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This document consists of the text and slides of an invited paper presented at the 112th Meeting of the Acoustical Society of America, held in Anaheim, California, from 8 to 12 December 1986. The following abstract was published in the program for the 112th Meeting.

The utility of wavevector-frequency spectral analysis for the description and interpretation of hydroacoustic and structural-acoustic fields has been amply demonstrated over the past decade in both theoretical and experimental applications. In the majority of these applications, the statistics of the random fields of interest were assumed to be stationary and homogeneous. While many of the hydroacoustic and structural-acoustic fields of practical interest can be considered stationary, few can be considered homogeneous.

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19. ABSTRACT (Cont'd.)

Structural-acoustic fields are nonhomogeneous owing to the space-varying nature (e.g., boundaries) of practical structures. In hydroacoustics, the natural growth of the turbulent boundary layer results in a nonhomogeneous pressure field at the boundary. This paper addresses the application of wavevector-frequency analysis to the description and interpretation of nonhomogeneous, but stationary, fields. Whereas only one definition of the wavevector-frequency spectrum exists for a homogeneous, stationary field, several alternative definitions of the wavevector-frequency spectrum are possible for the nonhomogeneous, stationary field. The utility of these various spectral forms for the analysis and interpretation of nonhomogeneous, stationary fields is assessed.

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Wavevector-Frequency Spectra Of Nonhomogeneous Fields

A Paper Presented at the 112th Meeting Of the Acoustical Society of America, 8-12 December 1986, Anaheim, California

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Preface

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WAVEVECTOR–FREQUENCY SPECTRA OF NONHOMOGENEOUS FIELDS

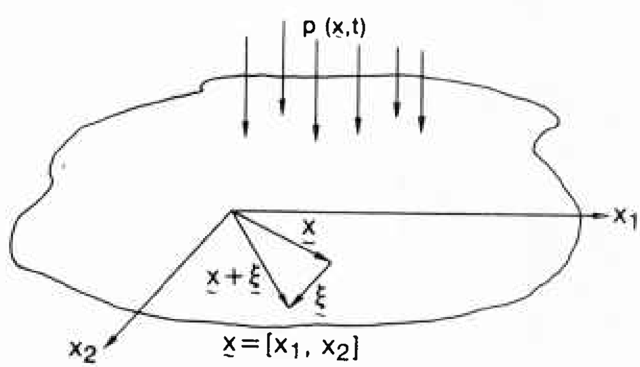
INTRODUCTION

The utility of the wavevector–frequency spectrum for description and interpretation of stationary and homogeneous fields has been amply demonstrated over the past decade. However, the fields associated with the flow–induced vibration of structures are generally nonhomogeneous. This nonhomogeneity can result from either the space–varying nature of practical structures or a nonhomogeneity of the turbulent flow field that excites the structure.

This paper presents alternative forms of wavevector–frequency spectra for nonhomogeneous, but stationary, fields and assesses, in a preliminary fashion, the practical utility of these spectral forms.

DEFINITIONS

Slide 1



SPACE-TIME CORRELATION OF THE PRESSURE FIELD (Q_{pp})

$$Q_{pp}(\underline{x}, \underline{\xi}, t, \tau) = E\left\{ p(\underline{x}, t)\, p(\underline{x}+\underline{\xi}, t+\tau) \right\}$$

STATIONARY FIELD (CORRELATION INDEPENDENT OF ABSOLUTE TIME, t)

$$Q_{pp}(\underline{x}, \underline{\xi}, t, \tau) = Q_{pp}(\underline{x}, \underline{\xi}, \tau)$$

HOMOGENEOUS FIELD (CORRELATION INDEPENDENT OF ABSOLUTE SPATIAL POSITION, $\underline{x}$)

$$Q_{pp}(\underline{x}, \underline{\xi}, t, \tau) = Q_{pp}(\underline{\xi}, t, \tau)$$

HOMOGENEOUS AND STATIONARY FIELD (CORRELATION INDEPENDENT OF $\underline{x}$ AND t)

$$Q_{pp}(\underline{x}, \underline{\xi}, t, \tau) = Q_{pp}(\underline{\xi}, \tau)$$

The field illustrated at the top of slide 1 is the pressure at the surface of the plane defined by the x_1 and x_2 axes. Although we use a pressure field for purposes of illustration, the field could just as well be displacement, stress, or any other quantity of interest.

The correlation of the pressure field over the plane of interest is defined as the average value, over many repetitions of the measurement, of the product of (1) the pressure at the vector location $\underset{\sim}{x}$ and time t and (2) the pressure at the vector location $\underset{\sim}{x} + \underset{\sim}{\xi}$ and the time $t + \tau$. The average over the ensemble of experiments is denoted by E.

In general, the correlation is a function of (1) the absolute spatial position and time of one observation of the pressure and (2) the spatial separation vector and the time difference between observations. A stationary field is one in which the correlation and all other statistical moments are independent of the absolute time of observation, t, and depend only on the time difference, τ, between observations. Similarly, a homogeneous field is one in which the correlation is independent of absolute spatial position, $\underset{\sim}{x}$, and depends only on the spatial separation vector, $\underset{\sim}{\xi}$, between observations.

In this paper, all fields are assumed to be stationary. Therefore, the correlation of nonhomogeneous fields will have the functional form of the second expression shown in this slide, and the correlation of homogeneous fields will have the form of the expression shown at the bottom of the slide.

Slide 2

DEFINITIONS OF WAVEVECTOR-FREQUENCY SPECTRA

STATIONARY HOMOGENEOUS FIELD

$$\Phi_p^H(\underset{\sim}{k}, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underset{\sim}{\xi}, \tau)\, e^{-i(\underset{\sim}{k}\cdot\underset{\sim}{\xi}+\omega\tau)}\, d\underset{\sim}{\xi}\, d\tau$$

WAVEVECTOR $\underset{\sim}{k} = [k_1, k_2]$; CIRCULAR FREQUENCY $\omega = 2\pi f$

STATIONARY NONHOMOGENEOUS FIELD

SPACE-VARYING SPECTRUM (K_p)

$$K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau)\, e^{-i(\underset{\sim}{k}\cdot\underset{\sim}{\xi}+\omega\tau)}\, d\underset{\sim}{\xi}\, d\tau$$

SPACE-AVERAGED SPECTRUM (Φ_p)

$$\Phi_p(\underset{\sim}{k}, \omega; A) = \frac{1}{A}\int_A K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega)\, d\underset{\sim}{x}$$

TWO-WAVEVECTOR SPECTRUM ($\mathcal{K}_p$)

$$\mathcal{K}_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega) = \int_{-\infty}^{\infty} K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega)\, e^{-i\underset{\sim}{\mu}\cdot\underset{\sim}{x}}\, d\underset{\sim}{x}$$

$$\underset{\sim}{\mu} = [\mu_1, \mu_2]$$

For the homogeneous, stationary field, the wavevector–frequency spectrum is defined as the multiple Fourier transform of the space–time correlation on the spatial separation vector, $\underset{\sim}{\xi}$, and the time difference, τ, as shown at the top of slide 2. Here, Φ_p is the wavevector–frequency spectrum and the superscript H designates the homogeneous field.

The wavevector, $\underset{\sim}{k}$, is the Fourier conjugate variable of the spatial separation vector, $\underset{\sim}{\xi}$, and the circular frequency, ω, is the conjugate variable of the time difference, τ.

By arguments similar to those used by Bendat and Piersol in defining frequency spectra for nonstationary fields, three alternative forms for the wavevector–frequency spectrum can be defined for nonhomogeneous, stationary fields.

The space-varying wavevector-frequency spectrum, designated by K_p, is defined by the same multiple Fourier transformation of the correlation field used for the homogeneous spectrum. However, because the nonhomogeneous correlation field is a function of the absolute spatial vector, $\underset{\sim}{X}$, the space-varying spectrum also varies with $\underset{\sim}{X}$.

The space-averaged spectrum, Φ_p, is simply the average value of the space-varying spectrum over some area, A, in absolute space.

The two-wavevector-frequency spectrum, designated by $\varkappa_p$, introduces a second wavevector, $\underset{\sim}{\mu}$, by an additional spatial Fourier transformation of the correlation field. This second wavevector is the conjugate variable of the absolute spatial vector, $\underset{\sim}{X}$.

Slide 3

PROPERTIES OF WAVEVECTOR-FREQUENCY SPECTRA

SPECTRAL TYPE	SYMBOL	MATHEMATICAL FORM	SYMMETRY PROPERTIES
HOMOGENEOUS	$\Phi_p^H(\underset{\sim}{k}, \omega)$	REAL	$\Phi_p^H(-\underset{\sim}{k}, -\omega) = \Phi_p^H(\underset{\sim}{k}, \omega)$
SPACE-VARYING	$K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega)$	COMPLEX	$K_p(\underset{\sim}{x}, -\underset{\sim}{k}, -\omega) = K_p^*(\underset{\sim}{x}, \underset{\sim}{k}, \omega)$ $K_p(\underset{\sim}{x}, -\underset{\sim}{k}, \omega) = K_p^*(\underset{\sim}{x}, \underset{\sim}{k}, \omega)$
SPACE-AVERAGED	$\Phi_p(\underset{\sim}{k}, \omega\ ;\ A)$	REAL	$\Phi_p(-k, -\omega\ ;\ A) = \Phi_p(\underset{\sim}{k}, \omega\ ;\ A)$
TWO-WAVEVECTOR	$K_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega)$	COMPLEX	$K_p(-\underset{\sim}{\mu}, -\underset{\sim}{k}, -\omega) = K_p^*(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega)$ $K_p(-\underset{\sim}{\mu}, -\underset{\sim}{k}, \omega) = K_p^*(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega)$

$\bigstar$ DENOTES COMPLEX CONJUGATE

Slide 3 presents, in tabular form, some of the properties of the spectral forms. Here we see that the homogeneous and space-averaged spectral forms are real functions of wavevector and frequency, whereas the space-varying and two-wavevector forms are complex. Bendat and Piersol refer to such complex spectra that result from Fourier transformation of the correlation field as "generalized spectra."

For the real fields associated with physical processes, all spectral forms possess conjugate symmetry in the wavevector-frequency domain. Therefore, in practice, measurement of any spectral form over positive frequencies is sufficient to define the spectrum over all frequencies. Symmetries of the correlation function provide additional symmetry properties for the various spectral forms. These additional symmetries ensure that the local frequency spectral density of the field is real.

Slide 4

RELATION BETWEEN FREQUENCY SPECTRUM AND WAVEVECTOR-FREQUENCY SPECTRA

DEFINITION OF LOCAL FREQUENCY SPECTRAL DENSITY

$$\Phi_p(\underset{\sim}{x}, \omega) = \int_{-\infty}^{\infty} Q_{pp}(\underset{\sim}{x}, \underset{\sim}{o}, \tau) e^{-i\omega\tau} \, d\tau$$

HOMOGENEOUS FORM

$$\Phi_p^H(\omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \Phi_p^H(\underset{\sim}{k}, \omega) \, d\underset{\sim}{k}$$

NONHOMOGENEOUS FORMS

SPACE VARYING:

$$\Phi_p(\underset{\sim}{x}, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) \, d\underset{\sim}{k}$$

SPACE-AVERAGED:

$$\frac{1}{A} \int_A \Phi_p(\underset{\sim}{x}, \omega) \, d\underset{\sim}{x} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \Phi_p(\underset{\sim}{k}, \omega; A) \, d\underset{\sim}{k}$$

TWO-WAVEVECTOR:

$$\Phi_p(\underset{\sim}{x}, \omega) = \frac{1}{(2\pi)^4} \iint_{-\infty}^{\infty} K_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega) e^{i\underset{\sim}{\mu}\cdot\underset{\sim}{x}} \, d\underset{\sim}{\mu} \, d\underset{\sim}{k}$$

The local frequency spectral density is the temporal Fourier transform of the correlation field at zero spatial separation, as shown at the top of slide 4.

By this definition and the definitions of the various spectral forms, the relationships, shown here, between the local frequency spectral density and the various wavevector-frequency spectra were established.

In all cases, the local frequency spectrum is related to an integral of the wavevector-frequency spectrum over the wavevector variable, $\underset{\sim}{k}$, the transform variable associated with the spatial separation vector, $\underset{\sim}{\xi}$. In the case of the two-wavevector spectrum, the relationship also involves a Fourier transformation on the wavevector, $\underset{\sim}{\mu}$, associated with the absolute spatial vector, $\underset{\sim}{x}$.

Note that the frequency spectrum of the homogeneous field is constant over all space and is thereby independent of the absolute spatial vector, $\underset{\sim}{x}$.

Note also that only a spatial average of the local frequency spectrum can be recovered from the space-averaged wavevector spectrum.

Recall that our objective is to determine the utility of the various wavevector-frequency spectral forms for description and interpretation of nonhomogeneous fields.

With regard to description, the space-varying and two-wavevector forms of wavevector-frequency spectra are Fourier transforms of the space-time correlation field and are therefore informationally equivalent to the correlation field. The space-averaged form is not informationally equivalent to the correlation field because the spatial averaging obscures the nonhomogeneous characteristics of the field. Therefore, we can eliminate the space-averaged form from further consideration.

It is now desirable to determine if the remaining two spectral forms offer advantages for analyzing or interpreting nonhomogeneous fields. In linear hydroacoustic and structural-acoustic systems, nonhomogeneous output fields result from only two sources: (1) a nonhomogeneous input (or excitation) to the system or (2) space-varying properties of the system. The utility of the remaining two spectral forms was evaluated on the basis of the mathematical simplicity afforded by these forms in the analysis of linear space-time systems.

LINEAR SPACE–TIME SYSTEMS

Slide 5

LINEAR SPACE-TIME SYSTEMS

$$\underset{\text{(INPUT)}}{f(\underset{\sim}{x}, \tau)} \longrightarrow \boxed{g(\underset{\sim}{x}, t; \underset{\sim}{\xi}, \tau)} \longrightarrow \underset{\text{(OUTPUT)}}{p(\underset{\sim}{x}, t)}$$

IMPULSE RESPONSE: $g(\underset{\sim}{x}, t; \underset{\sim}{\xi}, \tau)$ IS THE OUTPUT OF THE SYSTEM AT $\underset{\sim}{x}$ AND t DUE TO AN IMPULSIVE EXCITATION (INPUT) APPLIED AT $\underset{\sim}{x} - \underset{\sim}{\xi}$ AND $t - \tau$.

INPUT-OUTPUT RELATIONSHIPS

GENERAL

$$p(\underset{\sim}{x}, t) = \iint_{-\infty}^{\infty} g(\underset{\sim}{x}, t; \underset{\sim}{\xi}, \tau)\, f(\underset{\sim}{x} - \underset{\sim}{\xi}, t - \tau)\, d\underset{\sim}{\xi}\, d\tau$$

SPACE-VARYING, TIME-INVARIANT SYSTEM (g INDEPENDENT OF t)

$$p(\underset{\sim}{x}, t) = \iint_{-\infty}^{\infty} g_v(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau)\, f(\underset{\sim}{x} - \underset{\sim}{\xi}, t - \tau)\, d\underset{\sim}{\xi}\, d\tau$$

SPACE- AND TIME-INVARIANT SYSTEM (g INDEPENDENT OF $\underset{\sim}{x}$ AND t)

$$p(\underset{\sim}{x}, t) = \iint_{-\infty}^{\infty} g_i(\underset{\sim}{\xi}, \tau)\, f(\underset{\sim}{x} - \underset{\sim}{\xi}, t - \tau)\, d\underset{\sim}{\xi}\, d\tau$$

The concept of linear space–time systems is illustrated at the top of slide 5. Here, the system, characterized by the impulse response g, is excited by a space–time field, f, resulting in an output field designated by p. As an example in flow–induced vibrations, the system could be a plate, the input (or excitation) could be the pressure field of the turbulent boundary layer, and the output could be the resulting displacement field of the plate.

The impulse response of the system is defined as the output of the system at location $\underset{\sim}{x}$ and time t due to an impulsive input applied at $\underset{\sim}{x} - \underset{\sim}{\xi}$ and $t - \tau$. In general, the impulse response is a function of the four variables $\underset{\sim}{x}$, $\underset{\sim}{\xi}$, t, and τ.

By the principle of superposition for linear systems, the output field of the system can be expressed in terms of the input field and the impulse response according to the first expression in this slide.

A system with constant properties over time is called a time–invariant system. The impulse response of a time–invariant system is independent of the absolute time of observation, t, and depends only on the time difference, τ, between excitation and observation. For this study, we restricted our attention to time–invariant systems.

The expression shown in the middle of this slide defines the input–output relation for a time–invariant, but space–varying, system. The subscript "v" on the impulse response denotes the space–varying nature of the system. Note that the impulse response, in this case, is a function of both the absolute spatial position of observation, $\underset{\sim}{x}$, and the separation vector, $\underset{\sim}{\xi}$, between the points of excitation and observation.

A space–invariant system is one in which the system properties are constant over all space. A uniform, infinite plate is one example of a space–invariant system. In a space–invariant system, the impulse response is independent of the absolute position of observation, $\underset{\sim}{x}$, and depends only on the spatial separation, $\underset{\sim}{\xi}$, between excitation and observation.

The input–output relation for a space– and time–invariant system is shown at the bottom of this slide. Here, the subscript "i" is used to identify the space–invariant impulse response.

By use of these input–output relationships and the various descriptors of random fields presented previously, the response of space–varying and space–invariant systems to homogeneous and nonhomogeneous inputs can be expressed in several alternative forms. The next few slides compare the mathematical forms of these input–output relations for specific systems and inputs.

SPACE–INVARIANT SYSTEM, HOMOGENEOUS INPUT

Slide 6

INPUT-OUTPUT RELATIONS FOR SPACE- AND TIME-INVARIANT SYSTEM, HOMOGENEOUS INPUT

CORRELATION (SPACE-TIME):

$$Q_{pp}(\underset{\sim}{\xi}, \tau) = \iiiint_{-\infty}^{\infty} Q_{ff}(\underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \tau + \theta_1 - \theta_2)\, g_i(\underset{\sim}{\epsilon}, \theta_1)\, g_i(\underset{\sim}{\eta}, \theta_2)\, d\underset{\sim}{\epsilon}\, d\theta_1 d\underset{\sim}{\eta}\, d\theta_2$$

CROSS-SPECTRAL DENSITY (SPACE-FREQUENCY):

$$S_{pp}(\xi, \omega) = \iint_{-\infty}^{\infty} S_{ff}(\underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \omega)\, \Gamma_i(\underset{\sim}{\epsilon}, -\omega)\, \Gamma_i(\underset{\sim}{\eta}, \omega)\, d\underset{\sim}{\epsilon}\, d\underset{\sim}{\eta}$$

WHERE

$$\Gamma_i(\underset{\sim}{\xi}, \omega) = \int_{-\infty}^{\infty} g_i(\underset{\sim}{\xi}, \tau)\, e^{-i\omega\tau}\, d\tau$$

WAVEVECTOR-FREQUENCY SPECTRUM: (WAVEVECTOR-FREQUENCY)

$$\Phi_p^H(\underset{\sim}{k}, \omega) = \Phi_f^H(\underset{\sim}{k}, \omega)\, |G_i(\underset{\sim}{k}, \omega|^2$$

WHERE

$$G_i(\underset{\sim}{k}, \omega) = \iint_{-\infty}^{\infty} g_i(\underset{\sim}{\xi}, \tau)\, e^{-i(\underset{\sim}{k}\cdot\underset{\sim}{\xi} + \omega\tau)}\, d\underset{\sim}{\xi}\, d\tau$$

Slide 6 presents alternative input–output relationships for a space–invariant system excited by a homogeneous field. (Recall that all systems treated here are assumed to be time–invariant.)

In this case, the output field is also homogeneous and can be alternatively described in terms of the correlation, the cross–spectral density, or the homogeneous form of the wavevector–frequency spectrum. The cross–spectral density is the temporal Fourier transform of the correlation.

Note that the correlation of the output field is related to the correlation of the input and the impulse response of the system by a quadruple integral. The cross–spectral density of the output is related to that of the input and the space–frequency response of the system by a double integral. The space–frequency response, denoted by Γ_i, is the temporal transform of the impulse response and defines the response of the system to a point excitation that varies sinusoidally in time with frequency ω.

The wavevector–frequency spectrum of the output is related to the wavevector–frequency spectrum of the input and the wavevector–frequency response of the system by the simple product form shown at the bottom of the slide. The wavevector–frequency response, G_i, defines the response of the system to excitation by a plane wave characterized by the wavevector $\underset{\sim}{k}$ and the frequency ω.

Clearly, for a space–invariant system excited by a homogeneous field, the wavevector–frequency representation of the input and output fields results in the simplest mathematical description of the system. Given any two of the wavevector–frequency fields in this expression, the third can be determined by simple algebra.

SPACE–INVARIANT SYSTEM, NONHOMOGENEOUS INPUT

Slide 7

INPUT-OUTPUT RELATIONS FOR
SPACE- AND TIME-INVARIANT SYSTEM, NONHOMOGENEOUS INPUT

CORRELATION (SPACE-TIME):

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau) = \iiiint_{-\infty}^{\infty} Q_{ff}(\underset{\sim}{x} - \underset{\sim}{\epsilon}, \underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \tau + \theta_1 - \theta_2)\, g_i(\underset{\sim}{\epsilon}, \theta_1)\, g_i(\underset{\sim}{\eta}, \theta_2)\, d\underset{\sim}{\xi}\, d\theta_1\, d\underset{\sim}{\eta}\, d\theta_2$$

CROSS-SPECTRAL DENSITY (SPACE-FREQUENCY):

$$S_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \omega) = \iint_{-\infty}^{\infty} S_{ff}(\underset{\sim}{x} - \underset{\sim}{\epsilon}, \underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \omega)\, \Gamma_i(\underset{\sim}{\epsilon}, -\omega)\, \Gamma_i(\underset{\sim}{\eta}, \omega)\, d\underset{\sim}{\epsilon}\, d\underset{\sim}{\eta}$$

SPACE-VARYING WAVEVECTOR-FREQUENCY SPECTRUM: (SPACE-WAVEVECTOR—FREQUENCY)

$$K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} K_f(\underset{\sim}{z}, \underset{\sim}{k}, \omega)\, G_i(\underset{\sim}{k}, \omega)\, G_i(\underset{\sim}{\mu} - \underset{\sim}{k}, -\omega)\, e^{i\underset{\sim}{\mu}\cdot(\underset{\sim}{x} - \underset{\sim}{z})}\, d\underset{\sim}{\mu}\, d\underset{\sim}{z}$$

TWO-WAVEVECTOR-FREQUENCY SPECTRUM (WAVEVECTOR-FREQUENCY)

$$K_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega) = K_f(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega)\, G_i(\underset{\sim}{k}, \omega)\, G_i(\underset{\sim}{\mu} - \underset{\sim}{k}, -\omega)$$

Slide 7 compares alternative input–output relationships for a space–invariant system excited by a nonhomogeneous field. In this case, the output field is also nonhomogeneous, and it is evident, by comparison of the four descriptions offered, that the two–wavevector–frequency spectrum affords the simplest relation between the input and output fields.

By this simple algebraic form, the two–wavevector–frequency spectrum of either the input or output field is easily predicted, given the two–wavevector spectrum of one field and the wavevector–frequency response of the system. If the two–wavevector spectra of both input and output fields are known, only the particular product of the wavevector–frequency responses of the system shown in the last expression can be determined. However, from this product, the wavevector–frequency response can be specified to within a phase factor.

SPACE–VARYING SYSTEM, HOMOGENEOUS INPUT

Slide 8

INPUT–OUTPUT RELATIONS FOR SPACE-VARYING SYSTEM, HOMOGENEOUS INPUT

CORRELATION (SPACE-TIME):

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau) = \iiiint_{-\infty}^{\infty} Q_{ff}(\underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \tau + \theta, -\theta_2)\, g_v(\underset{\sim}{x}, \underset{\sim}{\epsilon}, \theta_1)\, g_v(\underset{\sim}{x} + \underset{\sim}{\xi}, \underset{\sim}{\eta}, \theta_2)\, d\underset{\sim}{\epsilon}\, d\theta_1\, d\underset{\sim}{\eta}\, d\theta_2$$

CROSS-SPECTRAL DENSITY (SPACE-FREQUENCY):

$$S_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \omega) = \iint_{-\infty}^{\infty} S_{ff}(\underset{\sim}{\xi} + \underset{\sim}{\epsilon} - \underset{\sim}{\eta}, \omega)\, \Gamma_v(\underset{\sim}{x}, \underset{\sim}{\epsilon}, -\omega)\, \Gamma_v(\underset{\sim}{x} + \underset{\sim}{\xi}, \underset{\sim}{\eta}, \omega)\, d\underset{\sim}{\epsilon}\, d\underset{\sim}{\eta}$$

WHERE Γ_v IS THE TEMPORAL FOURIER TRANSFORM OF g_v.

SPACE-VARYING WAVEVECTOR-FREQUENCY SPECTRUM:

$$K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \Phi_f^H(\underset{\sim}{x}, \omega)\, \mathcal{G}_v(\underset{\sim}{x}, -\underset{\sim}{\gamma}, -\omega)\, \mathcal{G}_v(\underset{\sim}{x} + \underset{\sim}{\eta}, \underset{\sim}{\gamma}, \omega)\, e^{i(\underset{\sim}{\gamma} - \underset{\sim}{k})\cdot\eta}\, d\underset{\sim}{\eta}\, d\underset{\sim}{\gamma}$$

WHERE $\mathcal{G}_v(\underset{\sim}{x}, \underset{\sim}{k}, \omega)$ IS THE DOUBLE FOURIER TRANSFORM OF $g_v(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau)$ ON $\underset{\sim}{\xi}$ AND τ.

TWO-WAVEVECTOR-FREQUENCY SPECTRUM:

$$K_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \Phi_f^H(\underset{\sim}{\gamma}, \omega)\, G_v(\underset{\sim}{\mu} + \underset{\sim}{\gamma} - \underset{\sim}{k}, -\underset{\sim}{\gamma}, -\omega)\, G_v(\underset{\sim}{k} - \underset{\sim}{\gamma}, \underset{\sim}{\gamma}, \omega)\, d\underset{\sim}{\gamma}$$

WHERE G_v IS THE TRIPLE FOURIER TRANSFORM OF $g_v(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau)$.

Slide 8 presents the various input–output relationships for a space–varying system excited by a homogeneous field. Here, the various descriptors of the nonhomogeneous output field are related to corresponding descriptors of the homogeneous input field and the system response. The mathematically simplest form of input–output relation is that shown at the bottom of the slide, which relates the two–wavevector–frequency spectrum of the nonhomogeneous output to the wavevector–frequency spectrum of the homogeneous input field and the two–wavevector–frequency response of the system by a single integral.

The two–wavevector–frequency response of the system, $G_V(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega)$, is the wavevector–frequency transform of the output field at the wavevector $\underset{\sim}{\mu} + \underset{\sim}{k}$ resulting from excitation of the system by a plane wave characterized by the wavevector $\underset{\sim}{k}$ and the frequency ω. Thus, the two–wavevector–frequency response defines the degree to which the input wavevector is scattered into other wavevectors by the space–varying properties of the structure.

Note that if the wavevector–frequency spectrum of the homogeneous input field and the two–wavevector–frequency response of the structure are specified, the two–wavevector–frequency spectrum of the output field can, in principle, be obtained by the integral at the bottom of this slide.

If the two–wavevector–frequency spectrum of the output and the two–wavevector–frequency response of the system are known, the determination of the wavevector spectrum of the homogeneous input requires solution of an integral equation.

Note, finally, that to determine the product of two–wavevector–frequency responses in the last expression on this slide, the system must be excited by a homogeneous field comprised of a single wavevector and a single frequency. By knowledge of this product, the corresponding two–wavevector–frequency response of the system can be determined to within a phase factor.

SPACE–VARYING SYSTEM, NONHOMOGENEOUS INPUT

Slide 9

INPUT-OUTPUT RELATIONS FOR SPACE-VARYING SYSTEM, NONHOMOGENEOUS INPUT

CORRELATION (SPACE-TIME):

$$Q_{pp}(\underset{\sim}{x},\underset{\sim}{\xi},\tau) = \iiiint_{-\infty}^{\infty} Q_{ff}(\underset{\sim}{x}-\underset{\sim}{\epsilon},\underset{\sim}{\xi}+\underset{\sim}{\epsilon}-\underset{\sim}{\eta},\tau+\theta_1-\theta_2)\, g_v(\underset{\sim}{x},\underset{\sim}{\epsilon},\theta_1)\, g_v(\underset{\sim}{x}+\underset{\sim}{\xi},\underset{\sim}{\eta},\theta_2)\, d\underset{\sim}{\epsilon}\, d\theta_1\, d\underset{\sim}{\eta}\, d\theta_2$$

CROSS-SPECTRAL DENSITY (SPACE-FREQUENCY):

$$S_{pp}(\underset{\sim}{x},\underset{\sim}{\xi},\omega) = \iint_{-\infty}^{\infty} S_{ff}(\underset{\sim}{x}-\underset{\sim}{\epsilon},\underset{\sim}{\xi}+\underset{\sim}{\epsilon}-\underset{\sim}{\eta},\omega)\,\Gamma_v(\underset{\sim}{x},\underset{\sim}{\epsilon},-\omega)\,\Gamma_v(\underset{\sim}{x}+\underset{\sim}{\xi},\underset{\sim}{\eta},\omega)\, d\underset{\sim}{\epsilon}\, d\underset{\sim}{\eta}$$

SPACE-VARYING WAVEVECTOR-FREQUENCY SPECTRUM:

$$K_p(\underset{\sim}{x},\underset{\sim}{k},\omega) = \frac{1}{(2\pi)^4}\iiiint_{-\infty}^{\infty} K_f(\underset{\sim}{z},\underset{\sim}{\gamma},\omega)\,\mathscr{G}_v(\underset{\sim}{x},\underset{\sim}{\mu},-\gamma)\,\mathscr{G}_v(\underset{\sim}{x}+\underset{\sim}{\xi},\underset{\sim}{\gamma},\omega)\, e^{-i(\underset{\sim}{k}-\underset{\sim}{\gamma})\cdot\underset{\sim}{\xi}}\, e^{i\underset{\sim}{\mu}\cdot(\underset{\sim}{x}-\underset{\sim}{z})}\, d\underset{\sim}{z}\, d\underset{\sim}{\xi}\, d\underset{\sim}{\gamma}\, d\underset{\sim}{\mu}$$

TWO-WAVEVECTOR-FREQUENCY SPECTRUM:

$$K_p(\underset{\sim}{\mu},\underset{\sim}{k},\omega) = \frac{1}{(2\pi)^4}\iint_{-\infty}^{\infty} K_f(\underset{\sim}{\alpha},\underset{\sim}{\gamma},\omega)\, G(\underset{\sim}{\gamma}-\underset{\sim}{k}+\underset{\sim}{\mu}-\underset{\sim}{\alpha},\underset{\sim}{\alpha}-\underset{\sim}{\gamma},-\omega)\, G(\underset{\sim}{k}-\underset{\sim}{\gamma},\underset{\sim}{\gamma},\omega)\, d\underset{\sim}{\alpha}\, d\underset{\sim}{\gamma}$$

Slide 9 defines alternative input–output relations for a space–varying system excited by a nonhomogeneous field. In this situation, note that both the cross–spectral density and two–wavevector spectrum descriptions of the system output are related to the corresponding description of the input by double integrations, whereas the other descriptions are in the form of quadruple integrals. Further, note that the integrands of the cross–spectral and two–wavevector descriptions are of comparable mathematical complexity.

In the cross–spectral representation, the system is characterized by the space–varying frequency response, denoted by Γ_V. The space–varying frequency response is the temporal Fourier transform of the space–varying impulse response. In the two–wavevector representation, the system is characterized by the two–wavevector–frequency response described previously.

Given the knowledge of the appropriate description of the input field and the system response, the cross–spectral density or the two–wavevector–frequency spectrum of the output field can be obtained by performing the required double vector integrations.

However, if only the description of the output field and system response are known, one is faced with solving a double vector integral equation for the cross spectrum or two-wavevector spectrum of the input field. The kernel of this integral equation is the product of the appropriate system responses.

The kernel of an integral equation cannot be determined by knowledge of any single input and output field. Therefore, knowledge of the cross spectra or two-wavevector spectra of both the input and output fields from any single experiment is not sufficient to determine the associated product of system responses. However, by knowledge of the input and output fields from a carefully designed sequence of experiments, a sampling of the desired product of system responses can be determined in both space variables ($\underset{\sim}{x}$ and $\underset{\sim}{\xi}$) or both wavevector variables ($\underset{\sim}{\mu}$ and $\underset{\sim}{k}$) and in frequency. From this sampling of the kernel, a sampling of the desired system response can be determined to within an unknown phase factor.

SUMMARY AND CONCLUSIONS

Slide 10

SUMMARY

THREE ALTERNATIVE FORMS OF WAVEVECTOR-FREQUENCY SPECTRA FOR NONHOMOGENEOUS, STATIONARY FIELDS HAVE BEEN DEFINED. THESE WERE

- **SPACE-VARYING SPECTRUM**
- **SPACE-AVERAGED SPECTRUM**
- **TWO-WAVEVECTOR SPECTRUM.**

EVALUATION OF THESE FORMS FOR DESCRIPTION AND INTERPRETATION OF NONHOMOGENEOUS FIELDS REVEALED

- **ONLY THE SPACE-VARYING AND TWO-WAVEVECTOR FORMS ARE INFORMATIONALLY EQUIVALENT TO THE CORRELATION FIELD.**
- **THE TWO-WAVEVECTOR FORM PROVIDES THE MATHEMATICALLY SIMPLEST INPUT-OUTPUT RELATIONS FOR LINEAR SYSTEMS.**

In summary, we have defined three alternative forms of wavevector-frequency spectra for nonhomogeneous, but stationary, fields: a space-varying form, a space-averaged form, and a two-wavevector form.

The utility of these spectral forms for the description and interpretation of nonhomogeneous fields was evaluated by comparison with other descriptors of such fields.

With regard to description, it was found that only the space-varying and two-wavevector forms were informationally equivalent to the correlation field.

The interpretive utility of the various spectral forms was assessed on the basis of the mathematical simplicity of the input-output relationships of linear systems giving rise to nonhomogeneous fields. This comparison showed that the two-wavevector form consistently provided the simplest input-output relationships.

CONCLUSIONS

Slide 11

In conclusion, we have demonstrated that, in theory, the two–wavevector–frequency spectrum offers advantages over other descriptors for the analysis and interpretation of nonhomogeneous fields.

However, to determine whether these advantages can be realized in practice, several problems must be addressed. These problems include

(1) Development of techniques to measure the two–wavevector–frequency spectrum,

(2) Development of techniques to measure the two–wavevector–frequency response of practical space–varying systems, and

(3) Inversion of integral equations in input–output relationships.

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