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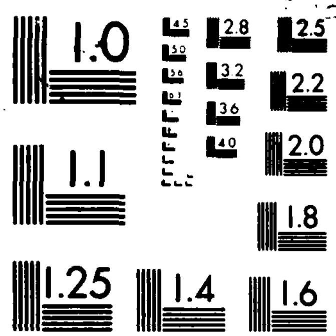

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COMPARISON OF

CHANNEL ESTIMATION TECHNIQUES

THESIS

Ralph M. Strother
Captain, USAF

AFIT/GE/ENG/86D-8

DEPARTMENT OF THE AIR FORCE
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COMPARISON OF CHANNEL ESTIMATION TECHNIQUES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Electrical Engineering

Ralph M. Strother, B.S.E.E

Captain, USAF

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List of Symbols

$B1$ . . . . . quadratic phase distortion

$B2$ . . . . . cubic phase distortion

e_{bk} . . . . . output of the lower portion of the lattice

e_{fk} . . . . . output of the upper portion of the lattice

E'_k . . . . . the difference between the input of the channel, y_k, and the predicted value, y'_k, generated by an estimator

G . . . . . the total number of stages of the baseband channel model

$G1$ . . . . . amplitude distortion

h_j . . . . . sampled impulse response of the system due to input X_k and also the unknown tap gain coefficients of the channel

H_k . . . . . (G)-component row vector containing the impulse response of the system

H'_k . . . . . (G)-component row vector containing the impulse response of the system contained in the estimator

$k_{k,j}$ . . . . adaptive lattice tap gains

n_k . . . . . white gaussian noise component of the channel

$n(t)$ . . . . time varying white gaussian noise

X_k . . . . . input symbols to the baseband channel

x'_k . . . . . output symbol from the detector

y_k . . . . . sampled output from the baseband channel

$v_{k,j}$ . . . . rough estimate of $h'_{k,j}$ fpr Feedback Estimator

$y(t)$ . . . . unsampled output of the baseband channel

Δ . . . . convergence factor for Feedback and Feedforward Estimators

Δ_1 . . . . convergence factor for upper portion of Lattice Estimator

Δ_2 . . . . convergence factor for lower portion of Lattice Estimator

Abstract

This ~~investigation~~ involved the comparison of three types of channel estimation techniques, the Feedforward Estimator, the Feedback Estimator, and the Lattice Estimator. A computer simulation of a communications channel was run involving varying levels of amplitude distortion, phase distortion, and Gaussian noise imposed on a data stream. The resulting output of the channel was fed to a receiver consisting of a detector and a channel estimator. The estimator took the output of the channel and the detector and used them to identify the impulse response of the channel.

Of the three channel estimators, the Feedback Estimator proved superior in terms of performance under varying levels of channel distortion and noise. Furthermore, the Feedback Estimator demonstrated the best error handling capabilities. Finally, the Feedback Estimator proved to be the simplest algorithm to implement of the three.

1. Introduction

Background

When signals are sent across communications channels such as telephone lines, the result is often a distorted version of the original transmitted message. To recover the original message, the receiver must combat the amplitude and phase distortion and the noise the signal has picked up from the channel. Complicating the problem is that the distortion may be time-varying.

Various methods of combatting the time-varying distortion and noise have been attempted but the most promising results have come from using adaptive techniques. These adaptive techniques continuously adjust the receiver to the changing conditions of the channel.

Two of the most widely used adaptive techniques are channel equalizers and channel estimators (1;2;3). Both channel equalizers and estimators form an estimate of the impulse response of the communication channel to use in adjusting the receiver for proper signal reception. The techniques differ in that while the channel equalizer uses the estimated impulse response directly to compensate for the noise and distortion, the channel estimator forms the estimated channel impulse response and then passes it onto a detector (see Figure 1). The detector uses the estimate with a probabilistic detection algorithm, such as the Viterbi algorithm (4), to compensate for the channel effects. Various types of estimators have been proposed but little data is available to make comparisons among them (1;2;3).

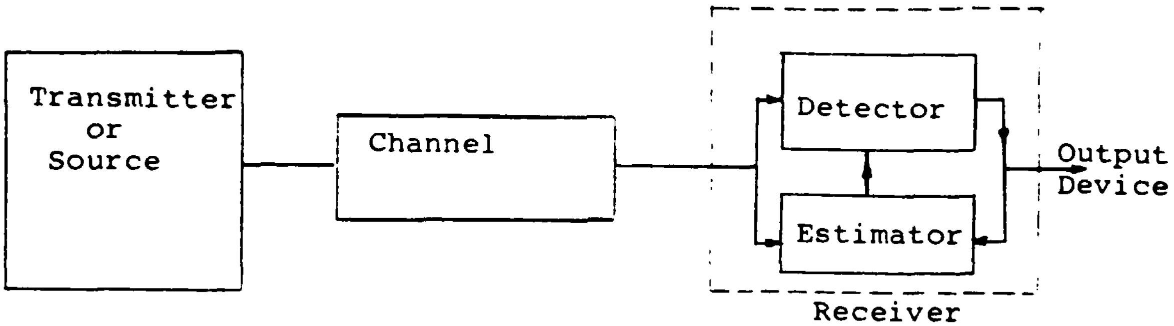

Figure 1. Communication System Utilizing a Channel Estimator

Problem

The problem to be investigated is the evaluation of the performance of various channel estimators, and the design and evaluation of an estimator based on lattice techniques. The latter is a method of using an adaptive lattice filter to estimate the impulse response (or transfer function) of a system. The goal of this study is to determine the best overall estimation method and to determine any special instances where one technique may be preferrable over another.

Scope

The following types of channel estimators were analyzed and tested:

1. Feedforward Estimator
2. Feedback Estimator
3. Lattice Estimator

These channel estimators represent feedforward, feedback, and hybrid types of estimators (1;2;3;4;5). Each estimator was evaluated by inputting a signal containing various combinations of amplitude and phase distortion, and noise caused by the channel.

Assumptions

To make the proposed problem manageable, the following assumptions must be made:

1. All data symbols emitted by the source will be statistically independent and have equal probability of being either 1 or -1. This assumption is based on the premise that in any random data stream either symbol may occur at any time and that the result of one symbol being received does not bias what the next symbol will be.
2. Each symbol will be spaced at an interval of time T, apart. This assumption allows for the measure of any phase errors that occur as a result of the channel.
3. No special encoding of the signal is assumed for this source.
4. The channel operates in an additive white Gaussian noise environment. Burst error noise will also be considered.
5. The communication system operates with a low probability of error.

2. Theory

System Model

The communications system of Figure 1 can be more usefully modeled as shown in Figure 2 (1). In this model, sampled input symbols X_k are fed into the baseband channel. The baseband channel then induces amplitude (G1) and phase distortion (B1 and B2) and adds a noise term according to the equation

$$Y(f) = X(f)\ [(1 + G1\cdot f)\ \exp(-j2\pi f + B1\cdot f^2 + B2\cdot f^3) \qquad (2.1)$$

in the frequency domain which in the time domain appears as

$$y(t) = x(t-T) + (G1/2\pi)\ x'(t-T) + (B1/4\pi^2)\ x''(t-T) + (B2/8\pi^3)\ x'''(t-T) \qquad (2.2)$$

Where x(t-T) is a delayed time version of the input symbol x_k (6;11).

The baseband channel can be modeled as a composite of three filters and a white gaussian noise source. Two of the filters represent the input and output characteristics of the transmitter and receiver respectively. A third filter represents the characteristics of the transmission path. An equivalent

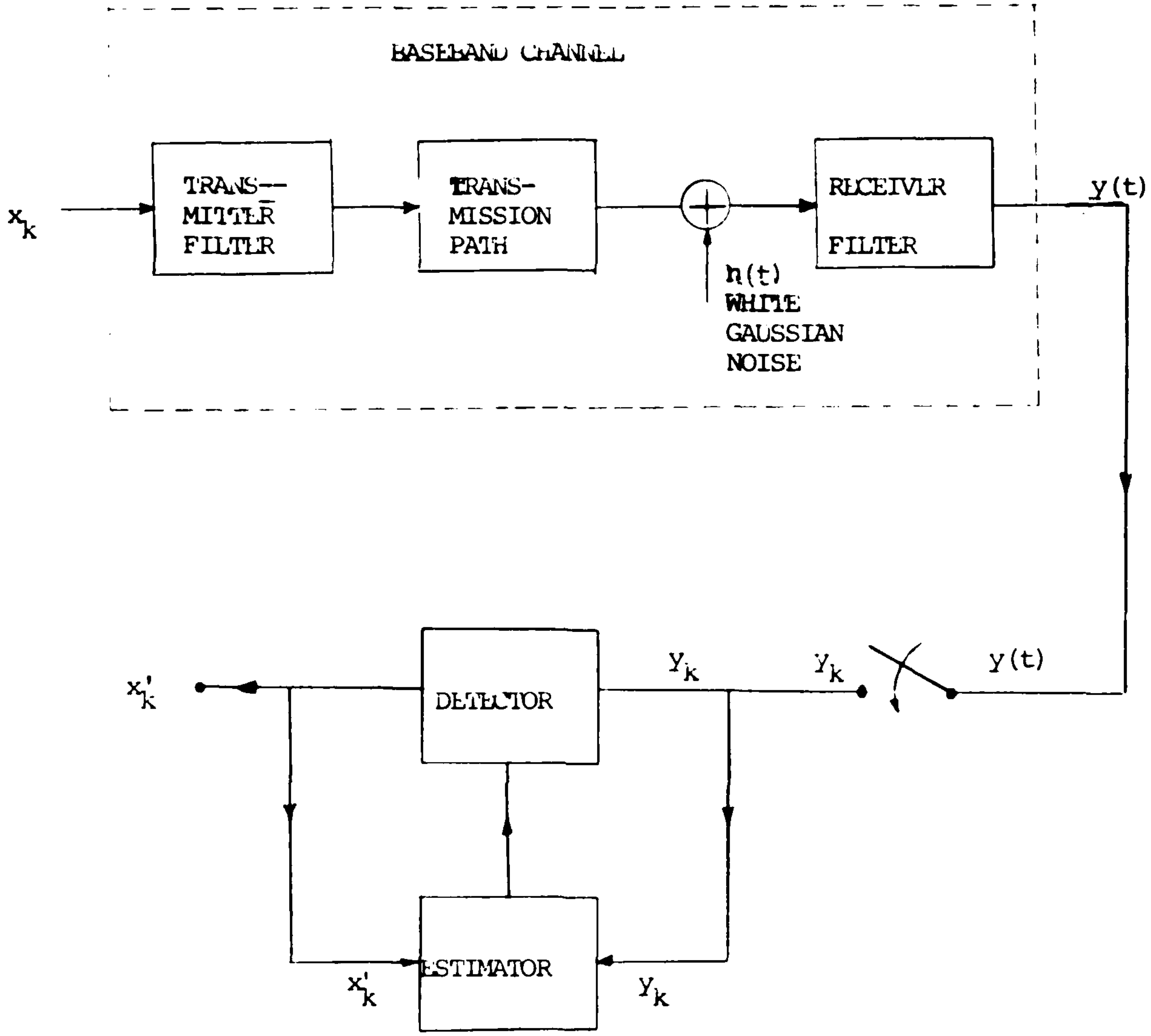

Figure 2. Idealized Model of a Synchronous Serial Binary System

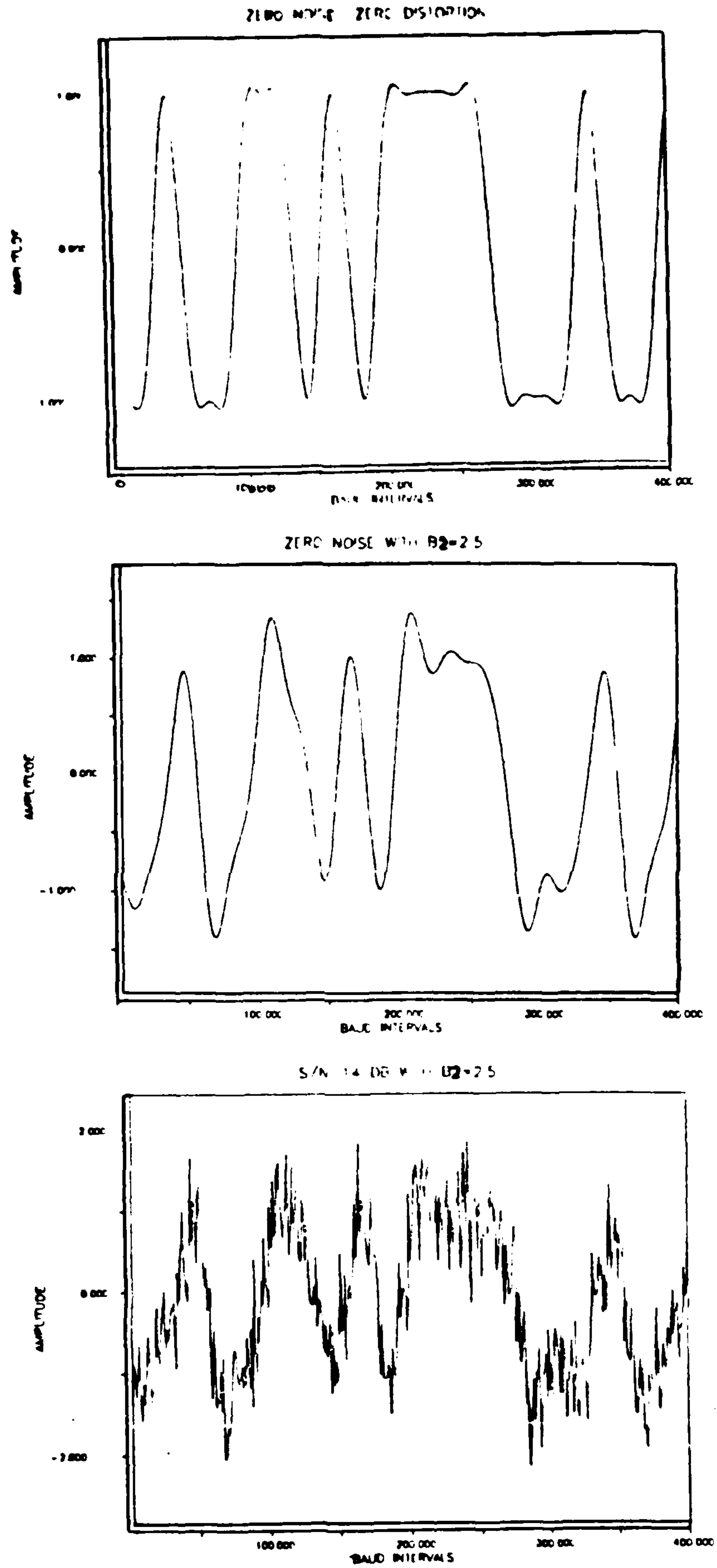

Figure 3. Baseband Output of the Distortion Channel Model

discrete-time filter is shown in Figure 4 (5) where the sampled output of the channel is found by

$$y_k = \sum_{j=1}^{G} h_j \, x_{k-j} + n_k \tag{2.3}$$

where

G = the total number of stages of the filter

h_j = unknown tap gain coefficients of the channel and also the sampled impulse response of the system

x_{k-j} = input symbol to baseband channel

n_k = sampled gaussian noise component of the channel

The overall *sampled* impulse response of the system to the symbols X_k can be written as the (G)-component row vector:

$$H = [\, h_1 \; h_2 \; \ldots \; h_G \,] \tag{2.4}$$

The discrete output y_k is received by both the detector and the channel estimator. The detector determines the symbols X_k', which corresponds to the transmitted symbols X_k, by using the sampled output y_k and an estimate of the channel H'_k from the estimator. H'_k is defined as the (G)-component row vector containing an estimate of the channel such that

$$H'_k = [\, h'_{k,1} \; h'_{k,2} \; \ldots \; h'_{k,G}] \tag{2.5}$$

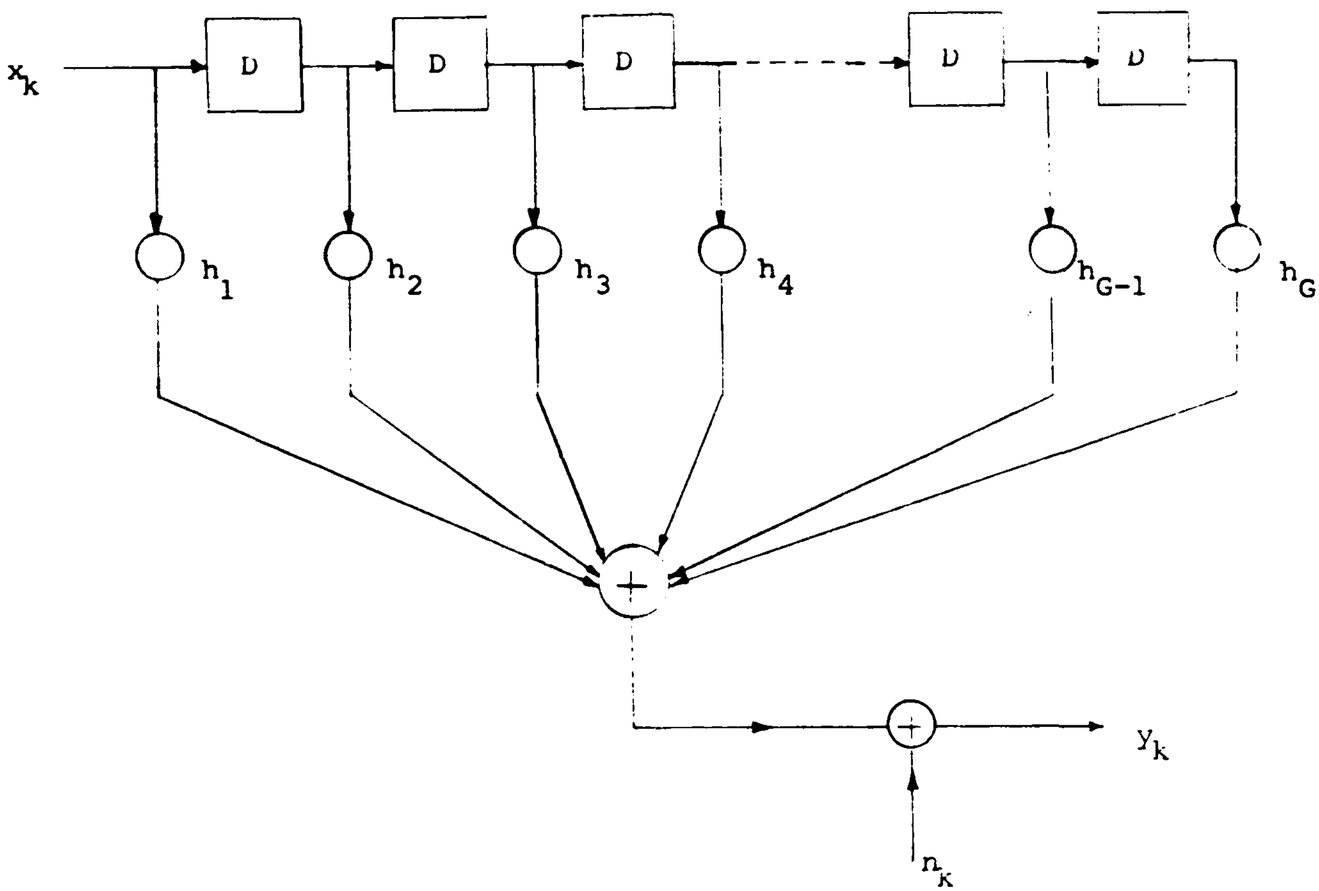

Figure 4. Equivalent Discrete-Time Filter

where $h'_{k,j}$ represents the j'th tap gain for the k'th sample.

The estimator takes the y_k output and uses it and the X'_k symbol to update the H'_k row vector to prepare for the y_{k+1} output of the channel.

Feedforward Estimator

The Feedforward Estimator is an application of an adaptive linear transversal filter in which the tap gains, H'_k, are updated automatically to minimize the mean squared error between the input to the detector y_k and an estimate or predicted value of the input to the estimator y'_k. The method employed to minimize the error between y_k and y'_k is a gradient search algorithm known as the Least-Mean-Squared (LMS) algorithm (3;7;8). In the LMS algorithm, future values of the tap gains, H'_{k+1}, are found using the past values, H'_k, and the current and some prior outputs of the detector, X'_k. The LMS algorithm can be expressed as:

$$H'_{k+1} = H'_k + \Delta E_k X'_{k-j} \tag{2.6}$$

where

H'_{k+1} = the updated G-component row vector containing the estimated impulse response of the system.

H'_k = the prior G-component row vector containing the estimated impulse response of the system.

$\Delta =$ the convergence factor which is a constant that determines how quickly the estimator responds to changes in the channel.

E'_k = the difference between the input of the channel, y_k, and the predicted value, y'_k, generated by the estimator.

$= [\ e_1, e_2, \ldots, e_k\ ]$

X'_{k-j} = the updated G-component row vector containing the outputs from the detector

$= [x'_{k-j},\ x'_{k-j+1},\ \ldots\ ,\ x'_{k-j+G}]$

The operation of the Feedforward Estimator is illustrated in Figure 5. The current output of the detector, X'_k, is fed into the estimator and, with the (G-1) prior outputs of the detector, is multiplied by the G-components of H'_k and the result is summed to form an estimate of the channel output y'_k . Subtracting y'_k from y_k forms the error signal E_k which is then weighted by the convergence factor Δ . This weighting by Δ controls how quickly the tap gains of H'_{k+1} are changing in response to a single input. The weighted error signal is then multiplied by the present and G prior inputs and the result added to the current tap gains of H'_k to form H'_{k+1}. This result is then fed

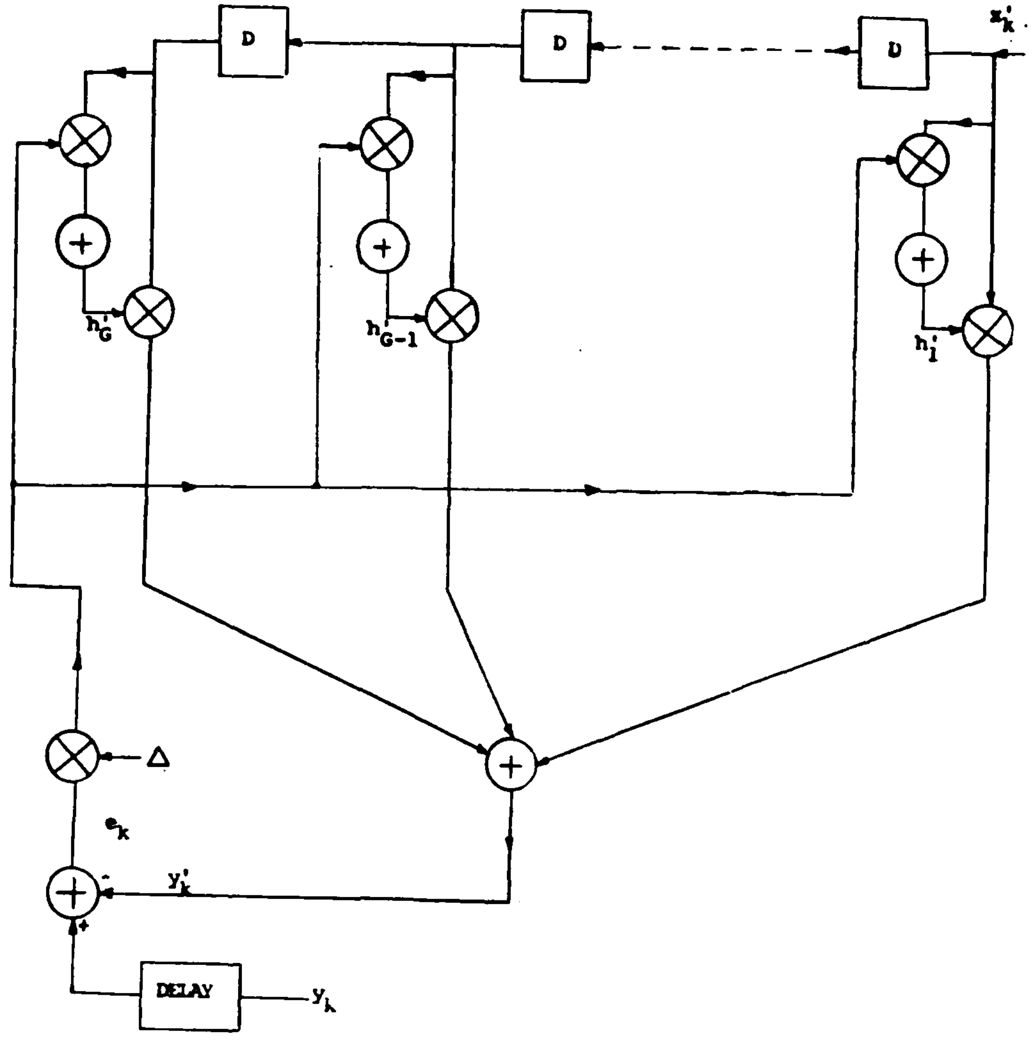

Figure 5. Feedforward Estimator

to the detector for use when the y'_{k+1} output of the channel occurs. The cycle then begins again. A total of G^2 multiplications are required for this technique.

Feedback Estimator

Unlike the Feedforward Estimator which uses X'_{k-j} and H'_k to update the H'_{kl} tap gain values, the Feedback Estimator uses X'_k and Y_k to derive the tap gain estimates of H'_k. The recursive algorithm used to do so can be expressed graphically as shown in Figure 6 (1).

Initially the first (G+1) outputs from the channel are shifted into the estimator. The y_k input from the channel is divided by the output of the estimator x'_k to form:

$$v_{k,j} = y_k \,/\, x'_k \qquad (2.7)$$

where $v_{k,j}$ is the rough estimate of $h'_{k,j}$. Using this rough estimate $v_{k,j}$, the convergence factor Δ, and the prior value of the tap gain $h_{k-1,j}$, we can obtain the updated value of $h_{k,j}$ by:

$$h'_{k,j} = \Delta v_{k,j} + (1-\Delta)\, h'_{k-1,j} \qquad (2.8)$$

This result is then fed to the multipliers and multiplied by X'_{k+1} where X'_{k+1} can be written as:

$$X'_{k+1} = [x'_{k+1},\ x'_{k+2},\ \cdots\ ,\ x'_{k+G+1}] \qquad (2.9)$$

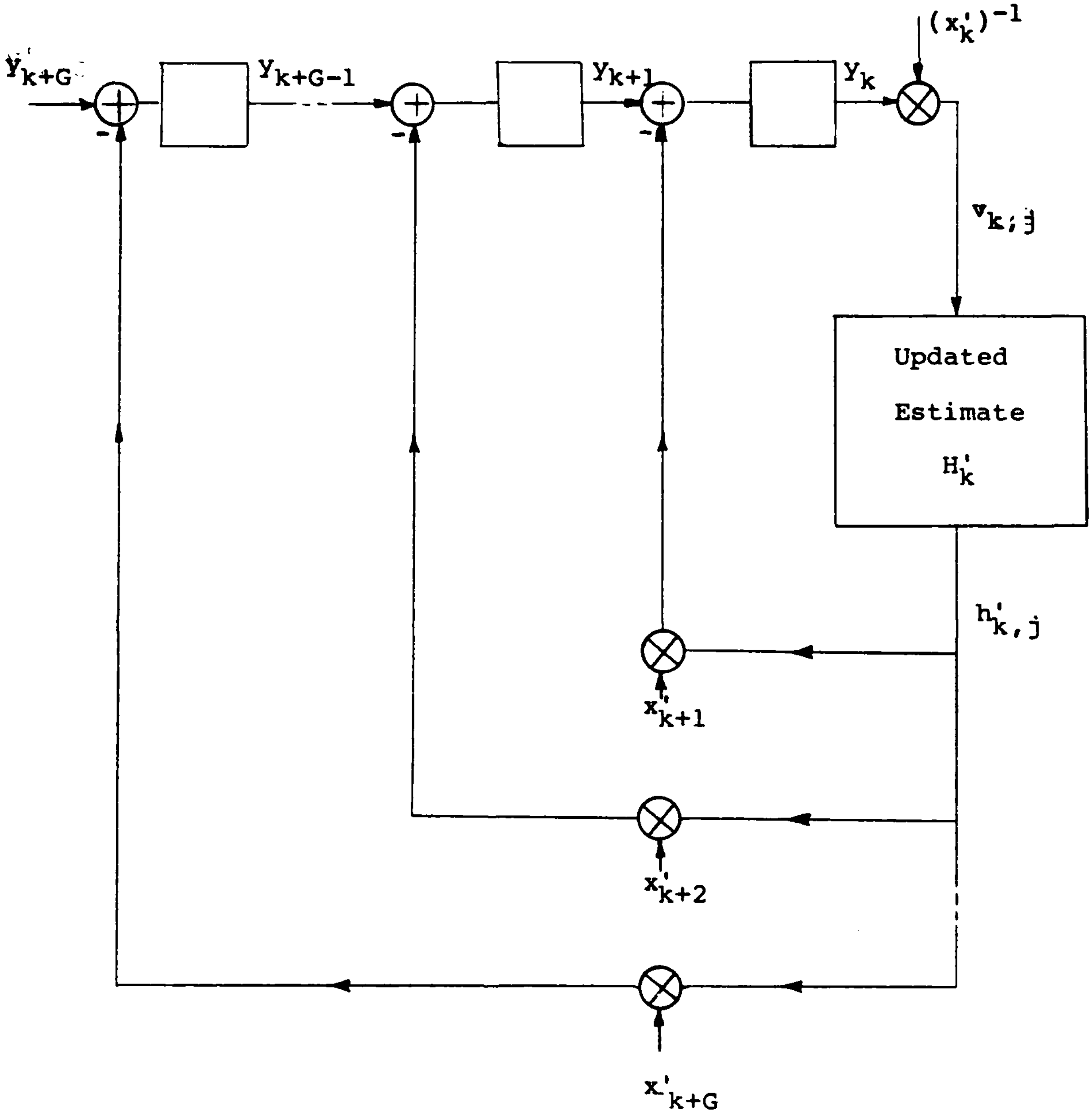

Figure 6. Feedback Estimator

The results from the multipliers are then fed to the adders and the results shifted according to:

$$y_k = y_{k+1} - (x'_{k+1})\ h'_{k,j} \qquad (2.10)$$

No new data points are fed into the estimator at this point. Equations (2.8) and (2.10) are repeated until all (G+1) original values have been shifted through, at which point H'_k will have been updated. This value of H'_k corresponds to what the estimator believes the channel's tap gains were at the time that the symbol x'_k was transmitted.

The algorithm is repeated with the (G+1) values that follow the previous y_k and the algorithm, expressed in equations (2.7), (2.8), and (2.10), begins again until the source stops sending symbols through the channel. Each time the algorithm goes through a complete cycle (1/2)(G+1)(G+2) multiplications are required (1).

Lattice Estimator

The adaptive lattice estimator (Figure 7) utilizes a lattice filter stage to orthogonalize the output signal from the detector before entering a transversal filter stage. The advantages of orthogonalizing the signal is in the possible increased speed of adaption and performance approaching that of much more computationally intensive exact least squared algorithms (8).

The general form of a lattice filter has been derived in Appendix B. The lattice of Figure 7 functions according to the following algorithm:

$$x'_k = x_{k,j} = x'_{k,j} \qquad j = \emptyset, G-1 \qquad (2.11)$$

$$x_{k,j+1} = x_{k,j} + k_{k-1,j}\, x'_{k-1,j} \qquad (2.12)$$

$$x'_{k,j+1} = k_{k-1,j}\, x_{k,j} + x'_{k-1,j} \qquad (2.13)$$

$$e_{fk} = x_{k,G} \qquad (2.14)$$

$$e_{bk} = x'_{k,G} \qquad (2.15)$$

$$k_{k,j} = k_{k-1,j} - \Delta_1\, x_{k,j+1}\, x'_{k-1,j} \qquad (2.16)$$

where

x'_k = k'th output from the detector

$x_{k,j}$ = k'th input at the upper j'th stage of the lattice

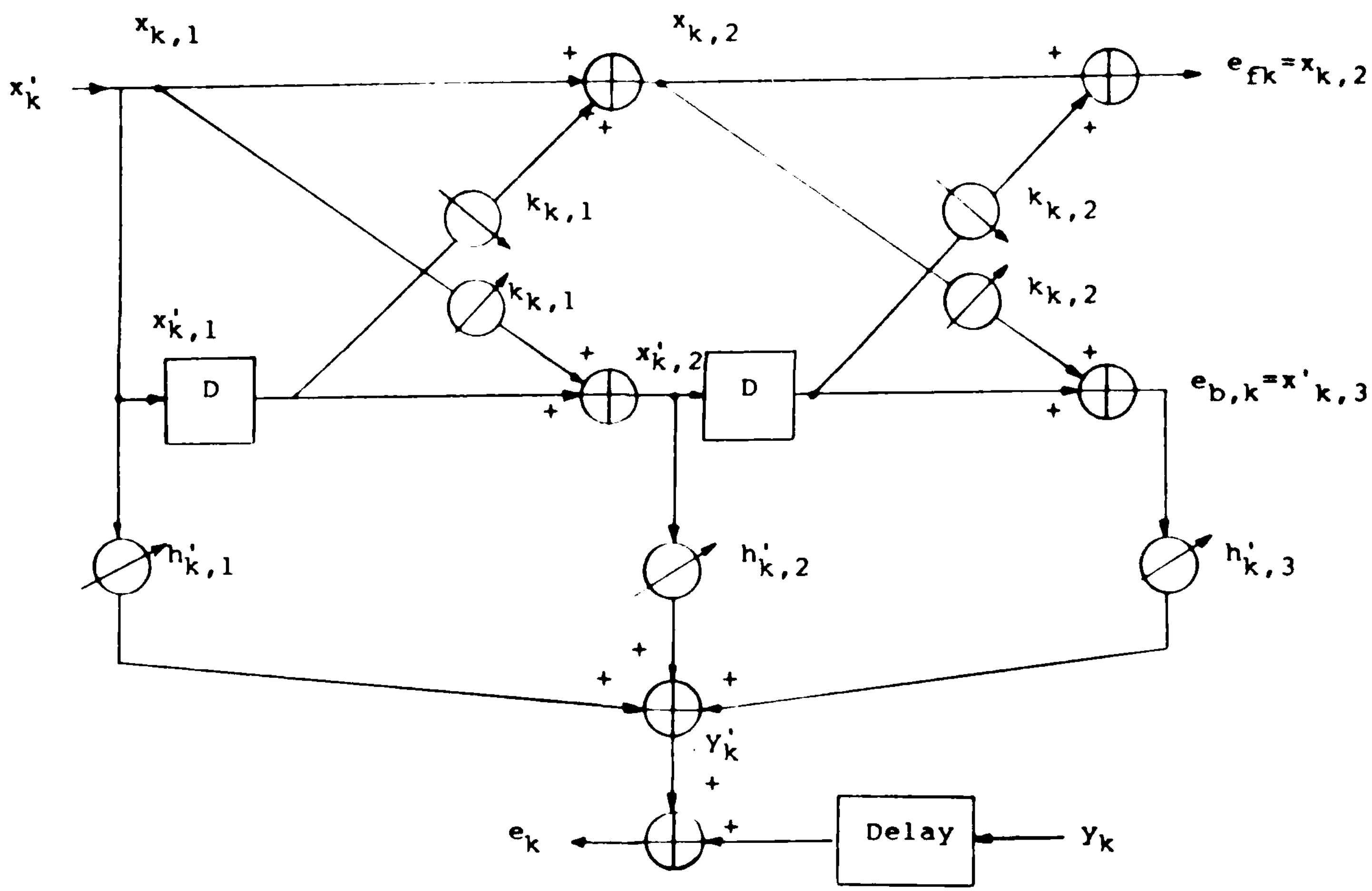

Figure 7. Two-Stage Lattice Estimator

$x'_{k,j}$ = k'th output at the lower j'th stage of the lattice

$k_{k,j}$ = adaptive lattice tap gains

e_{fk} = the output of the upper portion of the lattice from the (G-1)'th stage

e_{bk} = the output of the lower portion of the lattice from the (G-1)'th stage

Δ_1 = convergence factor for the adaptive lattice

G = number of stages in the lattice filter plus one

Eq (2.16) is a Least-Mean-Squared (LMS) algorithm used to update the lattice tap gains, k (8).

The adaptive transversal filter portion of the filter operates according to the algorithm:

$$y'_k = \sum_{j=1}^{G} h'_{k-1,j}\, x'_{k-1,j} \qquad (2.17)$$

$$e_k = y_k - y'_k \tag{2.18}$$

$$h'_{k,j} = h'_{k-1,j} + \Delta_2 \, e_k \, x'_{k,j} \tag{2.19}$$

where

y'_k = the predicted value of the input to the estimator

y_k = the actual value of the input to the estimator

$h'_{k,j}$ = the j'th estimated tap gain for the k'th input to the transversal filter

$x'_{k,j}$ = the k'th input at the lower j'th stage of the lattice

e_k = the difference between the actual value and the estimated value of the input to the estimator

Δ_2 = the convergence factor for the adaptive transversal filter

Eq (2.19) is also a Least-Mean-Squared (LMS) algorithm and is used to update the tap gain values, h of the transversal filter.

This is a slightly different form from the LMS algorithm used in the feedforward estimator (Eq (2.6)).

These two algorithms work in conjunction as shown in Figure 7 to form the impulse response of the system, H'_k of Eq (2.5). This estimated impulse response is then sent to the detector as with the other estimators. A total of $G^2(G-1)^2$ multiplications are required.

3. Results

To evaluate the three classes of estimators, plots were made to illustrate their various characteristic (See Appendix E for selected plots). The characteristics evaluated were:

1. The range of convergence factors (Δ or Δ_1 and Δ_2)
2. The rate of convergence for selected convergence factors with varying levels and combinations of amplitude (G1) and phase (B1 and B2) distortion and noise
3. The response and recovery of the estimators due to errors in the data stream
4. Response to change in channel characteristics

The resulting data from the tests on the estimators were plotted as the RMS value of the selected estimator parameter (such as weight1) versus the number of sample or iteration (See Figure 8). To allow comparisons among the different estimators only the values of the weights, H_k', were used from the various plots available. Data taken from their plots are summarized in Tables I - X.

Rate of Convergence for Selected Δ's

For comparison purposes, convergence factors of .1, .01 and .001 were selected for the Feedforward and the Feedback Estimators and for the Lattice Estimator $\Delta 1=.01$, $\Delta 2=.1$ and, $\Delta 1=.01$, $\Delta 2=.01$ and $\Delta 1=.001$ and $\Delta 2=.001$.

With no-noise and no-distortion on the chanel, the weights of the estimators converge to a constant value. Refering to Tables I - IX, all estimators tended to converge at approximately the same rate when $\Delta=.001$ ($\Delta 1=.001$ and $\Delta 2=.001$ in the Lattice case) regardles of the number of weights. In the case of $\Delta=.01$ ($\Delta 1=.01$, $\Delta 2=.01$), the Feedback Estimator converged in only 75 samples while the Lattice Estimator required 400 samples and the Feedforward estimator required 600 samples. When $\Delta=.1$ all estimators converged approximately in the same range of 70-100 samples depending upon the number of weights involved.

The addition of amplitude distortion (G1), quadratic phase distortion (B1), cubic phase distortion (B2) and/or Gaussian noise tended to effect all three types of estimators much the same. All estimators tended to deviate (or oscillate) around the optimum, no-noise, no-distortion value of H_k' with the percentage of deviation defined as the maximum value minus the minimum value of H_k' under the current channel conditions divided by the value of H_k' under the no-noise, no-distortion condition after the weights have converged. The greatest deviations were encountered

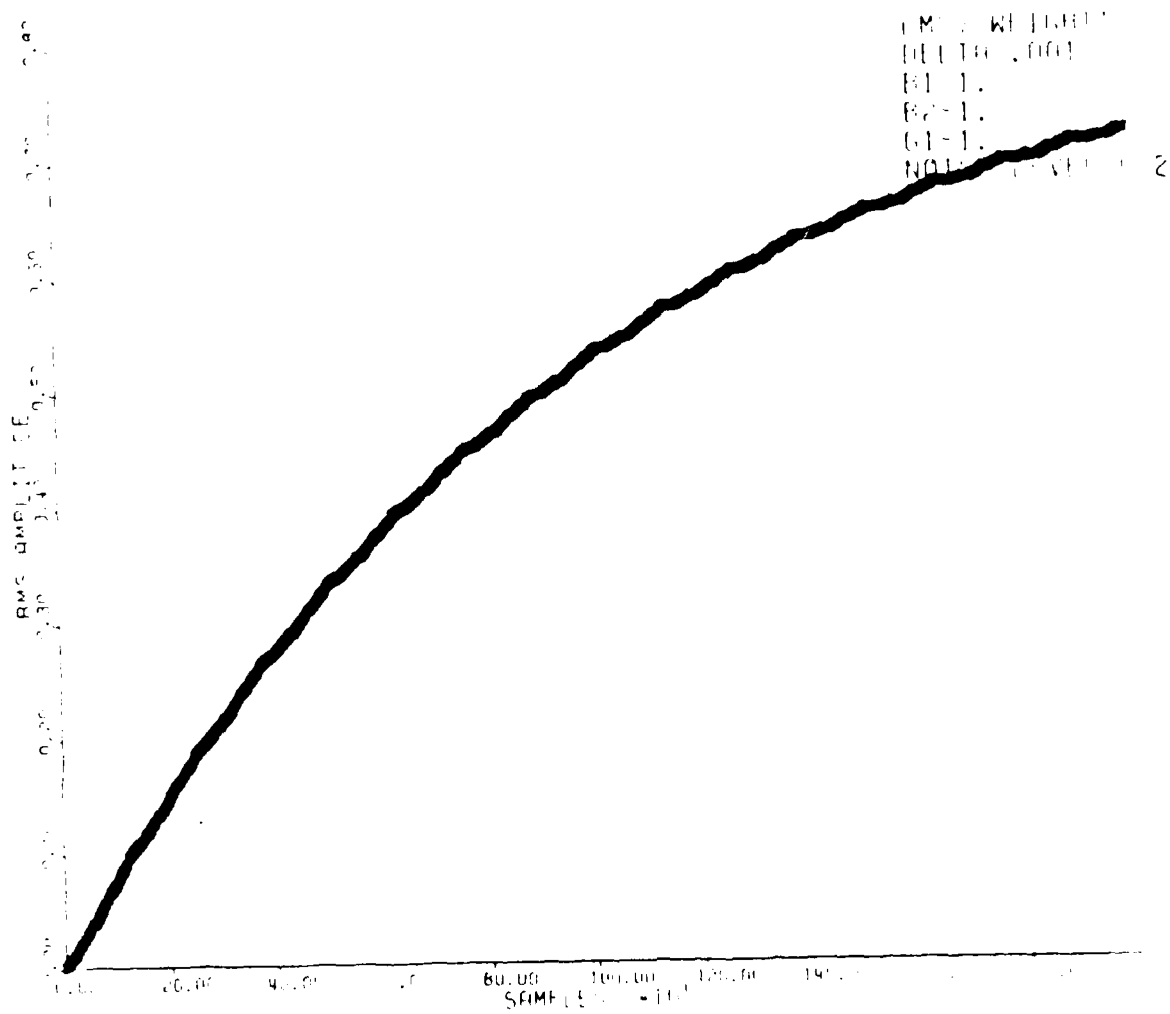

Figure 8. Typical Plot of Estimator Output

TABLE I

Feedforward Estimator Characteristics
For Three Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATION (PERCENT)
Feed-forward	3	.1	0	0	0	0	65	None
			1	0	0	0	65	4
			0	1	0	0	65	2
			0	0	1	0	65	2
			0	0	0	.2	65	4
			1	1	1	.2	65	8
		.01	0	0	0	0	600	None
			1	0	0	0	600	4
			0	1	0	0	600	2
			0	0	1	0	600	2
			0	0	0	.2	600	4
			1	1	1	.2	600	12
		.001	0	0	0	0	5000	None
			1	0	0	0	5000	<1
			0	1	0	0	5000	<1
			0	0	1	0	5000	<1
			0	0	0	.2	5000	<1
			1	1	1	.2	5000	<1
		.2	0	0	0	0	35	None
		.3	0	0	0	0	25	None
		.4	0	0	0	0	22	None
		.5	0	0	0	0	26	None
		.6	0	0	0	0	78	None
		.7	0	0	0	0	Unstable	100

TABLE II

Feedforward Estimator Characteristics
for Seven Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Feed-forward	7	.1	Ø	Ø	Ø	Ø	8Ø	None
			1	Ø	Ø	Ø	8Ø	5
			Ø	1	Ø	Ø	8Ø	2
			Ø	Ø	1	Ø	8Ø	2
			Ø	Ø	Ø	.2	8Ø	6
			1	1	1	.2	8Ø	1Ø
		.Ø1	Ø	Ø	Ø	Ø	6ØØ	None
			1	Ø	Ø	Ø	6ØØ	5
			Ø	1	Ø	Ø	6ØØ	<1
			Ø	Ø	1	Ø	6ØØ	2
			Ø	Ø	Ø	.2	6ØØ	4
			1	1	1	.2	6ØØ	1Ø
		.ØØ1	Ø	Ø	Ø	Ø	5ØØØ	None
			1	Ø	Ø	Ø	5ØØØ	<1
			Ø	1	Ø	Ø	5ØØØ	<1
			Ø	Ø	1	Ø	5ØØØ	<1
			Ø	Ø	Ø	.2	5ØØØ	<1
			1	1	1	.2	5ØØØ	<1
		.2	Ø	Ø	Ø	Ø	1ØØ	None
		.3	Ø	Ø	Ø	Ø	Unstable	1ØØ

TABLE III

Feedforward Estimator Characteristics
For Ten Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Feed-forward	10	.1	0	0	0	0	100	None
			1	0	0	0	100	2
			0	1	0	0	100	<1
			0	0	1	0	100	<1
			0	0	0	.2	100	8
			0	0	0	.2	100	10
		.01	0	0	0	0	650	None
			1	0	0	0	650	3
			0	1	0	0	650	<1
			0	0	1	0	600	2
			0	0	0	.2	650	4
			1	1	1	.2	600	8
		.001	0	0	0	0	5000	None
			1	0	0	0	5000	<1
			0	1	0	0	5000	<1
			0	0	1	0	5000	<1
			0	0	0	.2	5000	<1
			1	1	1	.2	5000	<1
		.2	0	0	0	0	Unstable	100

TABLE IV

Feeback Estimator Characteristics
for Three Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Feedback	3	.1	0	0	0	0	70	None
			1	0	0	0	70	3
			0	1	0	0	70	2
			0	0	1	0	70	<1
			0	0	0	.2	70	4
			1	1	1	.2	70	8
		.01	0	0	0	0	70	None
			1	0	0	0	70	5
			0	1	0	0	70	<1
			0	0	1	0	70	2
			0	0	0	.2	70	4
			1	1	1	.2	75	10
		.001	0	0	0	0	5000	None
			1	0	0	0	5000	<1
			0	1	0	0	5000	<1
			0	0	1	0	5000	<1
			0	0	0	.2	5000	<1
			1	1	1	.2	5000	<1
		.2	0	0	0	0	40	None
		.3	0	0	0	0	30	None
		.4	0	0	0	0	18	None
		.5	0	0	0	0	12	None
		.5	0	0	0	0	12	None
		.6	0	0	0	0	8	None
		.7	0	0	0	0	6	None
		.9	0	0	0	0	3	None
		1.0	0	0	0	0	1	None

TABLE V

Feedback Estimator Characteristics
for Seven Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Feedback	7	.1	0	0	0	0	70	None
			1	0	0	0	70	8
			0	1	0	0	70	1
			0	0	1	0	70	<1
			0	0	0	.2	70	2
			1	1	1	.2	70	10
		.01	0	0	0	0	70	None
			1	0	0	0	70	4
			0	1	0	0	70	<1
			0	0	1	0	70	2
			0	0	0	.2	70	4
			1	1	1	.2	70	6
		.001	0	0	0	0	5000	None
			1	0	0	0	5000	<1
			0	1	0	0	5000	<1
			0	0	1	0	5000	<1
			0	0	0	.2	5000	<1
			1	1	1	.2	5000	<1
		.2	0	0	0	0	40	None
		.3	0	0	0	0	30	None
		.4	0	0	0	0	18	None
		.5	0	0	0	0	12	None
		.6	0	0	0	0	12	None
		.7	0	0	0	0	8	None
		.8	0	0	0	0	6	None
		.9	0	0	0	0	3	None
		1.0	0	0	0	0	1	None

TABLE VI

Feedback Estimator Characteristics
for Ten Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR △	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Feedback	1Ø	.1	Ø	Ø	Ø	Ø	8Ø	None
			1	Ø	Ø	Ø	8Ø	4
			Ø	1	Ø	Ø	8Ø	<1
			Ø	Ø	1	Ø	8Ø	<1
			Ø	Ø	Ø	.2	8Ø	4
			1	1	1	.2	8Ø	8
		.Ø1	Ø	Ø	Ø	Ø	62	None
			1	Ø	Ø	Ø	62	6
			Ø	1	Ø	Ø	62	2
			Ø	Ø	1	Ø	62	2
			Ø	Ø	Ø	.2	62	2
			1	1	1	.2	62	8
		.ØØ1	Ø	Ø	Ø	Ø	5ØØØ	None
			1	Ø	Ø	Ø	5ØØØ	<1
			Ø	1	Ø	Ø	5ØØØ	<1
			Ø	Ø	1	Ø	5ØØØ	<1
			Ø	Ø	Ø	.2	5ØØØ	<1
			1	1	1	.2	5ØØØ	<1
		.5	Ø	Ø	Ø	Ø	1Ø	None
		1.Ø	Ø	Ø	Ø	Ø	1	None

TABLE VII

Lattice Estimator Characteristics
for Three Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	WEIGHTING FACTOR Δ_1	Δ_2	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Lattice	3	.01	.1	0	0	0	0	78	None
				1	0	0	0	78	10
				0	1	0	0	78	1
				0	0	1	0	78	1
				0	0	0	.2	78	1
				1	1	1	.2	78	10
		.01	.01	0	0	0	0	400	None
				1	0	0	0	400	4
				0	1	0	0	400	<1
				0	0	1	0	400	<1
				0	0	0	.2	400	4
				1	1	1	.2	400	12
		.001	.001	0	0	0	0	5000	None
				1	0	0	0	5000	1
				0	1	0	0	5000	<1
				0	0	1	0	5000	<1
				0	0	0	.2	5000	<1
				1	1	1	.2	5000	<1

TABLE VIII

Lattice Estimator Characteristics
for Seven Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ_1	Δ_2	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Lattice	7	.Ø1	.1	Ø	Ø	Ø	Ø	8Ø	None
				1	Ø	Ø	Ø	8Ø	12
				Ø	1	Ø	Ø	8Ø	5
				Ø	Ø	1	Ø	8Ø	5
				Ø	Ø	Ø	.2	8Ø	7
				1	1	1	.2	8Ø	24
		.Ø1	.Ø1	Ø	Ø	Ø	Ø	4ØØ	None
				1	Ø	Ø	Ø	4ØØ	5
				Ø	1	Ø	Ø	4ØØ	<1
				Ø	Ø	1	Ø	4ØØ	<1
				Ø	Ø	Ø	.2	4ØØ	2
				1	1	1	.2	4ØØ	12
		.ØØ1	.ØØ1	Ø	Ø	Ø	Ø	5ØØØ	None
				1	Ø	Ø	Ø	5ØØØ	<1
				Ø	1	Ø	Ø	5ØØØ	<1
				Ø	Ø	1	Ø	5ØØØ	<1
				Ø	Ø	Ø	.2	5ØØØ	<1
				1	1	1	.2	5ØØØ	<1

TABLE IX

Lattice Estimator Characteristics
for Ten Weight Case

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ_1	Δ_2	CHANNEL CHARACTERISTICS B1	B2	G1	NOISE (RMS)	CONVERGENCE (NUMBER OF SAMPLES)	DEVIATON (PERCENT)
Lattice	10	.01	.1	0	0	0	0	90	None
				1	0	0	0	90	20
				0	1	0	0	90	8
				0	0	1	0	90	8
				0	0	0	.2	90	16
				1	1	1	.2	90	40
		.01	.01	0	0	0	0	400	None
				1	0	0	0	400	4
				0	1	0	0	400	<1
				0	0	1	0	400	<1
				0	0	0	.2	400	2
				1	1	1	.2	400	12
		.001	.001	0	0	0	0	5000	None
				1	0	0	0	5000	<1
				0	1	0	0	5000	<1
				0	0	1	0	5000	<1
				0	0	0	.2	5000	<1
				1	1	1	.2	5000	<1

whenever quadratic distortion (B1) or Gaussian noise was added. The effects of cubic phase distortion (B2) and amplitude distortion (G1) caused less than half as much deviation in general.

All three estimators tended to react similarly to the distortions and/or noise to the channel regardless of the number of weights for the selected convergence factors with the exception of the Lattice Estimator with $\Delta 1=.01$ and $\Delta 2=.1$. In this case, the Lattice Estimator demonstrated increased distortion depending on the number of weights whenever quadratic phase distortion (B1) and/or noise was present. The other estimators exhibited no such behavior at $\Delta=.1$.

Range of Convergence Factors

The convergence factors control the rate of adaptation of the estimators. The weight was considered to have converged when the deviation is less than .01% from its final rms value. First, various of convergence factors were tried on the estimators with the channel having no-distortion and no-noise on the input symbols. As shown in Tables I - III, the Feedforward Estimator exhibits a range of convergence factors up to and including =.6 for the three weight case (G=3), up to and including Δ=.2 for the seven weight case (G=7) and Δ=.1 for tne ten weight (G=10) case indicating a limitation of the convergence factor based on the number of weights in the estimator. The Feedback Estimator will converge with up to and including $\Delta 1$=1.0 regardless of the number of weights (see Table IV - VI). The Lattice Estimator demonstrated a maximum set of convergence factors of $\Delta 1$=.01 and $\Delta 2$=.1 independent of the number of weights (see Tables VII - IX). No lower bound was found as all estimators exhibited similar convergences for convergence factors of .001 and below.

Response to Errors

To simulate the effects of errors made by the detector or the channel, single and multiple data errors were fed to each of the estimators. The results are shown in Table X. For single data point errors, all three Estimators show similar recovery times when compared at identical channel conditions. The only exception is at the Δ=.1 (Δ1=.01, Δ2=.1) case for no-noise, no-distortion where the Lattice Estimator recovered significantly more quickly than the Feedback Estimator which in turn recovered more quickly than the Feedforward Estimator (see Figures 47-59 and 55-57).

When multiple consecutive errors were introduced,such as with burst noise, both the Lattice and Feedback Estimators were almost 20 samples faster to reconverge than the Feedforward Estimator which required 80 samples at Δ=.1. At the other selected weighting factors, however, performance among the three was comparable (See Table X and Figures 50-54 and 58-60).

TABLE X

Error Behavior of the Estimators

ESTIMATOR TYPE	NUMBER OF STAGES	CONVERGENCE FACTOR Δ	$\Delta 1$	$\Delta 2$	NUMBER OF ERRORS	NUMBER OF SAMPLES TO RECOVER
Feed-forward	3	.1	--	--	1	65
		.Ø1	--	--	1	6
		.ØØ1	--	--	1	2
		.1	--	--	1Ø	8Ø
		.Ø1	--	--	1Ø	3Ø
		.ØØ1	--	--	1Ø	1Ø
Feedback	3	.1	--	--	1	5Ø
		.Ø1	--	--	1	4
		.ØØ1	--	--	1	1
		.1	--	--	1Ø	62
		.Ø1	--	--	1Ø	29
		.ØØ1	--	--	1Ø	13
Lattice	3	--	.Ø1	.1	1	38
		--	.Ø1	.Ø1	1	8
		--	.ØØ1	.ØØ1	1	2
		--	.Ø1	.1	1Ø	6Ø
		--	.Ø1	.Ø1	1Ø	28
		--	.ØØ1	.ØØ1	1Ø	11

Response to Changing Channel Characteristics

The final characteristic to be examined was the response of the estimators to a change in channel characteristics (See Figures 61-63). The channel switched from no-noise, no-distortion condition to one of maximum amplitude and phase distortion and noise. None of the estimators worked well with a weighting factor of Δ=.1 (Δ1=.01, Δ2=.1). At other weighting factors the estimators all adapted at approximately the same rate with no significant adaptation time.

4. Conclusions and Recommendations

All three types of estimators have similar characteristics as the convergence factors fall below .ØØ1 . The deviation from distortion and noise becomes smaller due to less emphasis on what any single value is but instead more on the general trend. It is only at convergence factors of .Ø1 and higher that any decision can be made as to the desireability of one estimator over the others made by its demonstrated performance.

At convergence factors of .Ø1 and higher, the clear choice of the best estimator is the Feedback Estimator. Not only does it have the largest range of convergence factors (up to Δ=1.Ø) but it is not effected by the number of weights it has as was the case for the Lattice and Feedforward Estimators under certain conditions. In addition, the Feedback Estimator converged more quickly at a lower convergence factor than did the others.

The Feedback Estimator demonstrated a single bit error recovery capability that was superior to the Feedforward Estimator under the test conditions and to the Lattice Estimator in two out of three test cases. Against multiple errors, it was again superior to the Feedforward Estimator and about equal to the Lattice Estimator in performance.

A final reason for choosing the Feedback Estimator, even if all performance had been the same, would have been the simplicity of implementation as opposed to the two other estimators. The Feedback Estimator requires only about one half as many multiplications as the next simplest.

Before implementing any of these estimators in conditions that require convergence factors of greater than .001 , the performance of the estimator would be greatly improved if there were an adaptive noise canceller used as a pre-filter. This would decrease the deviation due to the noise by a significant factor allowing the channel estimator free to deal with only the amplitude and phase distortion.

Appendix A

Sample Calculations for the Feedforward Adaptive Estimator

A simple case was chosen to illustrate the function of the Feedforward Estimator algorithm and to serve as a verification of the correct operation of the program. To simplify claculations, a three weight implmentation was chosen as is shown in Figure 9. The output of the detector X'_k was assumed to consist of the following sequence:

$$X'_k = [\ 1\ -1\ 1\ -1\ 1\ .\ .\ .\ ] \tag{A.1}$$

Furthermore, for these calculations, only the no-noise and no-distortion case will be considered so that $X'_k = Y'_k$. A convergence factor, Δ, was chosen to be Ø.5 . Utilizing Equation (2.6), where

$$H'_{k+1} = H'_k + \Delta E_k X'_{k-j} \tag{2.6}$$

the tap gain coefficients can be calculated.

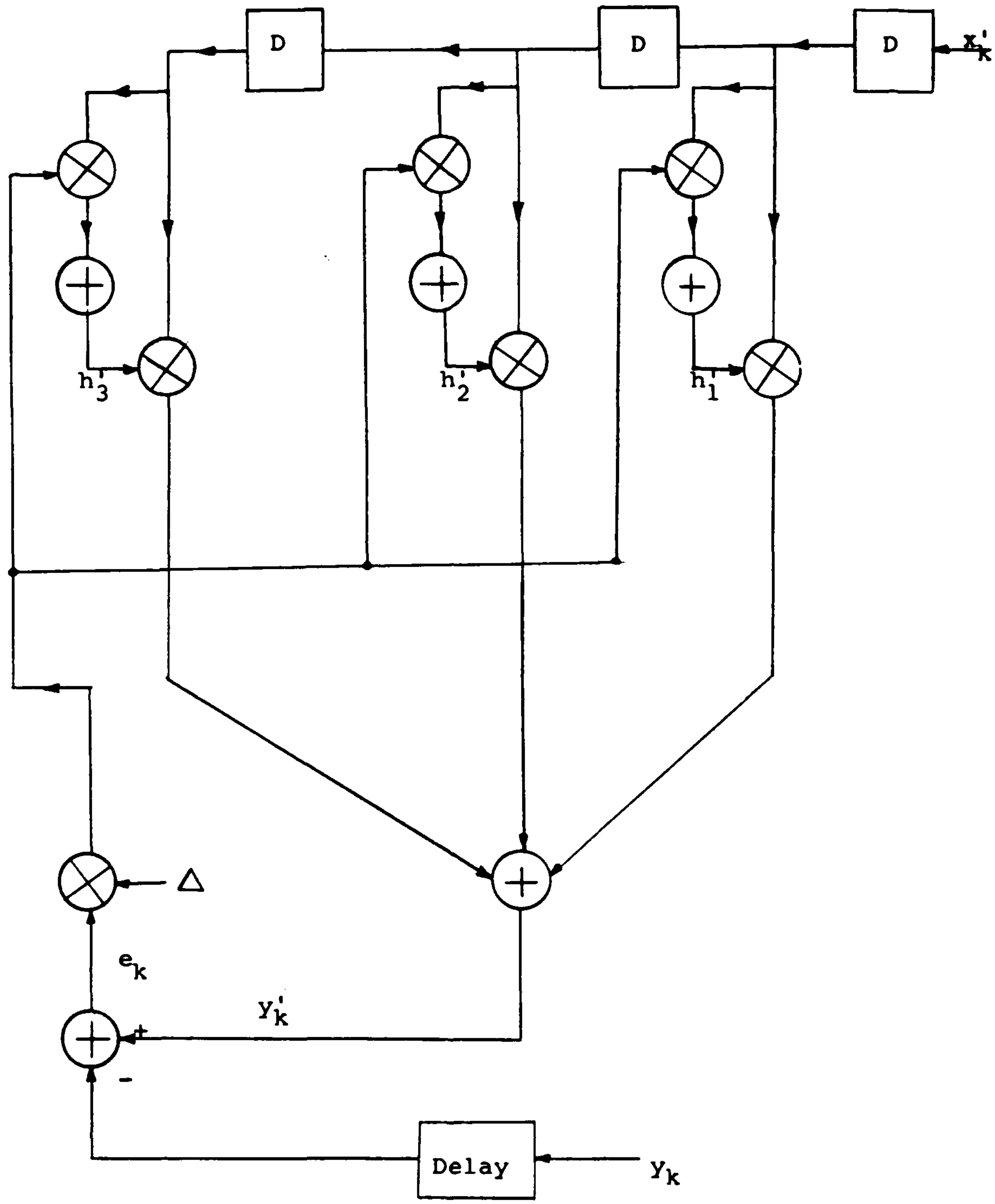

Figure 9. Feedforward Estimator

For the first input:

$$x'_1 = 1 \qquad x'_0 = 0 \qquad x'_{-1} = 0 \tag{A.2}$$

$$\begin{aligned} y'_1 &= (h'_{0,1})(x'_1) + (h'_{0,2})(x'_0) + (h'_{0,3})(x'_{-1}) \\ &= (0)(1) + (0)(0) + (0)(0) \\ &= 0 \end{aligned} \tag{A.3}$$

$$y_1 = 1 \tag{A.4}$$

$$\begin{aligned} e_1 &= y_1 - y'_1 \\ &= 1 - 0 \\ &= 1 \end{aligned} \tag{A.5}$$

$$\begin{aligned} h'_{1,1} &= h'_{0,1} + \Delta e_1 \; x'_1 \\ &= 0 + (.5)(1) \\ &= .5 \end{aligned} \tag{A.6}$$

$$\begin{aligned} h'_{1,2} &= h'_{0,2} + \Delta e_1 \; x'_0 \\ &= 0 + (.5)(0) \\ &= 0 \end{aligned} \tag{A.7}$$

$$\begin{aligned} h'_{1.3} &= h'_{0,3} + \Delta e_1 \; x'_{-1} \\ &= 0 + (.5)(0) \\ &= 0 \end{aligned} \tag{A.8}$$

For the second input:

$$x'_2 = -1 \qquad x'_1 = 1 \qquad x'_\emptyset = \emptyset \tag{A.9}$$

$$\begin{aligned} y'_2 &= (h'_{1,1})(x'_2) + (h'_{1,2})(x'_1) + (h'_{2,2})(x'_\emptyset) \\ &= (.5)(-1) + (\emptyset)(1) + (\emptyset)(\emptyset) \\ &= -.5 \end{aligned} \tag{A.1Ø}$$

$$y_2 = -1 \tag{A.11}$$

$$\begin{aligned} e_2 &= y_2 - y'_2 \\ &= -1 - (-.5) \\ &= -.5 \end{aligned} \tag{A.12}$$

$$\begin{aligned} h'_{2,1} &= h'_{1,1} + \Delta e_2\ x'_2 \\ &= (.5) + (.5)(-.5)(-1) \\ &= .75 \end{aligned} \tag{A.13}$$

$$\begin{aligned} h'_{2,2} &= h'_{1,2} + \Delta e_2\ x'_1 \\ &= (\emptyset) + (.5)(-.5)(1) \\ &= -.25 \end{aligned} \tag{A.14}$$

$$\begin{aligned} h'_{2,3} &= h'_{1,3} + \Delta e_2\ x'_\emptyset \\ &= (\emptyset) + (.5)(-.5)(\emptyset) \\ &= \emptyset \end{aligned} \tag{A.15}$$

For a third input:

$$x'_3 = 1 \qquad x'_2 = -1 \qquad x'_1 = 1 \tag{A.16}$$

$$\begin{aligned} y'_3 &= (h'_{2,1})(x'_3) + (h'_{2,2})(x'_2) + (h'_{2,3})(x'_1) \\ &= (.75)(1) + (-.25)(1) + (\emptyset)(1) \\ &= 1 \end{aligned} \tag{A.17}$$

$$y_3 = 1 \tag{A.18}$$

$$\begin{aligned} e_3 &= y_3 - y'_3 \\ &= 1 - 1 \\ &= \emptyset \end{aligned} \tag{A.19}$$

$$\begin{aligned} h'_{3,1} &= h'_{2,1} + \Delta e_3\, x'_3 \\ &= (.75) + (.5)(\emptyset)(1) \\ &= .75 \end{aligned} \tag{A.2\emptyset}$$

$$\begin{aligned} h'_{3,2} &= h'_{2,2} + \Delta e_3\, x'_2 \\ &= (-.25) + (.5)(\emptyset)(-1) \\ &= -.25 \end{aligned} \tag{A.21}$$

$$\begin{aligned} h'_{3,3} &= h'_{2,3} + \Delta e_3\, x'_1 \\ &= (\emptyset) + (.5)(\emptyset)(1) \\ &= \emptyset \end{aligned} \tag{A.22}$$

After the second input, the error went to zero during the next iteration of updating the estimated tap gain coefficients and stayed there as long as the pattern of alternating 1's and -1's continued. This turned out not to be the general case as a random pattern of 1's and -1's do not converge to the tap gains as shown but rather to gains of $h_{k,1} = 1$, $h_{k,2} = 0$, and $h_{k,3} = 0$. It does however illustrate the typical calculations carried on by this algorithm.

Sample Calculations for the Feedback Estimator

The simple case of three taps was selected to illustrate the workings of the Feedback Estimator algorithm as shown in Figure 10. A no-noise, no-distortion condition was placed on the channel to simplify calculations. The convergence factor, Δ, was chosen to be .5 and an input and output stream to/from the estimator was assumed to be:

$$Y_k = [\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1] \tag{A.23}$$
$$= X'_k$$

Shifting in the first three inputs gives:

$$y_1 = 1 \qquad y_2 = -1 \qquad y_2 = -1 \tag{A.24}$$

and

$$x'_1 = 1 \qquad x'_2 = -1 \qquad x'_3 = -1 \tag{A.25}$$

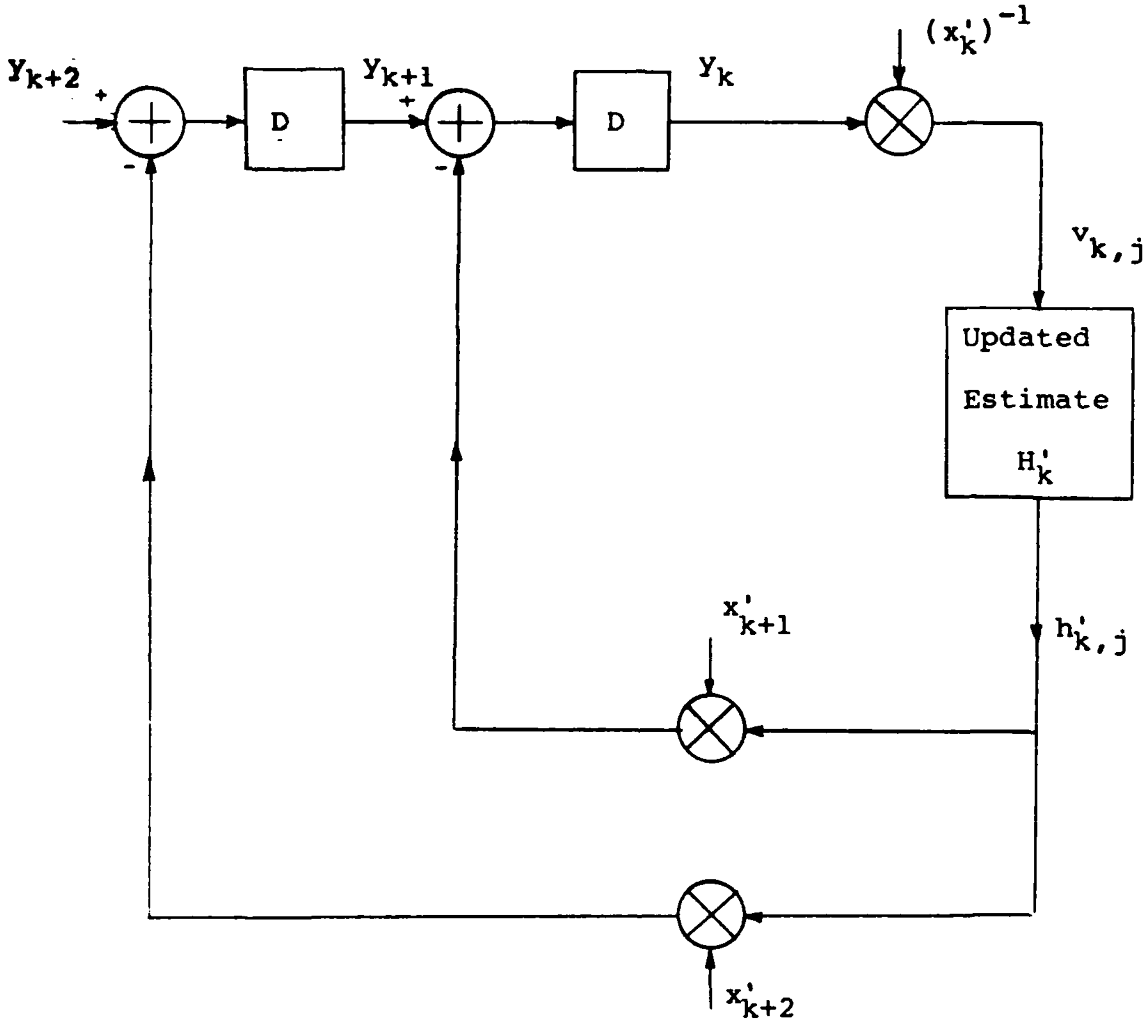

Figure 1Ø. Feedback Estimator

Calculating $h'_{1,1}$ (all previous values of $h'_{\emptyset,j}$ are zero to start):

$$v_{1,1} = y_1 / x'_1 \qquad \text{(A.26)}$$
$$= 1 / 1$$
$$= 1$$

$$h'_{1,1} = \Delta v_{1,1} + (1-\Delta)\, h'_{\emptyset,1} \qquad \text{(A.27)}$$
$$= (.5)(1) + (1-.5)\ (\emptyset)$$
$$= .5$$

The values are updated as shown in Figure and the result shifted to the right. No new data is input to the front of the estimator at this time, however.

$$y_1 = y_2 - (x'_2)(h'_{1,1}) \qquad \text{(A.28)}$$
$$= -1 - (-1)(.5)$$
$$= -.5$$

$$y_2 = y_3 - (x'_3)(h'_{1,1}) \qquad \text{(A.29)}$$
$$= -1 - (-1)(.5)$$
$$= -.5$$

Repeating the above process for the other two tap gains

$$v_{1,2} = y_1 \;/\; x'_1 \tag{A.30}$$
$$= -.5 \;/\; 1$$
$$= -.5$$

$$h'_{1,2} = \Delta v_{1,2} + (1-\Delta)\, h'_{0,2} \tag{A.31}$$
$$= (.5)(-.5) + (1-.5)(0)$$
$$= -.25$$

$$y_1 = y_2 - (x'_2)(h'_{1,2}) \tag{A.32}$$
$$= -.5 - (-1)(-.25)$$
$$= -.75$$

$$v_{1,3} = y_1 \;/\; x'_1 \tag{A.33}$$
$$= -.75 \;/\; 1$$
$$= -.75$$

$$h'_{1,3} = \Delta v_{1,3} + (1-\Delta)\, h'_{0,3} \tag{A.34}$$
$$= (.5)(-.75) + (1-.5)(0)$$
$$= -.375$$

Therefore for the first iteration the tap gains are:

$$H'_1 = [.5, -.25, -.375] \tag{A.35}$$

During the next and following iterations, a new point is read and set equal to y_{k+2} and the preceding two bits become y'_{k+1} and y'_{k+2} respectively so that

$$y_2 = -1 \qquad y_3 = -1 \qquad y_4 = -1 \tag{A.36}$$

and

$$x'_2 = -1 \qquad x'_3 = -1 \qquad x'_4 = -1 \tag{A.37}$$

Repeating the above calculations for the three tap gains:

$$\begin{aligned} v_{2,1} &= y_2 \,/\, x'_2 \\ &= (-1) \,/\, (-1) \\ &= 1 \end{aligned} \tag{A.38}$$

$$\begin{aligned} h_{2,1} &= \Delta v_{2,1} - (1-\Delta)\, h_{2,1} \\ &= (.5)(1) - (1-.5)(.5) \\ &= .75 \end{aligned} \tag{A.39}$$

$$y_2 = y_3 - (x'_3)\ h_{2,1} \tag{A.40}$$
$$= (-1) - (-1)(.75)$$
$$= -.25$$

$$y_3 = y_4 - (x'_4)\ h_{2,1} \tag{A.41}$$
$$= (-1) - (-1)(.75)$$
$$= -.25$$

$$v_{2,2} = y_2\ /\ x'_2 \tag{A.42}$$
$$= (-.25)\ /\ (-1)$$
$$= .25$$

$$h'_{2,2} = \Delta v_{2,2} + (1-\Delta)\ h_{1,2} \tag{A.43}$$
$$= (.5)(.25) + (1-.5)(-.25)$$
$$= \emptyset$$

$$y_2 = y_3 - (x'_3)\ h_{2,2} \tag{A.44}$$
$$= -.25 - (-1)(\emptyset)$$
$$= -.25$$

$$v_{2,3} = y_2 \;/\; x'_2 \tag{A.45}$$
$$= (-.25) \;/\; (-1)$$
$$= .25$$

$$h'_{2,3} = \Delta v_{2,2} + (1-\Delta)\; h_{1,3} \tag{A.46}$$
$$= (.5)(.25) + (1-.5)(-.375)$$
$$= .0625$$

Therefore, for the second iteration the tap gains are

$$H'_2 = [.75,\ 0,\ -.0625\ ] \tag{A.47}$$

The calculations can be continued until the tap gains become

$$H'_k = [1,\ 0,\ 0\ ] \tag{A.48}$$

which was experimentally verified.

Sample Calculations for the Lattice Estimator

The simple case of three tap gains and a two stage lattice filter was chosen to illustrate the algorithm used as a lattice estimator as shown in Figure 7. The no-noise, no-distortion case was chosen to simplify calculations. Both Δ_1 and Δ_2 were chosen to be 0.1 . The input and output data stream was assumed to be:

$$Y_k = [\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ ] \tag{A.49}$$

All weights (k's and h's) are assumed to be zero at the start as are any previous outputs (x's).

For the first input to the estimator:

$$x'_1 = x_{1,1} = x'_{1,1} = 1 \tag{A.50}$$

$$\begin{aligned} x_{1,2} &= x_{1,1} + k_{0,1}\, x'_{0,1} \\ &= 1 + (0)(0) \\ &= 1 \end{aligned} \tag{A.51}$$

$$\begin{aligned} x_{1,3} &= x_{1,2} + k_{0,2}\, x'_{0,2} \\ &= 1 + (0)(0) \\ &= 1 \end{aligned} \tag{A.52}$$

$$x'_{1,2} = k_{\emptyset,1}\, x_{1,1} + x'_{\emptyset,1} \tag{A.53}$$
$$= (\emptyset)\ (1) + (\emptyset)$$
$$= \emptyset$$

$$x'_{1,3} = k_{\emptyset,2}\, x_{1,2} + x'_{\emptyset,2} \tag{A.54}$$
$$= (\emptyset)\ (1) + (\emptyset)$$
$$= \emptyset$$

$$e_{f1} = x_{1,3} \tag{A.55}$$
$$= 1$$

$$e_{b1} = x'_{1,3} \tag{A.56}$$
$$= \emptyset$$

$$k_{1,1} = k_{\emptyset,1} - \Delta_1\, x_{1,2}\, x'_{\emptyset,1} \tag{A.57}$$
$$= (\emptyset) - (.1)(1)(\emptyset)$$
$$= \emptyset$$

$$k_{1,2} = k_{\emptyset,2} - \Delta_1\, x_{1,3}\, x'_{\emptyset,2} \tag{A.58}$$
$$= (\emptyset) - (.1)(1)(\emptyset)$$
$$= \emptyset$$

Updating the transversal filter portion for the first iteration:

$$y'_1 = h'_{0,1}\,x'_{1,1} + h'_{0,2}\,x'_{1,2} + h'_{0,3}\,x'_{1,3} \tag{A.59}$$
$$= (0)\ (1) + (0)\ (0) + (0)\ (0)$$
$$= (0)$$

$$e_1 = y_1 - y'_1 \tag{A.60}$$
$$= 1 - 0$$
$$= 1$$

$$h'_{1,1} = h'_{0,1} + \Delta_2\, e_1\, x'_{1,1} \tag{A.61}$$
$$= (0) + (.1)\ (1)\ (1)$$
$$= .1$$

$$h'_{1,2} = h'_{0,2} + \Delta_2\, e_1\, x'_{1,2} \tag{A.62}$$
$$= (0) + (.1)(1)(0)$$
$$= 0$$

$$h'_{1,3} = h'_{0,3} + \Delta_2\, e_1\, x'_{1,3} \tag{A.63}$$
$$= (0) + (.1)(1)(0)$$
$$= 0$$

Therefore for the first iteration the various gains are

$$K_1 = [\ 0,0\ ] \tag{C.64}$$

and

$$H_1 = [\ .1,\ 0,\ 0\ ] \tag{C.65}$$

When the next input to the estimator is read in the process begins again with the following results:

$$x'_2 = x_{2,1} = x'_{2,1} = -1 \tag{C.66}$$

$$\begin{aligned} x_{2,2} &= x_{2,1} + k_{1,1}\ x'_{1,1} \\ &= -1 + (0)(0) \\ &= -1 \end{aligned} \tag{C.67}$$

$$\begin{aligned} x_{2,3} &= x_{2,2} + k_{1,2}\ x'_{1,2} \\ &= -1 + (0)(0) \\ &= -1 \end{aligned} \tag{C.68}$$

$$\begin{aligned} x'_{2,2} &= k_{1,1}\ x_{2,1} + x'_{1,1} \\ &= (0)(-1) + (1) \\ &= 1 \end{aligned} \tag{C.69}$$

$$\begin{aligned} e_{f2} &= x_{2,3} \\ &= -1 \end{aligned} \tag{C.70}$$

$$e_{b2} = x'_{2,3} = \emptyset \tag{A.71}$$

$$\begin{aligned} k_{2,1} &= k_{1,1} - \Delta_1 \, x_{2,2} \, x'_{1,1} \\ &= (\emptyset) - (.1)(-1)(1) \\ &= .1 \end{aligned} \tag{A.72}$$

$$\begin{aligned} k_{2,2} &= k_{1,2} - \Delta_1 \, x_{2,3} \, x'_{1,2} \\ &= (\emptyset) - (.1)(-1)(\emptyset) \\ &= \emptyset \end{aligned} \tag{A.73}$$

Updating the transversal filter portion for the second iteration:

$$\begin{aligned} y'_2 &= h'_{1,1} \, x'_{2,1} + h'_{1,2} \, x'_{2,2} + h'_{1,3} \, x'_{2,3} \\ &= (.1)(-1) + (\emptyset)(1) + (\emptyset)(\emptyset) \\ &= -.1 \end{aligned} \tag{A.74}$$

$$\begin{aligned} e_2 &= y_2 - y'_2 \\ &= (-1) - (-.1) \\ &= -.9 \end{aligned} \tag{A.75}$$

$$\begin{aligned} h'_{2,1} &= h'_{1,1} + \Delta_2 \, e_2 \, x'_{2,1} \\ &= (.1) + (.1)(-.9)(-1) \\ &= .19 \end{aligned} \tag{A.76}$$

$$h'_{2,2} = h'_{1,2} + \Delta_2 \, e_2 \, x'_{2,2} \tag{A.77}$$
$$= (\emptyset) + (.1)(-.9)(1)$$
$$= -.\emptyset 9$$

$$h'_{2,3} = h'_{1,3} + \Delta_2 \, e_2 \, x'_{2,3} \tag{A.78}$$
$$= (\emptyset) + (.1)(-.9)(\emptyset)$$
$$= \emptyset$$

For the second iteration the various gains are

$$K_2 = [\ .1,\ \emptyset\ ] \tag{A.79}$$

and

$$H_2 = [\ .19,\ -.\emptyset 9,\ \emptyset\ ] \tag{A.8\emptyset}$$

For the third input the results are:

$$x'_3 = x_{3,1} = x'_{3,1} = -1 \tag{A.81}$$

$$x_{3,2} = x_{3,1} + k_{2,1} \, x'_{2,1} \tag{A.82}$$
$$= (-1) + (.1)(-1)$$
$$= -1.1$$

$$x_{3,3} = x_{3,2} + k_{2,2} \, x'_{2,2} \tag{A.83}$$
$$= (-1.1) + (\emptyset)(-1)$$
$$= (-1.1)$$

$$x'_{3,2} = k_{2,1}\, x_{3,1} + x'_{2,1} \tag{A.84}$$
$$= (.1)(-1) + (-1)$$
$$= -1.1$$

$$x'_{3,3} = k_{2,2}\, x_{3,2} + x'_{2,2} \tag{A.85}$$
$$= (\emptyset)(-1.1) + (1)$$
$$= 1$$

$$e_{f3} = x_{3,3} \tag{A.86}$$
$$= -1.1$$

$$e_{b3} = x'_{3,3} \tag{A.87}$$
$$= 1$$

$$k_{3,1} = k_{2,1} - \Delta_1\, x_{3,2}\, x'_{2,1} \tag{A.88}$$
$$= (.1) - (.1)(-1.1)(-1)$$
$$= -.\emptyset 1$$

$$k_{3,2} = k_{2,2} - \Delta_1\, x_{3,3}\, x'_{2,2} \tag{A.89}$$
$$= (\emptyset) - (.1)(-1.1)(1)$$
$$= .11$$

Updating the transversal filter portion for this iterations:

$$y'_3 = h'_{2,1}\, x'_{3,1} + h'_{2,2}\, x'_{3,2} + h'_{2,3}\, x'_{3,3} \qquad \text{(A.90)}$$

$$= (.19)\ (-1) + (-.09)(-1.1) + (0)(1.0)$$

$$= -.091$$

$$e_3 = y_3 - y'_3 \qquad \text{(A.91)}$$

$$= (-1) - (-.091)$$

$$= (-.909)$$

$$h'_{3,1} = h'_{2,1} + \Delta_2\, e_3\, x'_{3,1} \qquad \text{(A.92)}$$

$$= (.19) + (.1)(-.909)(-1.1)$$

$$= .281$$

$$h'_{3,2} = h'_{2,2} + \Delta_2\, e_3\, x'_{3,2} \qquad \text{(A.93)}$$

$$= (-.09) + (.1)\ (-.909)\ (-1.1)$$

$$= .01$$

$$h'_{3,3} = h'_{2,3} + \Delta_2\, e_3\, x'_{3,3} \qquad \text{(A.94)}$$

$$= (0) + (.1)(-.909)(1)$$

$$= -.091$$

After this iteration then the various gains are

$$K_3 = [\ -.01,\ .11\ ] \qquad \text{(A.95)}$$

and

$$H_3 = [\ .281,\ .010,\ -.091\ ] \qquad \text{(A.96)}$$

Appendix B

Derivation of the Lattice Filter

Of the various forms of filters, the direct, cascade and parallel forms are the best known. There are occassions however when a fourth form, the lattice, can be more effectively utilized such as the case of adaptive filtering.

The transfer function of a system can be rewritten in the Z-transform form of a filter. This is expressed as

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{A(z)}{1+B(z)} \\ &= \frac{a_0 + a_1 z^{-1} + \ldots + a_L z^{-1}}{1 + b_1 z^{-1} + \ldots + b_L z^{-1}} \end{aligned} \tag{B.1}$$

or alternately as

$$\begin{aligned} H(z) &= \frac{A_L(z)}{B_L(z)} \\ &= \frac{a_{L0} + a_{L1} z^{-1} + \ldots + a_{LL} z^{-L}}{b_{L0} + b_{L1} z^{-1} + \ldots + b_{LL} z^{-L}} \end{aligned} \tag{B.2}$$

where b_{L0} is always equal to one (9).

Starting with Eq (D.2) the following algorithm can be defined. For m = L, L-1, ... , 1 :

$$z\,C_m(z) = z^{-1}\,B_m(z^{-1}) \qquad \text{(B.3)}$$

$$k_{m-1} = b_{11} \qquad \text{(B.4)}$$

$$v_1 = a_{11} \qquad \text{(B.5)}$$

$$B_{m-1}(z) = \frac{B_m(z) - k_{m-1}\,z\,C_m(z)}{1 - k^2_{m-1}} \qquad \text{(B.6)}$$

$$A_{m-1}(z) = A(z) - v_m\,z\,C_m(z) \qquad \text{(B.7)}$$

where k_{m-1} and v_m are the coefficients used in the lattice. Using Eq (D.7) and substituting it into itself, $A_L(z)$ can be written as:

$$\begin{aligned}
A_L(z) &= A_{L-1}(z) + v_L\,z\,C_L(z) && \text{(B.8)}\\
&= A_{L-2}(z) + v_{L-1}\,z\,C_{L-1}(z) + v_L\,z\,C_L(z)\\
&\;\;\vdots\\
&= A_\emptyset(z) + v_1\,z\,C_1(z) + \dots + v_L\,z\,C_L(z)\\
&= a_\emptyset \quad + v_1\,z\,C_1(z) + \dots + v_L\,z\,C_L(z)\\
&= v_\emptyset \quad + v_1\,z\,C_1(z) + \dots + v_L\,z\,C_L(z)\\
&= \sum_{m=\emptyset}^{L} v_m\,z\,C_m(z) && \text{(B.9)}
\end{aligned}$$

Substituting for $A_L(z)$ in Eq (D.2) gives the transfer function

$$H(z) = \sum_{m=\emptyset}^{L} \frac{v_m \, z \, C_m(z)}{B_L(z)} \tag{B.1Ø}$$

so that

$$Y(z) = \sum_{m=\emptyset}^{L} \frac{v_m \, z \, C_m(z)}{B_L(z)} \; X(z) \tag{B.11}$$

Using the results developed by Itakura and Saito (1Ø), the following equations for $B_m(z)$ and $z\,C_m(z)$ can be developed:

$$B_m(z) = B_{m-1}(z) + k_{m-1} \, C_{m-1}(z) \tag{B.12}$$

and

$$zC_m(z) = k_{m-1} \, B_{m-1}(z) + z^{-1} \left[\, z \, C_{m-1}(z) \, \right] \tag{B.13}$$

Using Eqs (D.11), (D.12), and (D.13), the lattice element shown in Figure 12 can be created. The elements may be combined to form larger lattice structures as shown in Figure 13.

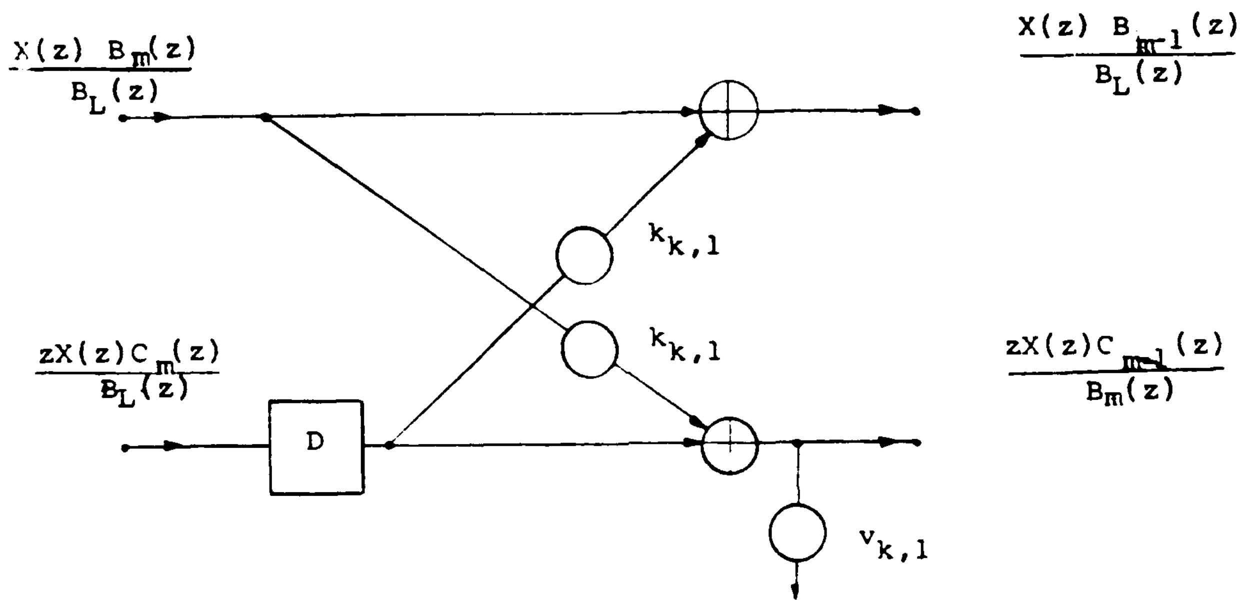

Figure 11. Single Stage Lattice

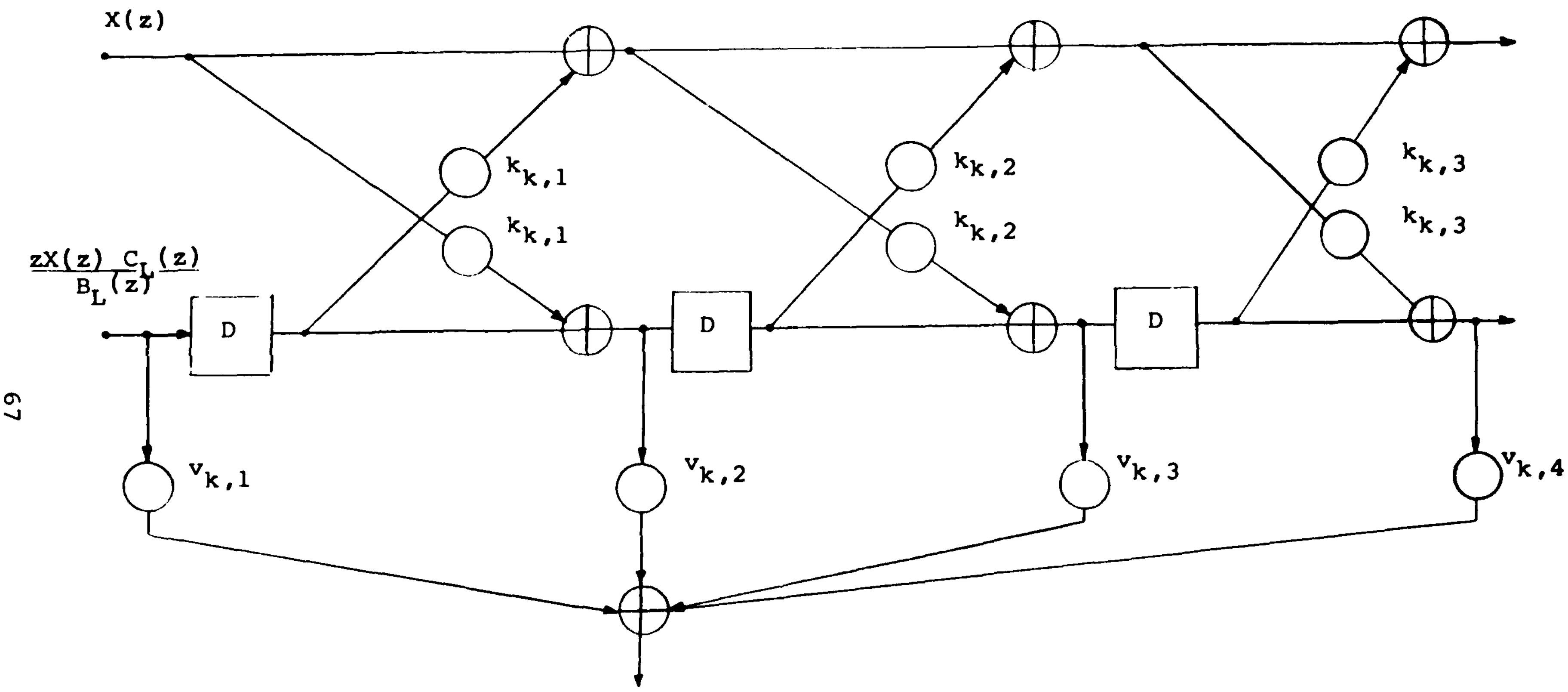

Figure 12. Multi-Stage Lattice

Appendix C

Results of Estimator Tests

This appendix contains selected outputs from the estimator tests. All graphs are of Weight 1, h_1 .

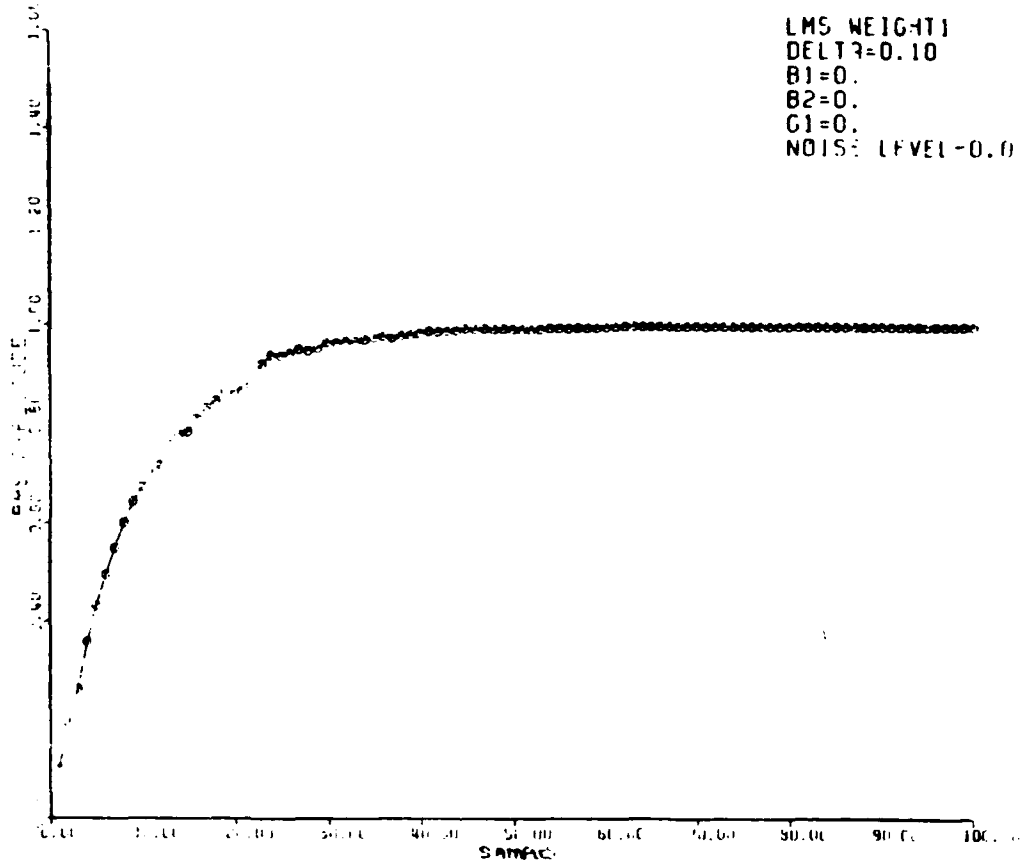

Figure 13. Feedforward Estimator, Three Weights, $\triangle$=.1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø

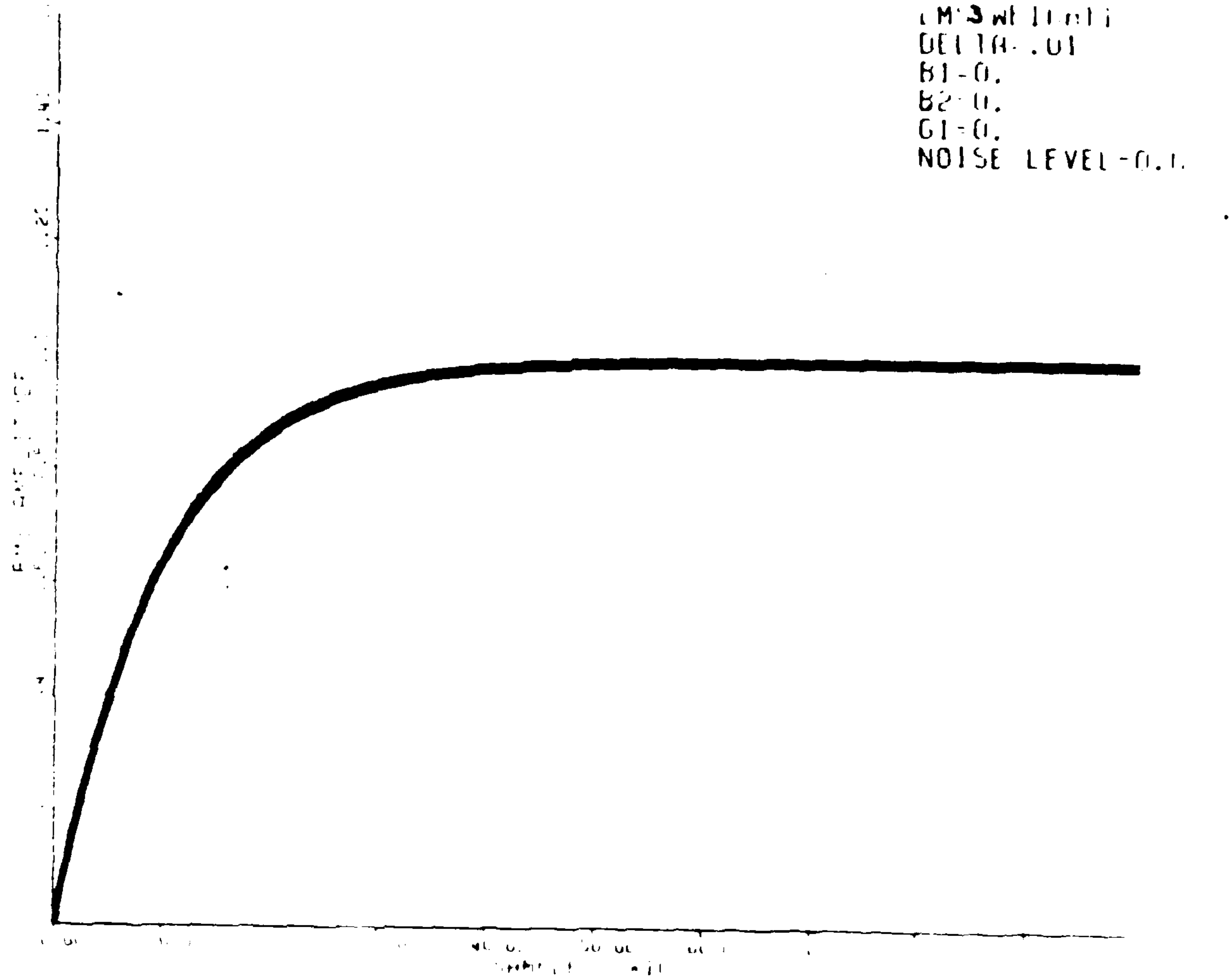

Figure 14. Feedforward Estimator, Three Weights, Δ=.01
B1=0, B2=0, G1=0, Noise=0

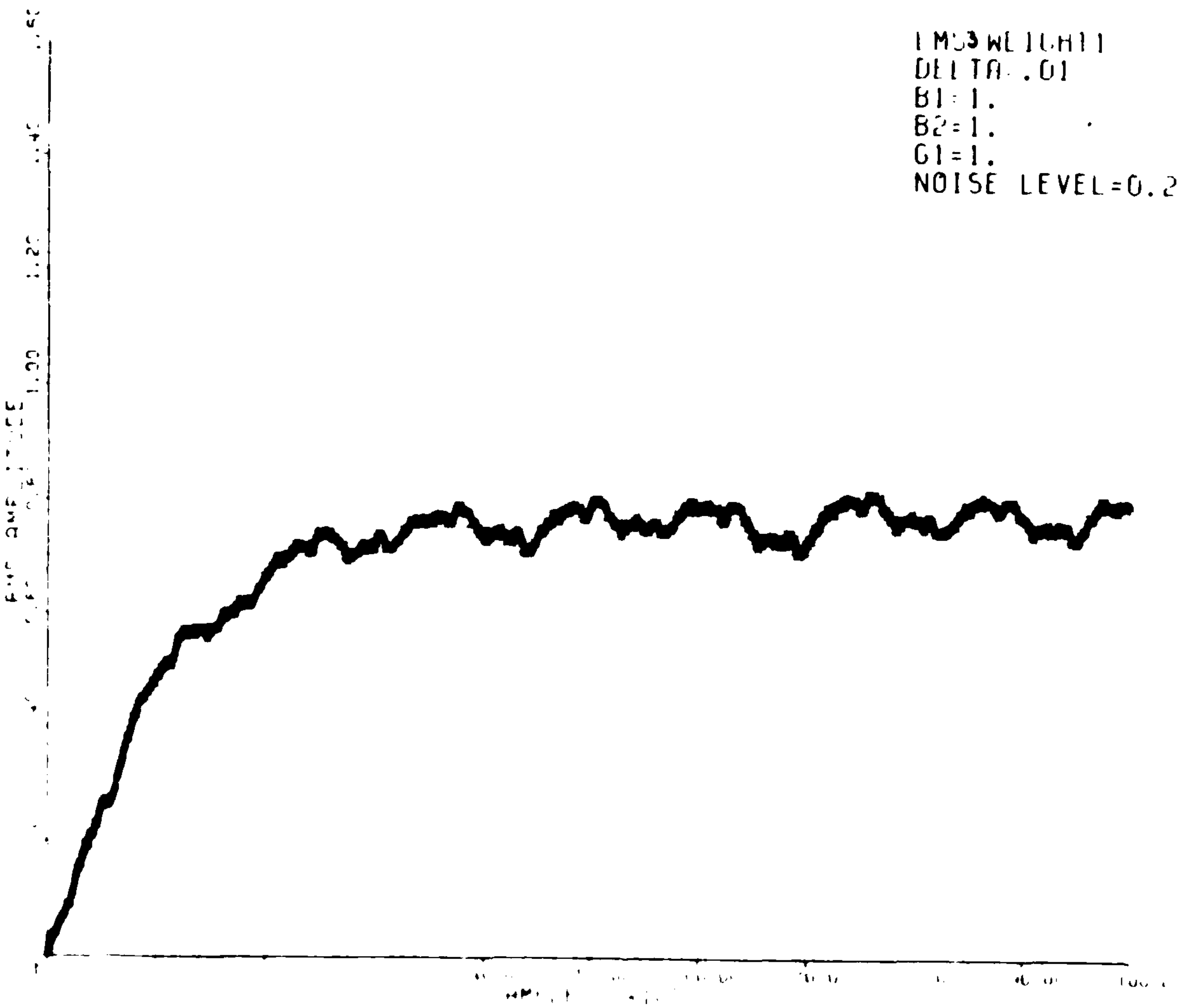

Figure 15. Feedforward Estimator, Three Weights, Δ=.01
B1=1, B2=1, G1=1, Noise=0.2

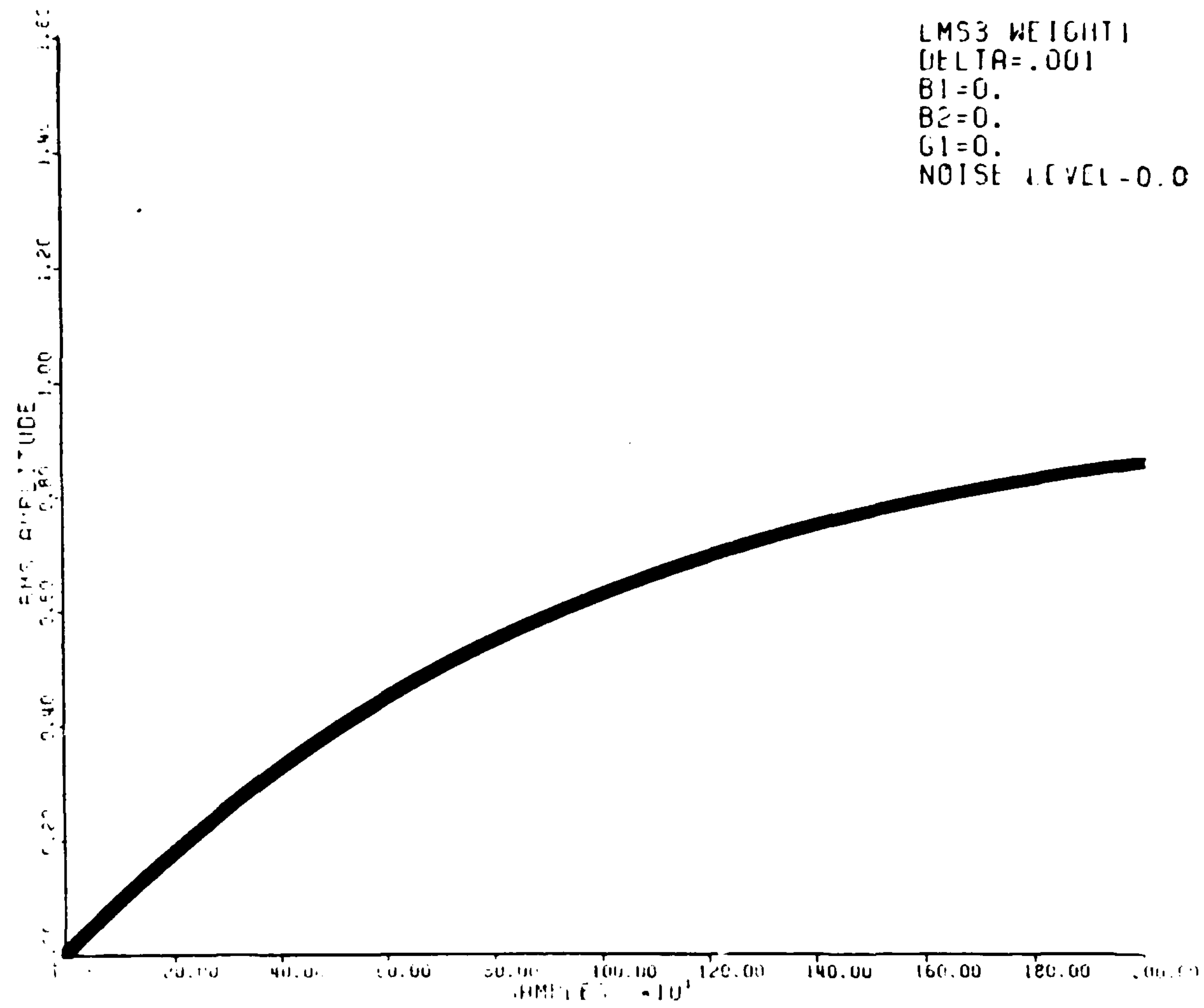

Figure 16. Feedforward Estimator, Three Weights, Δ=.001
B1=0, B2=0, G1=0, Noise=0

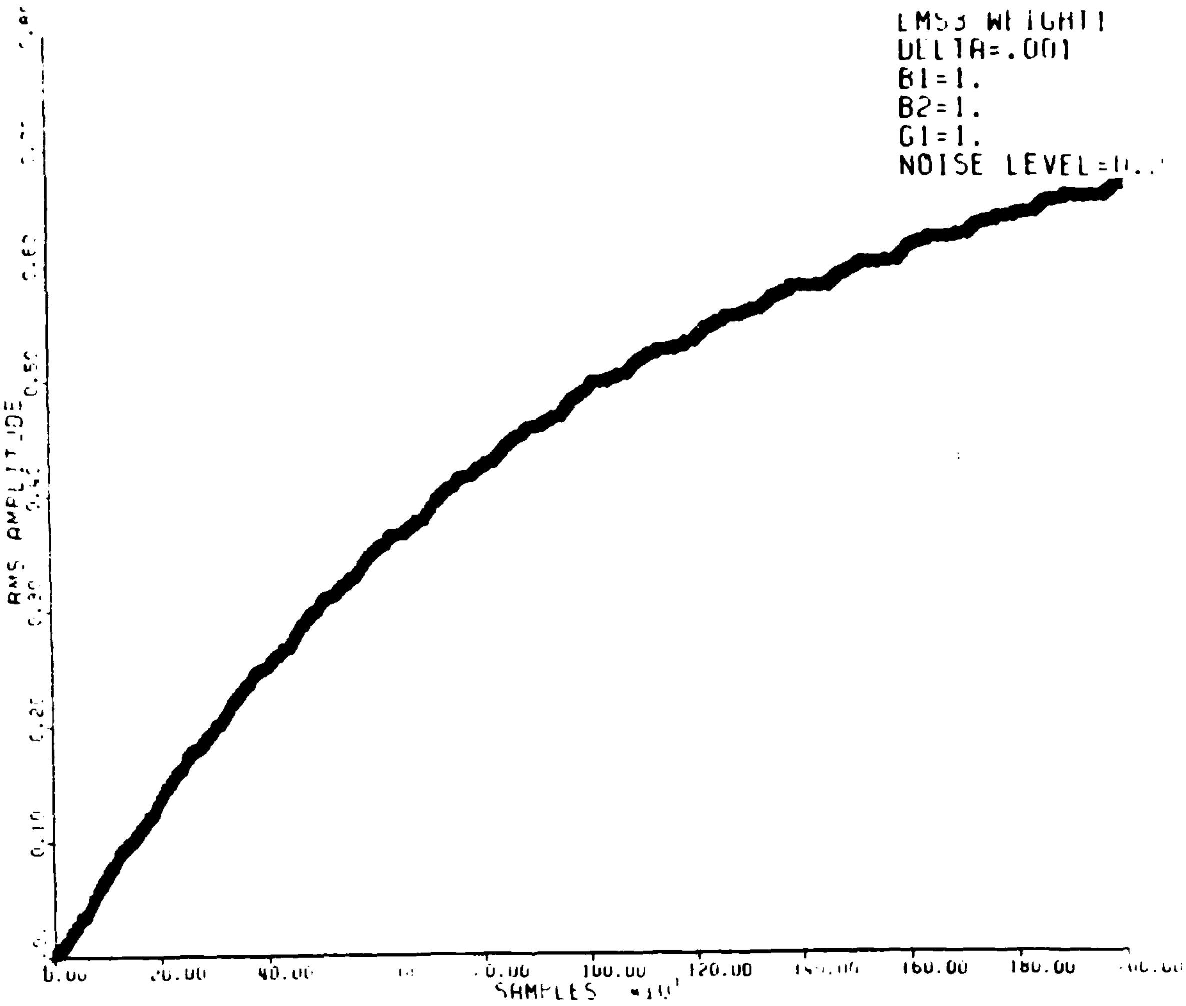

Figure 17. Feedforward Estimator, Three Weights, Δ=.001
B1=1, B2=1, G1=1, Noise=0.2

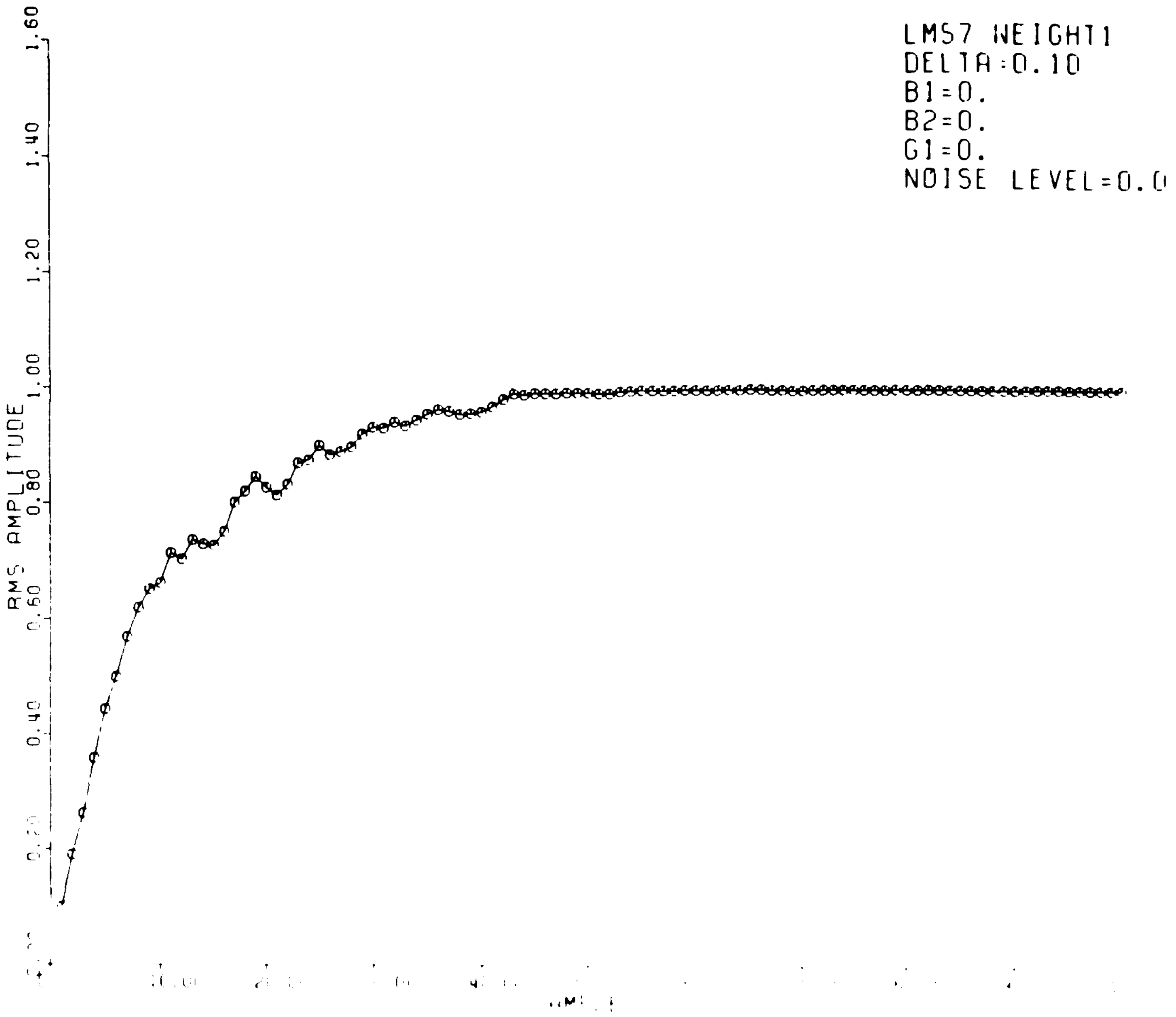

Figure 18. Feedforward Estimator, Seven Weights Δ=.1
B1=0, B2=0, G1=0, Noise=0

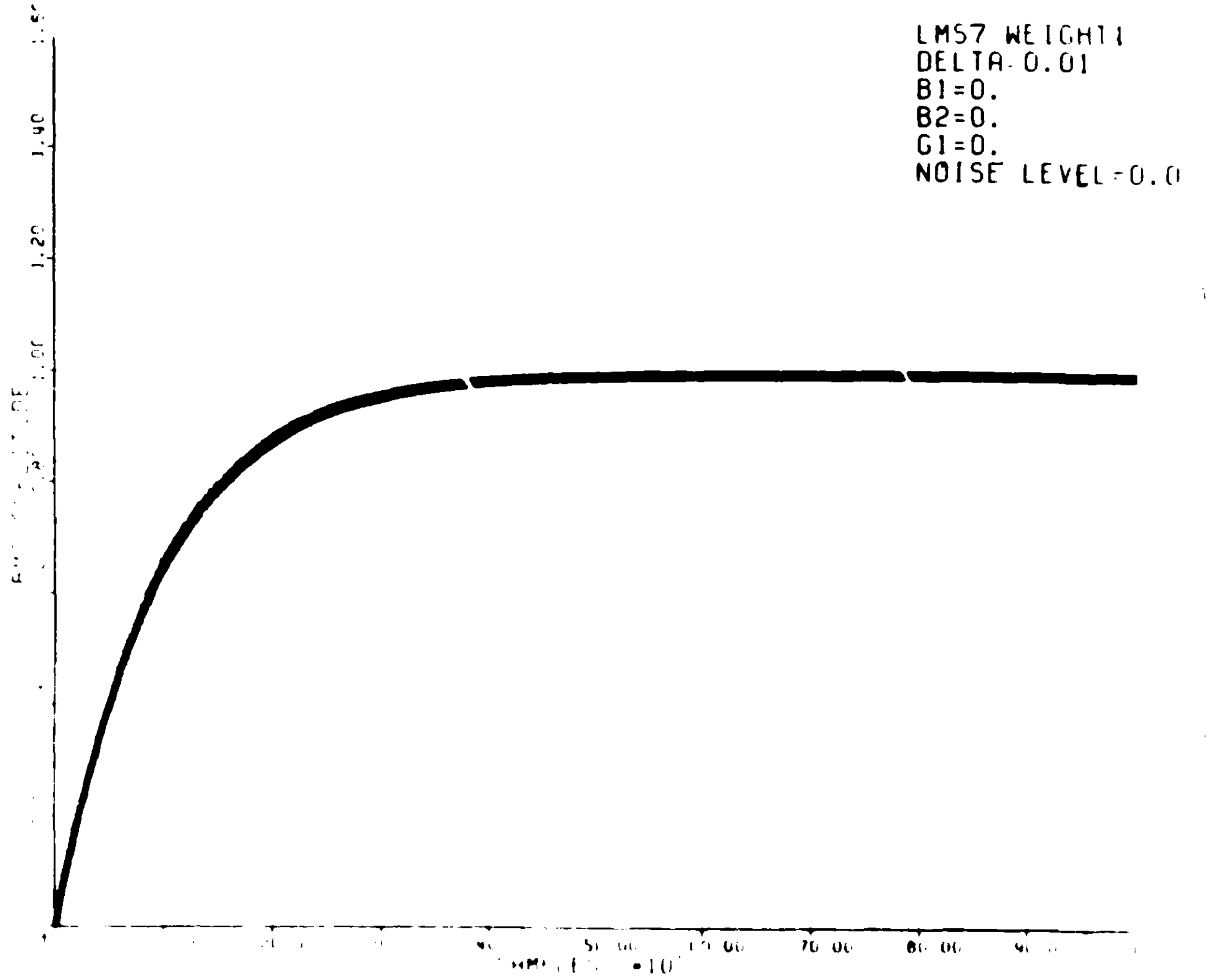

Figure 19. Feedforward Estimator, Seven Weights Δ =.01
B1=0, B2=0, G1=0, Noise=

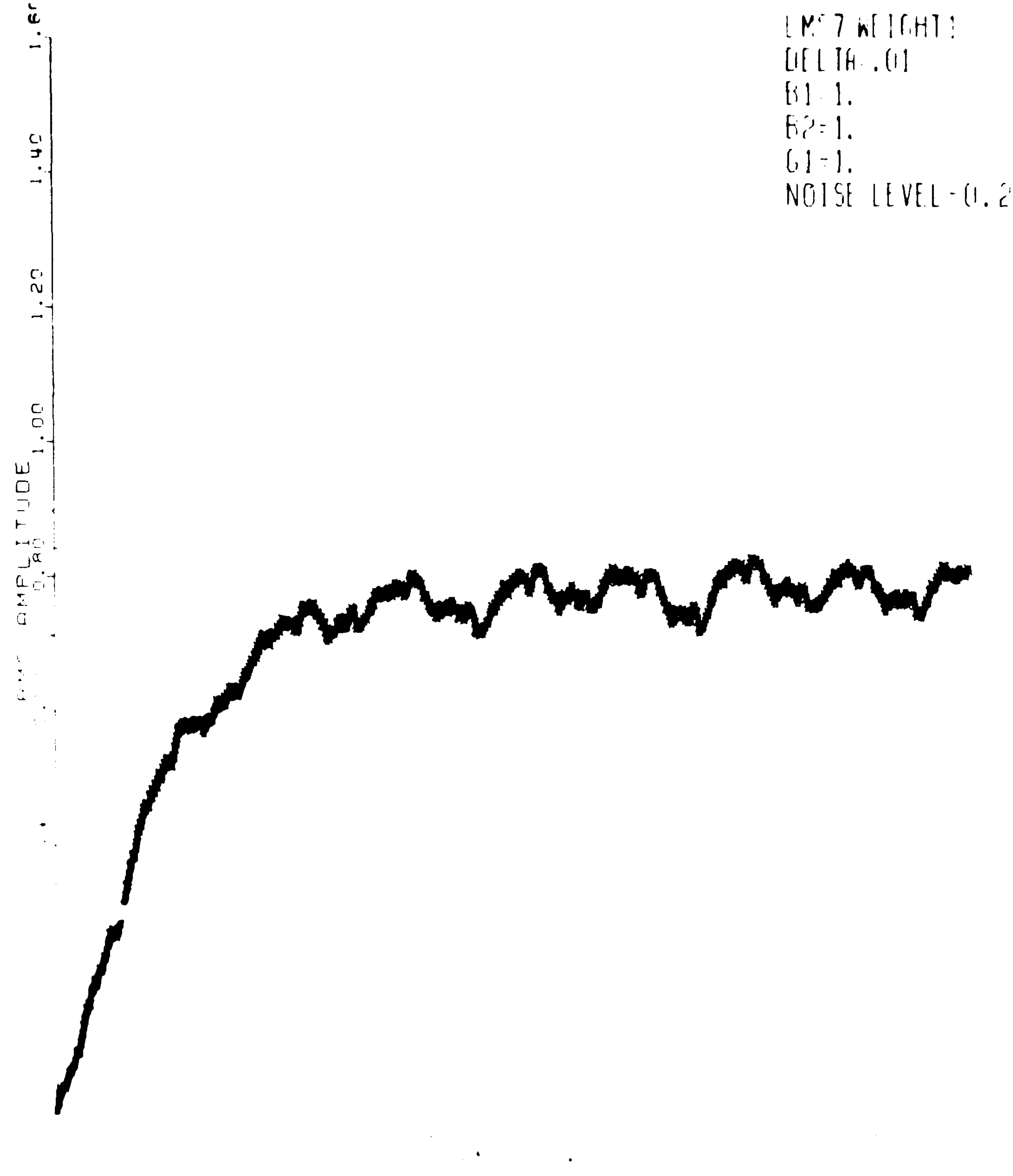

Figure 20. Feedforward Estimator, Seven Weights $\triangle$=.01
B1=1, B2=1, G1=1, Noise=0.2

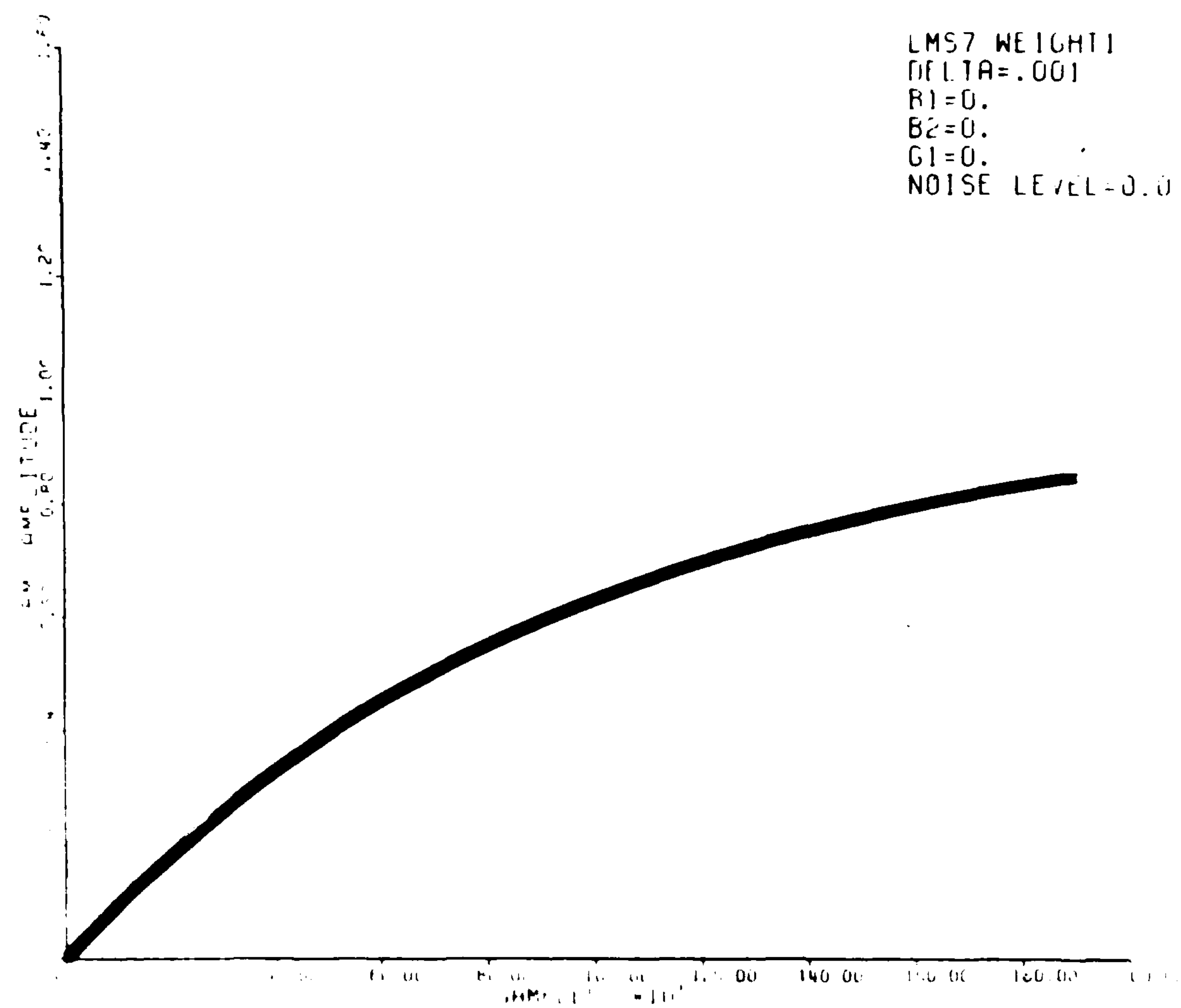

Figure 21. Feedforward Estimator, Seven Weights Δ=.001
B1=0, B2=0, G1=0, Noise=0

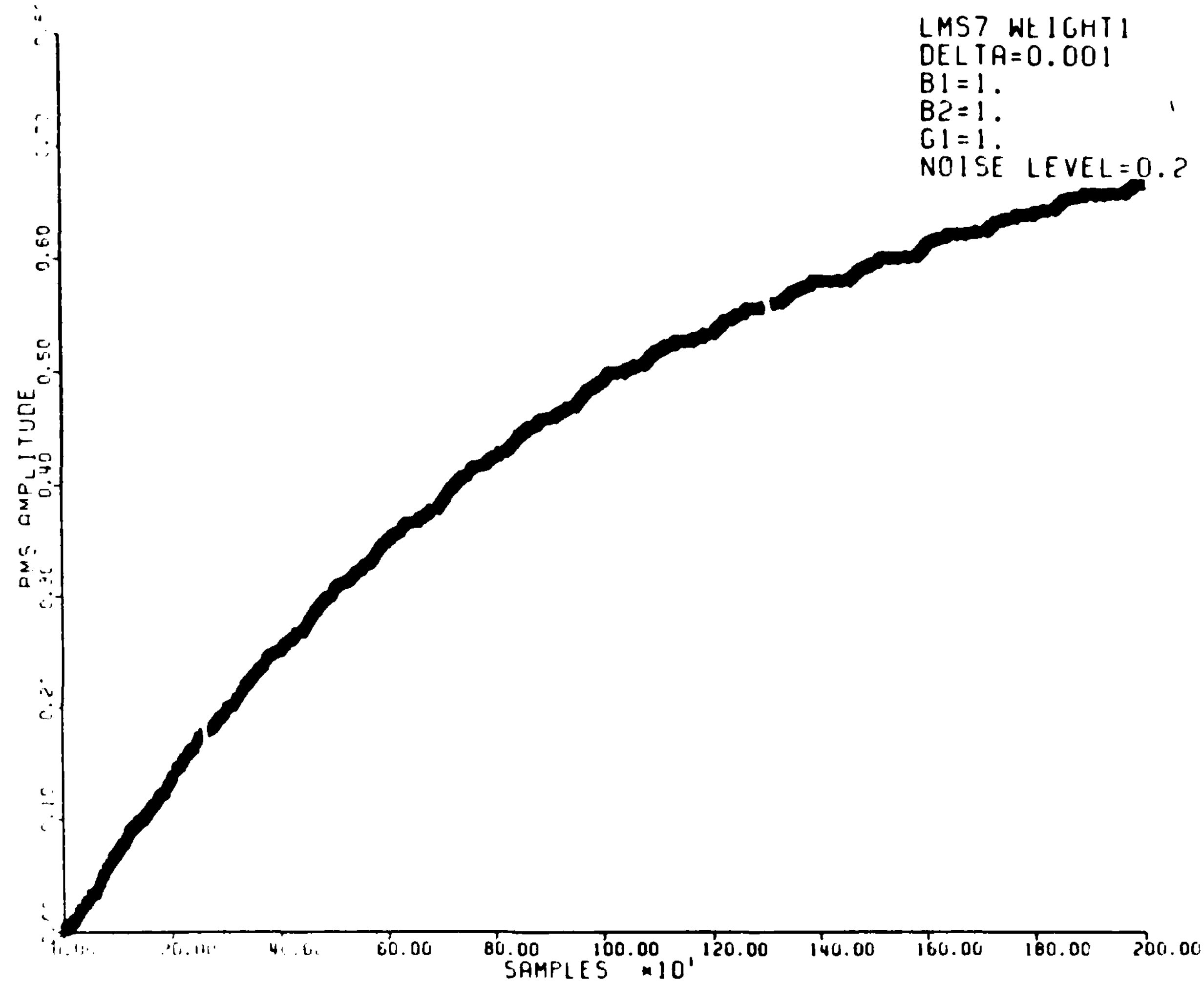

Figure 22. Feedforward Estimator, Seven Weights Δ=.001
B1=1, B2=1, G1=1, Noise=0.2

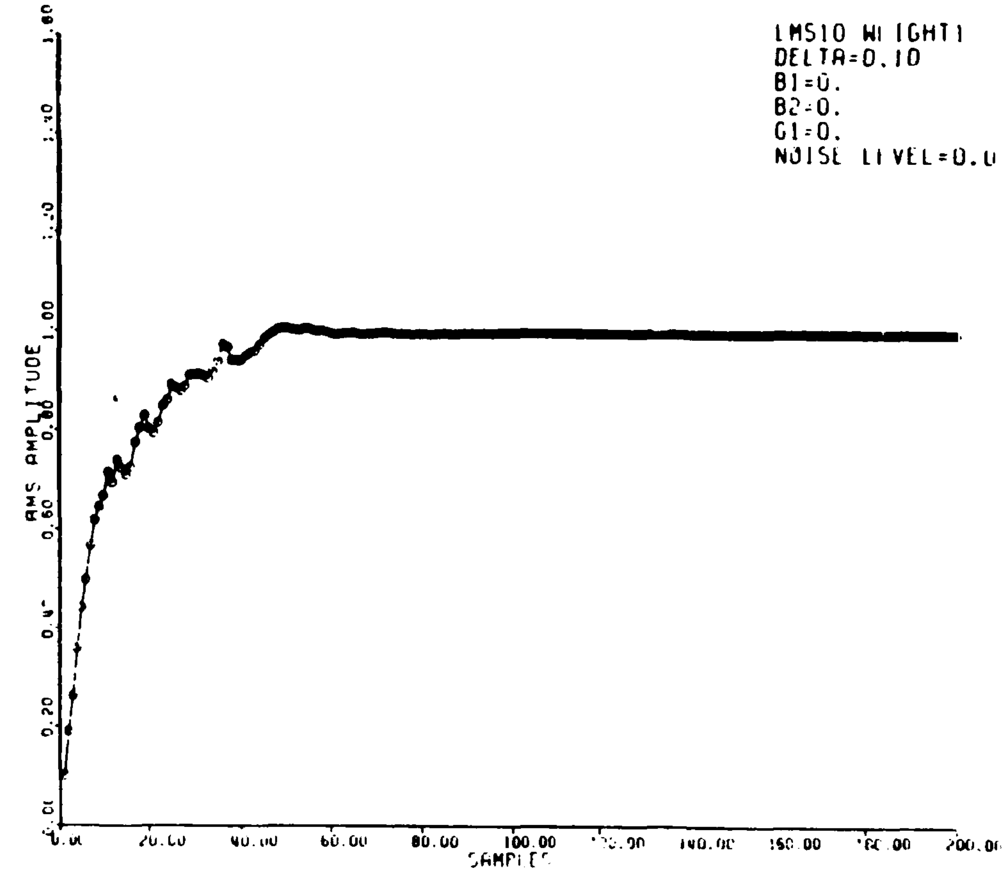

Figure 23. Feedforward Estimator, Ten Weights $\Delta = .1$
B1=0, B2=0, G1=0, Noise=0

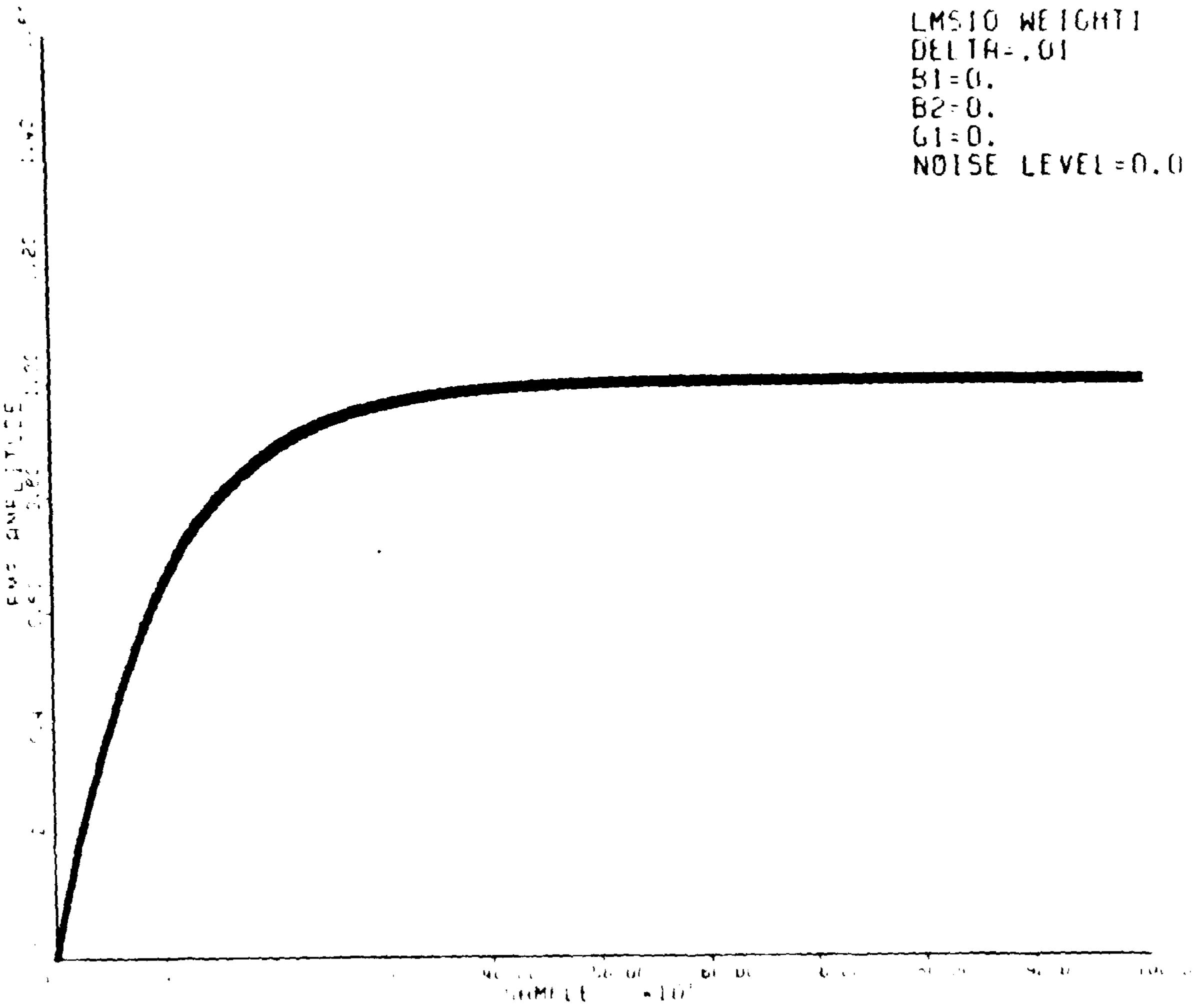

Figure 24. Feedforward Estimator, Ten Weights Δ =.01
B1=0, B2=0, G1=0, Noise=0

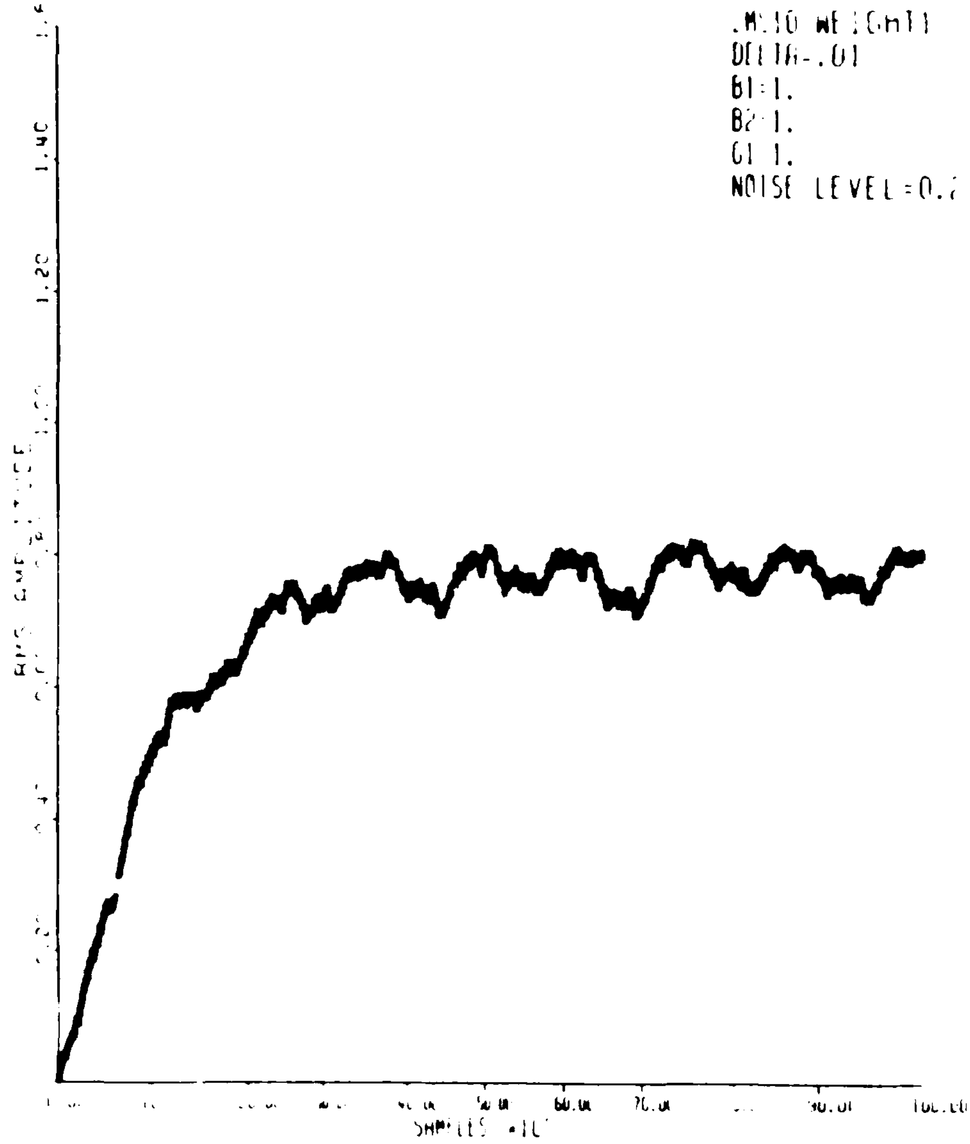

Figure 25. Feedforward Estimator, Ten Weights Δ =.Ø1
B1=1, B2=1, G1=1, Noise=Ø.2

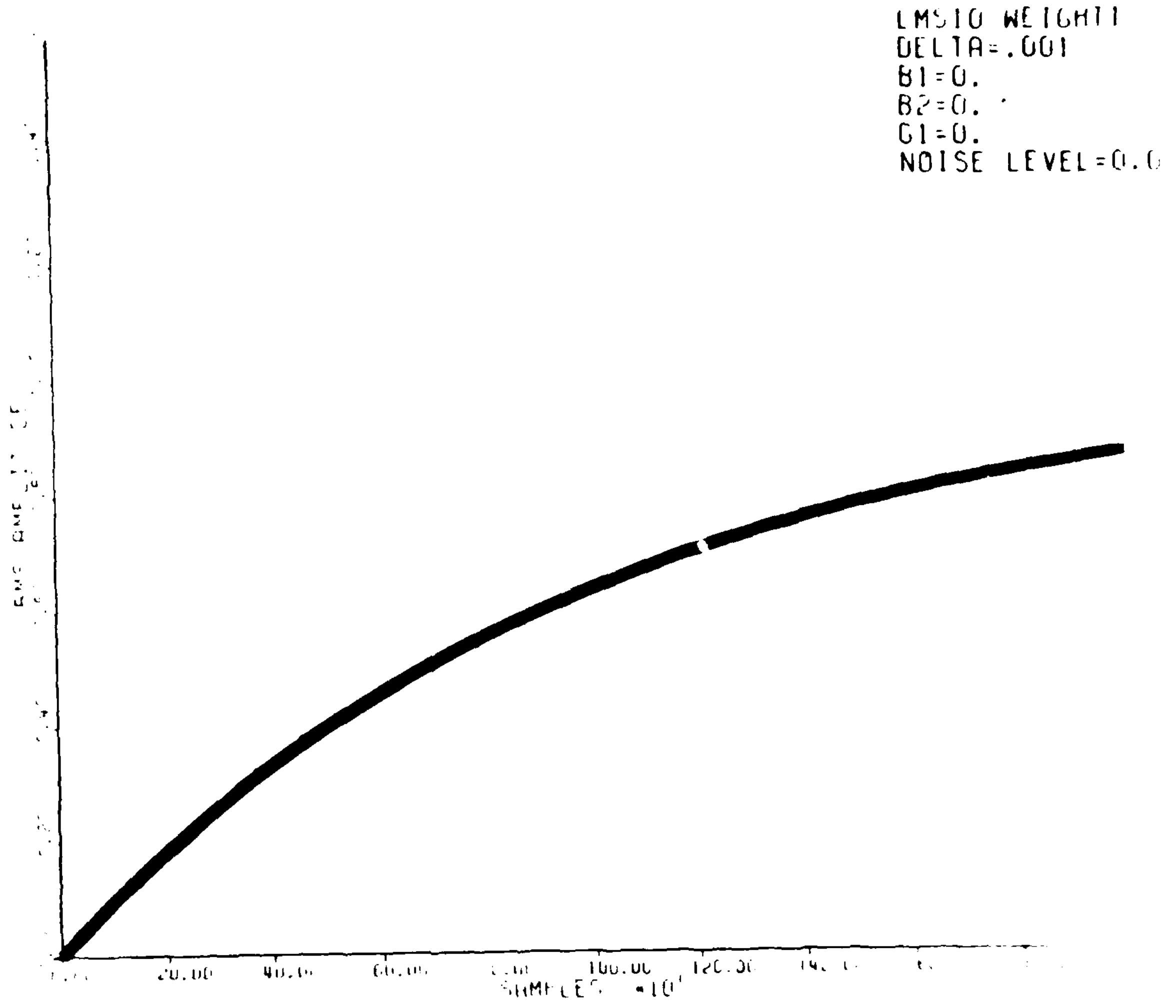

Figure 26. Feedforward Estimator, Ten Weights
B1=Ø, B2=Ø, G1=Ø, Noise=0

AD-A177 782 COMPARISON OF CHANNEL ESTIMATION TECHNIQUES(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING R M STROTHER DEC 86 AFIT/GE/ENG/86D-8 2/2

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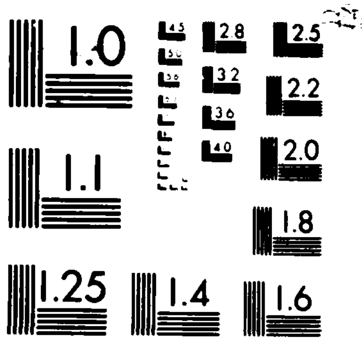

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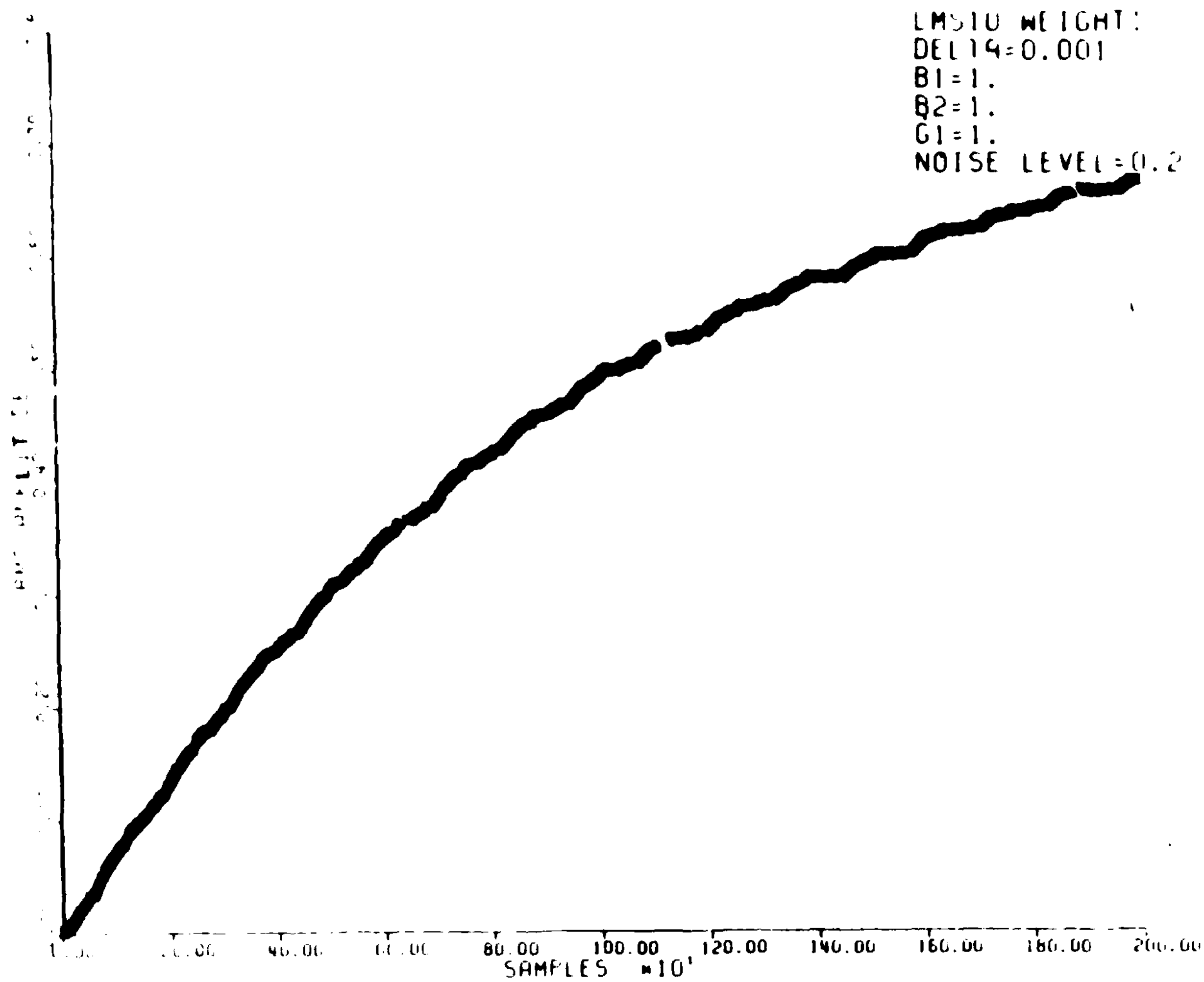

Figure 27. Feedforward Estimator, Ten Weights Δ =.001
B1=1, B2=1, G1=1, Noise=0.2

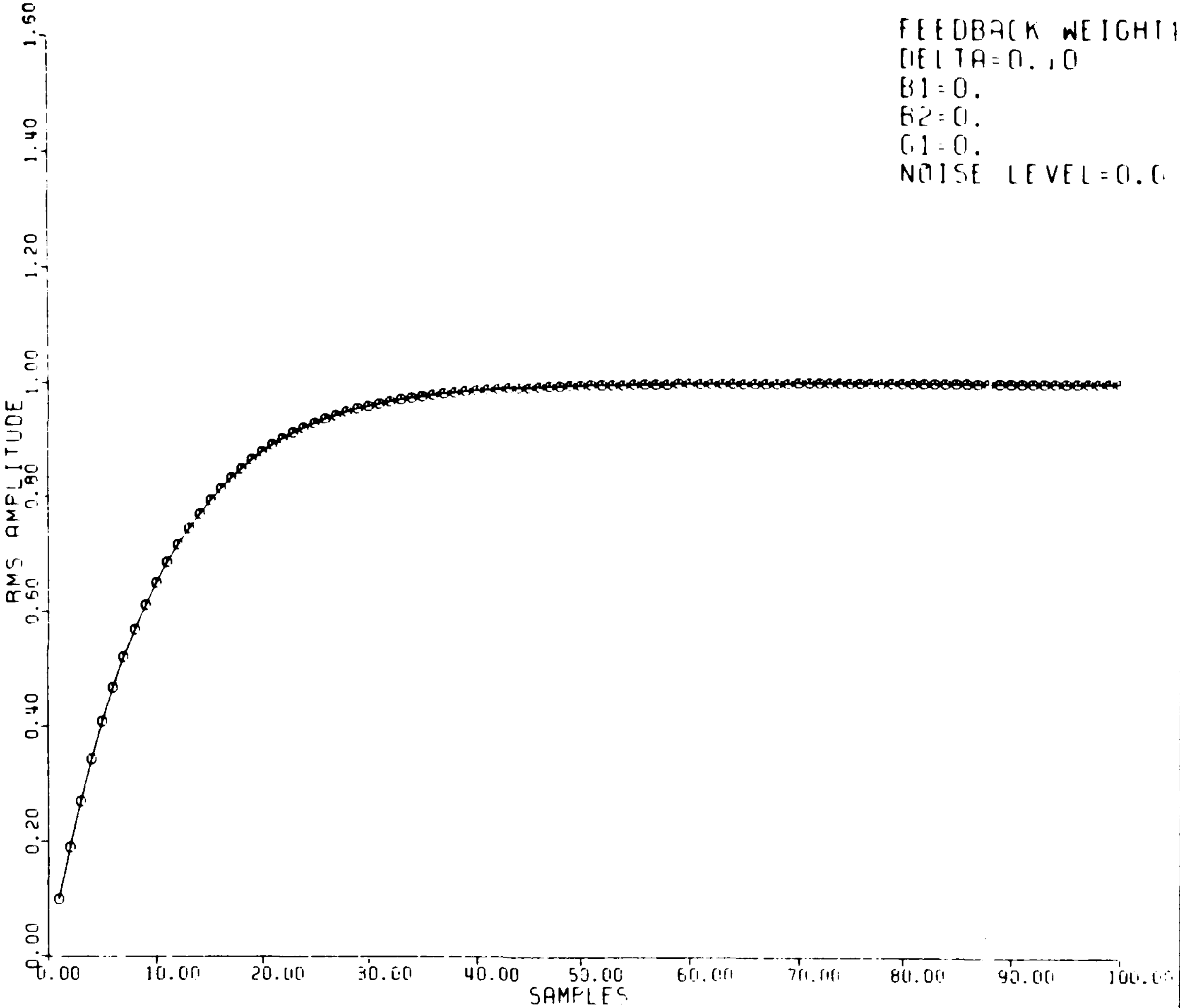

Figure 28. Feedback Estimator, Three Weights Δ=.1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø

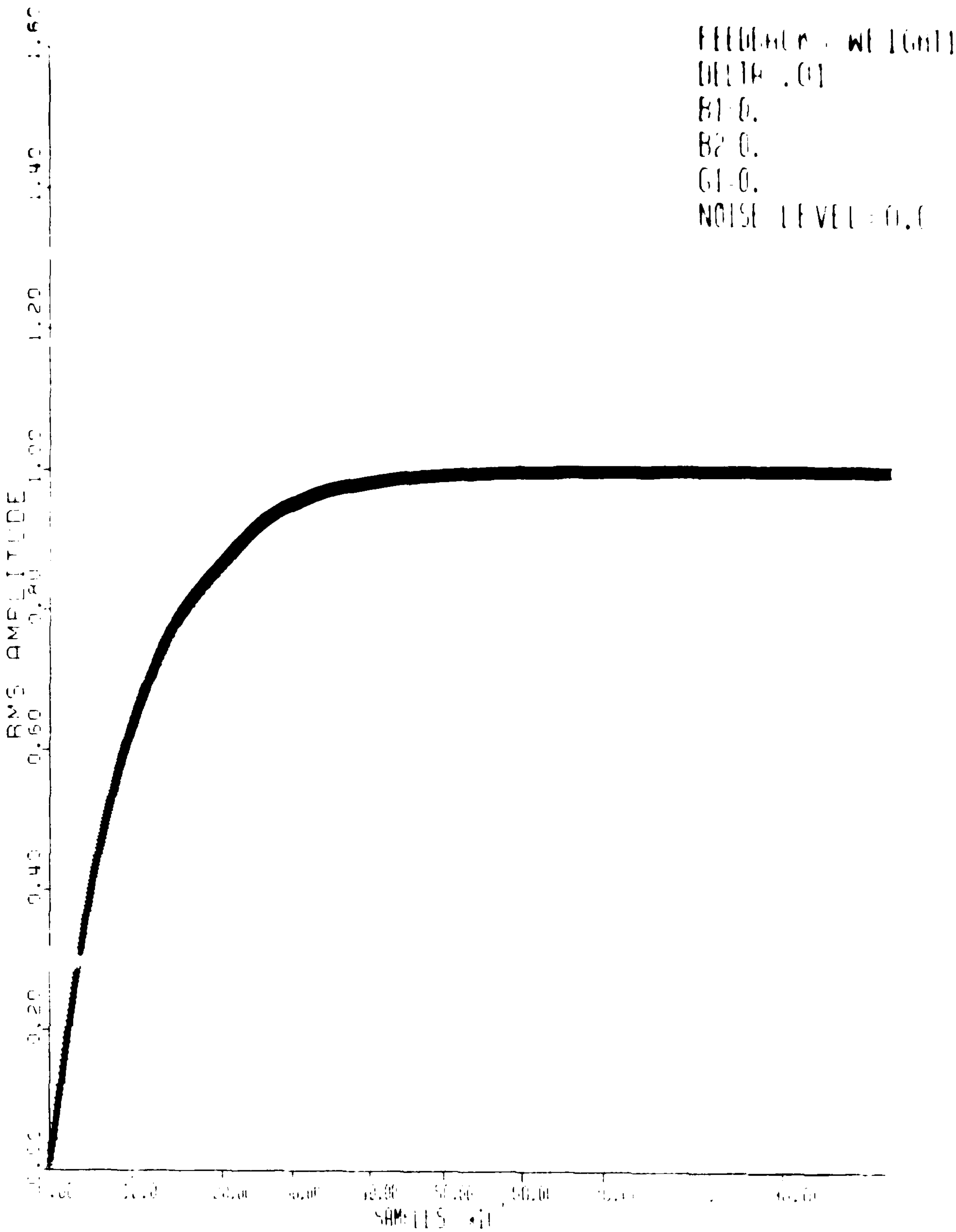

Figure 29. Feedback Estimator, Three Weights Δ=.Ø1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø

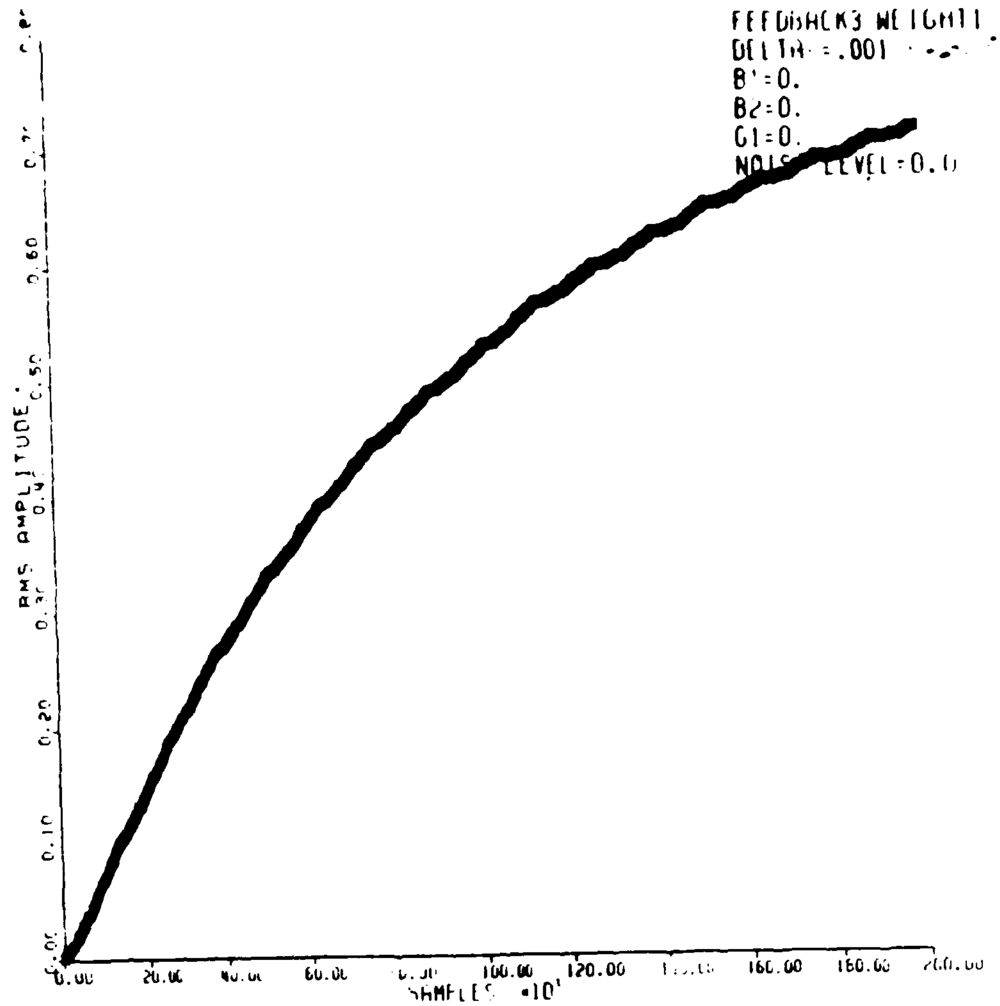

Figure 30. Feedback Estimator, Three Weights Δ=.001
B1=0, B2=0, G1=0, Noise=0

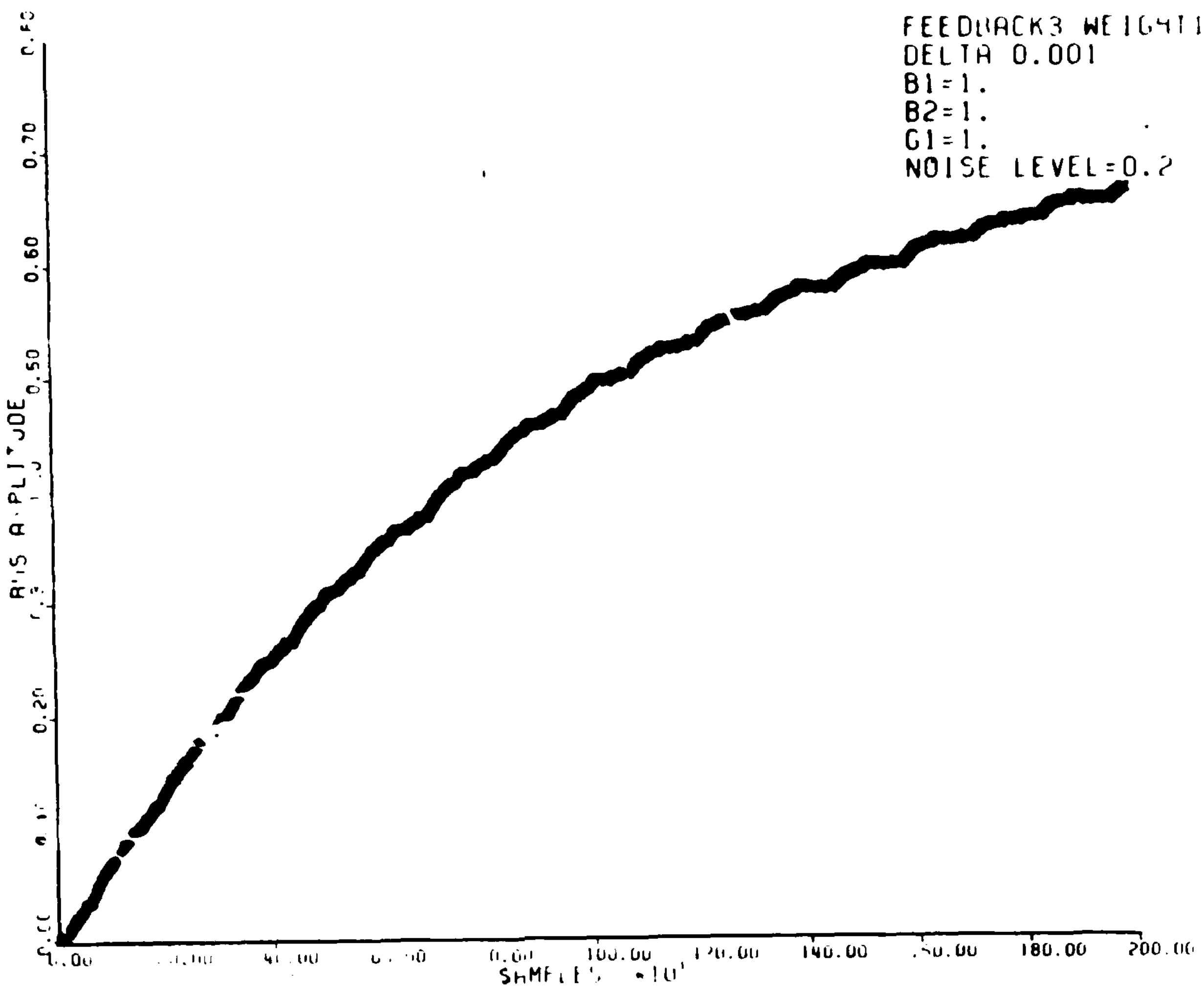

Figure 31. Feedback Estimator, Three Weights Δ =.001
B1=1, B2=1, G1=1, Noise=0.2

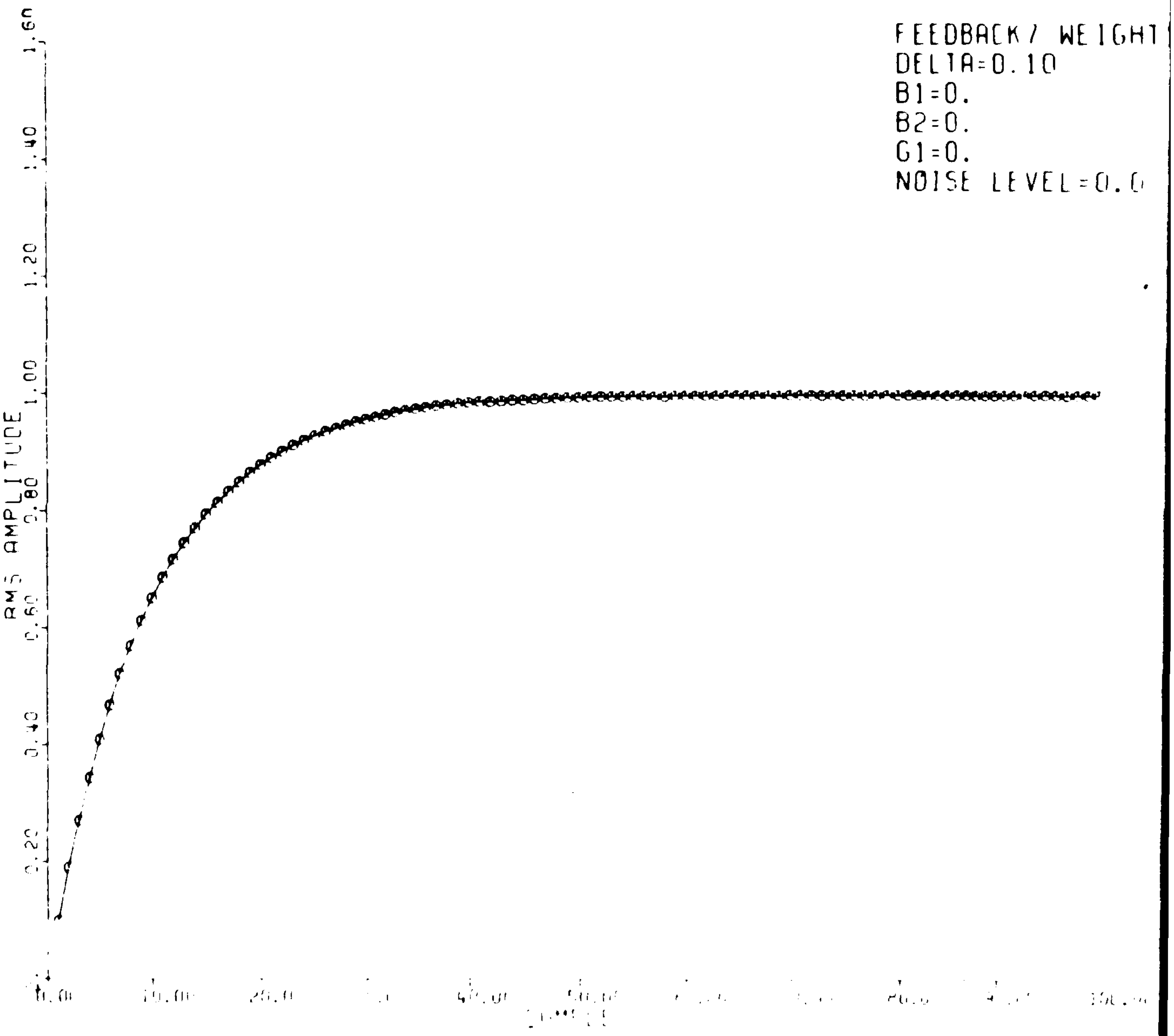

Figure 32. Feedback Estimator, Seven Weights Δ =.1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø

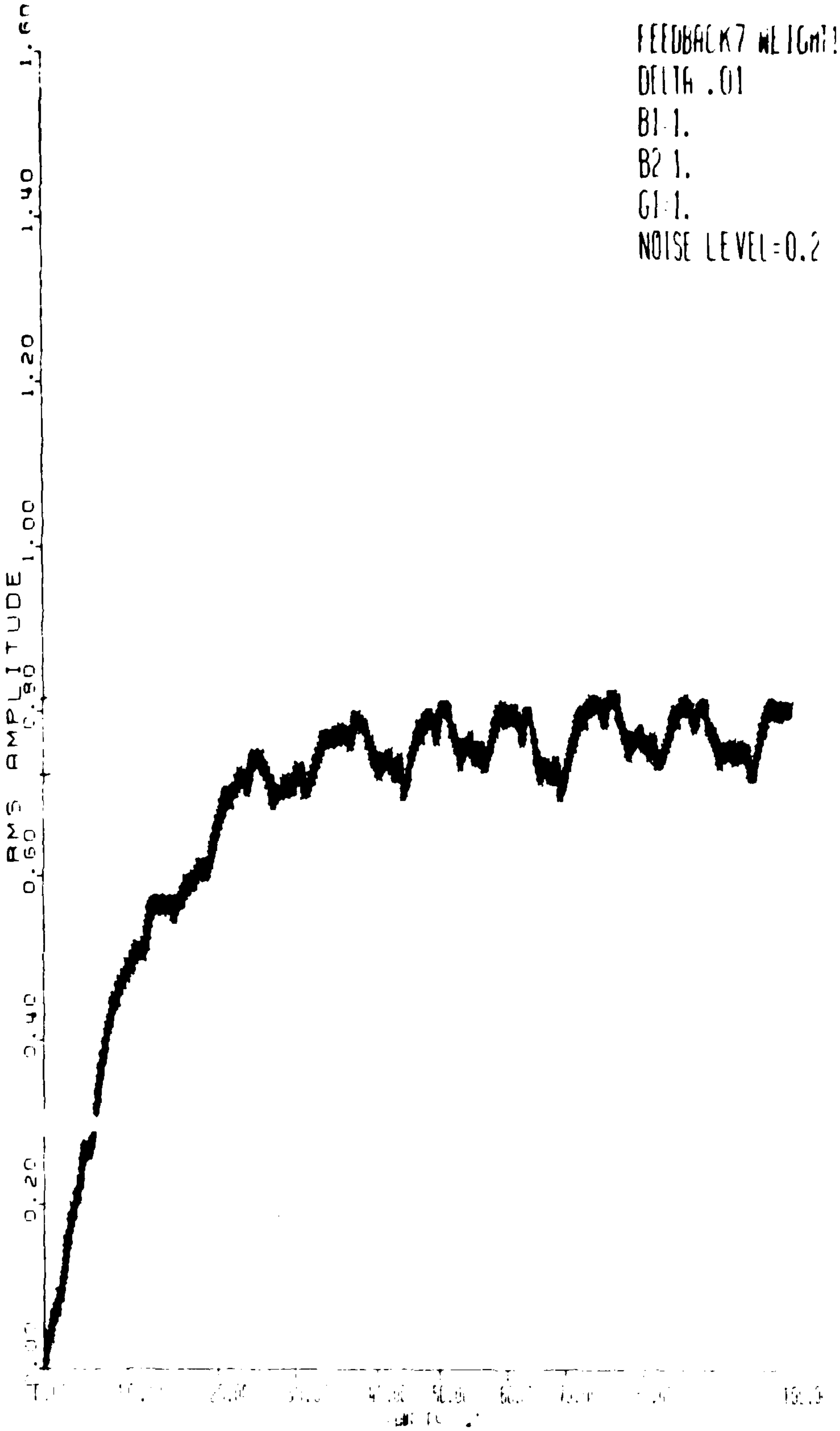

Figure 33. Feedback Estimator, Seven Weights Δ=.01 B1=1, B2=1, G1=1, Noise=0.2

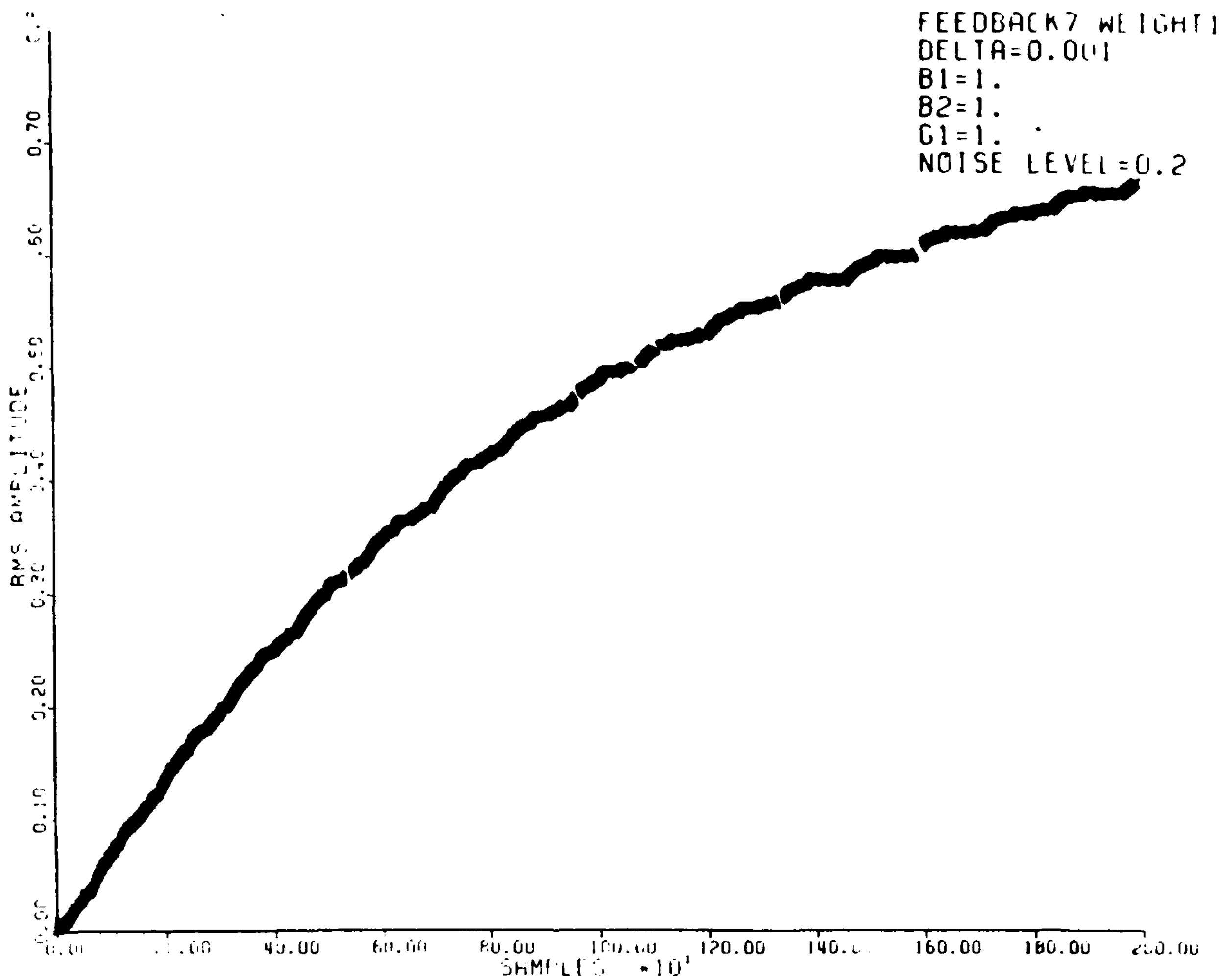

Figure 34. Feedback Estimator, Seven Weights Δ=.0Ø1
B1=1, B2=1, G1=1, Noise=Ø.2

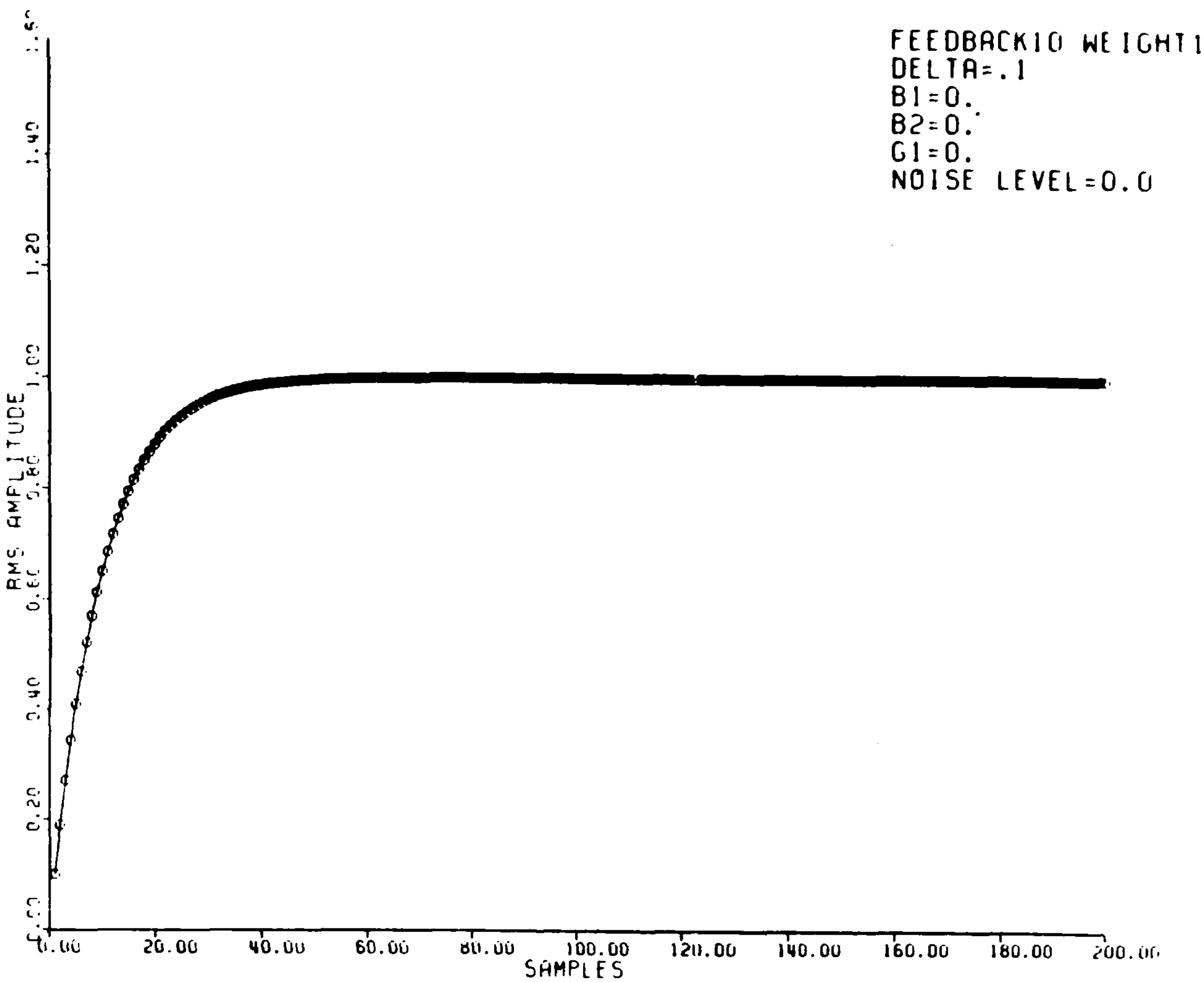

Figure 35. Feedback Estimator, Ten Weights Δ=.1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø

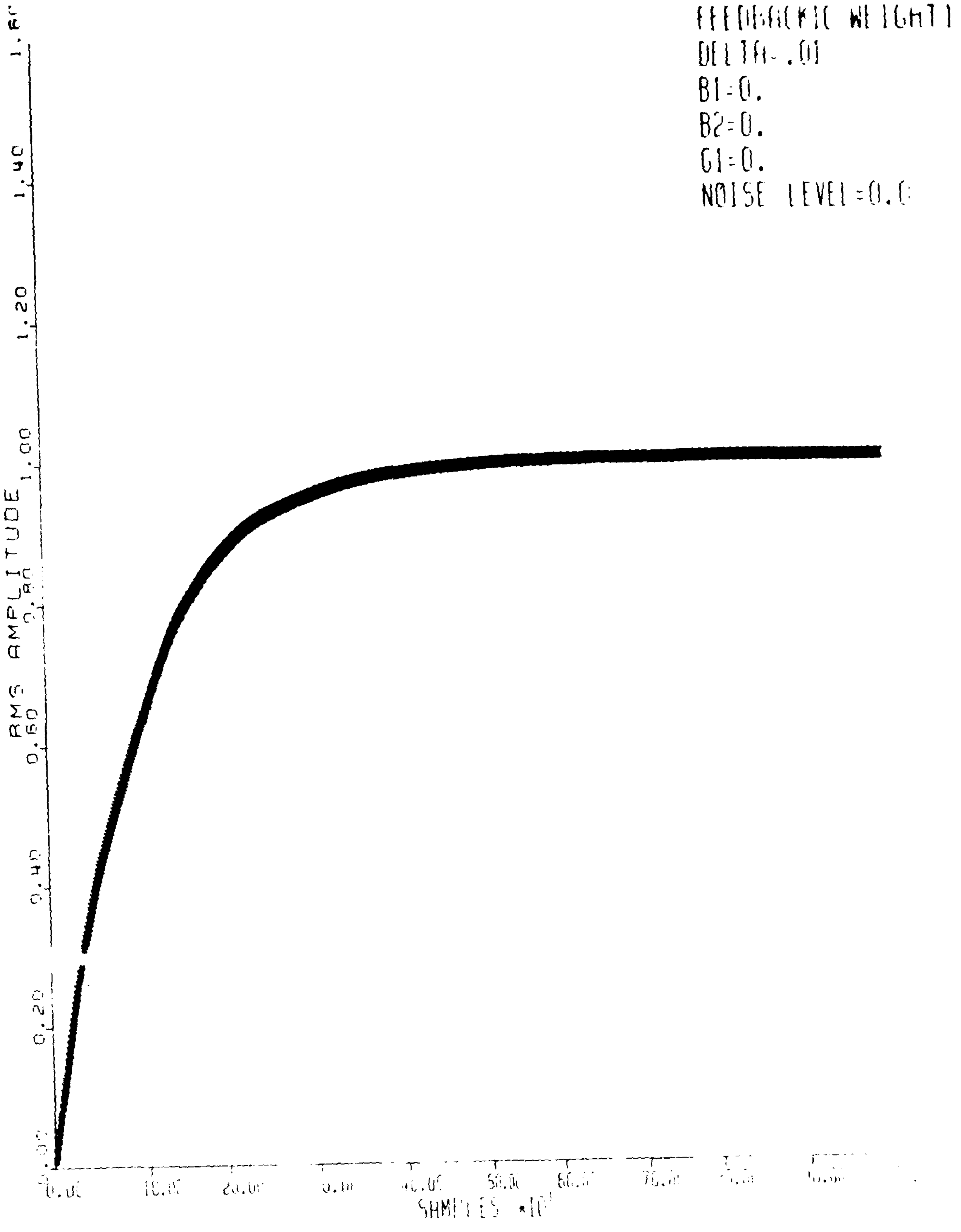

Figure 36. Feedback Estimator, Ten Weights Δ=.01
B1=0, B2=0, G1=0, Noise=0

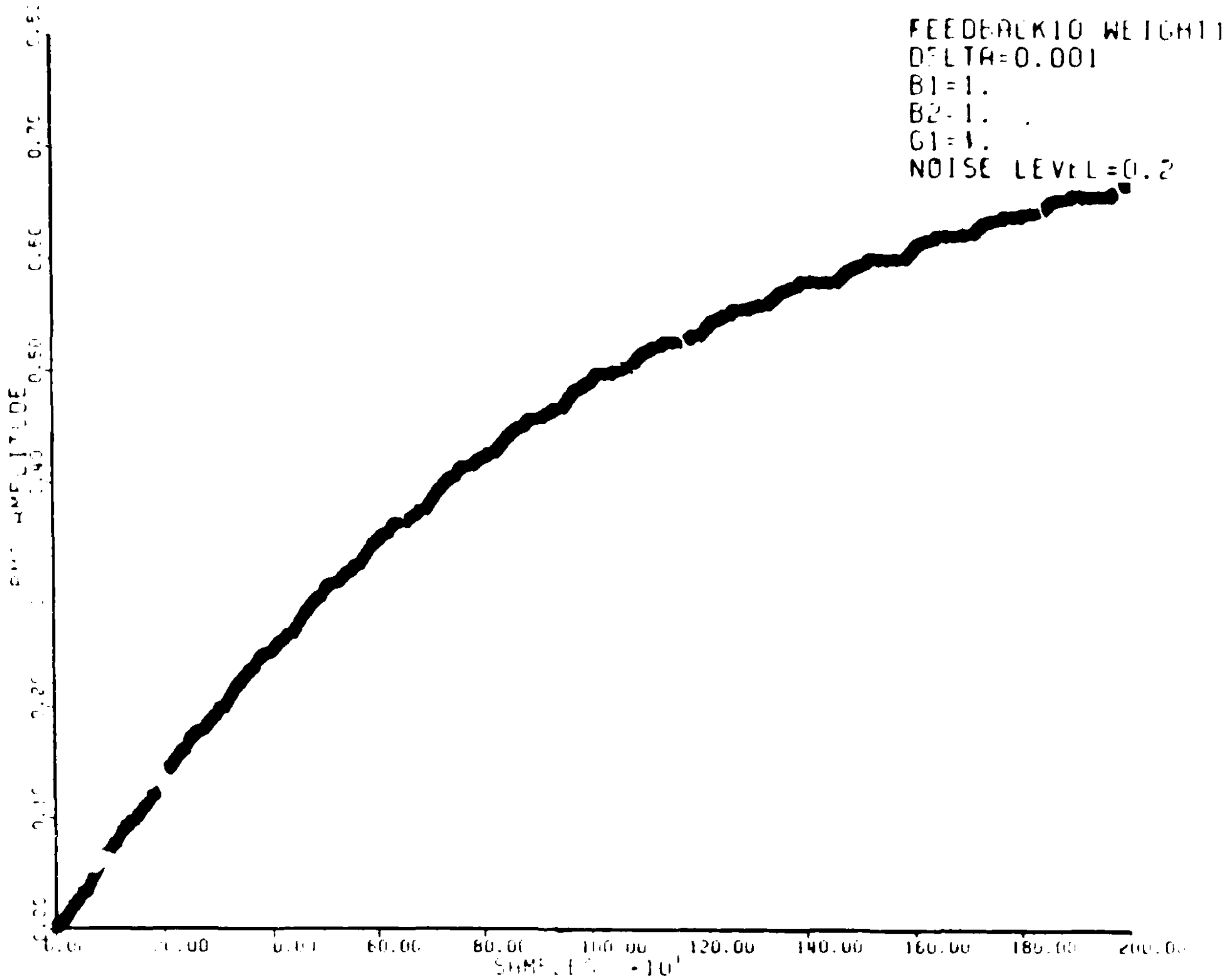

Figure 37. Feedback Estimator, Seven Weights Δ=.ØØ1
B1=1, B2=1, G1=1, Noise=Ø.2

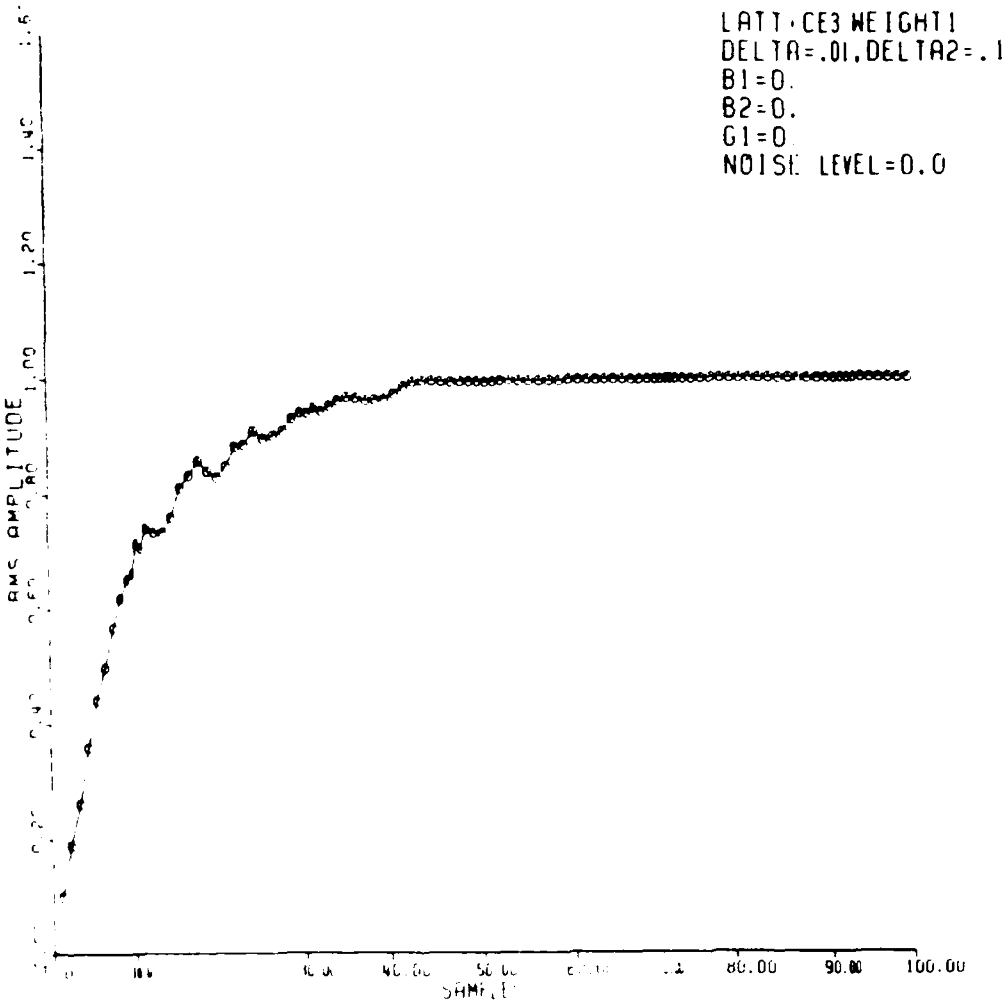

Figure 38. Lattice Estimator, Three Weights $\Delta_1=.01$
B1=0, B2=0, G1=0, Noise=0 $\Delta_2=.1$

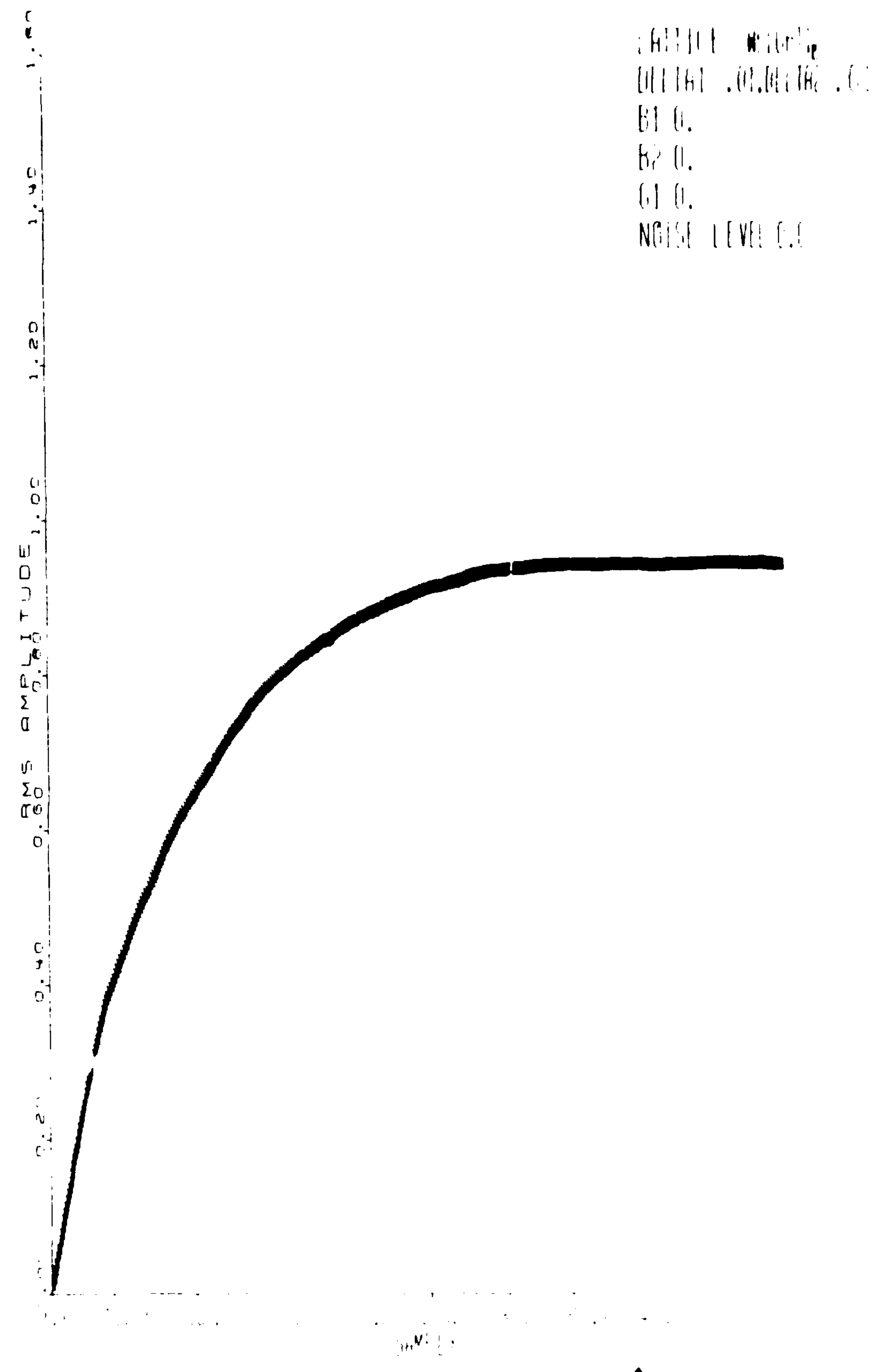

Figure 39. Lattice Estimator, Three Weights Δ_1=.Ø1
B1=Ø, B2=Ø, G1=Ø, Noise=Ø Δ_2=.Ø1

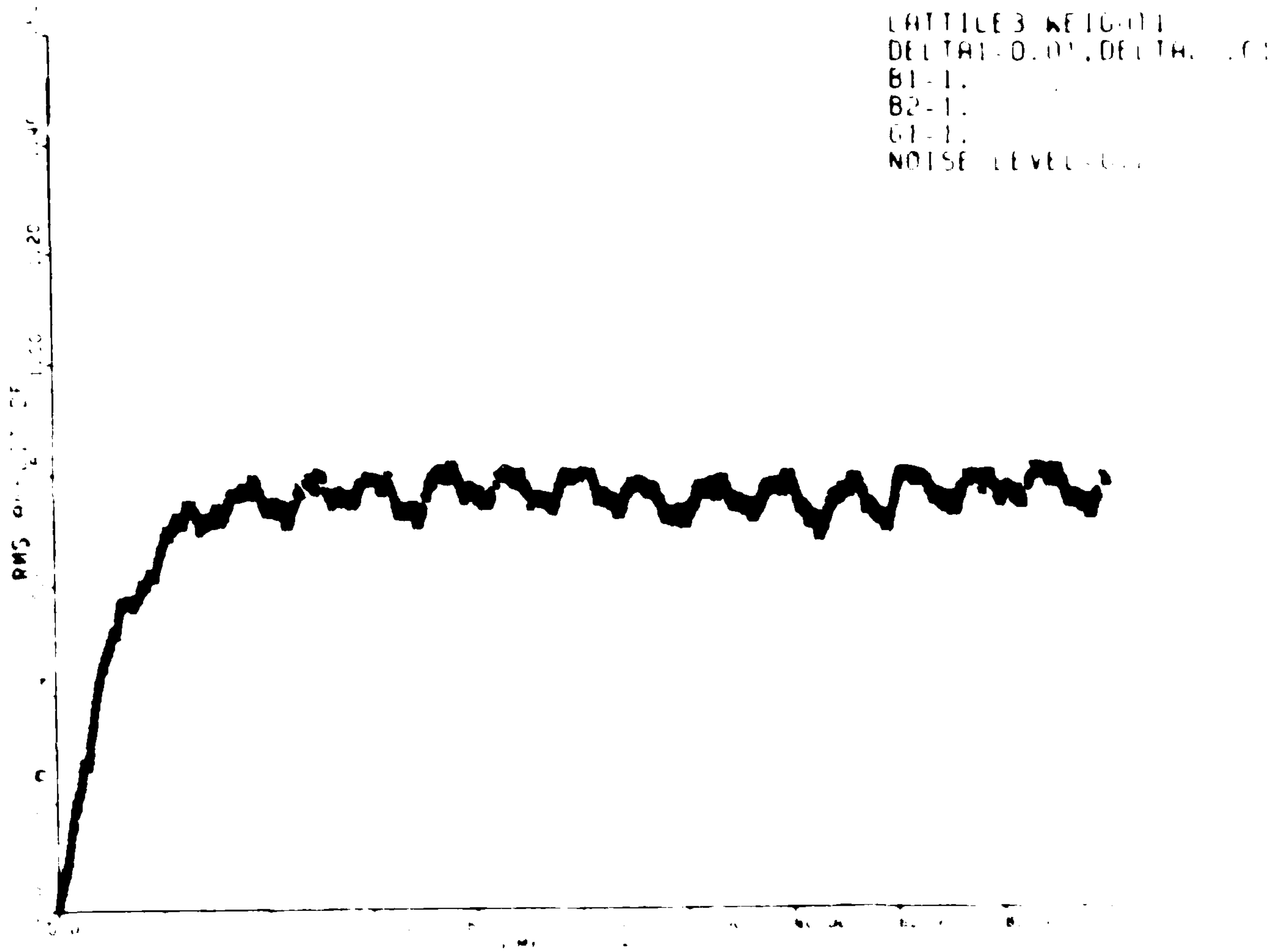

Figure 40. Lattice Estimator, Three Weights $\Delta_1 = .01$
B1=1, B2=1, G1=1, Noise=0.2 $\Delta_2 = .01$

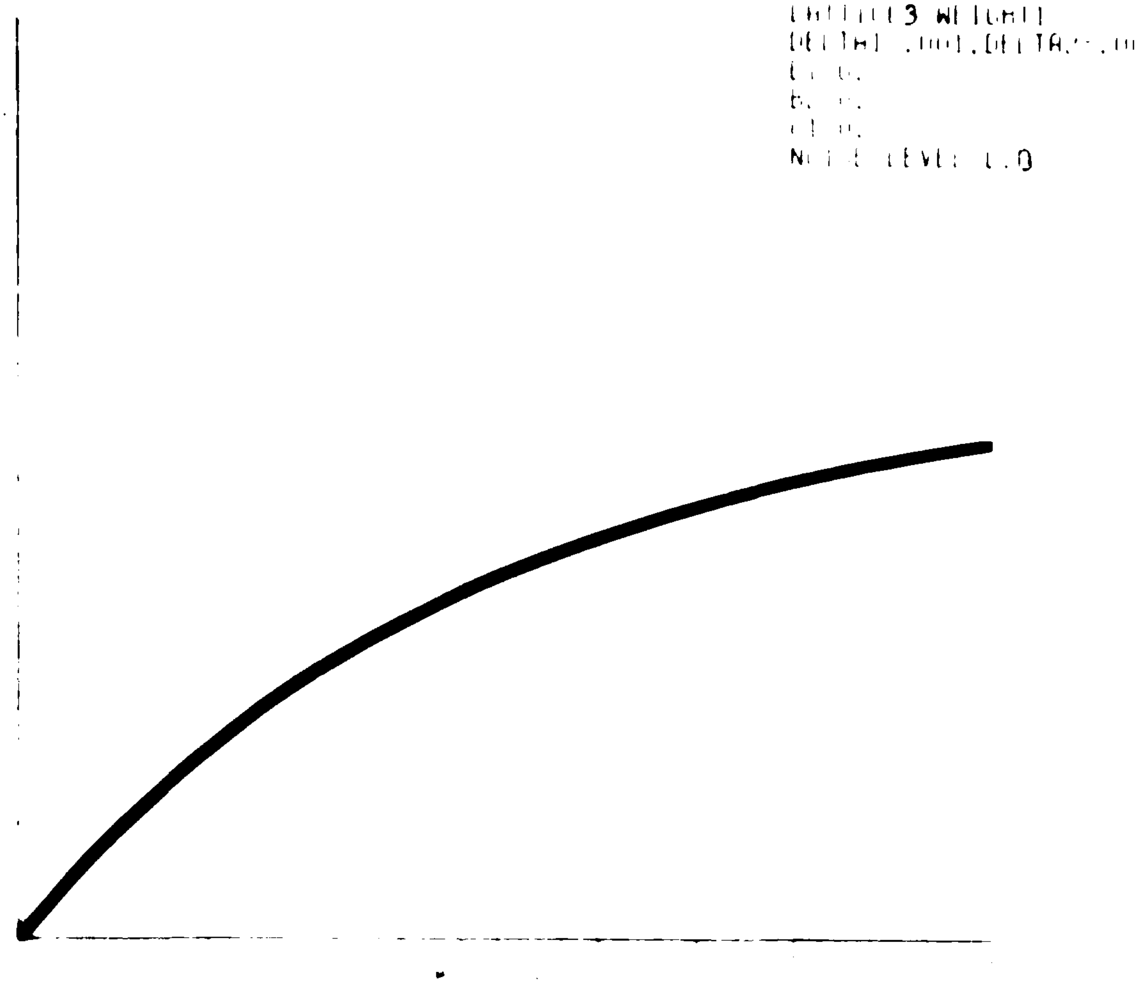

Figure 41. Lattice Estimator, Three Weights $\Delta_1 = .001$
B1=0, B2=0, G1=0, Noise=0 $\Delta_2 = .001$

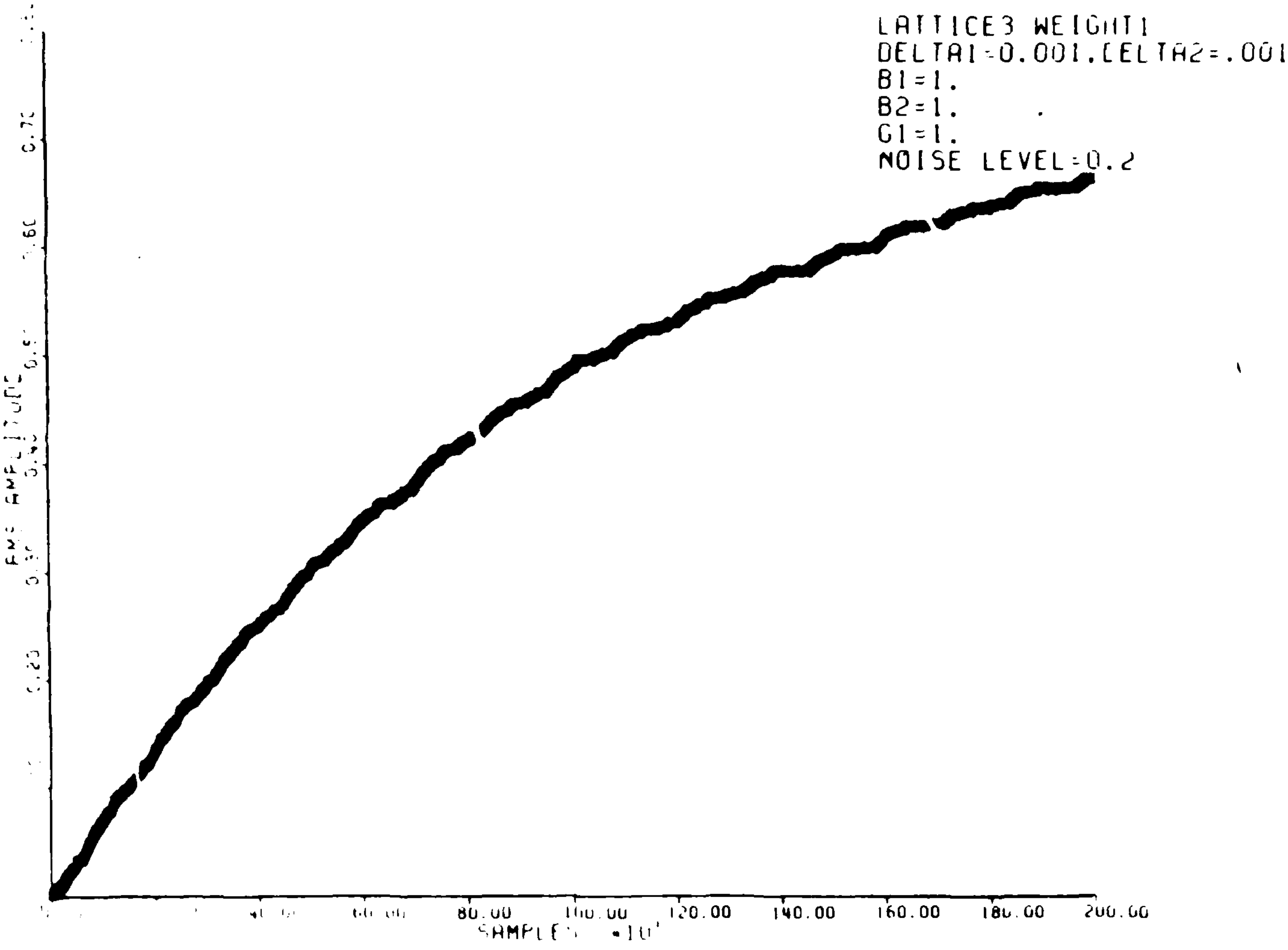

Figure 42. Lattice Estimator, Three Weights B1=1, B2=1, G1=1, Noise=0.2 Δ_1=.001 Δ_2=.001

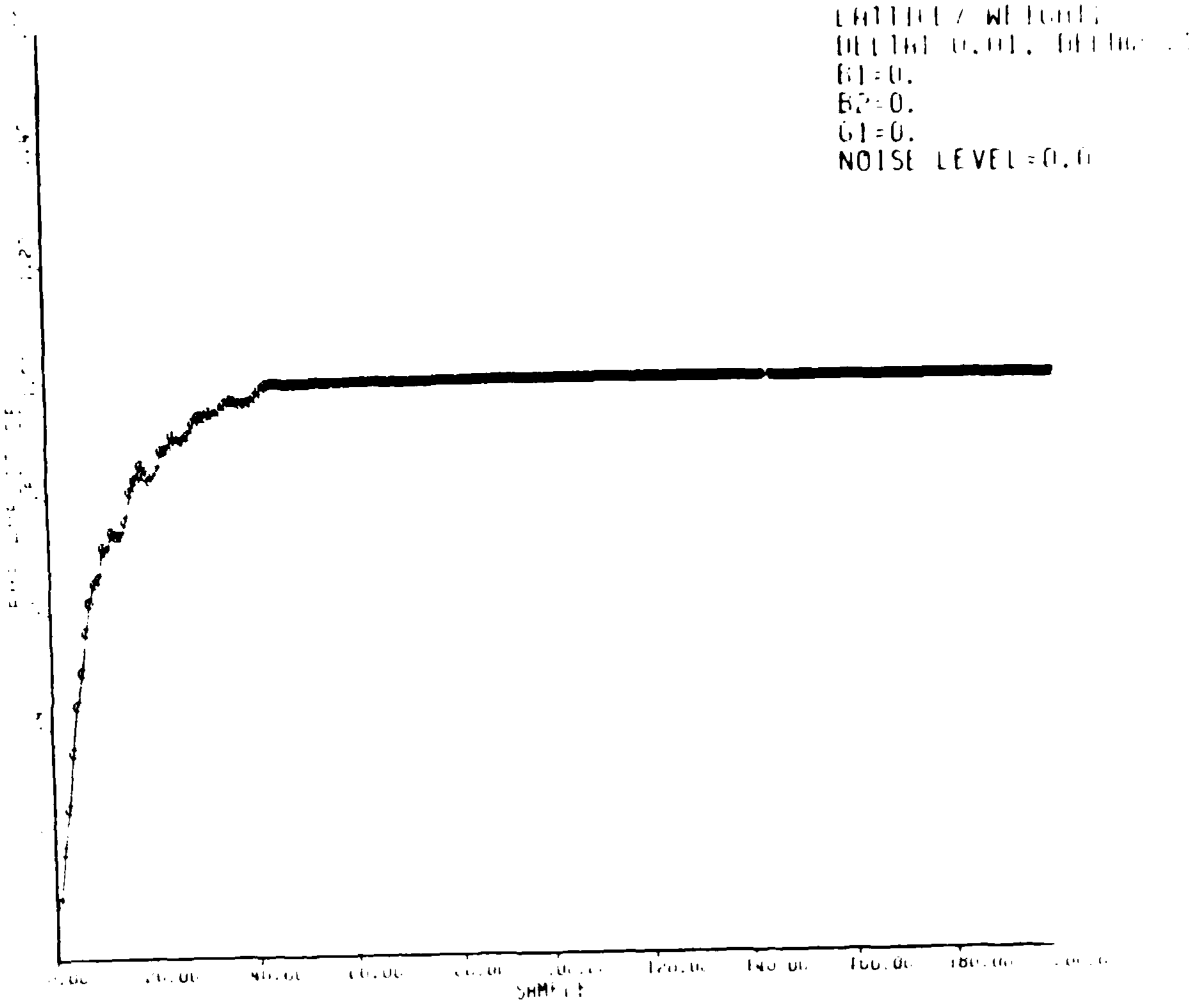

Figure 43. Lattice Estimator, Seven Weights $\Delta_1=.01$
Bl=0, B2=0, Gl=0, Noise=0 $\Delta_2=.1$

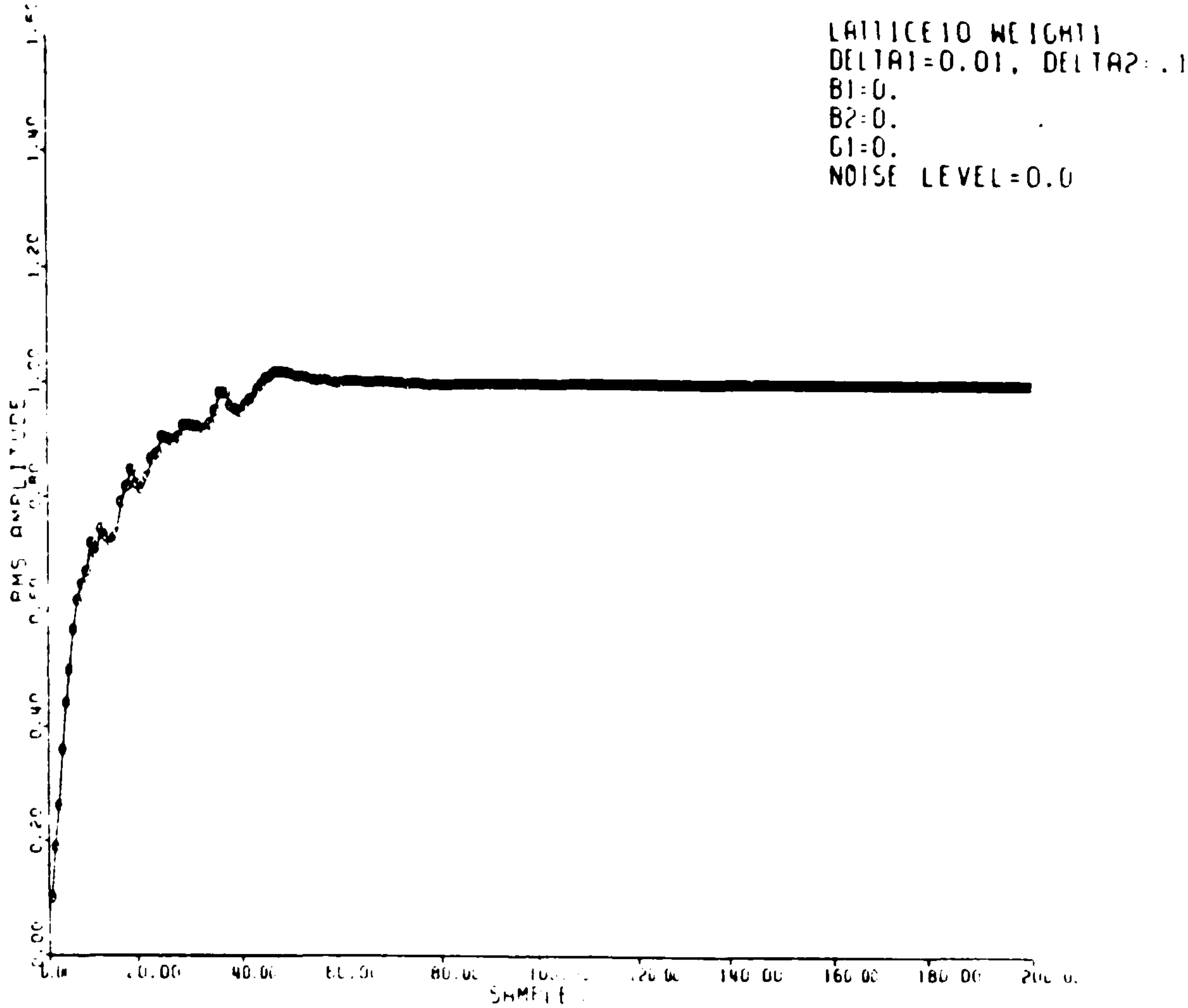

Figure 44. Lattice Estimator, Ten Weights Δ_1=.01
B1=0, B2=0, G1=0, Noise=0 Δ_2=.1

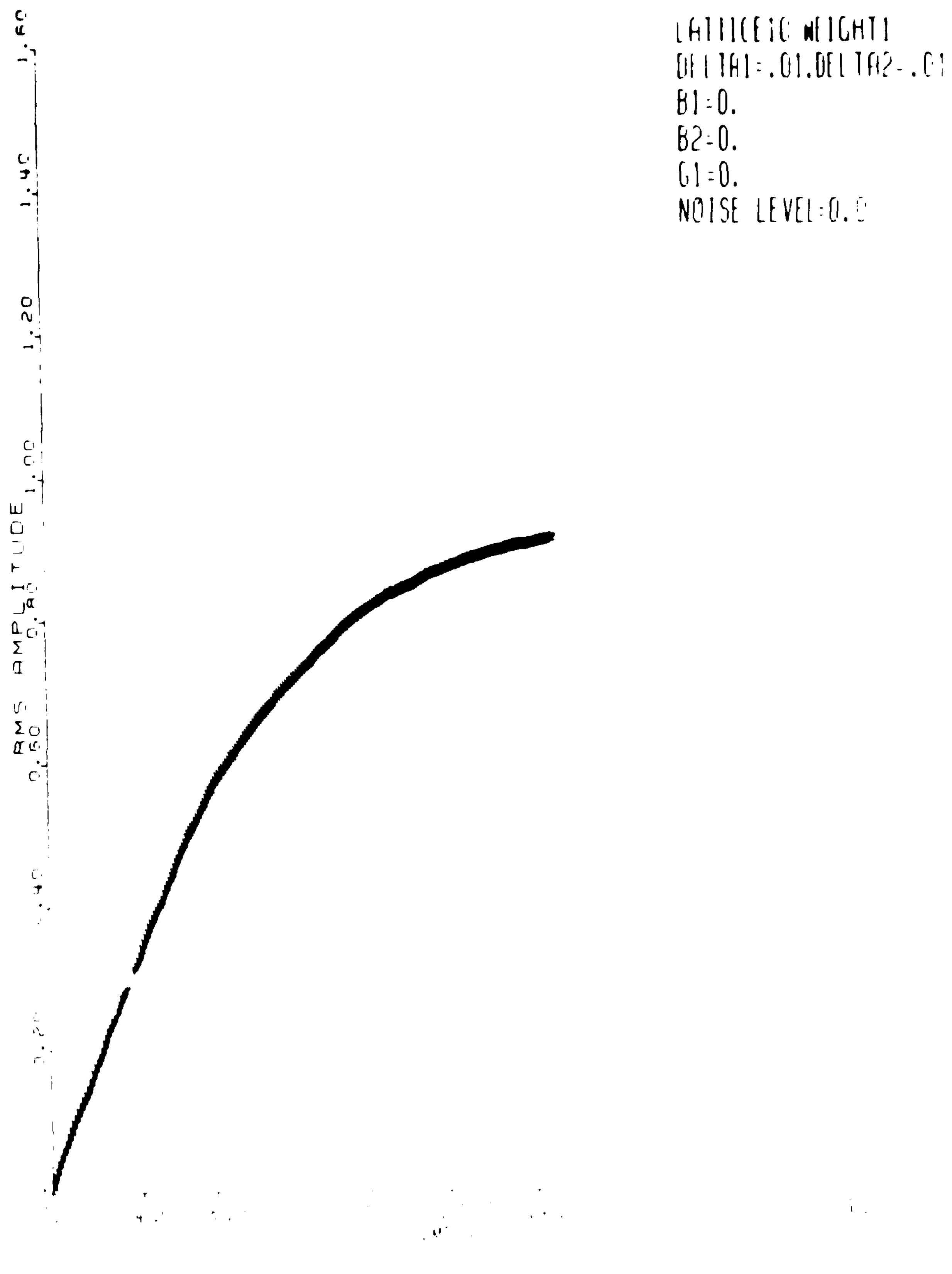

Figure 45. Lattice Estimator, Ten Weights B1=Ø, B2=Ø, G1=Ø, Noise=Ø Δ_1=.Ø1 Δ_2=.Ø1

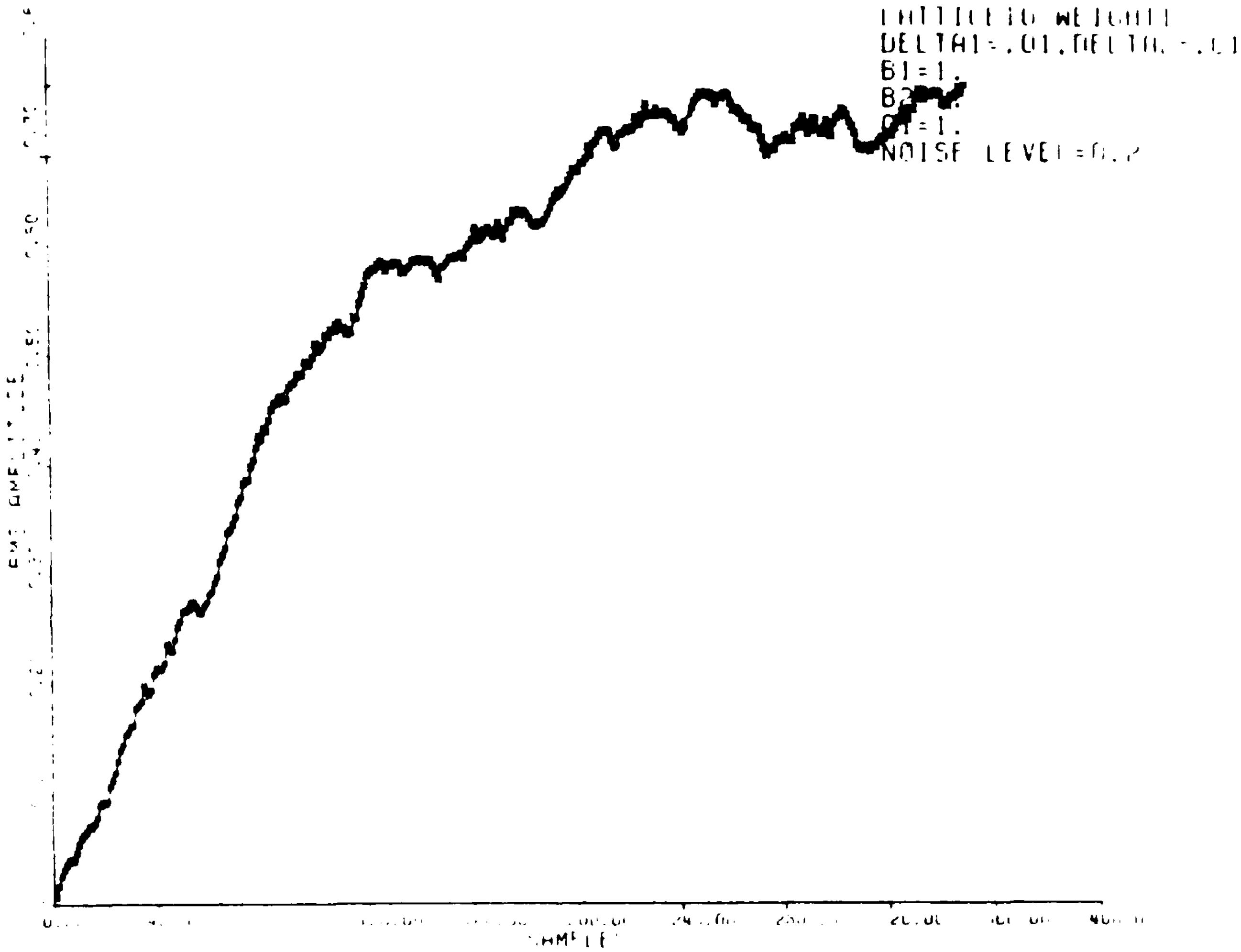

Figure 46. Lattice Estimator, Ten Weights Δ_1=.01
B1=1, B2=1, G1=1, Noise=0.2 Δ_2=.01

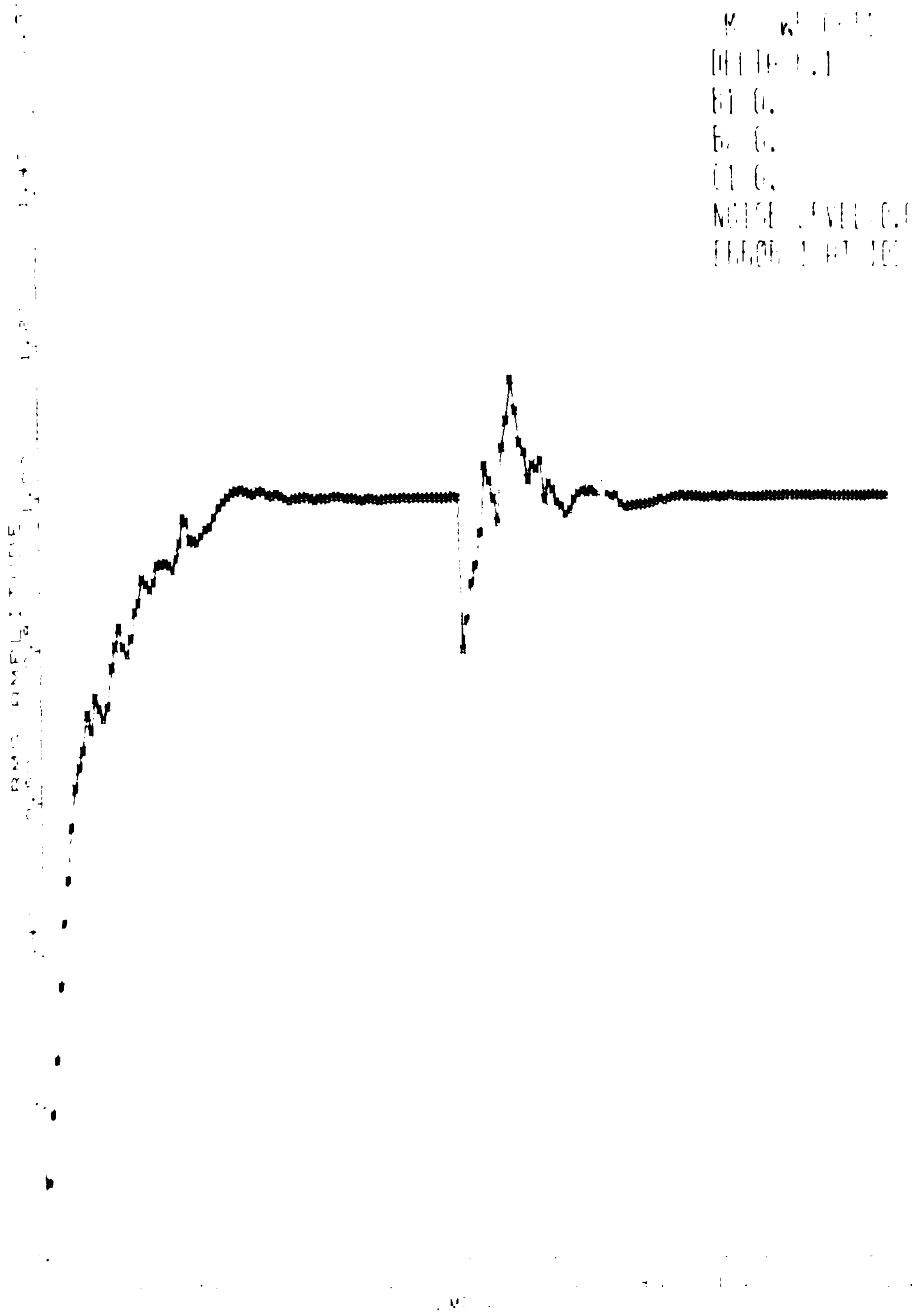

Figure 47. Feedforward Estimator, Three Weights, Δ=.1
One Error at 100

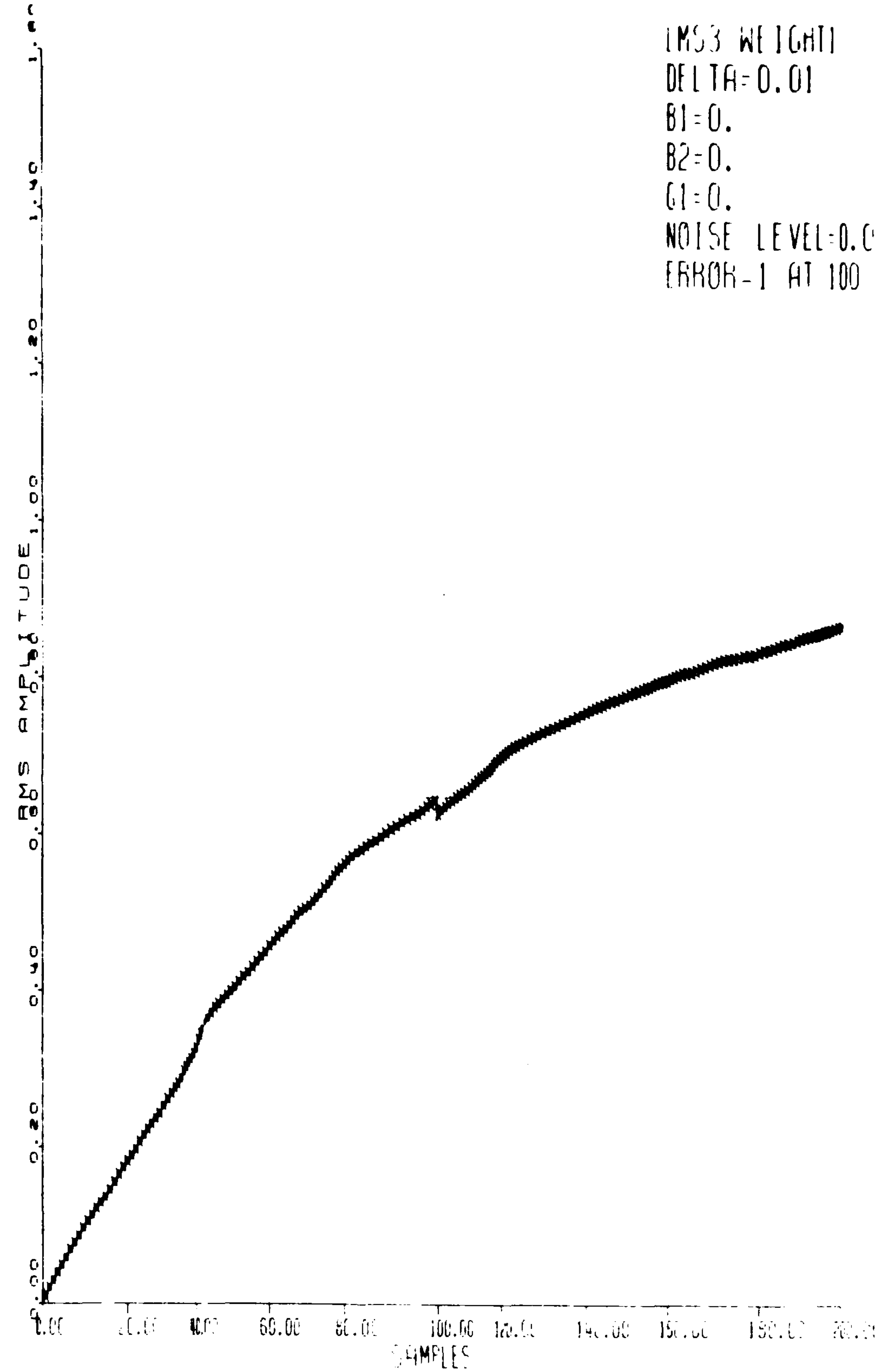

Figure 48. Feedforward Estimator, Three Weights, Δ=.01 One Error at 100

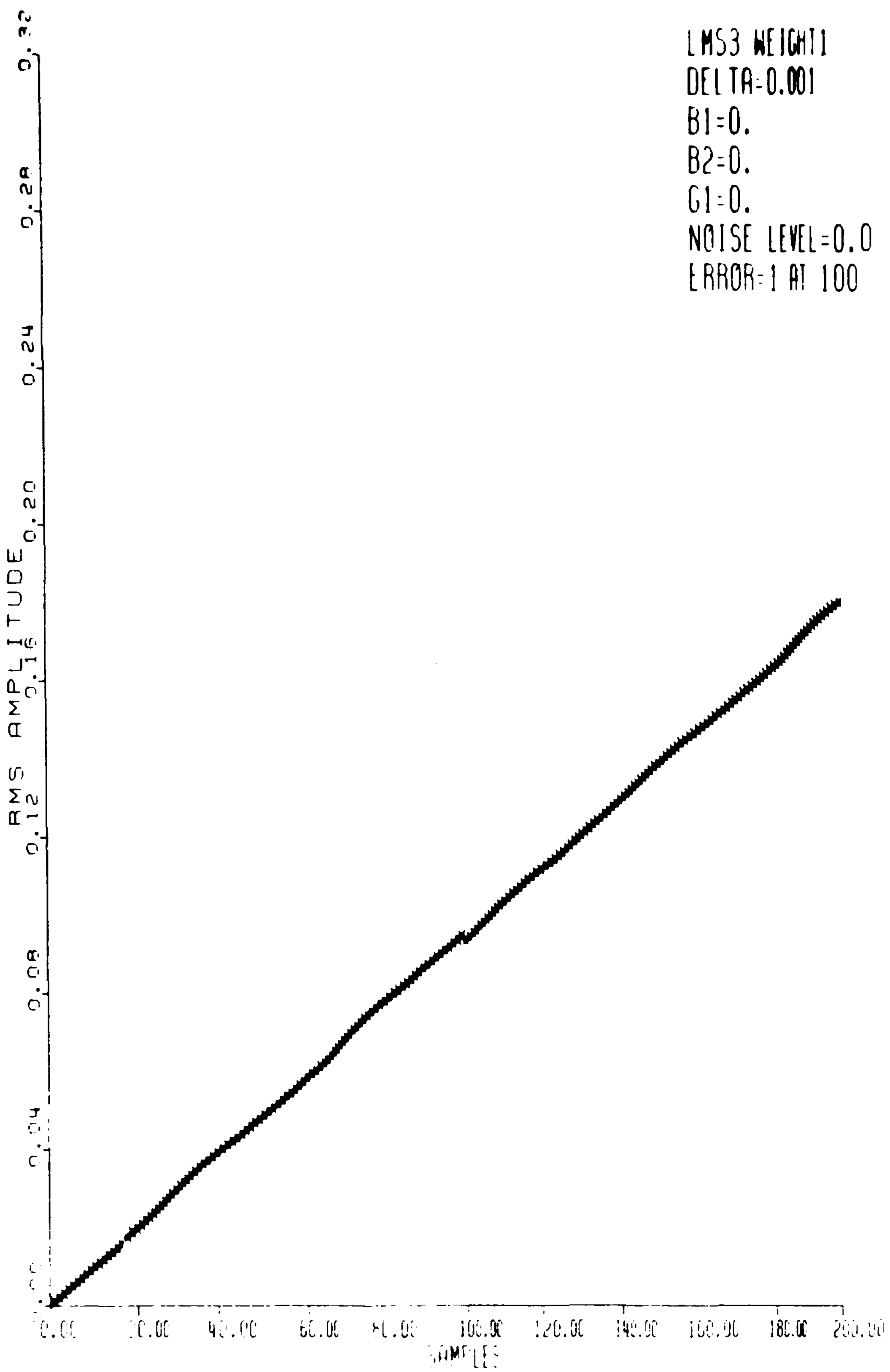

Figure 49. Feedforward Estimator, Three Weights, Δ=.001 One Error at 100

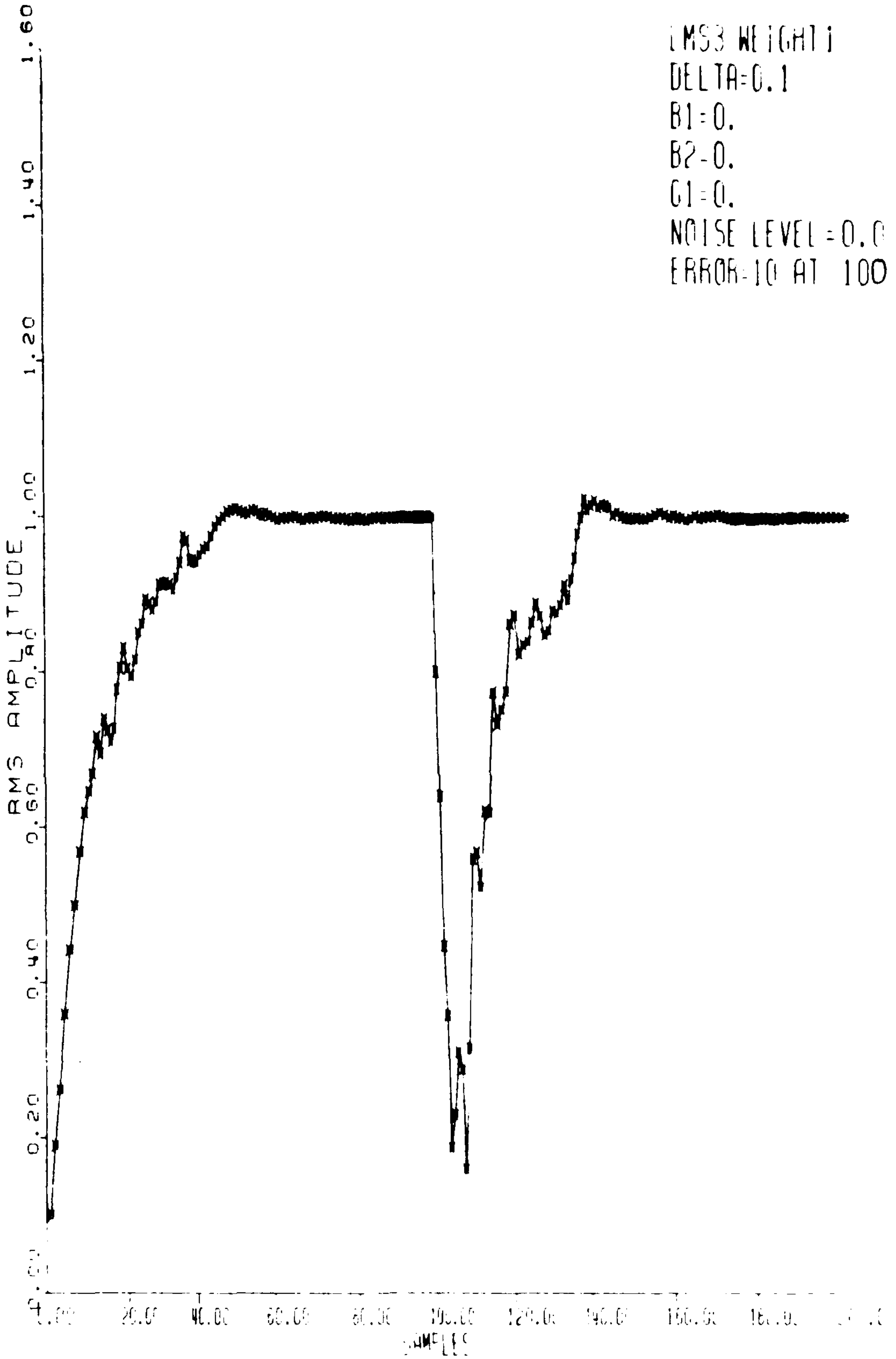

Figure 5Ø. Feedforward Estimator, Three Weights, Δ=.1 Ten Errors at 1ØØ

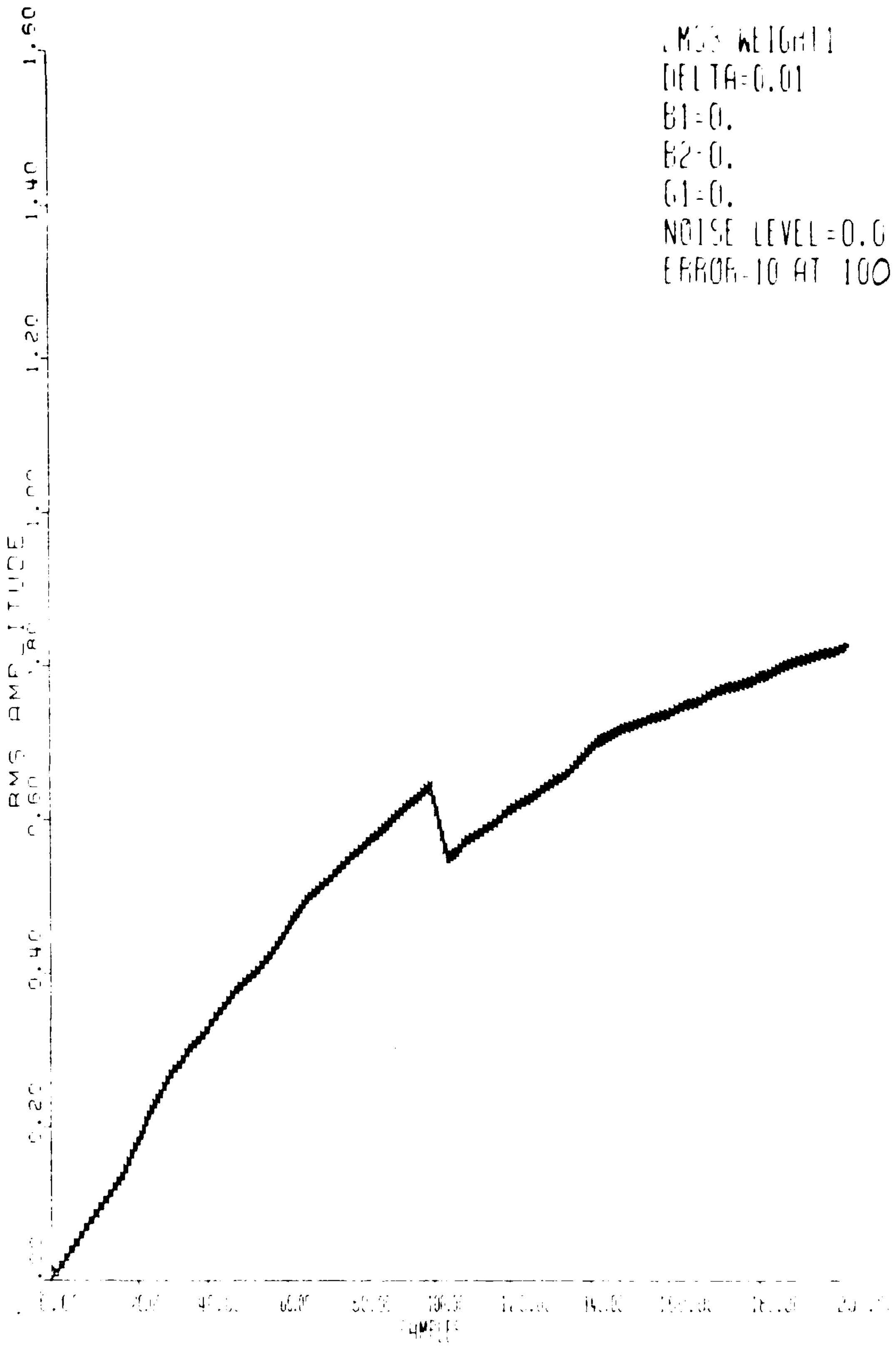

Figure 51. Feedforward Estimator, Three Weights, Δ =.01
Ten Errors at 100

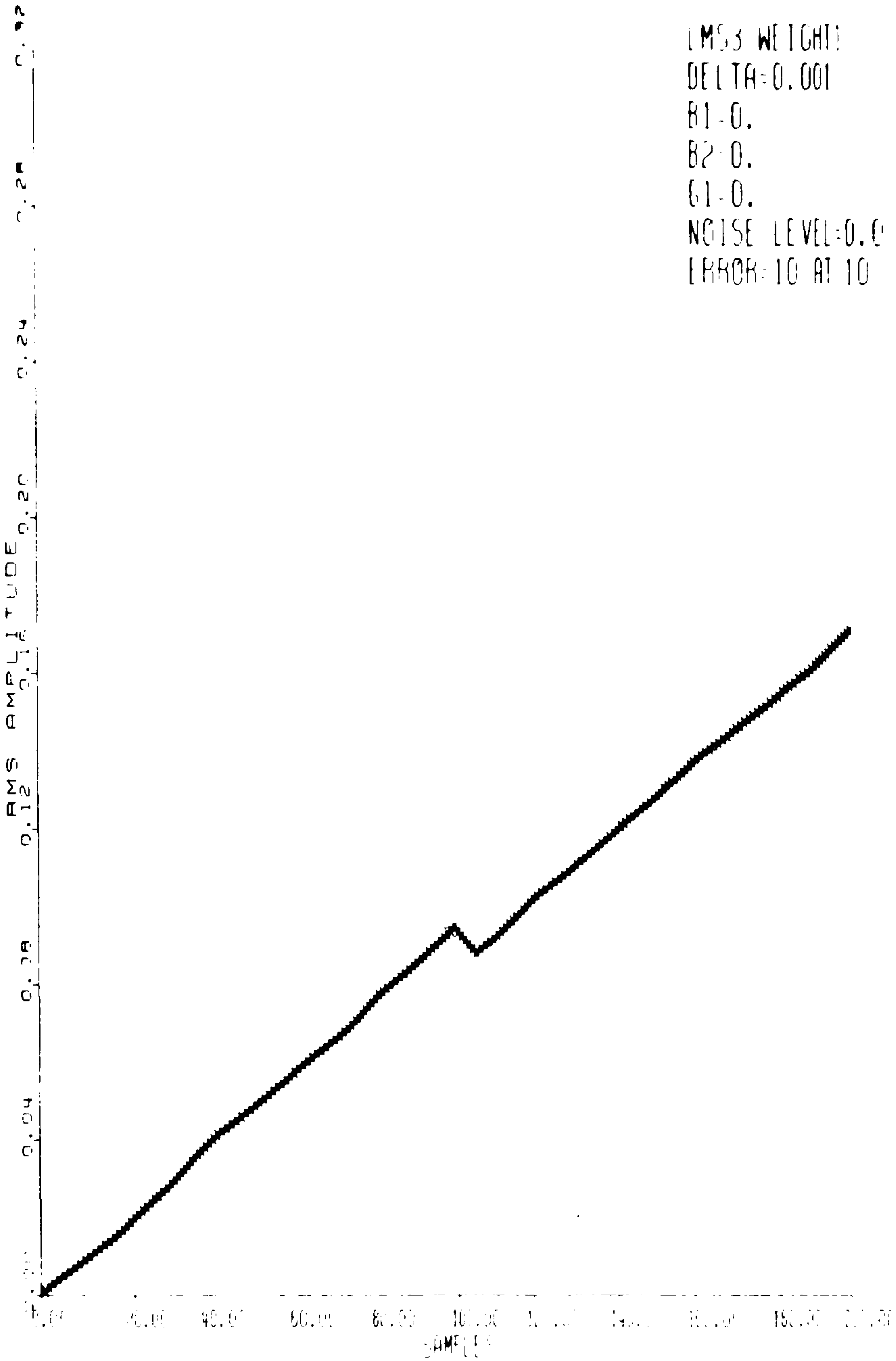

Figure 52. Feedforward Estimator, Three Weights, Δ=.001
Ten Errors at 100

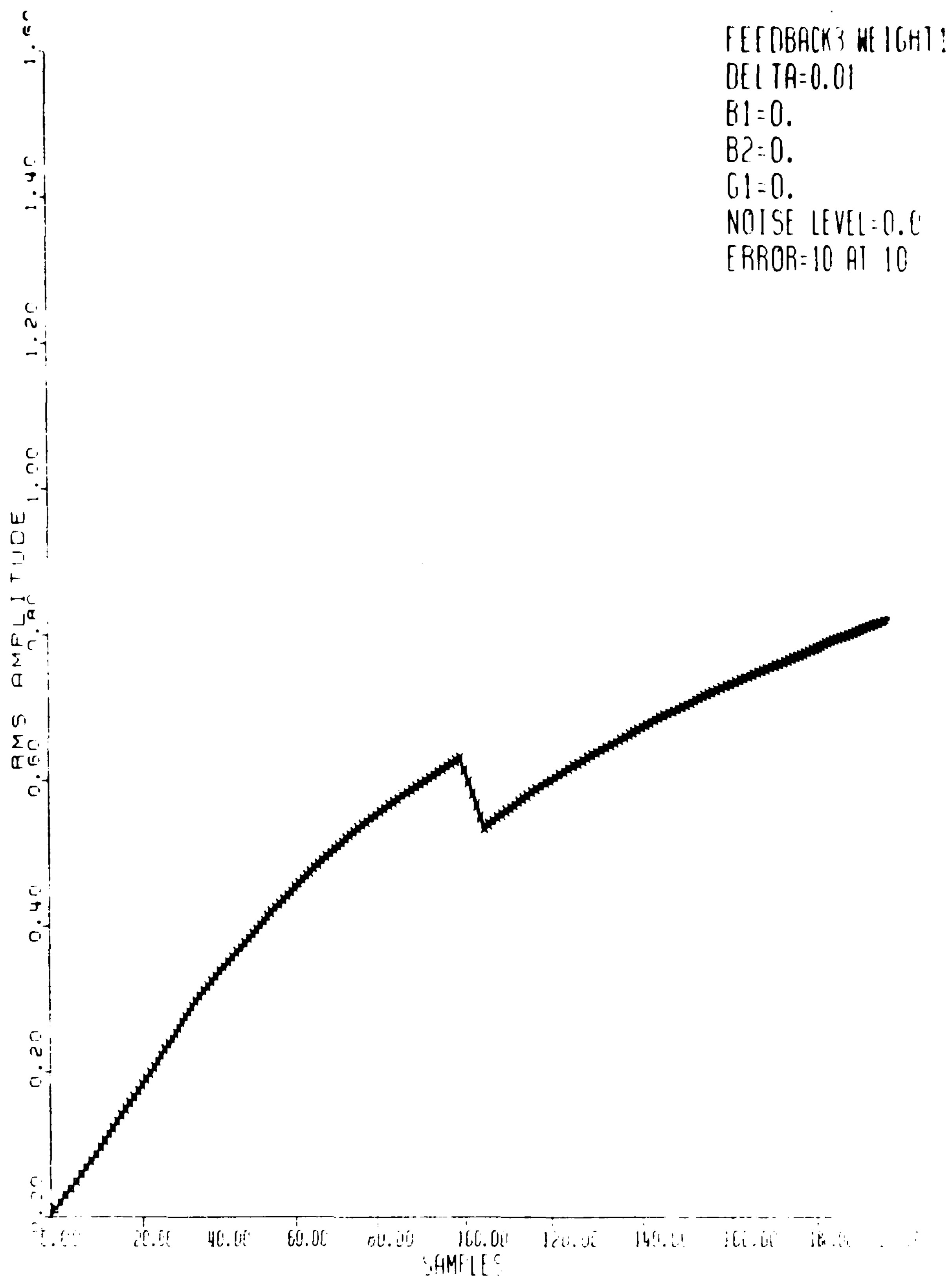

Figure 53. Feedback Estimator, Three Weights, Δ=.01 Ten Errors at 100

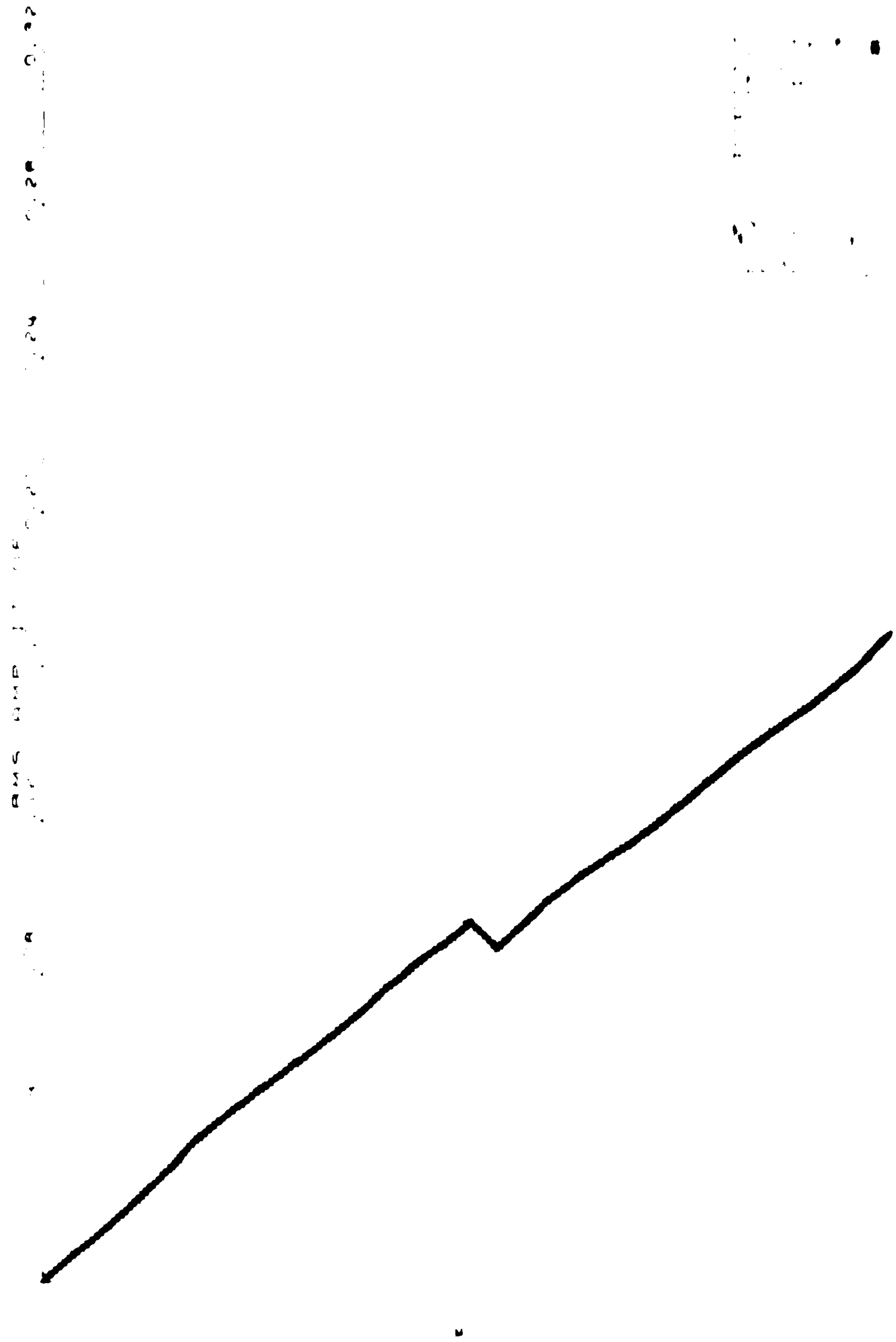

Figure 54. Feedback Estimator, Three Weights, $\Delta = .001$
Ten Errors at 100

Figure 55. Lattice Estimator, Three Weights, One Error at 100 Δ_1=.01 Δ_2=.1

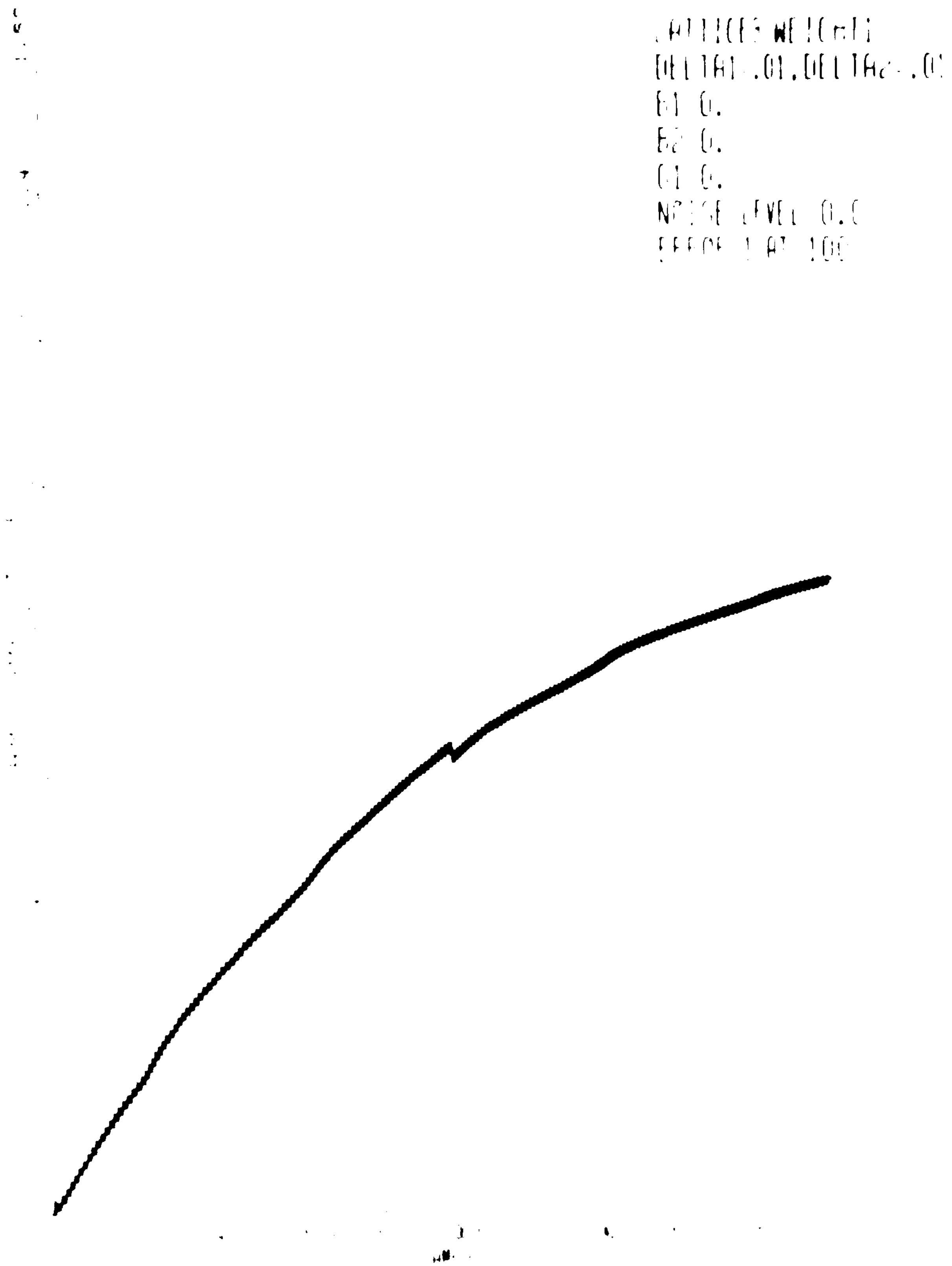

Figure 56. Lattice Estimator, Three Weights, One Error at 100 $\Delta_1 = .01$ $\Delta_2 = .01$

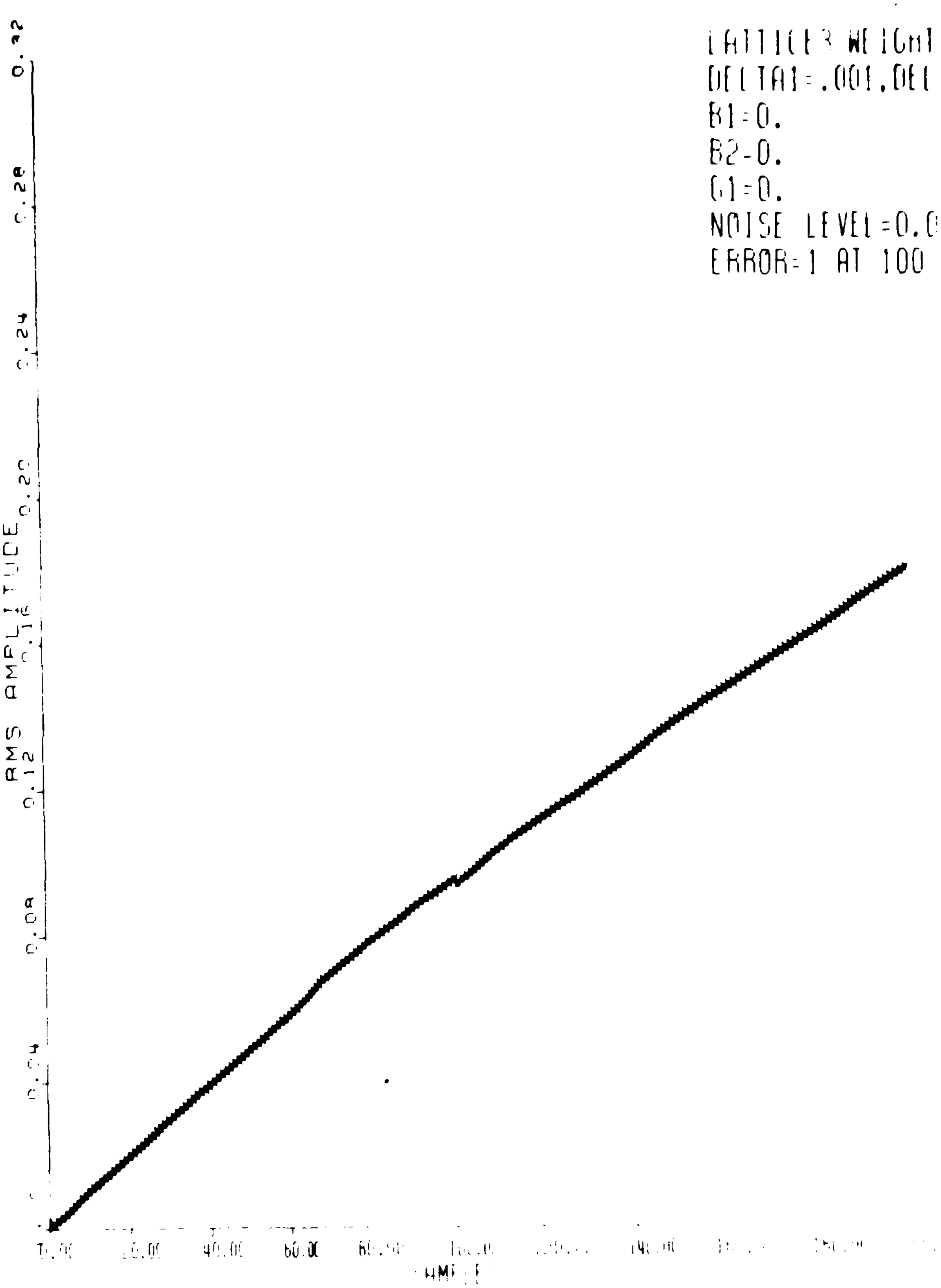

Figure 57. Lattice Estimator, Three Weights, One Error at 100 Δ_1=.001 Δ_2=.001

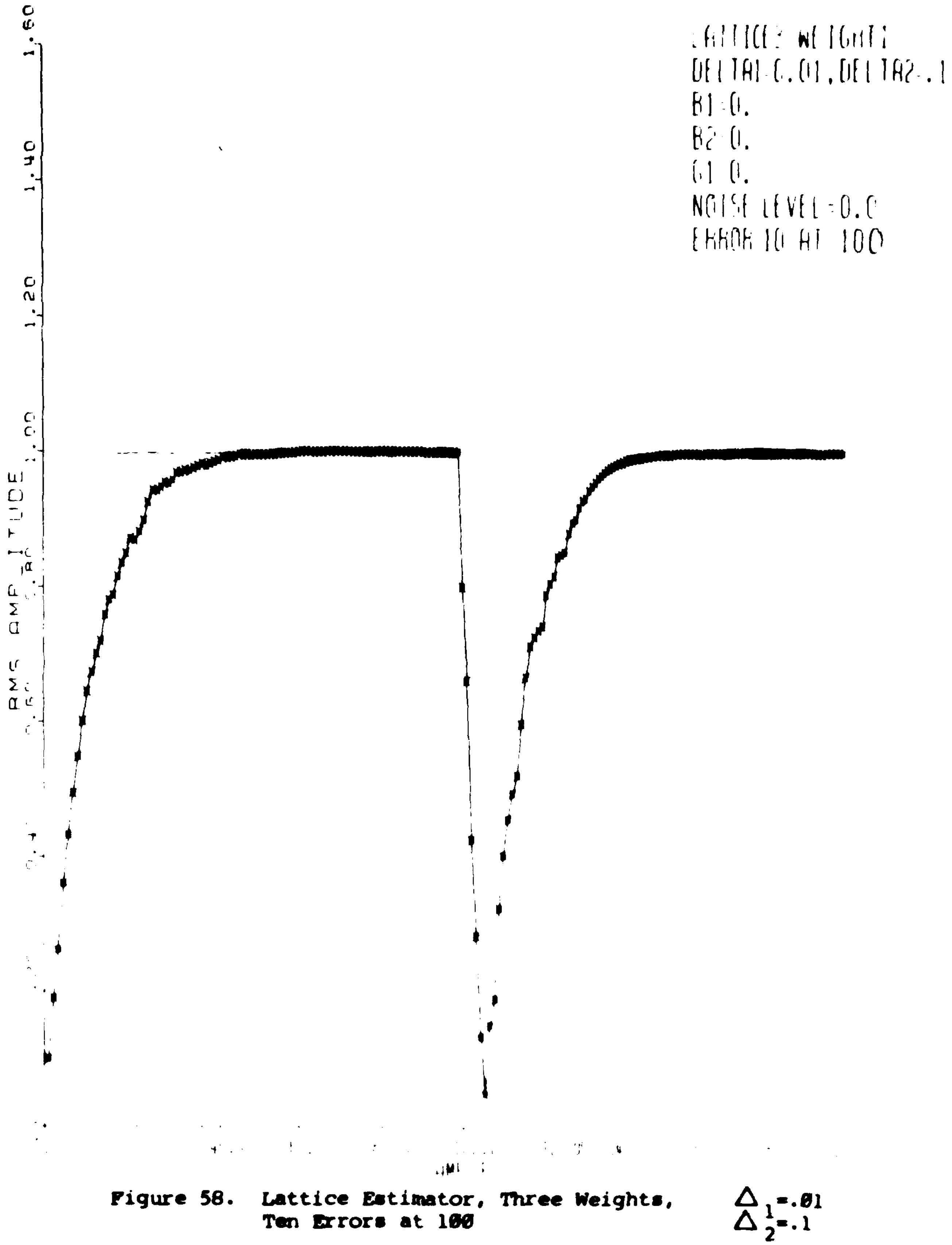

Figure 58. Lattice Estimator, Three Weights, Ten Errors at 100 $\Delta_1 = .01$ $\Delta_2 = .1$

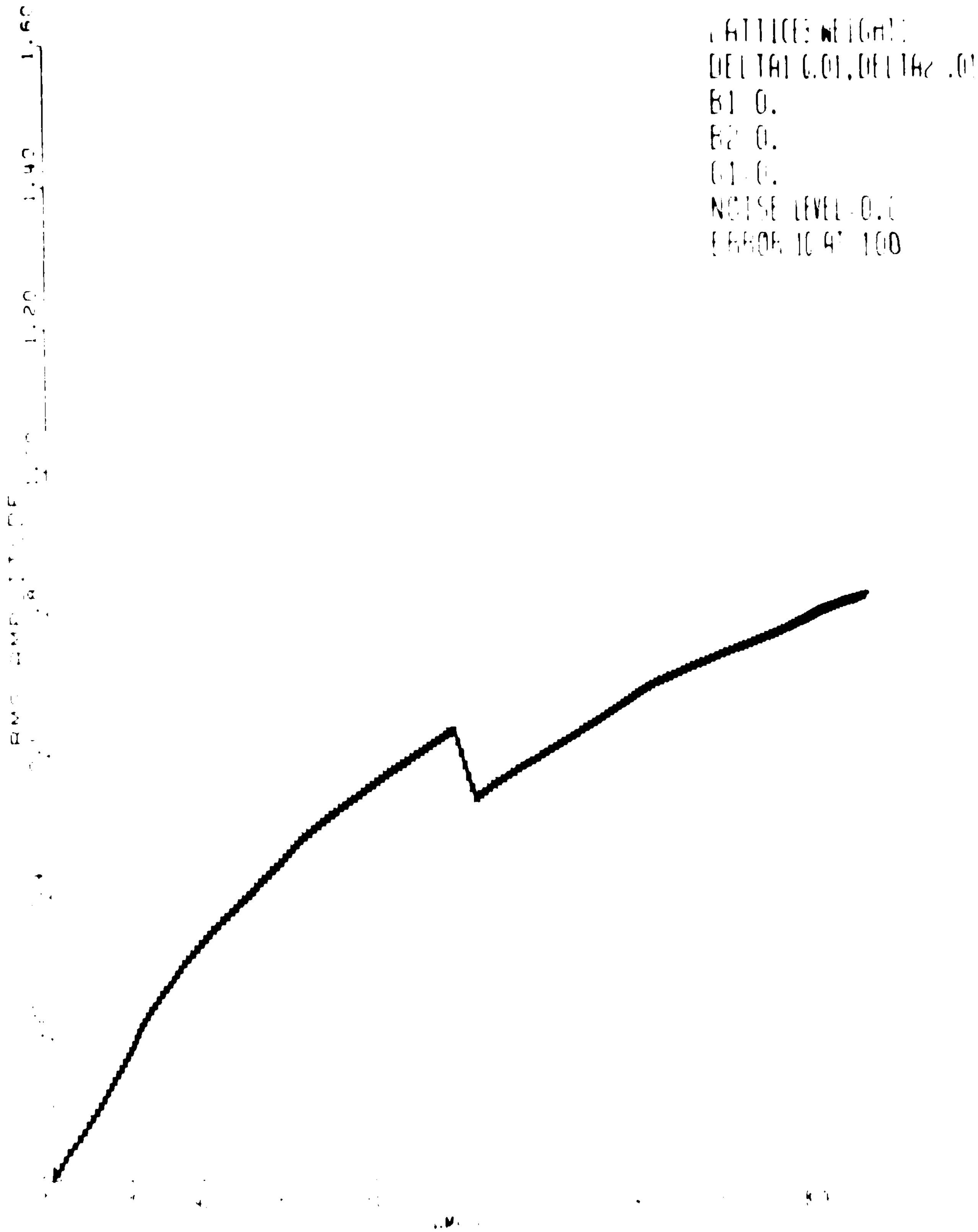

Figure 59. Lattice Estimator, Three Weights, Ten Errors at 100 $\Delta_1 = .01$ $\Delta_2 = .01$

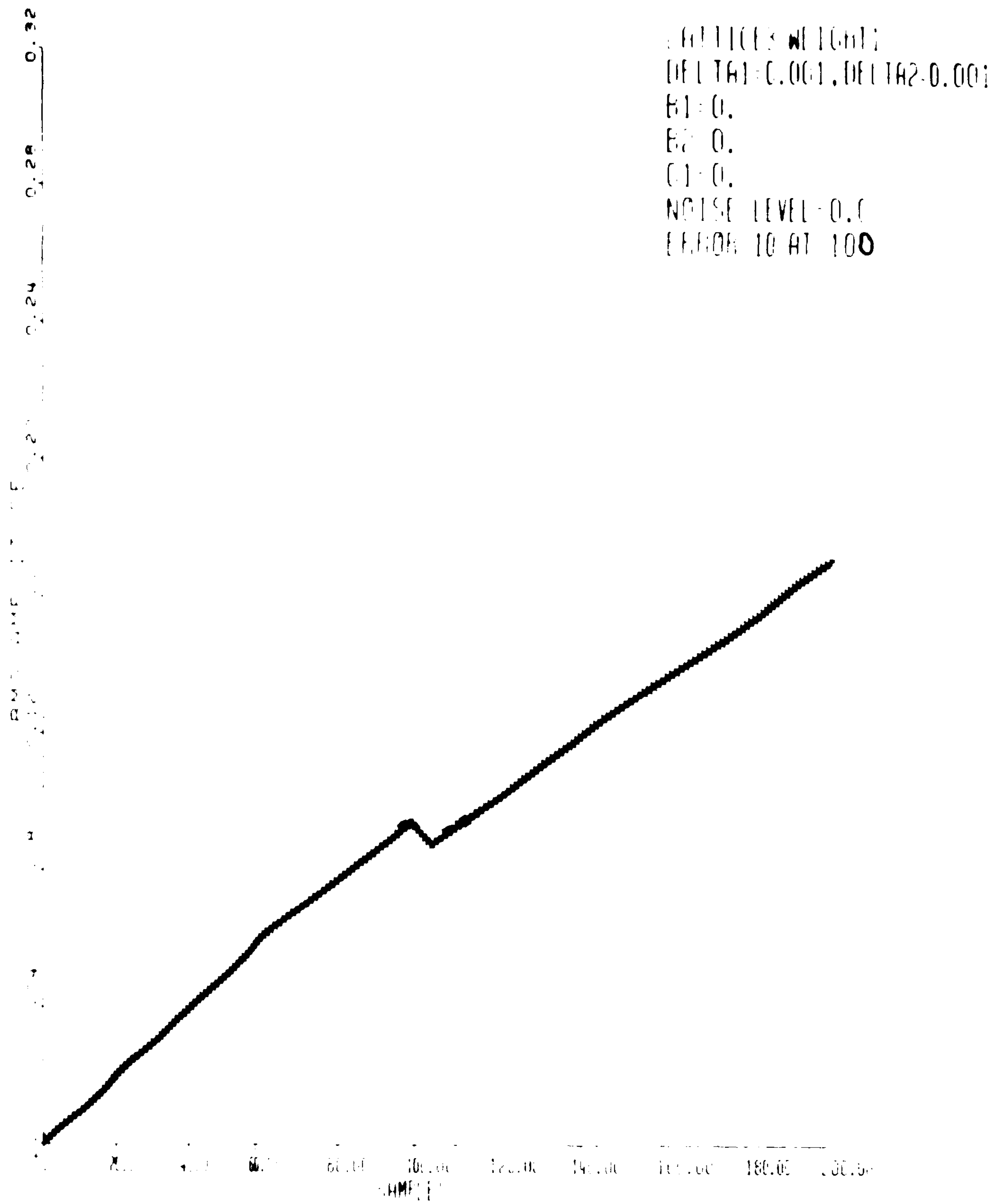

Figure 60. Lattice Estimator, Three Weights, Ten Errors at 100 Δ_1=.001 Δ_2=.001

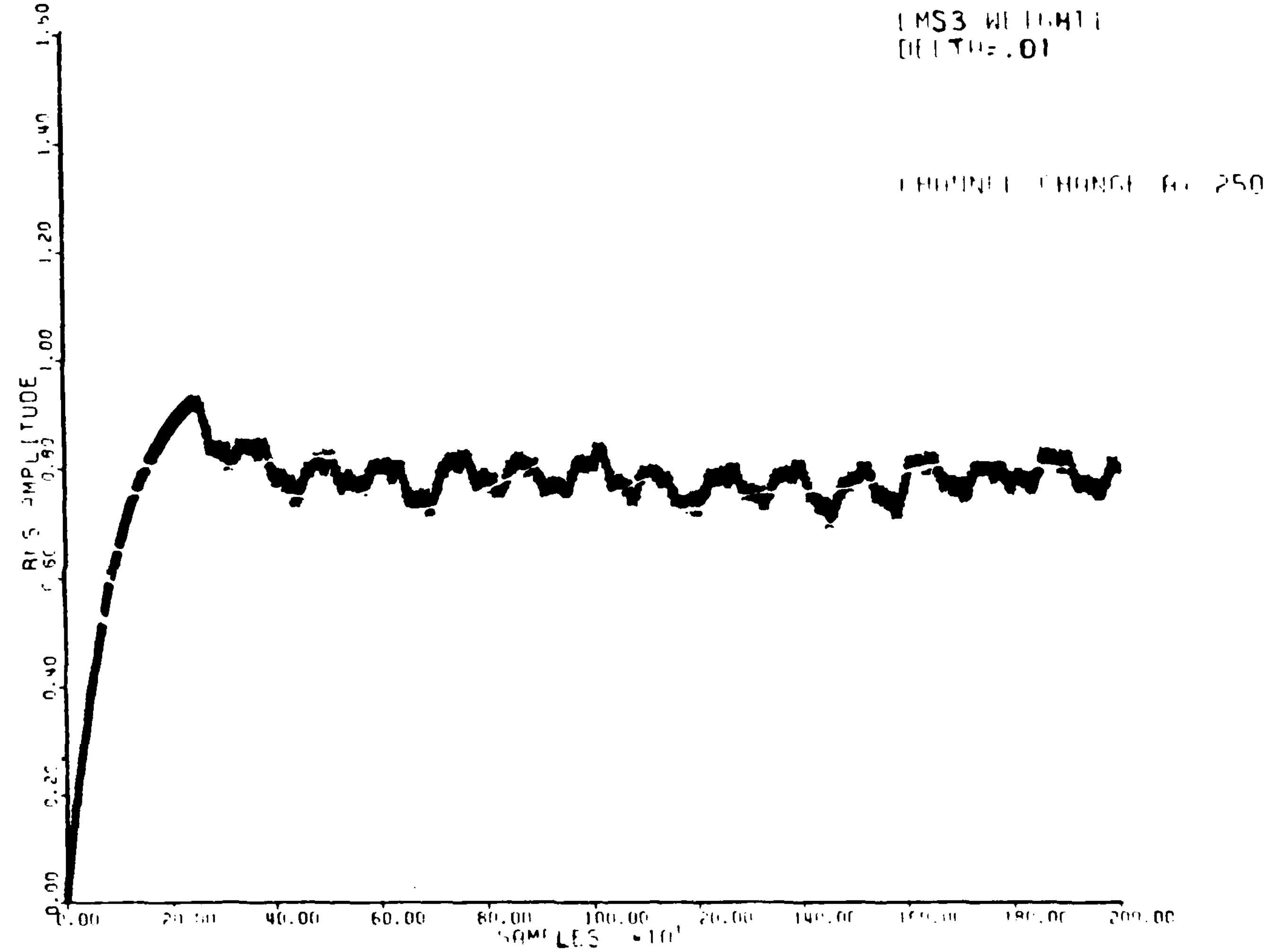

Figure 61. Feedforward Estimator, Three Weights Δ=.Ø1
Channel Change at 25Ø

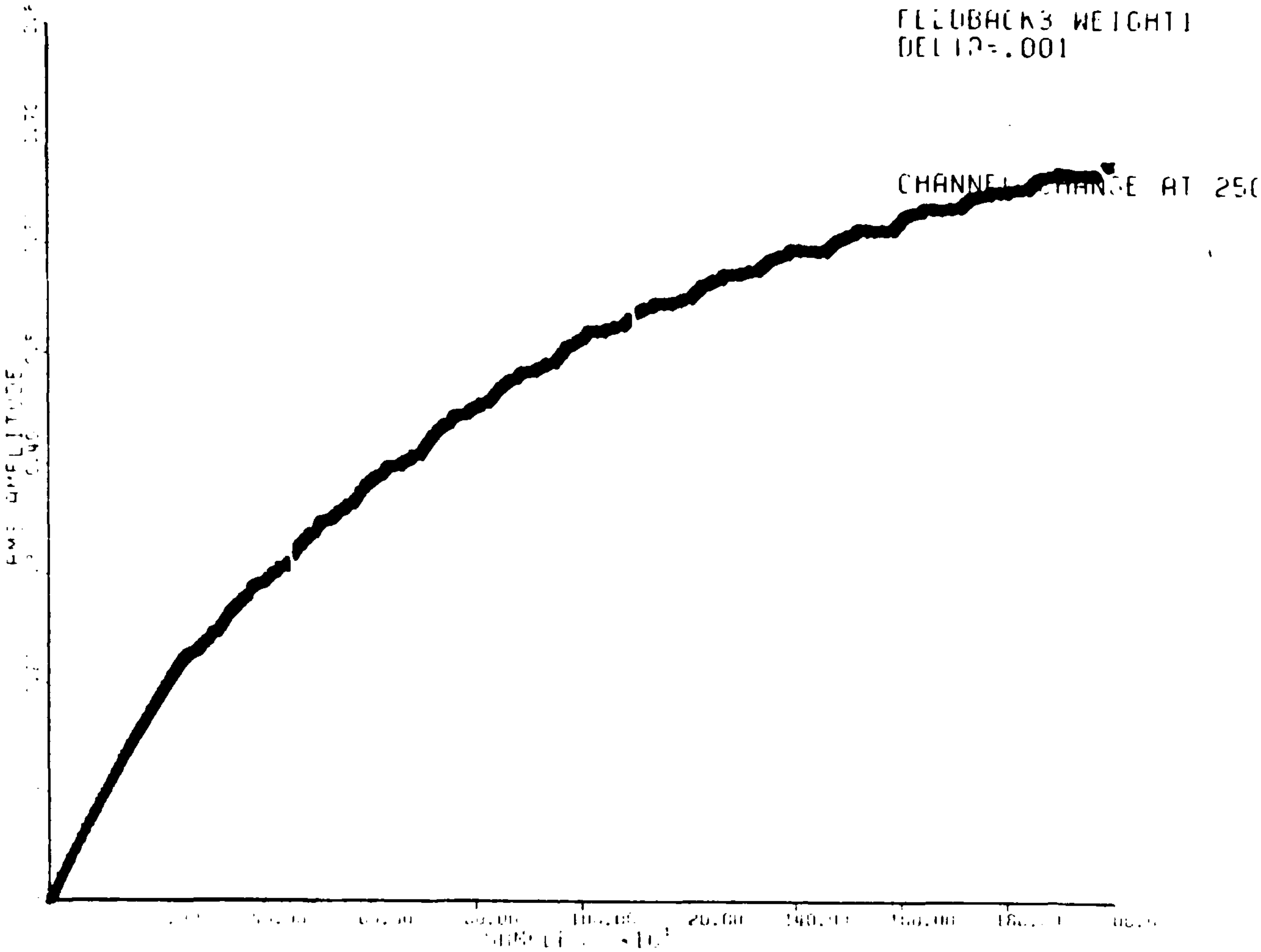

Figure 62. Feedback Estimator, Three Weights Δ=.001
Channel Change at 250

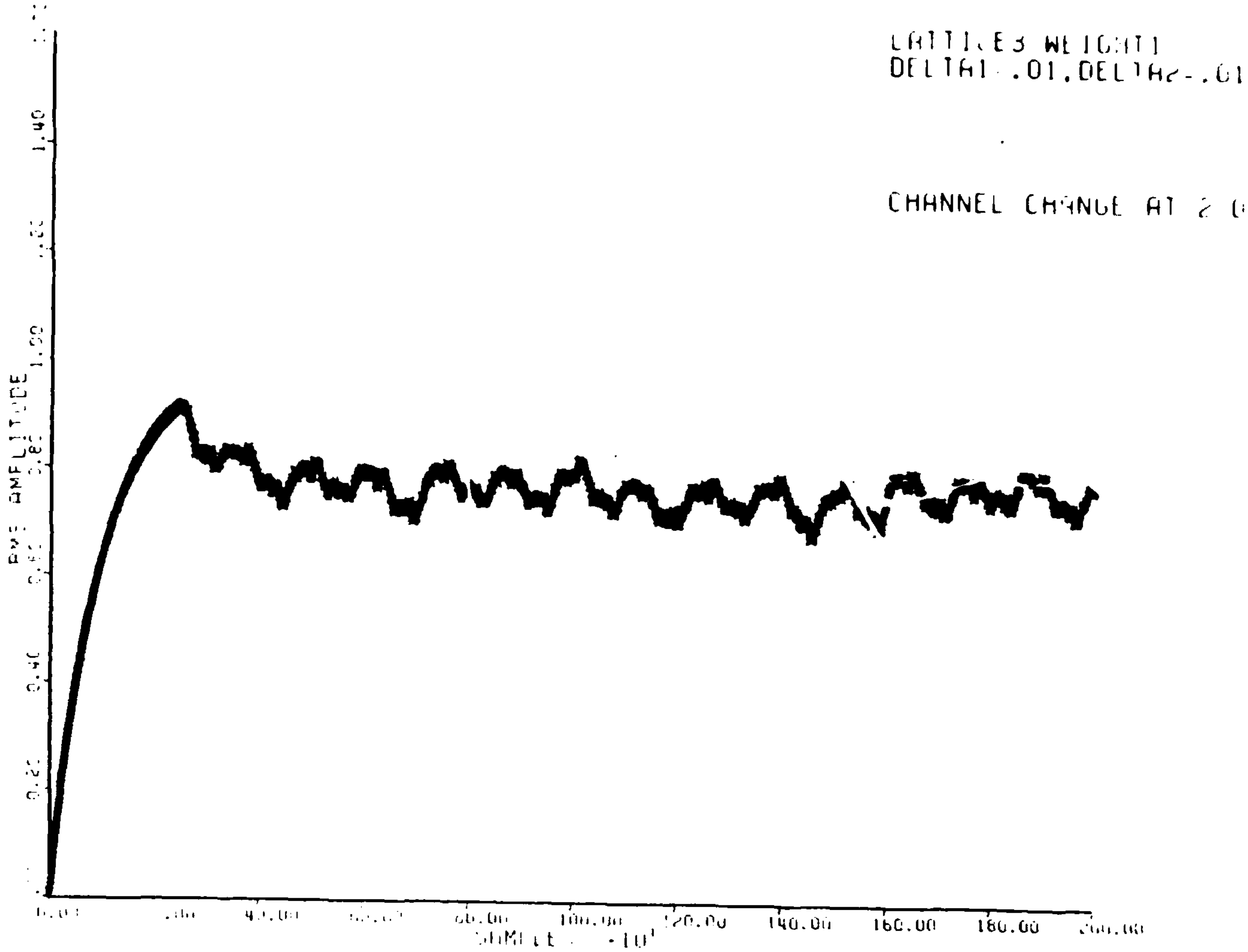

Figure 63. Lattice Estimator, Three Weights Δ_1=.01
Channel Change at 250 Δ_2=.01

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This investigation involved the comparison of three types of channel estimation techniques, the Feedforward Estimator, the Feedback Estimator, and the Lattice Estimator. A computer simulation of a communications channel was run involving varying levels of amplitude distortion, phase distortion, and Gaussian noise imposed on a data stream. The resulting output of the channel was fed to a receiver consisting of a detector and a channel estimator. The estimator took the output of the channel and the detector and used them to identify the impulse response of the channel.

Of the three channel estimators, the Feedback Estimator proved superior in terms of performance under varying levels of channel distortion and noise. Furthermore, the Feedback Estimator demonstrated overall better error handling capabilities. Finally, the Feedback Estimator proved to be the simplest algorithm to implement of the three.

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