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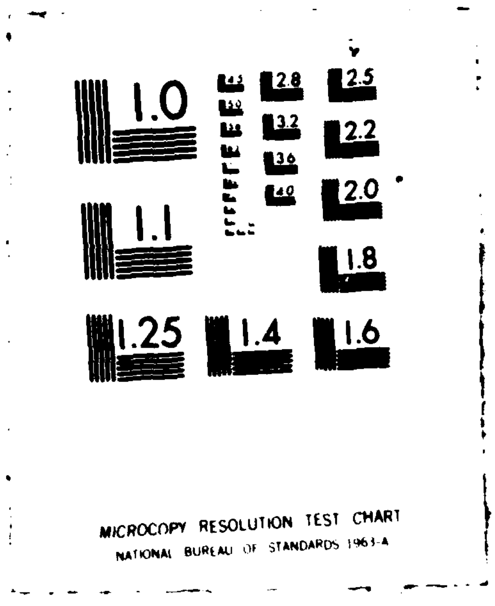
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ESTIMATION OF PARAMETERS IN LATENT CLASS MODELS WITH CONSTRAINTS ON THE PARAMETERS

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June 1986

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ABSTRACT

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INTRODUCTION

The purpose of this paper is twofold: first, to review the application of the EM algorithm of Dempster, Laird, and Rubin (1977) to marginal maximum likelihood estimation of parameters in the latent class model, and second, to extend this algorithm to the case where there are monotone homogeneity constraints on the item parameters.

Let us briefly review the elements of latent class models. The reader desiring a thorough introduction can consult Lazarsfeld and Henry (1968). The data to be accounted for are vectors of responses to items. In this paper we are only concerned with dichotomous item responses, although many of the ideas can be generalized to the case of polychotomous responses (cf. Goodman, 1974). It is assumed that every subject belongs to exactly one of a finite set of mutually exclusive and exhaustive latent classes. Theoretically, the distribution of the response vectors is to be accounted for by two sets of parameters and one key assumption. The two sets of parameters are the state probabilities, (ν_k), governing the multinomial distribution of subjects over the latent classes, and the conditional probabilities of correct response to each item, given the respective states, (p_{kj}). The key assumption is that the responses are _conditionally independent_, given the state of subject. This implies that any relationships between items must be explained in terms of differences in the p_{kj}'s between classes. Models are specified by stipulating the number of classes and by placing constraints on the matrix of conditional probabilities.

For some time the problem of estimating parameters in latent class structures presented a real obstacle in the application of the latent class framework. It is necessary to employ iterative procedures in which one selects a set of trial values, improves upon these values in the light of the data via some appropriate algorithm, and then repeats the process until (one hopes) the values stabilize at a good solution. McHugh (1956) derived the maximum likelihood estimators, but his solution applies only to the unconstrained model.

A great advance was achieved when Goodman (1974) described a particularly simple interative procedure which also has the virtue of automatically producing estimates of probabilities which fall in the unit interval; furthermore it is very easy to modify the procedure to satisfy a fair variety of other constraints on the parameters. There is one problem which Goodman's procedure shares with McHugh's, however. Both procedures take as their data the frequency counts in the cells of the multi-way item-by-item ---by-item contingency table. For a relatively small number of items this presents no problem. But the number of cells grows exponentially with the number of items, so calculations which require dealing with all these cells become impractical very quickly as the number of items is increased. For example, the contingency table for data from a 20 item test would have 2^{20} cells, which is more than a million. Fortunately, it is possible to formulate an algorithm which is logically equivalent to Goodman's, but which circumvents the problem of dealing with all possible cells in the n-way contingency table.

Goodman's algorithm and the modification of it to be presented here are just special cases of the EM algorithm applied to the latent class model. It will be useful to carefully review the rationale behind the application of the EM algorithm in the latent class model context, in order to lay the groundwork for the extension of the algorithm to cover monotone homogeneity constraints on the item parameters. The rationale will be developed in the next section and extended to monotonely homogeneous items in the section after that.

APPLICATION OF THE EM ALGORITHM TO THE LATENT CLASS MODEL

Estimation of parameters in the latent class model would be easy if we knew the state of each subject. The maximum likelihood estimates of the distribution of subjects over states would just be the sample proportions falling in the respective states. The estimates of conditional response probabilities to items, given state, would be the corresponding sample proportions of item responses.

The missing data about the states of the respective subjects turns an easy problem into a hard one. Problems with this general character, which would be manageable if only some crucial information were not missing, occur in many contexts. They have inspired numerous special algorithms, often of the following form:

1. Make an initial guess at the parameter values.

2. Using this guess, make an informed guess regarding the missing data.

3. Using this informed guess in place of the missing data, apply the procedure you would ordinarily use to estimate parameter values.

4. Replace the initial guess at the parameter values with the latter estimates and repeat the process until the parameter estimates in steps 1 and 3 no longer differ significantly.

Dempster, Laird, and Rubin (1977) synthesized these many special algorithms into a general approach, which they call the EM algorithm, and showed that under fairly general conditions, if maximum likelihood procedures are used at each iteration in Steps 2 and 3 above, the algorithm converges to marginal maximum likelihood estimates.

In describing how this process works in the case of the latent class model, let us use the following notation.

$$x_{ij} = \begin{cases} 1 & \text{if subject } i \text{ is correct on item } j \\ 0 & \text{if subject } i \text{ is incorrect on item } j; \end{cases}$$

n = the number of subjects;

J = the number of items;

ν_k = the probability of a subject being in class or state k;

s = the number of latent classes or states;

$\underline{\nu} = (\nu_1, \ldots, \nu_s)$, the vector of state probabilities;

$\underline{x}_i$ = the vector of responses of subject i to items $j = 1, .., J$;

p_{kj} = the conditional probability that a subject in state k will respond correctly to item j;

$P = (p_{kj})$, the states-by-items matrix of conditional response probabilities;

$\underline{e}_k$ = unit vector with 1 in $k^{\underline{th}}$ coordinate and 0's everywhere else, $k=1,\ldots,s$;

$\underline{z}_i$ = the unit vector $\underline{e}_k$ corresponding to the state of subject i.

Note that $\underline{e}_k'\underline{z}_i$ is 1 if subject i is in state k, and is 0 otherwise.

Let us denote the conditional probability of obtaining response vector $\underline{x}_i$, given that subject i is in state k, by

$$l_k(\underline{x}_i) = l_k(\underline{x}_i | P, \underline{\nu})$$

$$= \prod_{j=i}^{J} p_{kj}^{\,x_{ij}}(1-p_{kj})^{1-x_{ij}}. \tag{1}$$

The likelihood of the "complete" data, that is, the joint likelihood of responses and missing state membership vectors $\underline{z}_i$, is given by

$$L(\underline{x}_1,\ldots,\underline{x}_n;\ \underline{z}_1,\ldots,\underline{z}_n | P, \underline{\nu})$$

$$= \prod_{i=1}^{n} \prod_{k=1}^{s} [\nu_k l_k(\underline{x}_i)]^{\underline{e}_k' \underline{z}_i}$$

Let I_k be the set of indices for subjects in state k, i.e. those for whom $\underline{e}_k'\underline{z}_i = 1$. Then the likelihood of the complete data can be rewritten as

$$L(\underline{x}_1,\ldots,\underline{x}_n;\ \underline{z}_1,\ldots,\ \underline{z}_n | P,\ \underline{\nu}) \tag{2}$$

$$= \prod_{k=1}^{s} \prod_{i\varepsilon I_k} \nu_k l_k(\underline{x}_i)$$

$$=\left(\prod_{k=1}^{s} \nu_k^{n_k}\right)\left(\prod_{k=1}^{s}\prod_{j=1}^{J} p_{kj}^{r_{kj}}(1-p_{kj})^{n_k-r_{kj}}\right) \tag{3}$$

where n_k denotes the number of subjects in state k and

$$r_{kj} = \sum_{i\varepsilon I_k} x_{ij},$$

denotes the number of correct responses to item j from subjects in state k. Thus, the likelihood of the complete data is an exponential family and the set of r_{kj}'s and n_k's are sufficient statistics for the likelihood.

The likelihood function for the complete data is to be contrasted with the marginal likelihood function of the data actually observed. The marginal likelihood of the response vector $\underline{x}_i$ for a given subject is the average over states of the conditional probabilities of the response vector, given the states,

$$\begin{aligned} l^*(\underline{x}_i) &= l^*(\underline{x}_i | P,\underline{\nu}) \\ &= \sum_{k=1}^{s} \nu_k l_k(\underline{x}_i). \end{aligned} \tag{4}$$

The marginal likelihood of all the observed response vectors is given by

$$\begin{aligned} &L^*(\underline{x}_i,\ldots,\underline{x}_n | P,\nu) \\ &= E_z\{L(\underline{x}_1,\ldots,\underline{x}_n,z_1,\ldots,\underline{z}_n | P,\underline{\nu})\} \\ &= \prod_{i=1}^{n} l^*(\underline{x}_i). \end{aligned} \tag{5}$$

It is possible to attack the maximization of Equation 5 directly, but doing so leads to a cumbersome system of nonlinear equations. Another approach suggested by the relationship between the likelihood function for the complete data and the marginal likelihood can be used to maximize the latter indirectly.

Taking logarithms of the likelihood of the complete data in equation 3 yields

$$\log L = \sum_{k=1}^{S} n_k \log \nu_k + \sum_{k=1}^{S} \sum_{j=1}^{J} [r_{kj} \log p_{kj} + (n_k - r_{kj}) \log(1 - p_{kj})]. \quad (6)$$

If we knew the state of each subject, we could count the n_k's and r_{kj}'s and the standard ratios of these frequencies would be seen to be the maximum likelihood estimators of the ν_k's and p_{kj}'s. Suppose we symbolically calculate the conditional expectation of Equation 6, given the observable response vectors $\underline{x}_1, \ldots, \underline{x}_n$ and trial values of the parameters, P_0 and $\underline{\nu}_0$:

$$E\{\log L(\underline{x}_1, \ldots, \underline{x}_n, \underline{z}_1, \ldots, \underline{z}_n | P, \underline{\nu}) | \underline{x}_1, \ldots, \underline{x}_n, P_0, \underline{\nu}_0\}$$

$$= \sum_{k=1}^{S} E_0(n_k) \log \underline{\nu}_k + \sum_{k=1}^{S} \sum_{j=1}^{J} [E_0(r_{kj}) \log p_{kj} + E_0(n_k - r_{kj}) \log (1 - p_{kj})], \quad (7)$$

where $E_0(.)$ denotes the conditonal expectation, given the responses and trial parameter values, $E(. | \underline{x}_1, \ldots, \underline{x}_n, P_0, \underline{\nu}_0)$.

Let ν_{k0} denote the $k^{\underline{th}}$ coordinate of the trial state probability vector $\underline{\nu}_0$ and let ν_{ki} denote the conditional probability that subject i is in state k, given the subject's responses, $\underline{x}_i$, and the trial parameter values P_0 and ν_0. By Bayes' theorem,

$$\nu_{ki} = \frac{\nu_{ko} l_k(\underline{x}_i | P_o, \underline{\nu}_o)}{l^*(\underline{x}_i | P_o, \underline{\nu}_o)} \quad . \tag{8}$$

The conditional expectations $E_0(n_k)$ and $E_0(r_{kj})$ can be easily computed in terms of the ν_{ki}'s. Recall that the scalar product of the state vector for subject i, $\underline{z}_i$, and the unit vector corresponding to state k, $\underline{e}_k$, is 1 if subject i is in state k and 0, otherwise. Thus,

$$n_k = \sum_{i=1}^{n} \underline{e}_k' \underline{z}_i$$

and

$$r_{kj} = \sum_{i=1}^{n} x_{ij} \underline{e}_k' \underline{z}_i .$$

The expected number of subjects in state k, given the responses and trial values of the parameters, is

$$\begin{aligned} E_o(n_k) &= E\left(\sum_{i=1}^{n} \underline{e}_k' \underline{z}_i \,\middle|\, \underline{x}_1, \ldots, \underline{x}_n ;\ P_o, \underline{\nu}_o \right) \\ &= \sum_{i=1}^{n} P(\underline{z}_i = \underline{e}_k | \underline{x}_i ;\ P_o, \underline{\nu}_o) \\ &= \sum_{i=1}^{n} \nu_{ki} . \end{aligned} \tag{9}$$

The expected number of correct responses to item j from subjects in state k, given the responses and trial parameter values, is

$$\begin{aligned} E_0(r_{kj}) &= E\left(\sum^{n} x_{ij}\underline{e}_k'\underline{z}_i \,\middle|\, \underline{x}_1,\ldots,\underline{x}_n;\ P_0,\underline{v}_0\right) \\ &= \sum^{n} x_{ij}P(\underline{z}_i=\underline{e}_k \,|\, \underline{x}_i,\ldots,x\ ;\ P_0,\underline{v}_0) \qquad (10) \\ &= \sum_{i=1}^{n} x_{ij}v_{ki}. \end{aligned}$$

Equations 8, 9 and 10 enable us to compute numbers to use in Equation 7. Note that the trial values of the parameters P_0 and $\underline{v}_0$ used to computer the $E_0(n_k)$'s and $E_0(P_{kj})$'s are distinct from parameters P and $\underline{v}$ which are free variables in the likelihood functions given in Equations 5, 6 and 7.

It is often relatively easy to maximize Equation 7. If the resulting parameter estimates differ from the trial values P_0 and $\underline{v}_0$, they will yield higher values of the marginal likelihood function L* than P_0 and $\underline{v}_0$, though they may not maximize L*. If they do not differ from P_0 and $\underline{v}_0$, then P_0 and $\underline{v}_0$ are also solutions to the marginal likelihood equations which result from setting the partial derivatives of log L* equal to zero. This fact is established by Dempster, et. al (1977) for problems in which the likelihood of the complete data is an exponential family, as is the case here. Sometimes there are multiple possible solutions to the marginal maximum likelihood equations and the question arises whether a given solution is the global maximum of the likelihood function. Ways of dealing with this problem will be discussed later in this section.

Finding values of P and $\underline{\nu}$ to maximize Equation 7 breaks down conveniently into two subproblems: maximization of

$$L_\nu = \sum_{k=1}^{S} E_o(n_k)\log \nu_k \tag{11}$$

with respect to the vector of state probabilities, $\underline{\nu}$, and maximization of

$$L_p = \sum_{k=1}^{S} \sum_{j=1}^{J} [E_o(r_{kj})\log p_{kj} + E_o(n_k - r_{kj})\log (1-p_{kj})] \tag{12}$$

with respect to the matrix of conditional response probabilities, P. If no contraints are placed on the parameters, the solution to the first problem is given by

$$\nu_k' = \frac{E_o(n_k)}{n} \tag{13}$$

$$= \frac{\sum_{i=1}^{n}\nu_{ki}}{n} .$$

The solution to the second problem is given by

$$p_{kj}' = \frac{E_o(r_{kj})}{E_o(n_k)} \tag{14}$$

$$= \frac{\sum_{i=1}^{n} x_{ij}\nu_{ki}}{\sum_{i=1}^{n} \nu_{ki}} .$$

Let Θ represent a generic item parameter, possibly affecting several of the p_{kj}'s. In maximizing the part of Equation 7 which depends on the free item parameters, we set the partial derivative of Equation 11 with respect to Θ equal to zero; the resulting equation can be arranged to read

$$n \sum_{k=1}^{J} \sum_{k=1}^{s} \nu'_k \frac{(p'_{kj}-p_{kj})}{p_{kj}(1-p_{kj})} \cdot \frac{\partial p_{kj}}{\partial\Theta} = 0. \qquad (15)$$

Equality and complementarity constraints

In general, Equation 15 leads to a system of nonlinear equations which can be very difficult to solve. However, there are some special cases which are easy to handle. For example, if there are no constraints on the p_{kj}'s, then each p_{kj} is a distinct parameter affecting only one term in the sum in Equation 15. The partial derivative with respect to p_{kj} itself is 1, all the other partials are 0, and we obtain p'_{kj} as the solution.

More generally, solution is easy if we only wish to impose equality or complementarity constraints, so that we require $p_{kj} = \Theta$ for one set of p_{kj}'s, $p_{kj} = 1-\Theta$ for another set, and no p_{kj} outside of these sets depends on Θ. Then $p_{kj}(1-p_{kj})$ equals $\Theta(1-\Theta)$, independent of subscript, for all j,k such that the partial derivative $\partial p_{kj}/\partial\Theta$ is nonzero. The partial derivative is 1 for p_{kj}'s equal to Θ and -1 for those equal to $1-\Theta$. Let I_Θ be the set of indices j,k for which p_{kj} equals Θ and $I_{\bar\Theta}$ the set for which p_{kj} is $1-\Theta$. Then

Equation 14 reduces to a linear equation whose solution is the following weighted combination of p'_{kj}'s:

$$\Theta = \frac{\sum\limits_{j,k\varepsilon I_{\Theta}} \nu'_k p'_{kj} + \sum\limits_{j,k\varepsilon I_{\bar{\Theta}}} \nu'_k (1-p'_{kj})}{\sum\limits_{j,k\varepsilon I_{\Theta}} \nu'_k + \sum\limits_{j,k\varepsilon I_{\bar{\Theta}}} \nu'_k} \qquad (16)$$

The application of the EM algorithm to estimation of parameters in the latent class model with equality and complementarity constraints can be summarized as follows.

1. Select trial values of the parameters P_0 and $\underline{\nu}_0$.
2. Compute conditional state probabilities for all subjects, using Equation 8.
3. Revise the parameter estimates of $\underline{\nu}$ via Equation 13 and the estimates of P via Equations 14 and 16.
4. Repeat Steps 1 through 3, using the revised estimates as new trial values, until the trial values and the revised values no longer differ significantly.

The key computations in this algorithm involve ratios of counts or estimates of counts in which the denominators are always at least as big as the numerators. The constraint that all estimates lie in the unit interval is therefore automatically satisfied. This is a significant feature of the EM approach not shared by the Newton-Raphson algorithm when applied to the marginal likelihood function in Equation 5.

The most significant problem which this algorithm is likely to encounter in practice is one that it shares with all existing algorithms that would be practical to use on latent class model problems. It was noted earlier in the paper that the maximum likelihood equations can have multiple solutions. In problems where there are multiple solutions, any iterative algorithm will tend to go to a solution close to the trial values initially selected. The resulting solution may well not be the parameter values that truly maximize the likelihood, particularly if the starting values are selected arbitrarily. It is therefore a good idea to try a variety of plausible sets of starting values.

Goodman (1974) gives an algorithm for estimation of parameters in complex contingency tables where some of the variables are not observable. The specialization of his algorithm to the case of dichotomous responses is essentially equivalent to the algorithm given here. Since it is intended for analysis of contingency tables, it assumes that the joint response data for the subjects is summarized in that form. Latent class model estimation programs implementing Goodman's algorithm, such as Clogg and Sawyer (1981), are limited in terms of the number of items which they can accomodate, because the multi-item contingency table quickly becomes unmanageable as the number of items increases. The form of the algorithm given in this paper deals with each individual response vector, rather than cell counts in a contingency table. Hence, the effect of increasing the number of items has no effect on the algorithm beyond the increase in running time, which is directly proportional to the number of items. Actually, the effect on running time is more closely proportional to the square of the number of items in applications, such as scaling, in which the number of states in the model is also proportional to the number of items. Nevertheless, the effect is much more manageable than the exponential increase in the number of cells in the contingency table with which an algorithm for analysis of contingency tables must deal.

The EM algorithm in the form presented in this paper can cope with tests comprised of many items, while automatically satisfying the fundamental constraint that the parameter estimates all fall in the unit interval and any further equality and complementarity constraints the investigator may wish to impose on the parameters. This fact, together with computational simplicity at each iteration, makes the algorithm an attractive alternative to other approaches to the calculation of the maximum likelihood estimates for the latent class model. Two questions arise: one about how many models of interest can be formulated using only equality and complementarity constraints, and a second one about the possibility that there are other special kinds of constraints which would also yield easy solutions at each iteration of the algorithm.

The answer to the first question is that many latent class models of interest can be expressed in terms of equality and complementarity constraints on the parameters, including most of the models which have been proposed to date. The latent distance model of Lazarsfeld and Henry (1968) and the quasi-independence model of Goodman (1975), both of which are generalizations of the Guttman simplex model for scaling response patterns, fall in this category. Dayton and Macready (1976, 1980) have proposed anologs and extensions of these models for applications in the analysis of learning hierarchies; their extensions can also be expressed in terms of equality and complementarity constraints. Paulson (1985) has proposed models for signed-number addition test performance with one latent class for students who have mastered the concept and other latent classes corresponding to classes of subjects exhibiting certain systematic patterns of errors. These models are not scaling models, but they are expressable in terms of equality and complementarity constraints on the parameters.

If only equality and complementarity conditions constrain the item parameters, then each p_{kj} is influenced by exactly one parameter. This rules out models which characterize each p_{kj} in terms of conjoint effects of item and state parameters, as the Rasch model does, for example. It also rules out models that impose ordering constraints on the p_{kj}'s. Thus, while many interesting models can be cast in terms of equality and complementarity constraints, many others cannot. Fortunately, models involving conjoint item and state effects and models imposing ordering constraints can be formulated which lead to easily solved forms of Equation 15, preserving the computational simplicity of the EM algorithm.

EXTENSION TO MONOTONELY HOMOGENEOUS ITEMS

A set of items is said to be monotonely homogeneous if the probabilities of correct response in different subject states fall in the same order for all items. That is, for every pair of items j, j' and every pair of states k, k'

$$p_{kj} > p_{k'j} \Rightarrow p_{kj'} \geq p_{k'j'} \quad . \tag{16}$$

Any set of items conforming to a unidimensional item response theory which requires the probabilities of correct response to items be monotonically increasing functions of ability is monotonely homogeneous. All standard item response theory models impose this condition. On the other hand, if a set of items is monotonely homogeneous, then the averages of the conditional probabilities of correct response over all the items, given the respective states, must fall in the same order as the conditional probabilities for individual items. Let us define ability level for subjects in a given state to be the average of the conditional probabilities of correct response over all the items, i.e. the "true proportion correct". Consider the function associated with each item which is obtained when one plots the conditional probability of correct response to the item, given state, versus true proportion correct. This function is necessarily monotonically increasing for every item. That is, any monotonely homogeneous set of items is associated with a corresponding set of monotonically increasing item response functions. Thus, monotone homogeneity of a set of items is a necessary and sufficient condition for the items to be representable by an item response theory model with monotonically increasing item response functions.

The assumption of monotone homogeneity is of interest from a couple of different perspectives. Since it is the minimal assumption concerning the form of the item response function sufficient to yield a model with monotonically increasing tracelines, it is worth considering how far the theory can be developed with no further assumptions regarding the form of the functions. Mokken (1971), who first emphasized the importance of the assumption, Mokken and Lewis (1982), and Lewis (1985) have pursued this idea in developing a nonparametric approach to item response theory. A fundamental problem in this development is the estimation of item response functions. In this section we show how to obtain marginal maximum likelihood estimates for these functions in models restricted to a finite number of states. The restriction to a finite number of ability states would seem to be an extreme limitation on the value of such an approach, but Bock and Aitkin (1981) have shown that it is quite workable in application to standard item response theory models.

From another point of view, the assumption of monotone homogeneity is interesting because it provides a definitive criterion for deciding that a unidimensional representation of responses to a given set of items is inappropriate. Holland (1981) has derived from the assumption a series of necessary conditions observed data must satisfy in order to be capable of representation by a unidimensional item response theory model. Unlike the assumption itself, these conditions can be tested without estimating the item parameters. The simplest of the conditions is that interitem correlations must be nonnegative. Paulson (1985) has shown that this condition is violated in an analysis of signed-number addition test data from a study by Tatsuoka and Birenbaum (1979). Paulson describes a simple latent class model which does give a good account of this data. This model is not a scaling model: the states in the model correspond either to mastery of the concept or to one of a set of systematic misconceptions students fall into regarding the concept. The latter states are not ordered. The nonnegativity of interitem correlations is a simple but weak criterion for testing monotone homogeneity. Holland (1981) gives more stringent tests in terms of nonnegativity of correlations between indices based on combined item responses. We will describe a more direct approach later in this section - the likelihood ratio test of the goodness of fit of the monotonely homogeneous finite-state model compared to the fit of the corresponding latent class model without the monotone homogeneity constraint.

<u>Modification of the Algorithm to Provide Monotone Homogenity</u>

Recall that at each iteration of the EM algorithm, the problem of maximizing the conditional likelihood, given the responses and trial values of the parameters, reduces to maximization of two separate terms, one depending only on the state probability distribution parameters and the other only on item parameters, i.e. parameters affecting conditional response probabilities, given the subject's state. These terms were given above in Equations 10 and 11.

When the item parameters are unconstrained, each term in the sum in Equation 11 can be maximized separately. If the parameters are constrained, but the constraints apply separately to each item, then the set of terms involving each item can be maximized separately. Monotone homogeneity constraints are of this type: they specify the ordering of the conditional correct response probabilities to a particular item, given the respective states, but say nothing about relationships between response probabilities involving different items.

Thus, the maximization of Equation 11 can be written as

$$\max L_p = \sum_{j=1}^{J} \max \sum_{k=1}^{s} [E_o(r_{kj})\log p_{kj} + E_o(n_k - r_{kj})\log(1-p_{kj})]. \quad (17)$$

Each of the maximizations on the right hand side of Equation 17 corresponds to the maximium likelihood equation for estimating the success probabilities in $\underline{s}$ independent groups for a particular item. Carrying out the maximization under ordering constraints has a known solution which bears an interest relation to the algorithm given above for dealing with equality constraints.

Consider the problem of maximum likelihood estimation of proportions in $\underline{s}$ independent groups. Its solution is the familiar

$$\hat{p}_k = \frac{r_k}{n_k}, \quad \text{for } k=1,\ldots \underline{s}.$$

In our problem, r_k and n_k are replaced by $E_0(r_{kj})$ and $E_0(n_k)$, their conditional expectations for item j, given the observed responses and trial values of the parameters.

Now let us add the constraint that

$$p_1 \leq p_2 \leq \cdots \leq p_s \ .$$

Barlow, et al. (1972) have shown how to treat this problem in terms of <u>isotonic regression</u>. The solution is built upon the unconstrained maximum likelihood estimators just mentioned, which are referred to by Barlow, et al. as <u>basic estimates</u>. These basic estimates are amalgamated for <u>solution blocks</u> of adjacent groups within which each group's estimate is set equal to the weighted average of the $\hat{p}_k$'s for the groups comprising the solution block.

The solution blocks are formed as follows. At first each group forms its own block. If the basic estimates for all the groups fall in the right order, then the ordering constraint is not active and the constrained estimates coincide with the basic estimates. A group will continue to form its own solution block unless one or both of the following conditions hold:

a) its inclusion with the group or adjacent set of groups immediately above it in the hypothesized order would increase the average for the resulting block; or

b) its inclusion with the group or adjacent set of groups immediately below it in the hypothesized order would decrease the average for the resulting block.

The existence of either condition implies a violation of the ordering constraint which can be remedied by combining the groups involved and setting the estimate of probability correct in each of these groups equal to the weighted average of their basic estimates.

Let the weighted average of the basic estimates in the adjacent set of groups with indices running from $\underline{t}$ through $\underline{u}$ be denoted by

$$Av(\underline{t}, \underline{u}) = \frac{\sum_{k=t}^{u} E_0(n_k)\hat{p}_k}{\sum_{k=t}^{u} E_0(n_k)} . \tag{18}$$

The constrained maximum likelihood estimates can be expressed in terms of "max-min" formulas in four different but equivalent ways:

$$\begin{aligned} p^*_k &= \max_{t \le k} \min_{u \ge k} Av(\underline{t}, \underline{u}) \\ &= \min_{u \ge k} \max_{t \le k} Av(\underline{t}, \underline{u}) \\ &= \max_{t \le k} \min_{u \ge t} Av(\underline{t}, \underline{u}) \\ &= \min_{u \ge k} \max_{t \le u} Av(\underline{t}, \underline{u}). \end{aligned} \tag{19}$$

The result given by Equations 18 and 19 is what one would obtain using Equation 16 to impose the constraint that conditional probabilities of correct response in states belonging to the same solution block must be equal. The main difference between the algorithm to impose simple equality constraints and the algorithm necessary to provide monotone homogeneity is that the solution blocks and the equality constraints implicit in them are not given beforehand and can change from one iteration to the next. The latter algorithm must take this into account.

The Up-and-Down Blocks Algorithm. There are many ways one can determine the solution blocks needed to satisfy Equation 19. Barlow et al. (1972) recommend a procedure due to Kruskal (1964), called the "Up-and-Down Blocks" algorithm. Key terms in the tests used in the algorithm are defined as follows. Let B_-, B, B_+ be three consectuive blocks in order. Block B is said to be up-satisfied if Av $B <$ Av B_+. It is said to be down-satisfied if Av $B_- <$ Av B. At each stage of the algorithm one block is active; this may be amalgamated with an adjacent block or, if it is up-satisfied and down-satisfied, the next block become active. By convention, the first block in order is down-satisfied and the last block is up-satisfied. The exact sequence of events is as follows.

1. At the start, each state is a separate solution block. State 1 is initially specified to be the active block.

2. Test to see if the active block is up-satisfied. If it is, go to the next step. If it is not, pool the active block with the next higher block; the new block becomes active. Go to Step 3.

3. Test to see if the active block is down-satisfied. If it is, go to Step 4. If it is not, pool the active block with the next lower block; the new block becomes active. Go back to Step 2.

4. If the active block does not contain the highest state, make the next higher block active and go back to Step 2. If the active block contains the highest state, the algorithm is finished.

The sequence of tests and actions to determine the solution blocks is given for a hypothetical example in Table 1. In the example, there are five groups with equal sample sizes, so that unweighted averages are used. For the groups in their hypothesized order, the basic estimates are .50, .60, .70, .40, and .90, respectively. When the algorithm encounters the violation of monotone homogeneity in comparing the third and fourth groups, adjustments are made resulting in the final estimates .50, .57, .57, .57, .90.

Insert Table 1 about here

In summary, monotone homogeneity of items is provided by modifying the EM algorithm for unconstrained marginal maximum likelihood estimation as follows. At each iteration, compute the unconstrained estimates and then apply the Up-and-Down Blocks algorithm to the results for each item. Use these monotonely homogeneous values as trial values on the next iteration. Iterate until the stopping criterion you are using is satisfied.

A Test for Monotone Homogeneity

When there are J items on a test and one is fitting an unconstrained latent class model with s states, there are Js free item parameters to be estimated. Let m_j denote the number of level sets determined by the Up-and-Down Blocks algorithm for item j. The number of free item parameters in the model with the monotone homogeneity constraint is then $\sum m_j$. Let L_u and L_m denote the maxima of the marginal likelihood function evaluated under the unconstrained and monotonely constrained hypotheses, respectively. If the monotone homogeneity hypothesis is correct, then asymptotically the likelihood ratio test statistic

$$-2 \log \lambda = 2(\log L_u - \log L_m) \tag{20}$$

has a chi-squared distribution with $Js - \sum m_j$ degrees of freedom. This fact can be used to set up critical regions for tests of the hypothesis.

Example. Figure 1 gives graphs of item response functions for some signed-number addition test data obtained by Tatsuoka and Birenbaum (1979). Five pairs of response functions are depicted - one pair for each of five types of items on the test. Each pair consists of an unconstrained item response function and a function constrained to be monotonically homogeneous. The analysis refers to a special scoring of responses which only attends to whether the magnitude of the response is correct, disregarding the sign of the answer. The curves given are actually averages of four separate curves, because there were four items of each type. Within types, the curves are practically identical. The types vary in terms of whether the larger of the addends appears first or second in the sum, and in terms of the signs of the addends. An item such as "10+-5" would be of the type designated L+-S on Figure 1, for example.

Tatsuoka and Birenbaum found that if one examines the magnitude of the responses and the sign of the responses to these items separately, some very interesting patterns emerge. Some groups of subjects fall into systematic patterns of errors and correct response which correspond to use of erroneous rules. Paulson (1985) found that a five-state latent class model would give a good account of the magnitude responses. That is why five-state models were used to obtain the curves in Figure 1. Examination of the figures reveals that the unconstrained and monotonically homogeneous curves are very similar for four of the five item types. However, for the type -L+-S, the unconstrained curve is practically "U"-shaped. On the basis of these curves, we would expect to reject the hypothesis of monotone homogeneity. Since there are 20 items on the test and five states in the model, the unconstrained model has 100 free item parameters. It turns out that the total number of level sets in the monotonically constrained model is 81. Thus, there are 19 degrees of freedom for the chi-squared test. We do in fact reject the null hypothesis: $\chi^2(19)=82.30$, $p<.0001$.

Further insight can be obtained by examining the data in Figure 1 from another perspective. Figure 2 shows the profiles of responses to the different types of items for subjects in each of the five states. Subjects in State 4, the next to the highest state in terms of number correct, do well on all item types, except Type -L+-S. Subjects in the lowest state in terms of number correct, State 1, do well on Type -L+-S, but poorly on all the rest. Type -L+-S is the only type on the test for which one should add absolute values of the addends; one should subtract on all the rest. Subjects in State 1 appear to follow the rule, "Always add," whereas subjects in State 4 appear to follow the rule, "Always subtract." Clearly, clusters of subjects following erroneous rules of this sort can lead to violations of monotone homogeneity.

SUMMARY

This paper has reviewed the application of the EM algorithm to parameter estimation in the latent class model and shown how it can be used to extend existing algorithms to cover monotone homogeneity constraints on the item parameters. The assumption of monotone homogeneity is interesting from a couple of perspectives. Items on a test have monotonically increasing item response functions if and only if they are monotonely homogeneous, so the assumption leads to a minimally restrictive form of item response theory. If the assumption is violated, a *unidimensional* item response theory is clearly inappropriate for the data in question. The paper has shown that, if we restrict ourselves to finite-state latent-class models, we can use the EM algorithm to obtain marginal maximum likelihood estimates of the item response functions under the minimal monotone homogeneity assumption. These "nonparametric" estimates should be very useful when the assumption holds. On the other hand, if the assumption does not hold, we would certainly want to know about it. With the marginal maximum likelihood estimates in hand for both the monotonely homogeneous latent class model and the unconstrained model with the same number of states, we can calculate a direct likelihood ratio test of the monotone homogeneity hypothesis.

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Group Estimates	Up Satisfied	Down Satisfied	Last Block?	Action
.50 .60 .70 .40 .90	Yes	Yes	No	Make next block active
.50 .60 .70 .40 .90	Yes	Yes	No	Make next block active
.50 .60 .70 .40 .90	No			Pool up
.50 .60 .55 .55 .90		No		Pool down
.50 .57 .57. 57 .90	Yes	Yes	No	Make next block active
.50 .57 .57 .57 .90	Yes	Yes	Yes	Stop.

Table 1. Illustration of the "Up-and-Down Blocks" Algorithm.

Note: Each line indicates the outcomes of test made on an active block. The current estimate of the correct response probabilities for groups comprising the active block are underlined at the left. The action taken is given at the right.

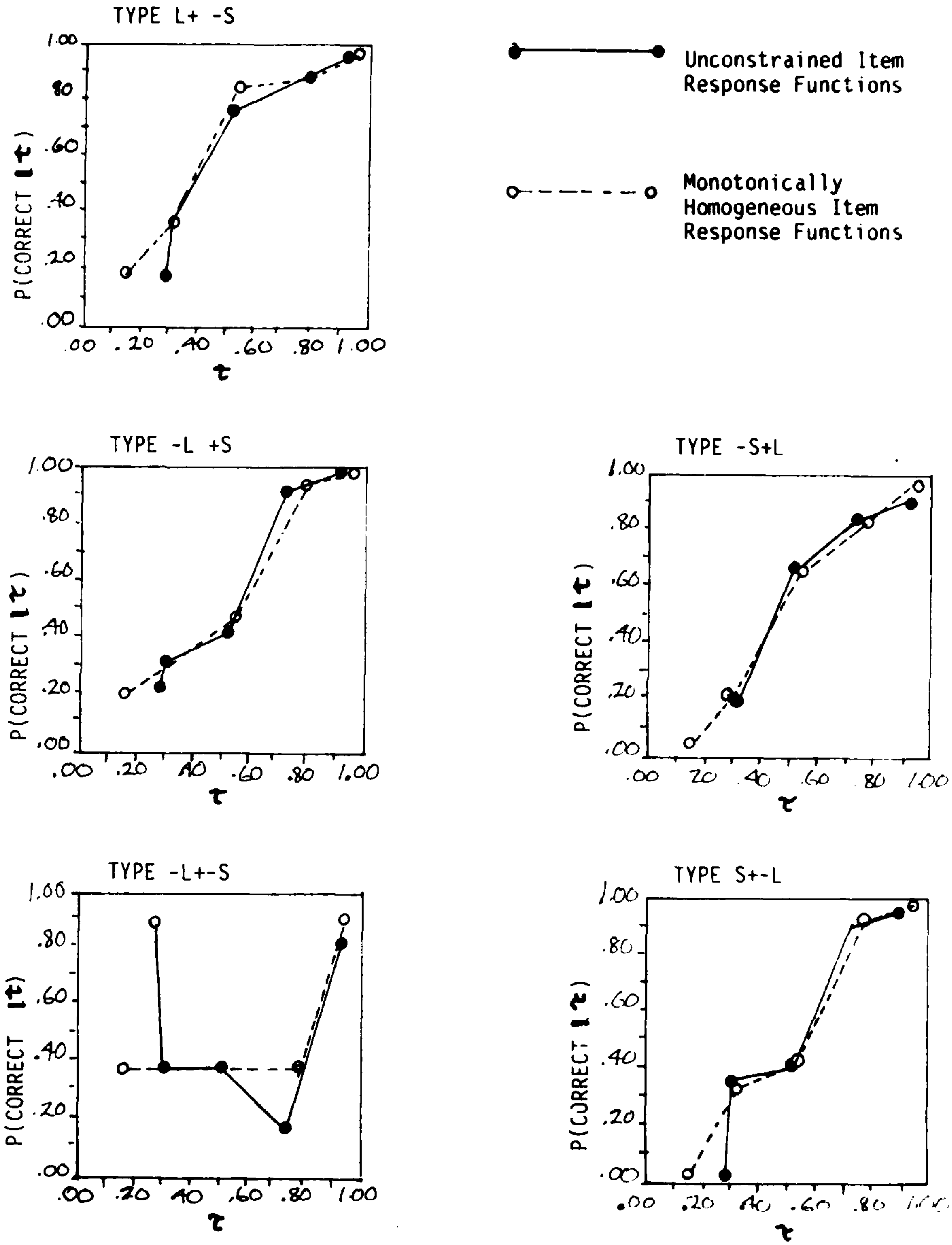

FIGURE 1. Comparison of monotically homogeneous and unconstrained item response funcitons.

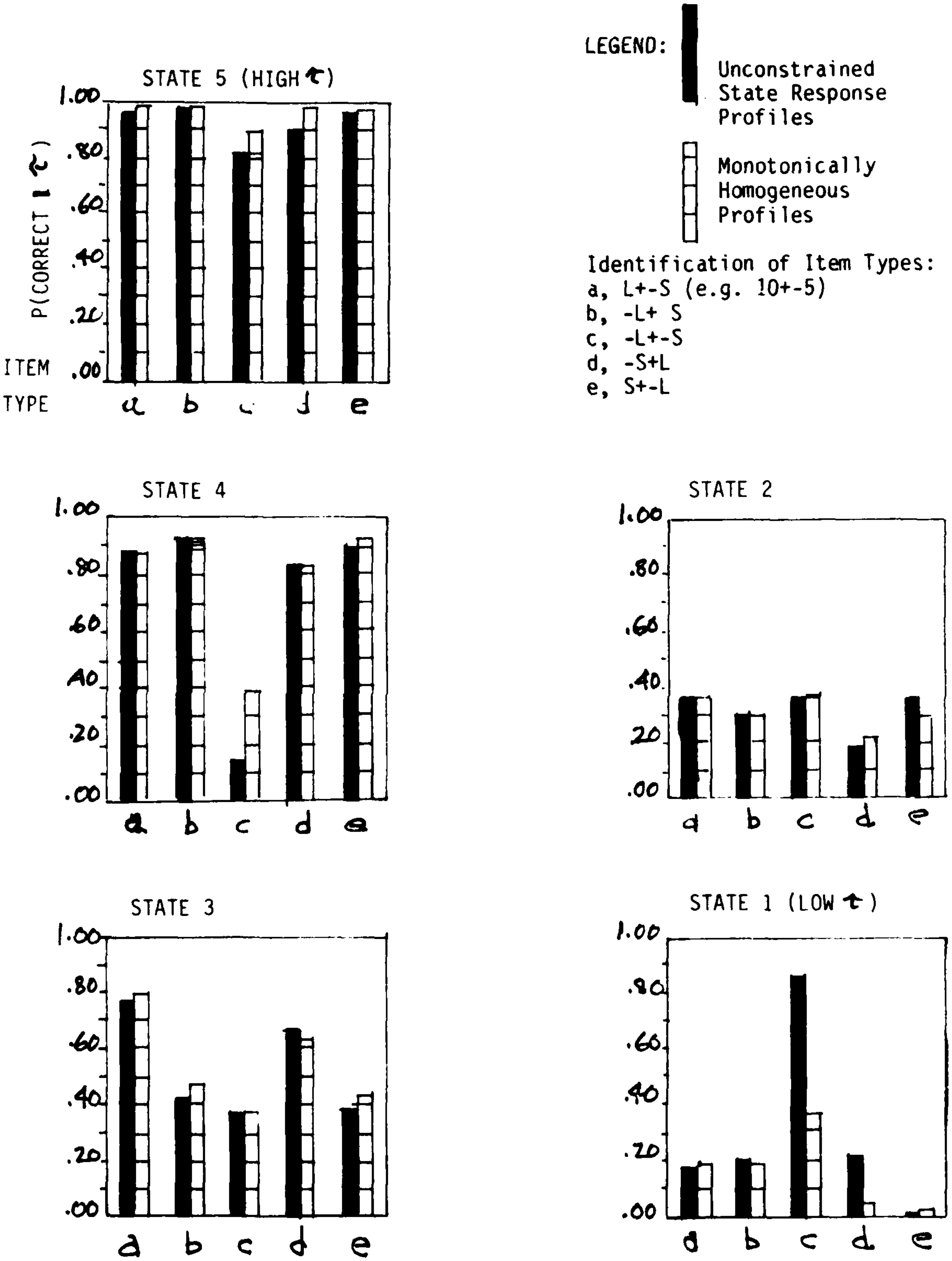

FIGURE 2. Comparison of monotonically homogeneous and unconstrained state response profiles.

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