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<p>A significant part of the present study has been the formulation of method for defining definitive error norms. Extensive literature searches have been completed. Analytical problems have been formulated to pre-test candidate error norms before they are implemented in computer codes. This approach has proved valuable in avoiding futile computer based experiments. We expect to see valuable 2-D results with trunkation error related error norms in future studies.</p>			
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ERROR NORM GUIDED FLOW ANALYSIS

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Final Report #1

Contract No. F49620-84-C-0037

for Period

1 March 1984 through 29 April 1985

Jim Wilson

Director of Fluid Mechanics Research

Office of Scientific Research/NA

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REFERENCES

## SUMMARY

The details of the work completed during the authorized study period are described in Sections 1 and 2 which contain contract Summary Report #1 of December 1984 and Summary Report #2 of March 1985.

Summary Report #1 discusses an elaborate 1-D eight grid level multigrid study to determine the minimum requirement for residual error control. This requirement is specified by the following ratio. It is the ratio of the sum of the absolute values of the residuals for all computational cells and the sum of the absolute values of the local truncation error estimates. It is called the residual contamination ratio. It is a measure of the contamination of the solution on any grid level by the residual error. Values of this ratio must be less than  $10^{-2}$  if reliable use of the truncation error estimates are to be made. This result is a key finding. It is expected to be valid for flow analysis problems of 1-D, 2-D or 3-D spatial extent and for flow analysis problems with or without discontinuities such as shocks, contact surfaces, free shear layers and wall boundary layers. Demonstration of this expectation is to be completed in future studies.

Summary Report #2 documents the implementation of one diagnostic error norm in 2-D for a co-flowing rocket nozzle plume and a Mach two free stream condition. The error norm of interest is the artificial diffusion ratio. This ratio measures the relative importance of the artificial diffusion terms in the local flux of mass, energy, and components of momentum. Values of this ratio should decay toward zero everywhere in the analysis domain except in shocks and unresolved viscous layers in which the ratio should remain near unity for any grid density. However, in the rocket nozzle investigated, values of this ratio failed to properly decay at the sharp corners of the blunt nozzle base. Inspection of the base pressure of the nozzle shows significantly lower values than required. Analysis of the flow turning shows that the Prandtl-Meyer expansion at the inner corner is inhibited by the artificial diffusion. The artificial diffusion ratio proved to be an effective indicator of inaccuracy in the 2-D Navier-Stokes analysis. Threshold values of this ratio must be yet correlated with the residual contamination ratio to determine levels of the artificial diffusion ratio at which accurate application of the dependent variable error norms can be made.

A significant part of the present study has been the formulation of methods for defining definitive error norms. Extensive literature searches have been completed. Analytical problems have been formulated to pre-test candidate error norms before they are implemented in computer codes. This approach has proved valuable in avoiding futile computer-based experiments. We expect to see valuable 2-D results with truncation error related error norms in future studies. The preparatory work for extending the studies in this direction is completed.

## INTRODUCTION

Clear statements of the nature of the flow equations and the character of numerical methods to model these equations have been set forth in References 1 and 2. During this contract period, considerable study and thought have been devoted to conceiving of approaches to compute the numerical errors in flow analysis codes. Some of the promising approaches have been tested. The implementation of these approaches has proved to be complicated yet informative. Section I describes the results of defining residual error properties. This is done in terms of a residual contamination ratio. Also studied were four error norms for the estimates of error in decoded values of the dependent variables. It is expected that these error norms will be valuable for 2-D and 3-D problems when blended with length scale error norms, the residual contamination ratio, and the artificial diffusion ratio. Section II describes the results of defining artificial diffusion. Artificial diffusion has the positive effect of enforcing the correct entropy constraint upon flow simulation. It has the negative effect of causing a loss of information and of causing serious numerical error in the numerical result in certain regions of the flow simulation. The artificial diffusion ratio is used to establish a measure of contamination by artificial diffusion.

Error Norm Guided Flow Analysis -  
Residual Contamination Ratio

Summary Report #1

for

December, 1984

Jim Wilson, Director of Fluid Mechanics Research

U. S. Air Force

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## Introduction

Flow analysis computer codes predict the behavior of the dependent variables for the imposed boundary conditions, for the selected initial conditions, for the selected residual error level, for the selected smoothing levels, and for the choice of the computational grid. These dependent variables include pressure, entropy, temperature, vorticity, velocity, Mach number, etc. There are numerical errors associated with these variables whose magnitude and behavior are not directly accessible. Error norms are being developed to remedy this problem. The error norms work from approximations of the formal definitions of residual and truncation error and from goals of accuracy that are set for the computations. This report summarizes the results of the first segment of a Boeing study for the AFOSR error norm study contract for the period of March '84 to November '84. Also included in this report is a brief description of the work that is proposed for the period between December '84 and March '85. Finally, several suggestions for March '85 efforts are provided.

## Summary of the March '84 to November '84 Period

A Boeing/NASA-Langley developed 1-D multi-grid computer code for the potential equation provides the test-bed for the error norm study during this time period. Six error norms are tested. To make the study unambiguous, eight grid levels (4, 8, 16, 32, 64, 128, 256, and 512 computational cells), three visit histories (the visit history is the frequency and the manner of returning to previously visited grid levels), and a wide range of residual tolerances ( $10^{-1}$  to  $10^{-9}$ ) are used on two test problems. The test problems are shown in figure 1a and 1b. The figures describe the change in cross sectional area of each channel flow problem. The first test problem has a gentle discontinuity at the juncture of a constant area and parabolic-shaped section. The second test problem has a step discontinuity at the juncture of a constant area and a cubic-shaped section. Except for these junctures, the geometric changes are analytically smooth.

There is a significant difference between the error behavior of the two test problems. The error in velocity decreases in magnitude as the size of the grid interval near the gentle discontinuity is reduced. Whereas, the error in velocity does not decrease as the size of the grid interval is reduced near

the severe discontinuity. Everywhere else in the analysis domain the error in velocity is much smaller than the largest errors near the discontinuities and these errors decay as the grid interval size is reduced.

The formulation of the six error norms is discussed as follows. The first five error norms compute the local velocity error from local velocity information. The sixth error norm is a global integral type. The first five error norms compare the local velocity on any grid level of interest with the local velocity generated by an independent means. These means include:

1. interpolating the finest grid velocity values to the grid of interest
2. decoding the local truncation error estimates for the reference value of local velocity
3. generating local values of velocity by neglecting the truncation error term from the multi-grid process
4. generating values of local velocity by a high order accurate method
5. interpolating the analytically exact solution to the grid levels of interest

The fifth error norm is the referee error norm by which the utility of the others is judged.

The behavior of the first four error norms is varied. The fourth error norm is the only one that can be used to get reliable error estimates without the multi-grid process. It is an approach that is promising for multi-dimensional flow analysis codes as well. The first three error norms require a multi-grid process. Moreover, a minimum of three grid level appears to be essential to obtaining reliable results. Viewed from the perspective of thorough error control, the grid must be changed anyway to achieve the resolution of interest. At this time it appears that any of the first four methods are usable. The use of all four error norms has been found to be valuable for removing errors from computer coding. Therefore, they are all viewed as complimentary.

The sixth error norm is defined as the ratio of the sum of the absolute values of the residuals for all computational cells and the sum of the absolute values of the local truncation error estimates. It is a measure of the contamination of the solution on any grid level by the residual error. Values of this ratio of less than  $10^{-2}$  are essential to reliable use of the first four error norms. This is viewed as a key finding of the work to date. It will allow further exploration of these error norms in a multi-variable context, seen as the next logical step in the error norms development.

#### Projected Effort between December '84 and March '85

The error norms in the above previous work have been tested on a single equation. This proposal effort will test the reliability of the error norms for coupled equations. A 1-D multi-grid code of the Euler equations will be used as a test-bed. The six error norms will be used to predict errors in velocity, static pressure and mass density for selected residual levels and grid densities. Three test problems are planned. They include two purely subsonic flow cases with and without analytically smooth geometries. The third test case involves transonic flow with a shock in a smooth geometry.

#### Project Effort between April '85 and April '87

The proper use of flow analysis codes for modeling the Euler and Navier-Stokes depends upon sorting through the computed results subject to the selection of a number of parameters. These parameters control the computational grid, the initial conditions, the boundary conditions, the residual tolerances, and the smoothing coefficients. The cost effectiveness of a grid is partly dependent upon the choice of the smoothing coefficients. Very smooth solutions require large grid sizes for a certain level of numerical accuracy. Non-smooth solutions may be unreliable irrespective of the grid size and distribution. Non-physical behavior of non-smooth solutions has to do with 'pits' and 'peaks' that develop in the flow field. The error norms must be able to detect 'pit' and 'peak' in solutions as well as solutions that are excessively smoothed. It is the goal of the work projected for this period to develop error norms in 2-D for the Euler and Navier-Stokes equations that can detect inadequate smoothness, excessive smoothness, correct level of residual errors and the required grid distribution for proper modeling the system of

differential equations. Moreover, the relationship between the local error and the true error is of constant interest. Test problems will be generated for 2-D domains. Nozzle and airfoil test problems have been found which will serve to test the error norms.

# FLOW TEST PROBLEMS

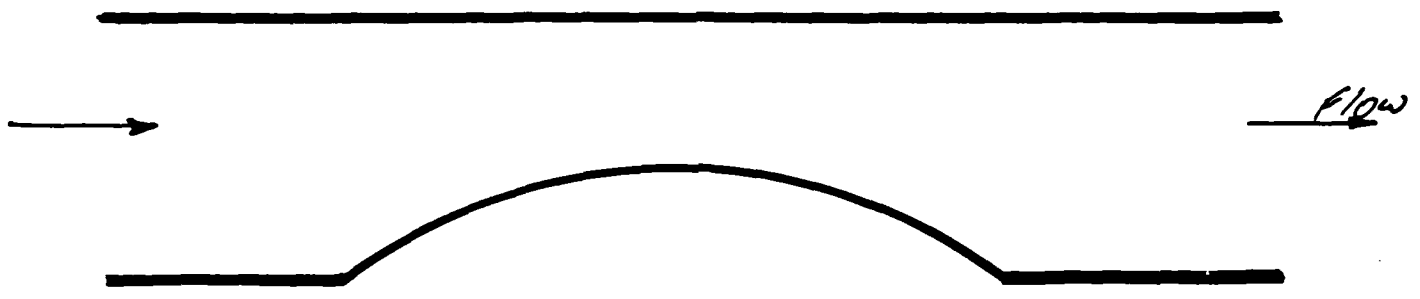


Figure 1a.



Figure 1b.

Error Norm Guide Flow Analysis -  
Artificial Diffusion Ratio

Summary Report #2

March, 1985

for

Jim Wilson, Director of Fluid Mechanics Research

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## Introduction

Considerable Computational Fluid Dynamics (CFD) work is engaged in at Boeing. Most of this work involves 2D and 3D with small disturbance theory, boundary layer theory, full potential theory, and strong interaction theory (such as the Navier-Stokes equations in suitable forms). It is of enduring interest how to apply earlier 1-D error norm studies to 2-D investigations in various on-going CFD activities. Of particular interest in December 1984: could the error norm activities enhance the understanding of the AGARD working group (WG-08) nozzle flow simulation assessment which discovered several discrepancies between various predictions and the benchmark test results utilized. The following report documents the application of an error norm concept to tracing problems with the accuracy of a 2D Navier-Stokes analysis of a rocket nozzle flow.

## Numerical Scheme

The numerical algorithm in the axisymmetrical/2D Navier-Stokes code, P367,<sup>(1)</sup> is the split MacCormack predictor - corrector finite volume scheme. Appropriate entropy constraints at shocks are enforced with artificial diffusion. Control of the Gibb's-effect errors in nonsmooth regions of the simulation is achieved with artificial diffusion as well. The character of the artificial diffusion is universal for each type of smoothing requirements. It is added explicitly to each face-oriented flux to augment the face flux. The augmented fluxes are generated from measurements of the curvature of the pressure field times the first difference of the dependent variable times a scale factor. This scale factor is called the smoothing coefficient. Considerable control over the need for the artificial diffusion is exercised by how the mesh is placed in the analysis domain. One control of the mesh shape that is available is to encourage the mesh to track the free shear layers to satisfy the constraint of nearly constant static pressure transverse to the free shear layer. In this process the mesh moves while the grid related equations are being solved. A converged solution is defined by suitably small values of flux balances for each cell in the computational grid. The grid point motion must cease in order for these residuals to be small.

## Description of the Rocket Nozzle Conditions

Figure 1 shows a cross section of an axisymmetric rocket nozzle in a longitudinal plane. The interior of the nozzle is of the convergent divergent type. The exterior is a cylinder/cone with a sharp intersection. It is perpendicular to the symmetry axis. The boattail angle is six degrees. The free stream conditions are for the Mach no. equal to 2.01, for the static freestream temperature equal to  $360^{\circ}\text{R}$  and for the internal to external pressure ( $P_i/P_e$ ) ratios of 1, 6, and 9. Figures 2, 3, and 4 show the flow vector fields, respectively, for the three pressure ratios. These results look very similar to the experimental data. However, significant errors exist in the predictions.

## Error Norm for Artificial Diffusion

The absolute value of the ratio of the smoothing flux to the total face flux is defined as the artificial diffusion ratio. This ratio measures the relative importance of the artificial diffusion term. A value of unity for this ratio means that the local flux is dominated by artificial diffusion. A value of zero for this ratio means that the local flux is free of artificial diffusion. Normalized integrals of this ratio over the analysis domain or over a region in the analysis domain show the neighborhood or global importance of the artificial diffusion. It is desirable that this integral decay toward zero as the length scales of the steep gradient regions approach a small fraction of the volume of the analysis domain. Example of such behavior are provided in the discussion of the results.

## Results of Axisymmetric Computations

Figure 5 shows a pressure coefficient plot of the boattail surface and base region for  $P_i/P_e$  equal 6. The agreement with test data is excellent except at the base where the pressure does not fully recover to the value that is expected. Careful inspection of the transverse velocity profiles of the free shear layers with respect to the longitudinal direction show abnormal growth rates near the base of the nozzle. Also the test data shows that a weak boundary layer separation is located upstream of the nozzle base in the external flow. This separation is preceded by a weak oblique shock. Neither

of these features is found in the numerical result. To enhance the strength of the separation, a test case with a pressure ratio of nine was selected. The numerical simulation should show a weak oblique shock/boundary layer interaction with reverse flow near the nozzle base in the external flow. Figure 6 shows the flow vectors for this case. Neither reverse flow nor the oblique precursor shock is evident. Grid refinement studies failed to show that the errors are related to grid density and length scales. Of course, it is always possible that the grid length scale near the nozzle base are much too coarse to be effective. If this is the case, the cost to solve the grid equations would be prohibitive because more than  $10^5$  iterations would be required to properly satisfy residual tolerance on grid densities that are expected to be necessary to the proper resolution of the corner flow. This would take many hours of CRAY computer time. A more efficient solver could improve this situation. Instead insight into the nature and source of the poor accuracy was sought.

Figures 6 and 7 show the result of changing the manner by which the mesh moving constraints are imposed. The constraint of constant distance between the plume edges is imposed for the results in Figure 6. This constraint is removed for the results which are shown in Figure 7. In Figure 7 the distance between plume edges is solved for the entire longitudinal span of the wake region. A comparison of Figures 6 and 7 shows that near the base of the nozzle the solutions are radically different. Figure 7 shows gross separation and an oblique shock/boundary layer interaction are obtained as expected. Figure 6 results are void of these required flow features. However, while the result shown in Figure 7 is considerably better than that of Figure 6, it also has flaws. Figures 8 and 9 show contour plots of the artificial diffusion ratio for cases of Figure 6 and 7. Note that near the inner base corner this ratio is still substantial for Figure 7 results but Figure 7 shows a more localized excessive diffusion. An attempt to reduce this ratio of the nozzle base corners was unsuccessful since when removing the artificial diffusion at the corners the numerical method became unstable. Some reduction of the artificial diffusion could be obtained by simply reducing the magnitude of the global smoothing coefficient. No great improvements were made in the results by this approach because again the numerical method became unstable.

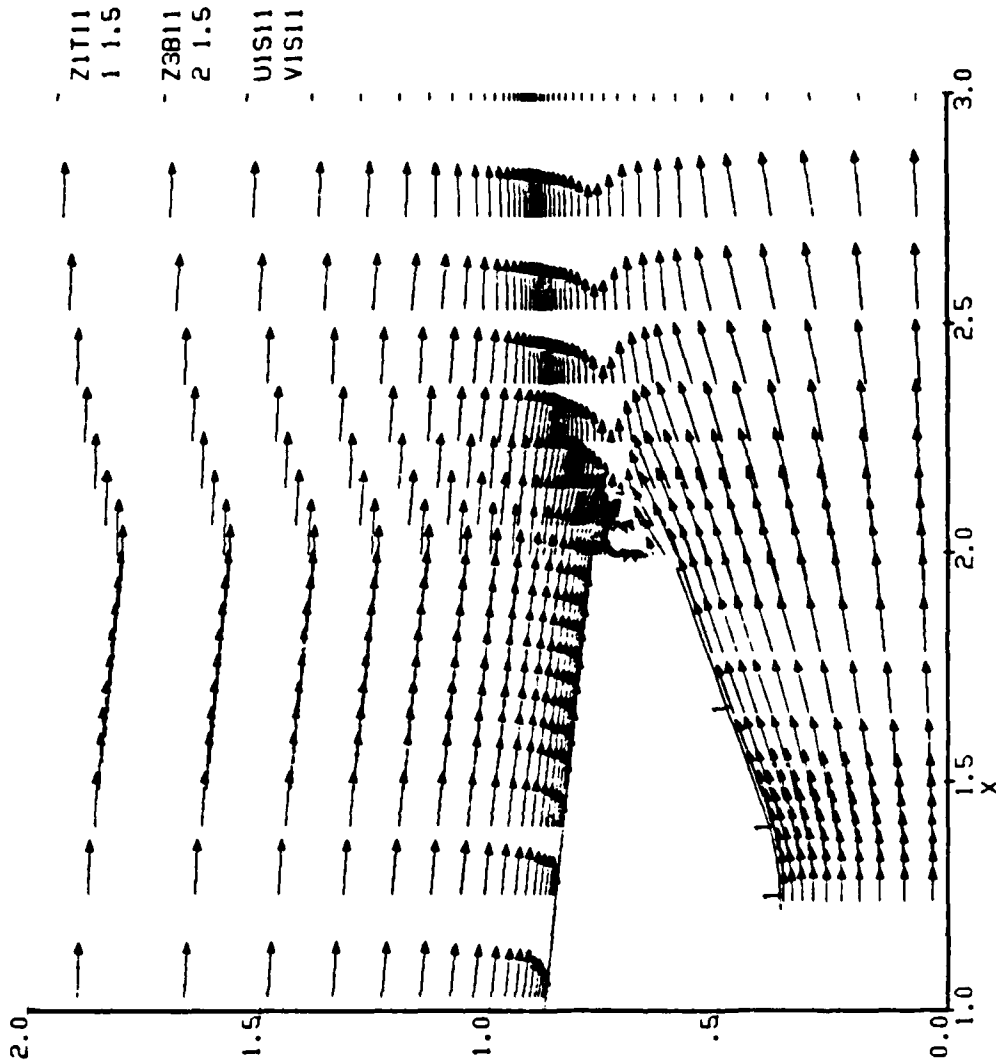
### Proposed Method of Improving the Corner Flow

It is clear that the artificial smoothing levels near the corners of the base of the nozzle dominates the inviscid flow structure. The artificial viscosity algorithm must be abandoned or suitably modified in this region to eliminate the excessive diffusion. It is conjectured that implementation of the Prandtl-Meyer conditions at the corners to get the correct mesh shape may be a viable means to removing the excessive diffusion. This approach seems plausible because it would eliminate of curvature of the pressure field at the corner which excites the excessive numerical diffusion. Future studies could verify these expectations.

## REFERENCES

1. Forester, C. K., "Error Norms for the Adaptive Solution of the Navier-Stokes Equations," NASA-langley Research Center Report NASA CR-165828, under contract NASI-16408, May 1982.
2. Forester, C. K., "Error Norm Guided Flow Analysis of Shock Wave/Boundary Layer/Free Layer Interactions," Technical Proposal submitted to: USAF, AFOSR/NA, Bolling Air Force Base, D. C., February 1985.

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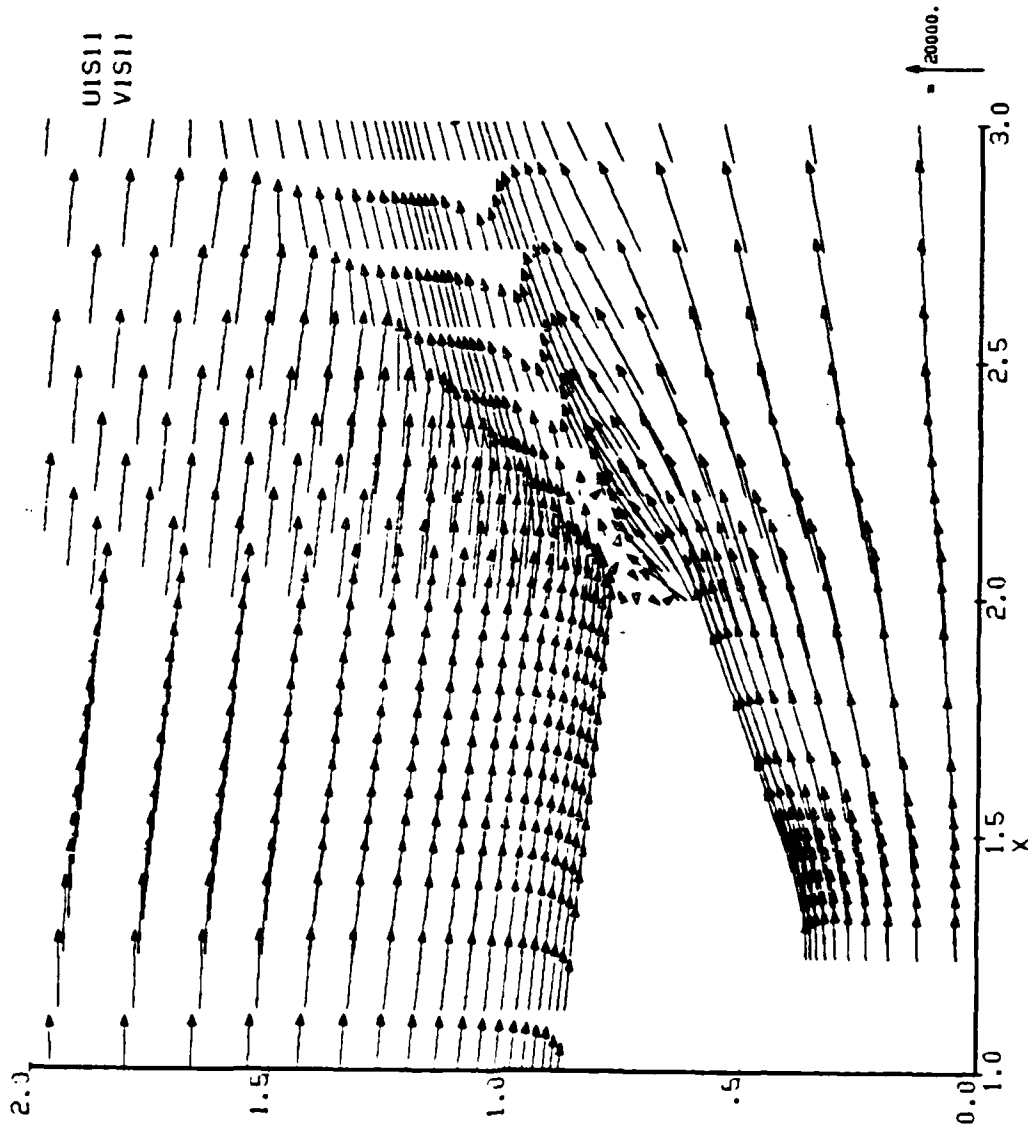
Fig 2

Case 12

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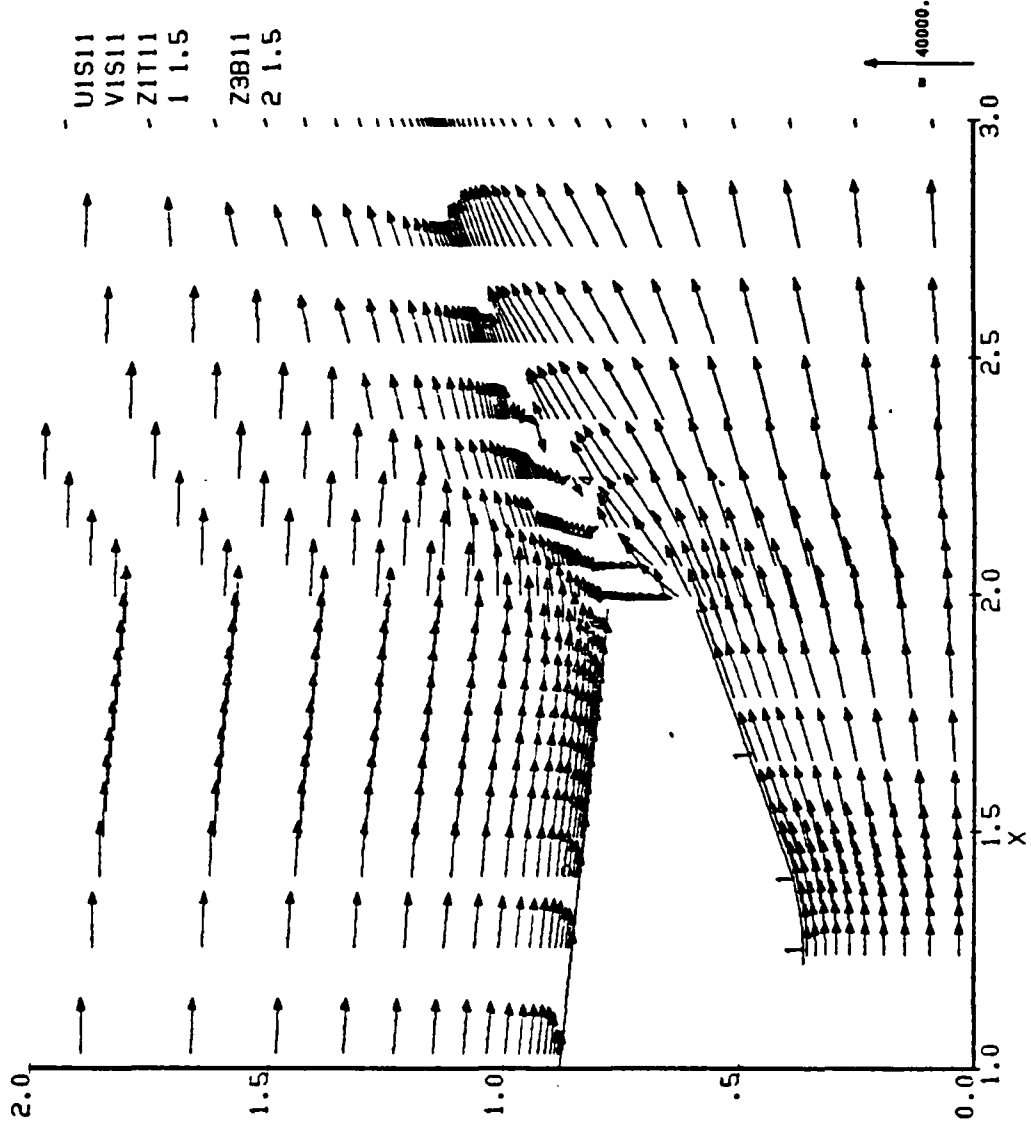
Scale .1

$P_z/P_e = 6.0$

$P_z/P_e = 6$

Fig 3

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SCALE .1

$P_I/P_E = 9.0$

$P_I/P_E = 9$   
Fig 8

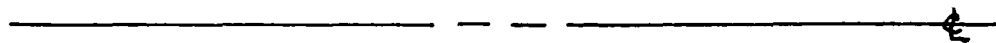
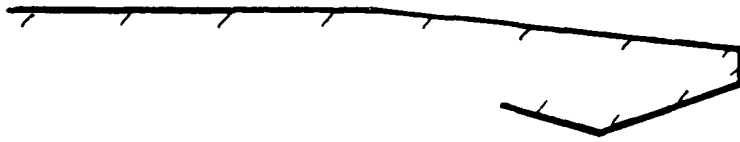


Fig 1 Cross Section of Rocket Nozzle

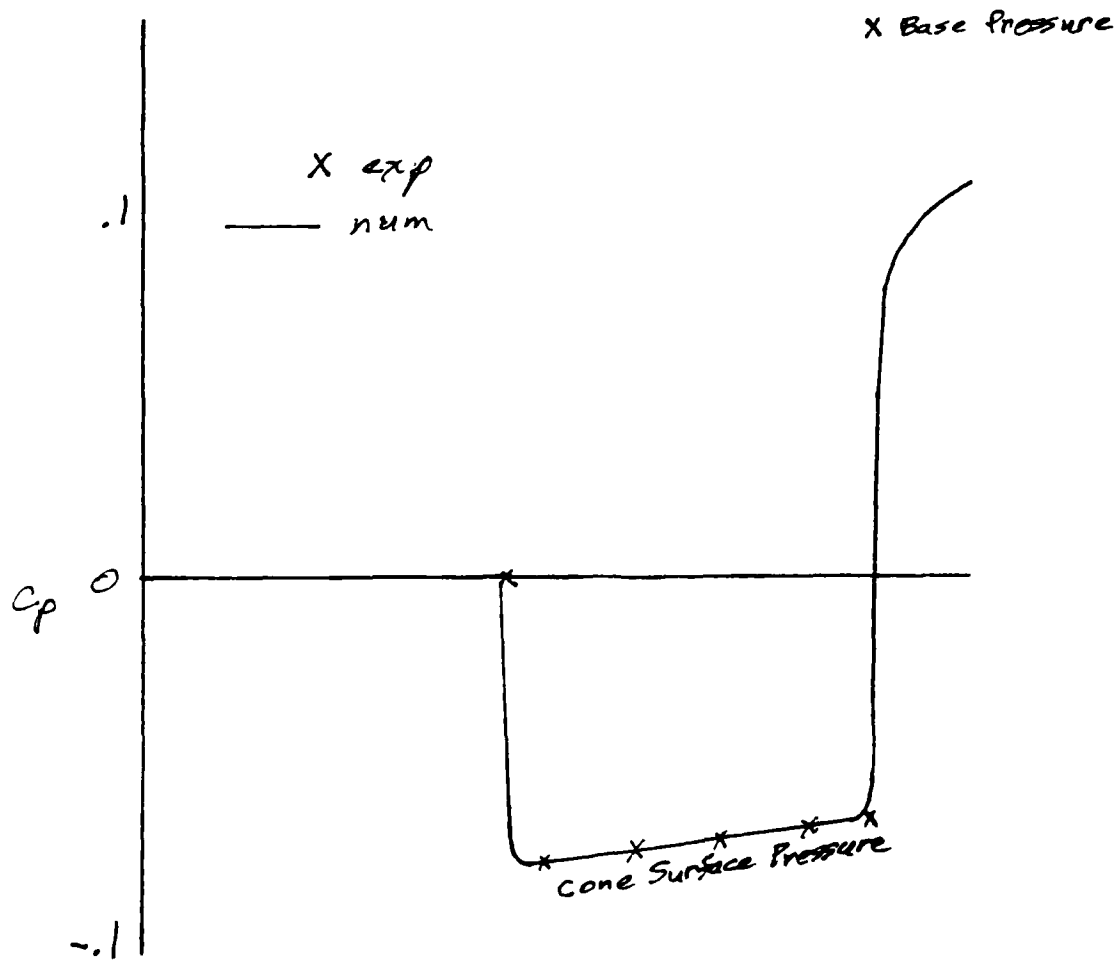
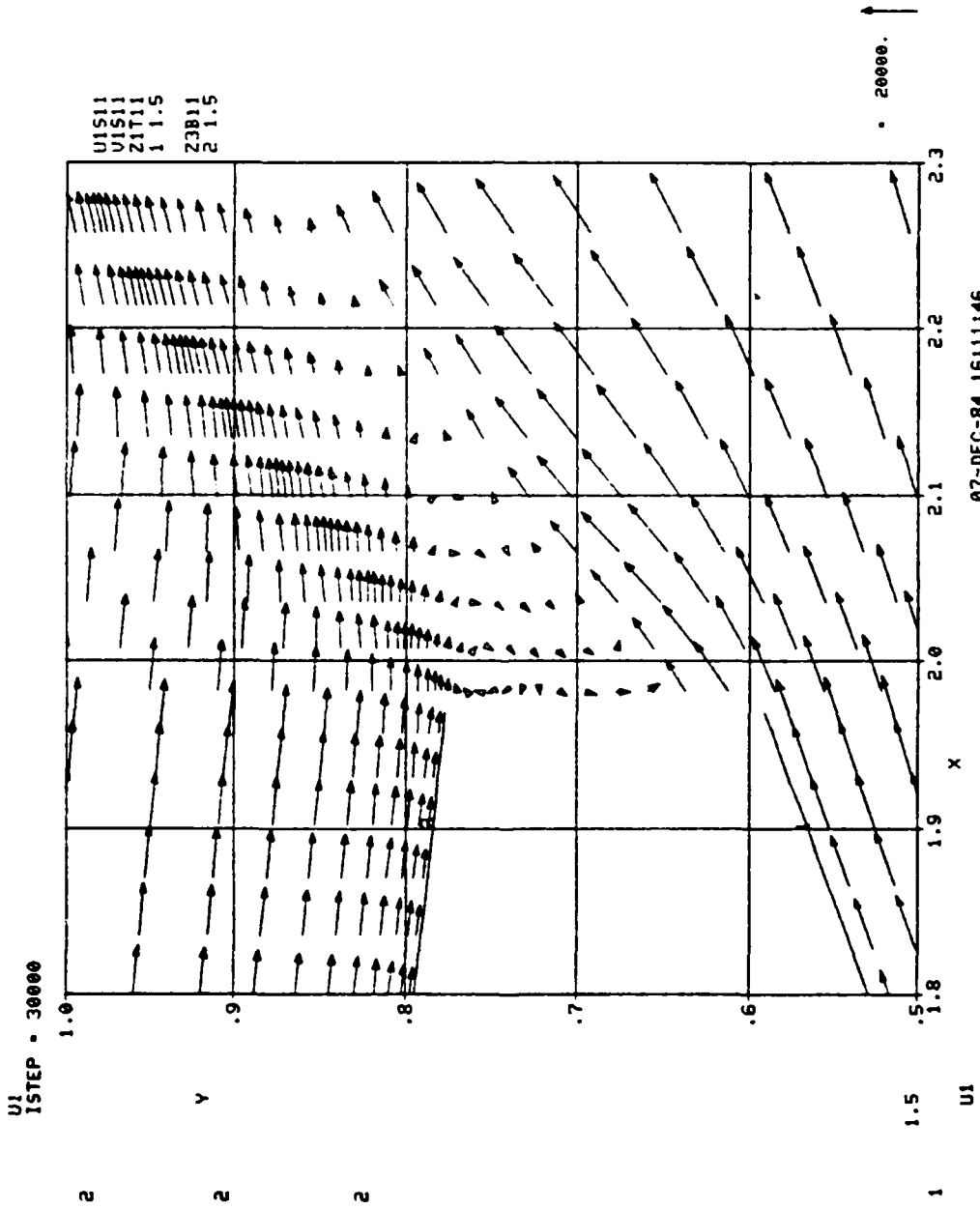


Fig 5  $P_i/P_e = 6$

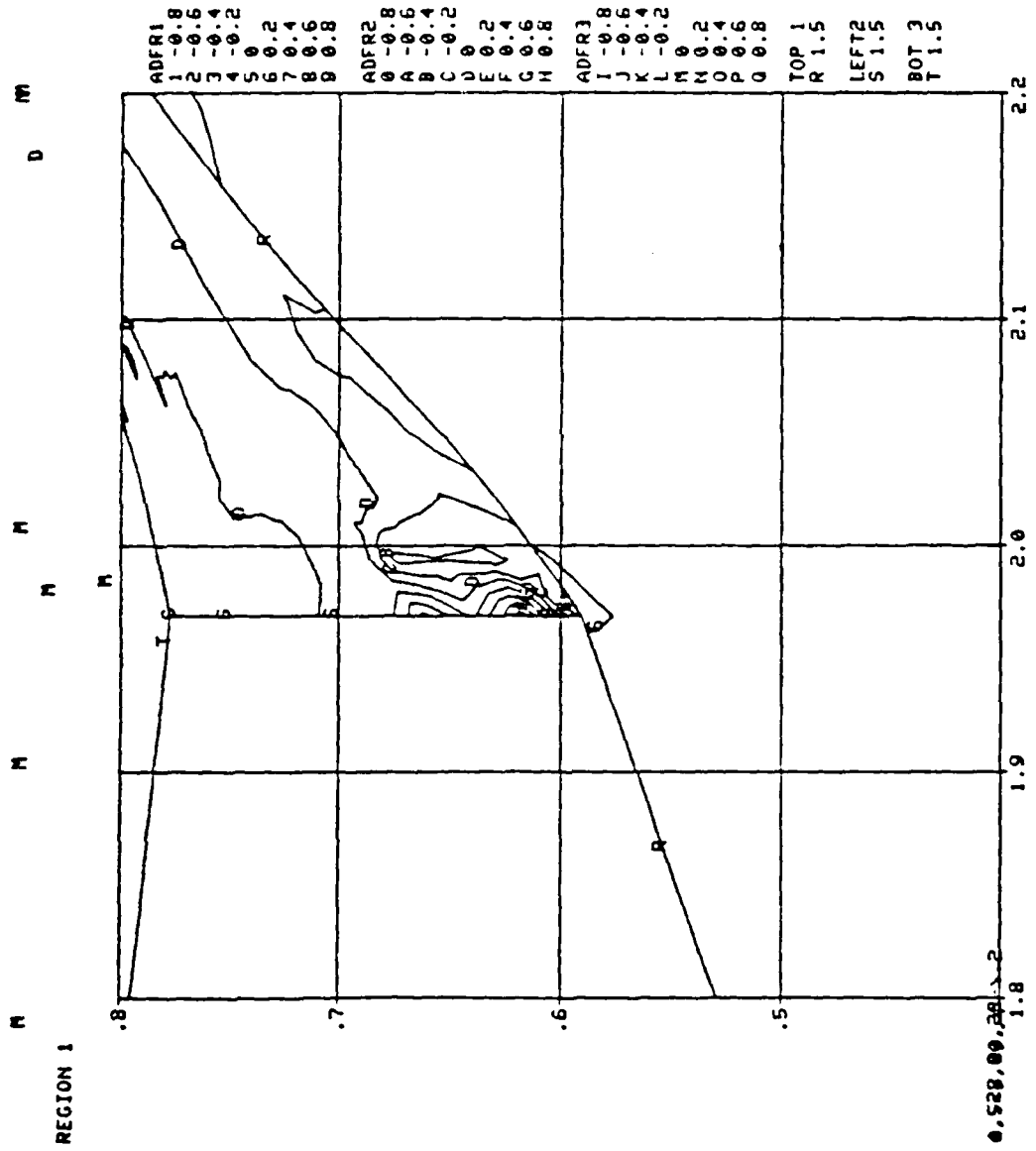


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Fig 6







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Fig 9

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