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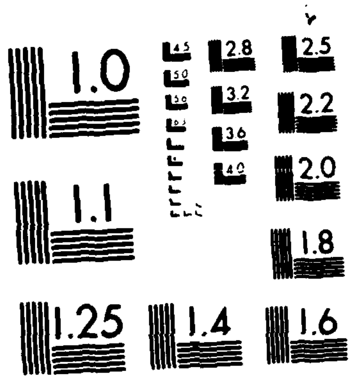

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Fluid Equations, Linear Wave Problems, Wave Propagation, Nonlinear Wave Problems, Pellet Fusion Process, von Karman Equations

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This research has three main parts. The first part consists of a systematic search, compilation and generation of exact explicit solutions for classes of practical nonlinear wave problems. The second part of the research concerns the construction of approximate solutions, with quantitative error estimates, which are generalizations of the explicit exact solutions. The third part of the project concerns input sets for both data and coefficients in wave problems.

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LINEAR OR NONLINEAR WAVE PROBLEMS WITH INPUT SETS

FINAL REPORT

PROF. WILLIAM F. AMES
SCHOOL OF MATHEMATICS

MARCH 6, 1987

U.S. ARMY RESEARCH OFFICE

CONTRACT NO. DAAG 29-84-K-0083

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A. Research Program

This research has three main parts. The first part consists of a systematic search, compilation and generation of exact explicit solutions for classes of practical nonlinear wave problems. These solutions, to be used in the second and third parts, will be developed, or have already been generated, using a wide variety of ad-hoc methods.

The second part of the research concerns the construction of approximate solutions, with quantitative error estimates, which are generalizations of the explicit exact solutions. The approximate solutions are obtained by a variety of analytical methods.

The third part of the project concerns input sets for both data and coefficients in wave problems. The purpose here is to develop a quantitative sensitivity analysis admitting finite deviations of inputs. The method consists chiefly in the application of truncated Taylor expansions with respect to input parameters followed by a quantitative estimate of the remainder using a Neumann series. Numerical implementation using deferred interval analysis will be carried out.

Applications have been made to several physical nonlinear wave problems. These are discussed in the sequel insofar as they are completed. The unfortunate interruption of funding for the third year means that some of these results are incomplete.

B. Summary of Results C. Publications (Given Together)

I. Analytical Studies

1. Nonlinear Waves in the Pellet Fusion Process

(V. J. Ervin, W. F. Ames and E. Adams)

A gas dynamic model of the pellet fusion process having a time-invariant source term is studied by means of group analysis. Some exact solutions of this nonlinear system are constructed for specific (physical) values of parameters. The development of multiple shock waves is demonstrated in several cases analytically. Additional numerical results illustrate the evolution of singularities.

Published as follows:

i) Transactions of the Third Army Conference on Applied Mathematics and Computing, ARO Rept. 86-1, pp. 605-618, 1986.

ii) Wave Phenomena: Modern Theory and Applications (Ed. T. B. Moodie and C. Rogers) North Holland, pp. 199-210, 1984.

iii) Ph.D. Thesis for V. J. Ervin - Sept. 1984, Georgia Institute of Technology.

2. Analysis of the Von Karman Equations by Group Methods (K. A. Ames and W. F. Ames)

One of the systems of equations approximating the large deflection of plates consists of two coupled nonlinear fourth order partial differential equations, known as the von Karman equations. The full symmetry group for the steady equations is a finitely generated Lie group with ten parameters. For the time-dependent system the full symmetry group is an infinite parameter Lie group. Several subgroups of the full group are used to generate exact solutions of the time-independent and the time-dependent systems. These include the dilatation group (similar solutions), rotation group, screw group and others. Physical implications and applications are discussed.

Published as follows:

i) Transactions of the First Army Conference on Applied Mathematics and Computing, ARO Rept. 84-1, pp. 289-300, 1985.

ii) International Journal of Nonlinear Mechanics, 20, 201-209, 1985.

3. Analysis of Fluid Equations by Group Methods (W. F. Ames and M. C. Nucci)

Using the machinery of Lie-group analysis several equations arising in fluid mechanics are studied. In particular, the Burgers' equation, the KdV equation, and the Lin-Tsien equation are analyzed. In all cases the particular group includes arbitrary functions of time which permit the transformation of time-dependent equations into the corresponding time-independent ones. Infinitely many time-dependent solutions are associated with each steady solution. Some solutions are constructed.

Published as follows:

i) Transactions of the Third Army Conference on Applied Mathematics and Computing, ARO Rept. 86-1, pp. 589-596, 1986.

ii) Journal of Engineering Mathematics, 20, pp. 181-187, 1985.

iii) An expanded version is to be published in the Proceedings of the Yale International Symposium on Computational Acoustics (1987).

4. Nonlinear Wave Propagation in Viscoplastic, Viscoelastic and Electrical Transmission Lines

(W. F. Ames, A. Holzmuller, E. Adams)

The exploration of the three equations for wave propagation in one-dimensional viscoplastic, viscoelastic and electrical transmission lines, described in the paper by W. F. Ames and I. Suliciu ("Some Exact Solutions for Wave Propagation in Viscoelastic, Viscoplastic and Electrical Transmission Lines," Int. J. Nonlinear Mechanics, 17, pp. 223-230, 1982), demonstrated that certain classes of realistic nonlinear media possessed exact invariant solutions. For others a solution algorithm was developed.

In an incomplete work (enclosed) this system is studied in general to discover what restrictions need be made on the physical constitutive relations in order for invariant solutions to exist. This work continues. It is related to Problem II - 2 of the Approximate Studies Section.

II. Construction of Approximate Solutions

1. Bounds for Spatially Nonhomogeneous Model Boltzmann Energy Equations

(E. Adams, J. Herod, H. Spreuer)

Nonlinear model Boltzmann equations for particle distribution functions f(t,r,v) are notoriously complicated.

Such equations describe the evolution in time t of distributions f of particles located at position r and having velocity v. The equations are often described by two types of evolution mechanism: a dispersion term and a collision term. It is necessary that reasonable assumptions be made about these two terms if the model is to remain physically realistic and if the model is to yield the mathematical analysis. Because spatially inhomogeneous distributions of the Boltzmann equations are of considerable interest, we suggest a more general Boltzmann energy equation which retains a dependence on the spatial variable r, as well as the energy variable x. The two particle collision term remains independent of space and time; it depends only on the energy of the colliding particles. We shall use a general collision kernel which includes the Tjon-Wu model as an example.

It is shown that disregarding the direction of the velocity vectors and using just its magnitude may lead to an uncountable family of model Boltzmann energy equations. Solutions of any of these equations will be dominated by bounds that are obtained through appropriate differential inequalities.

<u>Published as follows</u>:

i) Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 123-126, 1985.

ii) Submitted to the Journal of Nonlinear Analysis and Applications.

2. <u>Nonlinear Constitutive Equations and Uniform Boundedness of Perturbed Solutions of Evolution-Type</u>

(E. Adams, W. F. Ames and R. Lang)

This work considers the influences of nonlinear terms in the constitutive equations in the case of evolution type mathematical models, making use of a quantitative sufficient criterion ensuring stability. The application of this criterion rests on the construction of a very sharp componentwise enclosure of the set of perturbed solutions of the given or approximating system of differential equations, where these perturbations are due to initial data or parameters in the equations. If this criterion is satisfied, the following well-known difficulties of standard mathematical or numerical methods have been resolved in the problem at hand:

a) The influence of nonlinearities has been accounted for almost precisely and geared to the problem under investigation, i.e., not only by use of generally valid and, consequently, crude estimates. Consequently, a stabilizing influence of a nonlinearity can be ascertained.

b) Stability is confirmed quantitatively and with respect to known <u>finite</u> initial or parameter perturbations.

c) Finite sets of initial or parameter perturbations are admitted and their nonlinear influences are fully taken into account, without making use of any linearization.

d) Whereas the standard practical methods for nonlinear problems yield approximations without a quantitative <u>and</u> sharp error estimate, the subsequent treatment of ordinary differential equations accounts for every possible error, i.e., input deviations discretization errors, rounding errors, procedural errors in the determination of the values of standard functions, etc.

e) A sufficient criterion asserts stability for all positive t, even though its practical execution is confined to a finite time interval.

f) The execution of the test, leading to the fulfillment of the sufficient criterion, provides a rigorous sensitivity and/or safety analysis with respect to the non-enumerably many combinations of admissible initial and parameter perturbations.

Published as follows:

i) Approximate Practical Stability for Nonlinear Evolution Partial Differential Equations, Transactions of the 11th IMACS World Congress (Oslo, 1985). To appear in Journal Form, 1987.

ii) Untersuchung der praktischen Stabilitat von Losunger Nichtlinearer hyperbolischer Anfangsrandwertaufgaben, A. Angew. Math. u. Mech, 65, 1985, pp. T 76-T 78.

3. Nonlinear Membrane Models for Waves in Tires

(E. Adams, W. F. Ames, D. Weiler and R. Lohner) (see item III-1)

The numerical enclosure method of III-1 is being applied to study wave propagation in nonlinear membrane models of tires. The absence of standing waves indicated that the adopted models have to include a better simulation for the contact area with the road. Work initiated.

4. Nonlinear Ordinary Initial Value Problems

(E. Adams, R. Lang)

Applications of "mapping the initial intervals into themselves," a theory developed before this project is being applied to nonlinear ordinary initial value problems generated by a collocation method applied to nonlinear wave equations. Completed applications are concerned with the nonlinear transverse vibrations of beams under compression and with the one-dimensional wave propagation in media governed by classes of nonlinear constitutive equations. Here, free parameters in these equations are identified such that all perturbations of the trivial solutions are uniformly bounded provided the initial data are confined to prescribed intervals. Preliminary results have appeared and work continues.

Published as follows:

i) Mathematical Analysis of the Dependency of Dynamic Perturbations on Constitutive Equations, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 63-66, 1985.

III. Numerical Enclosure of Solutions

(E. Adams, W. F. Ames, R. Lohner, D. Cordes, A. Holzmuller)

Very modest

support by Army Grant

1. R. Lohner has completed his Ph.D. work on fully computer-implemented methods for the componentwise enclosure of solutions of nonlinear ordinary initial or boundary value problems with systems of explicity ODEs. Preliminary presentations of this work have been given.

Published as follows:

i) Enclosure of solutions of ordinary initial value problems and applications, to appear in "Discretization in Differential Equations and Enclosures," Akademie-Verlag, Berlin, 1987.

ii) (R. Lohner Only) Enclosing the solutions of ordinary initial and boundary value problems, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 99-102, 1985. An expanded version will appear in "Computer Arithmetic, Scientific Computation and Programming Languages," G. Teubner, 1987.

Currently, applications carried out by A. Holzmuller are concerned with problems in chemical kinetics, in particular, the enclosure and verification of the famous periodic solution of the Oregonator. Another application is concerned with a one-dimensional steady state analysis of thermochemical processes in rocket engines.

2. D. Cordes has completed his Ph.D. work on fully computer-implemented methods for the entirely safe verification of stability and the enclosure of quasistationary solutions of systems of linear ODEs with periodic coefficients. Applications are mainly concerned with gear drive vibrations. For preliminary presentations of this work see below. *In the case of linear ODEs,* Cordes employs a practical implementation of the Floquet theory or,

alternatively, a condition of mapping of initial intervals into themselves. In the case of nonlinear systems, only this mapping criterion is applicable and has been used for this purpose by Cordes.

Published as follows:

i) Cordes, D., Adams, E., Test for Uniform Boundedness for Problems Admitting Parameter Resonance, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Vol. 1, pp. 103-105, 1985.

ii) Cordes, E., Adams, E., Test for Uniform Boundedness for Dynamic Problems Admitting Parameter and Combination Resonance, will appear in "Computer Arithmetic, Scientific Computation, and Programming Languages," G. Teubner, Stuttgart, 1987.

D. Participating Scientific Personnel

I) Funded

a) Professor E. Adams, Professor of Applied Mathematics, University of Karlsruhe, 7500 Karlsruhe, Feberal Republic of Germany (Funded One Month).

b) Professor W. F. Ames, Regents' Professor of Mathematics and Director, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Five Months).

c) Dr. V. J. Ervin, Visiting Assistant Professor of Mathematics, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded One Month).

d) Mr. J. F. Froehlich, Research Assistant, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Six Months).

e) Mr. Armin Holzmuller, Research Assistant, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Nine Months).

II) <u>Unfunded But Associated</u>

a) Professor K. A. Ames, Iowa State University, Ames, Iowa

b) Dr. M. C. Nucci, University of Perugia, Italy

c) Professor J. Herod, Georgia Tech, Atlanta, GA

d) Dr. H. Spreuer, University of Karlsruhe, Germany

e) Mr. R. Lang, University of Karlsruhe, Germany, Assistant to Dr. E. Adams

f) Mr. R. Lohner, University of Karlsruhe, Germany, Assistant to Dr. E. Adams

g) Mr. D. Weiler, University of Karlsruhe, Germany, Assistant to Dr. E. Adams

h) Mr. D. Cordes, University of Karlsruhe, Germany, Assistant to Dr. E. Adams

E. <u>Remarks</u>:

The computations will continue apace in Germany under the auspices of my long time collaborator Prof. Dr. E. Adams of Karlsruhe. The preliminary results are very promising, but our association which has been modestly supported by the Army must cease. This international cooperation has been of enormous value far beyond the modest funds invested. A pity!

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