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CONSTRUCTION OF STATIONARY SETS VIA KUZNETSOV MEASURES

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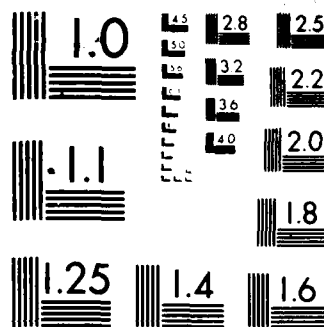
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CONSTRUCTION OF STATIONARY SETS VIA KUZNETSOV MEASURES

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Construction of Stationary Sets Via Kuznetsov Measures

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P. J. Fitzsimmons

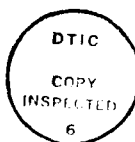
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ABSTRACT

In this paper we give a simple and comprehensive approach to the stationary regenerative sets, based on the Kuznetsov measure associated with an increasing process with independent increments and Lebesgue initial distribution. The range (closure of the image) of such a process with independent increments form a stationary regenerative set on the entire real line. We show that the underlying distribution of the regenerative set is finite iff the expectations of the increments are finite.

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1. Introduction.

A regenerative set is a set which form a probabilistic replica of itself after each stopping time which belongs to this set. The theory of regenerative sets was of a considerable interest for quite a while. Such sets are associated with visiting times of a point by a strong Markov process. Any such set can be obtained as a range, i.e., closure of the image, of a process with independent increments or a subordinator (see Maisonneuve [7], [8]).

Recently, there were several publications investigating stationary regenerative sets on a real line. They correspond to visiting times of a point by stationary strong Markov processes. In a seminal paper by Taksar [13] it was shown that all such sets are in one-to-one correspondence with the limiting ranges of the subordinators having finite expectations. The construction first employed consisted of taking the range of the process with uniform on $[-2n, -n]$ initial distribution and passing to a limit as $n \rightarrow \infty$.

Future generalizations and developments of the theory of regenerative sets were done in Maisonneuve [9], Fitzsimmons, Fristedt and Maisonneuve [1], Taksar [14]. In Maisonneuve [9] construction of stationary regenerative sets were done via obtaining a stationary distribution for the semigroup of the "residual life" process associated with the jumps of a subordinator.

The regenerative sets studied so far had finite underlying distribution P . In recent years, however, a new type of Markov processes emerged for which the underlying "probability measure" P is not finite but σ -finite. Accordingly, the visiting sets associated with such processes have infinite underlying distribution.

In this paper we present a simple method of construction of stationary regenerative sets which deals both with finite and infinite underlying distribution. We consider the Kuznetsov measure on the space of all trajectories Y , associated with

the transition function of a subordinator and Lebesgue invariant measure. Then we consider, the first hitting time T of the positive half line by this process. The joint distribution of T and the range M of Y is σ -finite and could be represented as a product of two measures. The second multiplier in this product gives us the distribution of a stationary regenerative set.

The paper structured as following. In section two we give general properties of subordinators which are necessary for a construction of stationary sets. In the third section we study the range of the stationary subordinator with Lebesgue one-dimensional distribution. We show how to obtain a stationary regenerative law via the distribution of the stationary subordinator. In the last section we prove that $-M$ has the same law as M .

2. Generalities.

In our notations and definitions we follow Maisonneuve [9], Fitzsimmons, Fristedt and Maisonneuve [1] with corrections made in Maisonneuve [10].

We denote by Ω a collection of all closed sets of $\mathbb{R}$. For $\omega \in \Omega$, $t \in \mathbb{R}$ put, assuming $\inf \emptyset = +\infty$.

$$d_t(\omega) = \inf\{s > t : s \in \omega\}, \quad r_t(\omega) = d_t(\omega) - t,$$

$$\tau_t(\omega) = \overline{(\omega - t) \cap]0, \infty[} = \overline{\{s - t : s \in \omega, s > t\}},$$

where the bar over a set stands for the closure of the set. We denote by G° (G_t° respectively) the σ -field generated by d_s , $s \in \mathbb{R}$ (d_s , $s \leq t$ respectively). The process d_t is an increasing cad-lag process optional with respect to G_t° , subject to $d_t \geq t$. Knowing d_t , one can reconstruct ω by the formula $\omega = \{t \in \mathbb{R} : d_{t-} = t\}$.

We will call a random set on a space $(\Omega, \mathcal{F})$ a measurable mapping $M: (\Omega, \mathcal{F}) \rightarrow (\Omega^\circ, G^\circ)$.

The process $D_t \triangleq d_t \circ M$ and $R_t \triangleq r_t \circ M$ are cad-lag and measurable as well as $M_t \triangleq \tau_t \circ M$, that is the mapping $(t, \omega) \rightarrow M_t(\omega)$, $t \leq +\infty$ is a measurable mapping of $((\mathbb{R} \cup \{\infty\}) \times \Omega, \mathcal{B}_{\mathbb{R} \cup \{\infty\}} \times \mathcal{F})$ into $(\Omega^0, \mathcal{G}^0)$.

Let $(\Omega, \mathcal{F}, P)$ be a probability space with σ -finite measure P and let G_t be a filtration of $\mathcal{F}$.

(2.1) Definition. A random set M on $(\Omega, \mathcal{F}, P)$ is regenerative if the process $D_t = d_t \circ M$ is adapted to G_t and there exists a probability measure P^0 on $(\Omega^0, \mathcal{G}^0)$ with $P^0(\Omega^0) = 1$ such that for all $t \in \mathbb{R}$ and all $f \in b\mathcal{G}^0$ (the set of bounded $\mathcal{G}^0$ -measurable functions).

$$(2.2) \quad P\{f \circ M_{D_t} | G_t\} = P^0\{f\}.$$

The measure P^0 is called the law of regeneration.

Let (X_t, P^X) be a subordinator on $(\Omega, \mathcal{F})$ that is an increasing process with independent increments with respect to a filtration H_t , and transition probabilities P^X . It is characterized by a nonnegative constant λ and measure Π on $]0, \infty[$ such that $\Pi(x \wedge 1) < \infty$. For such a process

$$(2.3) \quad P^0\{e^{-sX_t}\} = e^{-tg(s)},$$

where

$$(2.4) \quad g(s) = \lambda s + \int_0^\infty (1 - e^{-sx}) \Pi(dx).$$

The range $M(\omega) \triangleq \overline{X_{\mathbb{R}_+}}$ is a regenerative set with respect to the filtration $G_t \triangleq H_{T_t}$, where

$$T_t = \inf\{u > 0 : X_u > t\}$$

and the law of regeneration P^0 .

Let $U(x, \Gamma)$ be the potential kernel of the process X :

$$(2.5) \quad U(x, \Gamma) \triangleq P^x \left\{ \int_0^\infty 1_\Gamma(X_t) dt \right\}.$$

In view of obvious relation

$$(2.6) \quad P^x \{f(X_t)\} = P^0 \{f(X_{t-x})\},$$

we have that

$$(2.7) \quad U(x, \Gamma) = U(\Gamma - x),$$

where $U(\Gamma) \triangleq U(0, \Gamma)$. From (1.3) and (1.5), using Fubini's theorem, we get

$$(2.8) \quad \int_0^\infty e^{-s} U(dx) = g(s)^{-1}, \quad s > 0.$$

Let δ_x denote a unit measure concentrated at the point x , and m be the Lebesgue measure.

(2.9) Proposition. Let

$$(2.10) \quad \pi(\Gamma) = \lambda \delta_0(\Gamma) + \int_\Gamma \Pi]x, \infty[dx.$$

Then

$$(2.11) \quad \int_0^\infty U(x, \Gamma) \pi(dx) = m(\Gamma \cap \mathbb{R}_+).$$

Proof. Take Laplace transform of the measure in the right hand side of (2.11).

By (2.7) and (2.8)

$$\begin{aligned}
 \int_0^\infty e^{-sy} \int_0^\infty U(x, dy) \pi(dx) &= \int_0^\infty \pi(dx) \int_x^\infty e^{-sy} U(x, dy) \\
 &= \int_0^\infty \pi(dx) e^{-sx} \int_0^\infty e^{-sz} U(x, dz) \\
 &= g(s)^{-1} \int_0^\infty e^{-sx} \pi(dx) = g(s)^{-1} (\lambda + \int_0^\infty e^{-sx} \Pi]x, \infty[dx) \\
 &= g(s)^{-1} (\lambda + \int_0^\infty s^{-1} (1 - e^{-sx}) \Pi(ds)) = s^{-1}.
 \end{aligned}$$

Thus the Laplace transform of the right hand side of (2.11) equals the Laplace transform of the left hand side.

Let W be the set of all trajectories Y_t , $-\infty < t < +\infty$ endowed with the Kolmogorov σ -field F . Let P^X be a family of transition probabilities on $\mathcal{B}(Y_s, s \geq 0)$.

The next theorem is a particular version of the result of Kuznetsov [6].

(2.12) Theorem. Let η be a σ -finite measure on $\mathbb{R}$ invariant with respect to the transition probabilities P^X . There exists a σ -finite measure Q_η on (W, F) under which Y_s , $-\infty < s < +\infty$ is strong Markov and stationary with transition probabilities P^X and one-dimensional laws η . The measure Q_η is finite iff $\eta(\mathbb{R}) < \infty$.

We apply this theorem to Lebesgue measure m and transition probabilities P^X of a subordinator. We call Q_m the canonical Kuznetsov measure for the subordinator. This measure will be our main tool in description of stationary sets.

3. Construction of stationary regenerative sets.

Let (X, P^X) be a subordinator and $Q = Q_m$ be the canonical Kuznetsov measure on (W, F) associated with this subordinator. Let (W, F, F_t, Y_t, Q) be the corresponding stationary Markov process and σ_s be the shift operator in W such that $Y_t \circ \sigma_s = Y_{t+s}$.

Let T be a $\bar{F}$ -measurable random variable, where $\bar{F}$ is a Q -completion of F . We call T intrinsic if $T = u + T \circ \sigma_u$ for each $u \in \mathbb{R}$.

(3.1) Proposition. Let S and T be two intrinsic times and let $A \in F$ be an invariant event that is $\sigma_u^{-1} \circ A = A$ for all $u \in \mathbb{R}$. Let ϕ and ψ be any two nonnegative Borel functions on $\mathbb{R}$ such that

$$\int_{\mathbb{R}} \phi(t) dt = \int_{\mathbb{R}} \psi(t) dt$$

Then

$$Q\{\psi(S); A\} = Q\{\phi(T); A\}$$

Proof. Put

$$a = \int_{\mathbb{R}} \phi(t) dt \equiv \int_{\mathbb{R}} \psi(t) dt > 0.$$

then, using Febuini's theorem and the relation $Q \circ \sigma_u^{-1} = Q$,

$$\begin{aligned}
 a \ Q\{\psi(S); A\} &= Q\{\psi(S) \int_{\mathbb{R}} \phi(T+u); A\} \\
 &= \int_{\mathbb{R}} Q\{\psi(S) \phi(T+u); A\} du \\
 &= \int_{\mathbb{R}} Q\{\psi(S \circ \sigma_u) \phi(T \circ \sigma_u + u); \sigma_u^{-1} \circ A\} du \\
 &= \int_{\mathbb{R}} Q\{\psi(S-u) \phi(T); A\} du \\
 &= a \ Q\{\phi(T); A\}
 \end{aligned}$$

Put

$$T_s = \inf\{t > -\infty : Y_t > s\}$$

Since Y is a.e.Q increasing process, it is clear that T_s is an intrinsic time.

Consider a random set M on (W, F)

$$M(Y_*) = \bar{Y}_{\mathbb{R}_+},$$

(3.2) Theorem. There exists a σ -finite measure P_s on Ω^2 such that for each $A \in G^2$ and each $B \in \mathcal{B}_{\mathbb{R}}$

$$Q\{1_B(T_s) 1_A(M)\} = m(B) P_s\{A\}.$$

Proof. 1°. Let Q be the law on $\mathbb{R} \times \Omega^\circ$ of (T, M) under Q . Then for $f \in b\mathcal{B}_{\mathbb{R}}$ and $g \in b\mathcal{G}^\circ$, $g = h(r_{s_1}, r_{s_2}, \dots, r_{s_n})$. Then (we put $T = T_s$ below).

$$(3.3) \quad \begin{aligned} \tilde{Q}\{f(s)g(\omega^\circ)\} &= Q\{f(T)h(Y_{T_{s_1}}, Y_{T_{s_2}}, \dots, Y_{T_{s_n}})\} \\ &= Q\{f(T+u)h(Y_{T_{s_1}}, Y_{T_{s_2}}, \dots, Y_{T_{s_n}})\} = \tilde{Q}\{f(x+u)g(\omega^\circ)\}. \end{aligned}$$

The second equality in (3.3) is due to the stationarity of the process Y under the measure Q . By virtue of Gettoor [3], $\tilde{Q}$ can be represented as

$$(3.4) \quad \tilde{Q} = m \times P_s,$$

where P_s is a Σ -finite measure on Ω° (i.e., P_s is a countable sum of finite measures)

2°. Because Y_t is an increasing process and because $Q\{T=t\}=0$, we have

$$Q\{f(Y_t), T < t\} = Q\{f(Y_t), Y_t > 0\} = m\{1\}_{0, \infty}[f].$$

Let

$$\mu(\Gamma) = Q\{Y_T \in \Gamma, 0 \leq T < 1\}.$$

By virtue of (3.5) and Theorem (2.4)(iv) of [2]

$$m\{1_{\mathbb{R}_+} f\} = \mu U(f)$$

where U is the potential kernel given by (2.5).

In view of Proposition (2.9)

$$(3.6) \quad \mu = \pi$$

In view of (3.6) and Theorem (2.4)(i) of [2]

$$(3.7) \quad Q\{Y_T \in A, T \in \Gamma\} = m(\Gamma)\pi(A).$$

3. By virtue of (3.4) and Fubini's theorem

$$(3.8) \quad P_S\{A\} = Q\{\psi(T); M(Y_\bullet) \in A\}$$

for any $A \in \mathcal{G}$ and any $\psi \in b\mathbb{R}_+$, $\psi \geq 0$ such that

$$\int_{\mathbb{R}} \psi(t) dt = 1$$

Let $\Gamma \in \mathcal{B}_{\mathbb{R}_+}$ and $A = \{\omega : r_S(\omega) \in \Gamma\}$. Substituting in (3.8), we have

$$\begin{aligned} P_S\{r_S \in \Gamma\} &= Q\{1_\Gamma(Y_T)\psi(T) \\ (3.9) \quad &= \iint_{\mathbb{R} \times \mathbb{R}} 1_\Gamma(x)\psi(t)\pi(dx)dt = \pi(\Gamma) \end{aligned}$$

The second equality in (3.9) is due to (3.7) Relation (3.9) shows that P_S is a π -finite measure.

(3.10) Proposition. For any $s, t \in \mathbb{R}$

$$P_s = P_t$$

Proof. Let $P = P_0$. Since $M(Y \circ \sigma_s) = (M(Y.))$ we get that $A = \{M \in \Gamma\}$ is an invariant set. By virtue of (3.8) and Proposition (3.1)

$$\begin{aligned} P_s\{\Gamma\} &= Q\{\psi(T_s); A\} \\ &= Q\{\psi(T_0); A\} = P_0\{\Gamma\} = P\{\Gamma\}. \end{aligned}$$

(3.11) Theorem. The random set $M(\omega^\circ)$ on $(\Omega^\circ, F^\circ, P)$ is a stationary regenerative set with respect to the filtration $G_t = \sigma(r_s, s \leq t)$ with the law of regeneration P^0 .

The measure P is finite iff

$$(3.12) \quad \int_0^\infty x \Pi(dx) < \infty$$

Proof. 1°. Let $\theta_x: \Omega^\circ \rightarrow \Omega^\circ$, $\theta_x(\omega^\circ) = \omega^\circ - x$ and $\phi_x: W \rightarrow W$, $(\phi_x(Y.))_t = Y_t - x$. Then

$$(3.13) \quad M(\phi_x(Y.)) = \theta_x(M(Y.)),$$

$$T_s(\phi_x(Y.)) = T_{s+x}(Y.).$$

Since both the initial distribution (the Lebesgue measure) and the transition probabilities P^x are spatially invariant, we have

$$Q \circ \phi_x^{-1} = Q$$

for any $x \in \mathbb{R}$. Therefore, using (3.13),

$$\begin{aligned} (3.14) \quad P\{\theta_x^{-1}A\} &= Q\{\psi(T_0); M \circ \phi_x \in A\} = \\ &= Q\{\psi(T_{-x}) \circ \phi_x; M \circ \phi_x \in A\} \\ &= Q\{\psi(T_{-x}); M \in A\} = P_{-x}\{A\}. \end{aligned}$$

By virtue of Proposition (3.10), $P_{-x} = P$, therefore (3.14) equals $P(A)$. This shows that P is the law of a stationary set.

2°. Let $f \in bG^\circ$ and $A \in G^\circ$. Then $\{M(Y.) \in A\} \in F_{T_0}$. Using the strong Markov property for Y

$$\begin{aligned}
 (3.15) \quad P(f \circ M_{D_0}; A) &= Q\{\psi(T_0)f(M_{D_0}); M(Y.) \in A\} \\
 &= Q\{\psi(T_0)P^0\{f\}; M(Y.) \in A\} = P\{P^0\{f\}; A\} \\
 &= P\{A\}P^0\{f\}.
 \end{aligned}$$

In view of stationarity, (3.15) will hold if D_0 is replaced by T_t . The latter shows that P has regenerative property with the law of regeneration P^0 .

3°. In view of (3.9) and Proposition (3.10)

$$\begin{aligned}
 P\{\Omega^\circ\} &= P\{r_s \in \mathbf{R}\} = \pi(\mathbf{R}) \\
 &= \lambda + \int_0^\infty \Pi]x, \infty[dx = \lambda + \int_0^\infty x\Pi(dx).
 \end{aligned}$$

Thus $P\{\Omega^\circ\} < \infty$ iff (3.12) holds.

4. Reversability properties of regenerative sets

For simplicity we will consider here only perfect regenerative sets. Discrete case is treated similarly. For this sets the regenerative law P^0 is the law of a strictly increasing subordinator.

(4.1) Theorem. If M is a stationary regenerative set then $-M$ has the same law as M .

Proof 1°. Consider the process (W, F, F_t, Y, Q) which generates the set M .

Let $\hat{Y}_t = -(Y_{(-t)-})$. Put

$$\begin{aligned} T_0 &= \inf\{t: \hat{Y}_t > 0\} = -\sup\{t: Y_{t-} < 0\} \\ (4.2) \quad &= -\sup\{Y_t < 0\} = -\inf\{Y_t > 0\} = -T_0, \text{ a.s. } Q \end{aligned}$$

The third and the fourth equalities in (4.2) hold because Y is strictly increasing a.s. Q .

2°. Simple calculations show that $\hat{Y}_t$ is a Markov process with the same one-dimensional distributions and potential kernel U as Y_t . Thus $\hat{Y}_t$ has the same law as Y_t . Let $\psi(x) = .5 \exp -|x|$. Let $\hat{M} = M(\hat{Y} \cdot)$. Then $\hat{M} = -M$; and by virtue of (3.8)

$$\begin{aligned} P\{M \in A\} &= Q\{\psi(T_0); M(Y \cdot) \in A\} \\ &= Q\{\psi(-T_0); M(Y \cdot) \in A\} \\ &= Q\{\psi(\hat{T}); -M(\hat{Y}) \in A\} = P\{-M \in A\} \end{aligned}$$

The latter shows that M and $-M$ have the same distribution.

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