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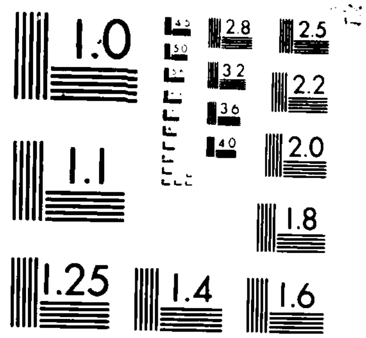

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A MATHEMATICAL MODEL FOR RANGE-GATE PULLOFF

by

B.M. Barry

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
TECHNICAL NOTE 86-22

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ABSTRACT

A mathematical model of the range-gate pulloff electronic countermeasure is developed based on a statistical model for the radar return from a slowly fluctuating point target in the presence of white noise. It is shown that the same model is also appropriate for describing velocity-gate pulloff. An optimization problem is formulated which determines the jammer delay inducing the maximum bias in the range estimation processor of the victim radar. These results are then applied to the specific case in which the transmitted signal is a gaussian pulse. The optimal delay and bias are calculated as functions of the signal-to-jammer power ratio and the pulse width of the transmitted signal.

RÉSUMÉ

Un modèle mathématique de contre-mesure électronique par rétablissement des portes distance est élaboré à partir d'un modèle statistique applicable aux échos radars provenant d'une cible ponctuelle qui fluctue lentement en présence de bruit blanc. Il est démontré que le même modèle peut également décrire le rétablissement des portes vitesse. Un problème d'optimisation est formulé pour déterminer le retard du brouilleur introduisant un biais maximal dans le calcul de la distance par le processeur du radar victime. Les résultats sont ensuite appliqués au cas particulier où le signal transmis est une impulsion gaussienne. Le delai maximum et le biais sont calculés en fonction du rapport des puissances signal/brouilleur et de la durée d'impulsion du signal transmis.

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1.0 INTRODUCTION

DREO has a continuing interest in simulating hypothetical engagements between fighter aircraft and one or more surface-to-air missiles. One of the aims in developing such simulations might be to discover whether overall performance of the aircraft can be enhanced by providing it with an electronic countermeasures (ECM) suite. In particular, two of the countermeasures which are frequently encountered in the open literature [1] are range-gate and velocity-gate pull-off. In this technical note we will propose a simple model which can be used to describe either of these phenomena. We will then use this model to derive some general "rules of thumb" regarding the possible performance and response time of a hypothetical ECM system which uses these countermeasures.

2.0 BASIC RADAR MODEL

We shall first outline a statistical model for a slowly fluctuating point target in the case of white bandpass noise. Since this model is described in Chapter 9 of [2] in some detail, we shall dispense with derivations and formal proofs, and restrict our presentation to a summary of the principal assumptions and results. In particular, we will make the following basic assumptions:

(i) the aircraft can be modelled as a number of reflecting surfaces, such that the return from each of these can be described as the product of an independent complex gaussian random variable and a time-varying (complex-valued) sinusoid. Moreover, we assume that there are sufficiently many reflecting surfaces that the central limit theorem can be applied to the sum of reflected signals;

(ii) the reflection process is linear and frequency-independent;

(iii) the change in time scale of the complex envelope due to non-zero target velocity can be ignored (i.e. only the Doppler shift needs to be considered);

(iv) all other stochastic variations in the radar returns can be modelled as additive white bandpass noise.

Subject to these assumptions, we can derive the following mathematical description of the received waveform in the presence of a moving target:

$$s(t) = \sqrt{2}\ \mathrm{Re}\ [r(t)\ \exp(jw_c t)] \tag{2.1}$$

where r(t) is defined by

$$r(t) = b_s \sqrt{E_s}\ f(t - u_s) \exp(jw_s t) + n(t) \tag{2.2}$$

and the following notation and conventions are adopted:

(1) $f(t)$ is the complex envelope of the transmitted signal

(2) w_c is the carrier frequency

(3) u_s is a delay proportional to target range

(4) w_s is Doppler shift due to relative target velocity

(5) E_s is the transmitted energy

(6) b_s is a zero-mean complex gaussian random variable

(7) $E(b_s b_s^*) = 2\,V_s^2$ (where the value of V_s depends on antenna gains, path losses, target radar cross-section, etc., "*" denotes complex conjugate, and E( ) indicates expected value).

(8) $n(t)$ is an independent zero-mean white complex gaussian random process

and

(9) $E(n(t)\, n^*(t)) = N_o\, \delta\, (t-s)$

where we use the notation

$$n^*(t) = (n(t))^*.$$

Clearly we can work with either (2.1) or (2.2); for convenience, we will usually use (2.2) in the remainder of this paper.

We shall restrict our attention to the case of transmission of a single pulse. For this case, u_s and w_s may be considered as constants; for algebraic simplicity, we can in fact assume without loss of generality that

$$u_s = w_s = 0 \tag{2.3}$$

Hence we can rewrite (2.2) as

$$r(t) = b_s \sqrt{E_s}\, f(t) + n(t) \tag{2.4}$$

According to [1], Range-Gate Pulloff is a self-screening ECM technique for use against pulsed, noncoherent, automatic range-tracking radars. The victim radar's signal is received, amplified, and then retransmitted with a minimum delay, in an attempt to provide a strong beacon signal which "captures" the victim's range-gate. The time delay in the repeated signal is then successively increased on a pulse-by-pulse basis, creating a series of false targets. When the victim range-gate has been moved sufficiently far away from the true target position, the ECM repeater is turned off, and the victim radar will break its range track.

In the context of our radar model (2.4), we can describe the deception pulse by adding an extra term consisting of a delayed and amplified version of the complex envelope f. Since the deception pulse will in general be out of phase with the transmitted signal, we will also include a random phase factor. Indeed, without increasing the computational complexity, we can generalize the random phase component to be a complex gaussian random variable. Hence we obtain.

$$r(t) = b_s \sqrt{E_s}\, f(t) + b_d \sqrt{E_d}\, f(t-T) + n(t) \tag{2.5}$$

where T is the delay in transmitting the deception pulse, and b_d is an independent, zero-mean complex gaussian random variable such that

$$E(b_d b_d^*) = 2\, V_d^2 \tag{2.6}$$

and

$$E(b_d b_d) = E(b_s b_s) = E(b_s b_d^*) = E(b_d b_s^*) = 0 \tag{2.7}$$

We can model the action of the radar receiver in estimating the true value of the return signal as a time-invariant linear filter with impulse response h(t). For the moment, we will only assume that h is an $L^2(R)$ function (i.e. is square-integratable). We will be more specific as to other properties of h later. Denoting the receiver output as $\tilde{r}(t)$, we obtain from (2.5)

$$\tilde{r}(t) = b_s \sqrt{E_s}\, \tilde{f}(t) + b_d \sqrt{E_d}\, \tilde{f}(t-T) + \tilde{n}(t) \tag{2.8}$$

where

$$\tilde{f}(t) = (f*h)(t) = \int_R f(u)\, h(t-u)\, du$$

$$\tilde{n}(t) = (n*h)(t)$$

Whether "*" denotes complex conjugate or convolution will usually be obvious.

Now, we define the expected power envelope of the filtered return signal to be

$$\tilde{P}(t;T) = E(\tilde{r}(t)\,\tilde{r}^*(t)) = (b_s\sqrt{E_s}\,\tilde{f}(t) + b_d\sqrt{E_d}\,\tilde{f}(t-T) + \tilde{n}(t))$$
$$\times\ (b_s\sqrt{E_s}\,\tilde{f}(t) + b_d\sqrt{E_d}\,\tilde{f}(t-T) + \tilde{n}(t))^*$$
$$= 2V_s^2\ \ E_s\,\tilde{f}(t)\tilde{f}^*(t) + 2V_d^2\ \ E_d\,\tilde{f}(t-T)\tilde{f}^*(t-T)$$
$$+\ E(\tilde{n}(t)\tilde{n}^*(t)) \tag{2.9}$$

This last follows from (2.6), (2.7), and assumption (7) above. We now wish to evaluate the last term in (2.9):

$$E(n(t)n^*(t)) = E\,[\int_R n(u)\,h(t-u)\,du][\int_R n(s)\,h(t-s)\,ds]^*$$
$$= \int_R\int_R h(t-u)\,h^*(t-s)\,E(n(u)n^*(s))\,duds$$
$$= \int_R h(t-s)\int_R N_o\,\delta(u-s)\,h^*(t-u)\,du\ \ ds \quad \text{(assumption (9))}$$
$$= N_o\int_R h(t-s)\,h^*(t-s)\,ds$$
$$= N_o\int_R h(s)\,h^*(s)\,ds \quad \text{(translation invariance of Lebesque integral)}$$
$$= N_o\,||\ h\ ||_2^2 \tag{2.10}$$

Substituting (2.10) into (2.9), we obtain

$$\tilde{P}(t;T) = 2V_s^2\,E_s\,\tilde{f}(t)\tilde{f}^*(t) + 2V_d^2\,E_d\,\tilde{f}(t-T)\tilde{f}^*(t-T) + N_o\,||\ h\ ||_2^2 \tag{2.11}$$

As is pointed out in [3], we can model the estimation of range by the tracking radar as a simple optimization problem. In effect, we claim that those values of t at which P(t;T) attains local maximal will correspond to estimates of target ranges. This models leading edge tracking in the sense that it analyzes centroid biasing caused by the addition of the deception pulse. We note in passing that this algorithm would not accurately model tracking gates with sophisticated statistical signal processing or threshold as well as differencing logic.

We have not thus far committed ourselves to a choice of the receiver impulse response h. If we wished to extend our analysis to include a more complex estimation model, then a matched filter or a filter based on estimates of the statistical properties of the interfering signals might be appropriate. However, given that we are using the simple estimation model described above, we will assume that the impulse response of the radar receiver is (nearly) a delta function. In effect, this corresponds to a receiver which estimates the target range based only on the return signal without doing any signal processing.

Taking h as a delta function appears to conflict with our earlier assumption that $h \in L^2(R)$. However, we will now show that there is a sequence of $L^2(R)$ functions, denoted $d_h(t)$, which "look like" delta functions for sufficiently small values of h.

Let $d_h(t)$ be defined by

$$d_h(t) = \frac{1}{h} a\left(\frac{t}{h}\right) \tag{2.12}$$

where a(t) is any non-negative bounded continuous function with compact support contained in the interval [-1,1]. Assume that the total mass of a is 1. Then following the same style of argument as is used in (4), Chapter 6, we can show that for any function $f \in L^2(R)$, $(d_h * f)$ converges to f both pointwise and uniformly in $L^2(R)$ as $h \to 0$. For consider:

$$|(d_h * f)(t) - f(t)|^2 = \left| \int_R \frac{1}{h} a\left(\frac{u}{h}\right) f(t-u)\, du - \int_R f(t)\, a(u)\, du \right|^2$$

$$= \left| \int_R a(s)\, f(t-hs)\, ds - \int_R f(t)\, a(s)\, ds \right|^2$$

(using the change of variable s = u/h)

$$\leq \left[\int_R |f(t-hs) - f(t)|\, a(s)\, ds \right]^2 \tag{2.13}$$

$$\leq \left[\int_R |f(t-hs) - f(t)|^2\, a(s)\, ds \right] \left[\int_R a(s)\, ds \right]$$

(by the Schwarz inequality)

$$\leq \|a\|_\infty \left[\int_R |f(t-hs) - f(t)|^2\, ds \right]$$

$$\leq \|a\|_\infty\, w^2(f,h) \tag{2.14}$$

where $w(f,h)$ is the oscillation of $f \in L^2(R)$, defined by

$$w(f,h) = \sup_{|s| \leq h} \left(\int_R |f(t-s) - f(s)|^2 \, ds \right)^{1/2}$$

and $\|a\|_\infty$ denotes the maximum value of a. We note from [4] that $w(f,h) \to 0$ as $h \to 0$. This proves that $(d_h * f)$ converges pointwise. Now, using (2.13) and integrating both sides we obtain

$$\begin{aligned} \int_R |(d_h * f)(t) - f(t)|^2 \, dt &= \int_R \int_R |f(t-hs) - f(t)|^2 \, a(s) \, dtds \\ &\leq w^2(f,h) \int_R a(s) \, ds \\ &\qquad \text{(using definition of oscillation)} \\ &\leq w^2(f,h) \end{aligned} \tag{2.15}$$

which proves convergence in $L^2(R)$.

We are now in a position to state the problem we wish to consider: given the model as described, choose the value of the deception delay T which achieves the maximum range deception while maintaining (statistically) non-resolvable signal and deception pulses. Intuitively this problem will admit one or more solutions. Since we can assume that the signal envelope f(t) has been designed to have only one maximum, very small values of T will still generate only one maximum. On the other hand, very large values of T will certainly result in a power envelope which resolves the return signal and deception pulse as two distinct targets. Clearly, there must be some "happy medium".

According to our model, the estimated target range(s) will correspond to the maximum or maximal of the function P(t;T) defined in (2.11). Since the last term in (2.11) is constant (depending only on the choice of the impulse response of the receiver filter and the spectral height of the noise), we can effectively disregard it. Moreover, we showed above that we can choose the impulse response of our receiver filter so as to make the function f(t) as close as we like (pointwise) to the function f(t), by choosing the parameter h sufficiently small. This implies that the maximum of the function P(t;T) defined by

$$P(t;T) = 2V_s^2 \, E_s \, f(t)f^*(t) + 2V_d^2 \, E_d \, f(t-T)f^*(t-T) \tag{2.16}$$

will also maximize $\ddot{P}(t;T)$. It is this function we will work with in section 4.0.

3.0 VELOCITY-GATE PULLOFF

As indicated in the introduction, we wish to extend our analysis to include the velocity-gate pulloff ECM as well. As described in (1), this is another self-screening ECM technique. The signal from the victim radar is received, amplified coherently, and retransmitted to provide a strong beacon which captures the velocity-gate of the radar. The Doppler frequency of the deception signal is then moved away from the true target Doppler frequency at a rate that does not exceed the victim radar's tracking capability. At some point the ECM repeater is turned off, causing the radar to break track. It should be noted that if velocity-gate pulloff is used in conjunction with range-gate pulloff, the rate at which the false range is changed must equal the false velocity. Although it may be a subject of future research, the analysis presented in this paper cannot be applied to simultaneous use of the two countermeasures.

Referring to the basic equations (2.2) and (2.4) of the last section, we see that a possible model for velocity-gate deception is given by

$$r(t) = b_s \sqrt{E_s}\, f(t) + b_d \sqrt{E_d}\, f(t) \exp(jw_d t) + n(t) \tag{3.1}$$

under the same assumptions as were posed in the last section.

Now, we will denote the Fourier transform of a function f(t) by F(jw) or by $\mathcal{F}(f)$. Recalling that

$$\mathcal{F}(f(t) \exp(jw_d t)) = F(jw - jw_d)$$

we obtain (formally) from (3.1)

$$R(jw) = b_s \sqrt{E_s}\, F(jw) + b_d \sqrt{E_d}\, F(jw - jw_d) + N(jw) \tag{3.2}$$

Now, from (3.1) and Plancherel's Theorem, the energy in the return signal is

$$\int_R E(r(t)r^*(t))\, dt = \int_R E(R(jw)R^*(jw))\, dw \tag{3.3}$$

Of course, the left hand side of (3.3) will not converge; however, using an argument similar to that of the previous section, we can overcome this technical difficulty. Hence, if we define

$$S(w) = E(R(jw)R^*(jw))$$
$$= 2V_s^2 E_s F(jw)F^*(jw) + 2V_d^2 E_d F(j(w-w_d))F^*(j(w-w_d)) + N_o \tag{3.4}$$

then we can interpret $S(w)$ as the expected spectral power density of the return signal. Following the same line as was taken in the last section, we now claim that those values of w at which $S(w)$ attains local maximal will correspond to estimates of target velocity. Since the constant term in (3.4) due to the spectral height of the additive noise will not affect the position of any maximum values, we can effectively ignore it. Consequently, the form of (3.4) is identical to that of (2.16).

In the next section we will show how our model can be used to determine the optimal bias for the range-gate problem, i.e. equation (2.16). However, as we have shown, the results may equally be applied to the velocity-gate problem, although we shall not explicitly do so in this paper.

4.0 DETERMINING OPTIMAL BIAS

In order to illustrate the utility of some of these ideas, let us assume that the transmitted signal is a gaussian pulse, i.e.

$$f(t) = \left(\frac{1}{\pi T_s^2}\right)^{1/4} \exp\left(\frac{-t^2}{2T_s^2}\right) \tag{4.1}$$

Thus from (2.16) we obtain

$$P(t;T) = 2V_s^2 E_s \left(\frac{1}{\pi T_s^2}\right)^{1/2} \exp\left(\frac{-t^2}{T_s^2}\right) + 2V_d^2 E_d \left(\frac{1}{\pi T_s^2}\right)^{1/2} \exp\left(-\frac{(t-T)^2}{T_s^2}\right)$$
$$= \tilde{E}_s \exp\left(\frac{-t^2}{T_s^2}\right) + \tilde{E}_d \exp\left(-\frac{(t-T)^2}{T_s^2}\right) \tag{4.2}$$

Now, for each value of T, the value(s) of t which maximize $P(t;T)$ must satisfy

$$\frac{\partial}{\partial t} P(t;T) = \frac{-2t}{T_s^2} \tilde{E}_s \exp\left(\frac{-t^2}{T_s^2}\right) - 2\frac{(t-T)}{T_s^2} \tilde{E}_d \exp\left(-\frac{(t-T)^2}{T_s^2}\right) \tag{4.3}$$
$$= 0$$

Simplifying (4.3) we obtain

$$\frac{t}{T-t} = \exp\left(\frac{t^2}{T_s^2} - \frac{(t-T)^2}{T_s^2}\right) = \exp\left\{\frac{1}{T_s^2}(2tT - T^2)\right\} \tag{4.4}$$

where

$$(\tilde{E}_s/\tilde{E}_d) = \alpha$$

We note that since the right-hand side of (4.4) is always strictly positive, and since we can assume without loss of generality that $t > 0$, we must have

$$T-t > 0 \quad \text{or} \quad t < T \tag{4.5}$$

Let us designate the value of t at which a local maximum occurs by t*. Then in order to guarantee that t* satisfying (4.4) <u>is</u> a local maximum (rather than a local minimum), we impose the further necessary condition

$$\frac{\partial^2 P}{\partial t^2}(t;T)\Big|_{t=t^*} < 0$$

$$\frac{2}{T_s^2}\exp\left(\frac{-t^{*2}}{T_s^2}\right)\left[-\tilde{E}_s + 2\tilde{E}_s\frac{t^{*2}}{T_s^2} - \tilde{E}_d\exp\left\{\frac{t^{*2}}{T_s^2} - \frac{(t^*-T)^2}{T_s^2}\right\}\right. \tag{4.6}$$

$$\left. + 2\tilde{E}_d\frac{(t^*-T)^2}{T_s^2}\exp\left\{\frac{t^{*2}}{T_s^2} - \frac{(t^*-T)^2}{T_s^2}\right\}\right] < 0$$

Substituting (4.4) into (4.6) and simplying, we obtain

$$\frac{2\tilde{E}_s}{T_s^2}\exp\left(\frac{-t^{*2}}{T_s^2}\right)\left[-1 + \frac{2t^{*2}}{T_s^2}\ \frac{-t^*}{T-t^*} + \frac{2(T-t^*)t^*}{T_s^2}\right] < 0$$

$$\frac{2E_s}{T_s^2}\exp\left(\frac{-t^{*2}}{T_s^2}\right)\left[\frac{-T}{T-t^*} + \frac{2Tt^*}{T_s^2}\right] < 0 \tag{4.7}$$

Since the leading factor in (4.7) is already strictly positive, it is clear that the sign of the left-hand side in (4.7) depends only on the expression in square brackets. Moreover, since T is a delay, we can also assume that $T > 0$, resulting in the following condition which is equivalent to (4.7):

$$\frac{-1}{T-t^*} + \frac{2t^*}{T_s^2} < 0 \tag{4.8}$$

$$2t^{*2} - 2t^{*}T + T_s^2 > 0 \qquad (4.9)$$

where we have used (4.5) in (4.8) and simplified. We note that for $(T/T_s) < \sqrt{2}$, then (4.9) will be true for all $t^{*}\varepsilon[0,T]$. Hence, for all choices of T such that $(T/T_s) < \sqrt{2}$, we are quaranteed there will be a single local maximum, hence t* will designate a global maximum. We can interpret this as meaning that only one target will be resolved by the victim radar. However, it is unclear how much larger T can be chosen such that the radar will continue to resolve only one target. This obviously will depend on the actual value(s) of t* which solve (4.4), which will in turn depend on the "power ratio" $(\tilde{E}_s/\tilde{E}_d)$, and on the "pulse width" T_s.

Now, in order to resolve two peaks, there must be some value of T which yields a "flat spot", i.e., a pair (t*,T) such that

$$\frac{\partial P(t^{*},T)}{\partial t} = \frac{\partial^2 P(t^{*},T)}{\partial t^2} = 0$$

From (4.4) and (4.9), this gives the pair of equations,

$$2t^{*2} - 2t^{*}T + T_s^2 = 0 \qquad (4.10)$$

and

$$\alpha \frac{t^{*}}{T-t^{*}} = \exp\left\{\frac{1}{T_s^2} \quad (2t^{*}T - T^2)\right\} \qquad (4.11)$$

To simplify the solution of this pair of equations, we introduce a "trick" based on (4.5). Since we know that $t^{*} < T$, there is a number F (possibly depending on T) such that

$$0 < F < 1 \quad \text{and} \quad FT = t^{*} \qquad (4.12)$$

Substituting (4.12) into (4.10) we obtain

$$2F^2T^2 - 2FT^2 + T_s^2 = 0$$

$$F(1-F) = \frac{T_s^2}{2T^2} \qquad (4.13)$$

Substituting (4.12) into (4.11) yields

$$\alpha \frac{F}{(1-F)} = \exp\{(2T^2/T_s^2)(F - \frac{1}{2})\} \tag{4.14}$$

Now combining (4.13) and (4.14) gives

$$\alpha = \frac{1-F}{F} \exp \frac{(F-\frac{1}{2})}{F(1-F)} = g_1(F) \tag{4.15}$$

Hence for a given value of α, we can solve for $F = g_1^{-1}(\alpha)$ in (4.15), and then use (4.13) to find the value of T, say $T_r(\alpha)$, at which targets will be resolved. Figure 1 shows a graph of g_1; we note that $g_1^{-1}(\alpha)$ is defined and one-to-one for all $\alpha > 0$.

However, what we are seeking is not necessarily $T_r(\alpha)$, but rather the value of T (less than or equal to $T_r(\alpha)$) which gives rise to a single target with the largest "bias" value t*. In order to find a necessary condition which defines these values of T and t*, let us differentiate (4.4) implicitly with respect to T:

$$\alpha \frac{t'}{T-t^*} - \frac{\alpha t^*}{(T-t^*)^2}(1-t') = \frac{1}{T_s^2}(2t'T + 2t^* - 2T)\exp\{\frac{1}{T_s^2}(2t^*T - T^2)\} \tag{4.16}$$

where $t' = \frac{d}{dT} t^*(T)$. Substituting (4.4) into (4.16) yields

$$\frac{\alpha\ t'T - \alpha t^*}{(T-t^*)^2} = \frac{2}{T_s^2}\{t'T + t^* - T\}\{\frac{\alpha\ t^*}{T-t^*}\}$$

$$t'\{T - \frac{2Tt^*}{T_s^2}(T-t^*)\} = t^*\{1 - \frac{2}{T_s^2}(T-t^*)^2\} \tag{4.17}$$

Hence, we deduce that

$$\frac{d}{dT} t^*(T) = \frac{t^*\{1 - (2/T_s^2)(T-t^*)^2\}}{[T - \frac{2}{T_s^2} T\ t^*(T-t^*)]} \tag{4.18}$$

To find a (local) maximum, we let $\frac{d}{dT} t^*(T) = 0$ in (4.18), obtaining

$$t^* = 0 \quad \text{or} \quad 1 - (2/T_s^2)(T-t^*)^2 = 0$$

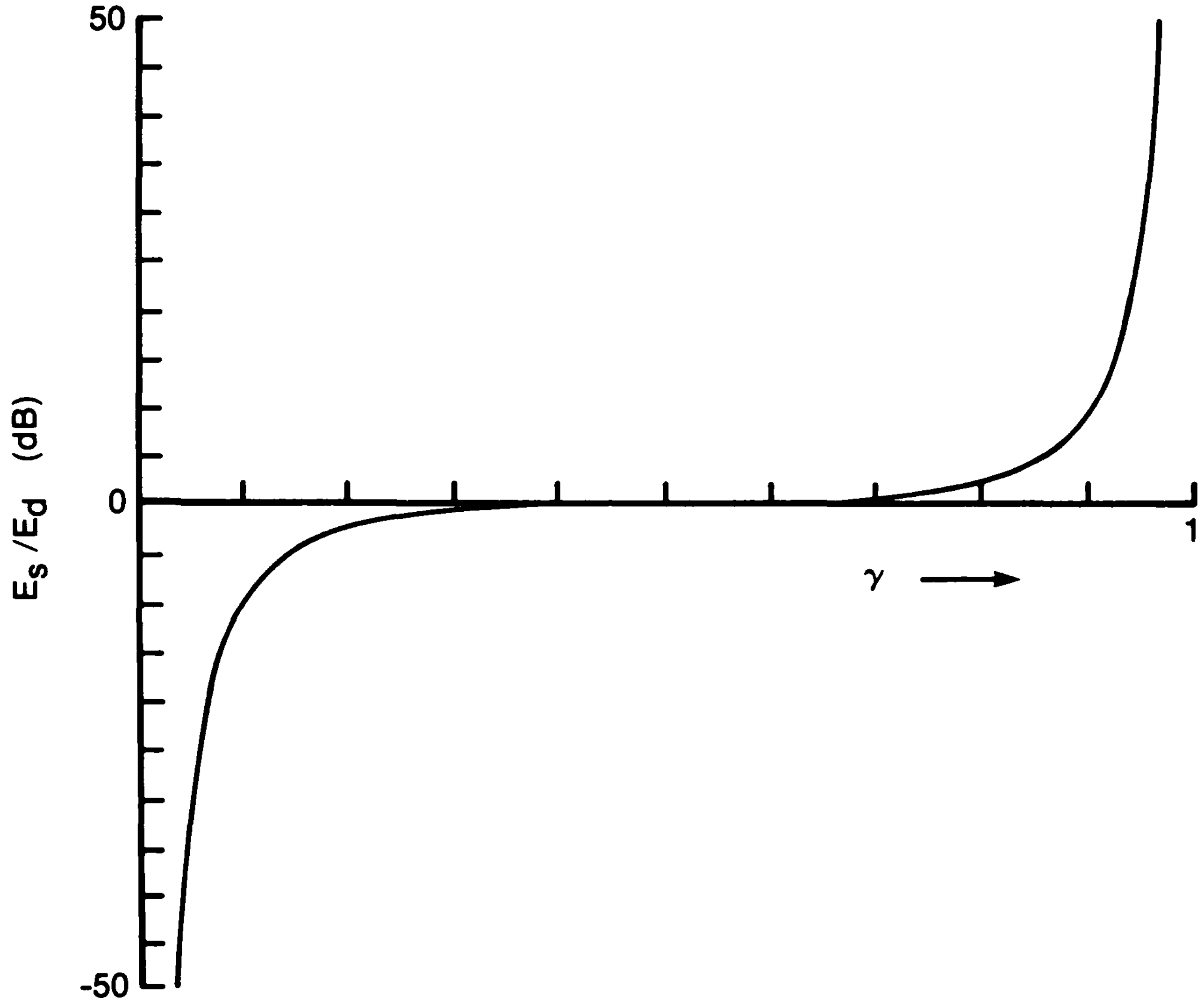

FIGURE 1: GRAPH OF FUNCTION g_1

Since we have assumed that $t^* > 0$, we are left with

$$T - t^* = T_s/\sqrt{2} \tag{4.19}$$

Now, combining (4.19) with the basic equation (4.4) yields

$$\alpha \sqrt{2} \frac{t^*}{T_s} = \exp\{(t^*/T_s)^2 - \frac{1}{2}\}$$

$$\alpha = \frac{1}{\sqrt{2}} \frac{(T_s)}{t^*} \exp\{(t^*/T_s)^2 - \frac{1}{2}\} = g_2\left(\frac{t^*}{T_s}\right) \tag{4.20}$$

For any value of $\alpha \geq 1$, there are two numbers t^* such that $t^* = g_2^{-1}(\alpha)$, as shown in Figure 2. For each of these (4.19) can be used to calculate corresponding values of T, say $T^*(\alpha)$ and $T_*(\alpha)$, such that $T_* < T^*$.

However, as shown in Figure 3, $T^*(\alpha)$ is actually greater than $T_r(\alpha)$, i.e., if $T = T^*(\alpha)$ then two targets will be resolved. Hence we only use that portion of g_2 to the left of the dotted line in Figure 2 to find t^*.

If $\alpha < 1$, then by inspection there is no solution for (4.20), and hence no value of T such that $\frac{d}{dT} t^*(T) = 0$. Referring to (4.18), if we assume that $T < \frac{1}{\sqrt{2}} T_s$, we have

$$t^*\{1 - \left(\frac{2}{T_s^2}\right)(T-t^*)^2\} > t^*\{1 - \left(\frac{2}{T_s^2}\right)T^2\} > 0 \tag{4.21}$$

and

$$T - \left(\frac{2}{T_s^2}\right) T\, t^*(T-t^*) > T\,\{1 - \left(\frac{2}{T_s^2}\right) T^2\} > 0 \tag{4.22}$$

recalling that $0 < t^* < T$. Hence,

$$\frac{d}{dT} t^*(T) > 0 \text{ for } T \in [0, \frac{T_s}{\sqrt{2}}] \tag{4.23}$$

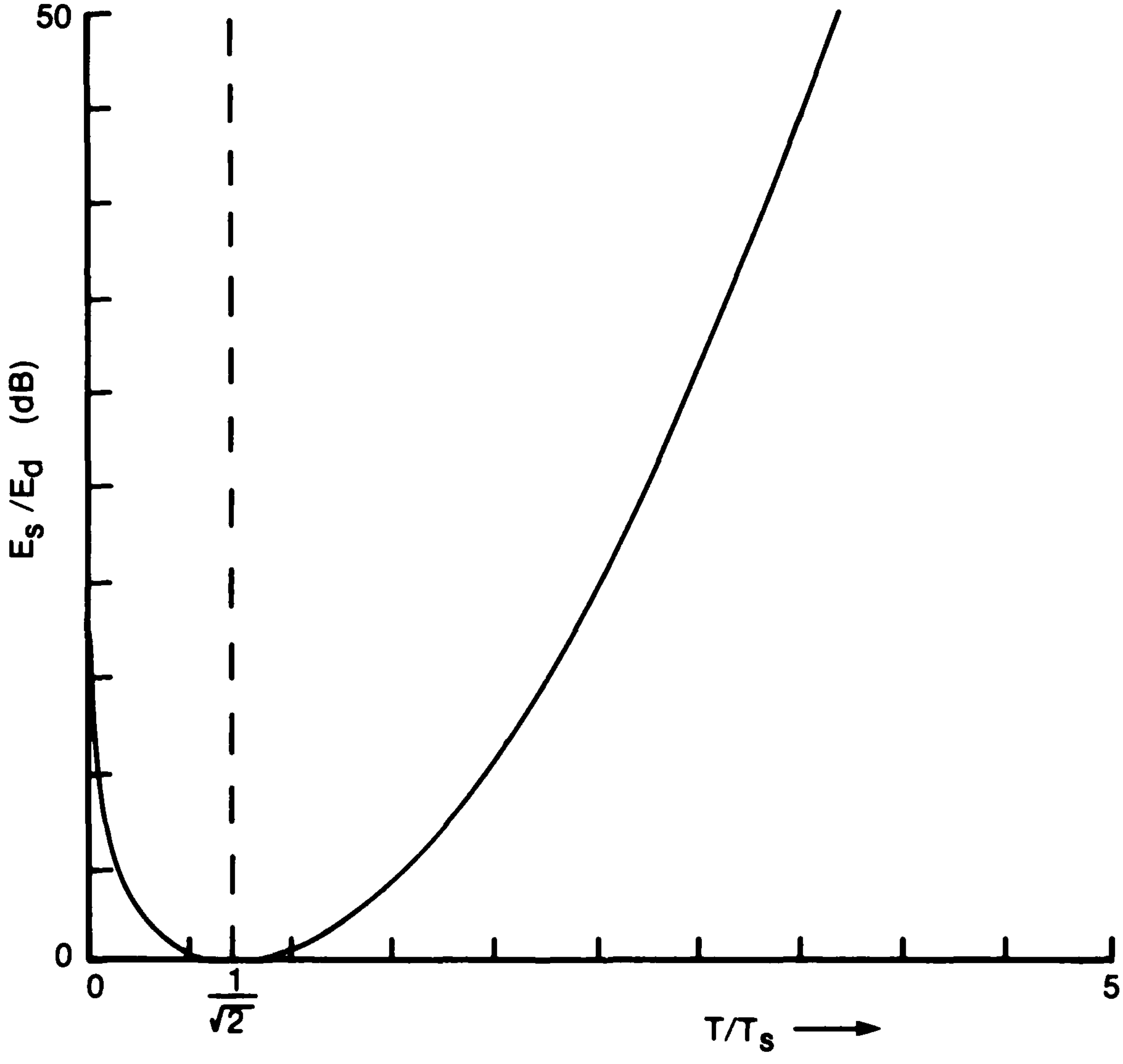

FIGURE 2: GRAPH OF FUNCTION g_2

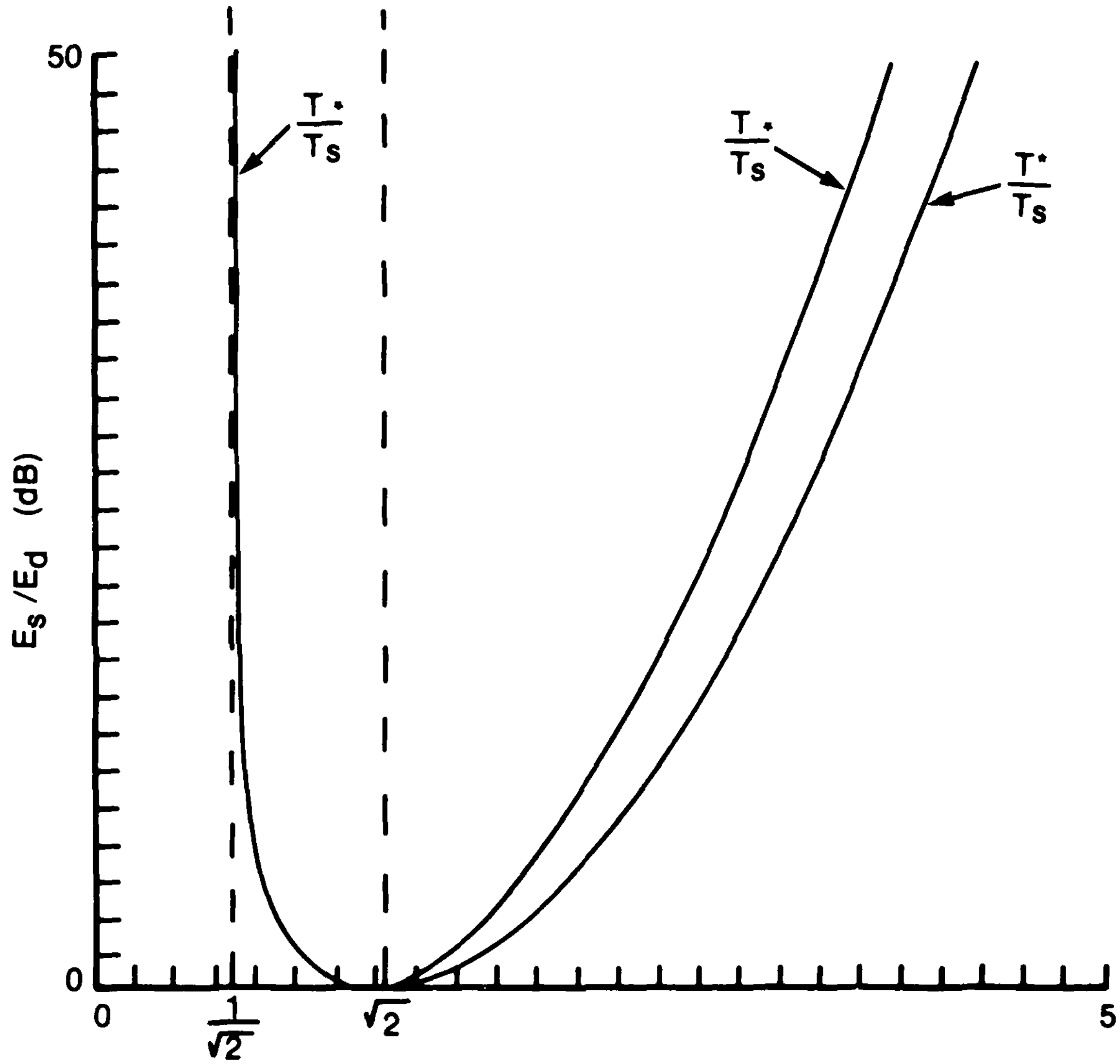

FIGURE 3: COMPARISON BETWEEN T_r, T^* AND $T*$

Now if $\alpha < 1$, since $\frac{d}{dT} t^*(T) \neq 0$, we can in fact deduce that

$$\frac{d}{dT} t^*(T) \quad 0 \text{ for } T \in [0, T_r(\alpha)], \; \alpha < 1 \tag{4.24}$$

and hence the value of T which maximizes $t^*(T)$ is $T_r(\alpha)$.

We will make one more observation before presenting a complete solution. Recall that $T_r(\alpha)$ for the case $\alpha < 1$ can be found by solving (4.13) and (4.15), that is,

$$(1- \;) = 1 \; (T_s/T_r)^2 \tag{4.25}$$

and

$$= \exp \frac{-1}{1-}$$

$$1 \; (1- \;) = \exp \frac{(1-)}{(1-)} \tag{4.26}$$

Now, let $\beta = \underline{1}$ and $s = 1-$ in (4.25) and (4.26), to obtain the pair of equations

$$(1-s)s = 1 \; (T_s/T_r)^2 \tag{4.27}$$

and

$$\frac{}{1-s} \quad \exp \frac{(s-1)}{s(1-s)}$$

$$= g_1(s) \tag{4.28}$$

But $\quad 1$, hence (4.27) and (4.28) are solved by $T_r(\alpha) = T_r(\beta)$ $= T_r(1/\alpha)$ from our previous work.

Now, in order to find the bias t^* induced by the delay $T_r(\alpha)$, we need to solve equation (4.14) with $T = T_r(\alpha)$. We note that there must be two values of $\mathcal{F}$ which solve (4.14), i.e. $\mathcal{F} = g_1^{-1}(\alpha)$, which corresponds to the "flat spot" which defines $T_r(\alpha)$, and the value of $\mathcal{F}$ which corresponds to the maximum bias t^*. Define the function g_3 by

$$\alpha = \frac{1 - \mathcal{F}}{\mathcal{F}} \exp 2(T_r/T_s)^2 \left(\mathcal{F} - \tfrac{1}{2}\right) = g_3(T_r;\mathcal{F}) \tag{4.29}$$

and define $g_4(\alpha)$ by

$$g_4(\alpha) = \{\mathcal{F} \mid \alpha = g_3(T_r(\alpha);\mathcal{F}),\ \alpha \neq g_1(\mathcal{F})\} \tag{4.30}$$

Then g_4 is defined and single-valued for all $\alpha < 1$. Figure 4 shows a graph of g_3 for representative values of (T_r/T_s) superimposed on a graph of g_1. Figure 5 shows a graph of g_4.

Hence from (4.12) we conclude

$$t^* = g_4(\alpha)\, T_r(\alpha) = g_4(\alpha)\, T_r(1/\alpha) \tag{4.31}$$

We have now obtained a complete solution for the problem of choosing a delay so as to maximize the bias induced in a victim radar. The solution divides naturally into two cases:

(i) $\alpha = E_s/E_d \leq 1$

Optimal bias $= t_{opt} = T_s\, g_2^{-1}(\alpha)$

Optimal delay $= T_{opt} = t_{opt} + T_s/\sqrt{2}$

$$= T_s \left(g_2^{-1}(\alpha) + \frac{1}{\sqrt{2}}\right) \tag{4.32}$$

(ii) $\alpha = E_s/E_d > 1$

Optimal delay $= T_{opt} = \dfrac{1}{\sqrt{2}}\, T_s \left[\dfrac{1}{\mathcal{F}(1-\mathcal{F})}\right]^{1/2}$

where $\mathcal{F} = g_1^{-1}(\alpha)$ or $\mathcal{F} = g_1^{-1}(1/\alpha)$

Optimal bias $= t_{opt} = g_4(\alpha)\, T_{opt}$

Graphs of the solutions (T_{opt}/T_s) and (t_{opt}/T_s) are shown in Figure 6.

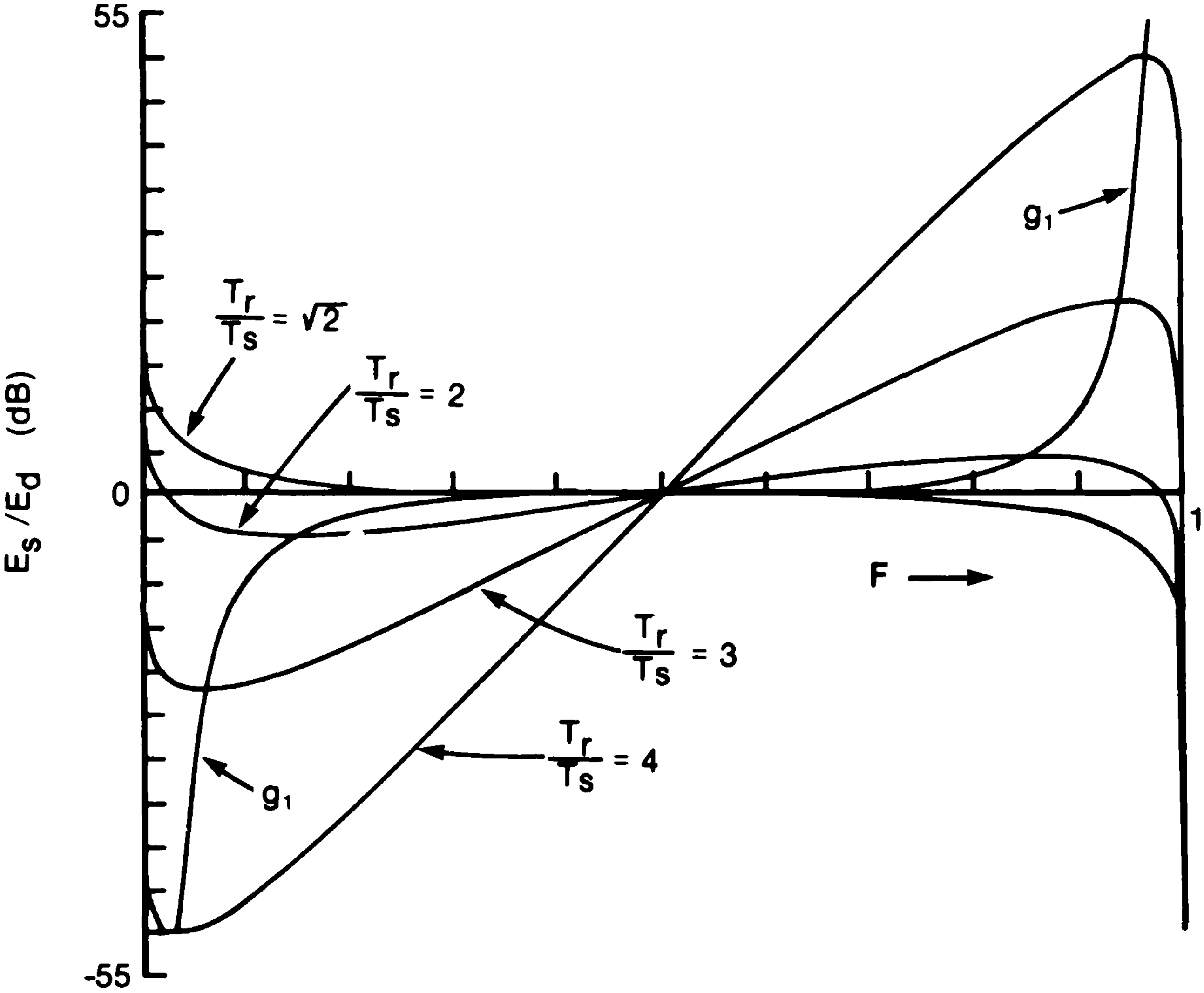

FIGURE 4: GRAPH OF g_4

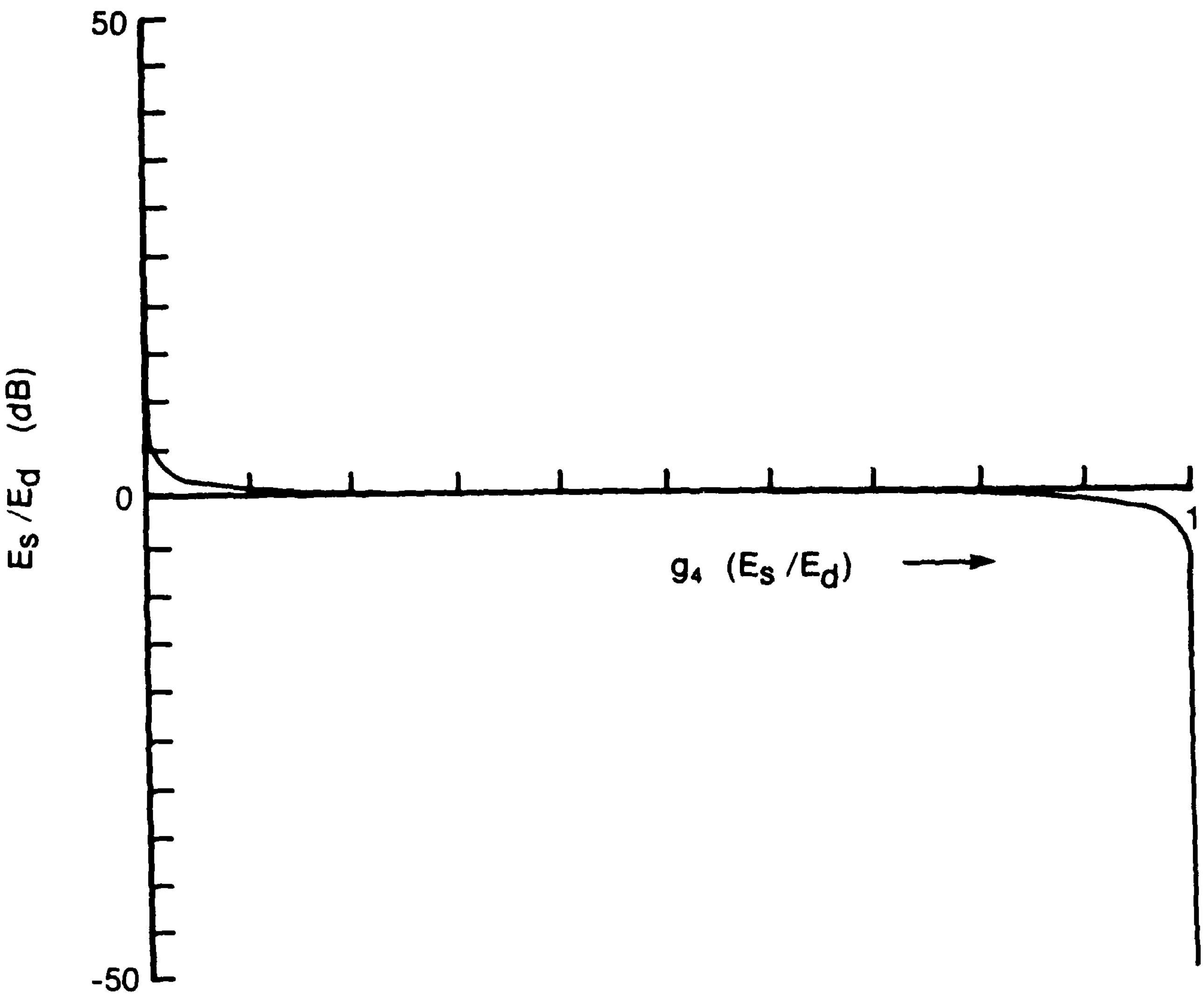

FIGURE 5: GRAPH OF g_3 FOR SEVERAL VALUES OF Tr/T_s
SUPERIMPOSED ON GRAPH OF g_1

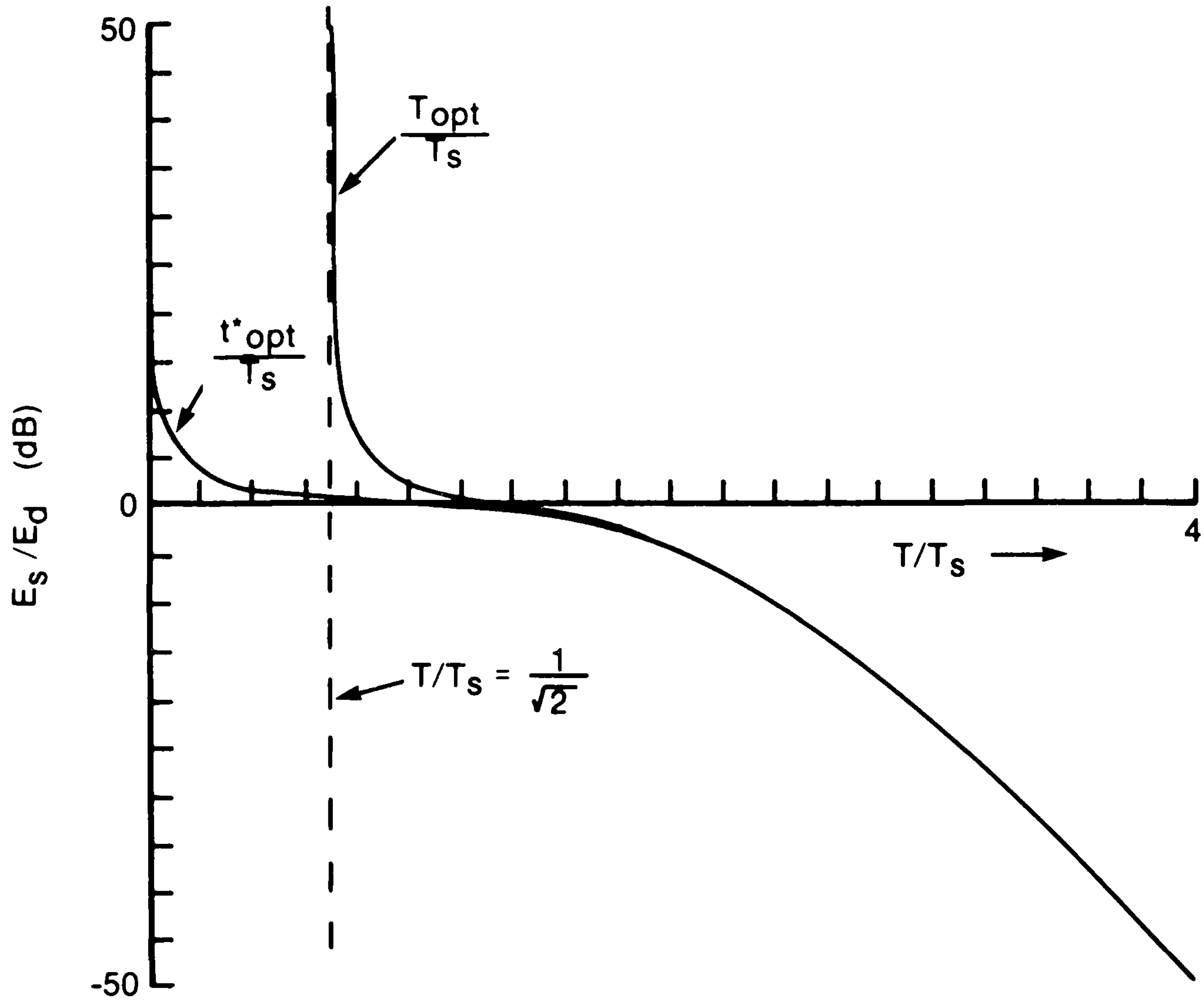

FIGURE 6: GRAPH OF SOLUTIONS T_{opt} AND t_{opt}

6.0 CONCLUSIONS

We set out to derive some basic "rules of thumb" regarding the possible performance and response time of a hypothetical ECM system which used the range-gate (or velocity-gate) pulloff countermeasure. Our analysis has been based on a standard statistical model for the radar return from a slowly fluctuating point target in the presence of white noise. Using our model, which also included a deception signal representing the range-gate pulloff ECM, we derived an expression for the expected power envelope of the filtered combined signal. We saw that target locations could be identified as local maximum of this power envelope. Moreover, we showed that the same basic model was also appropriate for the analysis of velocity-gate pulloff.

We then explored our model in more detail for the specific case in which the transmitted signal could be represented by a gaussian pulse. We saw that small deception delays resulted in biased estimates of target position; however, as the delay was increased, a threshold was reached beyond which the victim radar could resolve the jammer as a separate target. Now, in order for the countermeasure to be successful, one must be able to move the apparent target position sufficiently far from the true position that the victim radar cannot keep both the true and biased positions within its range-gate. As one might have expected, the determining factors as to whether this can be accomplished appear to be the pulse width of the transmitted signal, and the signal-to-jammer power ratio (calculated at the receiver).

A number of possibilities exist for extending or enhancing this model. For example, at the moment the additive deception signal is restricted to an amplified and delayed version of the signal transmitted by the radar. One could allow a more general type of deception, such as one which included variations in the pulse width. Indeed, one could go so far as to consider an adaptive jammer which constructs a jamming signal (based on the measured transmitted waveform) designed to induce the maximum deception in the victim radar. Needless to say, we suspect this problem would prove to be very difficult. Another direction might be to attempt a simultaneous solution of the range-gate and velocity-gate problems; reformulating the model in terms of the ambiguity function would be a possible approach. One might also wish to add a more realistic description of the dynamics of the victim radar, or include a model of the observer needed by the ECM system to measure the parameters of the transmitted signal.

We caution the reader once again that this paper is aimed primarily at those who want to do a "top level" analysis of the effectiveness of the range-gate pulloff ECM, particularly simulation designers. We should also point out that there are some obvious limitations which must be considered before applying these results to specific problems. For example, this model would certainly not be an appropriate one for describing a tracking gate which used sophisticated signal processing. Also, it is important to bear in mind that this model describes signals by their statistical averages; consequently, one should not attempt to draw conclusions about the pulse-by-pulse behaviour of such systems from this analysis. However, used carefully in the context for which it was intended, we believe that our model can provide a useful tool for the EW systems analyst.

7.0 REFERENCES

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APPENDIX A

COMPUTER PROGRAMS

The numerical results used in this paper (which appear primarily in the graphs) were generated by programs written in the APL programming language using a DEC-2020 minicomputer. In this appendix we will briefly describe each of these programs; these descriptions will be followed by a comprehensive listing.

(a) CHANGE: CHANGE accepts a boolean vector (i.e. one consisting of 0's and 1's), which we will denote by $(a_1, a_2, \ldots a_n)$, as an argument. It returns a vector of indices i corresponding to those a_i which satisfy $a_i \neq a_{i-1}$.

(b) INV: INV is an infix function which takes as its first argument the name of the function to be inverted. The second argument is a three element array composed of the value at which the inverse function is to be evaluated followed by the interval over which the inverse is to be calculated. That is,

$$f \text{ INV } (x,a,b) = f^{-1}(x) \quad \text{for} \quad x \in f([a,b])$$

$$= \phi \qquad \text{for} \quad x \notin f([a,b])$$

Note that $f^{-1}(x)$ may be set-valued. The basic algorithm used to compute the inverse function is quite simple. The interval $[a,b]$ is subdivided into a number of intervals $[a_i, b_i]$, and the program checks to see if $x \in f([a_i, b_i])$. If it is, the process is repeated until the size of the subinterval(s) is less than a pre-selected tolerance.

(c) G1 : G1 is the function defined by (4.15), i.e.

$$g_1(\text{[illegible]}) = \frac{1-\text{[illegible]}}{\text{[illegible]}} \exp \frac{(\text{[illegible]} - \text{[illegible]})}{\text{[illegible]}\,1-\text{[illegible]}}$$

Since g_1 has asymptotes at 0 and 1, traps are set to catch these values.

(d) G2: G2 is the function defined by (4.20), i.e.

$$g_2(x) = \frac{1}{\sqrt{2}\,x} \exp\left(x^2 - \frac{1}{2}\right)$$

Since g_2 has an asymptote at 0, a trap is set to catch small values of x.

(e) G3: G3 is an infix representation of the function defined by (4.29), i.e.

$$(T_r/T_s) \text{ G3 } F = g_3(T_r; F)$$

where

$$g_3(T_r; F) = \frac{1-F}{F} \exp\{2(T_r/T_s)^2 (F - \frac{1}{2})\}$$

Note that the asymptote at 0 is trapped.

(f) G4: G4 is the function defined by (4.30), i.e.

$$g_4(\alpha) = \{F \mid \alpha = g_3(T_r(\alpha); F), \alpha \neq g_1(F)\}$$

G4 can accept vector-valued arguments; however, since G4 works by invoking INV, the input vector is disassembled, the inverse is calculated for each scalar that results, and the answer is assembled into an output vector. Note that any function which invokes INV is accompanied by a SUSPEND block which returns control to the user if INV returns the empty set or multiple values. Often this happens because of numerical considerations even when the inverse function exists and is single-valued. Rather than provide complicated code to deal with such problems, it is easier when using an interpreter like APL to return control to the user and let him sort out the difficulty.

(g) SOLUTION: SOLUTION finds the values of T_{opt} and t_{opt} (as defined by (4.32)) which correspond to various values of $(\tilde{E}_s/\grave{E}_d)$ ranging from -50 to 50 dB. LOOPA solves part (i) of (4.32), and LOOPB solves part (ii). Note the presence of the SUSPEND block once again since INV is invoked frequently.

```
     ∇  INDEX←CHANGE MASK;I;J;SAVEMASK;FLAG
[1]    ⍝
[2]    ⍝ MASK IS A VECTOR OF 0'S AND 1'S
[3]    ⍝ CHANGE FINDS INDICES CONTAINING VALUES WHICH
[4]    ⍝ DIFFER FROM PRECEDING VALUE
[5]    ⍝
[6]     INDEX←⍳0;MASK←MASK,0;J←0;FLAG←0;SAVEMASK←MASK
[7]    LOOP:→(0=⍴MASK←(I←MASK⍳1)↓MASK)/EXIT
[8]     INDEX←INDEX,I+J+¯1↑INDEX
[9]     →(0=⍴MASK←(J←MASK⍳0)↓MASK)/EXIT
[10]    →LOOP
[11]   EXIT:→(0≠FLAG)/FINISH
[12]    J←0;INDEX←⍳0;SAVEIN←INDEX;MASK←(~SAVEMASK),0;FLAG←1
[13]    →LOOP
[14]   FINISH:INDEX←(0≠INDEX)/INDEX←SAVEIN,INDEX
[15]    INDEX←((⍴SAVEMASK)≠INDEX)/INDEX←INDEX[⍋INDEX]
[16]    G
     ∇
[18]
```

```
    ∇ T←FUNC INV VALUE;LEFT;RIGHT;UN;X;Y;MASK;INDEX;OK;NOTOK
[1]   ⍝
[2]   ⍝ INV FINDS INVERSE FUNCTION FOR FUNC EVALUATED
[3]   ⍝ AT FIRST ELEMENT OF ARRAY VALUE.  SECOND AND
[4]   ⍝ THIRD ELEMENTS OF VALUE DEFINE INTERVAL
[5]   ⍝ OF INTEREST.
[6]   ⍝
[7]   ⍝ CHECK TO SEE IF VALUE IS LEGAL FORMAT
[8]   ⍝
[9]    →((⍴VALUE)≠3)/ERROR
[10]  ⍝
[11]  ⍝ DEFINE LEFT AND RIGHT ENDPOINTS
[12]  ⍝
[13]   T←⍳0;LEFT←1↑1↓VALUE;RIGHT←1↑2↓VALUE;UN←((⍳101)÷101),1;⎕IO←0
[14]  YLOOP:X←⍳0
[15]  ⍝
[16]  ⍝ DEFINE X VECTOR; NOTE RIGHT, LEFT MAY BE VECTORS
[17]  ⍝
[18]  XLOOP:X←X,(1↑LEFT)+UN×((1↑RIGHT)-(1↑LEFT))
[19]   LEFT←1↓LEFT;RIGHT←1↓RIGHT
[20]   →((⍴LEFT)>0)/XLOOP
[21]   Y←⍎FUNC,' X'
[22]  ⍝
[23]  ⍝ USE CHANGE TO FIND INTERVALS WHOSE IMAGES
[24]  ⍝ CONTAIN FUNC(1↑VALUE)
[25]  ⍝
[26]   INDEX←CHANGE MASK←(1↑VALUE)≤Y
[27]  ⍝
[28]  ⍝ DEFINE NEW LEFT,RIGHT VECTORS;CHECK
[29]  ⍝ SIZE OF NEW INTERVALS AGAINST TOLERANCE
[30]  ⍝
[31]   RIGHT←X[INDEX];LEFT←X[¯1+INDEX]
[32]   OK←TOLERANCE≥|-/⍉(2,(⍴RIGHT))⍴(RIGHT,LEFT)
[33]   T←T,X[OK/INDEX]
[34]   NOTOK←(~OK)/⍳(⍴RIGHT)
[35]   RIGHT←RIGHT[NOTOK];LEFT←LEFT[NOTOK]
[36]   →(0<⍴RIGHT)/YLOOP
[37]   →0
[38]  ERROR:⍞←' WRONG FORMAT FOR INPUT'
    ∇
```

```
    ∇  T←G1 G;INDEX;MASK;SIZE
        ⍝
        ⍝ IMPLEMENTS FUNCTION G1
        ⍝ INCLUDES TRAPS FOR UNDEFINED VALUES NEAR 0 AND 1
        ⍝
         INDEX←(~MASK←((LOWER←G≤0.001)+(UPPER←G≥0.99)))/⍳SIZE←⍴G←,G
         G←G[INDEX]
         T←(~MASK)\((1-G)÷G)×*(¯1+2×G)÷(2×G×(1-G))
         T[(UPPER)/⍳SIZE]←1.00E10
         T[LOWER/⍳SIZE]←0
    ∇
```

```
    ∇  T←G2 X;INDEX;MASK;SIZE
        ⍝
        ⍝ IMPLEMENTS FUNCTION G2
        ⍝ INCLUDES TRAPS FOR UNDEFINED VALUE NEAR 0
        ⍝ AND FLOATING POINT OVERFLOW ≥5
        ⍝
         INDEX←(~MASK←((X≤1.00E¯10)+(X≥5)))/⍳SIZE←⍴X←,X
         X←X[INDEX]
         T←(~MASK)\(*(¯0.5+X×X))÷(X×2*0.5)
         T[(MASK)/⍳SIZE]←1.00E10
    ∇
```

```
    ∇  T←BETA G3 G;INDEX;MASK;SIZE;G1;ZERO
        ⍝
        ⍝ IMPLEMENTS FUNCTION G3; BETA = TR/TS
        ⍝ TRAPS 0 VALUES AND USES ASYMPTOTIC
        ⍝ APPROXIMATION NEAR 0
        ⍝
         INDEX←(~MASK←G≤0.0001)/⍳SIZE←⍴G←,G
         G1←G[INDEX]
         T←(~MASK)\((1-G1)÷G1)×*(¯1+2×G1)×BETA×BETA
         MASK←MASK×~ZERO←G≤0
         T[INDEX]←(1÷G[INDEX←MASK/⍳SIZE])×*BETA×BETA
         T[ZERO/⍳SIZE]←1.00E30
    ∇
```

```
    ∇ T←G4 ALPHA;A;FHALT;CRIT;TR;G;COM;JUMP;RCOM
       ⍝
       ⍝ IMPLEMENTS FUNCTION G4
       ⍝ ALPHA MAY BE VECTOR; IN ORDER TO USE
       ⍝ INV DISASSEMBLE ALPHA FIRST
       ⍝
        T←⍳0;ALPHA←1,ALPHA
       ⍝
       ⍝ FIRST INVERT G1 TO FIND TR; SAVE G
       ⍝
      LOOP:→(0=⍴ALPHA←1↓ALPHA)/0
        A←1↑ALPHA;FHALT←'G1'
        CRIT←'G1' INV(A,0,1)
        →(1≠⍴CRIT)/SUSPEND
      CONT1:TR←(2×(1-G)×G←CRIT)*¯0.5
       ⍝
       ⍝ NOW INVERT G3(TR;∘)
       ⍝
        FHALT←(⍕TR),' G3'
        CRIT←'TR G3' INV(A,0,1)
        →(1≠⍴CRIT)/SUSPEND
       ⍝
       ⍝ THROW AWAY ANY VALUES EQUAL G
       ⍝
      CONT3:T←T,(G≠CRIT)/CRIT
        →LOOP
       ⍝
       ⍝ GET HERE IF INV RETURNS EMPTY SET OR MULTIPLE VALUES
       ⍝
      SUSPEND:⎕ ←A;⍞←'NO SOLUTION FOR ⍺ = ';⎕ ←'PROGRAM HALTED INVERTING ',FHALT
       ⎕ ←'ASSIGN INVERSE FOR ',FHALT,' TO VARIABLE CRIT AND TYPE CONTINUE'
      COMMAND:COM←⍞;⍞←'HALT>>'
        JUMP←∊'CONT',¯1↑FHALT
        →(×/('CONTINUE'=RCOM←8⍴(COM←((' '≠COM)/COM)),8⍴' ')/JUMP
        →(×/('QUIT    '=RCOM))/0
        ∊COM
        →COMMAND
    ∇
```

```
∇ SOLUTION;R2;ALPHA CRIT;A;FHALT;COM;JUMP;RCOM
  ⍝
  ⍝ SOLUTION CALCULATES OPTIMAL DELAY AND BIAS VALUES
  ⍝ FOR THE RANGE-GATE PULLOFF PROBLEM
  ⍝
  ⍝ SET UP VECTOR OF SIGNAL/JAMMER POWER VALUES
  ⍝
   ALPHA1←((⌽(1÷ALPHA1)),ALPHA1←10*0.1×(⍳200)÷4);TSTAR←TOPT←⍳0
  ⍝
  ⍝ FIRST DO CASE α ≥ 1
  ⍝
   ALPHA←(ALPHA1≥1)/ALPHA1
  LOOPA:A←1↑ALPHA;FHALT←'G2'
   CRIT←('G2' INV(A,0,R2)),(A=1)/(R2←1÷2*0.5)
   →(1≠⍴CRIT)/SUSPEND
  CONT2:TSTAR←TSTAR,CRIT;TOPT←TOPT,(CRIT÷R2)
   →(0=⍴ALPHA←1↓ALPHA)/NEXT
  ⍝
  ⍝ PRINT MESSAGE EVERY 10 TIMES THROUGH LOOP
  ⍝
   →(0≠10|⍴ALPHA)/LOOPA
   ⎕←A;⍞←'LOOPING...α = '
   →LOOPA
  ⍝
  ⍝ NOW DO CASE α < 1
  ⍝
  NEXT:ALPHA←(ALPHA1<1)/ALPHA1
  LOOPB:A←¯1↑ALPHA;FHALT←'G1'
   CRIT←'G1' INV(A,0,1)
   →(1≠⍴CRIT)/SUSPEND
  CONT1:TR←R2×(CRIT×(1-CRIT))*¯0.5
   TOPT←TR,TOPT;TSTAR←((G4 A)×TR),TSTAR
   →(0=⍴ALPHA←¯1↓ALPHA)/0
  ⍝
  ⍝ PRINT MESSAGE EVERY 10 TIMES THROUGH LOOP
  ⍝
   →(0≠10|⍴ALPHA)/LOOPB
    ⎕←A;⍞←'LOOPING...α = '
   →LOOPB
  ⍝
  ⍝ STOP HERE IF INV RETURNS EMPTY SET OR MULTIPLE VALUES
  ⍝ TYPE CONTINUE TO GO BACK, OR QUIT TO GET OUT
  ⍝
  SUSPEND:⎕←'PROGRAM HALTED INVERTING ',FHALT
    ⎕←A;⍞←'NO SOLUTION FOUND FOR α = '
    ⎕←'ASSIGN INVERSE FOR ',FHALT,' TO VARIABLE CRIT AND TYPE CONTINUE'
  COMMAND:COM←⍞;⍞←'HALT>> '
   JUMP←⍎'CONT',¯1↑FHALT
   →(∧/('CONTINUE'=RCOM←8⍴(COM←((' '≠COM)/COM)),8⍴' '))/JUMP
   →(∧/('QUIT    '=RCOM))/0
   ⍎COM
   →COMMAND
∇
```

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13 ABSTRACT

(U) A mathematical model of the range-gate pulloff electronic countermeasure is developed based on a statistical model for the radar return from a slowly fluctuating point target in the presence of white noise. It is shown that the same model is also appropriate for describing velocity-gate pulloff. An optimization problem is formulated which determines the jammer delay inducing the maximum bias in the range estimation processor of the victim radar. These results are then applied to the specific case in which the transmitted signal is a gaussian pulse. The optimal delay and bias are calculated as functions of the signal-to-jammer power ratio and the pulse width of the transmitted signal.

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13 ABSTRACT: Enter an abstract giving a brief and factual summary of the document, even though it may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall end with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (TS), (S), (C), (R), or (U).

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