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Attention: Dr. Robert J. Barker
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Department of Physics
Coral Gables, Florida 33124

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Principal Investigator: Dr. Manuel A. Huerta
Professor of Physics
[redacted]
Telephone No. (305) 284-2323



Signed: Manuel A. Huerta
Manuel A. Huerta
Principal Investigator
Department of Physics

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Dr. W. Pardo spent an entire summer month at AFATL to work with Mr. Ken Cobb on railgun research. In addition, Dr Huerta visited AFATL twice, giving seminars on plasma instabilities of railgun armatures. Graduate student Ann Decker completed her Ph.D. thesis work by numerically solving a fourth-order eigenvalue equation which described the development of the Krusal-Schivarychild instability in armature plasmas with finite conductivity and compressibility. In addition, another student, Mr. Boynton wrote a 2-D flux-corrected MHD transport code to model the arc dynamics in a more detailed fashion.

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## Introduction

This is a report on the work done during the second year of of this project to develop theoretical models of several aspects of accelerating arc plasmas in electromagnetic railguns with plasma driven projectiles.

The second year of the grant provided two months of summer support for Dr. J. C. Nearing. Dr. Huerta was supported for three months for during the summer plus three more months during the academic year. Dr. Huerta went on sabbatical during the Spring 1986 semester, from January, 1986 to June, 1986. Support was also provided for two graduate students, Miss Ann M. Decker, and Mr. G. Christopher Boynton both of whose Ph. D. dissertations are based on problems derived from the project. The second year of the grant also provided support for Mr. John Fornashe, Mr. Peter Papavaritis, Mr. Spiros Skourtis, and Mr. Miguel Bernard who were undergraduate students involved in the project.

The cooperation with Eglin AFB continued during the second year. Dr. W. B. Pardo was supported during a one summer month visit to Eglin AFB. The P. I. also visited Eglin AFB twice and gave talks there. A group of researchers on railgun problems was developed at the University of Miami largely due to the support flowing from this project. There appeared a strong possibility of funding by the SDIO/IST office of a consortium proposed by Dr. Harry S. Robertson, the P. I., and other faculty at the University of Miami. By the end of the period covered in this report the funding had not yet been received.

Description of the Technical Work

The P. I. and Miss Decker studied the effects of introducing finite conductivity and compressibility, as well as allowing for a variety of equilibrium profiles, on the development of the Kruskal-Schwarzchild instability. The inclusion of finite conductivity effects produced a fourth order eigenvalue equation for the modes. A numerical solution of the equation was done and a dispersion relation obtained. Miss Decker wrote up her work as her doctoral dissertation and was granted the Ph. D. in the Spring of 1986. A draft of a paper we intend to submit is attached.

The work with Mr. Boynton centered on a numerical solution of the MHD equations for a model plasma. A two dimensional FCT code was written and tested on a variety of simple problems where the answers could be obtained by other means. A good deal of effort was also expended on developing graphical methods that would allow good display of the results. A sample output figure is attached. It is a plot of pressure vs. position for a state that evolved from an initial pressure spike. No magnetic effects are included in the case shown.

Other work was done. For example the P. I. and Dr. Nearing improved the results for the distribution of rail current to the case of arbitrary time dependence of the current and acceleration, as well as a finite thickness for the armature.

# FINITE CONDUCTIVITY AND THE RAYLEIGH-TAYLOR INTERCHANGE INSTABILITY

Ann M. Decker and Manuel A. Huerta
University of Miami Physics Department

The literature of the physics of fluids abundantly documents the phenomenon that has come to be known as the Rayleigh-Taylor[1,2], or interchange, instability. A fluid supported against gravity by a lighter fluid is found to be unstable, the potential energy of the system being lowered by the exchange of the two. When the gravitational field is opposite the density gradient of a fluid, any initial perturbation grows exponentially in time.

The hydromagnetic analogue of this instability was first explored by Kruskal and Schwartzschild.[3] A plasma supported against gravity by a magnetic field tangential to its plasma-vacuum interface is subject to an instability of mathematically identical form. A Rayleigh-Taylor instability is thus expected in the case of a plasma slab accelerated by a magnetic field.

Many references exists describing the dispersion relations of special cases of Rayleigh-Taylor (R-T), or Kruskal-Schwartzschild (K-S), instabilities in plasmas. In the case of a plasma slab described by the laws of ideal magnetohydrodynamics, a surface deformation of wave vector k perpendicular to the magnetic field grows with dispersion $\omega = (kg)^{1/2}$, where g is the acceleration of the slab.[4,5] Tsai, et al.[6] have detailed the dispersion relations numerically for a compressible plasma of infinite conductivity. Huerta and Decker[7] have shown that the growth of R-T instabilities is unaffected by the inclusion of finite conductivity to first order for the case of an incompressible plasma. For parameters typical of plasma armatures in rail-launch devices, even the longest wavelength disturbances would have ample time to develop fully at this rate. Such an instability in the plasma arc would destroy the type of one-dimensional equilibria used in earlier arc models.[10]

A fully developed R-T instability will result in the breakoff and expansion of plasma packets away from the main arc. Trailing masses of plasma may lead to the formation of secondary arcs and turbulent flow. Such effects have been reported by Parker, et al.[11], and are important in that they could limit the efficiency of plasma driven mass accelerators. A detailed study of the R-T instability in a plasma slab, including viscosity, compressibility, finite conductivity, etc. is warranted to see what factors can impede its growth.

The present discussion will include the effects of finite conductivity in both the equilibrium and perturbation equations. A general mode equation will be derived, which for an infinitely conducting, isothermal plasma will reduce to Tsai's result. An equilibrium profile consistent with a rail geometry and the assumption of a one-dimensional steady state will be inserted into a dimensionless form of the general mode equation. In the limits of both very high and very low conductivity, the resulting equation depends only on the ratio $\ell g_0/K$, where $\ell$ is the plasma tail length, $g_0$ is the arc acceleration, and K is the square of the ion acoustic velocity.

The governing equations for the plasma system will be Maxwell's equations, the momentum equation, the equation of continuity, and an equation of state. Displacement current is neglected. An adiabatic equation is assumed, $p/\rho^\gamma$ = const, where as yet $\gamma$ is unspecified. The effects of finite, constant conductivity come from $\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}/c = \underset{\sim}{J}/\sigma$. The equilibrium state of a neutral, stationary plasma is then

$$\nabla p_0 = \frac{1}{c}\underset{\sim}{J}_0 \times \underset{\sim}{B}_0 + \rho_0 \underset{\sim}{g}_0 , \quad \nabla \times \underset{\sim}{B}_0 = \frac{4\pi}{c}\underset{\sim}{J}_0 , \quad \nabla \cdot \underset{\sim}{B}_0 = 0 , \quad \underset{\sim}{E}_0 = \frac{\underset{\sim}{J}_0}{\sigma}$$

A plasma that has experienced a small perturbation from its equilibrium can be described by the variables $\underset{\sim}{B}_1$, $\underset{\sim}{J}_1$, $\underset{\sim}{E}_1$, $\rho_1$, $p_1$, taken to be small deviations from the equilibrium values, $\underset{\sim}{B}_0$, $\underset{\sim}{J}_0$, $\underset{\sim}{E}_0$, $\rho_0$, $p_0$. These, and the velocity $\underset{\sim}{v}$, are substituted into Maxwell's equations, the force, and the continuity equations. Neglecting terms of second order in the perturbing variables,

$$\nabla \times \underset{\sim}{B}_1 = \frac{4\pi}{c}\underset{\sim}{J}_1 , \quad \nabla \times \underset{\sim}{E}_1 = -\frac{1}{c}\frac{\partial \underset{\sim}{B}_1}{\partial t} , \quad \nabla \cdot \underset{\sim}{B}_1 = 0 ,$$

$$\underset{\sim}{E}_1 + \frac{1}{c}\underset{\sim}{v}_1 \times \underset{\sim}{B}_0 = \frac{\underset{\sim}{J}_1}{\sigma} , \quad \frac{\partial \rho_1}{\partial t} = -\nabla \cdot \rho_0 \underset{\sim}{v}_1 , \quad \frac{\partial p_1}{\partial t} = -\gamma p_0 \nabla\cdot\underset{\sim}{v}_1 - \underset{\sim}{v}_1\cdot\nabla p_0$$

$$\rho_0 \frac{\partial v_1}{\partial t} = -\nabla p_1 + \frac{1}{c}\underset{\sim}{J}_1 \times \underset{\sim}{B}_0 + \frac{1}{c}\underset{\sim}{J}_0 \times \underset{\sim}{B}_1 + \rho_1 \underset{\sim}{g}_0 \qquad (1)$$

Eliminate various dependent variables in these equations and the result is a general first order perturbation equation for a plasma of arbitrary initial current, mass density, magnetic field, acceleration, and constant conductivity.

$$\rho_o \frac{\partial^2 \underset{\sim}{v}_1}{\partial t^2} = \nabla \left[ \underset{\sim}{v}_1 \cdot \nabla p_o + \gamma p_o \nabla \cdot \underset{\sim}{v}_1 \right]$$

$$+ \frac{1}{4\pi}\left[\nabla \times \left[\nabla \times (\underset{\sim}{v}_1 \times B_o)\right] - \frac{c^2}{4\pi\sigma} \nabla \times \left[\nabla \times (\nabla \times \underset{\sim}{B}_1)\right]\right] \times \underset{\sim}{B}_o$$

$$+ \underset{\sim}{J}_o \times \left[\frac{1}{c}\nabla \times (\underset{\sim}{v}_1 \times \underset{\sim}{B}_o) - \frac{c}{4\pi\sigma}\nabla \times (\nabla \times \underset{\sim}{B}_1)\right] - (\nabla \cdot \rho_o \underset{\sim}{v}_1)\underset{\sim}{g}_o \tag{2}$$

The general equation above will now be applied to the specific cartesian geometry of interest. The equilibrium parameters are as $\underset{\sim}{B}_o = B_o(y)\hat{z}$, $\underset{\sim}{J}_o = -J_o(y)\hat{x}$, $\underset{\sim}{g}_o = -g_o\hat{y}$, $p_o(y)$, and $\rho_o(y)$, while the velocity and the perturbation quantities will be in the form $A_1(x,y)\ e^{-i\omega t}$.

For this geometry, equation (2) can readily be analyzed in terms of its x and y components,

$$-\omega^2 \rho_o\ v_{1x} = -\rho_o\ g_o \frac{\partial}{\partial x} v_{1y} + \frac{\partial}{\partial x}\left[(\gamma p_o + \frac{B_o^2}{4\pi})(\nabla \cdot \underset{\sim}{v}_1) - \frac{c^2}{16\pi^2\sigma} B_o \nabla^2 B_{1z}\right] \tag{3}$$

and

$$\omega^2 \rho_o\ v_{1y} = \rho_o\ g_o \frac{\partial}{\partial y} v_{1y}$$

$$+ \frac{\partial}{\partial y}\left[(\gamma p_o + \frac{B_o^2}{4\pi})(\nabla \cdot \underset{\sim}{v}_1) - \frac{c^2}{16\pi^2\sigma} B_o \nabla^2 B_{1z}\right] - \rho_o\ g_o \nabla \cdot \underset{\sim}{v}_1 \tag{4}$$

The parenthetical quantity can be removed between these two equations by taking the derivative with respect to the y coordinate of equation (3), and the derivative with respect to x of (4) and subtracting. The result is

$$\frac{\partial v_{1x}}{\partial y} + \frac{v_{1x}}{\rho_o}\frac{\partial \rho_o}{\partial y} - \left[1 + \frac{g_o}{\omega^2}\frac{1}{\rho_o}\frac{\partial \rho_o}{\partial y}\right]\frac{\partial v_{1y}}{\partial x} = \frac{g_o}{\omega^2}\frac{\partial}{\partial x}(\nabla \cdot \underset{\sim}{v}_1) \tag{5}$$

It should immediately be noted that all terms involving a $\sigma$ dependence have been removed, and this result is valid for a plasma of arbitrary conductivity.

To this point, the results have been applicable to a wide range of plasmas obeying a very general adiabatic equation of state. Now the discussion will narrow slightly to the consideration of plasmas obeying the isothermal relations, $p = K\rho$, $\gamma = 1$, as used in other references examining the same phenomenon.[6,12] Here $K$ is the square of the ion acoustic velocity, $K = (z+1)k'T/m_i$, where $z$ is the (constant) ionization state of the plasma ions, $k'$ is Boltzmann's constant, $T$ is the temperature in Kelvins, and $m_i$ is the mass of the ionic (or neutral) species.

Powell[12] has calculated the equilibrium parameters under the assumption of a finite conductivity, isothermal plasma. It is found that the equilibrium is described by the relations

$$B_0(y) = \frac{4\pi}{c} J_0(\ell - y) \ , \qquad \rho_0\ (y) = \frac{4\pi}{c^2}\frac{J_0^2}{g_0}\Big[\ell + \lambda - y - (\ell + \lambda)e^{-y/\lambda}\Big]\text{sqbrk} \tag{6}$$

where $J_0$ is constant and $\lambda$ is the ratio of $K$, the square of the ion acoustic velocity, to the acceleration $g_0$.

The governing relations for the plasma will be scaled in terms of the length $\ell$ of the plasma tail, and time $(\ell/g_0)^{1/2}$ for the projectile to accelerate this length. The following transformations will be used:

$$y = u\ell, \quad k = \frac{q}{\ell}, \quad \omega^2 = w^2\frac{g_0}{\ell}, \quad \frac{\sigma}{c^2} = s\,\frac{1}{4\pi\ell\sqrt{\ell g_0}} \tag{7}$$

The quantities $u$, $q$, $w$, and $s$ are the dimensionless coordinate, wave vector, perturbation frequency, and conductivity, respectively. Furthermore, the magnetic field and mass density can be expressed in terms of their dimensionless analogues, $b$ and $p$.

$$B_0(y) = \frac{4\pi}{c} J_0 \ell b \ , \qquad b = 1 - u$$
$$\rho_0(y) = \frac{4\pi}{c^2} J_0^2 \frac{\ell}{g_0} p \ , \qquad p = \Big[1 + \frac{1}{x}\Big]\Big[1 - e^{-ux}\Big] - u \ , \tag{8}$$

where $x$ is $\ell/\lambda$.

To explore the effects of R-T instability, assume that the total $x$ dependence of the flow variables is characterized by a wave vector $k$. Thus, the perturbation quantities are taken to be of the form $A_1(y)e^{i(kx-\omega t)}$. Further manipulation of the above equations, eliminating everything but $\rho_0 v_{1y}$, gives a general mode equation for an adiabatic, constant conductivity plasma of arbitrary initial current, density, and magnetic induction profiles. We use the isothermal equation of state and express the mode equation terms of R:

$$R = \rho_o v_{1y} \tag{9}$$

$$DER'''' + \left[\frac{q^2}{w^2}DE + CE + D'E - E'D\right]R''' + \left[\frac{q^2}{w^2}CE + BE + C'E - E'C\right]R''$$
$$+ \left[\frac{q^2}{w^2}BE + AE + B'E - E'B - \frac{iq}{w^2}E^2\right]R' + \left[\frac{q^2}{w^2}AE + A'E - E'A - iqE^2\right]R = 0 \tag{10}$$

All primes indicate derivatives with respect to the dimensionless variable u.

$$A = \frac{-2i}{w}bp^2\left[w^2 - \frac{q^2}{x}\right] + \frac{iq^2}{w^3}b^2p^2\left[w^2 - \frac{q^2}{x}\right] - sb^2p^2$$
$$- s(b-p)b^2p - sx(b-p)b^4 \tag{11}$$

$$B = \frac{s}{x}b^2p^2 + sb^4p - \frac{2i}{wx}p^2 - \frac{2i}{w^3}\left[w^2 - \frac{q^2}{x}\right]bp^2$$
$$- \frac{i}{w}\left[w^2 - \frac{q^2}{x}\right]\left[1 - \frac{q^2}{w^4}\right]b^2p^2 + \frac{iq^2}{wx}b^2p^2 \tag{12}$$

$$C = \frac{-2i}{wx}p^2b - \frac{i}{w^3}\left[w^2 - \frac{q^2}{x}\right]p^2b^2 \tag{13}$$

$$D = -\frac{i}{wx}p^2b^2 \tag{14}$$

$$E = \frac{is}{q}\left[w^2 - \frac{q^2}{x}\right]p^2b^2 - iqspb^4 + \frac{2}{wq}p^2\left[w^2 - \frac{q^2}{x}\right]$$
$$- \frac{2q}{w^3}\left[w^2 - \frac{q^2}{x}\right]p^2b - \frac{q}{w}\left[w^2 - \frac{q^2}{x}\right]\left[1 - \frac{q^2}{w^4}\right]p^2b^2 \tag{15}$$

Equations (10) through (15) show that the dispersion of q vs. w, to be derived through the integration of (10), can only vary with s and x. In both the limits of very high and very low conductivity, the factor s is eliminated, so that the behavior of the dispersion relation is characterized by the dimensionless parameter x. Powell[12] has already found this to be the case for the limit of large conductivity, and has done the numerical integration of for that limit.

In the limit of infinite conductivity, the mode equation reduces to the exact form of the mode equation derived by Tsai[6] under the initial assumptions of large conductivity and an isothermal equation of state.

The boundary conditions at the plasma-vacuum interface are largely determined by two factors: the continuity of components of the electromagnetic fields, and the fluid properties and physical constraints on the velocity, pressure, mass density, etc., and their derivatives. These constraints will place certain restrictions on the derivatives of the velocity, and on the derivatives of the function R for this density profile. The results of this analysis are, in their the dimensionless form, and for the specific density and magnetic induction profiles (8):

$$R(0) = 0 \quad , \quad R'(0) = \text{unspecified} \; , \quad R''(0) = \left[\frac{q^2}{w^2} - iws - 1 - x\right]R'(0)$$

$$R'''(0) = \left[iws\left[2 + x + \frac{iws}{2} - \frac{q^2}{2w^2}\right] + \frac{q^2}{w^2}\left[2w^2 - x - 1\right] + x\left[1 + x - w^2\right]\right]R'(0)$$

The value of R'(0) will be left as a free parameter, so that the solution of the integration of equation (10) will be correct to within a constant multiplicative factor.

Equation (10) was integrated subject to the boundary conditions derived above and the additional constraint that the fluid velocity, and hence R, must vanish at the plasma-projectile boundary. Thus, for a given wave vector q, the frequency can be deduced by finding those values of w that ensure R = 0 at u = 1. A Runge-Kutta algorithm was used for the integration.

As with equation (10), the boundary conditions vary only with the dimensionless parameters x and s. The dispersion relation q vs. w have been found for several values of these parameters.

It is of interest to compare the relative sizes of s and x for a tangible model. Powell[12] has shown that, for small x, $x \rightarrow 3\beta/2$, where $\beta$ is the arc mass to projectile mass ratio. McNab[14] has approximated the mass of the plasma arc in the Rashleigh-Marshall launch experiment to be 0.1 grams. For the 3 gram mass used in that experiment, the approximate value of x is 0.05. Spitzer[11] gives the conductivity of a fully ionized plasma in terms of the average ionization state, the temperature, and the electron number density. For a rail armature experiencing the peak current per rail height $5.76 \times 10^{14}$ statamp/cm reported by Rashleigh and Marshall, numerical calculations by Powell and Batteh[10] give these parameters. Combining them with the relation[11] for the conductivity, the acceleration, and the definition of s, one arrives at a value of s for the Rashleigh-Marshall plasma armature: $x = 0.05$, $s = 11.2$

The relative values of s and x derived for the Rashleigh-Marshall plasma armature seem to indicate that conductivity will play an important role in determining the growth rates of R-T instabilities. For small x, the dimensionless mass density is of order $x^{-1}$. Examination of equations (11) through (14) reveals that the largest terms in the coefficients A, B, C, D, and E, are of order $x^{-3}$, making them several orders of magnitude larger than some of the smaller terms involving a $\sigma$ dependence. If, because of this, the coefficients C and D cannot be neglected, the dispersion relation will depend on the solutions of a fourth order ordinary differential equation (ODE), instead of the second order ODE in Tsai's model.

The first graph, approximates the conditions of the Westinghouse EMACK rail-launch device. For the physical parameters reported by Deis and McNab[15], again employing the numerical calculations for temperature, electron number density, ionization state, and arc length of Powell and Batteh[10], the EMACK rail-launch device can be characterized by $x = 0.004$, $s = 1.97$ In this case, both s and x are an order of magnitude smaller than for the Rashleigh-Marshall plasma arc, and it is expected that finite conductivity effects would be more pronounced.

The dispersion graph I differs markedly from the ones derived under the assumption of infinite conductivity. Growth rates of short wavelength perturbations do not reach arbitrarily large values, but appear to have a finite limit. The long wavelength behaviour, however, appears to be similar irrespective of conductivity.

While the growth rate of the R-T instability is reduced with the inclusion of finite conductivity effects, it must be noted that even the longest wavelength modes permitted by the geometry of the gun have a value of w of order 1.0 (i.e., the e-folding time of the disturbance is of the order of the time it takes the plasma arc to accelerate its own length). Since a typical rail device is about 30 times longer than the best estimates for the plasma arc length, there is ample time in the course of an acceleration for R-T instabilities to set in and develop fully.

A data point was picked from Graph I, and the associated values of q and w were inserted in equation (10). Numerical integration revealed this dispersion relation to be for an n = 1 mode of R. Higher order modes were found for various valves of s and x, but they had smaller growth rates. This is in agreement with the work of both Tsai and Powell, who documented the existence of higher order, slower growing modes under the assumption of infinite conductivity.

Variations in s and x noticeably affect the growth rates derived by numerical integration of equation (10). Graph II gives the dispersion relation of the n = 1 mode derived for the values s = 0.1 and x = 0.15 on the same scale as Graph I for comparison.

Finite conductivity effects are even more pronounced as x and s approach each other in size. Graph II shows the dispersion relation for a system characterized by x = 1, s = 1. While the growth rate diminished for shorter wavelength disturbances, it is interesting to note that the real part of $\omega$ grew monotonically to an asymptotic value of about 4.3 (the mode found for the EMACK device was non-oscillatory).

Integration of equation (10) for a point from Graph II reveals that this graph is also for an n = 1 mode. Graph III plots the mass current density R as a function of arc position for this mode.

Powell and Batteh[10] have determined how temperature, plasma arc length, acceleration, etc. in a rail device scale with current, projectile mass, and other factors. Employing these scaling laws, and assuming a Spitzer conductivity, it is found that the Rashleigh-Marshall plasma parameters would scale to about s = 1, x = 1 for an initial current per rail height of roughly

$3 \times 10^{19}$ statamp/cm. While this may not be feasible at present, the scaling relations do seem to indicate that the growth rate for R-T instabilities is diminished slightly with higher currents.

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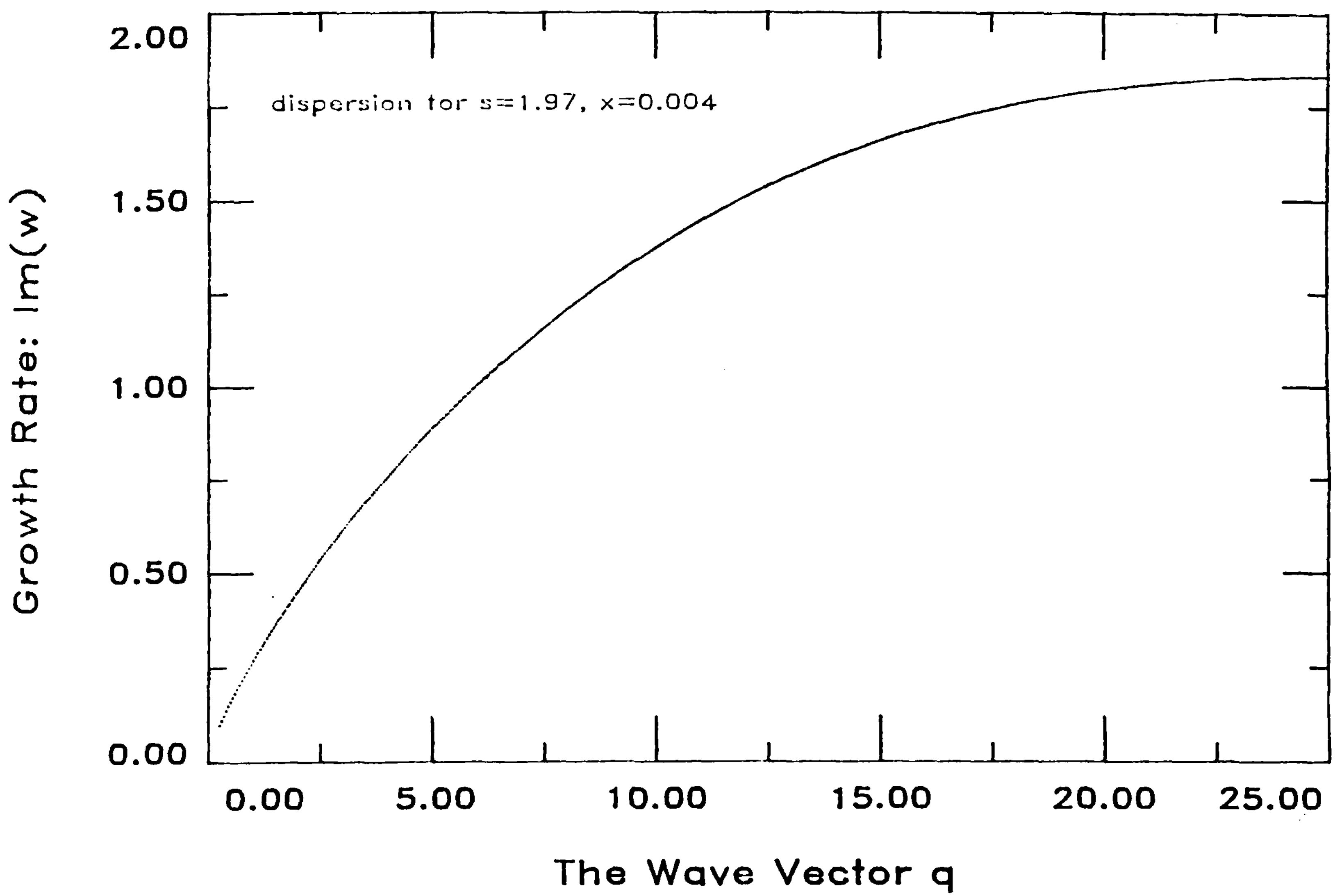
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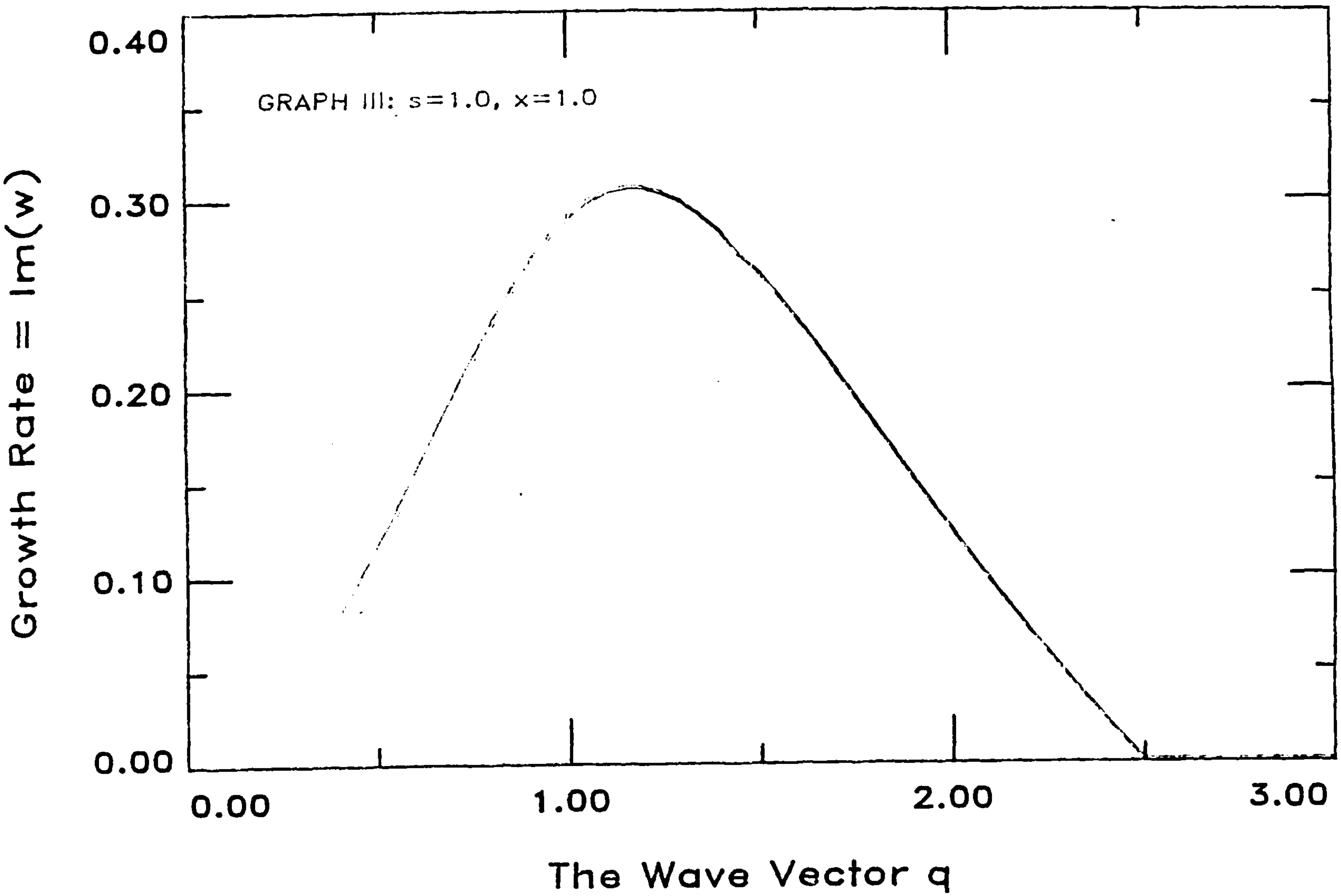
The Wave Vector q

(2)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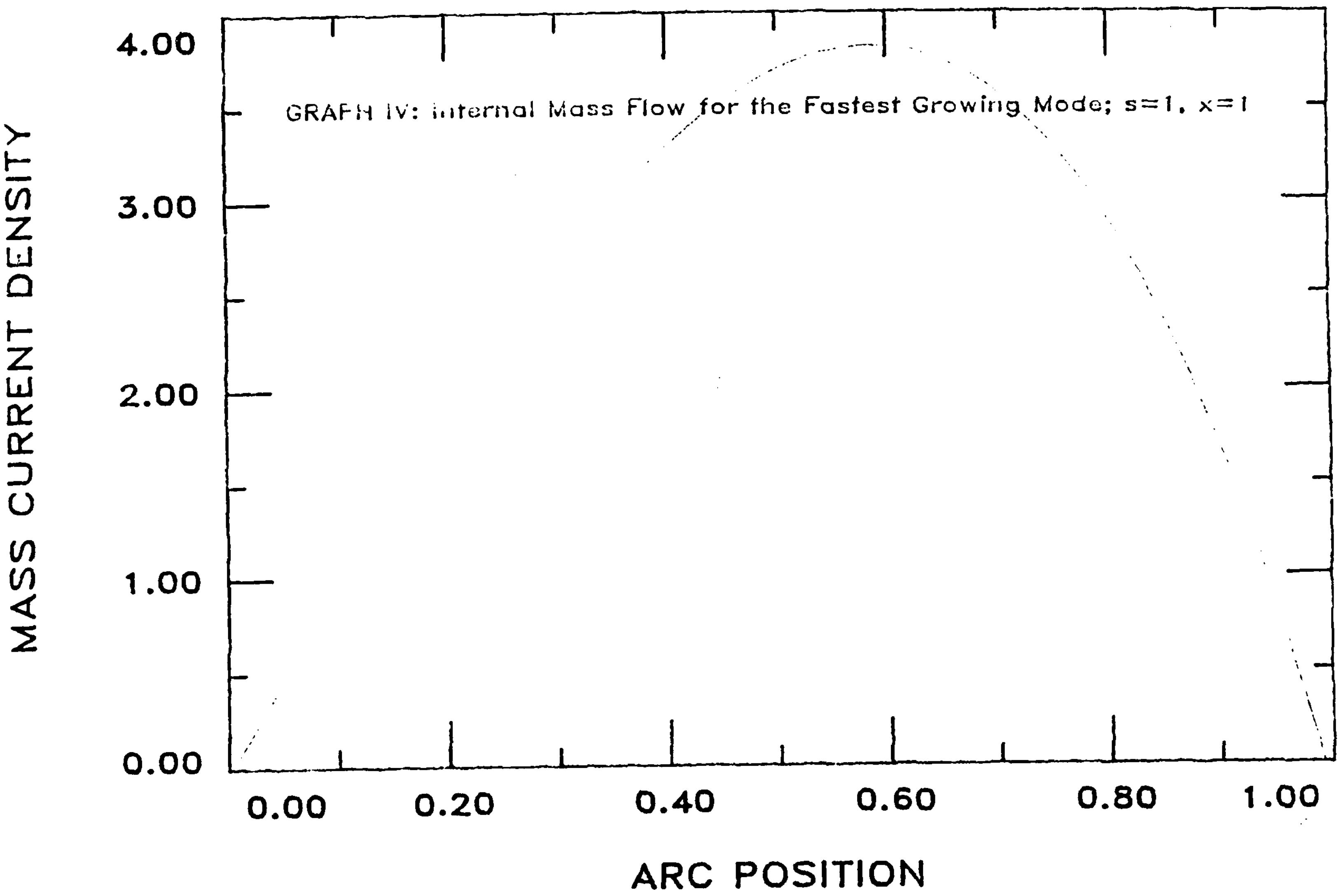
The Wave Vector q


GRAPH IV: Internal Mass Flow for the Fastest Growing Mode; s=1. x=1
MASS CURRENT DENSITY
4.00
3.00
2.00
1.00
0.00
0.00
0.20
0.40
0.60
0.80
1.00
ARC POSITION