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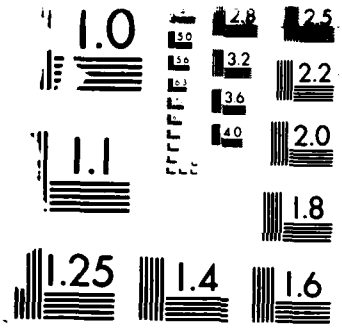
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optimal stochastic control, singular control, impulse control, variational inequalities, free-boundary problems, numerical solution, error estimates, wave propagation, turbulent media, progressing waves, travel time, stochastic partial differential equations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A number of problems in optimal stochastic control and random media have been studied. They include some problems in singular and impulse control, numerical approximation and error estimates for stochastic control. Furthermore, theory and application of stochastic partial differential equations to physical sciences are also investigated.

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FINAL TECHNICAL REPORT

Submitted to

U.S. ARMY RESEARCH OFFICE

DEPARTMENT OF THE ARMY

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TITLE OF PROPOSAL: Stochastic Partial Differential Equations in Physical and Systems Sciences

CONTRACT NUMBER: DAAG29-83-K-0014

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I. Statements of Problems

Research has been conducted in three major problem areas:

(1) Optimal control of dynamical systems under continuous and/or impulsive random perturbations.

(2) Numerical solution of stochastic control problems.

(3) Applications of stochastic partial differential equations in physical sciences.

In the period of three years ARO funding, we have emphasized our research work in the first problem area. In this area we have investigated several problems in singular and impulsive control problems which arise from applied problems in opertion research and engineering sciences. In order to implement the theory of optimal stochastic control, the numerical approximation scheme is indispensible. We have initiated a research program in the second problem area. The initial phase of investigation has been restricted to some optimal correction and optimal stopping problems. The third problem area has been our long-standing research interest in stochastic partial differential equations and their applications to engineering and physics. In this area we have studied in stochastic inverse problems in wave radiation problems, the wave-travel time in turbulent media, as well as the theory of stochastic partial differential equation.

II. Summary of principal results.

During the proposed period of investigation, a number of research findings have been summarized in a series of semi-annual reports, which have been submitted periodically to ARO. Instead of repeating the previously reported results, we will only outline the major results and refer the reader to the cited papers, where the details and other results may be found. The numbers below correspond to the paper numbers indicated in the list of publications given in the next section

(1). Singular stochastic control problems:

We introduced a powerful analytical method for solving this class of problems by means of variational and quasi-variational inequalities. This approach, in contrast with the usual probabilistic approach, allows an extension of one-dimensional results to higher-dimensional cases more easily. The papers [5, 8, 12] deal with the singular stochastic control theory in one dimension. The results are generalized to optimal correction problems [9, 11, 19] in random vibrations in two dimensions with full or partial observations.

(2). Control of diffusion processes with jumps:

To extend the range of applicability of stochastic control models the external disturbances are allowed to be random impulses in additon to the white noise. Analytical method and solution of such problems are given in the papers [7, 10, 16, 20].

(3). Impulse and other type of control problems:

Some impulse stochastic control problems have been treated in [1, 16, 20]. These problems arise commonly in the area of operation research. In the presence of some state constraints, one is lead to control of reflected diffusions. Several mathematical questions are resolved in the paper [2].

(4). Numerical approximation in stochastic control problems:

We have introduced a numerical algorithm for solving a certain type of singular control problems in [14, 18]. The method is an iterative scheme based on a discret maximum principle. It applies to the degenerate diffusion problem as well. Furthermore, error estimate for the numerical solution for a stochastic control problem is treated in [14, 22].

(5). Stochastic PDE's and applications:

A general method is proposed for reconstructing the mutual coherent function for a static or moving source from the random radiation data [4]. Several applications of stochastic differential equations to engineering science are discussed in [5]. The statistics of travel-time for waves in turbulent media is treated in [16,19]. Some theoretical aspects of stochastic PDE's are studied in [3,22].

III. List of Papers Published or Submitted under ARO Sponsorship

1. J.L. Menaldi and M. Robin. "On Some Cheap Control Problems for Diffusion Processes", Trans. A.M.S., 278 (1983), pp. 771-802.

2. J.L. Menaldi, "Stochastic Variational Inequality for Reflected Diffusions", Indiana Univ. Math. J. 32 (1983), pp. 733-744.

3. P.L. Chow and J.L. Menaldi, "Method of Regularization for Second-Order Stochastic Evolution Equations:, J. Stoch. Analy. and Appl. 1 (1983), pp. 353-376.

4. P.L. Chow, "Reconstruction of the Mutual Coherence Function for a Moving Source", SIAM J. Appl. Math., 43 (1983), pp. 1439-1445.

5. P.L. Chow, "On some stochastic problems in Engineering Sciences", in Recent Developments in Applied Mathematics, R.P.I. Press, (1983), pp. 31-43.

6. P.L. Chow, J.L. Menaldi and M. Robin, "Additive Control of Stochastic Linear Systems with Finite Horizon", Proc. 23rd IEEE Conf. on Decision Control (1984), pp. 1475-1478.

7. J.L. Menaldi and M. Robin, "On Singular Stochastic Control Problems for Diffusion with Jumps", IEEE Trans. Auto. Control, Vol. AC-29(1984), pp. 911-1004.

8. P.L. Chow and J.L. Menaldi, On the Control of a Linear Stochastic System with Finite Horizon, Trans. 1st Army Conference on Applied Math. and Computing, ARO Report 84-1, (1984) pp 301-310.

9. P.L. Chow and J.L. Menaldi, Optimal Corrections of a Damped Linear Oscillator under Random Perturbations, Trans. 2nd Army Conf. on Appl. Math. and Comput., ARO Report 85-1, (1985), pp. 149-158.

10. J.L. Menaldi and M. Robin, On Singular Stochastic Control Problems for Diffusion with Jumps, IEEE Trans. on Automatic Control, AC-29 (1984), pp. 991-1004.

11. J.L. Menaldi and M. Robin, On Optimal Correction Problems with Partial Information, J. Stoch. Analy. and Applications, 3 (1985), pp. 63-92.

12. P.L. Chow, J.L. Menaldi and M. Robin, "Additive Control of Stochastic Linear Systems with Finite Horizon", SIAM J. Control and Optim, 23 (1985), pp. 858-899

13. P.L. Chow and J.L. Menaldi, on the numerical solution of a stochastic optimal correction problem, Trans. 3rd Army Conf. on Appl. Math. and Comput., ARO Report 86-1, (1986), pp. 531-546.

14. ____, Error Estimate for the Numerical Solution of a Stochastic Control Problem, Trans. 3rd Army Conf. on Appl. Math. and Comput., Aro Report 86-1, (1986), 547-557.

15. P.L. Chow, Some Applications of the Method of Progressing Waves in Random Media, in Multiple Scattering of Waves in Random Media and Rough Surfaces, Technomic Pub. Co., Lancaster, PA. (1986) to appear.

16. J.L. Menaldi, Optimal Impulse Control Problem for Degenerate Diffusion with Jumps. Acta Appl. Math., (1986) to appear

17. M.C. Bancora-Imbert, P.L. Chow and J.L. Menaldi, On the Numerical Approximation of an Optimal Correction Problem, SIAM J. Numer. and Statist. Comput., (1986), Submitted.

18. P.L. Chow, "Travel-time Problems for Waves in Random Media", Proc. IMA Workshop on Random Media, Springer-Verlag (Submitted).

19. M. Sun and J.L. Menaldi, "Optimal Control of a Damped Linear Oscillator under Random Perturbations", Appl. Math. and Optim., (Submitted).

20. J.L. Menaldi, "Optimal Impulse Control Problems for Degenerate Diffusions with Jumps", Proc. Intern. Conf. on Nonlinear P.D.E.'s and Their Applications, (Submitted).

21. P.L. Chow, Expectational Functionals Associated with some Evolution Equations, IMA Reprint series #235, Univ. of Minnesota, (April, 1986); Stochastic PDE's and Applications, Lecture Notes in Math., Springer Verlag (1986), to appear.

22. J.L. Menaldi, Probabilitic View of Estimates for Finite Difference Methods. SIAM J. Control and Optim. (Submitted)

IV. Scientific Personnel Supported by the Project and Degrees Awarded.

During the reporting period, the following personnel has been supported by the ARO contract.

1. P.L. Chow (principal investigator) — 2 summer months x 3 (1982-85)
2. J.L. Menaldi (co-investigator) — 2 summer months x 3 (1982-85)

There are no advanced degrees awarded under the ARO support.

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