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RADC-TR-86-125
In-House Report
July 1986

CALCULATION OF RAY PATHS IN THE IONOSPHERE USING AN ANALYTIC RAYTRACING TECHNIQUE

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Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

AD-A178 888

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: Unclassified	1b RESTRICTIVE MARKINGS: N/A
2a SECURITY CLASSIFICATION AUTHORITY: N/A	3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release, distribution unlimited.
2b DECLASSIFICATION / DOWNGRADING SCHEDULE: N/A	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): RADC-TR-86-125	5 MONITORING ORGANIZATION REPORT NUMBER(S): N/A
6a NAME OF PERFORMING ORGANIZATION: Rome Air Development Center,	6b OFFICE SYMBOL (If applicable): EEPI
	7a NAME OF MONITORING ORGANIZATION: Rome Air Development Center (EEPI)
6c ADDRESS (City, State, and ZIP Code): Hanscom AFB, Massachusetts 01731	7b ADDRESS (City, State, and ZIP Code): Hanscom AFB, Massachusetts 01731
8a NAME OF FUNDING / SPONSORING ORGANIZATION: Rome Air Development Center	8b OFFICE SYMBOL (If applicable): EEPI
	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N/A
8c ADDRESS (City, State, and ZIP Code): Hanscom AFB, Massachusetts 01731	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO 61102F; PROJECT NO 2305; TASK NO J2; WORK UNIT ACCESSION NO 01

11 TITLE (Include Security Classification)

Calculation of Ray Paths in the Ionosphere Using an Analytic Raytracing Technique

12 PERSONAL AUTHOR(S): Stanford P. Yukon

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
In-House	FROM 2/85 TO 2/86	1986 July	20

16 SUPPLEMENTARY NOTATION: N/A

17 COSATI CODES		
FIELD	GROUP	SUB-GROUP
20	03	14

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

Raytracing, Ionosphere, Potential well, Ducted propagation, High frequency, IONCAP, Paraxial approximation, Parabolic approximation, Scalar wave equation

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A method for tracing rays in the ionosphere using analytic solutions to approximate path varying ionospheric potentials is outlined in this report. Using the ionospheric parameters generated by IONCAP and approximating the E-F_2 and ground-E layer potential wells by a scaled Morse potential and a linear potential respectively, known solutions to these potentials are promoted to final ray paths by using methods developed for solving the time dependent Schroedinger equation. The computer code necessary to fit the potentials, connect the solutions at the E layer peak, and trace arbitrarily launched rays is described.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL: Stanford P. Yukon	22b TELEPHONE (Include Area Code): (617) 377-2985 / 22c OFFICE SYMBOL: RADC/EECP

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

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1. INTRODUCTION

Raytracing in a physically realistic model ionosphere has generally been approached by numerically integrating the differential equations of motion (such as the Haselgrove Hamiltonian equations) that describe a propagating ray. The method developed here starts from the paraxial approximation to the scalar wave equation. Making the paraxial approximation allows the scalar wave equation to be written in the form of a time dependent Schroedinger equation where time is identified as distance along a great circle path. Neglecting diffraction effects and making the ray approximation further simplifies the description to Newton's equations of motion. The problem of ray tracing in a range dependent ionospheric potential is thus equivalent to solving Newton's equations of motion in a time varying potential and can be treated using methods developed for a certain class (those soluble by canonical transformation) of time varying potentials.[1] Upon approximating the variations in the ionospheric potentials such that they belong to this class, analytic solutions for ray paths may thus be obtained.

(Received for publication 18 July 1986)

1. Ray, J. R. (1982) Exact solutions to the time-dependent Schroedinger equation, Phys. Rev. A, 26:729.

The computer program described in this report uses known solutions to the equations of motion for the Morse potential, which is used to model the E-F_2 ionospheric potential well and for the linear potentials, which are used to model the ground-E layer potential well. By fitting the complete range dependent ionosphere along the great circle path using shift and scaling functions, solutions for the complete ray path are obtained.

2. OUTLINE OF THE PROGRAM

The analytic ray tracing program (RAY1) consists of two main parts: Part I computes the parameters for fitting a Morse (or harmonic oscillator) potential between the E and F_2 layers and a linear potential between the ground and the E layer using previously generated ionospheric data stored on a file (TAPE8). TAPE8 contains the E and F_2 layer values for electron density peaks, half widths, and heights at equally spaced range intervals (usually 1°) along a great circle path starting at the transmitter [which may be on the ground or elevated]. The results of fitting the Morse and linear potentials in Part I (DO loop 10) are stored in the vectors RHO(I), SIG(I), VMIN(I), and SIG(I)* where I counts the number of range intervals between I=1 at the transmitter to I=MAXR at the receiver.

In Part II, using the series of potential wells generated in Part I, trajectories are plotted for rays launched at altitude XSTART for a series of takeoff angles (DO loop 200). The ray trajectories for each initial angle THET are stored in X(J) and the time delay is then calculated as the quantity DLAY.

An outer loop encompassing loop 10 and loop 200 (DO loop 12) steps the frequencies from 6 to 28 MHz. After all of the frequencies have been stepped through, an ionogram may then be plotted by calling DLAYPL.

To produce an ionogram for a different path or time of day or season, a new TAPE8.DAT** file must be produced by running ION.COM. ION.COM generates the ionospheric parameters along the path selected using the model ionosphere generated by IONCAP with the path parameters fed in as data from the file PW.DAT.

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*In this report, words written in upper case type refer to items appearing in the computer code and in general are the counterparts of items in normal mathematical notation (which here are usually lower case or mixed upper case with subscripts).

**File name notation conforms to the operating system for the VAX, VMS 3.5.

3. CALCULATION OF FITTING PARAMETERS

The ionospheric potential is modeled (for a particular range slice $z = z_i$) by a Morse potential

$$V_M(x, z_i) = \gamma^2 \left[e^{\alpha[(x - \sigma(z_i))/\rho(z_i)]} - 1 \right]^2 \Big/ \rho(z_i)^2 + g(z_i) \qquad (1)$$

between the E and F_2 layers ($X_E \rightarrow X_{F_2}$) and by a linear potential between the ground and X_E as shown in Figure 1.

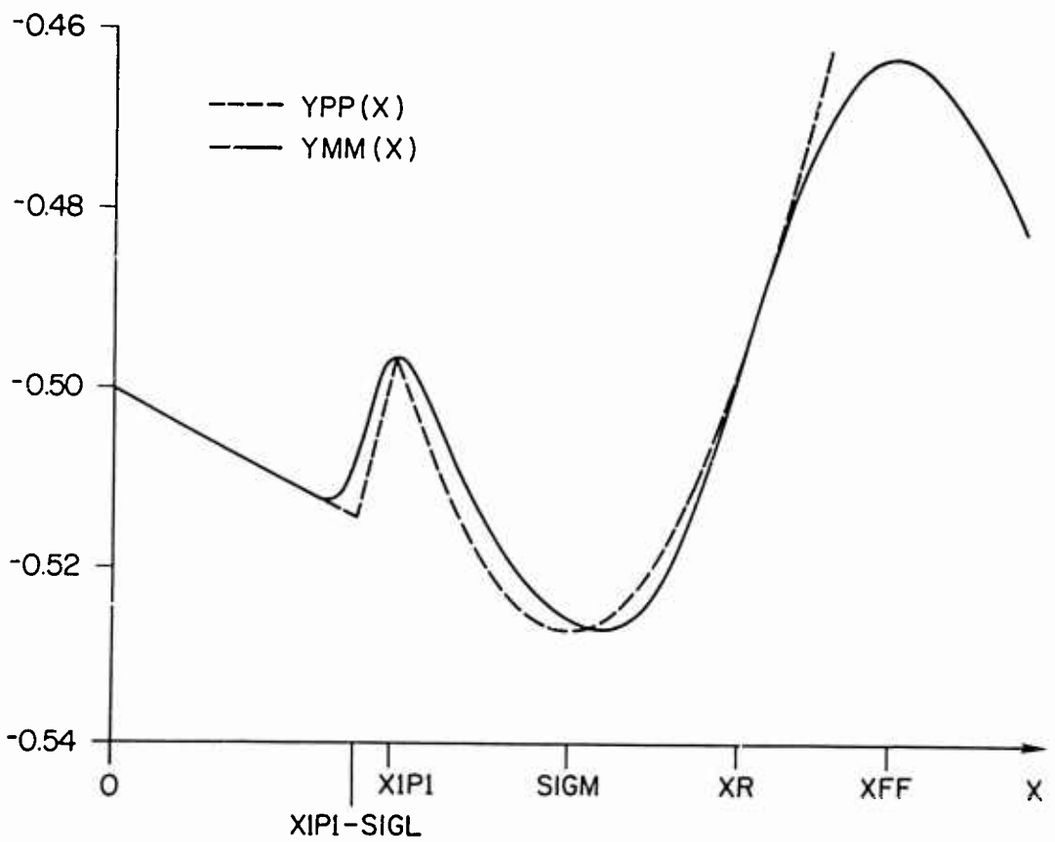

Figure 1. Plot of the Effective Ionospheric Potential Well ymm(x)———and the Fitted Morse Plus Linear Potential ypp(x) – – – – Versus Height x

From the expression (1) for the Morse potential it can be seen that $V_M(x, z_i)$ will have a minimum at $x = \sigma(z_i)$ equal to $g(z_i)$ and an asymptote $= g(z_i) + \gamma^2/\rho^2$ at $x = -\infty$. Normally all of the parameters α, γ^2, $\sigma(z_i)$, $g(z_i)$ and $\rho(z_i)$ are fit at range = 0 (I = 1). For range steps with i > 1 only the minimum $g(z_i)$ and its position $\sigma(z_i)$ and the scaling factor $\rho(z_i)$ are fit. If desired, a different range fitting point may be designated when initializing the program.

The analytic form for the ionospheric potential well is generated by using the Chapman function $f_c(x) = e^{1/2(1-x-e^{-x})}$ to represent the electron density in both the E and F_2 layers and by using a term $-1/2(1+2x/R_o)$ to approximately account for the effects of curvature due to a finite earth radius R_o. Its value at any given altitude x [in units of 100 km] can be found by evaluating the function ymm(x) which uses the current values for the ionospheric parameters and the wave frequency ν. The complete expression for ymm(x) in terms of the layer half widths y_e, y_f, heights x_e, x_f, and frequencies ν_e, ν_f at range point z_i is given by

$$\text{ymm}(x) = -\frac{1}{2}\left(1 + 2X/R_o\right) + \frac{\nu_f^2}{2\nu^2}\, e^{1/2[1-(x-x_f)y_f - e^{-(x-x_f)/y_f}]} + \frac{1}{2}\frac{\nu_e^2}{\nu^2}\, e^{1/2[1-(x-x_e)/y_e - e^{-(x-x_e)/y_e}]} \qquad (2)$$

To facilitate fitting the Morse potential parameters, various fiducial points are established between the E and F_2 layer peaks using the one dimensional minimizing subroutine ZXLSF.[2]

1. The first point is XEEP, the position where the ionospheric potential function YMM(X) is a maximum near the E layer peak. Since the position of the E layer peak is always given by IONCAP as $X_E = 1.1$, XEEP will always be a few percent smaller than 1.1. In order to simplify calculations of ray trajectories in the linear ground $-X_E$ potential well in Part II, a point X1P1 = 1.05 is designated as the peak of the fitted potential YPP(X) near X_E and its value there is defined to be YMM(XEEP).

2. The second point is SIG(I) = SIGM which is simply $\sigma(z_i)$, the minimum of the ionospheric potential function YMM(X).

3. The third point, XFF, is the maximum of the ionospheric potential function YMM(X) near the F_2 layer peak.

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2. ZXLSF and ZXPOWL are part of the IMSL Mathematical Library, IMSL Inc., 7500 Bellaire Blvd, Houston, Texas.

4. The next point to be determined is XR which is the point above the minimum X = SIGM where YMM(XR) = YMM (X1P1) as shown in Figure 1. The values of the potential at these points are defined as PEFL = YMM(X1P1) and PEFR = YMM(XR) and they should be equal to within the accuracy demanded in calling ZXLSF.

5. Under circumstances where the F_2 layer is weak or for higher frequencies, it is possible that the F_2 layer peak will be below that of the E layer. In this case YR is set equal to XFF and XEE is defined as the point where YMM(XEE) = YMM(XFF).

At this point it is now possible to compute the initial parameters γ^2, α, $\rho(0)$, $\sigma(0)$ and $g(0)$ needed for fitting the Morse potential to YMM(X) between X_E and X_{F2}. For the initial point I = 1 we may take $\rho(0) = 1$ leaving one nonlinear [α] and three linear parameters to be evaluated. As the best least squares fit will not necessarily have $g(0) = \min \{ymm(x)\}$ we will assume that g(0) along with γ^2 is one of the linear parameters to be evaluated. Thus for some initial choice for α and σ, the conditions for a least square fit for the remaining linear parameters γ^2 and g(0) are:

$$\frac{\delta}{\delta \gamma^2} \int_{x_\ell}^{x_r} \left[y_{mm}(x) - \gamma^2 (e^{\alpha(x-\sigma)} - 1)^2 - g \right]^2 dx = 0 \tag{3}$$

$$\Rightarrow \int_{x_\ell}^{x_r} ymm(x)\,(e^{\alpha(x-\sigma)} - 1)^2 = \gamma^2 \int_{x_\ell}^{x_r} (e^{\alpha(x-\sigma)} - 1)^4\, dx + g \int_{x_\ell}^{x_r} (e^{\alpha(x-\sigma)} - 1)^2\, dx$$

$$\frac{\delta}{\delta g} \int_{x_\ell}^{x_r} \left[ymm(x) - \gamma^2 (e^{\alpha(x-\sigma)} - 1)^2 - g \right]^2 dx = 0$$

$$\Rightarrow \int_{x_\ell}^{x_r} ymm(x)\, dx = \gamma^2 \int_{x_\ell}^{x_r} (e^{\alpha(x-\sigma)} - 1)^2 + g \int_{x_\ell}^{x_r} dx\ . \tag{4}$$

Defining

$$B_1 = \int_{x_\ell}^{x_r} ymm(x)\, dx \tag{5}$$

$$B_2 = \int_{x_\ell}^{x_r} ymm(x)\,(e^{\alpha(x-\sigma)} - 1)^2\, dx \tag{6}$$

and

$$a_n = \int_{x_\ell}^{x_r} (e^{\alpha(x-\sigma)} - 1)^{2n} \, dx \tag{7}$$

we then have the solutions

$$\gamma^2 = (a_0 B_2 - a_2 B_1)/\mathrm{Det} \tag{8}$$

and

$$g(0) = (a_4 B_1 - a_2 B_2)/\mathrm{Det} \tag{9}$$

where

$$\mathrm{Det} = (a_0 a_4 - a_2^2) \, .$$

The problem has thus been reduced to a two-dimensional minimization in α - σ space. At this point it might be feasible to use a multi-dimensional minimization subroutine. This was tried using ZXPOWL[2] and FIT.[3] It was found, however, that even with good starting values the subroutines tried would wander, producing non-global minima in the α - σ plane.

To go further we can make use of the relation satisfied by the Morse potential at the point X1P1.

$$\gamma^2 (e^{\alpha(\mathrm{X1P1} - \sigma)} - 1)^2 + g = \mathrm{PEFL0} \tag{10}$$

where the solution is given by

$$\alpha(\mathrm{X1P1} - \sigma) = \ell n \, (1 - [(\mathrm{PEFL0}-g)/\gamma^2]^{1/2} \, . \tag{11}$$

Thus for an initial choice of α, with X1P1 and PEFL0 given, a value of σ may be may be found as a function of γ^2 and g(0).

From the least squares condition we have a solution for γ^2 and g in terms of σ (and α). If we assume that a solution exists, a consistent set of the parameters

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3. FIT is part of the DATAPLOT Language developed by J. J. Filliben, Center for Applied Mathematics, National Bureau of Standards, U. S. Department of Commerce.

σ, g, and γ^2 may be determined by iterating the two sets of equations, Eqs. (8) and (11), using σ in Eq. (11) to determine g and γ^2 and using g and γ^2 in Eq. (8) to determine σ. The four parameter least squares fit has thus been reduced to a one-dimensional minimization as a function of α where for each value of α a consistent set of γ^2, σ, and g is found by iteration.

These procedures are implemented in the code by using ZXLSF to minimize the quantity SMINIT (ALF) which is the sum of squares

$$\sum_{x_i} \left\{ ymm(x_i) - \left[\gamma^2 \left(e^{\alpha(x_i-\sigma)} - 1 \right)^2 + g \right] \right\}^2 . \qquad (12)$$

SMINIT in turn calls GMINIT to get the self consistent set of values for γ^2, σ, and g, which are found by iterating the two relations (8) and (11) IFLAG times. The final values produced are stored as

SIG(1) (=σ), GIM^2 (=γ^2),

VMIN(1) (=g), and ALF (=α).

To obtain $\rho(z_i)$ and $\sigma(z_i)$ and $g(z_i)$ for fitting subsequent ionospheric profiles down range, the least squares sum in Eq. (12) is minimized as a function of $\rho(z_i)$ (= RHOM =RHO(I)). This is accomplished by calling ZXLSF to minimize SUMSQ (RHOM) which determines $\sigma(z_i)$ through the relation (12) using as inputs the new fiduciary points (for the range z_i), the value of YMM(X1P1) (=PEFL) and the already determined ALF and GIM2.

This procedure is performed MAXR-1 times within DO loop 10, thus fitting the E-F_2 potential along the entire path length by a varying Morse potential whose variation is constrained such that exact z dependent solutions to the equations of motion are possible.

For the potential well between the ground and the E layer peak, we have chosen to fit YMM(X) by a simpler linear potential consisting of a straight line segment from X = 0 to X=X1P1-SIGL(I) with a negative slope of magnitude GRAV = $1/R_o$ with initial value of -0.5, and a straight line segment from X = X1P1 - SIGL(I) to X1P1 with a slope GRVR $\cong$ (PEFL-VGMIN)/Y_E having a final value of EH above the minimum at X = X1P1-SIGL(I).

The value of SIGL(I) that will accomplish this is determined from the value of PEFE(I) [=PEFL-VGMIN] as

$$SIGL(I) = PEFE(I)/(GRVR + GRAV) . \qquad (13)$$

From Figure 2 we also have

$$EH = SIGL * GRVR \tag{14}$$

$$DEH = SIGL * GRAV .$$

Repeating this procedure at each range interval I, we will thus map out a varying ground-E layer potential constrained by the requirements $\sigma(x_i)$=X1P1-SIGL(I), $\rho(z_i)$=1, that allow exact z dependent solutions to the equations of motion between x = 0 and X1P1 to exist. The subroutine PLOTWELL may be called at every IPLWL'th range point to obtain a plot of YMM(X) and YPP(X) vs X where YMM(X)

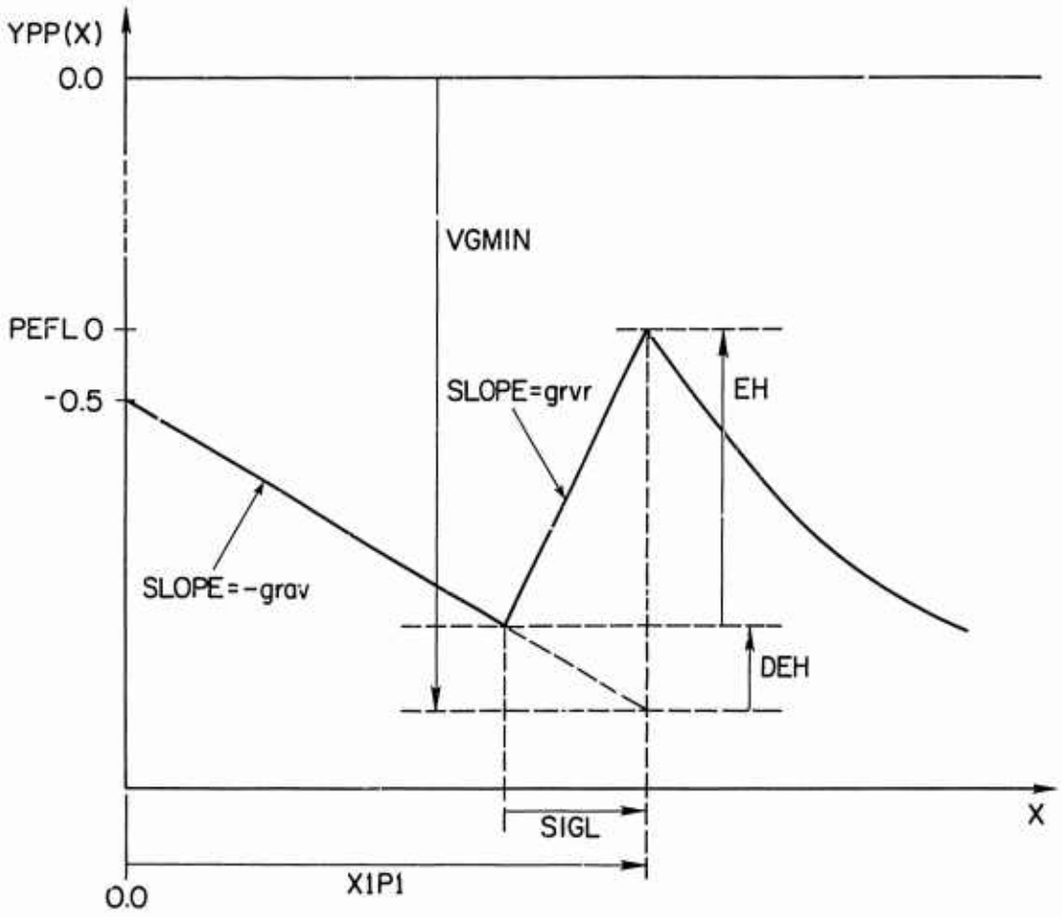

Figure 2. Plot of the Fitted Linear Ground-E Layer Potential Well ypp(x) Versus Height x

is the original ionospheric potential well and YPP(X) is the potential well fitted by a linear potential between $0<X<X1P1$ and a Morse potential between X1P1 and XFF.

4. CALCULATION OF EXACT RAY PATHS

In Part II of the program the values of $\sigma(z)$ and $\rho(z)$ are used to determine the global ray path x(z) from the relation

$$x'(z') = \frac{x(z) - \sigma(z)}{\rho(z)} \tag{15}$$

where $\sigma(z)$ and $\rho(z)$ have been previously determined for the linear and Morse potential wells respectively and x'(z') is the solution of the local equation of motion

$$\frac{d^2x'}{dz'^2} = -\nabla_{x'}\tilde{V}(x') \, . \tag{16}$$

Here $\tilde{V}(x')$ is given by the Morse potential $\tilde{V}_M(x') = \gamma^2(e^{\alpha x'} - 1)^2$ for rays in the E-F_2 potential well and by the linear potential

$$\tilde{V}_G(x') = \begin{cases} - \text{ grav} & x'<0 \\ + \text{ grvr} & x'\geq 0 \end{cases} \tag{17}$$

for rays in the ground $-X_E$ well.

The local range parameter z' is given by

$$z' = \int_0^z \frac{d\tau}{\rho^2(\tau)} \, . \tag{18}$$

Where z is related to the range $\zeta = R_o \int_0^\theta d\theta$ by

$$z = \int_0^\zeta \frac{d\zeta'}{n_o(\zeta')} \tag{19}$$

with $n_o(\zeta')$ the refractive index at the minimum of the corresponding potential well.

The relation between the potential and energy in the global and local frames is given by

$$\tilde{V}(x') = \left[V(x) - V_{MIN}\right] \cdot \rho(z)^2$$

and

$$E' = \left\{ E + \frac{1}{2} (\dot{\sigma} + \dot{\rho} x')^2 - \dot{x}(\dot{\sigma} + \dot{\rho} x') \right\} \cdot \rho^2 . \tag{20}$$

For a particle or ray moving in a Morse potential given by Eq. (13), the general solution may be written for the case $E' < U_o = \gamma^2$ as[4]

$$e^{-\alpha x'} = X_{co} - A(E') \cdot \cos(\omega_c(E') \cdot z') \tag{21}$$

where

$$X_{co} = U_o/(U_o - E'), \; A(E') = (U_o E')^{1/2}/(U_o - E')$$

and

$$\omega_c(E') = [2\alpha^2 (U_o - E')]^{1/2} \tag{22}$$

with the velocity

$$\frac{dx'}{dt'} = \frac{- A(E') \omega_c(E') \sin(\omega_c \cdot z')}{\alpha[X_{co} - A(E') \cos(\omega_c \cdot z')]} . \tag{23}$$

Thus starting the ray path at $\omega_c z' = 0$ implies that the ray is starting from the right-hand turning point at

$$x'_{MAX} = \frac{-\ell n (X_{co} - A(E')}{\alpha} \tag{24}$$

with velocity zero and moving to the left.

For the case $E' > U_o = \gamma^2$ the general solution is given by

$$e^{-\alpha x'} = X_{co} - A(E') - \text{ch}(\omega_b(E') \cdot z') \tag{25}$$

with $\omega_b = [2\alpha^2(E' - U_o)]^{1/2}$. The motion is unbounded for $x \to -\infty$ and there is a right-hand turning point

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4. Nieto, M. M., and Simmons, L. M. (1979) Coherent states for general potentials. III. Nonconfining one-dimensional examples, Phys. Rev. D, 20:1342.

$$X'_{MAX} = \frac{-\ell n\,(X_{co} - A(E'))}{\alpha} = \frac{-\ell n[\sqrt{U_o}/(\sqrt{U_o} + \sqrt{E'})]}{\alpha} \tag{26}$$

at $z' = 0$.

The velocity is given by

$$\frac{dx'}{dz'} = \frac{-\omega_b(E') \cdot \mathrm{sh}(\omega_b(E') \cdot z')}{\alpha\,[\mathrm{ch}(\omega_b(E') \cdot z') - \sqrt{U_o}/E']} \tag{27}$$

which is positive for $z' < 0$, zero at the turning point which is attained at $z' = 0$, and negative for $z' > 0$.

For a particle or ray moving in the linear potential for $X \leq X1P1$, the general solution is given by

$$x' = \frac{1}{2} g z'^2 + v_o z' + x'_o \tag{28}$$

where

$$g = \{\mathrm{grvr},\ (-\mathrm{grav})\} \tag{29}$$

for x' larger (or smaller) than X1P1-SIGL(I).

The program is written to allow a ray to be launched either from the ground by setting IGRND = 0 or in the E-F_2 duct by setting IGRND = 1, and specifying the starting altitude XSTRT (launching from $0 < X < X_E$ could be easily included if needed). The initial conditions for a ray are further specified by the launch angle θ(= THET) which runs from the initial angle in NTHE step of DTHE within the DO loop ending on label 200. Once a ray is launched, the loop ending on label 20 increases the range, by an increment DZ. The 'local' range ZP(J) is then determined using Eqs. (18) and (19).

The scaling function $\rho(z)$ is assumed to depend linearly on z

$$\rho(z_{j+1}) \equiv \rho_{j+1} = \rho_i + r_j dz_j \tag{30}$$

in order to satisfy the differential constraint equation

$$\ddot{\rho} + \Omega^2(z)\,\rho(z) = 0 \tag{31}$$

(since we have set the fitting function $\Omega^2(z) = 0$ everywhere, this can be satisfied by $\ddot{\rho} = 0$).

This yields

$$z_1' = \int_0^{z_1} \frac{d\tau}{[\rho_0 + r_0 \tau]^2} = \frac{1}{r_0} \left[\frac{1}{\rho_0} - \frac{1}{(\rho_0 + r_0 z_1)} \right] , \tag{32}$$

$$z_2' = \int_{\tau = t_1 - t_1 = 0}^{\tau = t_2 - t_1} \frac{d\tau}{[\rho_1 + r_1 \tau]^2} + \int_0^{\tau} \frac{d\tau}{[\rho_0 + r_0 \tau]^2} \tag{33}$$

$$= \frac{1}{r_1} \left[\frac{1}{\rho_1} - \frac{1}{\rho_2} \right] + \frac{1}{r_0} \left[\frac{1}{\rho_0} - \frac{1}{\rho_1} \right] .$$

Since

$$r_1 = (\rho_2 - \rho_1)/\Delta z_1 \tag{34}$$

this may be written as

$$z_2' = \left\{ \frac{\Delta z_2}{\rho_1 \rho_2} + \frac{\Delta z_1}{\rho_0 \rho_1} \right\} \tag{35}$$

and in general

$$z_{i+1}' = z_i' + \frac{\Delta z_{i+1}}{\rho_{i+1} \rho_i} . \tag{36}$$

This computation appears in the program before label 919 if the ray is propagating in the upper well, for which the indicator IFLAG is set = 0. If the ray if propagating in the lower well, IFLAG = 1 and z'_{i+1} is calculated at the line labeled 109.

For a ray launched in the E-F_2 duct, the initial energy with respect to the bottom of the potential well (VMIN0) is given as E = -VMIN0 + VSTRT + COS(θ)^2 where VSTRT is the potential at the point XSTRT. Depending on whether E is greater or less than U_o, z' = ZP(1) is set such that the ray will have positive or negative velocity according to whether θ is greater or iess than zero. The absolute value of the velocity is that which a particle released from the right-hand turning point where the line E = const. intersects YMM(X), would attain on arrival at XSTART. The initial value for ZP is calculated above label 1255 and then used to calculate the global position X(J) and local velocity XPDOT inside the range DO LOOP on label 20. These quantities are needed at each range step to evaluate the time delay for the entire path given by

$$\tau = \int_0^{S_{MAX}} \frac{ds}{n(x,\zeta)} = \int_0^{\zeta_{MAX}} \frac{\left[1 + \left(\frac{dx}{d\bar{\zeta}}\right)^2\right]^{1/2}}{n(x,\zeta)} \left(\frac{d\bar{\zeta}}{d\zeta}\right) d\zeta$$

$$= \int_0^{\zeta_{MAX}} \frac{\left[1 + \left(\frac{dx}{dz}\right)^2 \left(\frac{dz}{d\zeta} \frac{d\zeta}{d\bar{\zeta}}\right)^2\right]^{1/2}}{n(x,\zeta)} \left(\frac{d\bar{\zeta}}{d\zeta}\right) d\zeta \qquad (37)$$

$$= \int_0^{\theta_{MAX}} \left[1 + \left(\frac{dx}{dz}\right)^2 \left(\frac{1}{n_o} \frac{R_o}{R_o + x}\right)^2\right]^{1/2} \left(1 + \frac{x}{R_o}\right) d\theta$$

where

$$d\bar{\zeta} = (R_o + x)\, d\theta\,, \quad \frac{dx}{dz} = [\dot{\sigma} + \dot{\rho}\, x' + \frac{dx'}{dz'} \frac{1}{\rho}]\,,$$

and

$$x(z) = \rho(z) \cdot x'[z'(z)] + \sigma(z)\,. \qquad (38)$$

This is accomplished by using Simpson's N point integration with the integrand calculated at label 101 if X(J) is in the upper well and at label 201 if X(J) is in the lower well. The final sum is converted into milliseconds below label 20.

For ray paths in the upper well, situations may occur for high angle launches and at high frequencies where the fitted potential well prohibits a ray from escaping over the F_2 peak even though it is energetically possible. To remedy this situation a further fiducial point XFFI(I) is calculated at label 1001 to denote the point at which the line V(X) = PEAKF + DVMIN intersects YPP(X) where DVMIN is the difference between the minima of YPP and YMM. If a ray goes beyond this point then it is considered to have escaped, even though it would have been turned around by the fitted potential. To check whether this occurs it would be simplest to check each point in the path to see whether or not it exceeds XFFI(J). This will not work in every case as a ray may be situated at X(J) just before the turning point at range Z(J) and be situated at an essentially similar point X(J+1) after reflection at range Z(J+1). Both X(J) and X(J+1) may be less than XFFI(J) whereas the turning point XTURN could exceed XFFI(J). To account for this situation the quantity WSW = XPDOT(J)*XPDOT(J+1) is calculated. If WSW is negative, it indicates that the ray has been reflected between steps J and J+1. If this is true, XTURN is calculated and checked against XFFI(J) to see if the ray escaped. These operations occur in the program just before label 3030.

A ray with negative velocity that propagates to the left of X1P1 without being reflected is assumed to be propagating in the lower well as though it were launched at X1P1 with velocity

$$v_o = \left[\frac{dx'}{dt'} \frac{1}{\rho} + \dot{\sigma} + \dot{\rho}\, x'\right]\Big|_{z = z_b} \qquad (39)$$

where z_b is the range point at which the ray crossed X = X1P1. Due to the nature of the solutions of the equations of motion the energy E' measured with respect to the potential minimum $g_M(z_j)$ (=VMIN(J)) will be an invariant as long as the particle remains in the upper well and similarly E'_G will remain an invariant as long as the particle remains in the lower well (including ground reflections). E' may change if a ray returns to the upper well at range $z > z_b$ after having propagated in the lower well.

A similar argument can be made for E'_G being an invariant and changing only for a ray returning from the upper well. These changes are calculated at the entry point into the lower well (label 3031) and at the entry point into the upper well (label 301). For rays crossing X1P1 in either direction, the excess in z', that is, that which takes the ray beyond X1P1, is calculated and converted to the starting z' value for the ray in the adjacent well. This is done at label 1150 for rays coming from the right of X1P1 and above label 225 for rays coming from the left of X1P1.

For the case of a particle coming from the right of X1P1 with $E' > U_o$ the general solution written as

$$e^{\alpha x'} = u = \frac{[1 - (\gamma/\sqrt{E'})^2]}{(\gamma/\sqrt{E'})\,[\mathrm{ch}(\omega_b(E')\,z') - \gamma/\sqrt{E'}]} \tag{40}$$

may be solved for z' yielding

$$z' = -\frac{1}{\omega_b}\,\ell n \left\{ \frac{\sqrt{E'}\,\gamma\,u}{[E' - \gamma^2(1-u) + \sqrt{E' - \gamma^2}\,\sqrt{R}]} \right\}$$

where

$$R = E' - \gamma^2\,(1 - 2u + u^2)\,. \tag{41}$$

The crossing excess in z' in the lower well will thus be given by

$$\Delta z' = [z'(x'_j) - z'((X1P1 - \sigma_j)/\rho_j)] \cdot \rho_j \cdot \rho_{j-1}\,. \tag{42}$$

The general solution for a ray in the lower well is given by

$$\begin{aligned} x' &= z'\,(v_o + \tfrac{1}{2}\,z'\,\mathrm{grav}) \qquad x' \le 0 \\ x' &= z'\,(v_o - \tfrac{1}{2}\,z'\,\mathrm{grvr}) \qquad x' > 0\,. \end{aligned} \tag{43}$$

We assume that the starting value of z' for a ray in the lower well is zero when the ray starts from $x' = 0$ moving to the right or left with velocity $v = \pm v_o = \sqrt{2E'_G}$.

If no boundaries were interposed at x = 0 or removed at x = X1P1 the range interval z' required to reach the left and right turning points would be TG = |V0|/GRAV and TGR = |V0|/GRVR respectively. The interval in z' to go from x = 0 to the left-hand turning point is given by

$$DTL = [2*(+VGMIN + 0.5 + EG + DEH)]^{1/2}/GRAV \tag{44}$$

and for x' = X1P1 to the right-hand turning point the interval is given by

$$DTR = [2(EG - EH)]^{1/2}/GRVR . \tag{45}$$

For EG greater than (VGMIN + 0.5 + DEH), the ray will be reflected from x = 0 in classical ground hop fashion. In the program this is achieved by adding 2 x DTL to z' for a ray with negative velocity whose path goes below x = 0.

For rays trapped in the lower potential well, the motion is divided into four quadrants (denoted by IQUAD in the program: (I)SIGMA $\leq x <$ X1P1, $\frac{dx'}{dz'} < 0$; (II) $0 < x <$ SIGMA, $\frac{dx'}{dz'} < 0$; (III) $0 < x <$ SIGMA, $\frac{dx'}{dz'} > 0$; (IV) SIGMA $\leq x <$ X1P1, $\frac{dx'}{dz'} > 0$ where SIGMA = X1P1 − SIGL(J).

For motion in the lower well the calculation is directed at label 1050 to the quadrant in which it had previously been propagating (at range z_{j-1}) and is checked to determine whether X(J) is beyond the next quadrant boundary; if not it is allowed to propagate according to

$$\frac{dx'^2}{dz'^2} = -\begin{cases} \text{grvr in (I, IV)} \\ \text{-grav in (II, III)} \end{cases} . \tag{46}$$

If the ray crosses the I-II or III-IV boundary at x = SIGMA the excess range interval DZP is calculated and used as the initial range value in the next quadrant. A ray in quadrant IV with EG ≥ EH will not be turned around by the potential before reaching X = X1P1 and will cross into the E-F_2 potential well. The calculation is then directed to label 301 when the ray can be considered as propagating in the upper (Morse) potential well with energy

$$E = (EG + VGMIN + DEH - VMIN(J) \tag{47}$$

and with an invariant energy given by Eq. (20) or equivalently by

$$E' = \left[E - \frac{1}{2} \left(\dot{\sigma} + \dot{\rho}\, x' \right)^2 - \frac{dx'}{dt'} \left(\frac{\dot{\sigma} + \dot{\rho}\, x'}{\rho} \right) \right] \cdot \rho^2 \ . \tag{48}$$

Once launched a ray will propagate in one or the other of the two potential wells, crossing the boundary at X1P1 when energetically allowed. When J = MAXR the ray path is checked to determine whether the ray could have been intercepted by a receiver. For a receiver on the ground this is done at label 880 by checking whether the indicator for a ground reflection [VREV(MAXR) or VREV(MAXR-1)] set at label 222 is positive. If positive the point of reflection ZGRND is calculated and checked to see whether it is within the receiver's antenna beam (taken here to be 100 km). For rays which could have been intercepted, the delay time DLAY is stored in the array ADLAY (ITHET, IFREQ) which is subsequently plotted in the subroutine DLAYPL as soon as all frequencies in the DO loop 12 have been propagated. For a satellite receiver in the E-F_2 duct, an altitude bin is set at label 870 and all rays whose final height XX(MAXR) is within 10 km of the satellite height SXAT are accepted and stored in ADLAY for plotting.

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Printed by
United States Air Force
Hanscom AFB, Mass. 01731