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NUMERICAL INVERSION OF INTEGRAL EQUATIONS(U) UTAH UNIV
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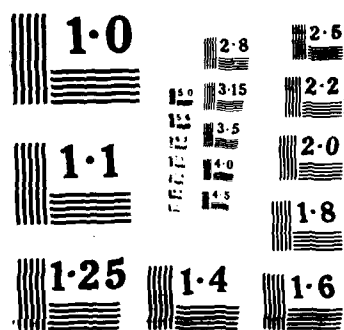
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NUMERICAL INVERSION OF INTEGRAL EQUATIONS

FINAL REPORT

AUTHOR OF REPORT: Frank Stenger

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- a.1. "Explicit, Nearly Optimal, Linear Rational Approximations with Preassigned Poles", Math. Comp. v.47 (1986) 225-252.
- a.2. "Overconvergence in Chebyshev Polynomial Approximation", (with A. Sharma and H.P. Dikshit) accepted for publication in J. Approx. Theory (to appear in 1988).
- a.3. "Multigrid-Sinc Methods", with S. Schaffer, ACM Special Issue on Multigrid Methods, 1986.
- a.4. "Ultrasound Inverse Scattering Solutions from Transmission and/or Reflection Data", with M.J. Berggren, S.A. Johnson, B.L. Carruth W.W. Kim and P.K. Kuhn. Submitted 5-3-86.
- a.5. "Inverse Scattering Solutions to the Exact Riccati Wave Equations by Iterative Rytov Approximations and Internal Field Calculations", with W.W. Kim, M.J. Berggren, S.A. Johnson, and C.H. Wilcox, in IEEE Ultrasonic Symposium (1985) 878-882.
- a.6. "Performance of Fast Inverse Scattering Solutions for the Exact Helmholtz Equation Using Multiple Frequencies and Limited Views", with M.J. Berggren, S.A. Johnson, B.L. Carruth, W.W. Kim and P.L. Kuhn, Submitted in 9-3-86.
- a.7. "A Method of Bisections for Solving a System of Nonlinear Equations", (with A. Eiger and K. Sikorski) ACM TOMS(1984) 367-377.
- a.8. "Algorithm 614: FORTAN Subroutine for the Method of Bisections in n Dimensions", (with A. Eiger and K. Sikorski) ACM TOMS (1984) 152-160.
- a.9. "Optimal Quadratures in H^P Spaces", (with K. Sikorski) ACM TOMS (1984) 140-151.
- a.10. "Rational Function Frequency Extrapolation in Ultrasonic Tomography" (with M.J. Berggren, S. Johnson and C. Wilcox), in Wave Phenomena, Modern Theory and Applications, C. Rogers and T.B. Moodie, eds. North-Holland (1984) 19-34.
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- a.12. "Fast Iterative Algorithms for Inverse Scattering Solutions of the Helmholtz and Riccati Wave Equations", (with S.A. Johnson, Y. Zhou, M.K. Tracy and M.J. Berggren) Acoustical Imaging, 13 (1984) 75-87.

- a.13. "Inverse Scattering Solutions by SINC Basis, Multiple Source Moment Method, --Part III: Fast Algorithms", with S.A. Johnson, Y. Zhou, M.L. Tracy, and M.J. Berggren, Ultrasonic Imaging, 6 (1984) 103-116.
- a.14. "Transient Electromagnetic Response of a Three-Dimensional Body in a Layered Earth", by G. Newman, G.W. Hohmann, and W.L. Anderson, in Geophysics v. 51 (1986) 1608-1627.
- a.15. "Integral Equation Solution for the Transient Electromagnetic Response of a Three-Dimensional Body in a Conductive Half-Space", by W.A. SanFilipo and G.W. Hohmann, in Geophysics v. 50 (1985) 798-809.
- a.16. "Sinc Method of Solution of Singular Integral Equations," with D. Elliott, IMACS Conference on CSIE, Philadelphia, P.A., 1984, pp.155-166.
- a.17. "Sinc Methods of Approximate Solution of Partial Differential Equations," in Contributions of Mathematical Analysis to the Approximate Solution of Partial Differential Equations, A. Miller Ed., Proceedings of the Centre for Mathematical Analysis, Australian National University, V. 7, 1984, pp.40-64.

WORK IN PROGRESS UNDER ARO SUPPORT

- b.1. "Laplace Transform Inversion Based on Using Only Values of the Laplace Transform on the Real Line", (with D.D. Ang and J.R. Lund), in preparation.
- b.2. "Rational Spline Interpolation", with H.P. Dikshit, in preparation.
- b.3. "Integral Equation Method of Solution of Problems in Potential Theory", with J. Schwing, in preparation.
- b.4. "Electromagnetic Scattering due to an Axially-Symmetric Body with Corners and Edges", in preparation.
- b.5. "A SINC Algorithm for the Solution of Cauchy-Type Singular Integral Equations", with B. Bialecki, in preparation.
- b.6. "Explicit Sinc Inversion of the Helmholtz Equation on a Half Space", in preparation.
- b.7. "A FORTRAN Algorithm for Solution of the Inverse Source Problem", with J. Hohmann, in preparation.
- b.8. "A Smooth Approximation Scheme on R^n ", with E. Cohen and R. Riesenfeld, in preparation.
- b.9. "A Poisson Solver In Two Dimensions", with R.B. Kearfott and K. Sikorski, in preparation.

SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED
STUDENTS WHO HAVE BEEN SUPPORTED BY THIS CONTRACT:

- DeeAnn Dorman, "Methods for Accelerating Convergence",
University of Utah, M.Sc. 1983.
- W. Faltenbacher, "Sinc Method of Solution of the Helmholtz
Equation on a Half Space", University of Utah Ph.D 1984.
- B. Bialecki, "SINC Methods of Solving CSIE and Inverse
Problems", to complete Ph.D. in 1987.
- M. Stromberg, "Solution of Shock Problems via Extrapolation
to the Limit", to complete Ph.D. in 1987.

BODY OF THE REPORT

1. THE PROBLEM

In this proposal we studied the inversion of the two
integral equations,

$$(1.1) \quad u(\vec{r}) = u^i(\vec{r}) + k^2 \iiint_V G(\vec{r} - \vec{r}') f(\vec{r}') u(\vec{r}') d\vec{r}'$$

and

$$(1.2) \quad \bar{E}(\vec{r}) = \bar{E}^i(\vec{r}) + k^2 \iiint_V [I - \nabla' \nabla' G(\vec{r} - \vec{r}')] f(\vec{r}') \bar{E}(\vec{r}') d\vec{r}'$$

Here, Eq. (1.1) (rsp. (1.2)) obtains from the Helmholtz differen-
tial equation (rsp. Maxwell differential equations). Inversion
involves the reconstruction of the complex scalar function $f(\vec{r})$
by applying inputs $u^i(\vec{r})$ (rsp. $\bar{E}^i(\vec{r})$) and then measuring $u(\vec{r})$
(rsp. $\bar{E}(\vec{r})$) on the boundary of V .

2. REMARK: In carrying out the solution to these problems,
nearly all of our effort was directed towards the inversion of
Eq. (1.1) for the following reasons:

- (a) Equation (1.1) is a scalar equation, and hence the amount of
work, the size of the computer program and storage, and the
amount of time required to run a program are all considerably
less than the corresponding ones for Eq. (1.2).
- (b) Equation (1.2) has higher order singularities than Eq. (1.1),
and the correct treatment of these requires additional

computer code. In particular, it requires the evaluation of integrals of the type in (1.1) as well as the evaluation of principal value-type integrals. In the references [a.16,b.5] above, we developed the tools which are required to handle the principal value-type singularities.

- (c) Except for the problem size and the evaluation of the principal value integrals, all other aspects of the inversion of (1.2) are the same as those for (1.1). That is, having developed an effective algorithm for the inversion of (1.1), we can, in effect, "write down" the corresponding one for the inversion of (1.2).

3. BRIEF SUMMARY OF RESULTS OBTAINED

During the duration of this proposal, we have developed some effective algorithms for inverting Eq. (1.1), which are described in the papers [a.4,a.5,a.6,a.10,a.11,a.12,a.13]. The other papers listed above, which describe work that was carried out with at least partial support of this proposal, involved the development of new mathematics and new algorithms that were required for the inversion of Eqs. (1.1) or (1.2).

4. MORE DETAILED SUMMARY OF RESULTS OBTAINED

Two main type of algorithms were developed for the inversion of Eqs. (1.1) and (1.2). We summarize these for our purposes of application to the inversion of Eq. (1.1) only.

4.1 Extrapolation to the Limit.

This method is described in the papers [a.10,a.11] above. It has the desirable feature that it does not require the computation of the field u . Since the computation of u is very time consuming, this, in my opinion, is by far the best method known to date. It has the disadvantage that it requires accurate data, but getting such data is, I believe, an engineering problem which can be resolved fairly simply.

The method is based on the geometric optics approximation to the solution of (1.1), which we can write in the form

$$(4.1) \quad \varphi(k) \equiv \frac{1}{ik} \log \frac{u(\bar{r})}{u'(\bar{r})} \Big|_{\bar{r}_s}^{\bar{r}_d} = \int_{\mathcal{P}} (1+f)^{1/2} ds + O(k^{-\sigma}), \quad k \rightarrow \infty.$$

Here $\bar{r}_s$ is the source point, i.e., the location where the source $u'(\bar{r})$ is generated, or where it enters the volume V , $\bar{r}_d$ is the detector point, where the output $u(\bar{r})$ is measured, $\mathcal{P}$ is the ray path of the sound wave, which connects $\bar{r}_s$ to $\bar{r}_d$, and σ is a positive number, which depends on the smoothness of $f(\bar{r})$ on $\mathcal{P}$ ($f \in \text{Lip}_{2\sigma}(\mathcal{P})$).

The reduction of $\varphi(k)$ to $\varphi(\infty)$ was accomplished via an extrapolation to the limit. The proof that this process actually works was far from trivial. In the past, the effective use of rational functions by applied mathematicians and other scientists was based on a "gut feeling", i.e., rational approximation or extrapolation to the limit based on rational approximation would at times work much better than polynomial approximation, or extrapolation to the limit based on polynomial approximation, but no a priori conditions were previously known which would enable us to determine exactly when one or the other actually works. The paper [a.1] above solves this problem, i.e., it is now possible to determine a priori when we can expect rational approximation to the limit to work well, or when we can expect polynomial approximation to the limit to work well. In order to show that rational extrapolation to the limit works well for purposes of reducing $\varphi(k)$ above to $\varphi(\infty)$, it was necessary to show that the function φ is a restriction to $(0, \infty)$ of a function which is analytic and bounded in a sector $\{k \in \mathbb{C}: |k| > k_0 > 0, |\arg(k)| < d, d > 0\}$. Then, under the condition that $\sigma > 0$ (indeed, we believe that in applications one always has $\sigma > 1/4$), we can carry out accurate extrapolation to the limit via the Thiele algorithm, and thus accurately evaluate $\varphi(\infty)$. For example, given $\varphi(k)$ accurate to 8 significant figures for 7 equi-spaced frequencies in the range $1 \text{ Mh} < \text{freq} < 4 \text{ Mh}$, ($k = 2(\pi)(\text{freq})/c_0$ where c_0 denotes the sound speed in the medium surrounding the body), we were able to find $\varphi(\infty)$ accurate to 8 significant figures. However, it was necessary to know $\varphi(k)$ in the above range of frequencies to at least four significant figures; otherwise it was not always possible to accurately determine $\varphi(\infty)$. Various methods were described in the papers [a.10, a.11] for combating the presence of noise in the data.

Once $\varphi(\infty)$ was known, it was possible to accurately reconstruct $f(\vec{r})$ via ray path algorithms.

4.2. Inversion via Galerkin's Method and Solution of Nonlinear Equations.

In these methods [a.4, a.5, a.6, a.12, a.13] Sinc approximations (most of these were summarized in ref. [c.1] below) which were developed by Stenger under previous ARO support and in [a.17] were used in algorithms developed by engineering co-workers, under consultation with Stenger, to reduce the integral equation (1.1) to a system of nonlinear algebraic equations. Both u and f were computed iteratively at each point of V , each increasing in accuracy after every iteration. Besides having obvious advantages for purposes of approximating solutions of differential and integral equations (see [c.1]) the Sinc approximation procedures have the additional desirable feature that they can tolerate an incredibly large amount of noise in the data, such as, for example, a 12% noise to signal ratio.

In future developments of these algorithms it is hoped that the efficient method of obtaining the forward solution to Eq. (1.1) which was developed in W. Faltenbacher's thesis (see above) may be used to further increase the efficiency of the iterative method of solution referred to in the above paragraph.

4.3. The Study of Transients in Forward Solutions

The work of the papers [a.14,a.15] above mainly involved the advancement of geophysicists art of "interpretation" via a new technique of studying the transient response. At this point, we have not attempted to combine the new results of these papers for purposes of developing new direct methods of inversion.

REFERENCE

- c.1 Numerical Methods Based on the Whittaker Cardinal, or Sinc Functions, SIAM Rev. 23 (1981) 165-224.

END

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