3}^a X_{\ell}$ is the number of RVs that survive the current engagement and $E[S(J, d-i, k-1)]$ is the expected fractional damage done to the target.

Now consider the alternative defense defined by $u'(1) = u(1) - 1$, $u'(2) = u(2) + 1$, and $u'(\ell) = u(\ell)$ for $\ell > 2$. Define the random variables $X'_1, \dots, X'_a$ in the same manner as before, and let $X'_{12} = X'_1 + X'_2$. Then $J' \equiv X'_{12} + \sum_{\ell=3}^a X'_{\ell}$ and $E[S(J', d-i, k-1)]$ are interpreted as above, but

for the alternative defense instead. $X'_{12}, X'_3, \dots, X'_a$ are mutually independent, as are $X_{12}, X_3, \dots, X_a$.

We now show that $X'_{12} \stackrel{st}{\leq} X_{12}$, i.e., X'_{12} is stochastically less than X_{12} [see Barlow and Proschan (1975), p. 110]. X_{12} and X'_{12} have possible values 0, 1 and 2, and

$$P(X_{12}=0) = (1-q_k^{u(1)}) (1-q_k^{u(2)}) = 1 - q_k^{u(1)} - q_k^{u(2)} + q_k^{u(1)+u(2)},$$

$$P(X_{12}=1) = q_k^{u(1)} + q_k^{u(2)} - 2 q_k^{u(1)+u(2)},$$

$$P(X_{12}=2) = q_k^{u(1)+u(2)}.$$

Corresponding expressions hold for the probability distribution of X'_{12} , with $u'(1) = u(1)-1$ and $u'(2) = u(2) + 1$ substituted. Since $u(1) + u(2) = u'(1) + u'(2)$, it follows that $P(X_{12} > 1) = P(X'_{12} > 1)$. Simple algebra yields

$$P(X'_{12} > 0) - P(X_{12} > 0) = q_k^{u(2)} (q_k - 1) (1 - q_k^{u(1)-u(2)-1}) < 0,$$

and this suffices to show that $X'_{12} \stackrel{st}{\leq} X_{12}$. Since $X'_\ell \stackrel{st}{=} X_\ell$ for $\ell > 2$, it follows from a simple extension of exercise 1 on page 176 of Barlow and Proschan (1975) that $J' \stackrel{st}{\leq} J$. Since $S(j, d-i, k-1)$, by virtue of its definition, is a nondecreasing function of j for fixed values of the other arguments, it follows from exercise 15 on page 151 of Barlow and Proschan (1975) that $E[S(J', d-i, k-1)] \leq E[S(J, d-i, k-1)]$. Thus the alternative defense does not increase the expected damage done to the target, and our verification is now complete.

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3. Equation for SEQAT1--Sequential Attack of Unknown Size with One or Two Opportunities to Shoot¹

There is a single target that may come under an attack by an unknown number of sequentially arriving RV's.

We are protecting this target with D interceptors, and we have sufficient time to observe the results of a first volley against an RV, and fire a second volley, if necessary.

Let

- q = probability that a defender targeted against an RV will miss in the first volley,
- r = probability that a defender targeted against a surviving RV will miss in the second volley,
- s = probability that an RV surviving both volleys will fail to destroy the target, and
- $p(A,D)$ = probability that the target protected by D defenders is destroyed by A weapons, given some firing doctrine.

Figure 1 illustrates a pair of possibilities when $D=1$. Here $(q,r,s) = (0.1, 0.2, 0.3)$, and the results of 3 firing doctrines are displayed. The doctrines are

- a) Don't fire at any RV (unprotected case $\equiv D=0$),
- b) Fire one defender in the first volley against the first arriving RV.
- c) Fire one defender in the second volley (none in the first) at the first arriving RV.

¹Source: James E. Falk, "Prim-Read Solutions with Shoot-Look-Shoot", unpublished memorandum, Institute for Defense Analyses, 31 July 1985.

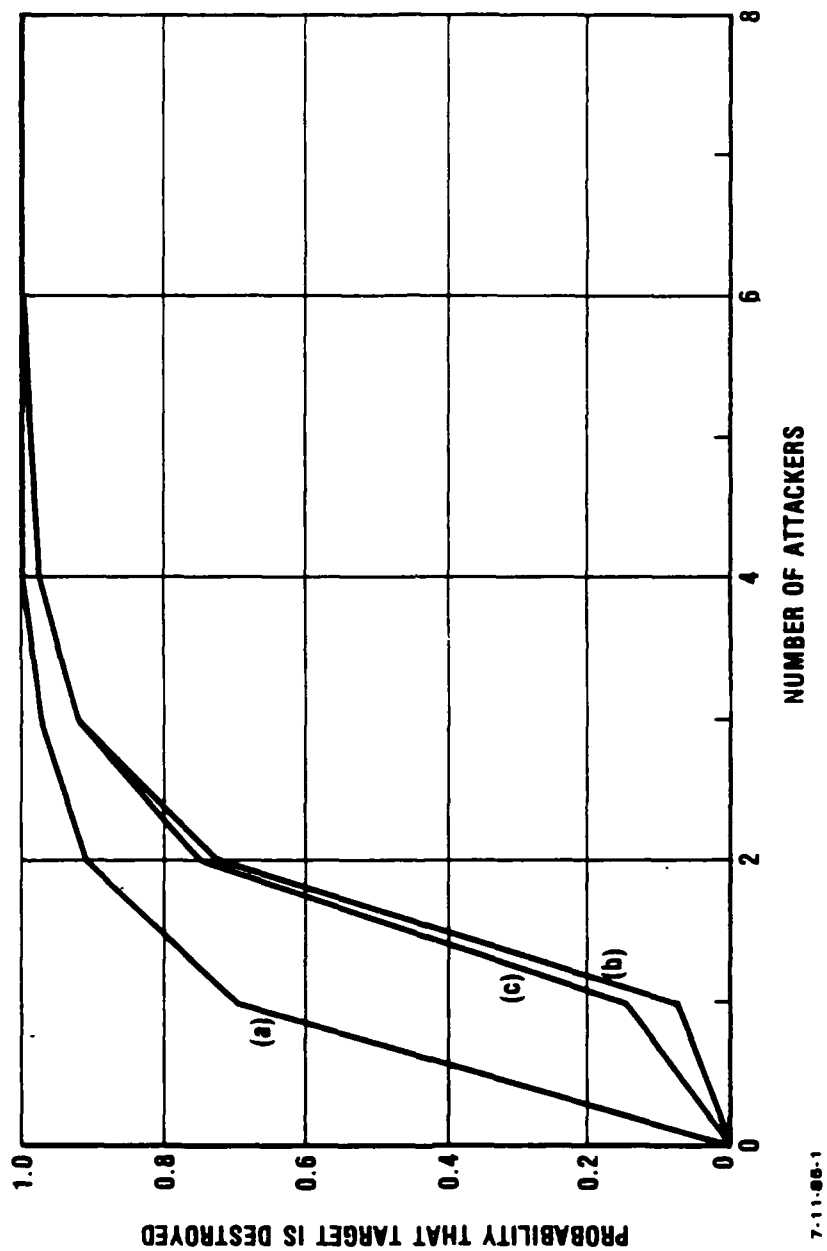


Figure 1. Expected Damage Curves for $D=1$

With only one defender, and with its chances of success being higher if used in the first volley, it is clear that it should be so used. With several defenders, and different q 's and r 's, the situation becomes less clear.

In keeping with the Prim-Read philosophy, we will use the maximum of the ratios $p(A,D)/A$ ($A=1,2,3,\dots$) as the measure of effectiveness of a firing doctrine. In Figure 1, we have

maximum ratio with doctrine (a) = 0.7 ($p_a(1,1)/1$)

maximum ratio with doctrine (b) = 0.361 ($p_b(2,1)/2$)

maximum ratio with doctrine (c) = 0.371 ($p_c(2,1)/2$)

so that firing doctrine (b) is the most effective in that its maximum ratio is the smallest.

In this note, we will assume that the defender will always decide on his first/second volley allocations in such a way as to minimize the maximum of those ratios, where the minimization takes place over all possible such allocations. Under this "behavioral assumption," we make the following definitions.

For each $D=0,1,2,\dots$, let

$Q_k(A,D)$ = probability that the target is destroyed by A attackers given that there are D defenders and k of them are sent against the first arriving RV in a first volley, with a second possible, if needed,

$R_k(A,D)$ = probability that the target is destroyed by A attackers given that there are D defenders, the defense has but a single (second) volley at the first arriving RV and sends k defenders at it,

when, in each case, k ranges from 0 to D .

For any given D , set

$$m(D) = \min_{0 \leq k \leq D} \max_{A \in I^+} Q_k(A, D)/A \quad (1)$$

and let $k^*(D)$ denote the smallest integer such that the above minimum is obtained. Given the functions $Q_0(\cdot, D)$, $Q_1(\cdot, D)$, ..., $Q_D(\cdot, D)$, both $m(D)$ and $k^*(D)$ are well-defined, and $k^*(D)$ represents the number of interceptors that the defender fires in his first volley against the first RV that he sees when he has D interceptors left.

In the event that a defender's first volley fails, he is left with a "reserve volley". Assuming that he now has D defenders left, if we have the functions $R_0(\cdot, D)$, $R_1(\cdot, D)$, ..., $R_D(\cdot, D)$, we define

$$n(D) = \min_{0 \leq k \leq D} \max_{A \in I^+} R_k(A, D)/A \quad (2)$$

and let $k^{**}(D)$ denote the smallest integer which minimizes the above. Then $k^{**}(D)$ represents the numbers of interceptors that the defender would fire if he had but one (second) volley to shoot at the first RV that he sees with D interceptors left (he has two volleys at any subsequent RV), and he wishes to choose a damage curve with the smallest slope.

Let

$$Q(A, D) = Q_{k^*(D)}(A, D) \quad (3)$$

and

$$R(A, D) = R_{k^{**}(D)}(A, D) \quad (4)$$

For each D , these represent the actual expected damage curves that the defender has selected.

Note that

$$Q(A,0) = R(A,0) = 1-s^A \quad A=0,1,\dots$$

(if the attackers are perfect, this holds if we define $0^0=1$).

The following recursions hold

$$R_k(A,D) = r^k(1-s) + (1-r^k(1-s)) Q(A-1,D-k) \quad (5)$$

$$Q_k(A,D) = q^k R(A,D-k) + (1-q^k) Q(A-1,D-k) \quad (6)$$

for $k=0,1,\dots,D$ and $D \geq 1$. They may be solved in the order:

$$Q(A,0) = R(A,0) \rightarrow R_1(A,1) = R(A,1)$$

$$R(A,1) \rightarrow Q_0(A,1), Q_1(A,1) \rightarrow Q(A,1)$$

$$Q(A,1) \rightarrow R_1(A,2), R_2(A,2) \rightarrow R(A,2)$$

$$R(A,2) \rightarrow Q_0(A,2), Q_1(A,2), Q_2(A,2) \rightarrow Q(A,2) \text{ etc.}$$

Note that $R_0(A,D)$ is never computed. The above recursion for $R_0(A,D)$ is

$$R_0(A,D) = (1-s) + s Q(A-1,D)$$

and the function $Q(\cdot,D)$ cannot be computed until $R(\cdot,D)$ has been computed. The function $R_0(\cdot,D)$ represents the expected damage when the defender decides not to use his second volley at an unintercepted RV.

In particular

$$R_0(1,D) = 1-s$$

i.e., the probability that the target is destroyed by the first RV when it is not engaged. We now show that $R_0(\cdot,D)$ does not

enter in the determination of $R(\cdot, D)$ as long as $r < 1$, and therefore need not be computed. The result is intuitive: If you have at least one interceptor left to engage an approaching RV, it is better to use it immediately (even if it has low reliability) instead of saving it for a possible subsequent RV (even if its first-shot reliability might be high).

Lemma. Assume $0 \leq r < 1$. Then for any $D \geq 1$

$$\frac{Q(A, D)}{A} \leq 1-s \quad A=1, 2, \dots \quad (7)$$

Proof. Fix $D \geq 1$. We will use induction on A .

Since $Q(0, D) = 0$, we have

$$R_k(1, D) = r^k(1-s) \quad \text{for any } k, \text{ and}$$

so

$$R(1, D) \leq 1-s$$

and

$$Q_k(1, D) = q^k R(1, D-k) \leq 1-s$$

and it follows that (7) is true for $A=1$.

For any k ,

$$\begin{aligned} Q_k(A, D) &= q^k R(A, D-k) + (1-q^k) Q(A-1, D-k) \\ &= q^k [r^k(1-s) + (1-r^k(1-s)) Q(A-1, D-k)] \\ &\quad + (1-q^k) Q(A-1, D-k) \\ &= q^k r^k(1-s) + (1-q^k r^k(1-s)) Q(A-1, D-k) \\ &\leq q^k r^k(1-s) + (1-q^k r^k(1-s)) (A-1)(s-1) \end{aligned}$$

where induction is used to get the last inequality. Thus

$$Q_k(A,D) \leq [q^k r^k + (1-q^k r^k)(1-s)(A-1)](s-1)$$

and the quantity in square brackets is clearly bounded above by the integer A. Thus

$$\frac{Q_k(A,D)}{A} \leq 1-s \quad \text{for any } k$$

and the result follows.

Theorem. If $0 \leq r, s < 1$, then $R_0(\cdot, D)$ need not be used to complete $R(\cdot, D)$.

Proof. To compute $R(\cdot, D)$, we first determine $n(D)$ and $k^{**}(D)$ from (2).

For $k=0$, we have

$$R_0(A,D) = (1-s) + sQ(A-1,D)$$

In particular

$$R_0(1,D) = 1-s$$

and we now show that

$$\frac{R_0(1,D)}{1} \geq \frac{R_0(A,D)}{A} \quad \text{for } A=2,3,\dots \quad (8)$$

i.e.,

$$A(1-s) \geq (1-s) + sQ(A-1,D)$$

i.e.,

$$(A-1) \geq s(A-1 + Q(A-1,D)). \quad (9)$$

But the lemma implies

$$Q(A-1, D) \leq (1-s)(A-1)$$

i.e.,

$$(A-1) \geq s(A-1 + Q(A-1, D))$$

so that (9) and hence (8) is true. Thus

$$\max_{A \in I^+} \frac{R_0(A, D)}{A} \leq 1-s = R_0(1, D).$$

We will now show that

$$\max_{A \in I^+} \frac{R_1(A, D)}{A} < 1-s$$

so that $k^{**}(D)$ of (2) is not 0. We need to show that

$$r(1-s) + (1-r(1-s)) Q(A-1, D-1) < A(1-s) \text{ for all } A \geq 1$$

but, again by the lemma,

$$Q(A-1, D-1) \leq (A-1)(1-s)$$

so that

$$\begin{aligned} r(1-s) + (1-r(1-s)) Q(A-1, D-1) &\leq r(1-s) + (1-r(1-s))(A-1)(1-s) \\ &= [r + (1-r(1-s))(A-1)](1-s) \end{aligned}$$

and since $r < 1$, the quantity in square brackets is strictly smaller than A and the theorem is proven.

Example. With $(q,r,s) = (.1,.2,.3)$. Table 1 exhibits the values $Q(A,D)$ for $A=0,1,\dots,10$ and $D=0,1,\dots,10$. The first column of the table ($Q(0,A)$) represents the probabilities of destruction in the non-defended case.

Table 1. Case $(q,r,s) = (.1,.2,.3)$

DESTRUCTION PROBABILITIES $Q(A,D)$

A	$Q(A,0)$	$Q(A,1)$	$Q(A,2)$	$Q(A,3)$	$Q(A,4)$	$Q(A,5)$	$Q(A,6)$	$Q(A,7)$	$Q(A,8)$	$Q(A,9)$	$Q(A,10)$
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.700	0.070	0.014	0.014	0.014	0.014	0.014	0.003	0.003	0.003	0.003
2	0.910	0.721	0.137	0.033	0.028	0.028	0.028	0.017	0.007	0.007	0.006
3	0.973	0.916	0.741	0.199	0.055	0.042	0.041	0.031	0.021	0.012	0.010
4	0.992	0.975	0.922	0.760	0.257	0.081	0.056	0.045	0.034	0.025	0.016
5	0.998	0.992	0.977	0.928	0.777	0.311	0.109	0.079	0.052	0.039	0.030
6	0.999	0.998	0.993	0.978	0.933	0.793	0.361	0.176	0.104	0.060	0.046
7	1.000	0.999	0.998	0.994	0.980	0.938	0.808	0.418	0.239	0.131	0.074
8	1.000	1.000	0.999	0.998	0.994	0.981	0.942	0.825	0.470	0.296	0.162
9	1.000	1.000	1.000	0.999	0.998	0.994	0.983	0.948	0.841	0.518	0.349
10	1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.984	0.952	0.855	0.561

THE NUMBER TO SHOOT AT NEXT RV IN 1ST VOLLEY:

0 1 1 1 1 1 1 1 1 1 1 1

IF MISS, THE NUMBER TO SHOOT IN 2ND VOLLEY:

0 0 1 1 1 1 1 2 2 2 2

SLOPES(D):

0.700 0.361 0.247 0.190 0.156 0.134 0.118 0.105 0.095 0.086 0.056

Also included in Table 1 is the firing doctrine which yields the $Q(A,D)$ values. To incorporate this doctrine, suppose, for example, that the defender has 10 interceptors at the target. If he sees an attacker coming in, he uses one of these 10 in the

first volley and, if he misses, he shoots two in the second volley. These values are read below column Q(A,10) of the Q(A,D) table. If the first volley is unsuccessful, but the second succeeds, the defender now has $10-1-2 = 7$ interceptors left to deal with any subsequent attackers. If indeed, a second RV attacks, the defender again uses a 1-2 firing doctrine as indicated below column D=7. If, again, his first volley fails but his second succeeds, he has $7-1-2 = 4$ interceptors left.

A possible (but unlikely*) battle history with D=10 is:

RV #	1st Volley	2nd Volley	# Interceptors Left if 1st Volley Misses
1	1	2	7
2	1	2	4
3	1	1	2
4	1	1	0
5	.	.	.

The slopes $m(D)$ of the lowest maximum damage curves which yield an upper bound on the expected damage are also given in Table 1. Thus, for example, with 7 interceptors, the expected damage is bounded above by the linear function $F(A) = 0.105A$.

Figure 2 exhibits the expected damage curves for increasing values of D.

Often one wishes to determine the minimum number of interceptors required to enforce a maximum damage function of a given slope. Figure 3 exhibits the curves both with and without an SLS capability when the interceptor failure probabilities are 0.3 and the RV failure probability is 0.1. For example, if one wishes to design a defense whose expected damage per attacking

*The probability that this would occur is

$$(.1)(1-.2^2)(.1)(1-.2^2)(.1)(1-.2)(.1)(1-.2) = 5.9 \times 10^{-5}$$

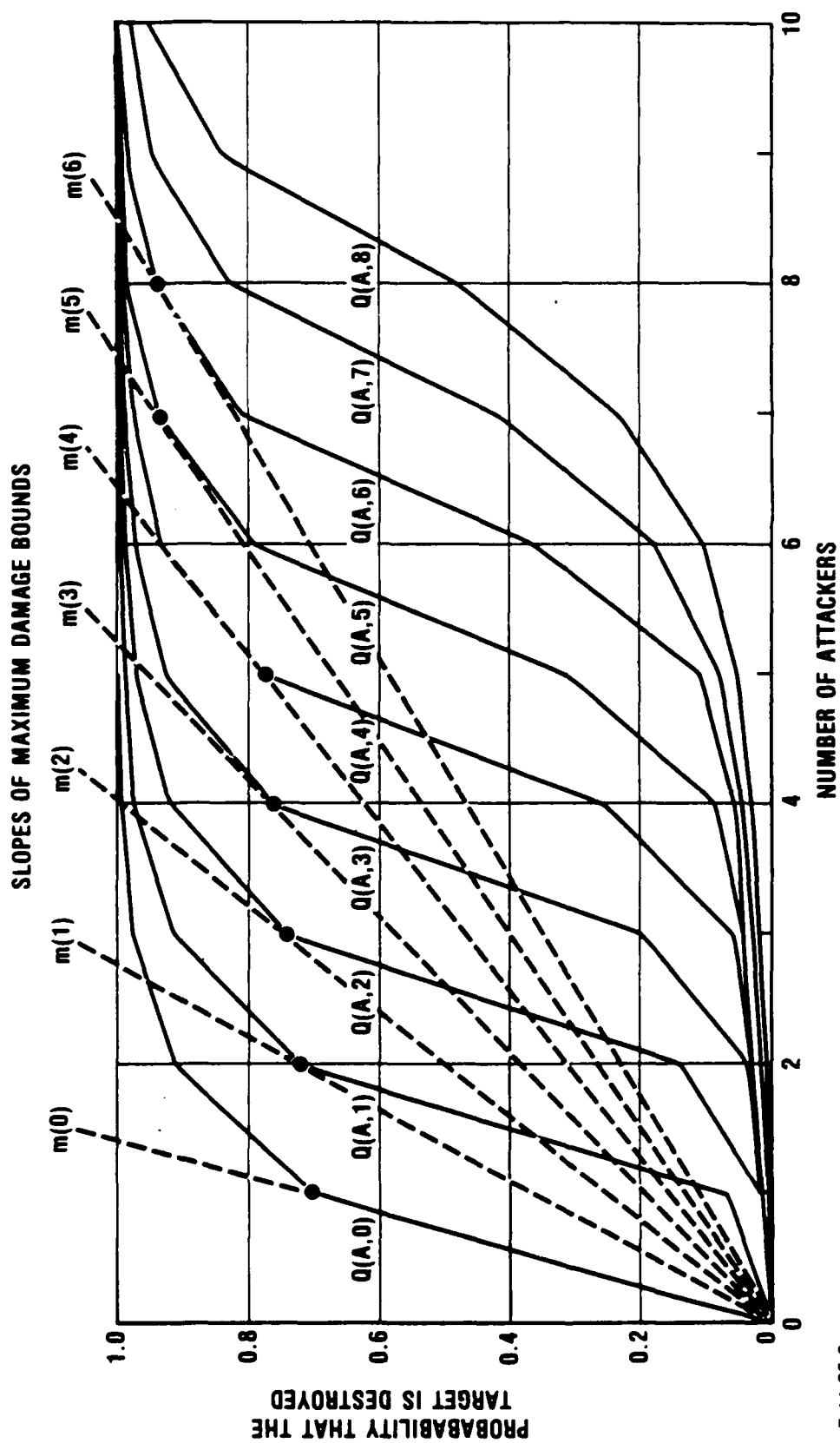


Figure 2. Expected Damage Curves and Maximum Damage Bounds for Varying D

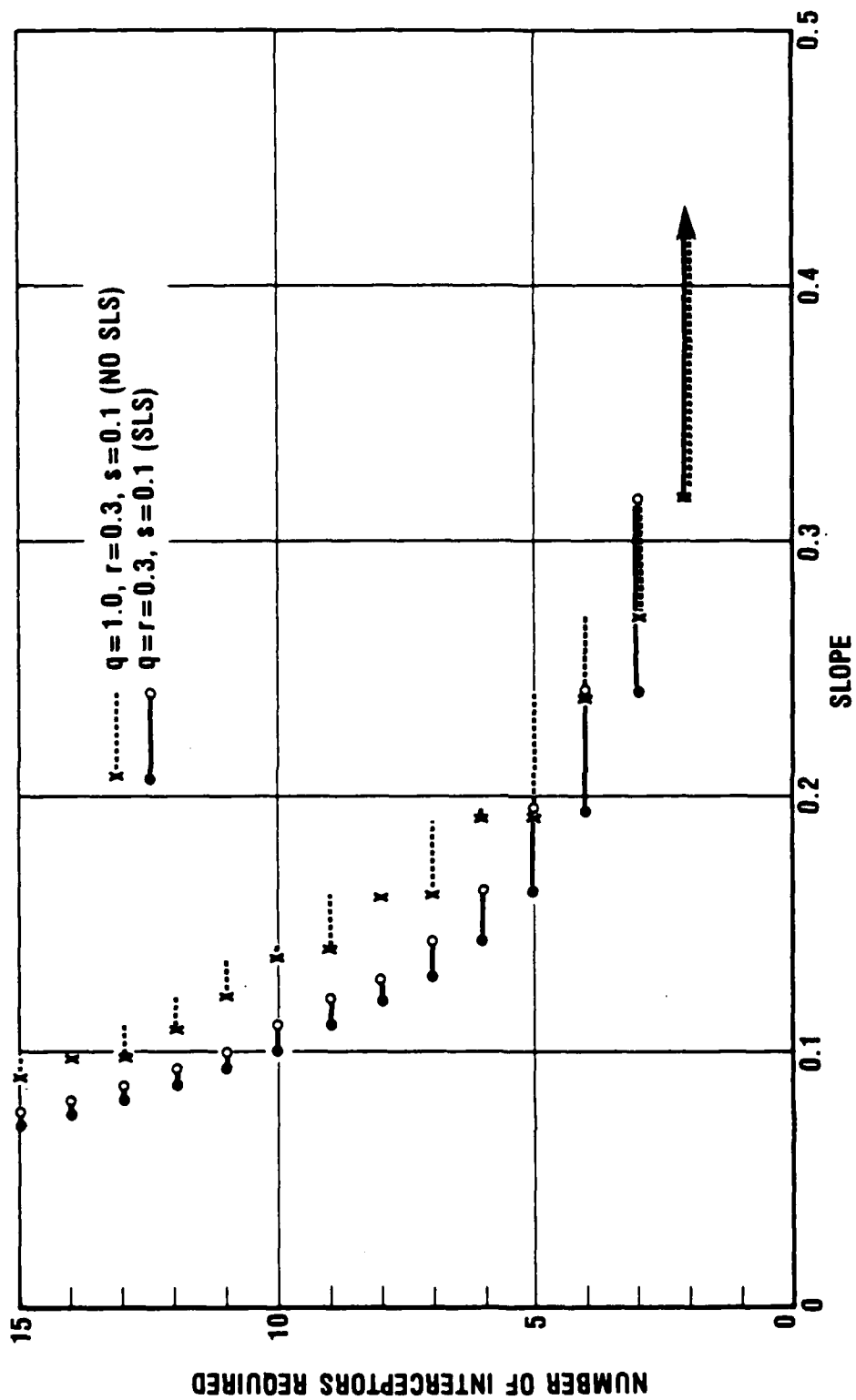


Figure 3. Number of Interceptors Required for a Given Slope

weapon is 0.15, one needs 6 interceptors with an SLS capability, and 9 interceptors without.

Extensions and Implementation

The case $q=1$ corresponds to no SLS capability. The case $s=1$ is a trivial case wherein the RV's are completely unreliable so no defense is needed.

The results of this note are easily extended to allow for additional volleys.

A FORTRAN program has been coded to generate the values $Q(A,D)$ for any range of A,D values.

4. Equation for SEQAT2--Sequential Attack of Known Size¹

Here we address the case where the attack is "sequential", i.e., there is enough time between successive attackers that they can be ordered and the attack size is known. We define, as before:

$P(A,D)$ = probability that the target survives, given that it is under attack by A missiles and is optimally defended by D defenders.

Obviously, if the defender knows the value of A, he will defend uniformly according to the result of Appendix A. (A simultaneous attack can be considered sequential by numbering the attackers in any order.)

However, if the defender has a shoot-look-shoot capability, and sufficient time between arrivals, he can choose to structure his defense in volleys, with the prospect of saving defenders for use against future attackers.

Suppose the defender has time for two volleys against each incoming attacker. Let a be, as before, the kill probability of an attacking missile. Let

d = probability that a defending interceptor will destroy an attacking missile in the first volley

and

e = probability that a defending interceptor will destroy an attacking missile in the second volley.

Let

$d(A)$ = number of interceptors to shoot at the first of A attacking missiles in the first volley

and

¹Source: Appendix C of Reference [1] on page R-1.

$e(A)$ = number of interceptors to shoot at the first of A attacking missiles in the second volley, given that the first volley has failed.

Then

$1-d(1-d)^{d(A)}$ is the probability that the first volley is successful,

$(1-d)^{d(A)}(1-(1-e)^{e(A)})$ is the probability that the first volley fails but the second is successful,

and

$(1-d)^{d(A)}(1-e)^{e(A)}(1-a)$ is the probability that both volleys fail and the attack also fails.

The following recursion holds:

$$P(A,D) = \max_{\substack{d(A), e(A) \in I^t \\ d(A) + e(A) \leq D}} \left\{ (1-(1-d)^{d(A)}) P(A-1, D-d(A)) + (1-d)^{d(A)}(1-a(1-e)^{e(A)}) P(A-1, D-d(A)-e(A)) \right\}$$

with

$$P(0,D) = 1 \quad \text{for all } D \in I^+.$$

Given a , d and e , the recursion can be solved by dynamic programming to determine $P(A,D) = p_{ij}$. Note that the solution of this recursion would agree with the results of Appendix A in the case where $e=0$.

Obviously, the above recursion could be extended if the defender had more than two opportunities to protect himself.

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