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STABILIZATION AND CONTROL PROBLEMS IN STRUCTURAL DYNAMICS

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During the first year of support of AFOSR Grant 85-0253, Dr. G. Chen and his collaborators made continuous progress on the research of control and stabilization problems in structural dynamics. His primary emphases were on the modelling, designs, placements and analysis of stabilizers and controllers. Two primary types of vibrating structures were treated: the second order wave equation such as strings and cables, and the fourth order equation such as beams. For those structures, he was able to classify the types of stabilizers, analyze their damping behavior, and determine the mechanical designs. Various methods such as characteristics, asymptotic estimation, energy, functional analysis and the Legendre spectral method were used to study the problems.

Some relevant theory and methods for general distributed systems and equilibrium problems were ~~also~~ developed.

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I. Summary

During the first year of support of AFOSR Grant 85-0253, Dr. G. Chen and his collaborators made continuous progress on the research of control and stabilization problems in structural dynamics. His primary emphases were on the modelling, designs, placement and analysis of stabilizers and controllers. Two primary types of vibrating structures were treated: the second order wave equation such as strings and cables, and the fourth order equation such as beams. For those structures, he was able to classify the types of stablizers, analyze their damping behavior, and determine the mechanical designs. Various methods such as characteristics, asymptotic estimation, energy, functional analysis and the Legendre spectral method were used to study the problems.

Some relevant theory and methods for general distributed systems and equilibrium problems were also developed.

II. Research Program

The research objectives of our research program last year consisted of the study of the following

i) Asymptotic spectral distribution and practical implementations of various damping conditions at boundary or joints for coupled strings or beams;

ii) Numerical computation of the spectrum of a boundary damped dynamic bi-harmonic equation modelling a plate;

iii) Stabilization and control of nonlinear vibrating systems.

So far, we have already completed the first phase of these objectives, as can be seen from our list of publications.

During the second year of support, we plan to enter the second phase of our research program by continuing the investigation of the above, focusing on more complicated systems. Meanwhile, we are also planning to research the following new objectives:

iv) Analysis and control of dynamic frame structures;

v) Study of new beam models such as the one recently proposed by D. L. Russell which exhibits better damping characteristics.

vi) Higher dimensional membrane and plate equations with nonlinearity.

They will constitute the major portion of our research program for the second-year.

III. Research Accomplishments and Work in Progress

We first describe our research findings under headings [A]-[D] below:

[A] Analysis and designs of stabilizers for second order systems-vibrating strings and cables.

The standard model is the wave equation

$$m \frac{\partial^2 y(x,t)}{\partial t^2} - T \frac{\partial^2 y(x,t)}{\partial x^2} = 0, \qquad 0 < x < L, \quad t > 0,$$

where $y(x,t)$ represents the vertical displacement at position x at time t, m is the mass density per unit length, and T is the tension coefficient. The energy of vibration at time t is

$$E(t) = \frac{1}{2} \int_0^1 [m y_t^2(x,t) + T y_x^2(x,t)]dx.$$

If a stabilizer is placed at the boundary $x = L$, then

$$\frac{d}{dt} E(t) = T y_t(L,t) y_x(L,t) \le 0, \tag{1}$$

assuming that the boundary condition at $x = 0$ is energy conserving. For (1) to hold, we must use proportional control

(2) $\quad Ty_x(L,t) = -k^2 y_t(L,t), \quad\quad k^2$=feedback gain >0, for all $t > 0$,

i.e.,

force is negatively proportional to velocity at $x = L$.

Therefore a standard viscous damper is sufficient for this purpose, as shown below:

Fig. 1 A vibrating string with a damping device at left end

The above fact is well known. Furthermore, if one chooses the feedback gain to be

$$k^2 = T\sqrt{T/m} \quad ,$$

then the damper action (2) becomes a so called characteristic impedance condition, which causes maximum energy loss to the vibrating system.

Now, let us consider the case when the stabilizer is installed at an in-span point, say at $x = \alpha$, $0 < \alpha < L$. Assume that the boundary conditions at $x = 0$ and $x = L$ are energy-conserving. Then

(3) $$\frac{d}{dt}\,[\text{Total energy at time } t] = \frac{d}{dt}\,\{\frac{1}{2}\int_0^{\alpha}[my_t^2(x,t)+Ty_x^2(x,t)]dx + \frac{1}{2}\int_{\alpha}^{L}[my_t^2(x,t) + Ty_x^2(x,t)]dx\} = Ty_x(\alpha^-,t)y_t(\alpha^-,t) - Ty_x(\alpha^+,t)y_t(\alpha^-,t) \le 0.$$

These are only two possibilities which can cause the negativity of the right hand side of (3):

(A1) Continuity of displacement (and velocity)

$$y(\alpha^-,t) = y(\alpha^+,t)$$

discontinuity of vertical force

$$Ty_x(\alpha^-,t) - Ty_x(\alpha^+,t) = -k_1^2 y_t(\alpha^-,t), \quad k_1^2\text{=feedback gain} > 0;$$

(A2) Discontinuity of displacement and velocity

$$y_t(\alpha^-,t) - y_t(\alpha^+,t) = k_2^2 Ty_x(\alpha^-,t), \quad k_2^2\text{=feedback gain} > 0,$$

continuity of vertical force

$$Ty_x(\alpha^-,t) = Ty_x(\alpha^+,t).$$

The mechanical designs of stabilizers of the above two types are given in Figures 2-5.

The mathematical analysis of damping behavior associated with the above designs is given in our paper [2], along with numerical results.

[B] <u>Analysis, designs and placements of stabilizers for fourth order systems-beams</u>

These include the Euler-Bernoulli, Rayleigh and Timoshenko beam models. We will use the Euler-Bernoulli beam

$$m\,\frac{\partial^2 y(x,t)}{\partial t^2} + EI\,\frac{\partial^4 y(x,t)}{\partial x^4} = 0, \qquad 0 < x < L, \quad t > 0,$$

to illustrate our designs and analysis. In the above, $y(x,t)$ is the vertical displacement, m is the mass density per unit length, and EI is the flexural rigidity coefficient. The energy of vibration at time t is

$$E(t) = \frac{1}{2}\int_0^L [m y_t^2(x,t) + EI y_{xx}^2(x,t)]dx.$$

Again, let us install a stabilizer at the right end $x = L$, and assume that the boundary conditions at $x = 0$ are energy conserving. Then

$$\frac{d}{dt} E(t) = EI[y_{xx}(L,t)y_{xt}(L,t) - y_{xxx}(L,t)y_t(L,t)] \leq 0. \tag{4}$$

Now, to achieve the negativity of the right hand side above, there are many different boundary conditions and corresponding designs which can serve this purpose. The following is a more or less exhaustive list of them:

$$\begin{cases} -EI\, y_{xxx}(L,t) = -k_1^2 y_t(L,t), \; k_1^2 > 0 , \\ -EIy_{xx}(L,t) = 0 \end{cases} \tag{5}$$

$$\begin{cases} -EIy_{xxx}(L,t) = 0 \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t), \quad k_2^2 > 0, \end{cases} \tag{6}$$

$$\begin{cases} y_x(L,t) = 0 \\ -EIy_{xxx}(L,t) = -k_1^2 y_t(L,t), \; k_1^2 > 0 , \end{cases} \tag{7}$$

$$\begin{cases} y(L,t) = 0 \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t), \quad k_2^2 > 0, \end{cases} \tag{8}$$

$$\begin{cases} -EIy_{xxx}(L,t) = -k_1^2 y_t(L,t) + c_1 y_{xt}(L,t) , \quad k_1^2 > 0 , \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t) + c_2 y_t(L,t), \quad k_2^2 > 0, \end{cases} \tag{9}$$

where in (9), c_1 and c_2 are real constants satisfying

$$(c_1-c_2)\zeta\eta - k_1^2\zeta^2 - k_2^2\eta^2 \le 0, \quad \text{for all } \zeta,\eta \in \mathbb{R}.$$

Note that throughout (5)-(9), all terms have physical meanings:

y_t = velocity

y_x = rotation, y_{xt} = angular velocity,

$-EIy_{xx}$ = bending moment,

$-EIy_{xxx}$ = shear.

Thus the feedback schemes (5)-(9) are all proportional controls. Their mechanical designs are indicated in Figures 6-10, cf. [3].

The analysis of eigenfrequencies of vibration has also been given in [3].

[C] Stability criterion for point stabilizers on coupled quasilinear vibrating strings

Consider two quasilinear wave equations

(10)
$$\begin{cases} \frac{\partial^2}{\partial t^2} w(x,t) - \frac{\partial}{\partial x}\left[\sigma_1\left(\frac{\partial w(x,t)}{\partial x}\right)\right] = 0, & 0 < x < 1, \\ \frac{\partial^2}{\partial t^2} w(x,t) - \frac{\partial}{\partial x}\left[\sigma_2\left(\frac{\partial w(x,t)}{\partial x}\right)\right] = 0, & 1 < x < 2, \end{cases}$$

where σ_1 and σ_2 are nonlinear functions satisfying $\sigma_i(0) = 0$, $\sigma_i'(u) > 0$, for $i = 1,2$. The boundary condition at $x = 0$ is

(11)
$$\sigma_1(w_x(0,t)) - k_0^2 w_t(0,t) = 0, \quad k_0^2 > 0,$$

and at $x = 2$ is

(12)
$$w(2,t) = 0.$$

The two strings are coupled at $x = 1$ through

$$(13)\qquad \begin{cases} \sigma_1(w_x(1^-,t)) = \sigma_2(w_x(1^+,t)) \\ w_t(1^-,t) - w_t(1^+,t) = -k_1^2\sigma_1(w_x(1^-,t)), \quad k_1^2 \geq 0. \end{cases}$$

Using some fairly recent theorems on quasilinear hyperbolic PDEs, we are able to show that if k_0^2 and k_1^2 in (11) and (13) are nonnegative, then the solution decays exponentially in the C^1-norm:

$$(14)\quad \|w(\cdot,t)\|_{C^1(0,1)} + \|w(\cdot t)\|_{C^1(1,2)} \leq Ke^{-\alpha t},\ K,\ \alpha > 0 \text{ for all } t \geq 0,$$

provided that

$$1 + K_1K_3 + K_2 + K_1K_2K_3 - 4c_1c_2\Delta^{-2}K_3 > 0$$

$$2(1-K_1K_2K_3 + 4c_1c_2\Delta^{-2}K_3) > 0$$

$$1 - K_1K_3 - K_2 + K_1K_2K_3 - 4\, c_1c_2\Delta^{-2}K_3 > 0$$

are satisfied, where

$$c_i \equiv \sqrt{\sigma_i'(0)}\ , \qquad i = 1,2, \qquad c_1 \geq c_2,$$

$$\Delta \equiv c_1 + c_2 + k_1^2c_1c_2$$

$$K_1 \equiv \Delta^{-1}(c_1 - c_2 + k_1^2c_1c_2)$$

$$K_2 = \Delta^{-1}|c_1 - c_2 - k_1^2c_1c_2|$$

$$K_3 = |c_1 - k_0^2|/(c_1 - k_0^2),$$

for solutions of (10)-(13) with sufficiently smooth small initial data. Note that if $k_0^2 = 0$ in (11), then generally (14) does not hold no matter how $k_1^2 > 0$ is chosen. This shows that installing a stabilizer in the middle of the span (without installing another one at the boundary) is <u>not robust</u> with respect to stability.

[D] The boundary element methods for optimal boundary control of elliptic partial differential equations

Elliptic PDEs often appear in continuum mechanics. Assume that the control appears on the boundary. Traditional numerical methods like the finite differences or finite elements require the discretization of the entire domain. The boundary element method only requires the discretization of the boundary which yields numerical solutions of optimal boundary control much more efficiently. In [6], we treated the Neumann type boundary control using the fundamental solution. In [7], Dirichlet boundary controls were studied using the Poisson representation of harmonic functions. Numerical solutions were computed which confirmed the theoretical error estimates.

Next, under headings [E]-[G], we describe the research that is presently in progress.

[E] Analysis, designs and behavior of dissipative joints for coupled beams.

In [1], [3], we have studied the case where a stabilizer is installed at boundary. What do we know if a stabilizer is installed at an in-span point? Let this point be $x = \alpha$, $0 < \alpha < L$. Assume that the boundary conditions at $x = 0$ and $x = L$ are energy conserving. Then

$$(15)\qquad \frac{d}{dt}[\text{Total energy at time } t] = \frac{d}{dt}\Big\{\frac{1}{2}\int_0^{\alpha}[my_t^2(x,t) + EIy_{xx}(x,t)]dx$$

$$+ \frac{1}{2}\int_{\alpha}^{L}[my_t^2(x,t) + EIy_{xx}^2(x,t)]dx\Big\}$$

$$= -[y_t(\alpha^-,t)\cdot EIy_{xx}(\alpha^-,t) - y_t(\alpha^+,t)\cdot EIy_{xx}(\alpha^+,t)]$$

$$+ [y_{xt}(\alpha^-,t)\cdot EIy_{xx}(\alpha^-,t) - y_{xt}(\alpha^+,t)\cdot EIy_{xx}(\alpha^+,t)]$$

$$\le 0\,,\quad \text{for all}\quad t > 0.$$

We collect all possible transmission conditions below which can cause energy dissipation in (15):

(E1) Continuity of bending moment and shear:

$$-EIy_{xx}(\alpha^-,t) = -EIy_{xx}(\alpha^+,t)$$

$$-EIy_{xxx}(\alpha^-,t) = -EIy_{xxx}(\alpha^+,t)$$

discontinuity of velocity (and displacement) and rotation:

$$y_t(\alpha^-,t) - y_t(\alpha^+,t) = k_1^2\cdot[-EIy_{xxx}(\alpha^-,t)] + c_1[-EIy_{xx}(\alpha^-,t)]$$

$$y_{xt}(\alpha^-,t) - y_{xt}(\alpha^+,t) = c_2[-EIy_{xxx}(\alpha^-,t)] + k_2^2[-EIy_{xx}(\alpha^-,t)]$$

(E2) Continuity of displacement and bending moment

$$y(\alpha^-,t) = y(\alpha^+,t)$$

$$-EIy_{xx}(\alpha^-,t) = -EIy_{xx}(\alpha^+,t)$$

discontinuity of shear and rotation

$$[-EIy_{xxx}(\alpha^-,t)] - [-EIy_{xxx}(\alpha^+,t)] = k_1^2y_t(\alpha^-,t) + c_1[-EIy_{xx}(\alpha^-,t)]$$

$$y_{xt}(\alpha^-,t) - y_{xt}(\alpha^+,t) = c_2 y_t(\alpha^-,t) - k_2^2[-EIy_{xx}(\alpha^-,t)].$$

(E3) Continuity of displacement and rotation

$$y(\alpha^-,t) = y(\alpha^+,t)$$

$$y_x(\alpha^-,t) = y_x(\alpha^+,t)$$

discontinuity of shear and bending moment

$$[-EIy_{xxx}(\alpha^-,t)] - [-EIy_{xxx}(\alpha^+,t)] = k_1^2 y_t(\alpha^-,t) + c_1 y_{xt}(\alpha^-,t)$$

$$[-EIy_{xx}(\alpha^-,t)] - [-EIy_{xx}(\alpha^+,t)] = c_2 y_t(\alpha^-,t) - k_2^2 y_{xt}(\alpha^-,t).$$

(E4) Continuity of rotation and shear

$$y_x(\alpha^-,t) = y_x(\alpha^+,t)$$

$$-EIy_{xxx}(\alpha^-,t) = -EIy_{xxx}(\alpha^+,t)$$

discontinuity of velocity (and displacement) and bending moment

$$y_t(\alpha^-,t) - y_t(\alpha^+,t) = k_1^2[-EIy_{xxx}(\alpha^-,t)] + c_1 y_{xt}(\alpha^-,t)$$

$$[-EIy_{xx}(\alpha^-,t)] - [-EIy_{xx}(\alpha^+,t)] = c_2[-EIy_{xxx}(\alpha^-,t)] - k_2^2 y_{xt}(\alpha^-,t).$$

Note that throughout (E1)-(E4), $k_1^2 \geq 0$, $k_2^2 \geq 0$ and

$$k_1^2 + k_2^2 > 0$$

(16) $$(c_1-c_2' \;, - k_1^2\zeta^2 - k_2^2\eta^2 \leq 0, \quad \text{for all } \zeta,\eta \in \mathbb{R}.$$

In each of these cases, if a physical variable is discontinuous, then its conjugate variable must be continuous.

So far, we have completed preliminary partial designs of stabilizer devices according to the above classifications (E1)-(E4). See Figures 11-14. In those designs, the choices of c_1 and c_2 are 0 and not as general as (16) indicates. We are presently making efforts to design devices with more general c_1 and c_2.

These mechanical designs included here were worked out jointly with

Professor Harry H. West of the Civil Engineering Department of The Pennsylvania State University, who is specialized in structural analysis and designs.

Eigenfrequency analysis along with the aforementioned designs will be presented in a forthcoming paper [5].

[F] Study of a new beam model proposed by D. L. Russell

In [9], D. L. Russell proposed the following beam model

$$mw_{tt}(x,t) + EIw_{xxxx}(x,t) - \frac{\partial}{\partial x}\int_0^1 h(x,\xi)[w_{xt}(x,t)-w_{\xi t}(\xi,t)]d\xi = 0,$$

$$0 < x < 1, \quad t > 0$$

with, say, clamped left end

$$\begin{cases} w(0,t) = 0 \\ w_x(0,t) = 0 \end{cases}$$

and free right end

$$\begin{cases} w_{xx}(1,t) = 0 \\ -EIw_{xxx}(1+t) + \int_0^1 h(1,\xi)[w_{xt}(1,t) - w_{\xi t}(\xi,t)]d\xi = 0. \end{cases}$$

The above model seems to exhibit damping characteristics of thin slender beams more closely to a realistic beam.

So far, we have completed the investigation of the case when

$$h(x,\xi) \equiv \text{constant} = h.$$

Our preliminary results are the following:

i) When

$$\frac{h}{EI} > 2\sqrt{\frac{m}{EI}} \; .$$

"overdamping" occurs.

ii) When

$$\frac{h}{EI} = 2\sqrt{\frac{m}{EI}} .$$

"critical damping" occurs.

iii) When

$$0 < \frac{h}{EI} < 2\sqrt{\frac{m}{EI}} .$$

"underdamping" occurs.

iv) There exists a holomorphic C_0-semigroup corresponding to the dynamic beam model.

This research is currently being continued. The results have potential applications to the modelling and control of structures, and provide some unifying ground between distributed parameter and finite element (i.e., lumped parameter) theories.

[G] Numerical computation of the spectrum of a boundary damped dynamic biharmonic equation modelling a plate

Recently, J. Lagnese [8] proved the exponential decay of energy of a dynamic plate using certain boundary stabilization scheme.

A Ph.D. student is writing computer codes to program the Legendre spectral method for the boundary damped plate. We expect to run the program at the John von Neumann Center using Cyber 205. The work involving vectorization and parallel algorithms is in good progress.

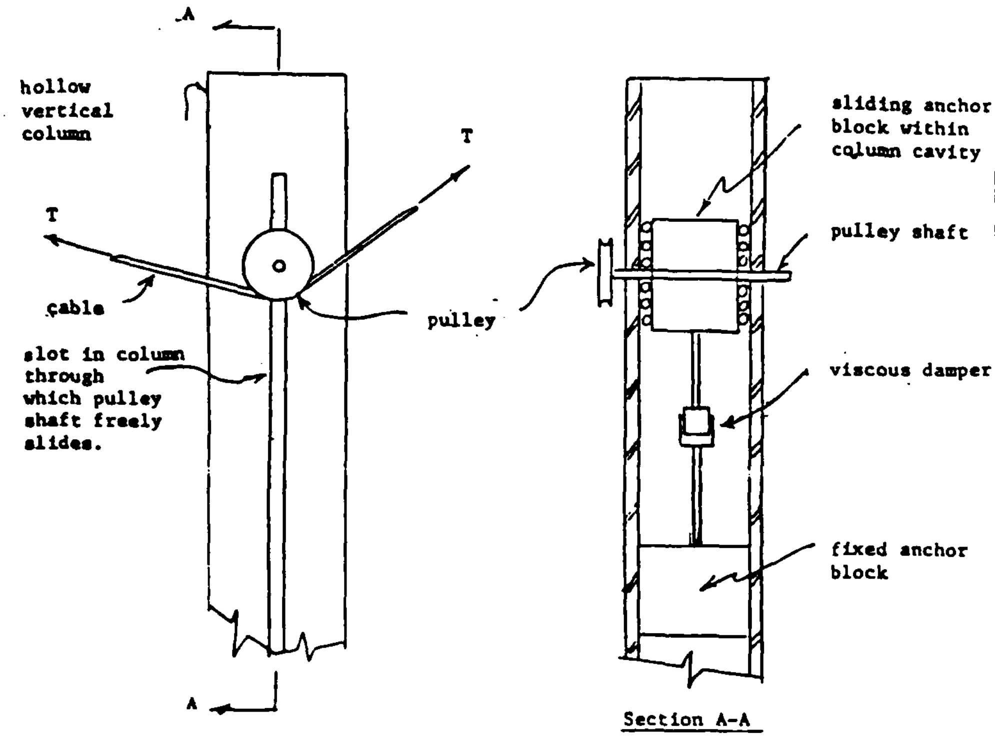


Fig. 2 Stabilizer device satisfying (A1)

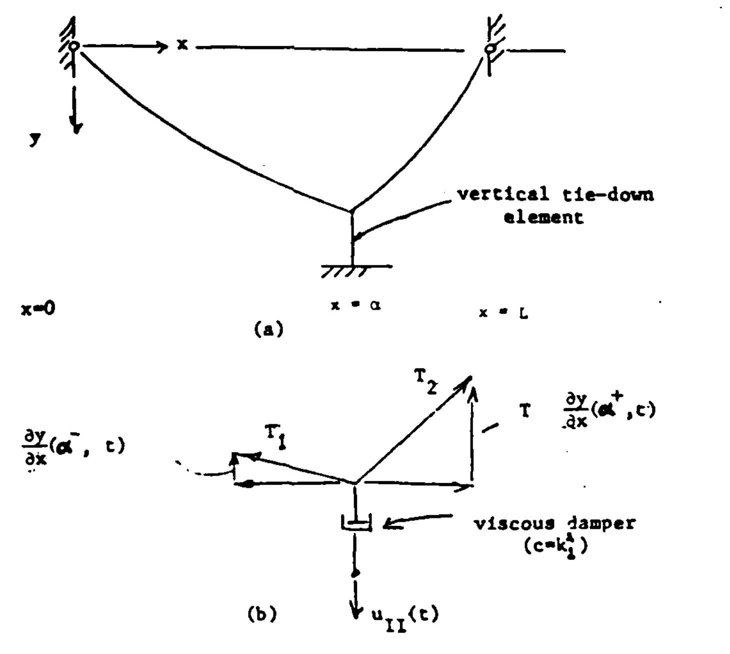


Fig. 3 Analysis of forces and velocities for the stabilizer device in Fig. 2

$$\text{(A1)} \quad y(\alpha^-,t) = y(\alpha^+,t)$$

$$Ty_x(\alpha^-,t) - Ty_x(\alpha^+,t) = -k_1^2 y_t(\alpha^-,t)$$

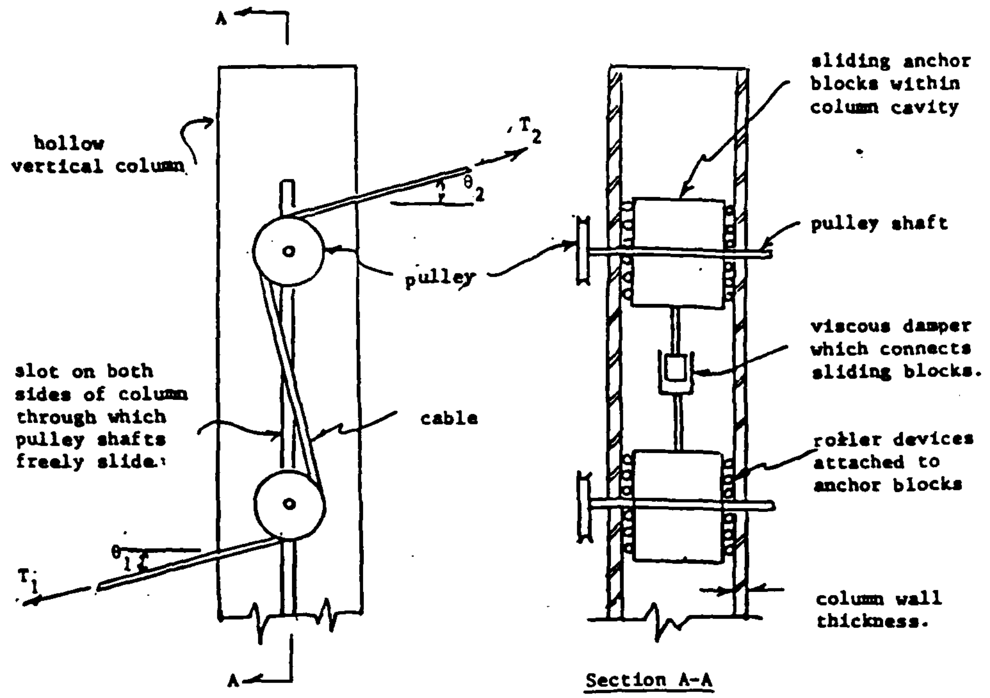


Fig. 4 Stabilizer device satisfying (A2)

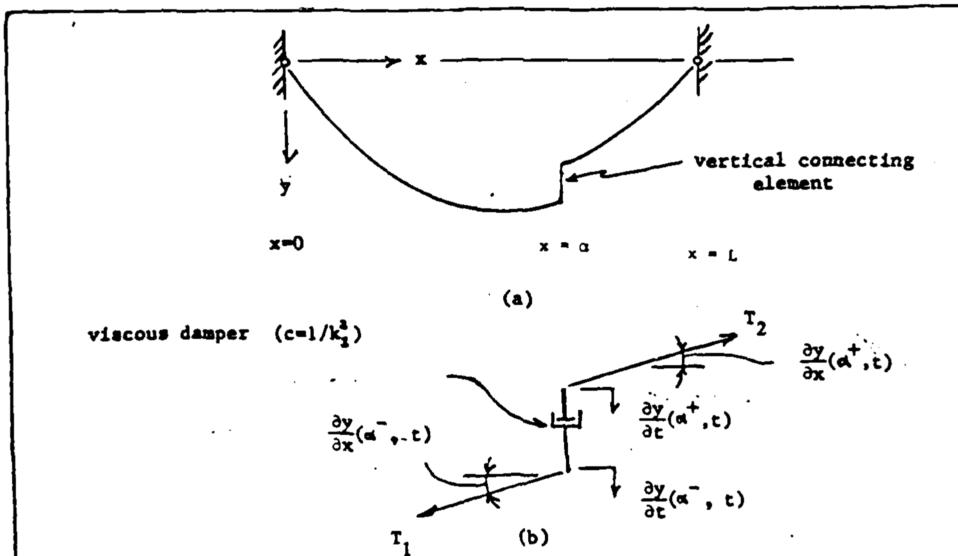
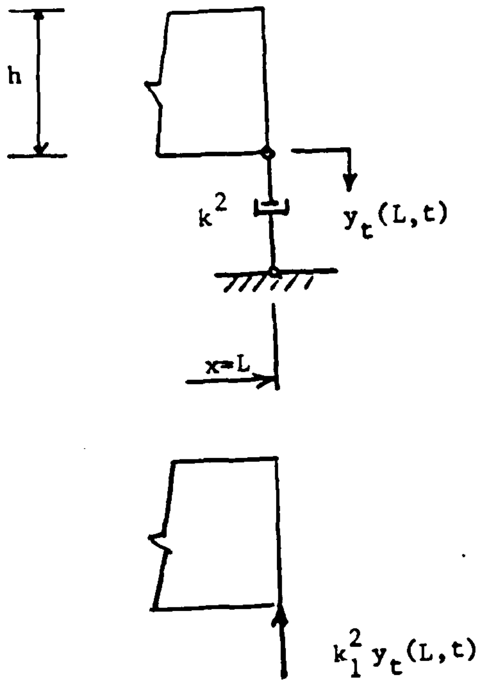


Fig. 5 Analysis of forces and velocities for the stabilizer device in Fig. 4

(A2) $y_t(\alpha^-,t)-y_t(\alpha^+,t)=-k_2^2Ty_x(\alpha^-,t)$

$Ty_x(\alpha^-,t)=Ty_x(\alpha^+,t)$

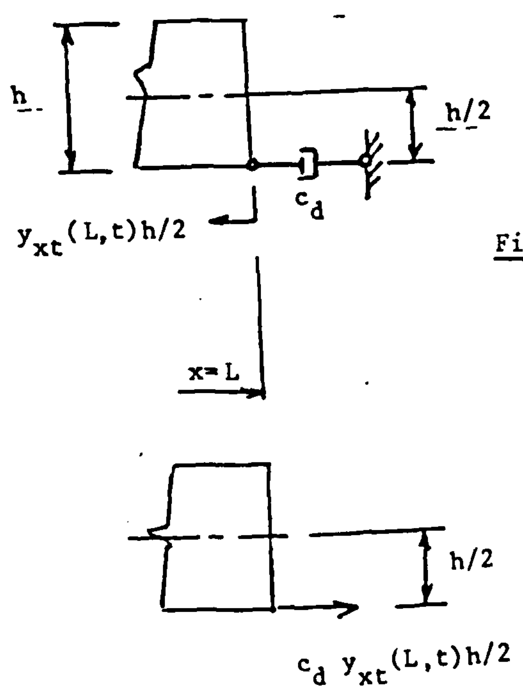


Shear(L,t) = $k_1^2\ y_t(L,t)$

Vertical Damper

<u>Fig. 5</u> <u>Stabilizer arrangement satisfying</u>

$$\begin{cases} -EI\ y_{xxx}(L,t) = -k_1^2 y_t(L,t),\ k_1^2 > 0\ . \\ -EIy_{xx}(L,t) = 0. \end{cases}$$



Moment(L,t) = $k_2^2\ y_{xt}(L,t)\ \ h^2/4$

Horizontal Damper

<u>Fig. 7</u> <u>Stabilizer arrangement satisfying</u>

$$\begin{cases} -EIy_{xxx}(L,t) = 0 \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t),\ \ k_2^2 > 0. \end{cases}$$

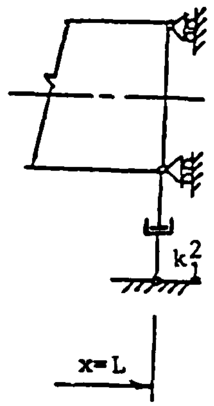


Fig. 8 Stabilizer arrangement satisfying

$$\begin{cases} y_x(L,t) = 0 \\ -EIy_{xxx}(L,t) = -k_1^2 y_t(L,t), \ k_1^2 > 0 \ . \end{cases}$$

$$\text{Shear}(L,t) = k_1^2 \, y_t(L,t)$$
$$\text{Slope}(L,t) = y_x(L,t) = 0$$

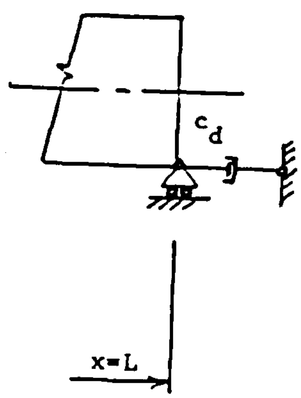
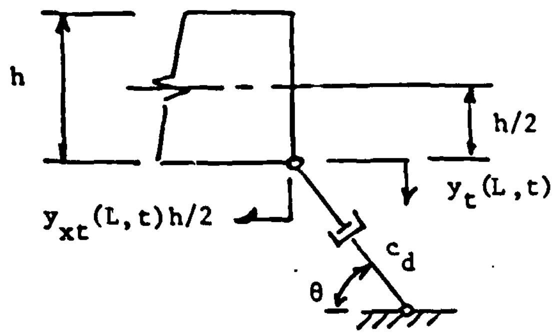


Fig. 9 Stabilizer arrangement satisfying

$$\begin{cases} y(L,t) = 0 \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t), \ \ k_2^2 > 0. \end{cases}$$

$$\text{Displ}(L,t) = y(L,t) = 0$$
$$\text{Moment}(L,t) = k_2^2 \, y_{xt}(L,t) \ h^2/4$$



Inclined Damper

Fig. 10 Stabilizer arrangement satisfying

$$\begin{cases} -EIy_{xxx}(L,t) = -k_1^2 y_t(L,t)+c_1 y_{xt}(L,t), \\ -EIy_{xx}(L,t) = k_2^2 y_{xt}(L,t)+c_2 y_t(L,t), \\ \text{with } c_1 = -c_2 = \frac{h}{2} c_d \sin\theta \cos\theta. \end{cases}$$

IV. References

[1] G. Chen, M.C. Delfour, A.M. Krall and G. Payre, Modelling, stabilization and control of serially connected beams, SIAM J. Cont. Opt., to appear.

[2] G. Chen, M. Coleman, H. H. West, Pointwise stabilization in the middle of the span for second order systems, nonuniform and uniform exponential decay of solutions, to appear in SIAM J. Appl. Math.

[3] G. Chen, S. G. Krantz, D.W. Ma, C.E. Wayne, and H.H. West, The Euler-Bernoulli beam equation with boundary energy dissipation, to appear in "Operator Methods for Optimal Control Problems", S.J. Lee, ed., Lecture Notes in Pure and Applied Mathematics Series, Marcel-Dekker, N.Y.

[4] G. Chen and H.K. Wang, Pointwise stabilization for coupled quasilinear and linear wave equations, presented at the International Conference on Identification and Control of Distributed Parameter Systems, Vorau, Austria, July, 1986. To appear in Springer Lecture Notes on Control and Information Sciences.

[5] G. Chen et. al, Analysis, designs and behavior of dissipative joints for coupled beams, in preparation.

[6] G. Chen and Y. L. Tsai, The boundary element numerical method for two dimensional linear quadratic elliptic problems: (I) Neumann control, preprint, submitted to Math. Comp., July, 1985.

[7] G. Chen, C. P. Chen and I. Aronov, A boundary element method based on Cauchy integrals for some linear quadratic boundary control problems on a circle, preprint, submitted to Optimal Control Applications & Methods, April, 1986.

[8] J. Lagnese, Uniform boundary stabilization of homonogeneous isotropic plates, preprint.

[9] D. L. Russell, On mathematical models for the elastic beam with frequency-proportional damping, preprint.

V. List of Publications

The following is a list of publications supported by the grant during September 1, 1985-August 31, 1986. Three preprint copies of each paper have been sent to the Program Manager. When they appear in journals in final form, reprints will be submitted to AFOSR immediately.

[1] G. Chen, M. C. Delfour, A. M. Krall and G. Payre, Modelling, stabilization and control of serially connected beams, accepted by SIAM J. Control Opt.

[2] G. Chen and Y. L. Tsai, The boundary element numerical method for two dimensinal linear quadratic elliptic problems: (I) Neumann control, preprint, submitted to Math. Comp., July, 1985.

[3] G. Chen, M. Coleman, and H. H. West, Pointwise stabilization in the middle of the span for second order systems, nonuniform and uniform decay results, accepted by SIAM J. Appl. Math..

[4] G. Chen and J. X. Zhou, Diagonal convexity conditions for problems in convex analysis and quasi-variational inequalities, preprint, submitted to J. Math. Anal. Appl., Jan. 1986.

[5] G. Chen, C. P. Chen and I. Aronov, A boundary element method based on Cauchy integrals for some linear quadratic boundary control problems on a circle, preprint, submitted to Optimal Control Applications & Methods.

April, 1986.

[6] G. Chen, S. G. Krantz, D. W. Ma, C. E. Wayne, and H. H. West, The Euler-Bernoulli beam equation with boundary energy dissipation, accepted in "Operator Methods for Optimal Control Problems", S. J. Lee, ed., Lecture Notes in Pure and Applied Mathematics Series, Marcel Dekker, Inc., New York.

[7] G. Chen and H. K. Wang, Pointwise stabilization for coupled quasilinear and linear wave equations, presented at the Conference on Control and Identification of Distributed Systems, Vorau, Austria, July 1986. To appear in Springer Lecture Notes on Control and Information Sciences.

[8] H. K. Wang and G. Chen, Asymptotic behavior of solutions of the one-dimensional wave equations with a nonlinear elastic dissipative boundary condition, submitted to Nonlinear Analysis, August, 1986.

VI. Ph.D. Thesis Supported by the Grant

Mr. J. Zhou completed his Ph. D. thesis entitled "Topics in Differential Games and Variational Inequalities" under the advisement of Dr. G. Chen in August 1986.

The abstract of his thesis is given in the attached letter as shown next page. Three copies of the thesis has been forwarded to the Program Monitor.

THE PENNSYLVANIA STATE UNIVERSITY
218 McALLISTER BUILDING
UNIVERSITY PARK, PENNSYLVANIA 16802

College of Science
Department of Mathematics

Area Code 814
865-7527

October 14, 1986

Professor Jim Crowley
Program Manager, AFOSR
AFSC Bldg. 410
Bolling AFB, D.C. 20332-6448

Dear Jim,

Enclosed are 3(three) copies of a Ph.D. thesis entitled "Topics in Differential Games and Variational Inequalities", written by my student J.Zhou, who has been partially supported by a research assistantship from AFOSR Grant 85-0253.

In his thesis he discusses theorems of existence and uniqueness of an optimal strategy for N-person differential games. He also studies similar problems for a solution to (quasi-) variational inequalities. He has formulated a class of weaker convexity (or concavity) conditions which require a functional Φ (x,y,) to be quasi-convex or convex for diagonal entries of certain type. He shows that many problems in convex analysis and (quasi-) variational inequalities can be treated by these generalized convexity conditions.

These theorems have potential applications to equilibrium PDE problems and multi-objective optimization.

Part of the thesis has been redacted and submitted for publication.

Thank you very much.

Sincerely,

Goong Chen

GC/jab

VII. Data on Scientific Collaborators

The following is a list of collaborators and their affiliations:

1. I. Aronov, graduate student, now at Cornell University.
2. C. P. Chen, graduate student, The Pennsylvania State University
3. M. Coleman, graduate student, The Pennsylvania State University
4. M. C. Delfour, Professor, Universite' de Montreal
5. A. M. Krall, Professor, The Pennsylvania State University
6. S. G. Krantz, Professor, Washington University, St. Louis, MO
7. D. W. Ma, Visitor, The Pennsylvania State University
8. G. Payre, Assistant Professor, Chemical Engineering Department Universite' de Sherbrooke
9. Y. L. Tsai, Graduate Student, The Pennsylvania State University
10. C. E. Wayne, Associate Professor, The Pennsylvania State University
11. H. K. Wang, Ph.D. student
12. H. H. West, Professor of Civil Engineering, The Pennsylvania State University
13. J. X. Zhou, former Ph.D. student, Research Associate, The Pennsylvania State University

VIII. Activities

Dr. Chen gave five invited talks at the following institutions and conferences:

1. Mathematics Department, Georgetown University, Washington, D.C., October 1985.
   Subject matter: point stabilizer for coupled wave equations
   Individual involved: Professor J. Lagnese

2. Mathematics Department, University of Maryland, College Park, MD November 1985.
   Subject matter: point stabilizer for coupled wave equations
   Individual involved: Professor T. P. Liu

3. Center for Control Science and Dynamical Systems, University of Minnesota, Minneapolis, MN, April 1986.
   Subject matter: theory, designs and applications of point stabilizers
   Individual involved: Professor E. Bruce Lee

4. MIPAC Facility, Mathematics Research Center, University of Wisconsin, Madison, WI, May 1986.
   Subject matter: theory, designs and applications of point stabilizers
   Individual involved: Professor David L. Russell

5. The International Conference on Identification and Control of Distributed Parameter Systems, Vorau, Austria, Organized by the Technical University of Graz, Austria, July 1986.
   Subject matter: pointwise stabilization for coupled quasilinear and linear wave equations
   Individual involved: Professor Karl Kunisch

Dr. Chen also organized a session on distributed parameter systems in the 22nd Annual Meeting of the Society of Engineering Science, held on October 7, 8, and 9, 1985 at The Pennsylvania State University. He presented a 30 minute lecture on the application of the boundary element method to compute optimal controls of certain linear quadratic elliptic problems.

IX. Personnel

The principal investigator of this project is Dr. G. Chen, Associate Professor of Mathematics. During September 1, 1985 - August 31, 1986 he was supported 75% during the summer, and 10% during the academic year.

Assisting the project is Dr. J. Zhou who is holding the postdoctoral appointment. AFOSR has approved $7,332 for his postdoctoral appointment during September 1, 1986 - August 31, 1987. The Department of Mathematics of The Pennsylvania State University has been able to allocate $14,668 to make a total of $22,000 for a full-time appointment for Dr. Zhou. His present title is Research Associate and he carries an 8 credit teaching load during the academic year. He is assisting Dr. Chen in carrying out numerical computations of partial differential equations.

Two Ph.D. students, M. Coleman and J. Zhou shared the research assistantship of allocated funds $6,817 during the past year.