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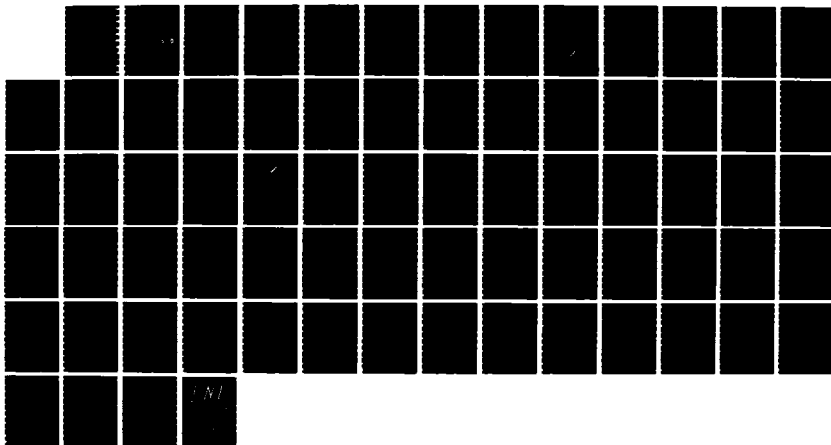
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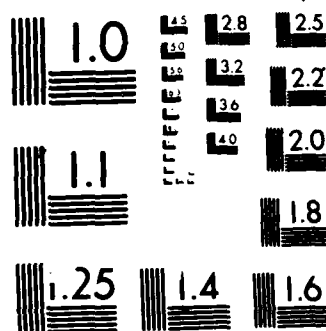
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CHARACTERIZATION OF STRIPLINE CROSSING BY TRANSVERSE
RESONANCE METHOD

TECHNICAL REPORT

TOMOKI UWANO AND TATSUO ITOH

JANUARY 1987

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CONTRACT NO. N00014-79-C-0553

UNIVERSITY OF TEXAS
DEPARTMENT OF ELECTRICAL ENGINEERING
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Characterization of Stripline Crossing by Transverse Resonance Method

Abstract

A method of analysis is described for characterizing the discontinuities made of orthogonally crossed two striplines on a suspended structure. The method of analysis is based on generalized transverse resonant technique extended here to 4-port configurations. The technique is used for determination of resonant structure at a given frequency and subsequently the equivalent circuit parameters of the discontinuities.

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1. INTRODUCTION OF THE PROBLEM

Stripline crossings of the multi-layer printed circuitboard are commonly used in digital circuit design. As the signal frequency gets higher due to high speed processors, an accurate wave analysis of the characteristics of the crossing becomes important. In addition, the crossing of strips on both sides of the suspended substrate often appears in microwave and millimeter wave integrated circuits [1]. To date little has been reported on the exact analysis of such structures.

The problem presented here is to characterize the discontinuities of orthogonally crossed two striplines. The structure to be analyzed is shown schematically in Fig. 1-1 along with the coordinate system. Two striplines are crossed orthogonally on opposite sides of the substrate. Auxiliary conducting planes are added to convert the structure to a closed one. Depending on which ground plane (at $z = -h_1$ or $z = c$) is associated with each strip, the structure may be regarded as consisting of either inverted or suspended striplines.

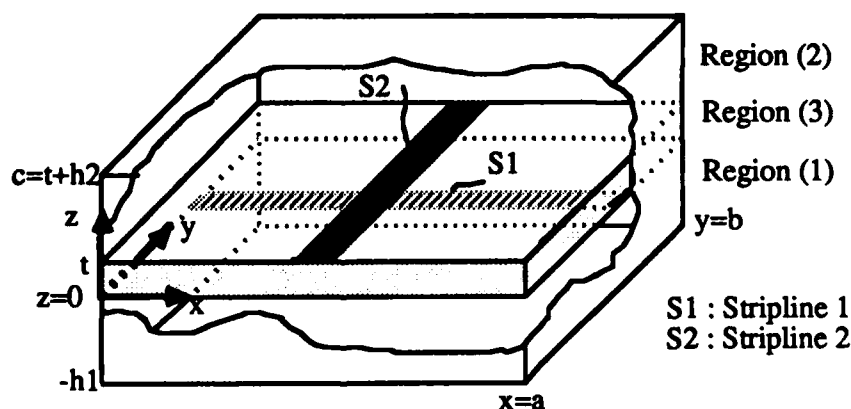


Fig. 1-1 Structure for the problem

It is assumed that each pair of opposing side walls does not influence electromagnetic fields guided by the strip parallel to the walls but only the field guided along the orthogonal directions. This assumption is valid as long as the field remains confined in the proximity of the two striplines. Thus, surface wave and radiation phenomena are excluded. For the sake of generality, two different dielectric layers with dielectric constant ϵ_1 and ϵ_2 are assumed in the region (1) and (2), respectively. The auxiliary walls are used for field analysis purpose. They permit the structure to be analyzed as a rectangular waveguide discontinuity problem.

The method for analysis is based on a generalized "transverse resonance technique" introduced for finline step discontinuity problem [2]. The technique is extended here to a 4-port configuration treated in this report. The method consists of two parts. First, the resonant structure created by auxiliary walls is described in terms of network representation containing a reactive 4-port. For a fixed resonant frequency, we try to find as many resonator sizes as required for extraction of 4-port matrix elements. The second part of analysis is a full-wave electromagnetic field analysis in which the resonant frequency is found as an eigenvalue problem. A unique feature of this second part of the analysis is to view the problem as the waveguide scattering for the waves traveling in the direction normal to the substrate surface.

2. CIRCUIT REPRESENTATION

2-1. Resonance Method

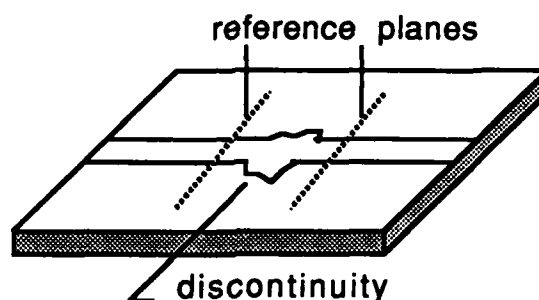


Fig. 2-1 Microstrip line with some discontinuities

In this section, the resonance method applied to 2-port network [3] is briefly described. Suppose we have a transmission line with some discontinuities as shown in Fig. 2-1. With appropriate reference planes for the input - output ports, the discontinuities can be represented as a 2-port network matrix form :

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \quad [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \quad (2-1)$$

where Z_{11} , Z_{12} , Z_{21} , Z_{22} are unknown parameters. In the assumption of lossless and reciprocal network, we have only three real unknowns in the matrix. If the two ports are reactively terminated, the entire circuit becomes a resonator. This is shown in Fig. 2-2 where Z_{0i} ($i = 1, 2$) is characteristic impedance, βl is the propagation constant and l is the length of the i th transmission line. In Fig. 2-2, the resonance condition is expressed as

$$(Z_{11} + Z_1)(Z_{22} + Z_2) - Z_{12}^2 = 0 \quad (2-2)$$

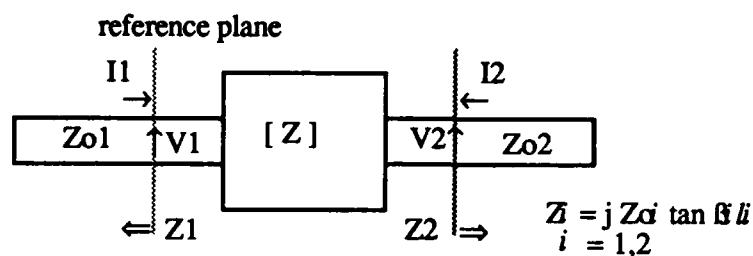


Fig. 2-2 Entire circuit as a resonator for a 2-port network

If the same resonant frequency is obtained with three different pairs of Z_1 and Z_2 , Eq. (2-2) yields three different equations and the Z parameters can be solved. The reference plane can be placed at any position as long as it is on the continuous part of the transmission line and the field disturbance due to the discontinuities remains the same.

The equivalent circuit of a 2-port network can be commonly expressed as shown in Fig. 2-3.

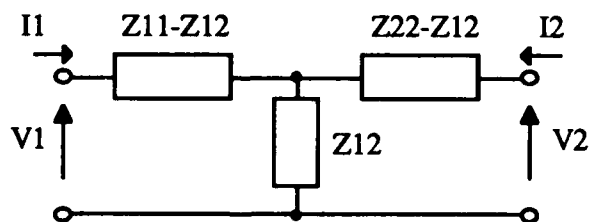


Fig. 2-3 Equivalent circuit of a 2-port network

Since the value of Z_{12} is solved with its square value in Eq. (2-2), the sign of the value must be chosen so that the equivalent circuit has physically proper frequency performance in the regarding frequency region. If the reference planes are placed close to the discontinuities, where the fields are much influenced by them, negative capacitors or inductors may appear in an equivalent circuit representation.

2-2. 4-Port Network

In this section, a procedure for a 2-port resonance method is extended to a 4-port configuration. The crossing between the two suspended striplines can be represented as a 4-port network at some reference planes sufficiently far from the discontinuity region. Each port is terminated with a reactance corresponding to the line section between the reference plane and the auxiliary wall as shown in Fig. 2-4.

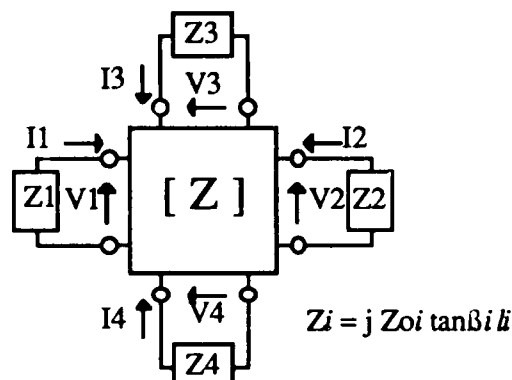


Fig. 2-4 4-port network for the problem

The network equations for the entire circuit is expressed in matrix form as

$$[V] = [Z][I] \quad (2-3a)$$

$$[V] = -\text{diag}[Z_i][I] \quad (2-3b)$$

thus
$$[[Z] + \text{diag}[Z_i]][I] = 0 \quad (2-3c)$$

where Z_i ($i = 1, 2, 3, 4$) are the terminal impedance: $Z_i = j Z_{oi} \tan \beta l_i$. The entire structure is assumed to be lossless so that the Z parameters of the 4-port network are imaginary and the resonant frequency is real. The resonant frequency is obtained from the condition that the voltages and currents are non-trivial in the absence of sources. From Eq. (2-3c), this condition is

$$\det || [Z] + \text{diag}[Z_i] || = 0 \quad (2-4)$$

The values of Z_i 's can be specified once the distance to the wall is fixed while the Z parameters are to be determined. The impedance matrix of a reciprocal 4-port lossless

network possesses in general 10 independent imaginary parameters. In the present case, however, because of the symmetry of the structure, only five parameters are needed to characterize the Z matrix :

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 & Z13 & Z13 \\ Z12 & Z11 & Z13 & Z13 \\ Z13 & Z13 & Z33 & Z34 \\ Z13 & Z13 & Z34 & Z33 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} \quad (2-5)$$

From Eq. (2-5), the resonant condition of Eq. (2-4) can be written in the following form :

$$\{ (Z11+Z1)(Z11+Z2)-Z12^2 \} \{ (Z33+Z3)(Z33+Z4)-Z34^2 \} \\ - 4 Z13^2 \{ Z11-Z12+(Z1+Z2)/2 \} \{ Z33-Z34+(Z3+Z4)/2 \} = 0 \quad (2-6)$$

The derivation of the equation is detailed in Appendix A. It is observed that, in the limit as Z13 equals zero, Eq. (2-6) reduces to

$$(Z11 + Z1) (Z11 + Z2) - Z12^2 = 0 \quad \text{or} \quad (2-7a)$$

$$(Z33 + Z3) (Z33 + Z4) - Z34^2 = 0 \quad (2-7b)$$

As expected, when the two striplines are uncoupled, the resonant condition splits into those of the individual lines. The unknown Z parameters as well as terminal impedances Zi's are frequency dependent. Eq. (2-6) can be regarded as a function of ω equated to zero. The Z parameters in Eq. (2-6) may be solved if the five different equations are derived corresponding to five different set of Zi's at the same resonant frequency.

Now we can show that by properly choosing the terminal impedance Zi's, the resonant conditions are simplified so that the problem is solved analytically. First, let us choose the terminal impedances in a symmetrical way, i.e., $I1=I2$ and $I3=I4$ so that

$$Z1 = Z2 \quad (2-8a)$$

$$Z3 = Z4 \quad (2-8b)$$

The above conditions correspond to the side walls located symmetrically with respect to the discontinuities. If Eqs. (2-8) are applied, Eq. (2-6) can be factorized in the form

$$(Z_{11}+Z_1-Z_{12})(Z_{33}+Z_3-Z_{34})\{(Z_{11}+Z_{12}+Z_1)(Z_{33}+Z_{34}+Z_3)-4Z_{13}^2\}=0. \quad (2-9a)$$

$$\text{Thus} \quad Z_{11} + Z_1 - Z_{12} = 0 \quad \text{or} \quad (2-9b)$$

$$Z_{33} + Z_3 - Z_{34} = 0 \quad \text{or} \quad (2-9c)$$

$$(Z_{11} + Z_{12} + Z_1)(Z_{33} + Z_{34} + Z_3) - 4Z_{13}^2 = 0. \quad (2-9d)$$

With each factor in Eq. (2-9a) equated to zero, the eigenvalues for the matrix in Eq. (2-3c) are obtained ; each eigenvalue ω is then the resonant frequency under the condition of the corresponding eigenvector of the currents. When the first factor in Eq. (2-9a) is equated to zero, i.e. Eq.(2-9b) is satisfied, the eigenvector of Eq. (2-4) is easily found as detailed in Appendix B :

$$I_1 = -I_2 \quad (2-10a)$$

$$I_3 = I_4 = 0. \quad (2-10b)$$

This condition corresponds to an odd resonance of the structure shown in Fig. 2-5(a).

The structure behaves as if an electric wall is placed symmetrically along at the center of stripline 2. For the given resonant frequency, the required resonance condition provides the quantity $Z_{11}-Z_{12}$ for a given value of Z_1 . $I_3 = I_4 = 0$ (therefore $V_3 = V_4 = 0$ unless Z_i is infinite) does not imply that the electromagnetic field is zero along the strip 2, but only that the electromagnetic field of the dominant mode is zero ; it is anticipated that the field is localized at the crossing region along the strip 2 as illustrated in Fig. 2-6(b).

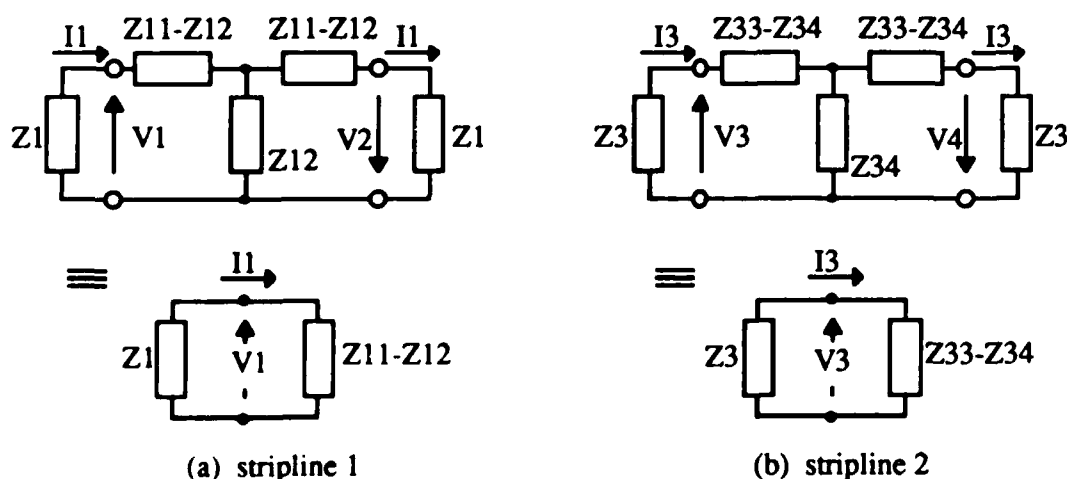


Fig. 2-5 Equivalent circuits for an odd resonance

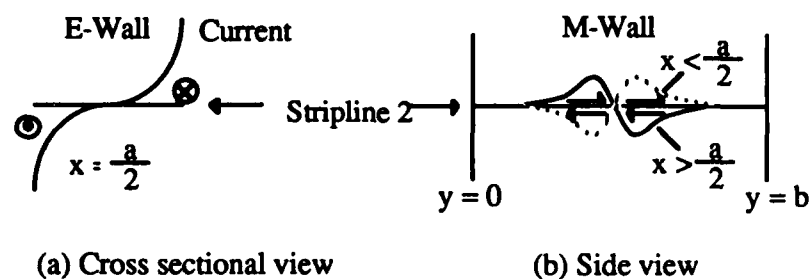


Fig. 2-6 Longitudinal current flow on stripline 2 with E-wall

Similary, when Eq. (2-9c) is satisfied, an odd resonance of stripline 2 is obtained

$$I_1 = I_2 = 0 \quad (2-11a)$$

$$I_3 = -I_4 \quad (2-11b)$$

The resonant circuit is expressed in Fig. 2-5(b). It is interesting to note that the individual parameters of the pairs of ports 1-2 and 3-4 are uncoupled in the odd resonance modes. Finally, from Eq. (2-9d), the eigenvector for an even resonance is obtained :

$$I_1 = I_2 \quad (2-12a)$$

$$I_3 = I_4 \quad (2-12b)$$

Substitution of this condition into Eq. (2-5) yields the 2-port network matrix equation

$$\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11}+Z_{12} & 2 Z_{13} \\ 2 Z_{13} & Z_{33}+Z_{34} \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} \quad (2-13)$$

whose equivalent circuit is illustrated in Fig. 2-7 together with terminal impedances.

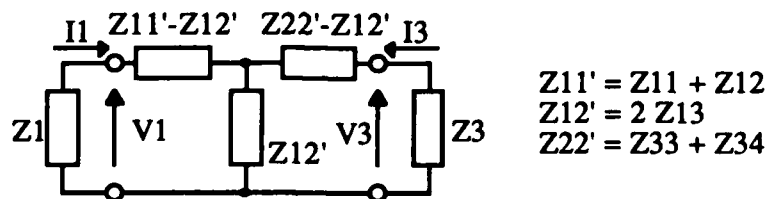


Fig. 2-7 Equivalent circuit for an even resonance

From the condition of Eqs. (2-12), it is postulated that the even resonance structure has two magnetic walls at each center of the striplines. The use of symmetry, therefore, has reduced the 4-port network problem to that of a 2-port. For a given resonant frequency, three different pairs of Z_1 and Z_3 (with $Z_1 = Z_2$ and $Z_3 = Z_4$) are used to compute three quantities Z_{11}' , Z_{12}' and Z_{22}' which denote the elements of the matrix in Eq. (2-13) :

$$Z_{11}' = Z_{11} + Z_{12} \quad (2-14a)$$

$$Z_{22}' = Z_{33} + Z_{34} \quad (2-14b)$$

$$Z_{12}' = 2 Z_{13} \quad (2-14c)$$

Combining the results with those for the two structures in Fig. 2-5, we obtain all five Z parameters. For the procedure illustrated above, we must know the propagation constants of the two isolated striplines, i. e., β_1 and β_3 . The quantities are necessary to obtain Z_1 and Z_3 :

$$z_1 = j \tan \beta_1 (a - w_2) / 2 \quad (2-15a)$$

$$z_3 = j \tan \beta_3 (b - w_1) / 2 \quad (2-15b)$$

where the impedances are normalized. These expressions are obtained for a specific choice of the reference planes as shown in Fig. 2-8. The field analysis can be also used to determine β_1 and β_3 . This can be done simply by setting $\beta_1 = \pi/a$ and $\beta_3 = \pi/b$ at a specified resonant frequency for the isolated stripline.

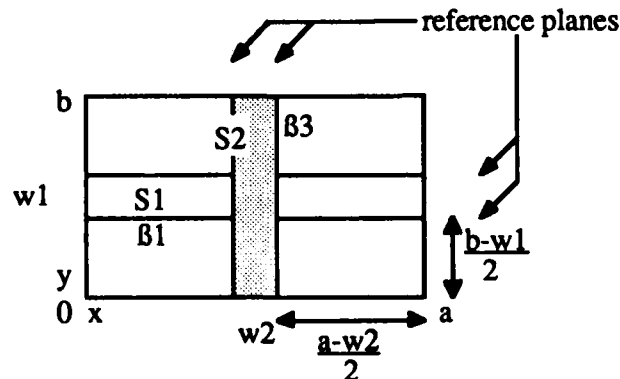


Fig. 2-8 A choice of reference planes (top view)

2-3. Equivalent Circuit

It is important to find an equivalent circuit representation of the 4-port structure so that it is most convenient for the present analysis. Intuitive insights are needed to figure out a good expression which well represents a structure in a broad frequency range. A basic idea for choosing proper elements is to use the knowledge of field distributions. If the E field originating from a stripline increases in the proximity of discontinuities, it may be represented by a parallel capacitive element. When the H field circling around a stripline changes, it may be represented by a series inductive element.

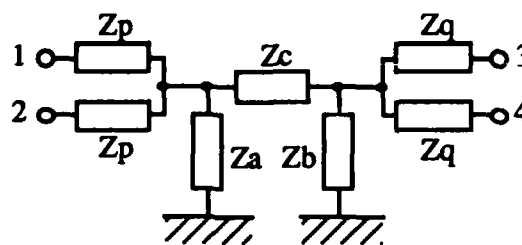


Fig. 2-9 Equivalent circuit for the problem

For the structure in Fig. 1-1, one possible equivalent circuit representation could be chosen as shown in Fig. 2-9. The circuit already takes the symmetry properties into account. Z_c is used to represent the coupling capacitance between the two strips. $Z_p + Z_p$ and $Z_q + Z_q$ represent the inductances associated with the two stripline sections, while Z_a and Z_b represent the two strip-to-ground capacitances. The Z matrix for the circuit in Fig. 2-9 is given as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} \\ Z_{12} & Z_{11} & Z_{13} & Z_{13} \\ Z_{13} & Z_{13} & Z_{33} & Z_{34} \\ Z_{13} & Z_{13} & Z_{34} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (2-16a)$$

where

$$Z_{11} = Z_p + Z_a (Z_b + Z_c) / (Z_a + Z_b + Z_c) \quad (2-16b)$$

$$Z_{12} = Z_a (Z_b + Z_c) / (Z_a + Z_b + Z_c) \quad (2-16c)$$

$$Z_{13} = Z_a Z_b / (Z_a + Z_b + Z_c) \quad (2-16d)$$

$$Z_{33} = Z_q + Z_b (Z_c + Z_a) / (Z_a + Z_b + Z_c) \quad (2-16e)$$

$$Z_{34} = Z_b (Z_c + Z_a) / (Z_a + Z_b + Z_c) \quad (2-16f)$$

From Eqs. (2-16), each element in the equivalent circuit is obtained :

$$Z_a = (Z_{13}^2 - Z_{12} Z_{34}) / (Z_{13} - Z_{34}) \quad (2-17a)$$

$$Z_b = (Z_{13}^2 - Z_{12} Z_{34}) / (Z_{13} - Z_{12}) \quad (2-17b)$$

$$Z_c = - (Z_{13}^2 - Z_{12} Z_{34}) / Z_{13} \quad (2-17c)$$

$$Z_p = Z_{11} - Z_{12} \quad (2-17d)$$

$$Z_q = Z_{33} - Z_{34} \quad (2-17e)$$

The derivation of Eqs. (2-16) and (2-17) are detailed in Appendix C. As computation for the Z parameters, accordingly the parameters in Eqs. (2-17), it is convenient to use normalized impedance parameters with respect to the characteristic impedance of each stripline. Thus, the matrix in Eq. (2-5) may be modified as follows :

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{13} \\ z_{12} & z_{11} & z_{13} & z_{13} \\ z_{13} & z_{13} & z_{33} & z_{34} \\ z_{13} & z_{13} & z_{34} & z_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (2-18a)$$

where

[v] : normalized voltage vector

[i] : normalized current vector

$$[v] = \text{diag} [\sqrt{Z_{oi}}]^{-1} [V] \quad (2-18b)$$

$$[i] = \text{diag} [\sqrt{Z_{oi}}] [I] \quad (2-18c)$$

$$z_{mn} = \frac{Z_{mn}}{\sqrt{Z_{om}} \sqrt{Z_{on}}} \quad \begin{matrix} m = 1, 2, 3, 4 \\ n = 1, 2, 3, 4 \end{matrix} \quad (2-18d)$$

Z_{oi} : Stripline characteristic impedance at the i-th port

The normalized terminal impedance z_i 's are given by Eqs. (2-15). Small letters z, v, i for Z, V, I denote normalized quantities. Accordingly, the equations in Eqs. (2-8), (2-9) and (2-13) are replaced by normalized ones, - replacing Z 's by z 's -, and solved with

respect to normalized parameters. Finally, equivalent circuit parameters in Eq. (2-17) are solved using the relation of Eq. (2-18d).

The S matrix for a 4-port network is obtained from the relation

$$[S] = [[z] + [U]]^{-1} [[z] - [U]] \quad (2-19)$$

where $[z]$ is a normalized impedance matrix and $[U]$ is a unit matrix. The matrix relations are detailed in Appendix D.

3. FIELD ANALYSIS

3-1. Modes in a Rectangular Waveguide

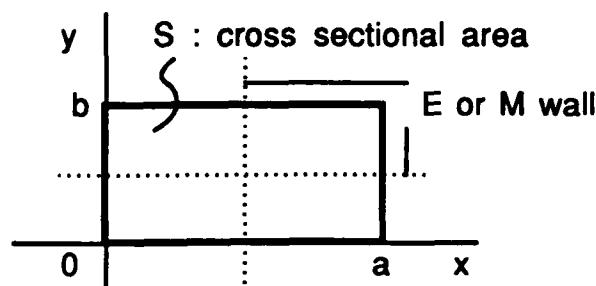


Fig.3-1 Rectangular waveguide

Fig. 3-1 shows a rectangular waveguide structure with the reference coordinate. With the scalar potentials, the general field expressions for TE to z and TM to z modes are as follows [4] :

TE	TM	
$E_t = -\nabla_t \Psi \times z$	$H_t = \nabla_t \Phi \times z$	
$H_t = -\frac{j\beta}{z^\wedge} \nabla_t \psi$	$E_t = -\frac{j\beta}{y^\wedge} \nabla_t \phi$	(3-1)
$H_z = \frac{kc^2}{z^\wedge} \psi$	$E_z = \frac{kc^2}{y^\wedge} \phi$	

where

$$kc^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

m, n : see Eqs. (3-2)

$$\beta^2 = k^2 - kc^2$$

$$y^\wedge = j\omega\epsilon, \quad z^\wedge = j\omega\mu, \quad k^2 = -y^\wedge z^\wedge = \omega^2\epsilon\mu$$

ϵ : permittivity μ : permeability

$$E_t = x E_x + y E_y$$

$$H_t = x H_x + y H_y$$

$$\nabla_t = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$$

x, y, z : unit vector in x, y, z direction

where z dependance $e^{-j\beta z}$ is assumed and subscript t denotes the transverse components of a quantity. Assuming an electric wall or a magnetic wall at each center of the wave guide, one can express scalar potentials including conductive guide boundary conditions as follows :

$$\Psi_{mn} = P_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \quad m,n = 0,1,2,3,-- \quad (3-2a)$$

$$\Phi_{mn} = P_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad m,n = 1,2,3, -- \quad (3-2b)$$

$$P_{mn} = \sqrt{\frac{\delta_m \delta_n}{a b}} \frac{1}{k_c} \quad \delta_i = \begin{cases} 1 & i=0 \\ 2 & i \neq 0 \end{cases}$$

where subscripts mn corresponds to the order of a harmonics, and P_{mn} is a coefficient to normalize the norm of scalar potential vector functions so as

$$\int_S |\nabla_t \Psi_{mn}|^2 dx dy = 1 \quad (3-3a)$$

$$\int_S |\nabla_t \Phi_{mn}|^2 dx dy = 1. \quad (3-3b)$$

Even symmetry solutions with respect to the x or y axis are obtained for m or n being an odd integer. Similarly, odd solutions are obtained for m or n being an even integer.

3-2. Field Expansions inside the Cavity

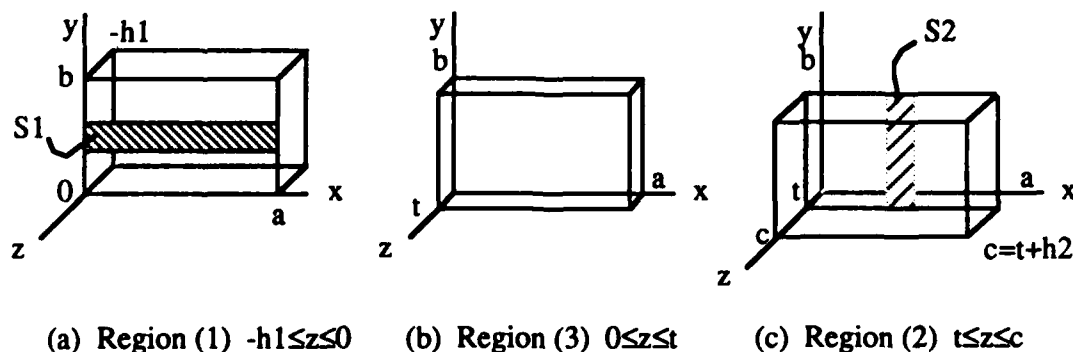


Fig. 3-2 Subregions for field analysis

The resonant structure of Fig. 1-1 can be subdivided into three homogeneous regions as shown in Fig. 3-2. In each region, we can expand the electromagnetic field in terms of TE-to-z and TM-to-z waveguide modes. Incorporating the boundary conditions for $z=-h1$ and $z=c$, the following expressions can be assumed :

region (1) : $-h1 \leq z \leq 0$

$$E_t^{(1)} = - \sum_{mn}^{MN} [A_{mn}^{(1)} \nabla_t \Psi_{mn} \times z + B_{mn}^{(1)} (\beta_{mn}^{(1)} / y^{\wedge}_1) \nabla_t \Phi_{mn}] \sin \beta_{mn}^{(1)}(z+h1) \quad (3-4a)$$

$$H_t^{(1)} = \sum_{mn}^{MN} [A_{mn}^{(1)} (\beta_{mn}^{(1)} / z^{\wedge}_1) \nabla_t \Psi_{mn} + B_{mn}^{(1)} \nabla_t \Phi_{mn} \times z] \cos \beta_{mn}^{(1)}(z+h1) \quad (3-4b)$$

$$\beta_{mn}^{(1)} = (k_1^2 - kc^2)^{1/2} \quad (3-4c)$$

region (2) : $t \leq z \leq c$

$$E_t^{(2)} = - \sum_{mn}^{MN} [A_{mn}^{(2)} \nabla_t \Psi_{mn} \times z + B_{mn}^{(2)} (\beta_{mn}^{(2)} / y^{\wedge}_2) \nabla_t \Phi_{mn}] \sin \beta_{mn}^{(2)}(z-c) \quad (3-5a)$$

$$H_t^{(2)} = \sum_{mn}^{MN} [A_{mn}^{(2)} (\beta_{mn}^{(2)} / z^{\wedge}_2) \nabla_t \Psi_{mn} + B_{mn}^{(2)} \nabla_t \Phi_{mn} \times z] \cos \beta_{mn}^{(2)}(z-c) \quad (3-5b)$$

$$\beta_{mn}^{(2)} = (k_2^2 - kc^2)^{1/2} \quad (3-5c)$$

region (3) : $0 \leq z \leq t$

$$E_t^{(3)} = - \sum_{mn}^{MN} [\{ C_{mn} \sin \beta_{mn}^{(3)} z + D_{mn} \sin \beta_{mn}^{(3)}(z-t) \} \nabla_t \Psi_{mn} \times z + \{ F_{mn} \sin \beta_{mn}^{(3)} z + G_{mn} \sin \beta_{mn}^{(3)}(z-t) \} (\beta_{mn}^{(3)} / y^{\wedge}_3) \nabla_t \Phi_{mn}] \quad (3-6a)$$

$$H_t^{(3)} = \sum_{mn}^{MN} [\{ C_{mn} \cos \beta_{mn}^{(3)} z + D_{mn} \cos \beta_{mn}^{(3)}(z-t) \} (\beta_{mn}^{(3)} / z^{\wedge}_3) \nabla_t \Psi_{mn} + \{ F_{mn} \cos \beta_{mn}^{(3)} z + G_{mn} \cos \beta_{mn}^{(3)}(z-t) \} \nabla_t \Phi_{mn} \times z] \quad (3-6b)$$

$$\beta_{mn}^{(3)} = (k_3^2 - kc^2)^{1/2} \quad (3-6c)$$

where $A_{mn}^{(1)}$, $A_{mn}^{(2)}$, $B_{mn}^{(1)}$, $B_{mn}^{(2)}$, C_{mn} , D_{mn} , F_{mn} , and G_{mn} are unknown coefficients, and superscripts in parentheses refer to respective regions. The boundary conditions at each interface are

Interface (I) : at $z = 0$

$$E_{t1} = E_{t3} \quad \text{on } S \quad (3-7a)$$

$$E_{t1} = E_{t3} = 0 \quad \text{on } S1 \quad (3-7b)$$

$$H_{t3} - H_{t1} = \begin{cases} \Delta H_{t1}^{(1)} & \text{on } S1 \\ 0 & \text{on } S-S1 \end{cases} \quad (3-7c)$$

Interface (II) : at $z = t$

$$E_{t2} = E_{t3} \quad \text{on } S \quad (3-8a)$$

$$E_{t2} = E_{t3} = 0 \quad \text{on } S2 \quad (3-8b)$$

$$H_{t2} - H_{t3} = \begin{cases} \Delta H_{t2}^{(2)} & \text{on } S2 \\ 0 & \text{on } S-S2 \end{cases} \quad (3-8c)$$

where $\Delta H_{ti}^{(i)}$ ($i = 1, 2$) are unknown functions of x, y . These functions are expanded in terms of a set of known orthogonal vector functions $\chi_{v(i)}$ defined over S_i with unknown coefficients $P_{v(i)}$ (see next section) :

$$\Delta H_{t1}^{(1)} = \sum_v P_{v(1)} \chi_{v(1)} \quad (3-9a)$$

$$\Delta H_{t2}^{(2)} = \sum_v P_{v(2)} \chi_{v(2)} \quad (3-9b)$$

Applying the boundary conditions of Eqs. (3-7), (3-8) to Eqs. (3-4), (3-5), (3-6) and making use of the orthogonal properties of $\nabla_t \Psi_{mn}$ and $\nabla_t \Phi_{mn}$, we obtain a homogeneous system of equations in terms of unknown coefficient P_v :

$$\sum_{v_1}^{v_1 M} P_{v(1)} U_{\mu v(11)} + \sum_{v_2}^{v_2 M} P_{v(2)} U_{\mu v(12)} = 0 \quad \mu = 1, 2, 3, \quad v_1 M \quad (3-10a)$$

$$\sum_{v_1}^{v_1 M} P_{v(1)} U_{\mu v(21)} + \sum_{v_2}^{v_2 M} P_{v(2)} U_{\mu v(22)} = 0 \quad \mu = 1, 2, 3, \quad v_2 M \quad (3-10b)$$

where

$$U_{\mu v(i)} = \sum_{mn}^{MN} S_i \{ a_{mn}^{(i)} \xi_{mn, \mu(i)} \xi_{mn, v(i)} - (\beta^{(i)} / y^{\wedge}_i) b_{mn}^{(i)} \theta_{mn, \mu(i)} \theta_{mn, v(i)} \} \quad (3-10c)$$

$i, j = 1, 2$

$$\xi_{mn,\mu}^{(i)} = \int_{S_i} \chi_{\mu}^{(i)} \cdot \nabla_t \Psi_{mn} ds \quad i = 1, 2 \quad (3-10d)$$

$$\theta_{mn,\mu}^{(i)} = \int_{S_i} \chi_{\mu}^{(i)} \cdot \nabla_t \Phi_{mn} \times z ds \quad i = 1, 2 \quad (3-10e)$$

$$a_{mn}^{(11)} = K_a (1/S_1) (B^{(2)}Ct_2 + B^{(3)}Ct_3) \quad (3-10f)$$

$$a_{mn}^{(12)} = K_a (1/(S_1 S_3)) B^{(3)} \quad (3-10g)$$

$$a_{mn}^{(21)} = -K_a (1/(S_2 S_3)) B^{(3)} \quad (3-10h)$$

$$a_{mn}^{(22)} = -K_a (1/S_2) (B^{(1)}Ct_1 + B^{(3)}Ct_3) \quad (3-10i)$$

$$K_a = z^\wedge / \{ B^{(3)2} - B^{(3)}Ct_3(B^{(2)}Ct_2 + B^{(1)}Ct_1) - B^{(1)}B^{(2)}Ct_1Ct_2 \}$$

$$b_{mn}^{(11)} = K_b (1/S_1) (B^{(3)}Ct_2 + (\epsilon_3/\epsilon_2)B^{(2)}Ct_3) \quad (3-10j)$$

$$b_{mn}^{(12)} = K_b (1/(S_1 S_3)) (\epsilon_3/\epsilon_2) B^{(2)} \quad (3-10k)$$

$$b_{mn}^{(21)} = -K_b (1/(S_2 S_3)) (\epsilon_3/\epsilon_1) B^{(1)} \quad (3-10l)$$

$$b_{mn}^{(22)} = -K_b (1/S_2) (B^{(3)}Ct_1 + (\epsilon_3/\epsilon_1)B^{(1)}Ct_3) \quad (3-10m)$$

$$K_b = B^{(3)} / [(\epsilon_3/\epsilon_1)(\epsilon_3/\epsilon_2)B^{(1)}B^{(2)} - B^{(3)}Ct_3 \{ (\epsilon_3/\epsilon_1)B^{(1)}Ct_2 + (\epsilon_3/\epsilon_2)B^{(2)}Ct_1 \} - B^{(3)2}Ct_1Ct_2]$$

$$B^{(i)} = \beta_{mn}^{(i)} \quad i = 1, 2, 3$$

$$C_1, C_2, C_3 = \cos B^{(1)}h_1, \cos B^{(2)}h_2, \cos B^{(3)}h$$

$$S_1, S_2, S_3 = \sin B^{(1)}h_1, \sin B^{(2)}h_2, \sin B^{(3)}h$$

$$Ct_1, Ct_2, Ct_3 = C_1/S_1, C_2/S_2, C_3/S_3$$

$$z^\wedge = j \omega \mu_0 \quad \mu_0 : \text{free space permeability} = 4\pi \times 10^{-7} [\text{H/m}]$$

The derivations of Eqs. (3-10) are detailed in Appendix E. The condition for non-trivial solution determines the characteristic equation of the given structure. This equation may be regarded as a function of ω , l_1 , l_3 equated to zero :

$$f(\omega, l_1, l_3) = 0 \quad (3-11)$$

For given value of $\omega = \omega_r$, Eq. (3-11) can be solved to evaluate the different pairs of l_1 and l_3 giving rise to the same resonant frequency ω_r . These values of l_1 and l_3 can be used for computing the discontinuity parameters discussed in the previous section.

3-3. H Field (Current) Expansion on the Strips

The H field on the strip are expanded in terms of known basis functions χ_v with unknown coefficients P_v . Actually what will be expanded is the H field discontinuity which is equal to $\mathbf{J}_t \times \mathbf{z}$, viz.

$$\Delta \mathbf{H}_t = \sum_v^{v_{\max}} P_v \chi_v = \mathbf{J}_t \times \mathbf{z} \quad (3-12)$$

The basis functions are chosen in such a way that the field is non-zero only on the strip. Additional boundary condition must be satisfied at the strip ends where the striplines are terminated with electric walls. The field may be expanded in terms of the basis functions of two different types. One of them is harmonic basis functions and another is singular basis functions. The singular behavior of the magnetic field component normal to the stripline edges is incorporated in the singular basis functions. They are therefore expected to provide a faster numerical convergence. For the H field expansion on S1 [see Fig. 3-3(a)], the following set is employed :

$$\chi_{x,rs(1)} = \sin \frac{\pi x}{a} \sin \left[\frac{s\pi}{w_1} \left\{ y - \frac{(b-w_1)}{2} \right\} \right] \quad (3-13a)$$

$$\chi_{y,rs(1)} = \cos \frac{\pi x}{a} \frac{\cos \left[\frac{s\pi}{w_1} \left\{ y - \frac{(b-w_1)}{2} \right\} \right]}{\sqrt{1 - \left\{ \frac{(y-b/2)}{w_1/2} \right\}^2}} \quad (3-13b)$$

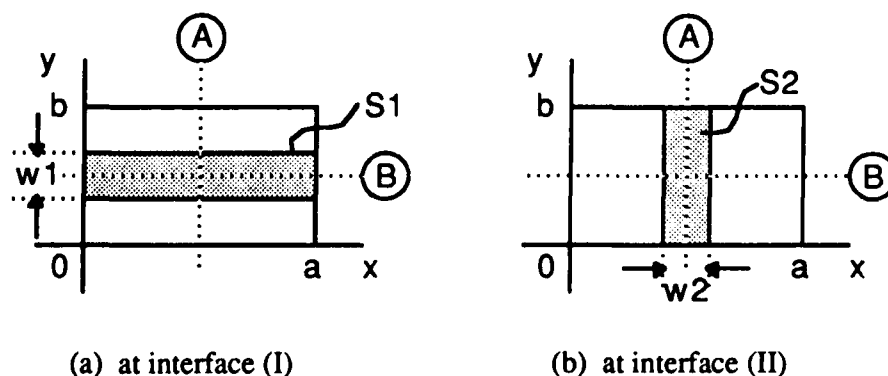


Fig. 3-3 Striplines at each interface

where subscript rs is used instead of v for representing the variations in the two orthogonal directions (see Appendix F). Similarly, for the H field on S2 in Fig. 3-3(b), the expansions are

$$\chi_{x,rs}^{(2)} = \frac{\cos \left[\frac{\pi}{w_2} \left\{ x - \frac{(a-w_2)}{2} \right\} \right]}{\sqrt{1 - \left\{ \frac{(x-a/2)}{w_2/2} \right\}^2}} \cos \frac{s\pi y}{b} \quad (3-14a)$$

$$\chi_{y,rs}^{(2)} = \sin \left[\frac{\pi}{w_2} \left\{ x - \frac{(a-w_2)}{2} \right\} \right] \sin \frac{s\pi y}{b} \quad (3-14b)$$

Depending on whether an electric or magnetic wall is assumed to be at (A) and (B) in Fig. 3-3(a), the numbers r and s in Eqs. (3-13) are classified to be odd or even as illustrated in Table 3-1.

Table 3-1 rs-table with respect to E M walls

(a) at interface (I)

	r	s
M wall	$2r'+1$	$2s'$
E wall	$2r'$	$2s'+1$

$$\chi_x^{(1)}: r, s = \begin{cases} 1, 3, 5, \dots \\ \text{or} \\ 2, 4, 6, \dots \end{cases}$$

$$\chi_y^{(1)}: r, s = \begin{cases} 1, 3, 5, \dots \\ \text{or} \\ 0, 2, 4, \dots \end{cases}$$

(b) at interface (II)

	r	s
M wall	$2r'$	$2s'+1$
E wall	$2r'+1$	$2s'$

$$\chi_x^{(2)}: r, s = \begin{cases} 1, 3, 5, \dots \\ \text{or} \\ 0, 2, 4, \dots \end{cases}$$

$$\chi_y^{(2)}: r, s = \begin{cases} 1, 3, 5, \dots \\ \text{or} \\ 2, 4, 6, \dots \end{cases}$$

4. COMPUTED RESULTS

In accordance with the technique described above, the electromagnetic fields in each region are expressed in terms of the series expansions. In the numerical computation, only a finite number of terms can be retained in the series expansions. In order to obtain a proper convergent behavior of the solution, the number of the terms was chosen in such a way that the highest spatial frequencies of the electromagnetic field are approximately the same in the transverse directions (x,y). The current on a stripline is expanded in such a way that the distributions of each component (J_x , J_y) in the transverse direction are expressed by only the first term and that the distributions in the longitudinal direction by a set of terms, the number of which are the same on each stripline.

The method is first tested by computing the resonant frequency of the isolated stripline with certain structural parameters. Fig. 4-1 shows the convergence of the resonant frequency with respect to the number of the terms of the field expansions in the homogeneous region of Fig. 3-2. The number MN was chosen to be the same in both x and y directions. The results exhibit very good agreement with those by the SDA [5] in which the resonant frequency was calculated from the equation

$$f_0 = \frac{c}{a} \frac{\beta_0}{\beta} \quad (4-1)$$

where c : speed of the light

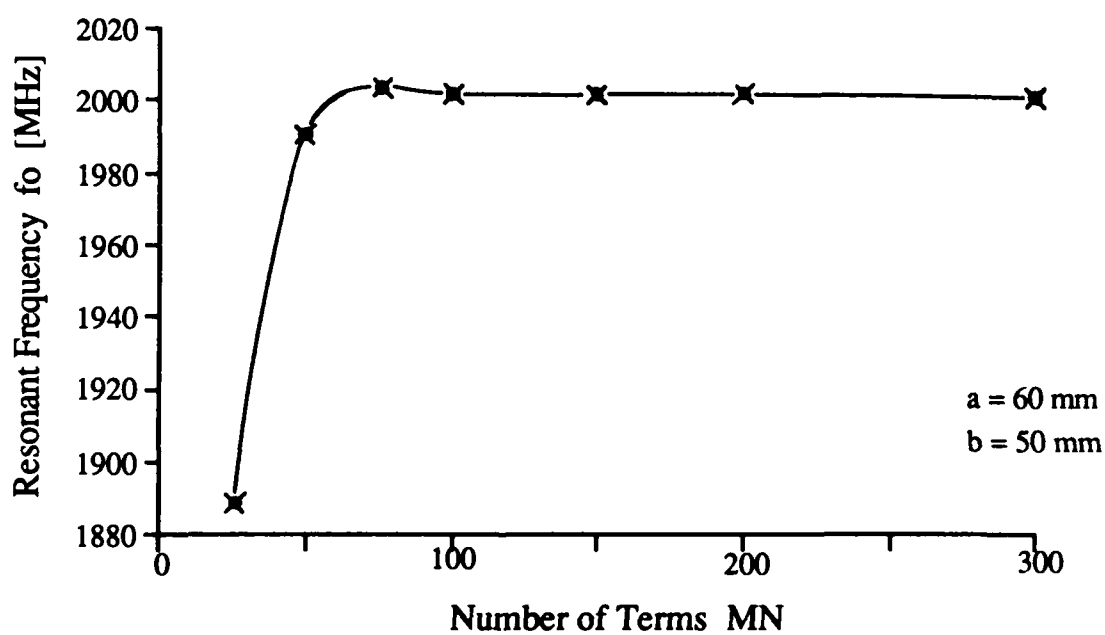
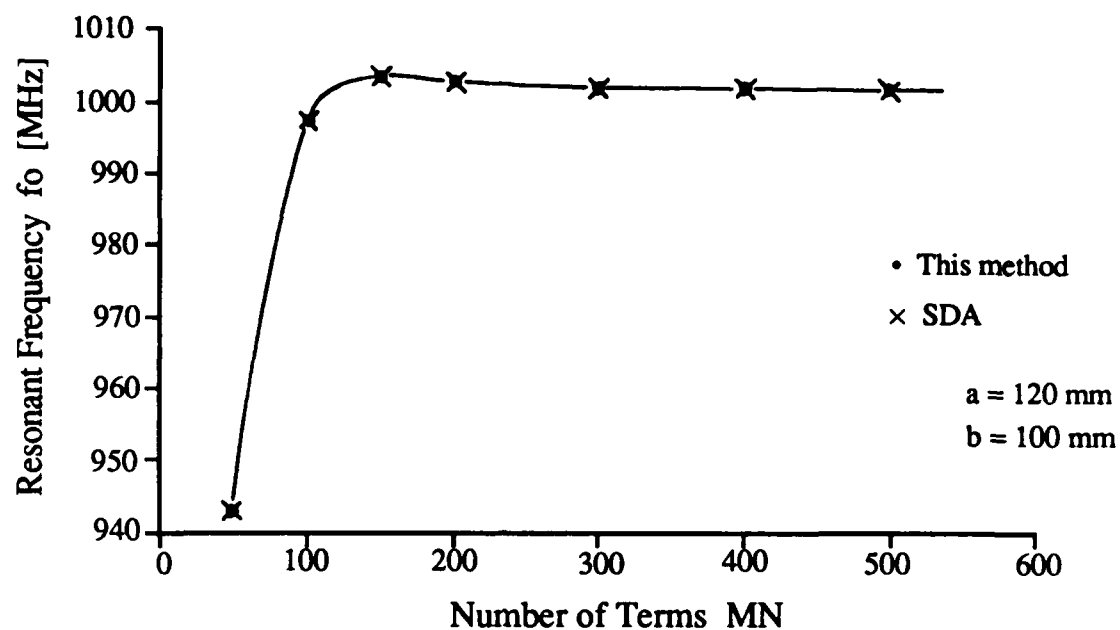
a : stripline length

β : propagation constant in proximity of f_0

β_0 : " " in free space

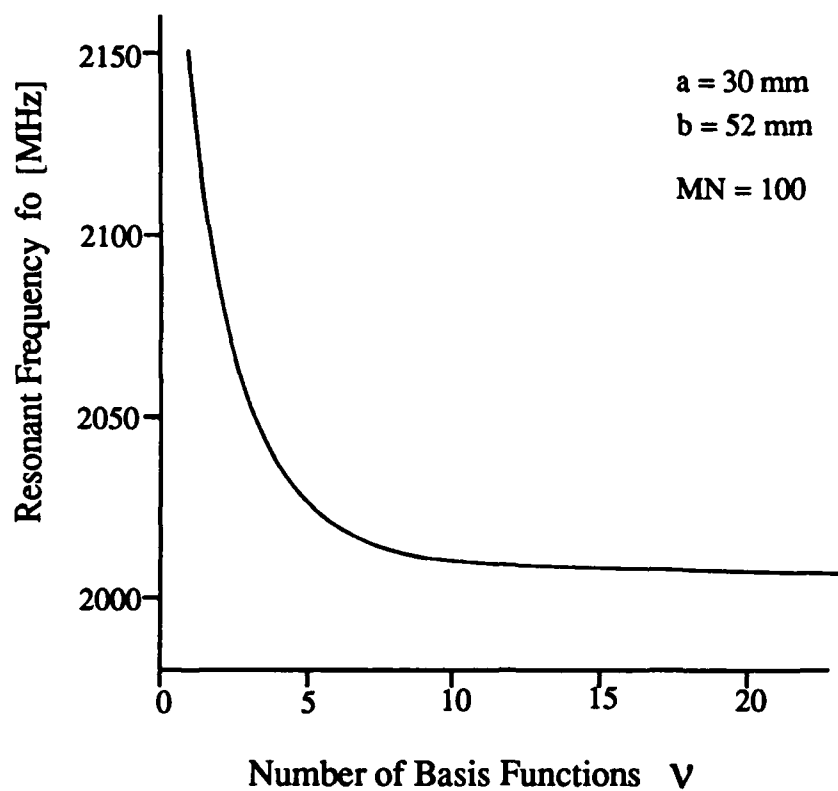
because computationally the field was represented by the same number of harmonics in both this method and SDA.

Fig. 4-2 shows the convergence of the resonant frequency of crossing stripline structure with respect to the number v of the current expansion terms in the longitudinal direction. At beginning, we expected that the number of the terms would be required as



$$h_1 = h_2 = 5 \text{ mm}, t = 1 \text{ mm}, w_1 = w_2 = 1 \text{ mm}, \epsilon_r = 3.8$$

Fig. 4-1 Convergence of the resonant frequency of an isolated stripline with respect to the number of terms for electromagnetic field in the x and y directions



$$h_1 = h_2 = 5 \text{ mm}, \quad t = 1 \text{ mm}$$
$$w_1 = w_2 = 1 \text{ mm}, \quad \epsilon_r = 3.8$$

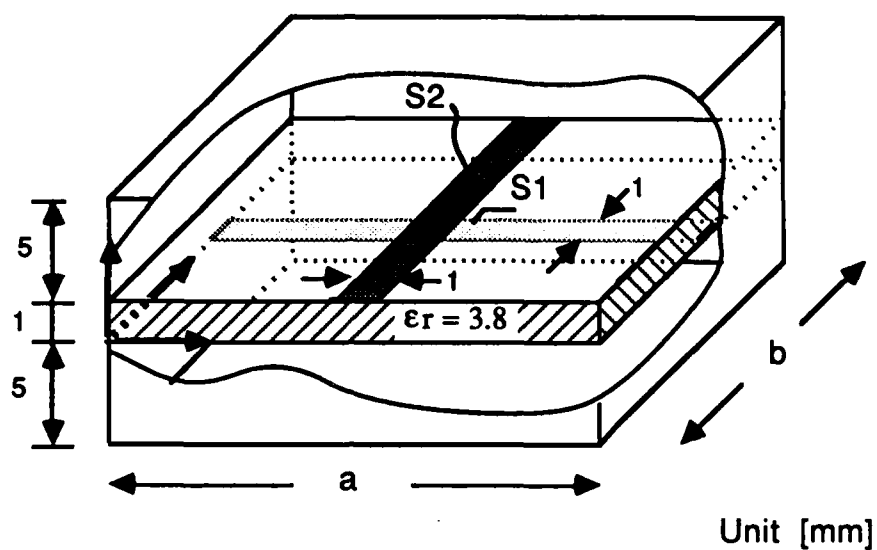
Fig. 4-2 Convergence of the resonant frequency of crossing striplines with respect to the number of basis functions

large as that of the electromagnetic field so that the longitudinal current would have the same highest spatial frequency and the results would have a good convergence. From Fig. 4-2, however, we could observe a fast convergence characteristic with a relatively small number of current expansions if the electromagnetic field is well expressed by an adequate number of harmonics.

Fig. 4-3 shows the parameters of the structure to be computed. The substrate was placed symmetrically between the top and bottom planes with both strips having the same widths. Hence, the impedance matrix representation of the discontinuities has $z_{11}=z_{33}$ and $z_{12}=z_{34}$, i.e. unknown parameters number is three: z_{11}, z_{12}, z_{13} . The computation was done at the three different frequencies: 0.5, 1, 2GHz. The approximate lengths of striplines, namely the size of cavity at each frequency, are also illustrated in Fig. 4-3. The reference planes are defined in the same manner as illustrated in Fig. 2-8.

Table 4-1 shows the element values for the equivalent circuit of the discontinuities shown in Fig. 4-4. Fig. 4-5 shows the corresponding S parameters of the discontinuities. Note that the elements for Z_a and Z_b ($=Z_a$) are capacitors with negative value. This is acceptable because they compensate the parallel distributed capacitance for the isolated stripline with the absence of the other stripline as follows. Assuming the absence of stripline 2 by making the port 3 and 4 open, the structure is nearly an isolated stripline. The section between port 1 and 2, therefore, would have an equivalent parallel capacitor which is corresponding to the distributed capacitor of the stripline. When the section is very short, the capacitance of the equivalent capacitor may be negative so that it would compensate the fringing capacitance due to the open ends at port 3 and 4 whose value could be larger than that of the distributed capacitor as a simple stripline section.

Z_c was also calculated with $h_1=10\text{mm}$, $h_2=100\text{mm}$, $t=5\text{mm}$, $w_1=w_2=0.4\text{mm}$, $\epsilon_r=1$ at 2 GHz simulating the crossing model of the work by Giri, et al [6]. The result for the value of C_c was 0.082pF, while the value of the coupling capacitor C_m was estimated to be 0.15pF by using the equivalent radius.



$a \approx 230 \sim 240$
 $b \approx 200 \sim 210$

@ 0.5 GHz

$a \approx 100 \sim 120$
 $b \approx 90 \sim 100$

@ 1 GHz

$a \approx 30 \sim 40$
 $b \approx 50 \sim 52$

@ 2 GHz

Fig. 4-3 Structural parameters for computation

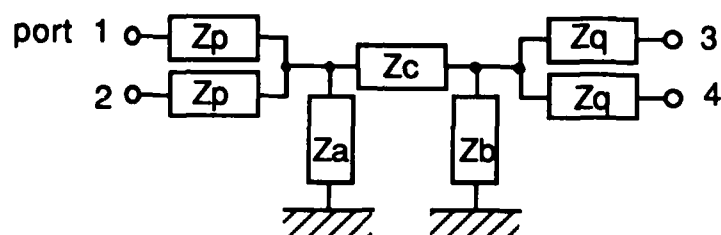


Fig. 4-4 Equivalent circuit of the discontinuities

Table 4-1 Values of elements

	0.5 GHz	1 GHz	2 GHz
$L_p = L_q$	0.331nH	0.331nH	0.329nH
$C_a = C_b$	-0.0885pF	-0.103pF	-0.101pF
C_c	0.249pF	0.272pF	0.258pF
MN	400	200	100
V	20	20	20

$$L_p = L_q = \frac{Z_o Z_p}{j \omega}$$

$$C_a = C_b = \frac{1}{j \omega Z_o Z_a}$$

$$C_c = \frac{1}{j \omega Z_o Z_c}$$

Z_p, Z_q, Z_a, Z_b, Z_c : Normalized impedance

Z_o : Stripline characteristic impedance
(159.2, 159.1, 158.3 Ω at 0.5, 1, 2 GHz)

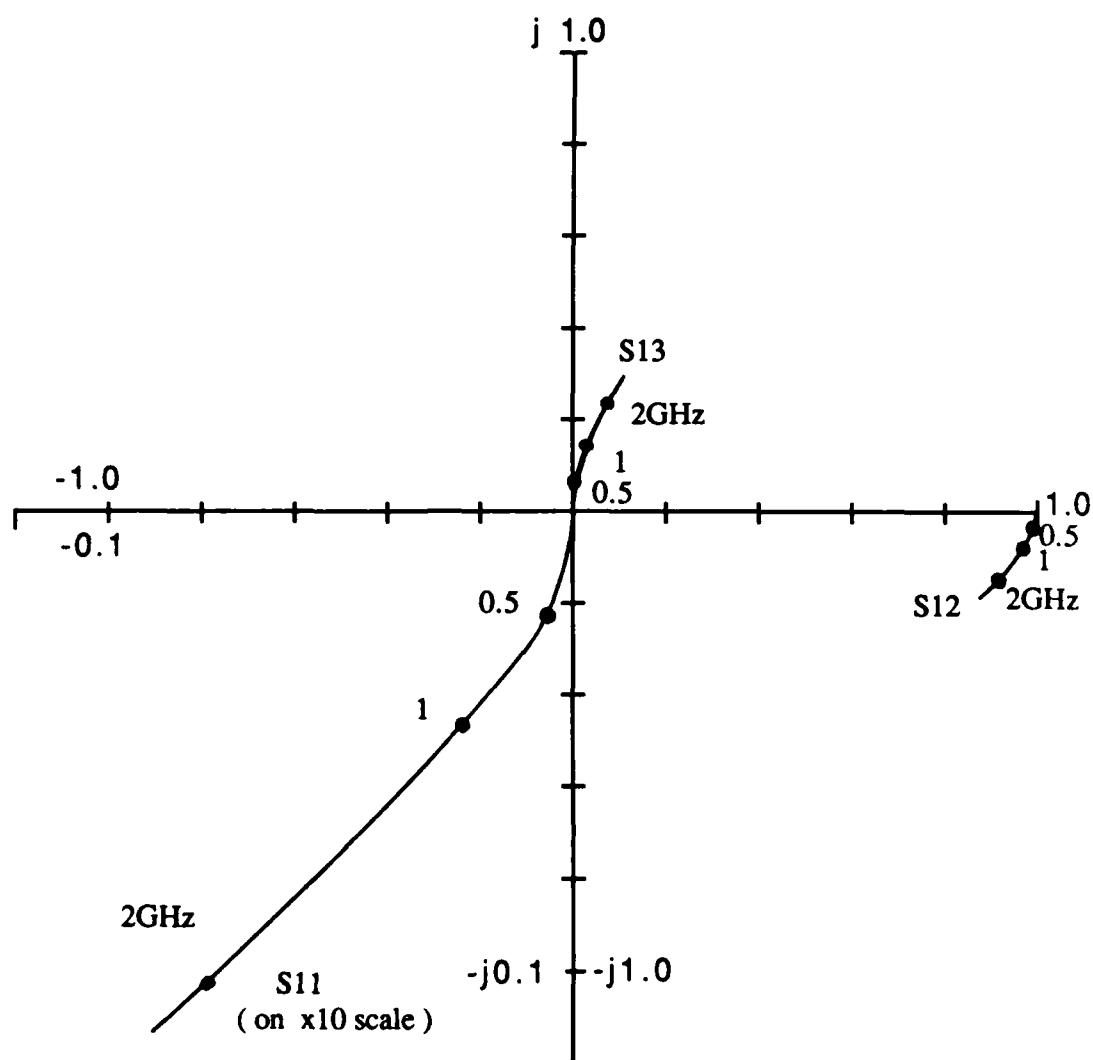


Fig. 4-5 S parameters of the discontinuities of the structure

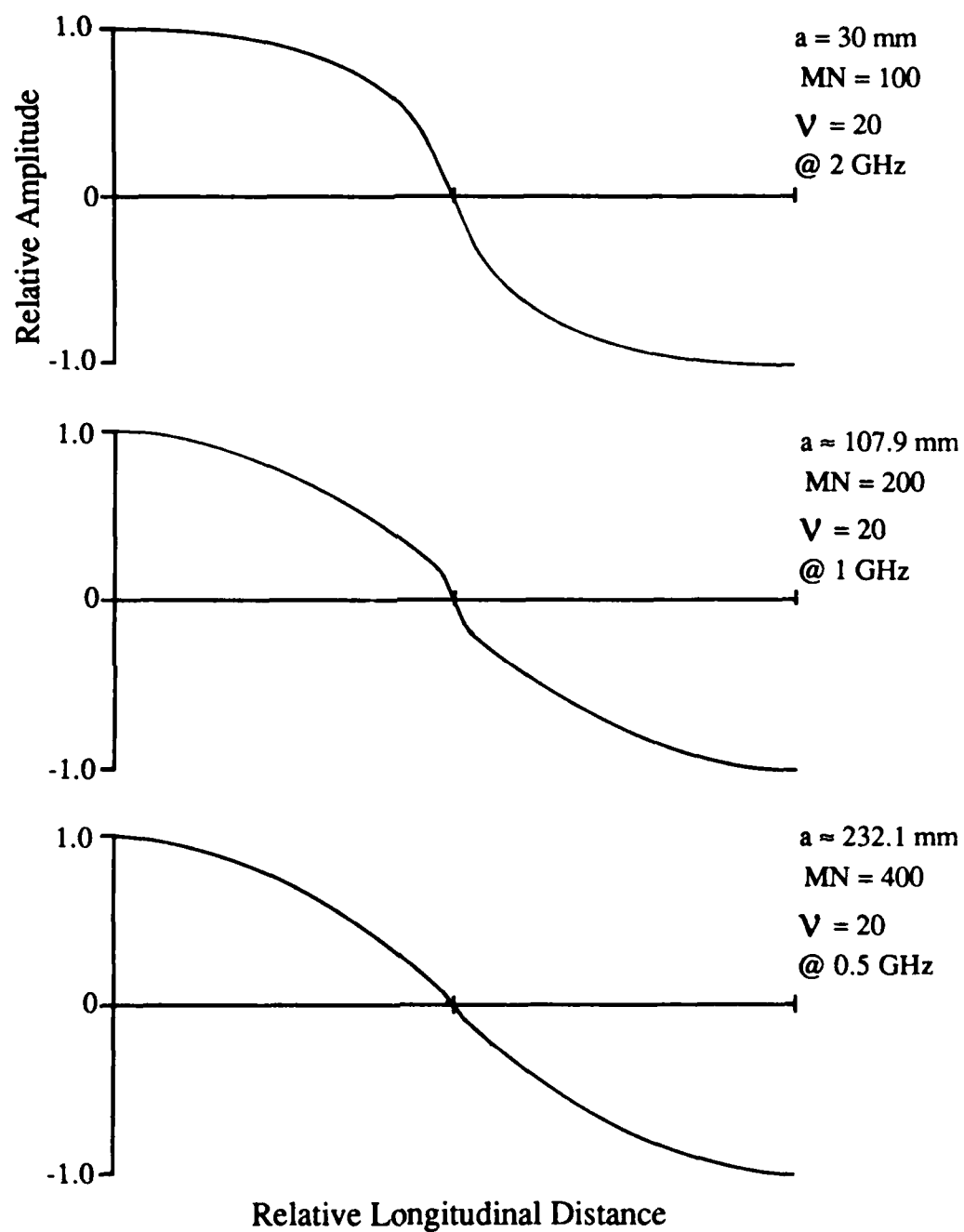
Longitudinal Currents (J_y on Stripline 1)

Fig. 4-6 Current distribution on the stripline

(a) longitudinal current

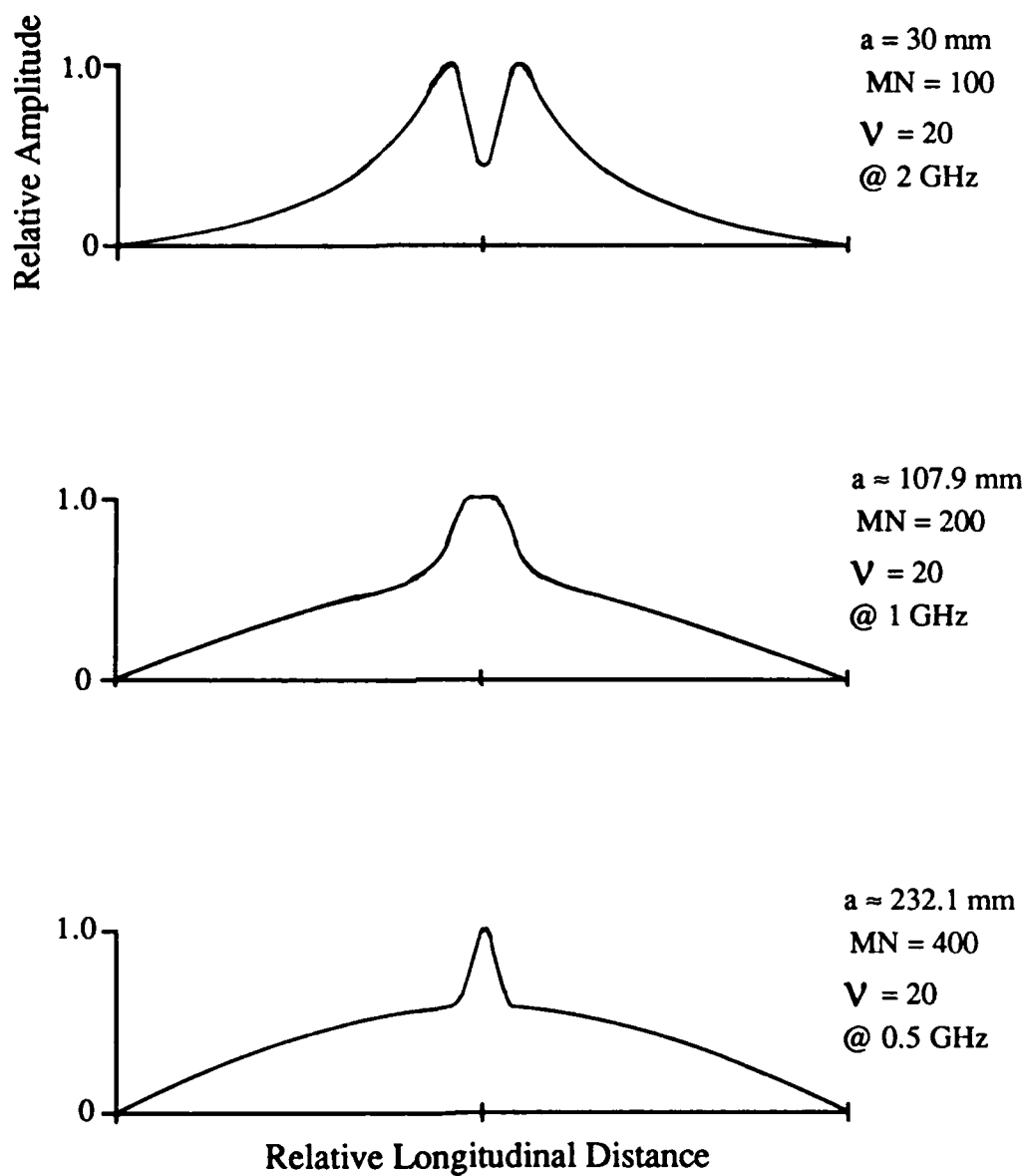
Transverse Currents (J_x on Stripline 1)

Fig. 4-6 Current distribution on the stripline

(b) transverse current

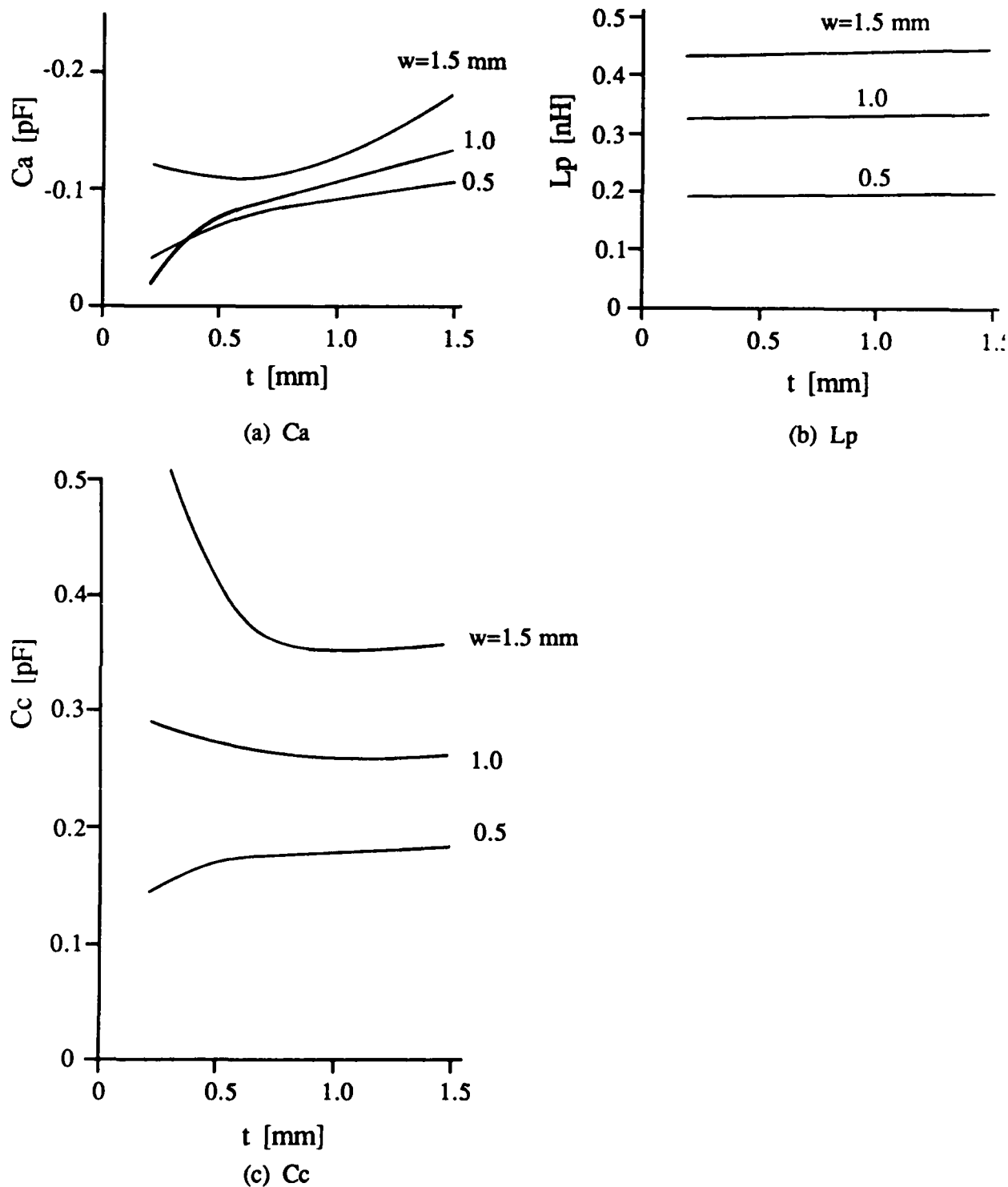


Fig. 4-7 Element values vs stripline width and substrate thickness

Fig. 4-6(a),(b) show the longitudinal and transverse components of the current densities at the center of the strip at the three different frequencies. In Fig. 4-6(a), it is observed that each figure was of perturbed cosine form and had larger absolute amplitude than cosine in the proximity of the center. This feature is anticipated from the insight that the stripline whose length is slightly shorter than a half wavelength turn out to be a resonator with capacitive loading at the center of the stripline. The longitudinal current, hence, may be represented by the combination of a cosine and an additional polynomial functions [7] so that the computation time could be reduced. As to the transverse current shown in Fig. 4-6(b), the figures were not readily characterized. This implies that the current function would require a number of terms of basis functions; the current function could not be easily replaced by few terms of special functions.

Fig. 4-7(a)(b)(c) show the computed results for different values of the stripline width and the substrate thickness.

5. CONCLUSIONS

A method of analysis has been described for characterizing the discontinuities of two crossed striplines. The method is based on a generalized transverse resonance technique for computing the resonant frequency of a resonator created by enclosing the crossing with auxiliary perfectly conducting walls. This resonator problem is analyzed as the waveguide scattering for waves traveling in the direction normal to the substrate surface. For a specified frequency, resonant structures are found by adjusting the lengths of the strips and hence the resonator size. These structures are used for deriving the equivalent circuit parameters characterizing the discontinuity.

This method can also be applied for the characterization of stripline-slotline transition.

APPENDIX

A. DERIVATION OF EQ. (2-6)

$$\det \| [Z] + \text{diag} [Z_i] \|$$

$$= \begin{bmatrix} Z_{11}+Z_1 & Z_{12} & Z_{13} & Z_{13} \\ Z_{12} & Z_{11}+Z_2 & Z_{13} & Z_{13} \\ Z_{13} & Z_{13} & Z_{33}+Z_3 & Z_{34} \\ Z_{13} & Z_{13} & Z_{34} & Z_{33}+Z_4 \end{bmatrix}$$

Subtract row 2 from row 1
Subtract row 3 from row 4

$$= \begin{bmatrix} Z_{11}+Z_1-Z_{12} & Z_{12}-Z_{11}-Z_2 & 0 & 0 \\ Z_{12} & Z_{11}+Z_2 & Z_{13} & Z_{13} \\ Z_{13} & Z_{13} & Z_{33}+Z_3 & Z_{34} \\ 0 & 0 & Z_{34}-Z_{33}-Z_3 & Z_{33}+Z_4-Z_{34} \end{bmatrix}$$

Subtract column 2 from column 1
Subtract column 3 from column 4

$$= \begin{bmatrix} 2Z_{11}-2Z_{12}+Z_1+Z_2 & Z_{12}-Z_{11}-Z_2 & 0 & 0 \\ Z_{12}-Z_{11}-Z_2 & Z_{11}+Z_2 & Z_{13} & 0 \\ 0 & Z_{13} & Z_{33}+Z_3 & Z_{34}-Z_{33}-Z_3 \\ 0 & 0 & Z_{34}-Z_{33}-Z_3 & 2Z_{33}-2Z_{34}+Z_3+Z_4 \end{bmatrix}$$

$$= \{ (2Z_{11}-2Z_{12}+Z_1+Z_2)(Z_{11}+Z_2) - (Z_{12}-Z_{11}-Z_2)^2 \}$$

$$\times \{ (2Z_{33}-2Z_{34}+Z_3+Z_4)(Z_{33}+Z_3) - (Z_{34}-Z_{33}-Z_3)^2 \}$$

$$- 4Z_{13}^2 \{ Z_{11}-Z_{12}+(Z_1+Z_2)/2 \} \{ Z_{33}-Z_{34}+(Z_3+Z_4)/2 \}$$

$$= \{ (Z_{11}+Z_1)(Z_{11}+Z_2)-Z_{12}^2 \} \{ (Z_{33}+Z_3)(Z_{33}+Z_4)-Z_{34}^2 \}$$

$$- 4Z_{13}^2 \{ Z_{11}-Z_{12}+(Z_1+Z_2)/2 \} \{ Z_{33}-Z_{34}+(Z_3+Z_4)/2 \}$$

where the following matrix relation is used :

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & c & 0 \\ 0 & c & b_{11} & b_{12} \\ 0 & 0 & b_{21} & b_{22} \end{bmatrix} = [A][B] - c^2 a_{11} b_{22}$$

where

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad [B] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

B. EIGENVECTOR FOR AN ODD OR EVEN RESONANCE

Insert the odd resonance condition

$$Z_{11} - Z_{12} + Z_1 = 0 \quad (B-1)$$

into Eq. (2-3) to obtain

$$Z_{12} (I_1 + I_2) + Z_{13} (I_3 + I_4) = 0 \quad (B-2a)$$

$$Z_{12} (I_1 + I_2) + Z_{13} (I_3 + I_4) = 0 \quad (B-2b)$$

$$Z_{13} (I_1 + I_2) + (Z_{33} + Z_3) I_3 + Z_{34} I_4 = 0 \quad (B-2c)$$

$$Z_{13} (I_1 + I_2) + Z_{34} I_3 + (Z_{33} + Z_3) I_4 = 0 \quad (B-2d)$$

Subtracting Eq. (B-2d) from Eq. (B-2c), we obtain

$$(Z_{33} + Z_3 - Z_{34}) I_3 - (Z_{33} + Z_3 - Z_{34}) I_4 = 0$$

$$\text{or} \quad I_3 = I_4 \quad (B-3)$$

Substitution of Eq. (B-3) into Eqs. (B-2) yields

$$\begin{bmatrix} Z_{12} & 2Z_{13} \\ 2Z_{13} & Z_{33}+Z_{34}+Z_3 \end{bmatrix} \begin{bmatrix} I_1+I_2 \\ I_3 \end{bmatrix} = 0 \quad (B-4)$$

As Eq. (B-4) is a homogeneous system and is valid for the arbitrary value of Z_3 , the current vector is identical to zero, i.e.,

$$I_1 + I_2 = 0$$

$$I_3 = 0$$

$$\text{hence,} \quad I_1 = -I_2 \quad (B-5a)$$

$$I_3 = (I_4) = 0 \quad (B-5b)$$

Similarly, the eigenvector for another odd resonance is obtained as

$$I_3 = -I_4 \quad (B-6a)$$

$$I_1 = (I_2) = 0 \quad (B-6b)$$

The eigenvector for the even resonance may be obtained from Eq. (2-3) as $Z_1 = Z_2$ and $Z_3 = Z_4$:

$$(Z_{11} + Z_1) I_1 + Z_{12} I_2 + Z_{13} (I_3 + I_4) = 0 \quad (\text{B-7a})$$

$$Z_{12} I_1 + (Z_{11} + Z_1) I_2 + Z_{13} (I_3 + I_4) = 0 \quad (\text{B-7b})$$

$$Z_{13} (I_1 + I_2) + (Z_{33} + Z_3) I_3 + Z_{34} I_4 = 0 \quad (\text{B-7c})$$

$$Z_{13} (I_1 + I_2) + Z_{34} I_3 + (Z_{33} + Z_3) I_4 = 0 \quad (\text{B-7d})$$

Subtracting Eq. (B-7b) from Eq. (B-7a) and so Eq. (B-7d) from Eq. (B-7c), we get

$$(Z_{11} + Z_1 - Z_{12}) (I_1 - I_2) = 0 \quad (\text{B-8a})$$

$$(Z_{33} + Z_3 - Z_{34}) (I_3 - I_4) = 0 \quad (\text{B-8b})$$

Since the even resonance condition does not include the odd resonance conditions at a time, the impedance factors in Eqs. (B-8) are not zero. Thus,

$$I_1 = I_2 \quad (\text{B-9a})$$

$$I_3 = I_4 \quad (\text{B-9b})$$

C. DERIVATION OF EQS. (2-17) AND (2-18)

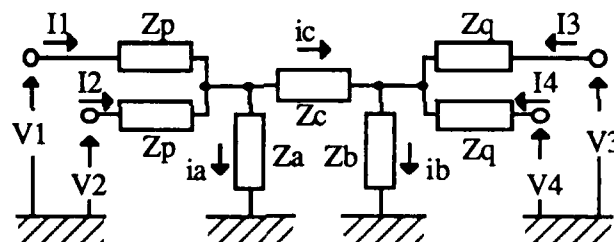


Fig. C-1 Equivalent circuit with V I definitions

Give the voltage and current definition references as in Fig. C-1 to obtain circuit equations :

$$I_1 + I_2 = i_a + i_c \quad (\text{C-1a})$$

$$I_3 + I_4 = i_a - i_c \quad (\text{C-1b})$$

$$Z_a i_a = Z_b i_b + Z_c i_c \quad (\text{C-1c})$$

$$V_1 = Z_p I_1 + Z_a i_a \quad (\text{C-1d})$$

$$V_2 = Z_p I_2 + Z_a i_a \quad (\text{C-1e})$$

$$V_3 = Z_q I_3 + Z_b i_b \quad (\text{C-1f})$$

$$V_4 = Z_q I_4 + Z_b i_b \quad (\text{C-1g})$$

From Eq. (C-1a) and (C-1b), we get

$$i_a + i_b = I_1 + I_2 + I_3 + I_4 \quad (C-2)$$

From Eq. (C-1b) and (C-1c), we get

$$Z_a i_a - (Z_b + Z_c) i_b = -Z_c (I_3 + I_4) \quad (C-3)$$

From Eq. (C-2) and (C-3), i_a and i_b are obtained :

$$i_a = \frac{Z_b + Z_c}{Z_a + Z_b + Z_c} I_1 + \frac{Z_b + Z_c}{Z_a + Z_b + Z_c} I_2 + \frac{Z_b}{Z_a + Z_b + Z_c} I_3 + \frac{Z_b}{Z_a + Z_b + Z_c} I_4 \quad (C-4a)$$

$$i_b = \frac{Z_a}{Z_a + Z_b + Z_c} I_1 + \frac{Z_a}{Z_a + Z_b + Z_c} I_2 + \frac{Z_c + Z_a}{Z_a + Z_b + Z_c} I_3 + \frac{Z_c + Z_a}{Z_a + Z_b + Z_c} I_4 \quad (C-4b)$$

Inserting Eqs. (C-4) into Eqs. from (C-1d) to (C-1g), system of V-I equations are obtained as follows :

$$V_1 = (Z_p + \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c}) I_1 + \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} I_2 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_3 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_4 \quad (C-5a)$$

$$V_2 = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} I_1 + (Z_p + \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c}) I_2 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_3 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_4 \quad (C-5b)$$

$$V_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_1 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_2 + (Z_q + \frac{Z_b(Z_c + Z_a)}{Z_a + Z_b + Z_c}) I_3 + \frac{Z_b(Z_c + Z_a)}{Z_a + Z_b + Z_c} I_4 \quad (C-5c)$$

$$V_4 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_1 + \frac{Z_a Z_b}{Z_a + Z_b + Z_c} I_2 + \frac{Z_b(Z_c + Z_a)}{Z_a + Z_b + Z_c} I_3 + (Z_q + \frac{Z_b(Z_c + Z_a)}{Z_a + Z_b + Z_c}) I_4 \quad (C-5d)$$

or

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} \\ Z_{12} & Z_{11} & Z_{13} & Z_{13} \\ Z_{13} & Z_{13} & Z_{33} & Z_{34} \\ Z_{13} & Z_{13} & Z_{34} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (C-5e)$$

where

$$Z_{11} = Z_p + \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} \quad (C-6a)$$

$$Z_{12} = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} \quad (C-6b)$$

$$Z_{13} = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \quad (C-6c)$$

$$Z_{33} = Z_q + \frac{Z_b(Z_c + Z_a)}{Z_a + Z_b + Z_c} \quad (C-6d)$$

$$Z_{34} = \frac{Z_b (Z_c + Z_a)}{Z_a + Z_b + Z_c} \quad (C-6e)$$

Next, we derive $Z_a, Z_b, \dots, Z_q$ as functions of Z parameters. For this, we solve Eqs. (C-6). Divide Eq. (C-6b) by Eq. (C-6c) to obtain

$$\frac{Z_{12}}{Z_{13}} = 1 + \frac{Z_c}{Z_b} \quad (C-7)$$

Divide Eq. (C-6e) by Eq. (C-6c) to obtain

$$\frac{Z_{34}}{Z_{13}} = 1 + \frac{Z_c}{Z_a} \quad (C-8)$$

Modify Eq. (C-6c) to get

$$1 + \frac{Z_c}{Z_b} = Z_a \left(\frac{1}{Z_{13}} - \frac{1}{Z_b} \right) \quad (C-9a)$$

$$1 + \frac{Z_c}{Z_a} = Z_b \left(\frac{1}{Z_{13}} - \frac{1}{Z_a} \right) \quad (C-9b)$$

Substitute Eqs. (C-9) into Eq. (C-7) and (C-8) to obtain

$$\frac{Z_{12}}{Z_{13}} = Z_a \left(\frac{1}{Z_{13}} - \frac{1}{Z_b} \right) \quad (C-10a)$$

$$\frac{Z_{34}}{Z_{13}} = Z_b \left(\frac{1}{Z_{13}} - \frac{1}{Z_a} \right) \text{ or } \frac{1}{Z_b} = \frac{Z_{13}}{Z_{34}} \left(\frac{1}{Z_{13}} - \frac{1}{Z_a} \right) \quad (C-10b)$$

Eliminate Z_b in Eq. (C-10a) by substituting Eq. (C-10b) to obtain

$$Z_a = \frac{Z_{13}^2 - Z_{12} Z_{34}}{Z_{13} - Z_{34}} \quad (C-11)$$

Inserting the result of Eq. (C-11) backward, all parameters are evaluated :

$$Z_b = \frac{Z_{13}^2 - Z_{12} Z_{34}}{Z_{13} - Z_{12}} \quad (C-12)$$

$$Z_c = - \frac{Z_{13}^2 - Z_{12} Z_{34}}{Z_{13}} \quad (C-13)$$

$$Z_p = Z_{11} - Z_{12} \quad (C-14)$$

$$Z_q = Z_{33} - Z_{34} \quad (C-15)$$

D. NORMALIZED IMPEDANCE MATRIX

For an N-port network, the impedance matrix is defined as

$$[V] = [Z][I] \quad (D-1)$$

where

$$[V] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad [I] = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ Z_{N1} & - & - & Z_{NN} \end{bmatrix}$$

Now, we introduce a normalized voltage vector and a normalized current vector defined as

$$[v] = \text{diag} [\sqrt{Z_{0i}}]^{-1} [V] \quad (D-2a)$$

$$[i] = \text{diag} [\sqrt{Z_{0i}}] [I] \quad (D-2b)$$

where Z_{0i} is the characteristic impedance at the i -th port.

Substitute Eqs. (D-2) into Eq. (D-1) to obtain

$$\begin{aligned} [v] &= \text{diag} [\sqrt{Z_{0i}}]^{-1} [Z] \text{diag} [\sqrt{Z_{0i}}] [i] \\ &= [z][i] \end{aligned} \quad (D-3)$$

where

$[z]$: normalized impedance matrix with each element as

$$z_{ij} = \frac{Z_{ij}}{\sqrt{Z_{0i}} \sqrt{Z_{0j}}} \quad i, j = 1, 2, \dots, N \quad (D-4)$$

$$* \text{diag} [a_i^{-1}] = \text{diag} [a_i]^{-1} \quad (D-5)$$

Meanwhile, voltages and currents are decomposed into forward wave component and backward wave component :

$$[v] = [v]^+ + [v]^- \quad (D-6a)$$

$$[i] = [i]^+ - [i]^- \quad (D-6b)$$

where superscript $(+)$ refers to forward quantity and $(-)$ refers to backward quantity. S matrix is defined commonly in terms of normalized voltage vectors as

$$[v]^- = [S][v]^+ \quad (D-7)$$

Substitute Eqs. (D-6) into Eq. (D-3) and make use of the relations that $[v]^+ = [i]^+$ and $[v]^- = [i]^-$ to obtain

$$\begin{aligned}
 [\mathbf{v}]^+ &= [[\mathbf{z}] + [\mathbf{U}]]^{-1} [[\mathbf{z}] - [\mathbf{U}]] [\mathbf{v}]^- \\
 \text{or} \quad [\mathbf{S}] &= [[\mathbf{z}] + [\mathbf{U}]]^{-1} [[\mathbf{z}] - [\mathbf{U}]] \quad (\text{D-8})
 \end{aligned}$$

where $[\mathbf{U}]$ is a unit matrix. $\mathbf{S}$ matrix, i.e., $\mathbf{S}$ parameters are calculated regarding to normalized z parameters as follows :

$$\begin{aligned}
 [[\mathbf{z}] + [\mathbf{U}]]^{-1} [[\mathbf{z}] - [\mathbf{U}]] &= \frac{\text{adj} \parallel [\mathbf{z}] + [\mathbf{U}] \parallel}{\det \parallel [\mathbf{z}] + [\mathbf{U}] \parallel} [[\mathbf{z}] - [\mathbf{U}]] \\
 &= \frac{1}{\det \parallel [\mathbf{z}] + [\mathbf{U}] \parallel} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{13} \\ a_{12} & a_{11} & a_{13} & a_{13} \\ a_{13} & a_{13} & a_{33} & a_{34} \\ a_{13} & a_{13} & a_{34} & a_{33} \end{bmatrix} \begin{bmatrix} z_{11}-1 & z_{12} & z_{13} & z_{13} \\ z_{12} & z_{11}-1 & z_{13} & z_{13} \\ z_{13} & z_{13} & z_{33}-1 & z_{34} \\ z_{13} & z_{13} & z_{34} & z_{33}-1 \end{bmatrix} \\
 &= \frac{1}{\det \parallel [\mathbf{z}] + [\mathbf{U}] \parallel} \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{13} \\ c_{12} & c_{11} & c_{13} & c_{13} \\ c_{13} & c_{13} & c_{33} & c_{34} \\ c_{13} & c_{13} & c_{34} & c_{33} \end{bmatrix} \quad (\text{D-10a})
 \end{aligned}$$

where

$$a_{11} = (z_{33}-z_{34}+1) \{ (z_{11}+1)(z_{33}+z_{34}+1) - 2z_{13}^2 \} \quad (\text{D-10b})$$

$$a_{12} = - (z_{33}-z_{34}+1) \{ z_{12}(z_{33}+z_{34}+1) - 2z_{13}^2 \} \quad (\text{D-10c})$$

$$a_{13} = - z_{13} (z_{11}-z_{12}+1) (z_{33}-z_{34}+1) \quad (\text{D-10d})$$

$$a_{33} = (z_{11}-z_{12}+1) \{ (z_{33}+1)(z_{11}+z_{12}+1) - 2z_{13}^2 \} \quad (\text{D-10e})$$

$$a_{34} = - (z_{11}-z_{12}+1) \{ z_{34}(z_{11}+z_{12}+1) - 2z_{13}^2 \} \quad (\text{D-10f})$$

$$c_{11} = a_{11} (z_{11}-1) + a_{12} z_{12} + 2 a_{13} z_{13} \quad (\text{D-10g})$$

$$c_{12} = a_{11} z_{12} + a_{12} (z_{11}-1) + 2 a_{13} z_{13} \quad (\text{D-10h})$$

$$c_{13} = (a_{11}+a_{12}) z_{13} + a_{13} (z_{33}+z_{34}-1) \quad (\text{D-10i})$$

$$c_{33} = 2 a_{13} z_{13} + a_{33} (z_{33}-1) + a_{34} z_{34} \quad (\text{D-10j})$$

$$c_{34} = 2 a_{13} z_{13} + a_{33} z_{34} + a_{34} (z_{33}-1) \quad (\text{D-10k})$$

$$\det \parallel [\mathbf{z}] + [\mathbf{U}] \parallel$$

$$= (z_{11}-z_{12}+1) (z_{33}-z_{34}+1) \{ (z_{11}+z_{12}+1)(z_{33}+z_{34}+1) - 4z_{13}^2 \} \quad (\text{D-10l})$$

E. DERIVATION OF EQS.(3-10)

For the notational convenience, the subscripts mn are mostly omitted in this section and some abbreviation forms are listed in advance :

$$\beta^{(i)} = \beta_{mn}^{(i)} \quad i = 1, 2, 3$$

$$A^{(i)} = A_{mn}^{(i)} \quad i = 1, 2$$

$$B^{(i)} = B_{mn}^{(i)} \quad i = 1, 2$$

$$D, E, F, G = D_{mn}, E_{mn}, F_{mn}, G_{mn}$$

$$\Psi, \Phi = \Psi_{mn}, \Phi_{mn}$$

$$\xi_{v^{(i)}} = \xi_{mn,v^{(i)}} \quad i = 1, 2$$

$$\theta_{v^{(i)}} = \theta_{mn,v^{(i)}} \quad i = 1, 2$$

$$S_1, S_2, S_3 = \sin \beta^{(1)} h_1, \sin \beta^{(2)} h_2, \sin \beta^{(3)} t$$

$$C_1, C_2, C_3 = \cos \beta^{(1)} h_1, \cos \beta^{(2)} h_2, \cos \beta^{(3)} t$$

$$Ct_1, Ct_2, Ct_3 = C_1/S_1, C_2/S_2, C_3/S_3$$

$$z^\wedge = j \omega \mu_0$$

Apply the boundary condition of Eq. (3-7a), i.e.,

$$Et_1 = Et_3$$

to Eq. (3-4a) and Eq. (3-6a) at $z = 0$, and take inner products over S with the orthogonal functions $\nabla t \Psi \times z$ and $\nabla t \Phi$ respectively to obtain

$$\int_S \nabla t \Psi \times z \cdot [Et_1 - Et_3] ds = 0 \quad \text{at } z = 0$$

$$\int_S \nabla t \Phi \cdot [Et_1 - Et_3] ds = 0 \quad \text{at } z = 0$$

$$-A^{(1)} S_1 = D S_3 \quad (E-1)$$

$$-B^{(1)} (\beta^{(1)}/y^\wedge_1) S_1 = G (\beta^{(3)}/y^\wedge_3) S_3 \quad (E-2)$$

Take the inner products over S with $\nabla t \Psi$, $\nabla t \Phi \times z$ for the boundary condition of Eq. (3-7c) respectively to obtain

$$\int_S \nabla t \Psi \cdot [Ht_3 - Ht_1] ds = \int_{s1} \nabla t \Psi \cdot \sum P_v^{(1)} \chi_v^{(1)} ds \quad \text{at } z = 0$$

$$\int_S \nabla t \Phi \times z \cdot [Ht_3 - Ht_1] ds = \int_{s1} \nabla t \Phi \times z \cdot \sum P_v^{(1)} \chi_v^{(1)} ds \quad \text{at } z = 0$$

$$\{ C + D C_3 \} (\beta^{(3)}/z^\wedge) - A^{(1)} (\beta^{(1)}/z^\wedge) C_1 = \sum P_{\mathbf{v}}^{(1)} \cdot \xi_{\mathbf{v}}^{(1)} \quad (\text{E-3})$$

$$F + G C_3 - B^{(1)} C_1 = \sum P_{\mathbf{v}}^{(1)} \cdot \theta_{\mathbf{v}}^{(1)} \quad (\text{E-4})$$

where

$$\xi_{\mathbf{v}}^{(1)} = \int_{S1} \chi_{\mathbf{v}}^{(1)} \cdot \nabla_t \Psi \, ds$$

$$\theta_{\mathbf{v}}^{(1)} = \int_{S1} \chi_{\mathbf{v}}^{(1)} \cdot \nabla_t \Phi \times \mathbf{z} \, ds$$

Note that the integral region for $\xi_{\mathbf{v}}^{(1)}$, $\theta_{\mathbf{v}}^{(1)}$ is reduced to $S1$.

In order to apply the boundary condition of Eq. (3-7b), we may take the inner product with function χ_μ so as to expect the same expressions as ξ_μ , θ_μ for convenience.

The H field function χ_μ , however, is orthogonal to E_t field. Hence, we use $\chi_\mu \times \mathbf{z}$ instead which is parallel to E_t and integration is done only over $S1$:

$$\int_{S1} \chi_\mu \times \mathbf{z} \cdot E_{t1} \, ds = 0 \quad \text{at } z = 0$$

MN

$$\sum_{mn} [A^{(1)} \int_{S1} \chi_\mu^{(1)} \times \mathbf{z} \cdot \nabla_t \Psi \times \mathbf{z} \, ds + B^{(1)} (\beta^{(1)}/y^\wedge_1) \int_{S1} \chi_\mu^{(1)} \times \mathbf{z} \cdot \nabla_t \Phi \, ds] S_1 = 0$$

MN

$$\text{or } \sum_{mn} [A^{(1)} \xi_\mu^{(1)} - B^{(1)} (\beta^{(1)}/y^\wedge_1) \theta_\mu^{(1)}] S_1 = 0, \quad \mu = 1, 2, 3, - \quad (\text{E-5})$$

where following vector identities are used :

$$\chi \times \mathbf{z} \cdot \nabla_t \Psi \times \mathbf{z} = \chi \cdot \nabla_t \Psi$$

$$\chi \times \mathbf{z} \cdot \nabla_t \Phi = - \chi \cdot \nabla_t \Phi \times \mathbf{z}$$

Similar process is taken for the boundary conditions at the interface (II) to extract unknown coefficients in Es. (3-5) and (3-6).

Take inner products over S with $\nabla_t \Psi \times \mathbf{z}$ and $\nabla_t \Phi$ respectively for the boundary condition of Eq. (3-8a) to obtain

$$\int_S \nabla_t \Psi \times \mathbf{z} \cdot [E_{t2} - E_{t3}] \, ds = 0 \quad \text{at } z = t$$

$$\int_S \nabla_t \Phi \cdot [E_{t2} - E_{t3}] \, ds = 0 \quad \text{at } z = t$$

$$A^{(2)} S_2 = - C S_3 \quad (E-6)$$

$$B^{(2)} (\beta^{(2)}/y^{\wedge}_2) S_2 = - F (\beta^{(3)}/y^{\wedge}_3) S_3 \quad (E-7)$$

Take the inner products over S with $\nabla_t \Psi$, $\nabla_t \Phi \times z$ for the boundary condition of Eq. (3-8c) respectively to obtain

$$\int_s \nabla_t \Psi \cdot [Ht_2 - Ht_3] ds = \int_{s2} \nabla_t \Psi \cdot \sum P_v^{(2)} \chi_v^{(2)} ds \quad \text{at } z = t$$

$$\int_s \nabla_t \Phi \times z \cdot [Ht_2 - Ht_3] ds = \int_{s2} \nabla_t \Phi \times z \cdot \sum P_v^{(2)} \chi_v^{(2)} ds \quad \text{at } z = t$$

$$A^{(2)} (\beta^{(2)}/z^{\wedge}) C_2 - [C C_3 + D] (\beta^{(1)}/z^{\wedge}) = \sum P_v^{(2)} \cdot \xi_v^{(2)} \quad (E-8)$$

$$B^{(2)} C_2 - [F C_3 + G] = \sum P_v^{(2)} \cdot \theta_v^{(2)} \quad (E-9)$$

where

$$\xi_v^{(2)} = \int_{s2} \chi_v^{(2)} \cdot \nabla_t \Psi ds$$

$$\theta_v^{(2)} = \int_{s2} \chi_v^{(2)} \cdot \nabla_t \Phi \times z ds$$

Take the inner product over S2 with $\chi_\mu \times z$ for the boundary condition of Eq. (3-8b) to obtain

$$\int_{s2} \chi_\mu \times z \cdot Et_2 ds = 0 \quad \text{at } z = t$$

MN

$$\sum_{mn} [A^{(2)} \int_{s2} \chi_\mu^{(2)} \times z \cdot \nabla_t \Psi \times z ds + B^{(2)} (\beta^{(2)}/y^{\wedge}_2) \int_{s2} \chi_\mu^{(2)} \times z \cdot \nabla_t \Phi ds] S_2 = 0$$

MN

$$\text{or } \sum_{mn} [A^{(2)} \xi_\mu^{(2)} - B^{(2)} (\beta^{(2)}/y^{\wedge}_2) \theta_\mu^{(2)}] S_2 = 0, \quad \mu = 1, 2, 3, - \quad (E-10)$$

Eliminate C_{mn} , D_{mn} , F_{mn} , and G_{mn} in Eqs. (E-3), (E-4), (E-8), and (E-9) by substitution of Eqs. (E-1), (E-2), (E-6), and (E-7) to obtain

$$-[\beta^{(1)} C_1 S_3 + \beta^{(3)} S_1 C_3] A^{(1)} - \beta^{(3)} S_2 A^{(2)} = S_3 z^{\wedge} \sum P_v^{(1)} \cdot \xi_v^{(1)} \quad (E-11a)$$

$$\beta^{(3)} S_1 A^{(1)} + [\beta^{(2)} C_2 S_3 + \beta^{(3)} S_2 C_3] A^{(2)} = S_3 z^{\wedge} \sum P_v^{(2)} \cdot \xi_v^{(2)} \quad (E-11b)$$

$$-[\beta^{(3)} C_1 S_3 + (\epsilon_3/\epsilon_1) \beta^{(3)} S_1 C_3] B^{(1)} - (\epsilon_3/\epsilon_2) \beta^{(2)} S_2 B^{(2)} = \beta^{(3)} S_3 \sum P_v^{(1)} \cdot \theta_v^{(1)} \quad (E-11c)$$

$$(\epsilon_3/\epsilon_1) \beta^{(1)} S_1 B^{(1)} + [\beta^{(3)} C_2 S_3 + (\epsilon_3/\epsilon_2) \beta^{(2)} S_2 C_3] B^{(2)} = \beta^{(3)} S_3 \sum P_v^{(2)} \cdot \theta_v^{(2)} \quad (E-11d)$$

Solve Eqs. (E-11) for $A_{mn}^{(1)}$, $A_{mn}^{(2)}$, $B_{mn}^{(1)}$, $B_{mn}^{(2)}$ to obtain

$$A^{(1)} = a^{(11)} \sum P_{V^{(1)}} \cdot \xi_{V^{(1)}} + a^{(12)} \sum P_{V^{(2)}} \cdot \xi_{V^{(2)}} \quad (E-12a)$$

$$A^{(2)} = a^{(21)} \sum P_{V^{(1)}} \cdot \xi_{V^{(1)}} + a^{(22)} \sum P_{V^{(2)}} \cdot \xi_{V^{(2)}} \quad (E-12b)$$

$$B^{(1)} = b^{(11)} \sum P_{V^{(1)}} \cdot \theta_{V^{(1)}} + b^{(12)} \sum P_{V^{(2)}} \cdot \theta_{V^{(2)}} \quad (E-12c)$$

$$B^{(2)} = b^{(21)} \sum P_{V^{(1)}} \cdot \theta_{V^{(1)}} + b^{(22)} \sum P_{V^{(2)}} \cdot \theta_{V^{(2)}} \quad (E-12d)$$

where

$$a_{mn}^{(11)} = K_a (1/S_1) (B^{(2)} C_{t2} + B^{(3)} C_{t3}) \quad (3-12e)$$

$$a_{mn}^{(12)} = K_a (1/(S_1 S_3)) B^{(3)} \quad (3-12f)$$

$$a_{mn}^{(21)} = -K_a (1/(S_2 S_3)) B^{(3)} \quad (3-12g)$$

$$a_{mn}^{(22)} = -K_a (1/S_2) (B^{(1)} C_{t1} + B^{(3)} C_{t3}) \quad (3-12h)$$

$$K_a = z^{\wedge} / \{ B^{(3)2} - B^{(3)} C_{t3} (B^{(2)} C_{t2} + B^{(1)} C_{t1}) - B^{(1)} B^{(2)} C_{t1} C_{t2} \}$$

$$b_{mn}^{(11)} = K_b (1/S_1) (B^{(3)} C_{t2} + (\epsilon_3/\epsilon_2) B^{(2)} C_{t3}) \quad (3-12i)$$

$$b_{mn}^{(12)} = K_b (1/(S_1 S_3)) (\epsilon_3/\epsilon_2) B^{(2)} \quad (3-12j)$$

$$b_{mn}^{(21)} = -K_b (1/(S_2 S_3)) (\epsilon_3/\epsilon_1) B^{(1)} \quad (3-12k)$$

$$b_{mn}^{(22)} = -K_b (1/S_2) (B^{(3)} C_{t1} + (\epsilon_3/\epsilon_1) B^{(1)} C_{t3}) \quad (3-12l)$$

$$K_b = B^{(3)} / [(\epsilon_3/\epsilon_1)(\epsilon_3/\epsilon_2) B^{(1)} B^{(2)} - B^{(3)} C_{t3} \{ (\epsilon_3/\epsilon_1) B^{(1)} C_{t2} + (\epsilon_3/\epsilon_2) B^{(2)} C_{t1} \} - B^{(3)2} C_{t1} C_{t2}]$$

Finally, substitute Eqs. (E-12) into Eq. (E-5) and Eq. (E-10) to obtain

$$\sum_{V_1}^{v_1 M} P_{V^{(1)}} U_{\mu V^{(11)}} + \sum_{V_2}^{v_2 M} P_{V^{(2)}} U_{\mu V^{(12)}} = 0 \quad \mu = 1, 2, 3, v_1 M \quad (E-13a)$$

$$\sum_{V_1}^{v_1 M} P_{V^{(1)}} U_{\mu V^{(21)}} + \sum_{V_2}^{v_2 M} P_{V^{(2)}} U_{\mu V^{(22)}} = 0 \quad \mu = 1, 2, 3, v_2 M \quad (E-13b)$$

where

$$U_{\mu V^{(ij)}} = \sum_{mn}^{MN} S_i [a^{(ij)} \xi_{\mu^{(i)}} \xi_{V^{(j)}} - (\beta^{(i)}/y^{\wedge}_i) b^{(ij)} \theta_{\mu^{(i)}} \theta_{V^{(j)}}] \quad (E-13c)$$

$i, j = 1, 2$

Eqs. (E-13) are referred to as a homogeneous system of equations in terms of unknown coefficients $Pv^{(i)}$ ($i = 1, 2$), and are expressed in the matrix form as follows :

$$[U][P] = 0$$

or

$$\begin{array}{c} \mu = 1 \\ 2 \\ \vdots \\ v1M \\ \mu = 1 \\ \vdots \\ v2M \end{array} \begin{array}{c} v = 1 \quad 2 \quad \dots \quad v1M \quad v = 1 \quad \dots \quad v2M \\ \left[\begin{array}{cccccc} U_{11}^{(11)} & U_{12}^{(11)} & \dots & U_{1v1M}^{(11)} & U_{11}^{(12)} & \dots \\ U_{21}^{(11)} & U_{22}^{(11)} & & & & \\ \vdots & & \ddots & & & \\ U_{v1M1}^{(11)} & \dots & & U_{v1Mv1M}^{(11)} & \dots & U_{v1Mv2M}^{(12)} \\ \hline U_{11}^{(21)} & \dots & & & U_{11}^{(22)} & \dots \\ \vdots & & \ddots & & & \\ U_{v2M1}^{(21)} & \dots & & U_{v2Mv1M}^{(21)} & \dots & U_{v2Mv2M}^{(22)} \end{array} \right] \begin{array}{c} P_1^{(1)} \\ P_2^{(1)} \\ \vdots \\ P_{v1M}^{(1)} \\ \hline P_1^{(2)} \\ P_2^{(2)} \\ \vdots \\ P_{v2M}^{(2)} \end{array} \end{array} = 0$$

(E-14)

Integral values of $\xi_{v^{(i)}}$, $\theta_{v^{(i)}}$ are derived as follows. First, we get the scalar potential vector functions as

$$\nabla_t \Psi = -P \left(x \frac{m\pi}{a} \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y + y \frac{n\pi}{b} \cos \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \right) \quad (E-15a)$$

$$\nabla_t \Phi \times z = P \left(x \frac{n\pi}{b} \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y - y \frac{m\pi}{a} \cos \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \right) \quad (E-15b)$$

where

$$P = \sqrt{\frac{\delta m \delta n}{a b}} \frac{1}{kc} \quad \delta i = \begin{cases} 1 & i = 0 \\ 2 & i \neq 0 \end{cases}$$

$$kc^2 = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2$$

Take integrals of the product of Eqs. (E-15) and Eqs. (3-13), (3-14), we get

$$\begin{aligned}
\xi_{x,mn,rs}^{(1)} &= \int_{s1} \chi_{x,rs}^{(1)} \cdot \nabla_t \Psi_{mn} ds \\
&= -P \frac{m\pi}{a} \int_0^a \sin \frac{m\pi}{a} x \sin \frac{n\pi}{a} x dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \cos \frac{n\pi}{b} y \sin \frac{s\pi}{w1} (y - \frac{b-w1}{2}) dy \\
&= -P \frac{m\pi}{a} \frac{a}{2} \delta_{rm} HI(s,n,w1,b)
\end{aligned} \tag{E-16}$$

where

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\begin{aligned}
HI(s,n,w1,b) &= \frac{w1}{2} \sin \frac{(s-n)\pi}{2} && \text{for } \frac{n}{b} = \frac{s}{w1} \\
&= \frac{(s\pi/w1)}{(s\pi/w1)^2 - (n\pi/b)^2} \left[\cos \frac{n\pi(b-w1)}{2b} - (-1)^s \cos \frac{n\pi(b+w1)}{2b} \right] && \text{for } \frac{n}{b} \neq \frac{s}{w1}
\end{aligned} \tag{E-17}$$

$$\begin{aligned}
\xi_{y,mn,rs}^{(1)} &= \int_{s1} \chi_{y,rs}^{(1)} \cdot \nabla_t \Psi_{mn} ds \\
&= -P \frac{n\pi}{b} \int_0^a \cos \frac{m\pi}{a} x \cos \frac{n\pi}{a} x dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \sin \frac{n\pi}{b} y \frac{\cos \frac{s\pi}{w1} (y - \frac{b-w1}{2})}{\sqrt{1 - \left\{ \frac{y-b/2}{w1/2} \right\}^2}} dy \\
&= -P \frac{n\pi}{b} \frac{a}{\delta_m} \delta_{rm} SI(s,n,w1,b)
\end{aligned} \tag{E-18}$$

where

$$\begin{aligned}
SI(s,n,w1,b) &= 0 && n+s : \text{even} \\
&= \frac{\pi w1}{4} \left[(-1)^2 J_0 \left(\left| \frac{n\pi}{b} - \frac{s\pi}{w1} \right| \frac{w1}{2} \right) + (-1)^2 J_0 \left(\left| \frac{n\pi}{b} + \frac{s\pi}{w1} \right| \frac{w1}{2} \right) \right] && n+s : \text{odd}
\end{aligned} \tag{E-19}$$

J_0 : Zero order Bessel function of the first kind

$$\begin{aligned}
\theta_{x,mn,rs}^{(1)} &= \int_{s1} \chi_{x,rs}^{(1)} \cdot \nabla_t \Phi_{mn} \times z ds \\
&= P \frac{n\pi}{b} \int_0^a \sin \frac{m\pi}{a} x \sin \frac{n\pi}{a} x dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \cos \frac{n\pi}{b} y \sin \frac{s\pi}{w1} (y - \frac{b-w1}{2}) dy \\
&= P \frac{n\pi}{b} \frac{a}{2} \delta_{rm} HI(s,n,w1,b)
\end{aligned} \tag{E-20}$$

$$\begin{aligned}
\theta_{y,mn,rs(1)} &= \int_{s1} \chi_{y,rs(1)} \cdot \nabla_t \Phi_{mn} \times z \, ds \\
&= -P \frac{m\pi}{a} \int_0^a \cos \frac{m\pi}{a} x \cos \frac{n\pi}{a} x \, dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \sin \frac{n\pi}{b} y \frac{\cos \frac{s\pi}{w1} \{y - \frac{(b-w1)}{2}\}}{\sqrt{1 - \{\frac{y-b/2}{w1/2}\}^2}} \, dy \\
&= -P \frac{m\pi}{a} \frac{a}{\delta m} \delta_{rm} SI(s,n,w1,b) \quad (E-21)
\end{aligned}$$

$$\begin{aligned}
\xi_{x,mn,rs(2)} &= \int_{s2} \chi_{x,rs(2)} \cdot \nabla_t \Psi_{mn} \, ds \\
&= -P \frac{m\pi}{a} \int_0^b \cos \frac{n\pi}{b} y \cos \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \sin \frac{m\pi}{a} x \frac{\cos \frac{r\pi}{w2} \{x - \frac{(a-w2)}{2}\}}{\sqrt{1 - \{\frac{x-a/2}{w2/2}\}^2}} \, dx \\
&= -P \frac{m\pi}{a} \frac{b}{\delta n} \delta_{sn} SI(r,m,w2,a) \quad (E-22)
\end{aligned}$$

$$\begin{aligned}
\xi_{y,mn,rs(2)} &= \int_{s2} \chi_{y,rs(2)} \cdot \nabla_t \Psi_{mn} \, ds \\
&= -P \frac{n\pi}{b} \int_0^b \sin \frac{n\pi}{b} y \sin \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \cos \frac{m\pi}{a} x \sin \frac{r\pi}{w2} \{x - \frac{(a-w2)}{2}\} \, dx \\
&= -P \frac{n\pi}{b} \frac{b}{2} \delta_{sn} HI(r,m,w2,a) \quad (E-23)
\end{aligned}$$

$$\begin{aligned}
\theta_{x,mn,rs(2)} &= \int_{s2} \chi_{x,rs(2)} \cdot \nabla_t \Phi_{mn} \times z \, ds \\
&= P \frac{n\pi}{b} \int_0^b \cos \frac{n\pi}{b} y \cos \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \sin \frac{m\pi}{a} x \frac{\cos \frac{r\pi}{w2} \{x - \frac{(a-w2)}{2}\}}{\sqrt{1 - \{\frac{x-a/2}{w2/2}\}^2}} \, dx \\
&= P \frac{n\pi}{b} \frac{b}{\delta n} \delta_{sn} SI(r,m,w2,a) \quad (E-24)
\end{aligned}$$

$$\begin{aligned}
\theta_{y,mn,rs(2)} &= \int_{s2} \chi_{y,rs(2)} \cdot \nabla_t \Phi_{mn} \times z \, ds \\
&= -P \frac{m\pi}{a} \int_0^b \sin \frac{n\pi}{b} y \sin \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \cos \frac{m\pi}{a} x \sin \frac{r\pi}{w2} \{x - \frac{(a-w2)}{2}\} \, dx \\
&= -P \frac{m\pi}{a} \frac{b}{2} \delta_{sn} HI(r,m,w2,a) \quad (E-25)
\end{aligned}$$

Here are summary of $\xi_v^{(i)}$, $\theta_v^{(i)}$:

$$\xi_x^{(1)} = -P_{mn} \frac{m\pi}{a} \frac{a}{2} \delta_{rm} HI(s, n, w1, b) \quad (E-26a)$$

$$\xi_y^{(1)} = -P_{mn} \frac{n\pi}{b} \frac{a}{2} \delta_{rm} SI(s, n, w1, b) \quad (E-26b)$$

$$\xi_x^{(2)} = -P_{mn} \frac{m\pi}{a} \frac{b}{2} \delta_{sn} SI(r, m, w2, a) \quad (E-26c)$$

$$\xi_y^{(2)} = -P_{mn} \frac{n\pi}{b} \frac{b}{2} \delta_{sn} HI(r, m, w2, a) \quad (E-26d)$$

$$\theta_x^{(1)} = P_{mn} \frac{n\pi}{b} \frac{a}{2} \delta_{rm} HI(s, n, w1, b) \quad (E-26e)$$

$$\theta_y^{(1)} = -P_{mn} \frac{m\pi}{a} \frac{a}{2} \delta_{rm} SI(s, n, w1, b) \quad (E-26f)$$

$$\theta_x^{(2)} = P_{mn} \frac{n\pi}{b} \frac{b}{2} \delta_{sn} SI(r, m, w2, a) \quad (E-26g)$$

$$\theta_y^{(2)} = -P_{mn} \frac{m\pi}{a} \frac{b}{2} \delta_{sn} HI(r, m, w2, a) \quad (E-26h)$$

$$\delta_i = \begin{cases} 1 & i = 0 \\ 2 & i \neq 0 \end{cases}$$

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$HI(s, n, w1, b)$

$$\begin{aligned} &= \frac{w1}{2} \sin \frac{(s-n)\pi}{2} && \text{for } \frac{n}{b} = \frac{s}{w1} \\ &\left\{ \begin{aligned} &= \frac{(s\pi/w1)}{(s\pi/w1)^2 - (n\pi/b)^2} \left[\cos \frac{n\pi(b-w1)}{2b} - (-1)^s \cos \frac{n\pi(b+w1)}{2b} \right] && \text{for } \frac{n}{b} \neq \frac{s}{w1} \end{aligned} \right. \end{aligned}$$

$SI(s, n, w1, b)$

$$\begin{aligned} &= 0 && n + s : \text{even} \\ &\left\{ \begin{aligned} &= \frac{\pi w1}{4} \left[(-1)^2 J_0 \left(\frac{n\pi}{b} - \frac{s\pi}{w1} \frac{w1}{2} \right) + (-1)^2 J_0 \left(\frac{n\pi}{b} + \frac{s\pi}{w1} \frac{w1}{2} \right) \right] && n + s : \text{odd} \end{aligned} \right. \end{aligned}$$

F. CHOICE OF rs COMBINATION

The numbers for r and s and the choice of x or y component for ξ_μ and θ_μ are assigned according to the number v in Eq. (E-14). An example of rs combination is presented here ; actually, the results in chapter 4 are calculated by this combination.

Table F-1 shows the example. In Table F-1, as M-M wall case with $v = 1$ (odd number), for instance, $\chi_y^{(1)}$ is chosen at interface (I) for ξ_μ and θ_μ calculations so that the first combination of $r' = 0$ and $s' = 0$ is assigned. Next $r' s'$ combination for $v = 3$ is that $r' = 1$ and $s' = 0$. With v for even number, χ_x and is chosen.

Table F-1 rs -table

Wall		at Interface(I)		
(A)	(B)	$v=\text{even}$ $\chi_x \text{ (Jy)}$	$v=\text{odd}$ $\chi_y \text{ (Jx)}$	
M	M	$r'=0,1,2,--$ $s'=1$	$r'=0,1,2,--$ $s'=0$	$r=2r'+1$ $s=2s'$
E	M	$r'=1,2,3,--$ $s'=1$	$r'=0,1,2,--$ * $s'=0$	$r=2r'$ $s=2s'$
M	E	$r'=0,1,2,--$ $s'=0$	$r'=0,1,2,--$ $s'=0$	$r=2r'+1$ $s=2s'+1$

at Interface(II)

Wall		$v=\text{odd}$ $\chi_x (Jy)$	$v=\text{even}$ $\chi_y (Jx)$	
(A)	(B)			
M	M	$r'=0$ $s'=0,1,2,--$	$r'=1$ $s'=0,1,2,--$	$r=2r'$ $s=2s'+1$
E	M	$r'=0$ $s'=0,1,2,--$	$r'=0$ $s'=0,1,2,--$	$r=2r'+1$ $s=2s'+1$
M	E	$r'=0$ $s'=0,1,2,-- *$	$r'=1$ $s'=1,2,3,--$	$r=2r'$ $s=2s'$

* The fundamental resonance mode is described as $r'=1(\text{interface(I)})$ or $s'=1(\text{interface(II)})$ instead of the first combination with $r'=0$ or $s'=0$

Note that transverse distribution of each current, i.e., longitudinal or transverse current, is represented only with one term in Table F-1; transverse distribution may be well expressed only with one term while many terms are needed for the expansion of longitudinal distribution because of discontinuities.

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PROGRAM LIST

PROGRAM CROSS2

```

*
* =====
* Subroutines in IMSL are required in LINKing
* =====
*

Real      Ea(180)
Common    /size/a,b,w1,w2,h1,h2,t
Common    /Para/Freq,Er1,Er2,Er3
Common    /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
Common    /Mat /Ea

*
External  RSNTR1

*
OPEN(FILE='CROSS2.DAT',UNIT=7,STATUS='OLD')

*
  write(*,901)
901 format(
- '          Cross-Striplines                      1986-0227,1986-0404'
- '/'                                           by Tom Uwano'/
- '/'      y                                           '
- '/'          w2                                           '
- '/'      b ----- z '
- '/'          1      i      i      1
- '/'          1      i      i      1      ----- h2'
- '/'          1      i      i      1      1      Er2      (II)  1 '
- '/'          1-----i--i-----1      1      w2      1 '
- '/' b/2      1      i      i      lw1 (III) 1-----***-----1 t '
- '/'          1-----i--i-----1      Er3 1-----1 0 '
- '/'          1      i      i      1      1      1 '
- '/'          1      i      i      1      1      Er1      (I)  1 '
- '/'          1      i      i      1      ----- -h1'
- '/'      z @----- '
- '/'          0          a/2          a      x
- '/'                                           Frequency [MHz]  '/')

*
*
  write(*,*) 'Input 0(MMwall),1(EMwall,x=a/2) or 2(MEwall,y=b/2)'
  read(*,*) MEwall
  if(MEwall.EQ.0)
- write(*,*) 'Input 0(combination),1(region(I)) or 2(region(II))'
  if(MEwall.EQ.1) write(*,*) 'Input 0(combination) or 1(region(I))'
  if(MEwall.EQ.2) write(*,*) 'Input 0(combination) or 2(region(II))'
  read(*,*) Nreso
  if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
    write(*,*) 'Input approximate a in [mm]'
    read(*,*) a
    write(*,*) 'Input b(fixed) in [mm]'
    read(*,*) b
  else
    write(*,*) 'Input approximate b in [mm]'

```

```

        read(*,*) b
        write(*,*) 'Input a(fixed) in [mm]'
        read(*,*) a
    endif
    write(*,*) 'Input h1,t,h2 in [mm]'
    read(*,*) h1,t,h2
    write(*,*) 'Input w1,w2 in [mm]'
    read(*,*) w1,w2
    write(*,*) 'Input frequency in [MHz]'
    read(*,*) freq
    write(*,*) 'Input Er1,Er2,Er3'
    read(*,*) Er1,Er2,Er3
*
    a = a*1.e-3
    b = b*1.e-3
    h1=h1*1.e-3
    t = t*1.e-3
    h2=h2*1.e-3
    w1=w1*1.e-3
    w2=w2*1.e-3
    freq=freq*1.e-3
    if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
        ax=a
    else
        ax=b
    endif
*
    write(*,*) 'Input m,n max'
    read(*,*) Nmax
    write(*,*) 'Matrix size (2N x 2N). Input N'
    read(*,*) NHmat
    Np=2*NHmat-1
*
    q=freq*1.e3
    ME=MEwall
    write(7,902) a,b,h1,t,h2,w1,w2,q,Er1,Er2,Er3,Nmax,NHmat,ME,Nreso
902 format('a =',E13.6/'b =',E13.6/'h1=',E13.6
-        /'t =',E13.6/'h2=',E13.6/'w1=',E13.6/'w2=',E13.6
-        /'freq=',E13.6,' MHz'/'Er1,Er2,Er3=',3F7.2
-        /'m,n max =',I5
-        /'Matrix 2N x 2N, N=',I3/'MEwall=',I3/'Nreso =',I3)
    icount=0
    coef1=1.
    Call ZERO2(RSNTRL,ax,ier)
    if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
        write(*,*) ' final a in [mm] =',ax*1.e3,ier
        write(7,*) ' final a in [mm] =',ax*1.e3,ier
    else
        write(*,*) ' final b in [mm] =',ax*1.e3,ier
        write(7,*) ' final b in [mm] =',ax*1.e3,ier
    endif
*
    write(*, '(4E16.7)') 1., (Ea(i),i=1,Np)
    write(7, '(4E16.7)') 1., (Ea(i),i=1,Np)

```

```
stop
end
```

```
FUNCTION RSNTR1(ax)
```

```
Cross-Striplines
```

```
1986-0226,1986-0401
by Tom Uwano
```

```

      Y
      b -----
      1      i  i      1
      1      i  i      1
      1      i  i      1
      1-----i--i-----1
      b/2 1      i  i      lw1 (III) 1-----***-----1 t
      1-----i--i-----1      Er3 1=====1 0
      1      i  i      1
      1      i  i      1
      1      i  i      1
      1      i  i      1
      z @-----
      0      a/2      a      x

```

```
Frequency [GHz]
```

```

Integer*4      MNmax,mn
Real      mu0,KK0,Kk1,Kk2,Kk3
Real      U(2,2),E1(32400),Ed(180),wk(180),Es1(8100),Es2(8100)
Real      Ep(32041),Ea(180)
Complex    JJ
Complex    zhat,yhat1,yhat2
Complex    Btmn1,Btmn2,Btmn3
Complex    a11,a12,a21,a22,b11,b12,b21,b22
Complex    SI1,SI2,SI3,CO1,CO2,CO3,TA1,TA2,TA3
Complex    dela,delb,zdela,b3delb
Complex    x,FARG,CMCOS,CMSIN

```

```

Common      /size/a,b,w1,w2,h1,h2,t
Common      /Para/Freq,Er1,Er2,Er3
Common      /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
Common      /Mat /Ea

```

```

Parameter    (JJ=(0.,1.),pai=3.1415926)
Parameter    (epsi0=8.855e-12,mu0=pai*4.e-7)

```

```
*----- Function definition -----
```

```

FARG(X)=REAL(X)+JJ*SIGN(1.,AIMAG(X))*AMIN1(70.,ABS(AIMAG(X)))
CMCOS(X)=CCOS(FARG(X))
CMSIN(X)=CSIN(FARG(X))

```

```

if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
  a=ax

```

```

    else
      b=ax
    endif
    Nmat=2*NHmat
    Np=Nmat-1
    NHp=NHmat-1
*
    Er31=Er3/Er1
    Er32=Er3/Er2
    Omega=2.*pai*Freq*1.e9
    Kk0=Omega*Omega*mu0*epsi0
    Kk1=Er1*Kk0
    Kk2=Er2*Kk0
    Kk3=Er3*Kk0
    zhat=JJ*Omega*mu0
    yhat1=JJ*Omega*epsi0*Er1
    yhat2=JJ*Omega*epsi0*Er2
*
*   ----- Matrix Size   N x N   (N= 2 x Mumax)
    Mumax=NHmat
    Numax=Mumax
*   -----
    Nmax=Nmax
    MNmax=Nmax*Nmax
*
    do 201 mu=1,Mumax
      do 201 nu=1,Numax
*
        Call RSTBL(ir1,is1,ixy1,mu,1)
        Call RSTBL(ir2,is2,ixy2,mu,2)
        Call RSTBL(jr1,js1,jxy1,nu,1)
        Call RSTBL(jr2,js2,jxy2,nu,2)
*
*   -----Beginning of Summation in terms of M,N---
*
        U(1,1)=0.
        U(1,2)=0.
        U(2,1)=0.
        U(2,2)=0.
*
        do 202 mn=1,MNmax
          Call MNTBL(m,n,mn)
*
          if(ir1.NE.m.AND.jr1.NE.m.AND.is2.NE.n.AND.js2.NE.n) go to 1
*
          GGmn=(Float(m)*pai/a)**2*(Float(n)*pai/b)**2
          Btmn1=CSQRT(CMPLX(Kk1-GGmn))
          Btmn2=CSQRT(CMPLX(Kk2-GGmn))
          Btmn3=CSQRT(CMPLX(Kk3-GGmn))
          CO1=CMCOS(Btmn1*h1)
          CO2=CMCOS(Btmn2*h2)
          CO3=CMCOS(Btmn3*t)
          SI1=CMSIN(Btmn1*h1)
          SI2=CMSIN(Btmn2*h2)

```

```

SI3=CMSIN(Btmn3*t )
TA1=SI1/CO1
TA2=SI2/CO2
TA3=SI3/CO3
dela=Btmn3*Btmn3*TA1*TA2*TA3-Btmn3*(Btmn2*TA1+Btmn1*TA2)
-      -Btmn1*Btmn2*TA3
delb=Btmn1*Btmn2*Er31*Er32*TA1*TA2*TA3
-      -Btmn3*(Btmn1*Er31*TA1+Btmn2*Er32*TA2)-Btmn3*Btmn3*TA3
zdela=zhat/dela
a11= zdela*(Btmn2*TA3+Btmn3*TA2)
a12= (1./CO3)*zdela*Btmn3*TA2
a21=-(1./CO3)*zdela*Btmn3*TA1
a22=-zdela*(Btmn1*TA3+Btmn3*TA1)
b3delb=Btmn3/delb
b11= b3delb*(Btmn3*TA3+Btmn2*Er32*TA2)
b12= (1./CO3)*b3delb*Btmn2*Er32*TA2
b21=-(1./CO3)*b3delb*Btmn1*Er31*TA1
b22=-b3delb*(Btmn3*TA3+Btmn1*Er31*TA1)
*
Call  GSITH1 (Gmu1,Tmu1,ixy1,m,n,ir1,is1)
Call  GSITH2 (Gmu2,Tmu2,ixy2,m,n,ir2,is2)
Call  GSITH1 (Gnu1,Tnu1,jxy1,m,n,jr1,js1)
Call  GSITH2 (Gnu2,Tnu2,jxy2,m,n,jr2,js2)
*
U11=AIMAG( TA1*Gmu1*a11*Gnu1-TA1*Btmn1/yhat1*Tmu1*b11*Tnu1)
U12=AIMAG( TA1*Gmu1*a12*Gnu2-TA1*Btmn1/yhat1*Tmu1*b12*Tnu2)
U21=AIMAG( TA2*Gmu2*a21*Gnu1-TA2*Btmn2/yhat2*Tmu2*b21*Tnu1)
U22=AIMAG( TA2*Gmu2*a22*Gnu2-TA2*Btmn2/yhat2*Tmu2*b22*Tnu2)
*
U(1,1)=U(1,1)+U11
U(1,2)=U(1,2)+U12
U(2,1)=U(2,1)+U21
U(2,2)=U(2,2)+U22
*
1 continue
*
202 continue
*
----- End of Summation -----
*
do 203 i=1,2
do 203 j=1,2
Call MXTBL(k,1,i,j,nu,mu,Numax)
kl=k+Nmat*(1-1)
El(kl)=U(i,j)
203 continue
*
201 continue
*
-----process to avoid overflow in the determinant
*
icount=icount+1
if(icount.EQ.1) then
sum=0.

```

```

do 205 i=1,Nmat
  k1=(i-1)*Nmat+i
205  sum=sum+ALOG(ABS(E1(k1)))
  coef1=EXP((10.-sum)/FLOAT(Nmat))
  else
  endif
*
do 206 i=1,Nmat*Nmat
206  E1(i)=coef1*E1(i)
*
* ----- end of process
*
do 204 i=1,NHmat
do 204 j=1,NHmat
  ij=i+NHmat*(j-1)
  k11=i+Nmat*(j-1)
  k12=i+NHmat+Nmat*(j+NHmat-1)
  Es1(ij)=E1(k11)
  Es2(ij)=E1(k12)
204 continue
*
* -----Finding Pnu--
*
  if(Nreso.EQ.0) then
    do 207 i=1,Np
207  Ea(i)=-E1(i)
    do 208 i=1,Np*Np
      k1=Nmat+i+(i-1)/Np
208  Ep(i)=E1(k1)
      Call LINV3F(Ep,Ea,2,Np,Np,d1,d2,wk,ier)
    endif
    if(Nreso.EQ.1.AND.NHmat.NE.1) Ea(1)=-Es1(1)/Es1(NHmat+1)
    if(Nreso.EQ.2.AND.NHmat.NE.1) Ea(1)=-Es2(1)/Es2(NHmat+1)
*
* -----
*
  Call LINV3F(Es1,Ed,4,NHmat,NHmat,d1,d2,wk,ier)
  det1=d1*2.**d2
  Call LINV3F(Es2,Ed,4,NHmat,NHmat,d1,d2,wk,ier)
  det2=d1*2.**d2
  Call LINV3F(E1,Ed,4,Nmat,Nmat,d1,d2,wk,ier)
  dett=d1*2.**d2
  if(Nreso.EQ.1) RSNT1=det1
  if(Nreso.EQ.2) RSNT1=det2
  if(Nreso.EQ.0) RSNT1=dett
  if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
    write(*,*)a*1.e3,dett,det1,det2
    write(7,*)a*1.e3,dett,det1,det2
  else
    write(*,*)b*1.e3,dett,det1,det2
    write(7,*)b*1.e3,dett,det1,det2
  endif
*
return

```

```

      END
*
* ----- END of RSNTR1
*
* ----- Beginning of Gsi ans Theta calculation
*
      SUBROUTINE GSITH1(G1,T1,ixy,im,in,ir,is)
*
*   Gsai and Theta on the interface btwn (I) and (III)
*                                     1986-0226,1986-0402
*
      Real      m,n
      Common    /size/a,b,w1,w2
      Parameter (pai=3.1415926)
      DL(I)=FLOAT(MIN(I+1,2))
      DLIJ(I,J)=FLOAT(1/(IABS(I-J)+1))
*
      m=im
      n=in
      r=ir
      s=is
*
      if(ixy.EQ.1) go to 1
      if(ixy.EQ.2) go to 2
*
*----- x and interface (I)
*
      1 continue
      if(ir.EQ.im) then
        pma=m*pai/a
        pnb=n*pai/b
        GGmn=pma*pma+pnb*pnb
        Gmn=SQRT(GGmn)
        Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
        DD=DLIJ(ir,im)
        Fsn=FI1(is,in,w1,b)
        G1=-Pmn*pma*(a/2.)*DD*Fsn
        T1= Pmn*pnb*(a/2.)*DD*Fsn
      else
        G1=0.
        T1=0.
      endif
      return
*
*----- y and interface (I)
*
      2 continue
      if(ir.EQ.im) then
        pma=m*pai/a
        pnb=n*pai/b
        GGmn=pma*pma+pnb*pnb
        Gmn=SQRT(GGmn)
        Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
        D1=DL(im)
        DD=DLIJ(im,ir)

```

```

      Fsn=FI2(is,in,w1,b)
      G1=-Pmn*pnb*(a/D1)*DD*Fsn
      T1=-Pmn*pma*(a/D1)*DD*Fsn
    else
      G1=0.
      T1=0.
    endif
  return
end

*
*
SUBROUTINE GSITH2(G2,T2,ixy,im,in,ir,is)
*
*   Gsai and Theta on the interface btwn (II) and (III)
*   1986-0226,1986-0402
*
  Real      m,n
  Common    /size/a,b,w1,w2
  Parameter (pai=3.1415926)
  DL(I)=FLOAT(MIN(I+1,2))
  DLIJ(I,J)=FLOAT(1/(IABS(I-J)+1))
*
  m=im
  n=in
  r=ir
  s=is
*
  if(ixy.EQ.1) go to 1
  if(ixy.EQ.2) go to 2
*
*----- x and interface (II)
*
  1 continue
  if(is.EQ.in) then
    pma=m*pai/a
    pnb=n*pai/b
    GGmn=pma*pma+pnb*pnb
    Gmn=SQRT(GGmn)
    Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
    D1=DL(in)
    DD=DLIJ(is,in)
    Fsn=FI2(ir,im,w2,a)
    G2=-Pmn*pma*(b/D1)*DD*Fsn
    T2= Pmn*pnb*(b/D1)*DD*Fsn
  else
    G2=0.
    T2=0.
  endif
  return
*
*----- y and interface (II)
*
  2 continue
  if(is.EQ.in) then
    pma=m*pai/a

```

```

    pnb=n*pai/b
    GGmn=pma*pma+pnb*pnb
    Gmn=SQRT(GGmn)
    Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
    DD=DLIJ(is,in)
    Fsn=FI1(ir,im,w2,a)
    G2=-Pmn*pnb*(b/2.)*DD*Fsn
    T2=-Pmn*pma*(b/2.)*DD*Fsn
  else
    G2=0.
    T2=0.
  endif
  return
end

```

```

*
* ----- end of Gsi and Theta
*

```

```

FUNCTION FI1(is,in,w,b)

```

```

  Real n
  pai=3.1415926

  s=is
  n=in
  psw=s*pai/w
  pnb=n*pai/b
  if(pnb.EQ.psw) then
    FI1=-w/2.*SIN((n-s)*pai/2.)
  else
    h= psw/(pnb*pnb-psw*psw)
    c=COS(pnb*(b-w)/2.)-(-1.)**is*COS(pnb*(b+w)/2.)
    FI1=-h*c
  endif
  return
end

```

```

FUNCTION FI2(is,in,w,b)

```

```

  Real n
  pai=3.1415926

  s=is
  n=in
  if(MOD((in+is),2).EQ.0) then
    FI2=0.
  else
    arg1=ABS(n*w*pai/(2.*b)+s*pai/2.)
    arg2=ABS(n*w*pai/(2.*b)-s*pai/2.)
    h=BSJ0(arg1)*(-1.)**((in+is-1)/2)
    c=BSJ0(arg2)*(-1.)**((in-is-1)/2)
    FI2=w*pai/4.*(h+c)
  endif
  return

```

```

end
*
*
SUBROUTINE MXTBL(k,l,i,j,Nu,Mu,Nmax)
k=Mu+Nmax*(i-1)
l=Nu+Nmax*(j-1)
return
end
*
*
SUBROUTINE RSTBL(ir,is,infoxy,NNu,i)
Common /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
*
if(i.EQ.1) go to 1
if(i.EQ.2) go to 2
*
1 continue
*
-----Interface (I)-----
if(MOD(NNu,2).EQ.0) then
*
r,s for the Xx(1)      NNu:even
infoxy=1
n=NNu/2
iir=n-1
if(MEwall.EQ.1) iir=iir+1
iis=1
if(MEwall.EQ.2) iis=0
if(MEwall.EQ.0) then
ir=2*iir+1
is=2*iis
endif
if(MEwall.EQ.1) then
ir=2*iir
is=2*iis
endif
if(MEwall.EQ.2) then
ir=2*iir+1
is=2*iis+1
endif
endif
else
*
r,s for the Xy(1)      NNu:odd
infoxy=2
n=(NNu+1)/2
iir=n-1
iis=0
if(MEwall.EQ.0) then
ir=2*iir+1
is=2*iis
endif
if(MEwall.EQ.1) then
irdum=iir
if(irdum.EQ.0) iir=1
if(irdum.EQ.1) iir=0
ir=2*iir
is=2*iis

```

```

        endif
        if (MEwall.EQ.2) then
            ir=2*iir+1
            is=2*iis+1
        endif
    endif
    return
*
2 continue
*
-----Interface (II)-----
*
if (MOD(NNu,2).EQ.0) then
*
                                r,s for the Xy(2)      NNu:even

    infoxy=2
    n=NNu/2
    iis=n-1
    if (MEwall.EQ.2) iis=iis+1
    iir=1
    if (MEwall.EQ.1) iir=0
    if (MEwall.EQ.0) then
        ir=2*iir
        is=2*iis+1
    endif
    if (MEwall.EQ.1) then
        ir=2*iir+1
        is=2*iis+1
    endif
    if (MEwall.EQ.2) then
        ir=2*iir
        is=2*iis
    endif
else
*
                                r,s for the Xx(2)      NNu:odd

    infoxy=1
    n=(NNu+1)/2
    iis=n-1
    iir=0
    if (MEwall.EQ.0) then
        ir=2*iir
        is=2*iis+1
    endif
    if (MEwall.EQ.1) then
        ir=2*iir+1
        is=2*iis+1
    endif
    if (MEwall.EQ.2) then
        isdum=iis
        if (isdum.EQ.0) iis=1
        if (isdum.EQ.1) iis=0
        ir=2*iir
        is=2*iis
    endif
endif
return
end

```

```

*
*
SUBROUTINE MNTBL(m,n,MN)
Common /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
*
*   M - M   Wall   case
Integer*4 MN
n=(MN-1)/Nmax
m=MN-Nmax*n-1
if (MEwall.EQ.0) then
    m=2*m+1
    n=2*n+1
endif
if (MEwall.EQ.1) then
    m=2*m
    n=2*n+1
endif
if (MEwall.EQ.2) then
    m=2*m+1
    n=2*n
endif
return
end
*
*
*
FUNCTION BSJ0(x)
                                Bessel function          1986-0226
*
ax=ABS(x)
if(ax.GT.3.) go to 1
x2=x*x/9.
x4=x2*x2
x6=x2*x4
x8=x4*x4
x10=x2*x8
x12=x6*x6
BSJ0=1.-2.2499997d0*x2+1.2656208d0*x4-0.3163866d0*x6
-      +0.0444479d0*x8-0.0039444d0*x10+0.00021d0*x12
return
1 continue
x1=3./ax
x2=x1*x1
x3=x1*x2
x4=x1*x3
x5=x1*x4
x6=x1*x5
f= 0.79788456d0-0.00000077d0*x1-0.0055274d0*x2-0.00009512d0*x3
-  +0.00137237d0*x4-0.00072805d0*x5+0.00014476d0*x6
t=ax-0.78539816d0-0.04166397d0*x1-0.00003954d0*x2+0.00262573d0*x3
-  -0.00054125d0*x4-0.00029333d0*x5+0.00013558d0*x6
BSJ0=f*cos(t)/SQRT(ax)
return
end
*

```



```

      x2=x3
      x3=xnew
      y1=y2
      y2=y3
      y3=Func(SNGL(x3))
      xlast=x3
      ylast=SNGL(y3)
      if(y2*y3.LE.0.) go to 100
201  continue
      ier=1
      x=x3
      return
      else
*
*                                backward step
      x2=x1
      y2=y1
      x1=x1-(x3-x1)
      if(xnew.GT.x1) x1=xnew-0.01*DABS(xnew)
      y1=Func(SNGL(x1))
      xlast=x1
      ylast=SNGL(y1)
      if(y2*y1.LE.0.) go to 100
      do 202 i=1,5
      xnew=x1-(x3-x1)
      x3=x2
      x2=x1
      x1=xnew
      y3=y2
      y2=y1
      y1=Func(SNGL(x1))
      xlast=x1
      ylast=SNGL(y1)
      if(y2*y1.LE.0.) go to 100
202  continue
      ier=1
      x=x1
      return
      endif
      endif
*
100  continue
      if(ylast.EQ.0.) then
      x=xlast
      return
      endif
*
      do 203 i=1,10
      if(i.NE.1) then
      y2=Func(SNGL(x2))
      xlast=x2
      ylast=SNGL(y2)
      if(ylast.EQ.0.) then
      x=xlast
      return

```

```

        endif
    endif
    Call ABCNEW(xnew,x1,x2,x3,y1,y2,y3)
    if(y1*y2.GT.0.) then
        x1=x2
        y1=y2
    else
        x3=x2
        y3=y2
    endif
    x2=xnew
    epsi=1.d-5
    dela=DABS((x2-xlast)/x2)
    if(dela.LT.epsi) then
        x=x2
        return
    endif
203 continue
    ier=2
    x=x2
    return
end

*
*
*

SUBROUTINE ABCNEW(xnew,x1,x2,x3,y1,y2,y3)
Implicit Real*8(a-z)
Call ABC(a,b,c,x1,x2,x3,y1,y2,y3)
bb=b
if(bb.EQ.0.) bb=1.d-30
if(DABS(a/bb).LT.1.e-10) then
    xnew=-c/b
else
    ac=b*b-4.*a*c
    anx1=(-b-DSQRT(ac))/(2.*a)
    anx2=(-b+DSQRT(ac))/(2.*a)
    xnew=anx1
    if(anx2.GE.x1.AND.anx2.LE.x3) xnew=anx2
endif
return
end

SUBROUTINE ABC(a,b,c,x1,x2,x3,y1,y2,y3)
Implicit Real*8(a-z)
a11=-1./((x2-x1)*(x1-x3))
a12=-1./((x2-x1)*(x3-x2))
a13=-1./((x3-x2)*(x1-x3))
a21=-a11*(x3+x2)
a22=-a12*(x1+x3)
a23=-a13*(x2+x1)
a31=a11*x2*x3
a32=a12*x1*x3
a33=a13*x1*x2
a=a11*y1+a12*y2+a13*y3

```

```
b=a21*y1+a22*y2+a23*y3  
c=a31*y1+a32*y2+a33*y3  
return  
end
```

*

END

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DTIC