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SIEVES AND SIGNAL EXTRACTION(U) WISCONSIN
UNIV-MILWAUKEE DEPT OF MATHEMATICAL SCIENCES J H BEDER
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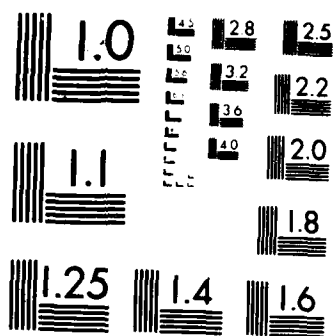
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19. ABSTRACT (Continue on reverse if necessary and identify by block number) Given a Gaussian signal with known mean and covariance, and given a second covariance kernel R, the PI has proved several results concerning the probability that the sample path of the signal will fall in the reproducing kernel Hilbert space with kernel R. This is applied to optimal extraction of an unobservable signal based on its conditional mean, and to the generalization of a zero-one law given by Kallianpur and by Driscoll. For an observable Gaussian process with unknown mean and covariance, an extension of previous work shows that simultaneous consistent estimation of both the mean and the covariance is possible by the method of sieves. In both this and the signal extraction problem, no assumption is made about the nature of the "time" parameter of the process. Some work on the axiomatic theory of confounding in experimental designs is also reported.				
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Sieves and Signal Extraction (Unclassified)

1. Summary: See DD 1473, item 19.
- 2,3. Research objectives, status of work: Part of this project is to generalize the work of Driscoll [1973, 1975] on signal extraction. A brief description is given below; more detail may be found in the original project proposal.

To describe the current status of this work, let $\{S_t, t \in \mathcal{T}\}$ be a Gaussian process (signal) with mean 0 and known covariance K . Here $\mathcal{T}$ is an arbitrary set. The sample path of the process is denoted $S_\cdot$. Let R be any positive definite kernel with reproducing kernel Hilbert space (RKHS) $\mathcal{H}(R) = \mathcal{H}(R, \mathcal{T})$, and assume that R dominates K ($K \ll R$), i.e. that $\mathcal{H}(K) \subset \mathcal{H}(R)$. Using the concept of n -domination ($K \prec^n R$) (Fortet [1974]), we have:

Theorem A. If $K \ll R$ but not $K \prec^n R$, then $P(S \in \mathcal{H}(R)) = 0$.

This generalizes a result of LePage [1973], for $K = R$.

The desired converse of Theorem A is: $K \prec^n R \Rightarrow P(S \in \mathcal{H}(R)) = 1$, where the completion-measurability of the event $\{S \in \mathcal{H}(R)\}$ is part of the conclusion, as in Theorem A. Driscoll's proof of this result appears to be incomplete. Thus far we can show the following: Let $\mathcal{T}$ be the family of finite subsets of $\mathcal{T}$, and for each $T \in \mathcal{T}$ let

$$Z_T = \|S_\cdot\|_T^2$$

(norm in $\mathcal{H}(R, T)$, where $S_\cdot$ is restricted to T). Call $\{Z_T, T \in \mathcal{T}\}$ the derived process of S . Then by Fortet [1973] we see that the sets

$\{S_\cdot \in \mathcal{H}(R)\}$ and $\{\sup_{T \in \mathcal{T}} Z_T\}$
are equal - call the set A - and that on A we have $\|S_\cdot\|^2 = \sup Z_T$
(norm in $\mathcal{H}(R)$). The set A may not be measurable. Consider the
weak separability assumption

$$(*) \quad \sup_{T \in \mathcal{T}} Z_T = \ell\text{-th.} \sup_{T \in \mathcal{T}} Z_T \quad \text{a.s.}$$

(The lattice-theoretic supremum is defined in Tucker [1967].)

We have

Theorem B. If $K <^n R$ and if (*) holds then $P(S \in \mathcal{H}(R)) = 1$.

It is not yet clear just how weak or strong (*) is. It certainly holds in at least one of the cases Driscoll considers, so that we have his [1973] zero-one law. In fact, as long as (*) holds we have Kallianpur's zero-one law by letting $K = R$. However, our formulation also characterizes the "zero" and "one" parts of the law separately.

A second issue is optimal extraction of the signal S with regard to the norm of $\mathcal{H}(R)$. Let $\mathcal{A}_x$ be any σ -algebra of observable events such that $\hat{S}_t = E^{\mathcal{A}_x}(S_t)$ is Gaussian. The covariance C of $\hat{S}$ is dominated by K , and we can show:

Theorem C: If $K <^n R$, then $C <^n R$.

Corollary: If $K <^n R$ and if the derived process of $\hat{S}$ satisfies (*), then $P(\hat{S} \in \mathcal{H}(R)) = 1$. It is not yet known whether it is sufficient to assume merely that S satisfies (*) in the Corollary. If both S and $\hat{S}$ satisfy (*) and if $K <^n R$, we wish to show

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$$(1) E^{\lambda_x} \|S_t - \hat{S}_t\|^2 \leq E^{\lambda_x} \|S_t - h(\cdot)\|^2 \text{ for all } h \in \mathcal{H}(R),$$

and to evaluate the left hand side of (1).

Write $V_t = S_t - \hat{S}_t$, with covariance D , say (so $C + D = K$). Associate to K , C , and D the operators θ , θ_C , θ_D on $\mathcal{H}(R)$ (as in Fortet [1973]), all assumed nuclear, and satisfying $\theta = \theta_C + \theta_D$.

Theorem D. $E^{\lambda_x} \|S_t - \hat{S}_t\|^2 = \text{tr } \theta_D.$

(The value $\text{tr } \theta_C$ given in Driscoll [1975] is incorrect.)

Finally, to prove the inequality (1) we will need to show that a cross-product term $E^{\lambda_x} (S_t - \hat{S}_t, \hat{S}_t - h)$ vanishes a.s. It is not yet clear how to do this.

We may mention one further result:

Theorem E. If $K <^n R < R_1$, then $K <^n R_1$. Thus there are many RKHS's $\mathcal{H}(R)$ whose norm could be used to define optimal signal extraction.

In Driscoll's original formulation, λ_x was the σ -algebra of an observed signal X of form $X_t = S_t + N_t$, where N_t is independent Gaussian noise. The present formulation generalizes this, so that the unobservable signal S_t may be corrupted by noise in other ways.

The other part of this project involves sieve estimation for the mean and covariance of a Gaussian process. It has now been shown, in a joint paper with A. Antoniadis, that the two sieves

can be combined to give simultaneous consistent estimation of the mean and the covariance functions. The original paper on mean estimation has been revised and will appear in Annals of Statistics. The paper on covariance estimation is being revised for that journal.

A joint paper with M. O'Laughlin gives an axiomatic theory of confounding which may be applicable to a variety of experimental designs including those in which the set of factor levels is arbitrary, possibly a continuum. In such cases one may view the "vector" of observations as a stochastic process.

References

- Driscoll, M. F. (1973). The reproducing kernel Hilbert space structure of the sample paths of a Gaussian process. Z. Wahrscheinlichkeitsth. 26: 309-316.
- _____, (1975). The signal-noise problem - a solution for the case that signal and noise are Gaussian and independent. J. Appl. Prob. 12: 183-187.
- Fortet, R. (1973). Espaces à noyau reproduisant et lois de probabilités des fonctions aléatoires. Ann. Inst. Henri Poincaré, B, IX: 41-58.
- _____, (1974). Espaces à noyau reproduisant et lois de probabilités de fonctions aléatoires. C. R. Acad. Sci. Paris, A, t.278: 1439-1440.
- LePage, R. (1973). Subgroups of paths and reproducing kernels. Ann. Prob. 1: 345-347.
- Tucker, H. (1967). A Graduate Course in Probability. Academic Press, N.Y.

4. Written Publications:

"A sieve estimator for the mean of a Gaussian process," Annals of Statistics (to appear in March 1987).

"A sieve estimator for the covariance of a Gaussian process," Annals of Statistics (in revision).

(with A. Antoniadis) Joint estimation of the mean and the covariance of a Banach valued Gaussian vector. Submitted to Journal of Applied Probability.

(with M. O'Laughlin) The cell-means interpretation of confounding. In manuscript.

5. Professional Personnel:

None besides the PI.

6. Interactions:

(a) Paper presented:

"Sieve estimation for Gaussian Processes," Colloquium, Dept of Mathematics, University of California, Irvine, June 10, 1986.

(b) Consultative and advisory functions: Joint research with A. Antoniadis on sieve estimation. Dept. of Mathematics, UC-Irvine, June 9-11, 1986.

7. New discoveries, inventions, patents: none.

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