Structure Theories and Applications of

Multivariable Control Systems Described by Block Canonical Forms

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Several block canonical forms for describing a class of multivariable control systems have been developed in this project. Based on these canonical structures, some algebraic and geometric theories have been developed for analysis and design of multivariable control systems. Several matrix-valued functions have been defined and computational algorithms have been established for computing these matrix-valued functions. These matrix-valued functions and associated computational algorithms have been utilized for analysis and design of general large-scale continuous-time and discrete-time systems. Based on these research results, twenty-two papers have been published in the referred journals.

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Summary of Research Results

Four basic block companion forms have been constructed in the time domain via the newly developed similarity block transformations. These canonical forms have been used to generate other block canonical structures, such as the parallel and cascaded block canonical forms, for state-feedback block controller design, block partial fraction expansion, model reduction, optimal control design, block modal controller design and stability test of multivariable control systems. The algebraic and geometric theorems of matrix polynomials and matrix-fraction descriptions of multivariable control systems have been developed in this project.

Several matrix-valued functions, such as the matrix sign functions, the matrix sector functions and the matrix logarithmic functions, have been established and computational algorithms, such as the matrix sign algorithm, the matrix sector algorithm and the matrix principal qth-root algorithm have been developed. These newly developed algorithms have been applied to solve Lyapunov equation, Riccati equation and Diophantine equation. The developed matrix-valued functions and associated computational algorithms have been employed for analysis and design of general large-scale continuous-time and discrete-time systems.

Based on the research results in the period of 1983-1986, twenty-two papers have been published in the referred journals and listed as follows:

1. Shieh, L.S., Y.T. Tsay, and R.E. Yates, "Block Transformations of a Class of Multivariable Control Systems to Four Basic Block Companion Forms," Computers and Mathematics with Applications, Pergamon Press, Vol. 9, No. 6, pp. 703-714, 1983.

Abstract

Various block transformations are presented for transforming a class of MIMO state equations in general coordinates to four basic block companion forms so that the analysis and control system design of a MIMO system in the time and frequency domains can easily be performed, and the classical lines of thought for SISO systems can be extended to MIMO systems. The invariant structure and the invariant characteristic matrix polynomial of four basic block companion forms are investigated.

2. Shieh, L.S. and Y.T. Tsay, "An Algebra-Geometric Approach for the Model Reduction of Large-Scale Multivariable Systems," Proceedings of the IEE (England) Part D. Control Theory and Applications, Vol. 131, Pt. D, No. 1, pp. 23-36, January 1984.

Abstract

The paper presents an algebra-geometric approach for

determining the time-domain reduced-order models and frequency-domain reduced-degree models of large-scale multivariable systems. First, the structures of the canonical state-space representations and corresponding matrix fraction descriptions of general multivariable systems are introduced and the associated characteristic λ-matrices are defined. Next, the divisors and spectral decomposition theorems for the nonsingular characteristic λ-matrices, which may not be regular or monic, are developed by using the algebraic and geometric properties of multivariable-system structures. Then, the derived algebra-geometric theorems are used to develop a frequency-domain aggregation method and a time-domain aggregation method for the model reduction of large-scale multivariable systems. Finally, the newly developed matrix sign functions, in conjunction with the aggregation method, are used to obtain the reduced-order and reduced-degree models of large-scale multivariable systems, without assuming that the eigenvalues of the original systems are known and/or the singularly-perturbed models are available.

3. Tsay, Y.T. and L.S. Shieh, "Block Modal Control Via Cascade Multivariable Control Structures," International Journal of Systems Science, Vol. 15, No. 2, pp. 199-214, February 1984.

Abstract

The paper deals with the algebraic theory and state-space theory of cascade multivariable control structures described by matrix fraction descriptions and state-space formulations. A block modal control via factorization of matrix polynomials and solvents placement is developed for multivariable control systems design.

4. Shieh, L.S., C.H. Chen and N. Coleman, "Stability of a Class of Multivariable Control Systems Described by the Nested Matrix Continued-Fraction Descriptions," International Journal of Systems Science, Vol. 15, No. 4, pp. 439-457, April 1984.

Abstract

This paper deals with the stability of a class of multivariable control systems which can be described by a right (left) matrix fraction description and associated right (left) matrix continued-fraction description in the nested Cauer form. Some simple sufficient conditions and some simple necessary conditions are developed for stability determination of the class of multivariable systems via the Lyapunov stability and instability theorems. By simple testing of the (positive or negative) definiteness of a set of matrix qotients obtained from the matrix Sturm algorithms, the stability of the class of multivariable systems can be determined. The matrix fraction descriptions of interest may be unsymmetric and reducible.

5. Shieh, L.S., Y.F. Chang and R.E. Yates, "Model Simplifications and Digital Design of Multivariable Sampled-Data Control Systems Via a Dominant Data Matching Method," _Applied Mathematical Modelling_, Vol. 8, pp. 355-364, October 1984.

Abstract

A dominant-data matching method is developed for model simplification and design of digital multivariable sampled-data control systems. A mixed method combining dominant-data matching and the dominant-pole technique is also derived for determining a stable reduced-degree multivariable digital controller. A real semiactive terminal homing missile system is used as an illustrative example.

6. Shieh, L.S., Y.T. Tsay, S.W. Lin, and N. Coleman, "Block-diagonalization and Block-triangularization of a Matrix Via the Matrix Sign Functions," _International Journal of Systems Science_, Vol. 15, No. 11, pp. 1203-1220, November 1984.

Abstract

A matrix sign function in conjunction with a geometric approach is utilized to construct a block modal matrix and a (scalar) modal matrix of a system map, so that the system map can be block-diagonalized and block-triangularized, and that the Riccati-type problems can be solved.

7. Shieh, L.S., Y.T. Tsay, and C.T. Wang, "Matrix Sector Functions and Their Applications to Systems Theory," _Proceedings of the IEE (England) Part D. Control Theory and Applications_, Vol. 131, Pt. D., No. 5, pp. 171-181, September 1984.

Abstract

The paper presents a new matrix function, the matrix sector function of a square complex matrix A, and its applications to systems theory. Firstly, based on an irrational function of a complex variable λ, a scalar sector function of λ, $(\lambda/\sqrt[n]{\lambda^n})$, is defined. Next, a fast algorithm is developed with the help of a circulant matrix for computing the scalar sector function of λ. Then, the scalar sector function of λ is extended to a matrix sector function of A, $A(\sqrt[n]{A^n})^{-1}$, and to associated partitioned matrix sector functions of A. Finally, applications of these matrix sector functions to the separation of matrix eigenvalues, the determination of A-invariant space, the block diagonalisation of a matrix, and the generalised block partial fraction expansion of a rational matrix are given. It is shown that the well-known matrix sign function of A is a special class of the newly developed matrix sector function of A. It is also shown that the Newton-Raphson type algorithm cannot, in general, be applied to determine the matrix sector function of A.

3

8. Shieh, L.S., C.T. Wang, and Y.T. Tsay, "Multivariable State-Feedback Self-tuning Controllers," *Stochastic Analysis and Applications*, Vol. 3, pp. 189-212, June 1985.

Abstract

    This paper describes a state-space approach for self-tuning control of a class of multivariable stochastic systems having the same number of inputs as outputs. A multivariable state-feedback self-tuning controller, based on pole-assignment concepts, is derived. The developed multivariable self-tuner can be applied to stable/unstable and minimum/non-minimum phase linear time-invariant multivariable systems. A multivariable reduced-order self-tuner and a state-feedback minimum-variance self-tuner are also derived. The simplicity and flexibility of the proposed state-space approach facilitate the practical applications of self-tuning control concepts to real systems.

9. Shieh, L.S., F.R. Chang, and R.E. Yates, "The Generalized Matrix Continued-Fraction Descriptions in the Second Cauer Form," *IEEE Trans. Automatic Control*, Vol. AC-30, No. 8, pp. 813-818, August 1985.

Abstract

    This note presents two generalized matrix continued-fraction descriptions in the second Cauer form obtained from a nonsingular rational matrix and/or polynomial matrix fraction descriptions. Computational algorithms are developed for the expansion and inversion of the generalized matrix continued-fraction descriptions which are useful in developing applications to multivariable control systems.

10. Shieh, L.S., Y.T. Tsay, and R.E. Yates, "Computation of the Principal nth Roots of Complex Matrices," *IEEE Trans. Automatic Control*, Vol. AC-30, No. 6, pp. 606-608, June 1985.

Abstract

    A new computational algorithm for finding the principal nth roots and associated roots of complex matrices is developed via a recursive solution of a block discrete state equation.

11. Zhu, J., L.S. Shieh, and R.E. Yates, "Fitting Continuous-time and Discrete-time Models Using Discrete-time Data and Their Applications," *Applied Mathematical Modelling*, Vol. 9, pp. 93-98, April 1985.

Abstract

    An exponential fnction scheme, which is an extension of the

time-domain prony method, and a mixed-matching method are developed for fitting the coefficients of both continuous-time and discrete-time transfer functions, using the discrete-time data of either continuous-time or discrete-time systems. When the discrete-time data are obtained from a continuous-time (discrete-time) system and the discrete-time (continuous-time) models are desirable, the proposed method can be applied to perform the model conversions. If the discrete-time data are obtained from a high-degree system, the proposed method can be applied to determine the reduced-degree models.

12. Shieh, L.S., Y.T. Tsay and S. Barnett, "A Review of Some Matrix Continued-fraction Descriptions and their Applications to the Stability of Multivariable Control Systems," IMA Journal of Mathematical Control & Information, Vol. 2, pp. 1-23, 1985.

Abstract

A review is given of some aspects of the matrix continued-fraction approach to the theory of multivariable linear control systems, as developed by Shieh and various co-workers over the past decade. These basic matrix continued-fraction descriptions (MCFDs) associated with matrix fraction descriptions (MFDs) of a square transfer-function matrix are described. A matrix Euclidean algorithm and a matrix Sturm algorithm are developed, and applications made in the determination of the greatest common divisor of polynomial matrices, and the transformation between right and left irreducible MFDs. Using the relevant linear matrix equation obtained from Liapunov theory, the structure of the matrix continued-fraction description allows the derivation of various sufficient conditions for stability or instability of a class of linear control systems.

13. Shieh, L.S., M.M. Mehio and R.E. Yates, "Cascade Decomposition and Realization of Multivariable Systems Via the Block-pole and Block-zero Placement," IEEE Trans. Automatic Control, Vol. AC-30, No. 11, pp. 1109-1112, November 1985.

Abstract

A technique is presented for state-feedback and state-feedforward block decomposition of a class of multivariable systems into a block-cascaded form structure having the assigned block poles and block zeros. A state-space technique is also presented for minimal realization of a multivariable compensator, represented by a voltage transfer function matrix, using the decoupled RC cascaded networks and summers.

14. Tsay, Y.T., L.S. Shieh and S. Barnett, "State-space Minimal Realizations and Latent Structures of Column Reduced and Nonsingular Polynomial Matrices," IMA Journal of Mathematical

Control and Information, Vol. 2, pp. 123-138, 1985.

Abstract

The concept of a column-reduced polynomial matrix is an important one in the theory of linear systems. The theory of Jordan chains and minimal realizations is developed for such matrices. Also, the relationships between generalized latent vectors of the nonsingular polynomial matrix and their associated generalized eigenvectors of the system map are explored in this paper. This permits the spectral analysis of an arbitrary nonsingular polynomial matrix, extending previous work for the monic case.

15. Shieh, L.S., S.J. Tsai and R.E. Yates, "The Generalized Matrix Sector Function and the Separation of Matrix Eigenvalues," IMA Journal of Mathematical Control and Information, Vol. 2, pp. 251-258, 1985.

Abstract

The matrix sector function of A is introduced and generalized to the matrix sector function of g(A), where the complex matrix A may have a real or complex characteristic polynomial and g(A) is a matrix function of a conformal mapping. The generalized matrix sector function of A is employed to separate the matrix eigenvalues relative to a sector, a circle, and a sector of a circle in the complex plane without actually seeking the characteristic polynomial and the matrix eigenvalues themselves. Also, the generalized matrix sector function of A is utilized to carry out the block-diagonalization and block-triangularization of a system matrix, which are useful in developing applications to mathematical science and control-system problems.

16. Shieh, L.S., F.R. Chang and R.E. Yates, "The Generalized Matrix Continued-fraction Descriptions and their Application to Model Simplification," International Journal of Systems Science, Vol. 17, No. 1, pp. 1-19, January 1986.

Abstract

This paper presents two generalized matrix continued-fraction descriptions in the second Cauer form from a transfer function matrix and/or matrix fraction descriptions which may contain non-square and/or non-regular polynomial matrices. It also presents two model reduction techniques for model simplification of high-degree non-square transfer function matrices and matrix fraction descriptions. The expansion algorithms as well as the inversion algorithms for the generalized matrix continued-fraction descriptions are derived. Two model reduction techniques, which are the matrix continued-fraction

expansion method and the mixed method of the matrix continued-fraction expansion method and the equivalent-dominant pole method, are developed for model simplification of multivariable control systems.

17. Shieh, L.S., F.R. Chang and B.C. McInnis, "The Block Partial Fraction Expansion of a Matrix Fraction Description with Repeated Block Poles," IEEE Trans. Automatic Control, Vol. AC-31, No. 3, pp. 236-239, March 1986.

Abstract

A state-space technique is developed for finding the block partial fraction expansion of an irreducible right matrix fraction description with distinct and/or repeated block poles. The developed method is the generalized version of the existing methods which consider only distinct block poles.

18. Shieh, L.S., H.M. Dib and B.C. McInnis, "Linear Quadratic Regulators with Eigenvalues Placement in a Vertical Stop," IEEE Trans. Automatic Control, Vol. AC-31, No. 3, pp. 241-243, March 1986.

Abstract

A computational method is presented for finding the linear quadratic regulator such that the optimal closed-loop system has eigenvalues lying within a vertical strip in the complex plane. The proposed method is suitable for the two-stage optimal design of two-time scale systems.

19. Shieh, L.S., H.M. Dib and R.E. Yates, "Separation of Matrix Eigenvalues and Structural Decomposition of Large-Scale Systems," Proceedings of the IEE (England) Part D. Control Theory and Applications, Vol. 133, Pt. D. No. 2, pp. 90-96, March 1986.

Abstract

The paper presents the separation of matrix eigenvalues relative to strips, sectors, trapezoids and circles in a complex plane without actually seeking the characteristic polynomial and the matrix eigenvalues themselves. The system matrix of interest is a real or complex matrix which may have a real or complex characteristic polynomial. Also, the paper develops a technique for block-diagonalisation and block-triangularisation of the system matrix according to the characteristics of the system eigenvalues. As each block-decomposed submatrix contains the matrix eigenvalues lying within a specific subregion of a complex plane, the existing design methods, such as the multi-stage design methods, can effectively be applied to the substructural models of large-scale systems for attaining a desired overall system behaviour. The fast matrix sign function, which has quick

convergence property and convergence speed independent of the
dimension of the system map, is used for the derivations.

20. Tsay, Y.T., L.S. Shieh, and S.J. Tsai, "A Fast Method for
Computing the Principal nth Roots of Complex Matrices," _Linear
Algebra and Its Application_, Vol. 76, pp. 205-221, April 1986.

Abstract

Based on the generalized continued-fraction method for
finding the nth roots of real numbers, this paper presents a fast
computation method for finding the principal nth roots of complex
matrices. Computation algorithms with high convergence rates are
developed, and their global convergence properties are
investigated from the viewpoint of systems theory.

21. Shieh, L.S., S.J. Tsai and S.R. Lian, "Determining Continuous-
time State Equations from Discrete-time State Equations Via the
Principal qth Root Method," _IEEE Trans. Automatic Control_, Vol.
AC-31, No. 5, pp. 454-457, May 1986.

Abstract

Fast computational methods are developed for finding the
equivalent continuous-time state equations from discrete-time
state equations. The computational methods utilize the direct
truncation metho , the matrix continued fraction method, and the
geometric-series method in conjunction with the principal qth root
of the discrete-time system matrix for quick determination of the
approximants of a matrix logarithm function. It is shown that
the use of the principal qth root of a matrix enables us to
enlarge the convergence region of the expansion of a matrix
logarithm function and to improve the accuracy of the
approximants of the matrix logarithm function.

22. Chang, F.R., L.S. Shieh and J.M. Navarro, "A Simple Division
Method for Solving Diophantine Equations and Associated Problems,"
International Journal of Systems Science, Vol. 17, No. 6, pp.
953-968, June 1986.

Abstract

Based on the transfer function matrices or the matrix
fraction descriptions of multivariable systems, a simple division
method in conjunction with the method owing to Wolovich and
Antsaklis (1984) is presented in this paper for obtaining the
observability and controllability indices, the irreducible right
(left) matrix fraction descriptions from reducible or irreducible
left (right) matrix fraction descriptions, the greatest common
divisors of two polynomial matrices and the solution of a
Diophantine equation, without utilizing the state-space

realization of the matrix fraction description.

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DISC