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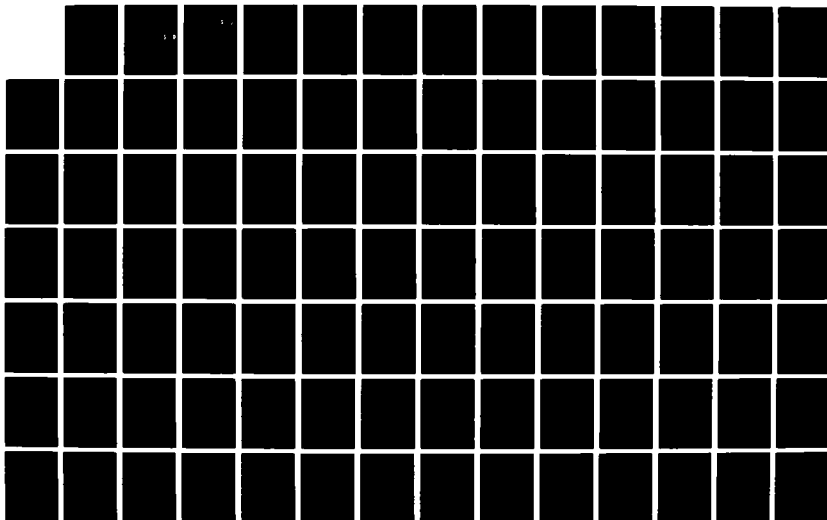
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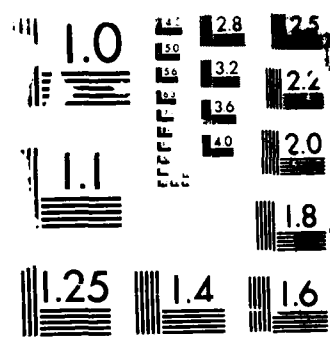
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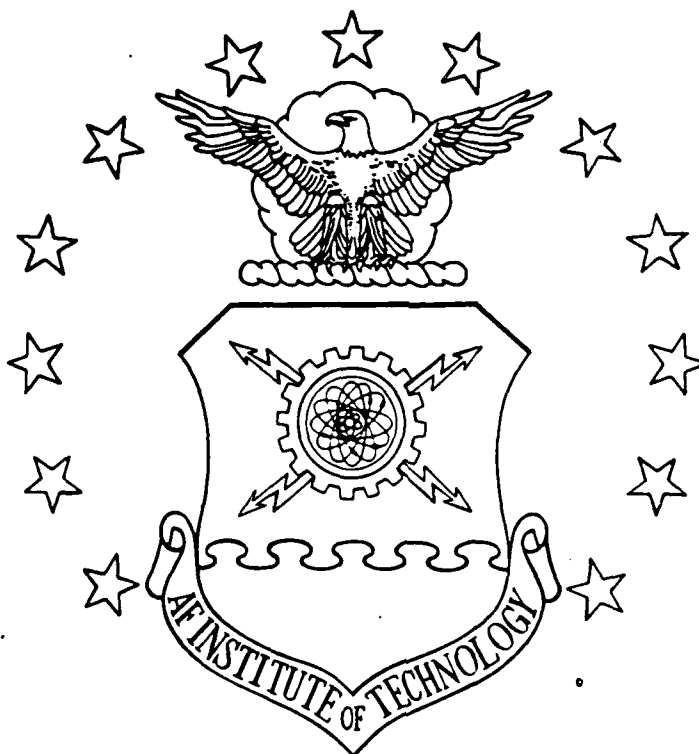


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THESIS

Bruce J. Sutherland
Captain, USAF

AFIT/GAE/AA/86D-17

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Aeronautical Engineering

Bruce J. Sutherland, B.S.
Captain, USAF

December 1986



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Acknowledgements

The experimental portion of this investigation was performed at facilities of the Air Force Institute of Technology, Wright-Patterson AFB, Ohio.

In performing this investigation I have had a great deal of help from others. I am deeply indebted to my faculty advisor, Dr. James Hitchcock, whose continuing patience and guidance were essential for the completion of this study. In addition, I would like to thank my committee members, Dr. William Elrod and Dr. Milton Franke, for their enthusiasm and support of this investigation.

I would also like to thank Leroy Cannon, Jay Anderson, and Nick Yardich for their fantastic support in operating and maintaining the experimental apparatus. Also a word of thanks is owed to the people of the Model Fabrication Shop, in particular, Joe Hofele, for their excellent job of preparing the materials used in this experiment.

I extend my deepest appreciation to my wife Patricia and daughter Lauren for their unwavering support and encouragement during this ordeal.

Computer used: ATARI 130XE

Software: PAPERCLIP by Batteries Included

Printer: PANASONIC KX-P1091

Bruce J. Sutherland

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List of Symbols

A	Duct cross-sectional area
AE	Area of Duct - area of modelling circle
As	Duct surface area
C _f	Fanning friction factor
D _h	Hydraulic diameter
D _{h,1}	New hydraulic diameter
D _m	Modelling circle diameter
L*	Hydrodynamic entrance length, non-dim. form
$\dot{m}$	Mass flow rate
P	Static pressure
PE	Perimeter of duct + perimeter of modelling circle
Per	Duct perimeter
Re	Reynolds number
Re _{cr}	Transition Reynolds number
V	Average duct velocity
X	Hydrodynamic entrance length
α	Aspect ratio
δ	Boundary layer thickness
λ	Darcy friction factor
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Fluid density
τ_0	Shear stress at the duct wall

Abstract

For laminar, steady flow in ducts, the current definition of hydraulic diameter, D_h , does not accurately depict the non-uniform wall shear stress distribution around the perimeter of non-circular duct shapes. In this investigation, a new hydraulic diameter, $D_{h,1}$, was empirically determined. It correlated friction factor data for many non-circular shapes to within approximately 2.4 % of the circular duct value.

An experiment, using the AFIT Oil Flow Rig Set-up, was run to determine the effect on transition Reynolds number, Re_{tr} , and hydrodynamic entrance length, L^* , of replacing D_h with $D_{h,1}$. Transition Reynolds number and L^* were determined, based on D_h and $D_{h,1}$, for a circular, square, and concentric annular duct.

Transition Reynolds numbers, based on D_h , for the square and concentric annular ducts were approximately 12.5 % lower than the circular duct Re_{tr} . The Re_{tr} , based on $D_{h,1}$, did not correlate well for the concentric annulus, but did correlate for the square duct.

Hydrodynamic entrance lengths, based on D_h and $D_{h,1}$, were experimentally determined for the circular and square duct only. The square duct L^* , and the analytic concentric annular duct L^* , based on $D_{h,1}$, did not correlate to the

to the circular duct value.

Although the Re_* and L^* for the square and concentric annular ducts did not correlate well when based on $D_{h,1}$, the data obtained is still useful to the engineering community since it provides an addition data base for experimental L^* , and provides experimental data on Re_* for square and concentric annular ducts.

A REDEFINED HYDRAULIC DIAMETER FOR LAMINAR FLOW

I. Introduction

Background

For flow in circular ducts, where there is a uniform wall shear stress around the perimeter and uniform distance from the center to the surface, duct radius, or diameter, is a logical choice for the characteristic dimension in flow correlations. For non-circular ducts a characteristic dimension, the hydraulic diameter (D_h), has been defined as $4A/P$ where A is the duct cross-sectional area and P is the duct perimeter. The hydraulic diameter is unambiguously defined for any cross-sectional shape, and for consistency is used in both turbulent and laminar duct flows. Experimentation has verified that, for turbulent flow, where the wall shearing stress is fairly uniform around the perimeter, the hydraulic diameter is the appropriate characteristic dimension for many non-circular duct shapes (1). For fully developed, laminar flow, however, the wall shearing stress distribution along the perimeter of non-circular ducts is not uniform and the hydraulic diameter is, perhaps, not the best characteristic dimension. For example, for a circular duct, $C_{f,Re} = 16.0$, while for a triangular duct, $C_{f,Re} = 13.333$, where Re is

defined as VD_h/ν . At present there is no characteristic dimension that correlates laminar flow data in non-circular ducts.

Some current uses of hydraulic diameter as a characteristic dimension are in transition Reynolds number and hydrodynamic entrance length calculations. Reynolds number, which is a ratio of inertia to viscous forces, is very much associated with the stability of laminar flows. Viscous forces tend to restore laminar flow after a disturbance, while inertia forces tend to amplify disturbances, therefore, transition from laminar to turbulent flow might be characterized by a transition Reynolds number, Re_{tr} . For circular duct flow, the Re_{tr} , based on diameter and duct averaged velocity, is approximately 2000 (2). This author has not found any Re_{tr} data for non-circular ducts, and since the wall shearing stress is not uniformly distributed around the perimeter of non-circular ducts, one would speculate that the Re_{tr} , based on hydraulic diameter, for these ducts would be different from the circular duct value of 2000.

Another use of hydraulic diameter has been in hydrodynamic entrance length (X) calculations. One of the most common definitions of X is the duct length required to achieve a duct centerline velocity 99% of the fully developed value (3). These results are often given in the non-dimensional form, $L^* = (X/D_h)/Re$. Several different

analytical solutions to the L^* problem exist for circular as well as non-circular duct shapes (14). Based on hydraulic diameter, L^* varies significantly for each duct shape. Little, if any, experimental L^* data has been reported.

Objectives

This investigation had four objectives:

1. Empirically determine a new hydraulic diameter, " $D_{h,1}$ ", that better represents the mean center to surface distance for laminar flow in non-circular ducts, and which correlates $C_f Re$ data.
2. Experimentally determine the Re_{cr} , based on $D_{h,1}$, for a circular duct, square duct, and concentric annulus with an aspect ratio that simulates flow between infinite parallel plates.
3. Experimentally determine L^* for the above shapes, based on $D_{h,1}$.
4. Examine the effect on Re_{cr} and L^* of replacing the traditional D_h with $D_{h,1}$.

The first objective will be accomplished using friction factors ($C_f Re$), for fully developed duct flow, as the data base for the $D_{h,1}$ calculations, with an attempt to converge the data to the circular duct value.

The remaining objectives will be accomplished using

the Air Force Institute of Technology's instrumented oil flow rig set-up. Pressure and mass flow data will be taken for the different duct shapes to determine Re_c and L^* .

II. Duct Theory

Circular Ducts

The circular duct is the most common duct shape in use today. For constant-property, steady, fully-developed, laminar flow through circular ducts, as in Figure 1, the equation of motion can be written as:

$$(\mu/r)[(\partial/\partial r)(r(\partial u/\partial r))] = dP/dx \quad (1)$$

Since the pressure is independent of r , Equation (1) can be integrated directly with respect to r to obtain the velocity profile. Applying boundary conditions:

At	$r = 0$	axis of the tube
	$\partial u/\partial r = 0$	velocity gradient is zero
At	$r = r_0$	duct wall
	$u = 0$	no slip at the wall

yields

$$u = (r_0^2/4\mu)(-dP/dx)(1-(r^2/r_0^2)) \quad (2)$$

It proves more convenient to express this velocity in terms of a mean duct velocity, V , rather than the pressure gradient. For incompressible flow, the mean velocity is defined as:

$$V = (\iint u dA)/A$$

resulting in,

$$V = (r_0^2/8\mu)(-dP/dx) \quad (3)$$

thus,

$$u = 2V(1-(r/r_0)^2) \quad (4)$$

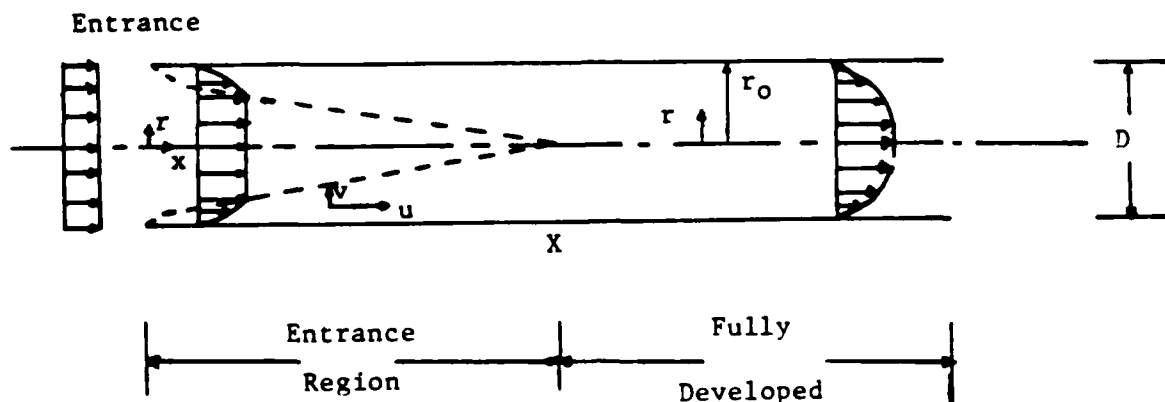


Figure 1. Flow in a Circular Duct

The gradient of this velocity profile is used to evaluate the shear stress at the wall.

$$\tau_0 = \mu (\partial u / \partial r)_{r=r_0} \quad (5)$$

Using Equation (4)

$$\tau_0 = \mu [2V(-2r/r_0^2)]_{r=r_0} = -4V\mu/r_0 \quad (6)$$

Since V is constant for steady, fully-developed flow, τ_0 is constant. The friction factor can now be evaluated. The Fanning friction factor (C_f) is defined as the ratio of wall shearing stress to the dynamic head of the flow.

$$C_f = \tau_0 / (\rho V^2 / 2) \quad (7)$$

thus,

$$C_f = (4V\mu/r_0) / (\rho V^2 / 2) = 8\mu / \rho V r_0 \quad (8)$$

For a circular duct, Re is defined as

$$Re = \rho V D / \mu$$

where $D = 2r$. Rewriting Equation (8)

$$C_f = 16/Re \quad (9)$$

Data from Stanton and Pannell (6:199) and Senecal and Rothfus (7:533) verifies this equation up to a Re of 2000.

Non-Circular Ducts.

Although a majority of ducts in use today are of circular cross-section, many non-circular duct applications exist and require further flow analysis. For constant property, fully developed, laminar flow through a non-circular duct, the equation of motion is:

$$\partial^2 u / \partial y^2 + \partial^2 u / \partial z^2 = (1/\mu)(\partial P / \partial x)$$

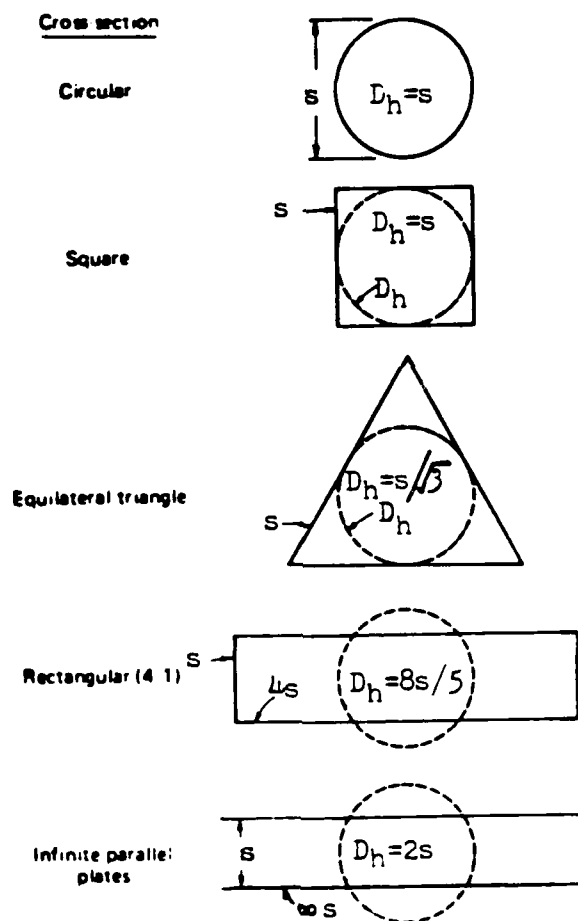
The friction forces are seen to be functions of the cross-sectional dimensions in both y and z directions. Traditionally, a single length dimension, the hydraulic diameter (D_h), has been used to characterize the cross-section. Hydraulic diameter is related to the hydraulic radius, R_h , which is the ratio of cross-sectional area, A, to wetted perimeter, Per, by:

$$D_h = 4R_h = 4A/Per$$

Hydraulic diameter for a non-circular duct is intended to represent the diameter of an equivalent circular duct which has the same $C_f Re$ as the non-circular duct. Table I illustrates D_h for several non-circular shapes.

TABLE I.

Five Ducts With the Same Hydraulic Diameter (5:77)



Turbulent flow through non-circular ducts, using D_h as the characteristic dimension, was investigated by Shiller (8) and Nikuradse (9), who verified that D_h provided satisfactory results. For laminar flows, however, the Shiller and Nikuradse data, as well as that of Koch and Feind (10), showed limitations to the use of hydraulic diameter. This data is shown in Figure 2 from Schlichting (2:576). Here λ is the Blasius, or Darcy, friction factor where,

$$\lambda = 64/Re = 4C,$$

for circular ducts. In Figure 2, λ has been multiplied by a constant, for each duct shape, in order to fit all the data onto one graph. For laminar flow, the hashed lines represent a circular duct. Note the difference in λ between circular and non-circular ducts based on D_h .

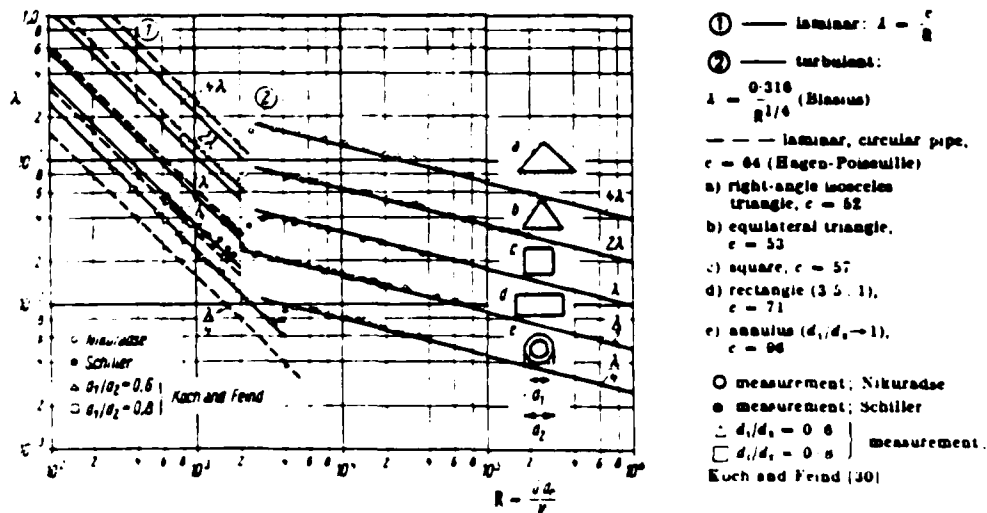


Figure 2. Friction Factor for Several Ducts

Exact solutions to the equations of motion, for many non-circular ducts, based on D_h , yield,

$$C, Re = C = \text{constant}$$

for each duct shape, instead of the desired circular duct solution, $C, Re = 16$. Several values of C, Re are shown in Table II (11).

TABLE II.

C, Re for Non-Circular Ducts

<u>DUCT SHAPE</u>	<u>$C = C, Re$</u>
SINE	12.630
TRIANGLE	13.333
SQUARE	14.227
HEXAGON	15.054
RECTANGLE 2:1	15.548
SEMI-CIRCLE	15.600
RECTANGLE 4:1	18.233
RECTANGLE 8:1	20.585
INFINITE PARALLEL PLATES	24.000

These values of C, Re represent exact solutions, based on D_h , most having been verified by experimental data (8,9,10).

By examining the definition of hydraulic radius, an

insight as to why these laminar flow values do not correlate with the equivalent circular duct data can be gained. The reciprocal of hydraulic radius is the wetted perimeter per unit of flow cross-section and describes the amount of wall area in contact with the moving fluid. The hydraulic radius is a good index if the resistance to momentum transfer is predominantly dependent on the amount of wall surface area, as it is in turbulent flow. In laminar flows this resistance to momentum transfer takes place throughout the cross-sectional flow area and is not primarily associated with the wall surfaces. Thus reliable conversions from circular to non-circular shapes cannot be expected.

Another explanation for poor correlation with laminar flows is seen by examining the surface area per unit of flow volume, As/Vol , for individual ducts. The friction factor is directly related to the amount of wall shear stress and this stress is related to the amount of surface area it acts upon. For ducts with As/Vol different than that of a circular duct, the shearing stress is acting on a different amount of surface area, per unit volume, and a different C_f can be expected. For example, a rectangular duct with an aspect ratio of 8:1, has more surface area, per unit volume, for the shearing stress to act upon than does the circular duct with the same D_h , and hence a larger C_f is expected.

Flow in the corners of non-circular ducts also influences C_r . Consider a square duct and a circular duct, each with the same D_h . Each duct has approximately the same As/Vol ratio. In the corners of the square duct, the velocity is low compared to V , and with this low velocity is low shearing stress. This low velocity, in essence, reduces the effective flow area. Low effective flow area produces a lower C_r for the square duct compared to the circular duct.

Since laminar flow $C_r Re$ values do not correlate well for non-circular ducts, it becomes desirable to have exact solutions for non-circular shapes, based on a new $D_{h,1}$, where the value of $C_r Re$ for these ducts equals that of the circular duct. Individual $C_r Re$, as in Table II, would no longer be necessary.

Exact $C_r Re$ values vary non-linearly for rectangular ducts, from the square duct value of 14.227, to the infinite parallel plate duct value of 24.0, as illustrated in Figure 3 (1). Since the corners of rectangular ducts have the same influence on C_r , from duct to duct, the $C_r Re$ variance can be directly attributed to the difference in As/Vol . Once again, the values in Figure 3 are based on D_h . As noted in Figure 3, a rectangular duct with an aspect ratio of approximately 2.3 has a $C_r Re$ equal to that of a circular duct, and D_h becomes an appropriate dimension.

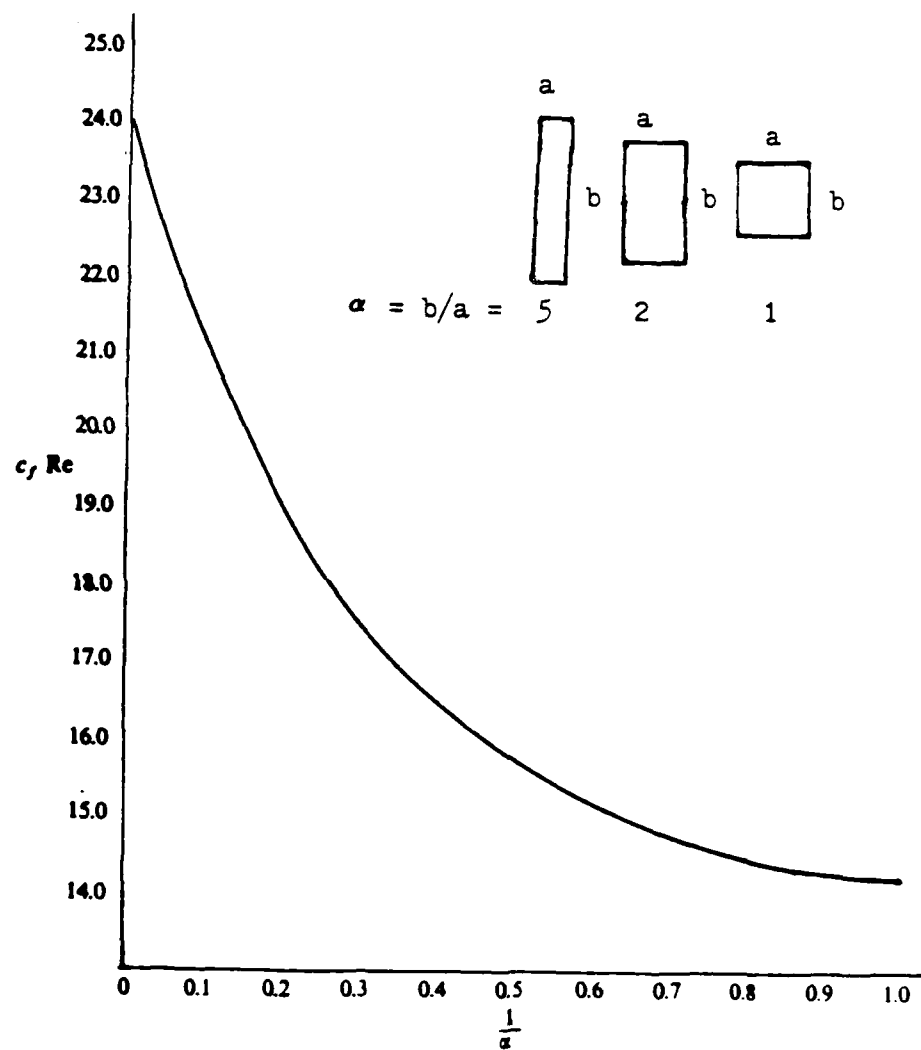


Figure 3. Friction Coefficients for Rectangular Ducts

III. Redefined Hydraulic Diameter

Approach

Currently, each non-circular duct has a unique exact solution for C_{Re} , based on D_h . This author seeks a new parameter, $D_{h,1}$, that would produce a non-circular duct value of C_{Re} equal to the circular duct value of 16. The first step is to examine the C_{Re} data for several non-circular duct shapes, where

$$Re = \rho V D_h / \mu$$

Table II and Figure 3 summarize C_{Re} , based on D_h , for many of the most common non-circular shapes. An empirical equation for $D_{h,1}$ will be derived that will correlate C_{Re} data to the circular duct. To do this, first the data from square, triangular, hexagonal, and semi-circular ducts are examined, since these shapes are similiar, in that they are fairly compact. Next the rectangular data from Figure 3 is approximated, to within 0.05%, by the following equation:

$$C_{Re} = 24[1 - (1.3553/\alpha) + (1.9467/\alpha^2) - (1.7012/\alpha^3) + (.9564/\alpha^4) - (.2537/\alpha^5)] \quad (3:199)$$

The rectangular duct will be examined first for α from 1.0 to 10., in increments of 0.25. Most manufactured rectangular ducts are in this range, so small increments are taken to fully evaluate these ducts. Aspect ratios of 25, 50, 100, and ∞ were examined next. These ducts

represent flow that is beginning to simulate infinite parallel plate flow.

The hydraulic diameter describes the diameter of an equivalent circular duct which has the same mean shear stress as the non-circular duct. Since D_h under-estimates the correct diameter in some cases, such as square and triangular ducts, and over-estimates it in others, such as infinite parallel plates and elongated rectangular ducts, a new D_h , $D_{h,1}$, is envisioned. $D_{h,1}$ is determined using a "modelling circle" diameter, D_m , plus a correction factor. This "modelling circle" represents the area of "main flow" in the duct. It, in essence, separates the corner areas from the "main area" in the duct. Table III illustrates the modelling circles for several duct shapes. For fairly compact shapes, such as square, triangular, and hexagonal ducts, the modelling circle area is identically the D_h area, and $D_m = D_h$. For elongated ducts, such as infinite parallel plates or rectangular ducts, where the flow extends out, the modelling circle is an ellipse where the elliptic aspect ratio is the same as that of the duct. For simplicity, D_m is the smaller length, a , for elliptical modelling circles. Comparing the flow area of circles with diameter D_h and their associated perimeter to those of the elliptical modelling circles, shows that the elliptical modelling circles better represents the actual flow area, and the perimeter upon which the mean shear stress may act.

TABLE III.

Modelling Circles for Several Duct Shapes

	$D_o = 1.000 S$ $D_h = 1.000 S$
	$D_o = .57735 S$ $D_h = .57735 S$
	$D_o = 1.732 S$ $D_h = 1.732 S$
$2b/2a = 1.3$	$D_o = .500 S$ $D_h = .611 S$
$2b/2a = 2$	$D_o = 1.000 S$ $D_h = 1.333 S$
$2b/2a = 4$	$D_o = 1.000 S$ $D_h = 1.600 S$
$2b/2a = \infty$	$D_o = 1.000 S$ $D_h = 2.000 S$

This can be seen, for example, for the rectangular duct of $\alpha = 4:1$, by comparing the sketches in Table I and Table III. In determining a correction factor for D_h , the area between the duct walls and the modelling circle, and its associated perimeter, is utilized.

Results

For laminar flow in non-circular ducts:

$$D_{h,1} = 3.2(AE/PE)(D_h/D_m) + D_m \quad (10)$$

where

AE = area of the duct - area of the "modelling circle"

PE = perimeter of the duct + perimeter of the
"modelling circle"

D_h = hydraulic diameter, $4A/P$

D_m = smallest diameter of elliptic "modelling circle"

Figure 4 shows an example of AE and its associated perimeter. Equation (10) is used for all ducts except rectangular ones. An additional term is needed for rectangular ducts to account for the changing D_h/D_m ratio and the aspect ratio, α . For rectangular ducts:

$$D_{h,1} = 3.2(AE/PE)(D_h/D_m)[1 + \{4(D_h - D_m)^{.5}/\alpha\}] + D_m \quad (11)$$

For example, using Figure 4 and Equation (11),

area of the duct = $3S^2$

area of the "modelling circle" = $\pi(.5S)(1.5S)$

perimeter of the duct = $8S$

perimeter of "modelling circle" = $2\pi[(1.5^2 + .5^2)/2]^{.5}S$

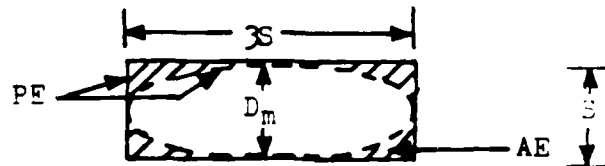


Figure 4. AE for a Rectangular Duct

$$AE = .644S^2$$

$$PE = 15.02S$$

$$\text{aspect ratio} = 3.0$$

$$D_h = 1.5S$$

$$D_n = 1.0S$$

and,

$$D_{n,1} = 1.4S$$

Here, for an ellipse:

$$\text{Area} = \pi ab$$

and the perimeter can be approximated by,

$$\text{Perimeter} \approx 2\pi[(a^2 + b^2)/2]^{.5}$$

Note that equation (11) reduces to equation (10) for the limiting cases of the square duct ($D_h = D_n$) and the infinite parallel plate duct ($\alpha = \infty$).

Table IV and V present $D_{n,1}$ and C_f data, based on $D_{n,1}$. Friction factors for all duct shapes investigated, are within 2.4% of the circular duct value of 16.0. Appendix A presents a breakdown of the $D_{n,1}$ calculations for each duct shape.

TABLE IV.

Redefined Hydraulic Diameter, $D_{h,1}$

<u>SHAPE</u>	<u>$C_f Re (D_h)$</u>	<u>D_h</u>	<u>D_m</u>	<u>$D_{h,1}$</u>	<u>$C_f Re (D_{h,1})$</u>	<u>$\Delta\%$</u>
SQUARE	14.227	1.000S	1.000S	1.098S	15.62	-2.40
TRIANGLE	13.333	0.577S	0.577S	0.693S	15.94	-0.04
HEXAGON	15.054	1.732S	1.732S	1.800S	15.64	-2.20
SEMI-CIR	15.600	0.611S	0.500S	0.633S	16.15	0.94
INF.PAR.PL.	24.000	2.000S	1.000S	1.345S	16.12	0.75

TABLE V.

Redefined Hydraulic Diameter, $D_{h,1}$ Rectangular Ducts

<u>Aspect Ratio</u>	<u>$C_{fRe}(D_h)$</u>	<u>D_h</u>	<u>$D_{h,1}$</u>	<u>$C_{fRe}(D_{h,1})$</u>	<u>$\Delta\%$</u>
1.00	14.230	1.000S	1.098S	15.620	- 2.4
1.25	14.382	1.111S	1.239S	16.039	0.2
1.50	14.715	1.200S	1.294S	15.863	- 0.9
1.75	15.127	1.273S	1.328S	15.786	- 1.3
2.00	15.557	1.333S	1.352S	15.770	- 1.4
2.25	15.978	1.385S	1.368S	15.783	- 1.4
2.50	16.377	1.429S	1.379S	15.810	- 1.2
2.75	16.749	1.467S	1.387S	15.843	- 1.0
3.00	17.095	1.500S	1.393S	15.877	- 0.8
3.25	17.414	1.529S	1.397S	15.909	- 0.6
3.50	17.710	1.556S	1.400S	15.940	- 0.4
3.75	17.982	1.579S	1.402S	15.969	- 0.2
4.00	18.234	1.600S	1.404S	15.995	- 0.0
4.25	18.467	1.619S	1.404S	16.019	0.1
4.50	18.684	1.636S	1.405S	16.040	0.3
4.75	18.885	1.652S	1.405S	16.060	0.4
5.00	19.072	1.667S	1.405S	16.078	0.5
5.25	19.246	1.680S	1.405S	16.095	0.6
5.50	19.409	1.692S	1.405S	16.110	0.7
5.75	19.561	1.704S	1.404S	16.124	0.8
6.00	19.705	1.714S	1.404S	16.136	0.9
6.25	19.839	1.724S	1.403S	16.148	0.9
6.50	19.965	1.733S	1.403S	16.159	1.0
6.75	20.084	1.742S	1.402S	16.170	1.1
7.00	20.197	1.750S	1.402S	16.180	1.1
7.25	20.303	1.758S	1.401S	16.189	1.2
7.50	20.404	1.765S	1.401S	16.197	1.2
7.75	20.499	1.771S	1.400S	16.206	1.3
8.00	20.590	1.778S	1.400S	16.214	1.3
8.25	20.676	1.784S	1.399S	16.221	1.4
8.50	20.758	1.789S	1.399S	16.229	1.4
8.75	20.836	1.795S	1.399S	16.236	1.5
9.00	20.910	1.800S	1.398S	16.242	1.5
9.25	20.981	1.805S	1.398S	16.249	1.6
9.50	21.049	1.810S	1.397S	16.255	1.6
9.75	21.114	1.814S	1.397S	16.262	1.6
10.00	21.176	1.818S	1.397S	16.268	1.7
25.00	22.770	1.923S	1.370S	16.218	1.4
50.00	23.370	1.961S	1.358S	16.178	1.1
100.00	23.680	1.980S	1.351S	16.154	1.0
∞	24.000	2.000S	1.343S	16.120	0.8

IV. Experimentation

Apparatus

The AFIT Oil Flow Rig Set-Up, shown in Figure 5, was used to determine Re_c , L^* , and C , for a circular, square, and concentric annular duct. Oil is pumped from the reservoir, up to the duct entrance, down the duct, and either into a weighing tank or back into the reservoir, depending on the selector valve setting. A mercury manometer board displays individual pressures for 10 static pressure stations along the duct.

SAE 10W-10 oil is the working fluid for this experiment. To determine Re_c , L^* , and C , the density (ρ) and the viscosity (μ) of the oil are needed. The density of the oil is determined by,

$$\rho_{oil} = \rho_{H_2O}(\text{specific gravity of oil})$$

The density of water is 62.4 lbm/ft^3 , and the specific gravity of oil is the ratio of the mass of a given volume of oil to that of an equal volume of water. Six measurements were taken with an average specific gravity, for this oil, of .863 (see Appendix B.). Thus, the density of the oil is 53.85 lbm/ft^3 .

The viscosity of the oil can be determined by using the Hagen-Poiseuille law (20:114), or by use of a viscometer. Lieutenant J. C. Ghiglieri (29), determined ν to be:

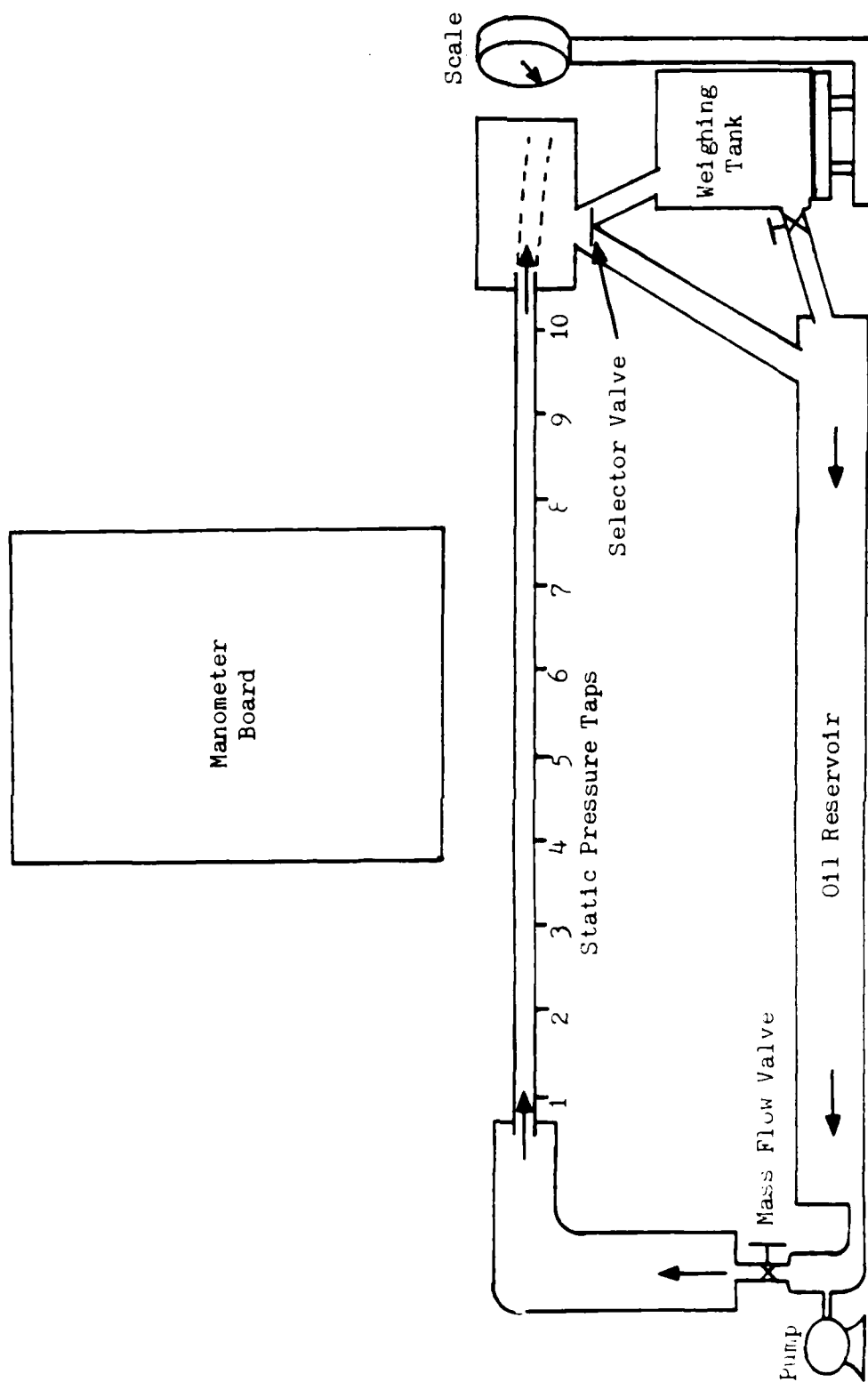


Figure 5. Oil Flow Rig Set-up

100 F - 10.18 centistokes

210 F - 2.53 centistokes

Using a Viscosity-Temperature chart, like that shown in Figure 6, these values can be linearized and μ at other temperatures extrapolated. Table VI shows some extrapolated values. These values were verified from the circular duct data using the Hagen-Poiseuille law,

$$dP/dx = (128 \mu Q) / (\pi D^4)$$

where,

$$Q = \dot{m} / \rho = AV$$

Circular Duct

Figure 7 shows a diagram of the circular duct that was used for this experiment. The duct is 21 feet long, with an inside diameter of 0.8125 inches. Ten static pressure taps are mounted along the duct with the first tap 8 inches from the entrance, and the remaining 9 taps located every 2 feet downstream. The cross-sectional flow area is 0.5185 square inches.

Square Duct

The square duct, shown in Figure 8, is 21 feet long, with an inside width of 0.75 inches. Pressure taps are mounted identical to those of the circular duct. The cross-sectional flow area for this duct is 0.5625 square inches.

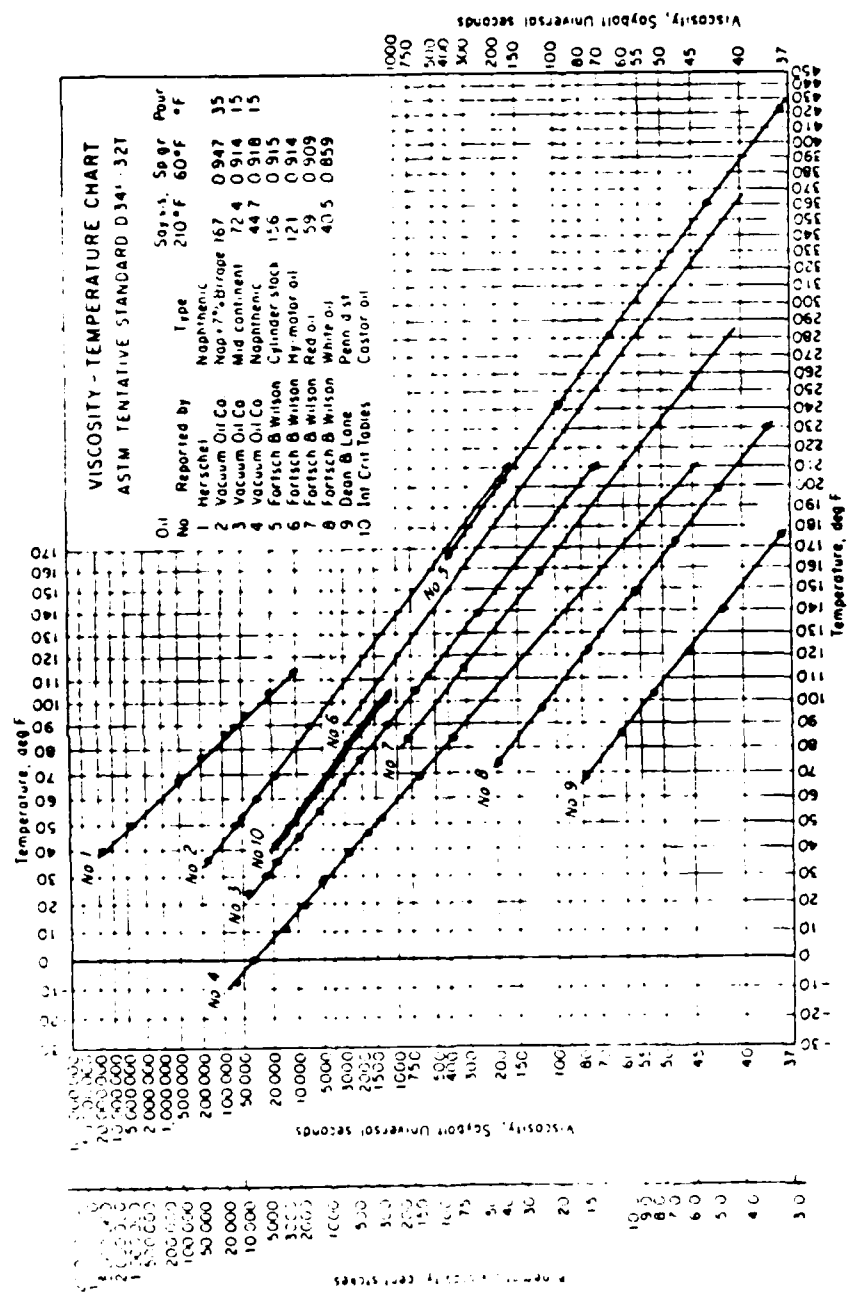
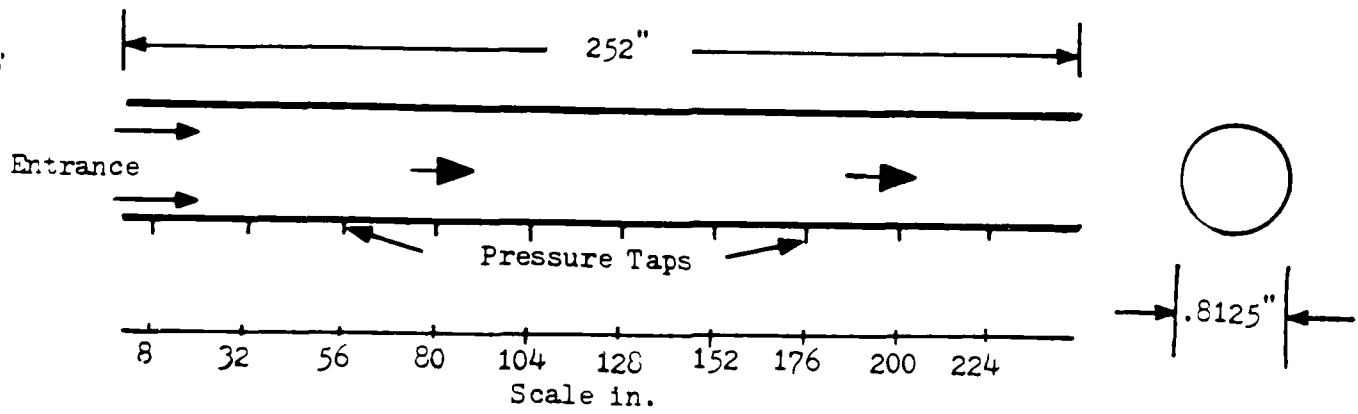


Figure 6. Viscosity-Temperature Chart Showing Typical Experimental Curves

TABLE VI.

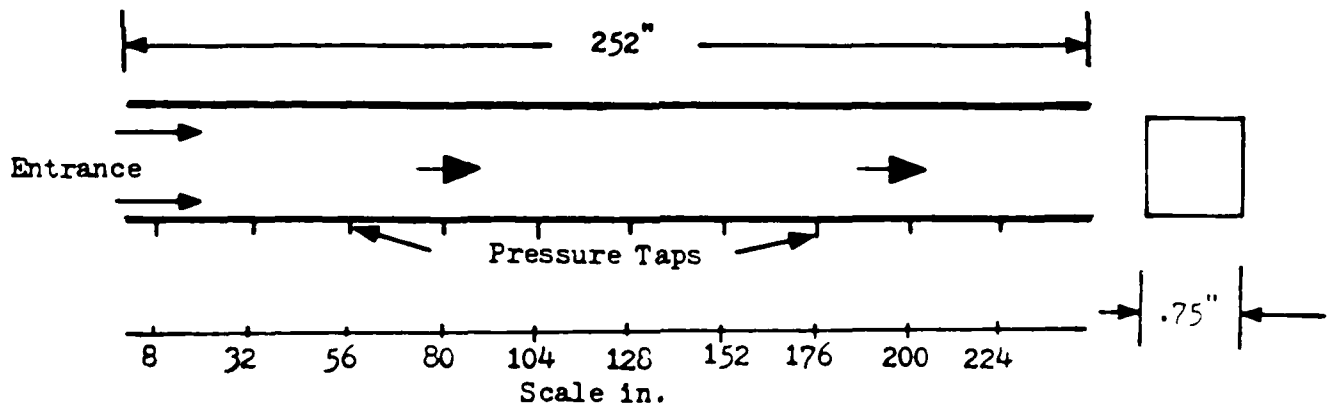
AFIT SAE 10W10 Oil Viscosity

<u>Temp.-F</u>	<u>ν - Centistokes</u>	<u>μ - (lbf-sec)/ft²</u>
70	18.80	3.39×10^{-4}
75	17.50	3.15
80	15.75	2.84
85	13.92	2.51
90	12.86	2.32
95	12.07	2.18
100	10.18	1.83
105	9.50	1.71
110	9.00	1.62
115	8.20	1.48
120	7.50	1.35
125	6.90	1.24



$$A = .5185 \text{ in.}^2$$

Figure 7. Circular Duct Diagram



$$A = .5625 \text{ in.}^2$$

Figure 8. Square Duct Diagram

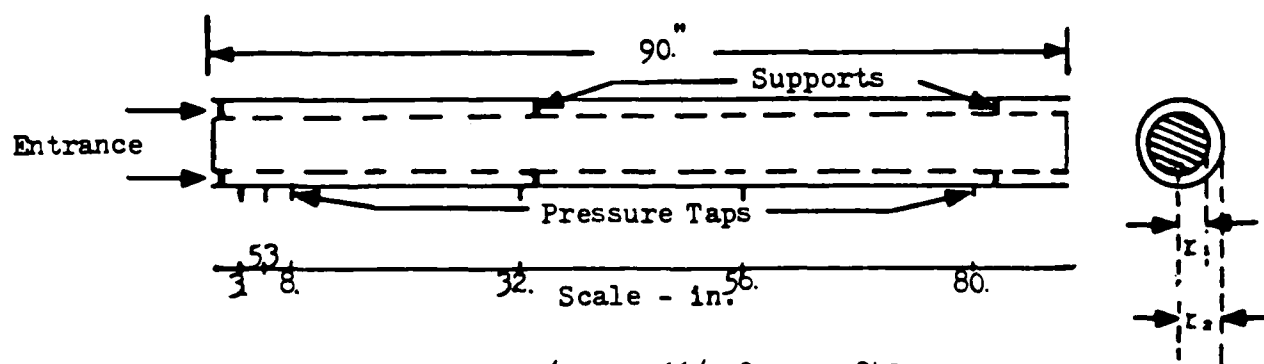


Figure 9. Concentric Annulus Diagram

Concentric Annulus

The concentric annulus, shown in Figure 9, consists of two ducts, one mounted within the other, with $r_1/r_2 = .8435$. The center duct is blocked at both ends to allow flow between the two ducts only. The concentric annulus was designed to simulate infinite parallel plates flow, which represents the upper bound of C, Re . Figure 10 shows that infinite parallel plates can be simulated with concentric annuli having r_1/r_2 greater than approximately 0.7 (3:286). This duct is 7.5 feet long with a flow height of .1225 inches. This results in a cross-sectional flow area of .555 square inches. Six pressure taps are mounted on the duct with the first three taps located 3.0, 5.3, and 8.0 inches, respectively, downstream of the entrance. The remaining three taps are located every two feet downstream from tap number 3. Since the entrance region develops

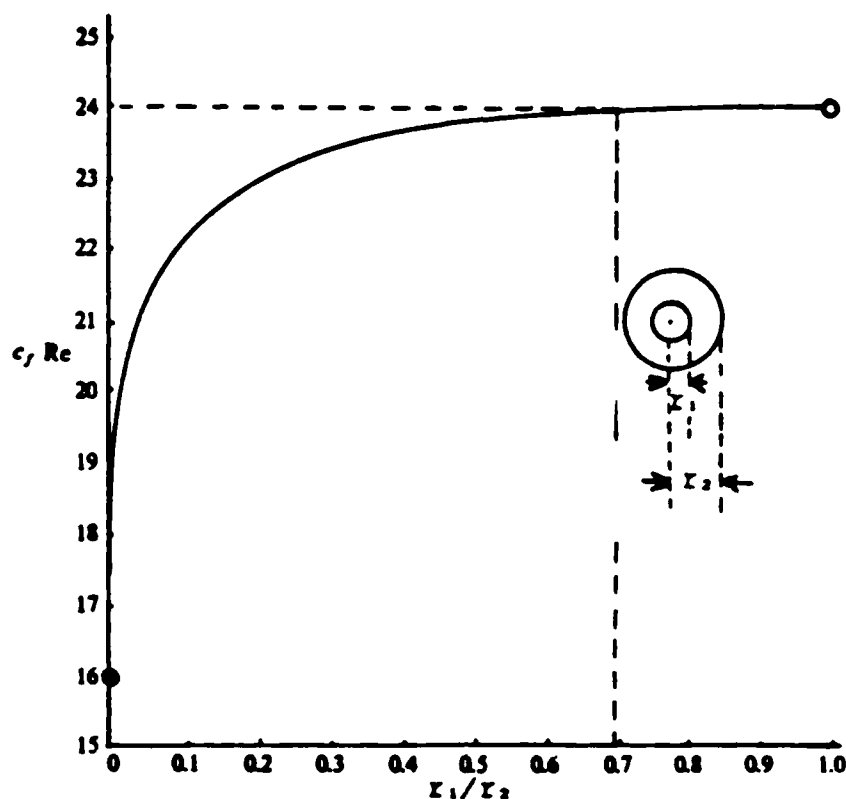


Figure 10. Friction Factors for Concentric Annuli

quickly for an infinite parallel plate duct, the first three pressure taps are located as close to the entrance as possible.

Procedure

The AFIT Oil Flow Rig allows measurement of static pressure drop along a duct, and the average mass flow rate through that duct. With these measurements, and the oil properties, $C_f Re$, Re_{crit} , and L^* can be determined. The static pressure for each measuring station along the duct

is presented on the mercury manometer board. The weighing tank is used to measure the mass flow of oil for a specific time period. By dividing the total mass weighed by the elapsed time, a mass flow rate can be determined.

$$\dot{m} = \text{lbm oil/time}$$

For friction factor measurements, steady, laminar flow is established in the duct, and static pressure is recorded for each of the 10 stations, from the manometer board.

dP/dx is determined from the fully developed (linear) portion of the pressure curve (see Figure 11). The average velocity (V), is determined from mass flow rate data where,

$$V = \dot{m}/\rho A$$

Friction factors are then determined from,

$$4C_f = (-dP/dx)(2gD/\rho V^2)$$

Transition from laminar to turbulent flow was determined visually at the duct exit, with the aid of a strobotac. Once laminar flow is established in the duct, the flow velocity was increased until turbulent eddies become noticable at the duct exit. The strobotac is a significant aid for this, especially with the concentric annular duct, where exit flow interaction occurs shortly past the duct exit. Once this transition point is located, the selector valve is turned to allow oil to flow into the weighing tank. The mass flow rate is then determined, and Re_{cr} calculated as,

$$Re_{cr} = \dot{m}D/A\mu$$

The hydrodynamic entrance length was determined graphically from the pressure vs. position plots for each duct. In fully developed, laminar flow, dP/dx is constant. In the hydrodynamic entrance region, where the velocity profile is still developing, the pressure drop is increasingly greater toward the entrance and thus, non-linear. Where the non-linear portion of the pressure drop curve becomes linear defines X (see Figure 11), and knowing the duct D_h , or $D_{h,1}$, and Re , L^* can be determined.

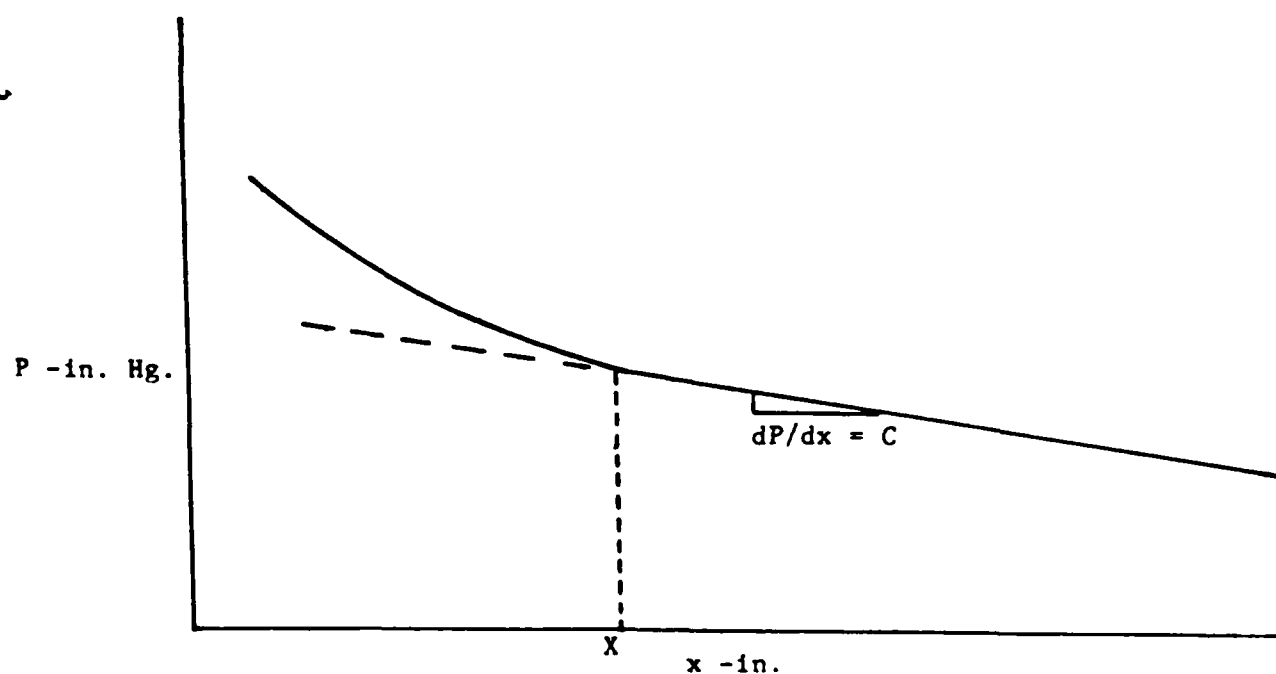


Figure 11. Pressure Drop for Fully Developed, Laminar Flow

V. Results and Discussion

Friction Factor

Friction factors for each duct can be determined from the pressure drop in the fully developed, laminar flow region. Friction factor data was used to verify exact solutions, based on D_h . Friction factors were determined from,

$$4C_f = (-dP/dx)(2gD/\rho V^2)$$

where D is D_h or $D_{h,1}$, depending on the application.

Friction factor data from the circular, square, and concentric annular ducts are compared to the exact solutions, based on D_h , in Figures 11, 12, and 13, respectively. Pressure and mass flow data are presented in Appendix C, for each duct. The experimental C_f 's agree very well with the exact solutions for all three ducts. This is no surprise, since experimental C_f data that verifies the exact solutions is abundant in the literature. More importantly, however, this data provides a high degree of confidence in the data taking and reduction techniques, which carries over to the Re_c and L^* investigations. Since the C_f values, based on D_h , match the analytic exact solutions, by definition, C_f , based on $D_{h,1}$, for the square and concentric annular duct will equal the circular duct value (within the $D_{h,1}$ limitations discussed in section III).

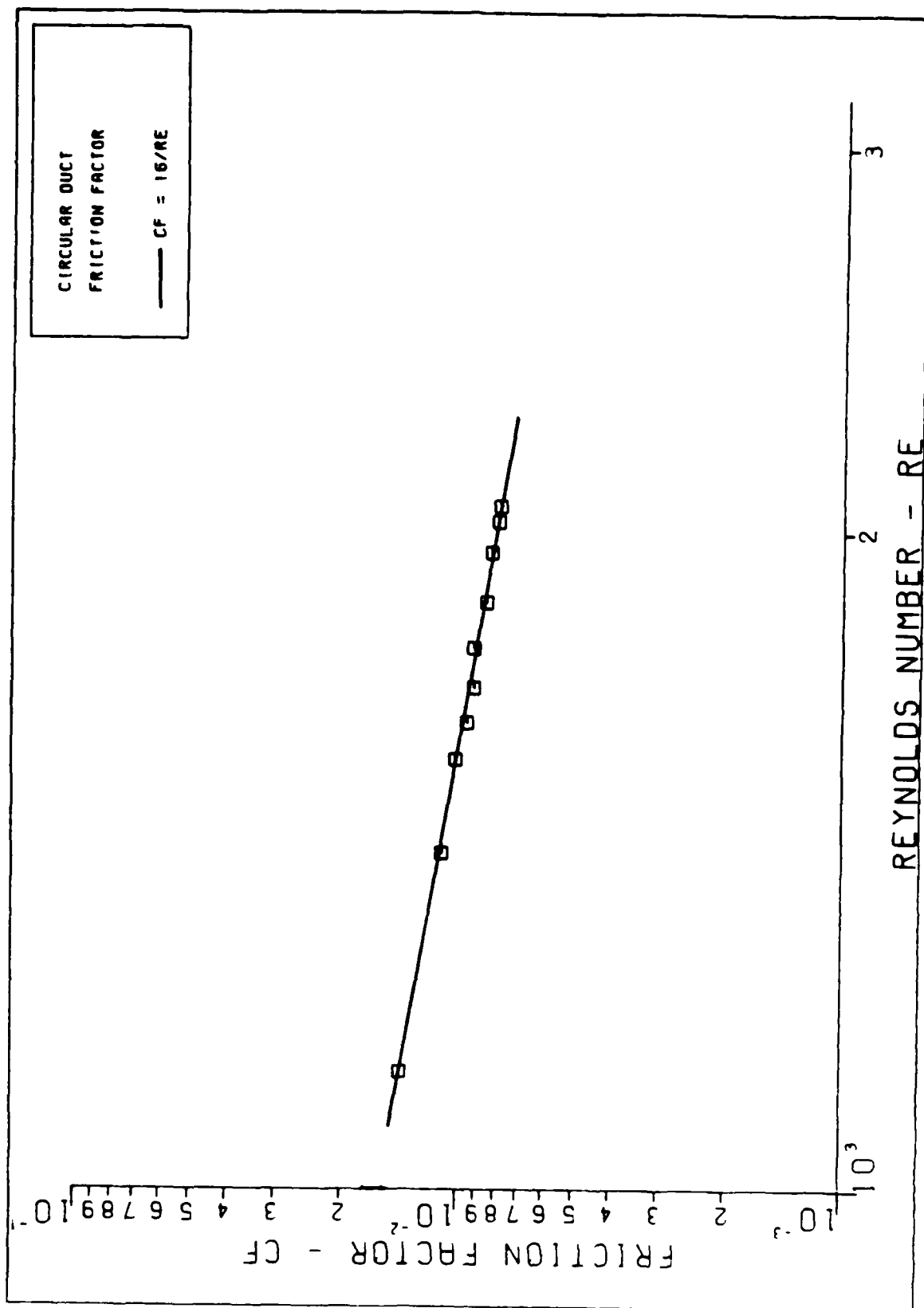


Figure 12. Circular Duct Friction Factors for Fully Developed, Laminar Flow

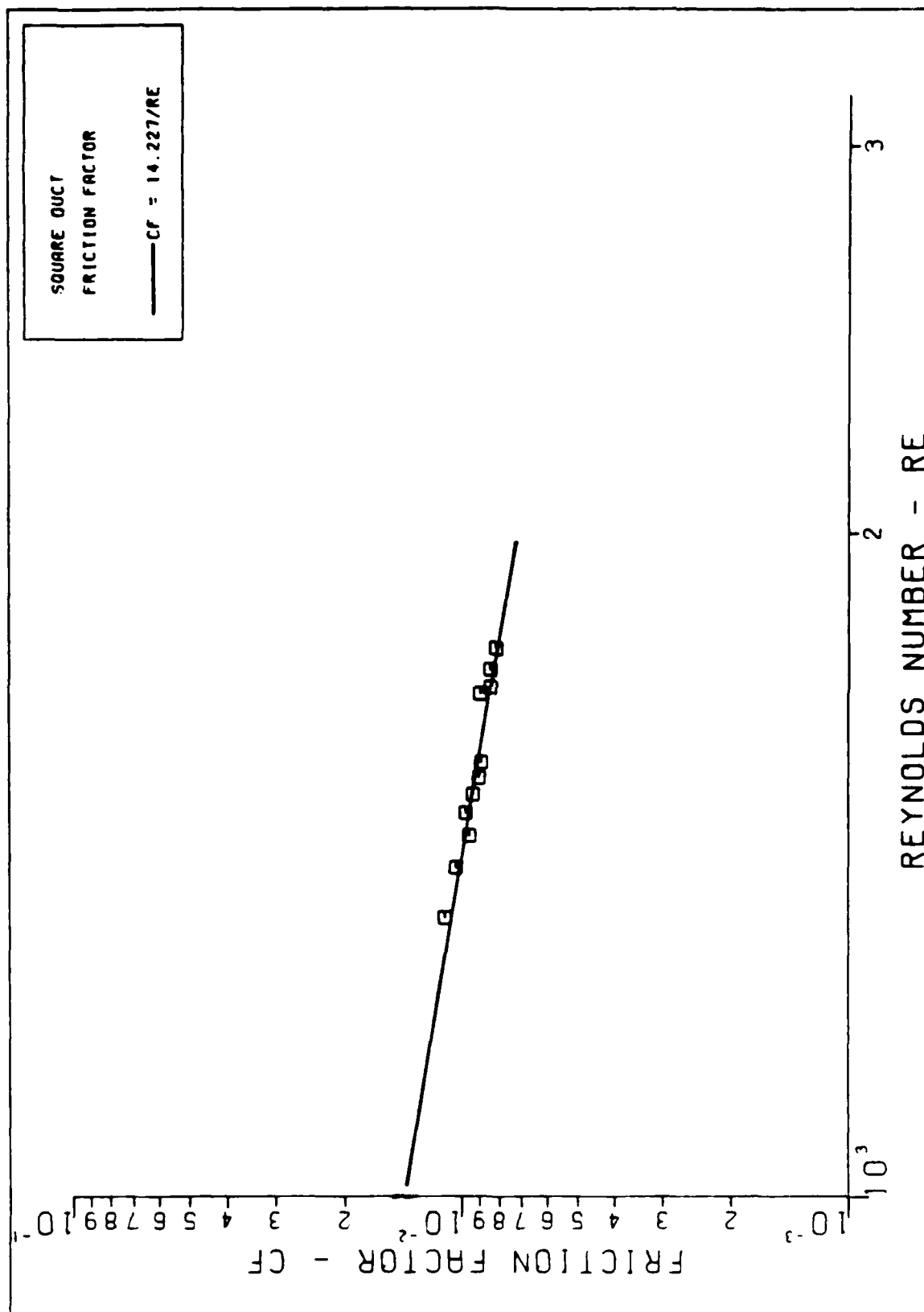


Figure 13. Square Duct Friction Factors for Fully Developed, Laminar Flow

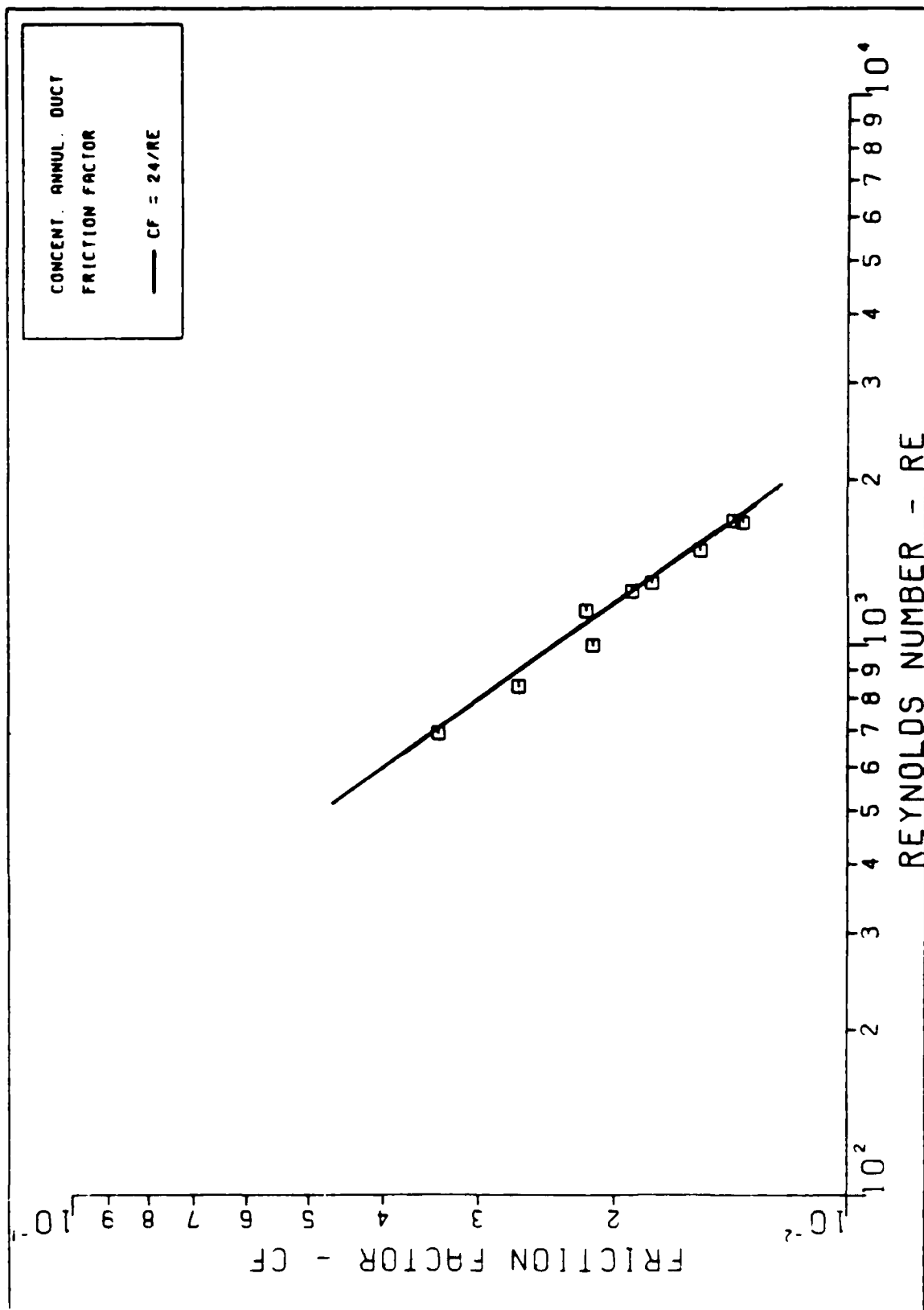


Figure 14. Concentric Annular Friction Factors for Fully Developed, Laminar Flow

Transition Reynolds Number

Transition from laminar to turbulent flow can be predicted, or correlated, by a transition Reynolds number, Re_{tr} . The value for Re_{tr} for circular ducts is approximately 2300. This is only under normal conditions, however. If disturbances in the duct entrance region, as well as the approach to it, are minimized, Re_{tr} can be increased to significantly higher values, the upper limit of which has not been established. V.W. Ekman, for example, reached a Re_{tr} of 40,000 with an exceptionally disturbance free entrance (15). The lower bound is approximately 2000, below which laminar flow is maintained, even with very strong disturbances present (2:433)

This author has found no data on Re_{tr} for non-circular ducts. If the Re_{tr} , based on D_h , is, in fact, dissimilar for different duct shapes, a desirable trait of D_h would be to correlate the Re_{tr} for all duct shapes to the circular duct value of 2000.

The three ducts each have abrupt entrances which induce a large disturbance at the entrance. With such a disturbance, transition should take place at the minimum Re_{tr} for each duct. For example, historical data indicates that this will occur at $Re_{tr} = 2000$ for a circular duct. The abrupt entrance should eliminate the Re_{tr} dependence on duct roughness.

Transition Reynolds number, based on $D_{h,1}$, is defined as:

$$Re_{t,1} = (\rho V D_h / \mu) (D_{h,1} / D_h)$$

Transition Reynolds number were first calculated based on D_h to compare to the circular duct $Re_{t,1}$. This was done because this author found no data on $Re_{t,1}$ for non-circular ducts. The $D_{h,1}$ correction was then applied to try to correlate the data to the circular duct value. For these calculations, μ was taken from Table VI for all three ducts. This was done after the extrapolated values from Table VI were verified from the circular duct data, using the Hagen-Poiseuille law (see Appendix D).

Ten to 15 data runs at transition were made for each of the three ducts, with the average $Re_{t,1}$, based on both D_h and $D_{h,1}$, shown in Table VII. Individual $Re_{t,1}$ values can be found in Appendix C. The circular duct $Re_{t,1}$ was 2028, which matches the historical value extremely well. The square and concentric annular ducts had $Re_{t,1}$ s, based on D_h , approximately 12.5% below the circular duct $Re_{t,1}$. What is interesting to note here is that the square and concentric annular ducts, each have similar $Re_{t,1}$, even though they are significantly different duct shapes. Recall, these two ducts were chosen because they represented two extremes when considering C_{Re} , based on D_h . The new hydraulic diameter appears to work well correlating the square duct

TABLE VII

Transition Reynolds Numbers

<u>Duct</u>	<u>Re_{tr} - D_n</u>	<u>Re_{tr} - D_{n,i}</u>
Circle	2028	2028
Square	1750	1925
Concentric Annulus	1733	1165

Re_{tr} to that of the circular duct. However, for the concentric annular duct, the Re_{tr} diverges from the circular duct value. This data indicates that Re_{tr}, based on D_n, remains the best choice.

Hydrodynamic Entrance Length

Hydrodynamic entrance length is defined as the duct length required to achieve a duct centerline velocity 99% of the fully developed value. The non-dimensional form is

$$L^* = (X/D)/Re$$

where D is either D_h , or $D_{h,i}$, depending on the application. Hydrodynamic entrance length is important in that it represents a position where fully developed flow can be assumed. For a circular duct, Liu (16) calculated $L^* = .0541$, Heaton (17) predicted $L^* = .0575$, and Langhaar (18) calculated $L^* = .058$. For equilateral triangular ducts, Shah (19) reported $L^* = .0398$. Table VIII and Table IX present analytic L^* for rectangular and concentric annular ducts, respectively. From these data it is easily seen that L^* varies considerably from duct shape to shape. In fact, as Table VIII and Table IX show, authors often disagree on L^* for a particular duct geometry. Data in Table VIII and Table IX represent analytic results. Little experimental data exists to verify the most accurate analytic method. This demonstrates the need for a parameter such as $D_{h,i}$ which can correlate data for different duct shapes to the circular duct value.

For each of the three ducts, 10 to 15 laminar flow data runs were made, at various Re , to determine L^* (see Appendix E).

TABLE VIII.
Rectangular Duct L^* for Fully Developed Laminar Flow

$1/\alpha$	L^*		
	Wiginton Dalton (21)	Fleming Sparrow (22)	Han (23)
1	0.090	-	0.0752
0.750	-	-	0.0735
0.500	0.085	0.095	0.0660
0.250	0.075	-	0.0427
0.200	0.080	0.080	-
0.125	-	-	0.0227
0	-	-	0.0099

TABLE IX.

Concentric Annular Duct

 L^* for Fully Developed Laminar Flow

r_1/r_2	L^*					
	Liu (16)	Manohar (25)	Roy (26)	Heaton (17)	Coney (27)	Sparrow (28)
0	.0541	-	-	.0575	-	-
0.001	-	-	-	.0296	-	.0375
0.01	-	-	-	-	-	.0303
0.02	-	-	-	.0206	-	-
0.05	.0206	-	-	.0172	.0329	.0241
0.10	.0175	.0164	.0180	.0146	.0253	.0210
0.20	-	-	.0158	-	.0214	.0171
0.25	-	-	-	.0118	.0204	-
0.30	-	.0122	.0140	-	.0194	-
0.40	-	-	.0128	-	.0178	.0131
0.50	.0116	.0110	.0121	.0103	.0168	-
0.60	-	-	.0114	-	.0161	-
0.70	-	.0103	-	-	.0156	-
0.75	.0109	-	-	-	.0152	-
0.80	-	-	.0113	-	.0150	.0118
1.00	.0108	-	-	.0099	.0147	-

TABLE X

Experimental Hydrodynamic Entrance Length

Duct	$L^* - D_h$	$L^* - D_{h,1}$
Circle	.0574	.0574
Square	.0917	.0758
Concentric Annulus	- (.011 - Anal.)	- (.0163 - Anal.)

The experimental L^* values, based on D_h and $D_{h,1}$, are shown in Table X. The hydrodynamic entrance length for the circular and square ducts were easily determined from plots like those shown in Figures 15 and 16, respectively. The concentric annular duct, however, posed some problems. By examining Figure 17, it is seen that the pressure drop stays constant, for the concentric annular duct, from station 1 to 6, even at high Re . A hydrodynamic developing region is indeterminable from the data, thus making it impossible to graphically determine L^* . By examining the analytic L^* data from Table IX, a possible explanation is found. For $r_1/r_2 = .84$, L^* is approximately .011. For a Re of 1700, just before transition, X , for this duct, is approximately 4.6 inches. Recall the first two pressure taps were located at 3.0 and 5.3 inches downstream of the entrance. The flow is fully developed before it reaches the second pressure tap! To graphically determine L^* , with only the first pressure tap within the hydrodynamic

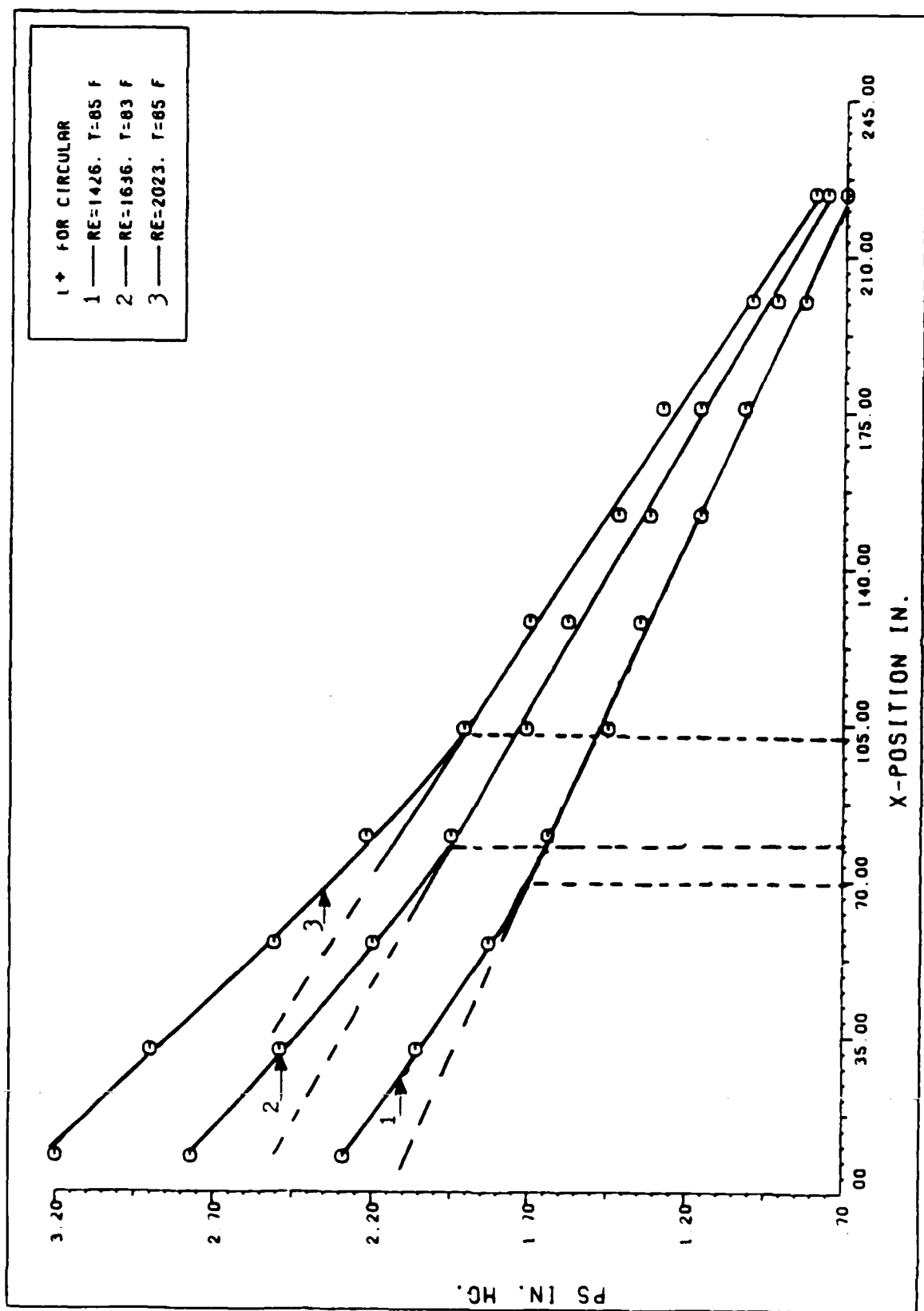


Figure 15. Hydrodynamic Entrance Length for Circular Duct

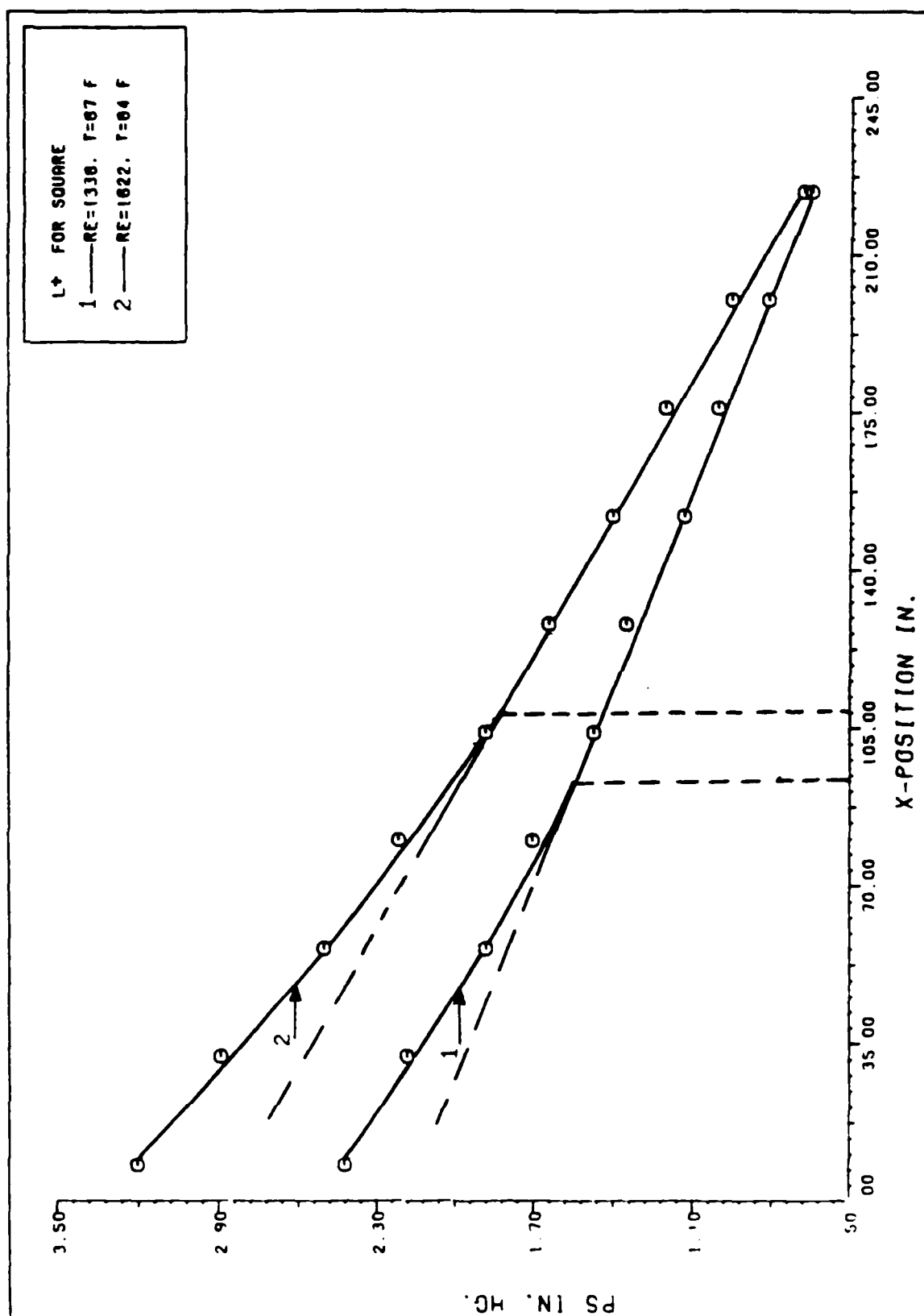


Figure 16. Hydrodynamic Entrance Length for Square Duct

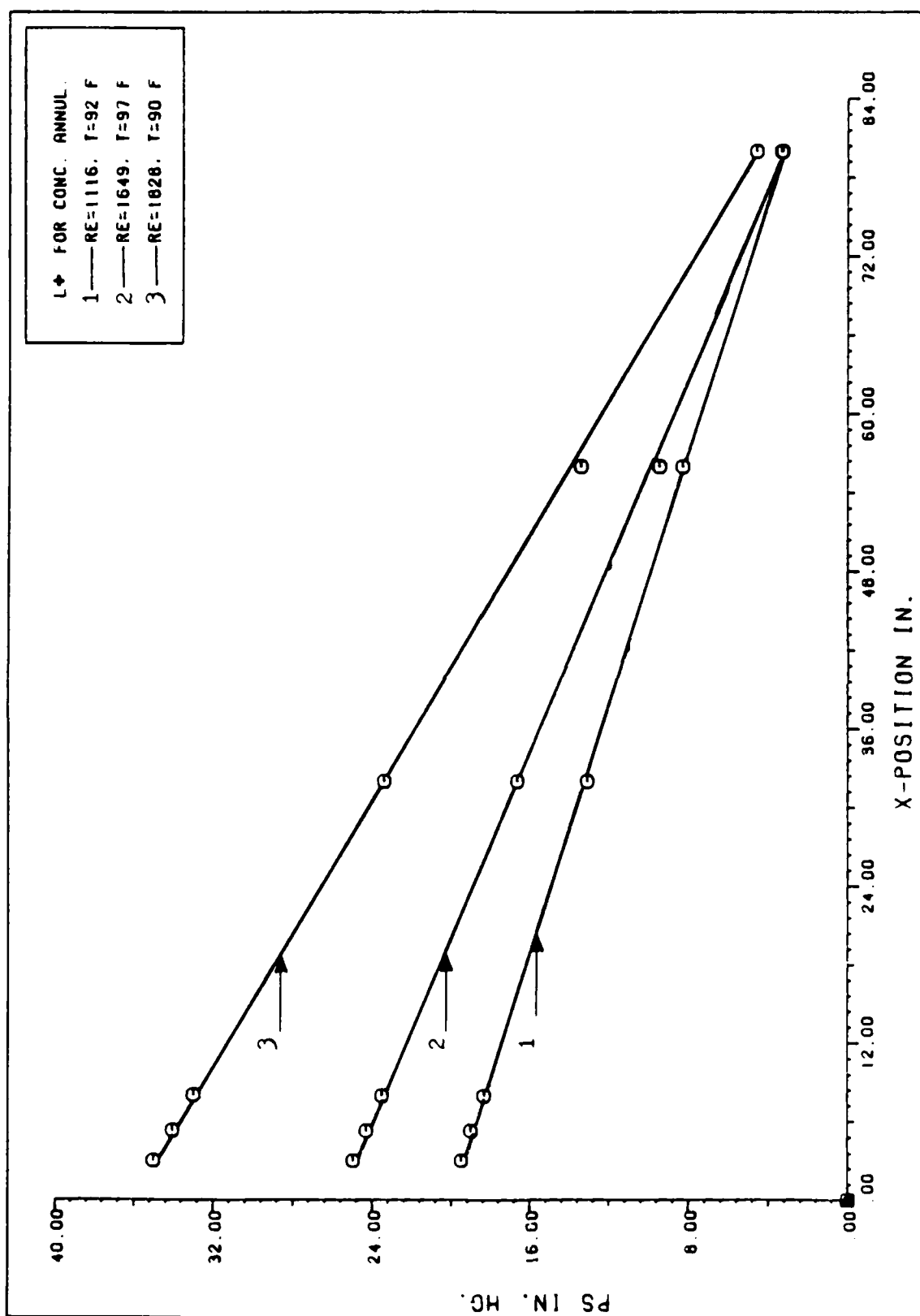


Figure 17. Hydrodynamic Entrance Length for Concentric Annular Duct

entrance region, and close to the end, at that, is virtually impossible. The first three pressure taps were located as close to the entrance as physically possible, so a L^* cannot be graphically determined for this particular duct.

The hydrodynamic entrance length, based on D_h , for the circular duct was 0.0574 which indicates that Heaton (17) and Langhaar (18) have good analytic techniques in predicting L^* . The square duct had a L^* of 0.0917. This value indicates that Wiginton and Dalton (21) have the best analytic technique for determining L^* for square ducts.

Using $D_{h,1}$ as the characteristic dimension instead of D_h , does not correlate the data to the circular duct value. The correlation trend, however, is favorable. The square duct L^* is reduced toward the circular duct value, while the concentric annular duct L^* is increased.

Since the correlation of L^* , based on $D_{h,1}$, produced unsatisfactory results, perhaps a different approach to the L^* problem is needed. Consider flow through a circular duct as shown in Figure 18. Here the boundary layer growth, δ , is generally described as,

$$\delta = C(\nu t)^{.5} \quad (2:83) \quad (10)$$

where C is a function of the pressure gradient, dP/dx , and the transverse velocity, v . We will let time, t , be equal to x/V , for this problem. (It should be noted that V is not the actual velocity that the boundary sees from

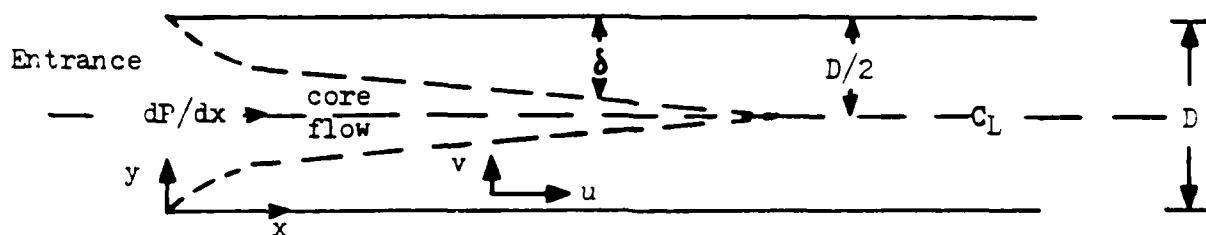


Figure 18. Boundary Layer Growth in the Entrance Region

the entrance to position X . The velocity actually varies, non-linearly, from V , at the entrance, to $2V$, at the centerline, for a circular duct, but we are using V here for convenience and to obtain "ballpark" expressions for L^* .) The hydrodynamic entrance length, X , becomes approximately the position where the boundary layer meets the duct centerline, $\delta = D/2$. Equation (10) becomes,

$$D/2 = C(\nu X/V)^{1/2}$$

which can be rewritten,

$$X/D = (VD/\nu)(1/4C^2) \quad (11)$$

where $X \propto D^2$.

$$L^* = (X/D)/Re = 1/4C^2 \quad (12)$$

For the circular duct, $L^* = .0574$, and C is approximately 2.1.

Now let us keep C constant at 2.1, and examine L^* for non-circular ducts, where δ becomes the average distance a disturbance must travel to reach the duct centerline. Let

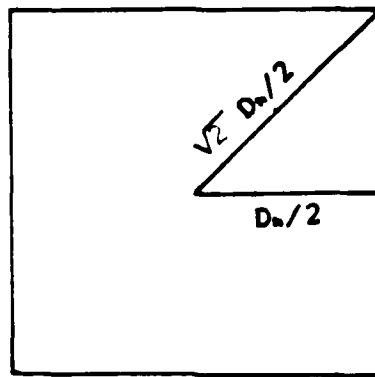


Figure 19. A Square Duct

us examine a square duct, shown in Figure 19, for instance, where, from the corners, a disturbance must travel $(2^{.5})(D_h/2)$ to reach the centerline. For non-circular ducts, X is proportional to D_h^2 , so,

$$L^* = (2)(.0574) = .1148$$

This would be the predicted L^* if all the disturbances in a square duct, travelled $(2^{.5})(D_h/2)$ to the center.

Obviously this is not the case. Those in the corners travel $(2^{.5})(D_h/2)$, but those on the sides only travel $(D_h/2)$ to the centerline. So, perhaps, the average distance a disturbance travels in a square duct is somewhere between these two values, say at $(1.5)^{.5}(D_h/2)$, for instance. If this is the case,

$$L^* = (1.5)(.0574) = .0861$$

This L^* is very close to our experimental value, $L^* = .0917$! Likewise, for an infinite parallel plate duct, the average disturbance travel distance is $.5(D_h/2)$, so,

$$L^* = .01435$$

This also is quite close to the analytically predicted L^* for the infinite parallel plate duct!

So, perhaps, a better way to correlate L^* data would be to scale the constant, C , with a new hydraulic diameter that estimates the average boundary layer growth to the centerline of non-circular ducts.

VI. Conclusions and Recommendations

Conclusions

In this investigation, a new hydraulic diameter, $D_{h,1}$, that better represented the mean center to surface distance for laminar flow in non-circular ducts was empirically determined. Experimentation was performed to examine the effect on Re_c and L^* of replacing the traditional hydraulic diameter, D_h , with $D_{h,1}$. The experiment was performed using a circular duct, square duct, and concentric annular duct that simulated flow between infinite parallel plates.

The new hydraulic diameter equation correlated C, Re data to the circular duct value for many non-circular duct shapes. All the shapes examined correlate within 2.4 percent of the circular duct value, with most within 1.5 percent.

Experimental Re_c were obtained for the circular, square, and concentric annular ducts. The circular duct Re_c , based on D_h , was very close to 2000 with the square and concentric annulus Re_c approximately 12.5 percent lower than this value. Square duct Re_c , based on $D_{h,1}$, correlated very close to the circular duct, but the concentric annulus Re_c did not. Concentric annulus Re_c actually diverged from the circular duct Re_c . Based on this data, it appears that $D_{h,1}$, which correlates C, Re data

very well, is not a good characteristic dimension for Re_{ν} . Perhaps an entirely different hydraulic diameter is needed if Re_{ν} is to be correlated to the circular duct value.

Hydrodynamic entrance lengths were determined for only the circular and square ducts. The concentric annular duct flow became fully developed before the second static pressure tap, making accurate prediction of L^* almost impossible. The analytic L^* value for the concentric annulus was thus used for $D_{h,1}$ correlations. Hydrodynamic entrance lengths, based on $D_{h,1}$, did not correlate well to the circular duct value. The correlation did, however, display the correct trend toward the circular duct value, and, perhaps, multiplication of $D_{h,1}$ by some constant might bring the L^* values in line.

The new hydraulic diameter is quite accurate in correlating $C_f Re$ data, but cannot be relied upon to correlate Re_{ν} and L^* data. Transition Reynolds numbers, based on D_h , were determined for the ducts, and will be useful, since no data for non-circular duct, Re_{ν} , has been reported. Likewise, experimental circular and square duct L^* values, based on D_h , will add to the limited data base available.

Recommendations

The new hydraulic diameter did not correlate Re_{ν} and L^* data. Separate hydraulic diameters are needed to correlate both Re_{ν} and L^* data. The L^* correlation

appears to require only a multiplier applied to the $D_{h,i}$ definition.

Instead of using $D_{h,i}$ directly in the L^* correlation, perhaps determining a $D_{h,i}$ that correlated the constant, C , in the boundary layer growth equation (Equation (10)), would yield better results, and should be investigated further.

Pressure gages, located close to the static pressure taps, should be installed, instead of relying on the 10-foot pressure lines that currently lead to the manometer board. This would provide better response indications to the pressure fluctuations which indicate the onset of transition. In addition, a static pressure tap should be added to each duct at the exit. This would aid in the detection of flow transition.

Static pressure taps should be installed very close to the entrance of the concentric annular duct to help in the L^* determination. This will require a major material overhaul.

Unusual duct shapes, such as triangular, hexagonal, or semi-circular, should be used in the investigation of C , Re , Re_{τ} , and L^* , to increase the experimental data base used in the $D_{h,i}$ correlations. It would be interesting to see where the data for these ducts fall, compared to that of the square and concentric annulus.

Appendix A.

New Hydraulic Diameter Calculations

Notation:

A = Duct area

P = Duct perimeter

Ac = Area of the modelling circle

Pc = Perimeter of the modelling circle

Ae = A - Ac

Pe = P + Pc

W = Width of infinite parallel plates (approaches infinity)

TABLE XI.

Compact Duct Hydraulic Diameters

<u>Shape</u>	<u>A</u>	<u>P</u>	<u>Ac</u>	<u>Pc</u>	<u>Ae</u>	<u>Pe</u>	<u>Ae/Pe</u>	<u>D_{h,1}</u>	<u>D_h</u>
Circle	.78S ²	3.14S	.78S ²	3.14S	.00S ²	6.28S	.000S	1.00S	1.00S
Square	1.0S ²	4.00S	.78S ²	3.14S	.21S ²	7.14S	.031S	1.10S	1.00S
Triang	.43S ²	3.00S	.26S ²	1.81S	.17S ²	4.81S	.036S	.693S	.577S
Hexag	2.6S ²	6.00S	2.4S ²	5.44S	.24S ²	11.4S	.021S	1.80S	1.73S
Semi-Circle	.40S ²	2.57S	.26S ²	1.80S	.14S ²	4.37S	.033S	.633S	.611S

Infinite A = WS P = 2W + 2S

Parallel

Plates Ac = .79WS Pc = 2W

Ae = .21WS Pe = 4W + 2S Ae/Pe = .21WS/(4W+2S)

As W goes to infinity, Ae/Pe reduces to .05365S

D_{h,1} = 1.34S D_h = 2S

TABLE XII.

Rectangular Hydraulic Diameters

<u>a</u>	<u>A</u>	<u>P</u>	<u>Ac</u>	<u>Pc</u>	<u>Ae</u>	<u>Pe</u>	<u>Ae/Pe</u>	<u>D_{h1}</u>	<u>D_h</u>
1.00	1.00S ²	4.0S	0.79S ²	3.0S	0.21S ²	7.0S	.031S	1.10S	1.00S
1.25	1.25S ²	4.5S	0.98S ²	3.4S	0.27S ²	7.9S	.034S	1.24S	1.11S
1.50	1.50S ²	5.0S	1.18S ²	3.9S	0.32S ²	8.9S	.036S	1.29S	1.20S
1.75	1.75S ²	5.5S	1.37S ²	4.3S	0.38S ²	9.8S	.038S	1.33S	1.27S
2.00	2.00S ²	6.0S	1.57S ²	4.8S	0.43S ²	10.8S	.040S	1.35S	1.33S
2.25	2.25S ²	6.5S	1.77S ²	5.3S	0.48S ²	11.8S	.041S	1.37S	1.38S
2.50	2.50S ²	7.0S	1.96S ²	5.7S	0.54S ²	12.7S	.042S	1.38S	1.43S
2.75	2.75S ²	7.5S	2.16S ²	6.2S	0.59S ²	13.7S	.043S	1.39S	1.47S
3.00	3.00S ²	8.0S	2.36S ²	6.7S	0.64S ²	14.7S	.044S	1.39S	1.50S
3.25	3.25S ²	8.5S	2.55S ²	7.2S	0.70S ²	15.7S	.044S	1.40S	1.53S
3.50	3.50S ²	9.0S	2.75S ²	7.7S	0.75S ²	16.7S	.045S	1.40S	1.56S
3.75	3.75S ²	9.5S	2.95S ²	8.2S	0.80S ²	17.7S	.045S	1.40S	1.58S
4.00	4.00S ²	10.0S	3.14S ²	8.7S	0.86S ²	18.7S	.046S	1.40S	1.60S
4.25	4.25S ²	10.5S	3.34S ²	9.2S	0.91S ²	19.7S	.046S	1.40S	1.62S
4.50	4.50S ²	11.0S	3.53S ²	9.7S	0.97S ²	20.7S	.047S	1.40S	1.64S
4.75	4.75S ²	11.5S	3.73S ²	10.2S	1.02S ²	21.7S	.047S	1.41S	1.65S
5.00	5.00S ²	12.0S	3.93S ²	10.7S	1.07S ²	22.7S	.047S	1.41S	1.67S
5.25	5.25S ²	12.5S	4.12S ²	11.2S	1.13S ²	23.7S	.048S	1.40S	1.68S
5.50	5.50S ²	13.0S	4.32S ²	11.6S	1.18S ²	24.6S	.048S	1.40S	1.69S
5.75	5.75S ²	13.5S	4.52S ²	12.1S	1.23S ²	25.6S	.048S	1.40S	1.70S
6.00	6.00S ²	14.0S	4.71S ²	12.6S	1.29S ²	26.6S	.048S	1.40S	1.71S
6.25	6.25S ²	14.5S	4.91S ²	13.1S	1.34S ²	27.6S	.049S	1.40S	1.72S
6.50	6.50S ²	15.0S	5.11S ²	13.6S	1.39S ²	28.6S	.049S	1.40S	1.73S
6.75	6.75S ²	15.5S	5.30S ²	14.0S	1.45S ²	29.5S	.049S	1.40S	1.74S
7.00	7.00S ²	16.0S	5.50S ²	14.5S	1.50S ²	30.5S	.049S	1.40S	1.75S
7.25	7.25S ²	16.5S	5.69S ²	15.0S	1.56S ²	31.5S	.049S	1.40S	1.76S
7.50	7.50S ²	17.0S	5.89S ²	15.5S	1.61S ²	32.5S	.050S	1.40S	1.76S
7.75	7.75S ²	17.5S	6.09S ²	15.9S	1.66S ²	33.4S	.050S	1.40S	1.77S
8.00	8.00S ²	18.0S	6.28S ²	16.4S	1.72S ²	34.4S	.050S	1.40S	1.78S
8.25	8.25S ²	18.5S	6.48S ²	16.8S	1.77S ²	35.3S	.050S	1.40S	1.78S
8.50	8.50S ²	19.0S	6.68S ²	17.3S	1.82S ²	36.3S	.050S	1.40S	1.79S
8.75	8.75S ²	19.5S	6.87S ²	17.8S	1.88S ²	37.3S	.050S	1.40S	1.79S
9.00	9.00S ²	20.0S	7.07S ²	18.2S	1.93S ²	38.2S	.051S	1.40S	1.80S
9.25	9.25S ²	20.5S	7.26S ²	18.7S	1.99S ²	39.2S	.051S	1.40S	1.80S
9.50	9.50S ²	21.0S	7.46S ²	19.1S	2.04S ²	40.1S	.051S	1.40S	1.81S
9.75	9.75S ²	21.5S	7.66S ²	19.6S	2.09S ²	41.1S	.051S	1.40S	1.81S
10.00	10.00S ²	22.0S	7.85S ²	20.0S	2.15S ²	42.0S	.051S	1.40S	1.82S
25.00	25.00S ²	52.0S	9.64S ²	50.0S	5.37S ²	102.0S	.053S	1.37S	1.92S
50.00	50.00S ²	102.0S	39.3S ²	100.0S	10.7S ²	202.0S	.053S	1.36S	1.96S
100.0	100.0S ²	202.0S	78.5S ²	200.0S	21.5S ²	402.0S	.053S	1.35S	1.98S

Appendix B.

Specific Gravity Measurements

TABLE XIII.

AFIT SAE 10W10 Oil Specific Gravity Measurements

<u>Measurement</u>	<u>Water (grams)</u>	<u>Oil (grams)</u>	<u>Sp. Gr.</u>
1	198.3	168.3	.849
2	377.0	325.6	.864
3	432.0	373.0	.8635
4	482.1	417.0	.8652
5	482.8	423.0	.876
6	541.2	466.4	<u>.862</u>
AVERAGE			.863

Appendix C.

Data Reduction

TABLE XIV.

Data for Circular Duct, Laminar Flow

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	3.20	2.77	2.29	1.92	2.14	2.24
P_2	2.83	2.49	2.06	1.78	1.92	2.00
P_3	2.53	2.20	1.83	1.60	1.72	1.78
P_4	2.25	1.95	1.64	1.45	1.50	1.55
P_5	1.97	1.71	1.45	1.25	1.35	1.39
P_6	1.78	1.58	1.35	1.19	1.28	1.29
P_7	1.48	1.32	1.16	1.01	1.09	1.10
P_8	1.28	1.16	1.02	0.97	0.98	0.99
P_9	1.02	0.92	0.83	0.78	0.80	0.80
P_{10}	0.76	0.74	0.70	0.69	0.70	0.70
Temp - F	81.0	83.0	85.0	87.5	92.0	93.0
$\dot{m}$ - lbm/sec	0.84	0.74	0.61	0.49	0.61	0.65
V - ft/sec	4.33	3.82	3.16	2.53	3.15	3.35
C_f	.0366	.0380	.0440	.0560	.0405	.0365
RE	1770	1636	1426	1132	1574	1698

TABLE XV.

Data for Circular Duct, Transition Flow
Data Set 1

	<u>Run Number</u>					
	1	2	3	4	5	6
P ₁ - in. Hg	3.20	2.89	2.90	2.80	2.51	2.30
P ₂	2.90	2.60	2.61	2.50	2.23	2.04
P ₃	2.51	2.27	2.29	2.22	2.00	1.81
P ₄	2.22	2.02	2.02	1.96	1.78	1.60
P ₅	1.91	1.75	1.73	1.73	1.56	1.40
P ₆	1.70	1.60	1.61	1.55	1.40	1.30
P ₇	1.42	1.32	1.34	1.30	1.10	1.11
P ₈	1.28	1.19	1.18	1.15	1.03	1.01
P ₉	1.00	0.91	0.93	0.92	0.86	0.83
P ₁₀	0.80	0.79	0.79	0.74	0.72	0.71
Temp - F	84.8	86.7	88.0	90.0	92.1	94.8
$\dot{m}$ - lbm/sec	0.88	0.82	0.81	0.80	0.79	0.74
V - ft/sec	4.51	4.23	4.15	4.13	4.05	3.83
C _r	.0316	.0330	.0339	.0324	.0307	.0295
RE	2023	1958	1962	2014	2027	1976

TABLE XVI.

Data for Circular Duct, Transition Flow
Data Set 2

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	4.00	3.90	3.40	3.22	3.12	2.15
P_2	3.56	3.50	3.04	2.91	2.85	1.95
P_3	3.15	3.05	2.66	2.57	2.50	1.75
P_4	2.83	2.70	2.40	2.20	2.17	1.58
P_5	2.41	2.32	2.05	1.91	1.85	1.39
P_6	2.14	2.00	1.82	1.75	1.70	1.28
P_7	1.78	1.67	1.55	1.45	1.46	1.10
P_8	1.47	1.43	1.32	1.25	1.2	1.00
P_9	1.15	1.11	1.04	1.00	1.00	0.82
P_{10}	0.85	0.81	0.80	0.80	0.73	0.70
Temp - F	81.0	82.5	85.5	89.0	91.0	96.5
$\dot{m}$ - lbm/sec	1.02	1.00	0.90	0.83	0.80	0.66
V - ft/sec	5.26	5.16	4.62	4.28	4.13	3.40
C_f	.0321	.0323	.0322	.0312	.0316	.0340
RE	2149	2183	2098	2056	2038	1858

TABLE XVII.

Data for Square Duct, Laminar Flow
Data Set 1

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	2.42	1.71	2.49	2.05	2.73	2.33
P_2	2.18	1.55	2.21	1.86	2.44	2.10
P_3	1.88	1.35	1.93	1.61	2.12	1.81
P_4	1.70	1.25	1.79	1.50	1.95	1.70
P_5	1.47	1.11	1.50	1.26	1.64	1.42
P_6	1.35	1.06	1.39	1.17	1.50	1.31
P_7	1.13	0.90	1.16	1.00	1.25	1.13
P_8	1.00	0.85	1.04	0.90	1.11	1.02
P_9	0.81	0.73	0.85	0.76	0.90	0.86
P_{10}	0.65	0.66	0.70	0.65	0.67	0.68
Temp - F	87.0	97.3	88.0	94.0	83.5	90.0
$\dot{m}$ - lbm/sec	0.66	0.52	0.70	0.65	0.74	0.68
V - ft/sec	3.11	2.47	3.33	3.07	3.52	3.23
C_f	.0441	.0378	.0381	.0343	.0412	.0381
RE	1338	1281	1453	1453	1410	1458

TABLE XVIII.

Data for Square Duct, Laminar Flow
Data Set 2

	<u>Run Number</u>		
	<u>1</u>	<u>2</u>	<u>3</u>
P ₁ -in. Hg	1.71	1.77	3.50
P ₂	1.55	1.60	3.12
P ₃	1.38	1.41	2.69
P ₄	1.31	1.30	2.39
P ₅	1.13	1.13	2.04
P ₆	1.09	1.08	1.81
P ₇	0.92	0.90	1.50
P ₈	0.86	0.87	1.30
P ₉	0.74	0.73	1.04
P ₁₀	0.62	0.62	0.72
Temp - F	100.	100.	78.0
$\dot{m}$ - lbm/sec	0.56	0.57	0.89
V - ft/sec	2.66	2.71	4.23
C _f	.0373	.0360	.0389
RE	1522	1549	1493

TABLE XIX.

Data for Square Duct, Transition Flow
Data Set 1

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	3.00	3.00	3.20	3.00	2.90	2.73
P_2	2.71	2.67	2.89	2.65	2.60	2.45
P_3	2.32	2.31	2.50	2.30	2.25	2.11
P_4	2.05	2.11	2.21	2.08	2.02	1.90
P_5	1.75	1.75	1.88	1.72	1.68	1.61
P_6	1.59	1.58	1.64	1.55	1.52	1.41
P_7	1.28	1.30	1.40	1.30	1.28	1.20
P_8	1.13	1.14	1.20	1.14	1.12	1.08
P_9	0.91	0.93	0.95	0.93	0.91	0.88
P_{10}	0.77	0.70	0.68	0.67	0.70	0.60
Temp - F	90.5	89.0	84.0	86.8	88.0	92.7
$\dot{m}$ - lbm/sec	0.83	0.84	0.84	0.83	0.82	0.80
V - ft/sec	3.96	3.99	3.99	3.95	3.90	3.80
C_f	.0286	.0337	.0397	.0356	.0336	.0325
RE	1797	1772	1622	1691	1702	1773

TABLE XX.

Data for Square Duct, Transition Flow
Data Set 2

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	4.16	3.31	2.94	2.49	2.08	2.16
P_2	3.70	2.95	2.63	2.24	1.87	1.94
P_3	3.25	2.60	2.23	1.97	1.63	1.72
P_4	2.88	2.30	2.04	1.77	1.50	1.58
P_5	2.40	1.90	1.72	1.49	1.25	1.31
P_6	2.19	1.72	1.56	1.33	1.18	1.20
P_7	1.70	1.45	1.28	1.14	1.00	1.05
P_8	1.47	1.21	1.13	1.02	0.91	0.91
P_9	1.13	0.98	0.90	0.85	0.76	0.78
P_{10}	0.80	0.70	0.65	0.65	0.62	0.65
Temp - F	80.0	86.0	90.0	95.0	100.	100.
$\dot{m}$ - lbm/sec	1.02	0.90	0.83	0.76	0.66	0.69
V - ft/sec	4.85	4.28	3.95	3.61	3.14	3.28
C_f	.0359	.0361	.0356	.0337	.0340	.0327
RE	1786	1811	1779	1734	1794	1875

TABLE XXI.

Data for Concentric Annular Duct, Laminar Flow
Data Set 1

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	28.58	30.24	19.48	23.18	20.94	24.70
P_2	27.80	29.41	19.00	22.60	20.44	24.09
P_3	26.61	28.18	18.32	21.80	19.75	23.40
P_4	19.23	20.26	13.11	15.50	14.15	16.60
P_5	11.12	11.50	8.30	8.30	7.99	9.27
P_6	3.79	3.97	2.23	3.29	2.90	3.36
Temp - F	90.0	92.0	91.8	93.5	96.3	88.0
$\dot{m}$ - lbm/sec	2.12	2.33	1.54	1.86	1.89	1.80
V - ft/sec	10.2	11.2	7.42	8.94	9.10	8.67
C_v	.0639	.0558	.0853	.0660	.0594	.0760
RE	1503	1693	1116	1373	1486	1236

TABLE XXII.

Data for Concentric Annular Duct, Laminar Flow
Data Set 2

	<u>Run Number</u>					
	1	2	3	4	5	6
P_1 - in. Hg	20.64	22.39	25.13	28.38	30.41	31.40
P_2	20.16	21.85	24.53	27.65	29.65	30.65
P_3	19.50	21.16	23.74	26.79	28.65	29.68
P_4	13.88	15.10	16.83	19.00	20.30	21.08
P_5	8.18	8.48	9.44	10.64	11.75	12.30
P_6	2.87	3.06	3.40	3.80	4.04	4.18
Temp - F	88.0	88.2	88.7	89.2	89.8	88.0
$\dot{m}$ - lbm/sec	1.56	1.70	1.92	2.14	2.31	2.28
V - ft/sec	7.52	8.19	9.25	10.3	11.1	11.0
μ	.0842	.0775	.0677	.0616	.0569	.0605
RE	1071	1170	1332	1500	1627	1565

TABLE XXIII.

Data for Concentric Annular Duct, Transition Flow
Data Set 1

	<u>Run Number</u>				
	1	2	3	4	5
P ₁ - in. Hg	34.10	33.30	31.35	30.02	28.85
P ₂	33.16	32.40	30.45	29.20	28.10
P ₃	31.96	31.25	29.38	28.18	27.05
P ₄	23.15	22.50	21.24	20.30	19.50
P ₅	13.08	12.56	12.22	11.70	11.28
P ₆	4.49	4.38	4.15	3.99	3.85
Temp - F	90.0	90.2	92.0	93.0	94.0
$\dot{m}$ - lbm/sec	2.54	2.49	2.42	2.37	2.33
V - ft/sec	12.2	12.0	11.7	10.8	11.2
C _r	.0538	.0543	.0544	.0606	.0536
RE	1801	1770	1758	1645	1736

TABLE XXIV.

Data for Concentric Annular Duct, Transition Flow
Data Set 2

	<u>Run Number</u>				
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
P ₁ - in. Hg	27.95	24.96	34.00	33.20	35.00
P ₂	27.20	24.30	33.06	32.30	34.05
P ₃	26.28	23.51	31.96	31.30	33.00
P ₄	18.20	16.66	22.73	22.20	23.42
P ₅	10.70	9.46	13.05	12.90	13.43
P ₆	3.70	3.35	4.46	4.40	4.60
Temp - F	95.0	96.5	90.0	88.3	89.5
$\dot{m}$ - lbm/sec	2.20	2.08	2.55	2.43	2.60
V - ft/sec	10.6	10.0	12.3	11.7	12.5
C _r	.0557	.0572	.0523	.0565	.0518
RE	1660	1649	1808	1676	1828

Appendix D

Viscosity Comparison

This appendix compares viscosity calculated using the Hagen-Poiseuille Law verses extrapolated values from Table VI. The Hagen-Poiseuille Law is used with circular duct flow data at various temperatures. Table VI values are extrapolated from two data points using a Viscosity-Temperature chart.

TABLE XXV.

Viscosity Comparison

<u>Re</u>	<u>dP/dx</u> <u>in.Hg/ft</u>	<u>V</u> <u>ft/sec</u>	<u>μ - H-P calcs</u> <u>lbf-sec/ft²</u>	<u>μ - Table VI</u> <u>lbf-sec/ft²</u>
1132	.0625	2.527	2.500x10 ⁻⁴	2.420x10 ⁻⁴
1426	.0770	3.162	2.470	2.510
1574	.0700	3.146	2.254	2.264
1636	.1000	3.817	2.654	2.642
1698	.0700	3.352	2.120	2.240

Since the extrapolated values from Table VI predict those calculated, using the Hagen-Poiseuille Law, they will be used in all other calculations, for the sake of convenience.

Appendix E.

Hydrodynamic Entrance Length Plots

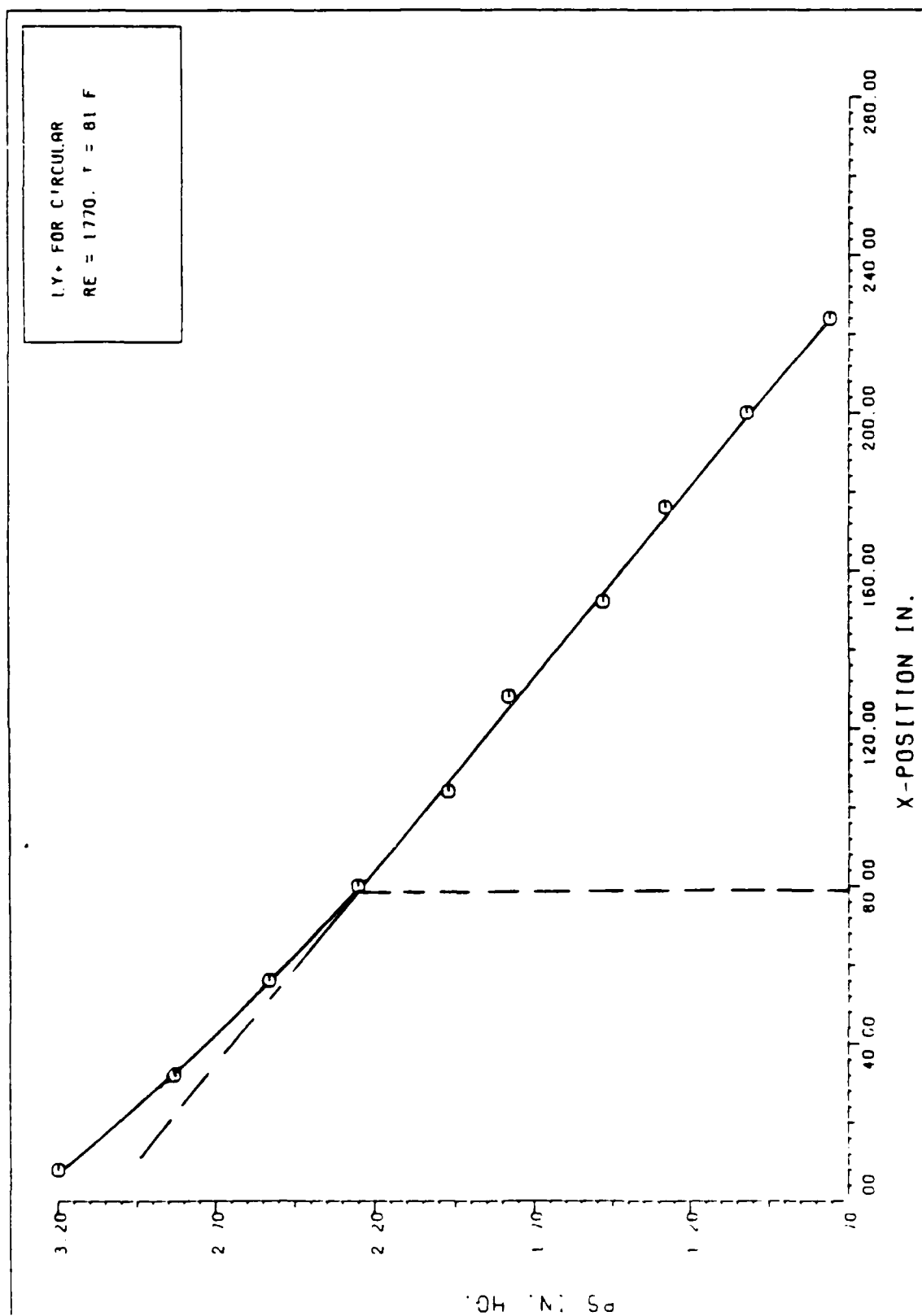


Figure 20. Hydrodynamic Entrance Length for Circular Duct, $Re = 1770$.

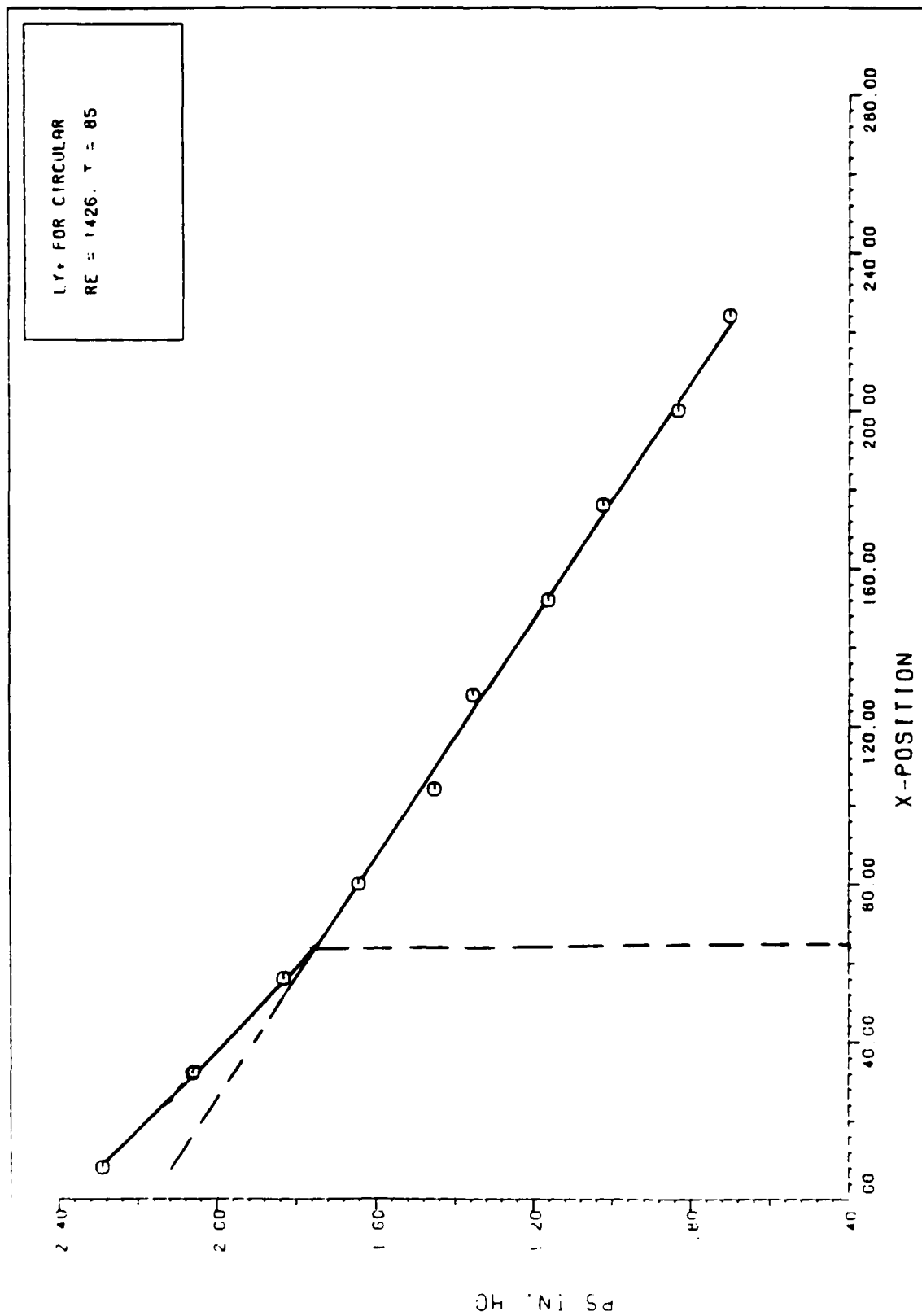


Figure 21. Hydrodynamic Entrance Length for Circular Duct, $Re = 1426$.

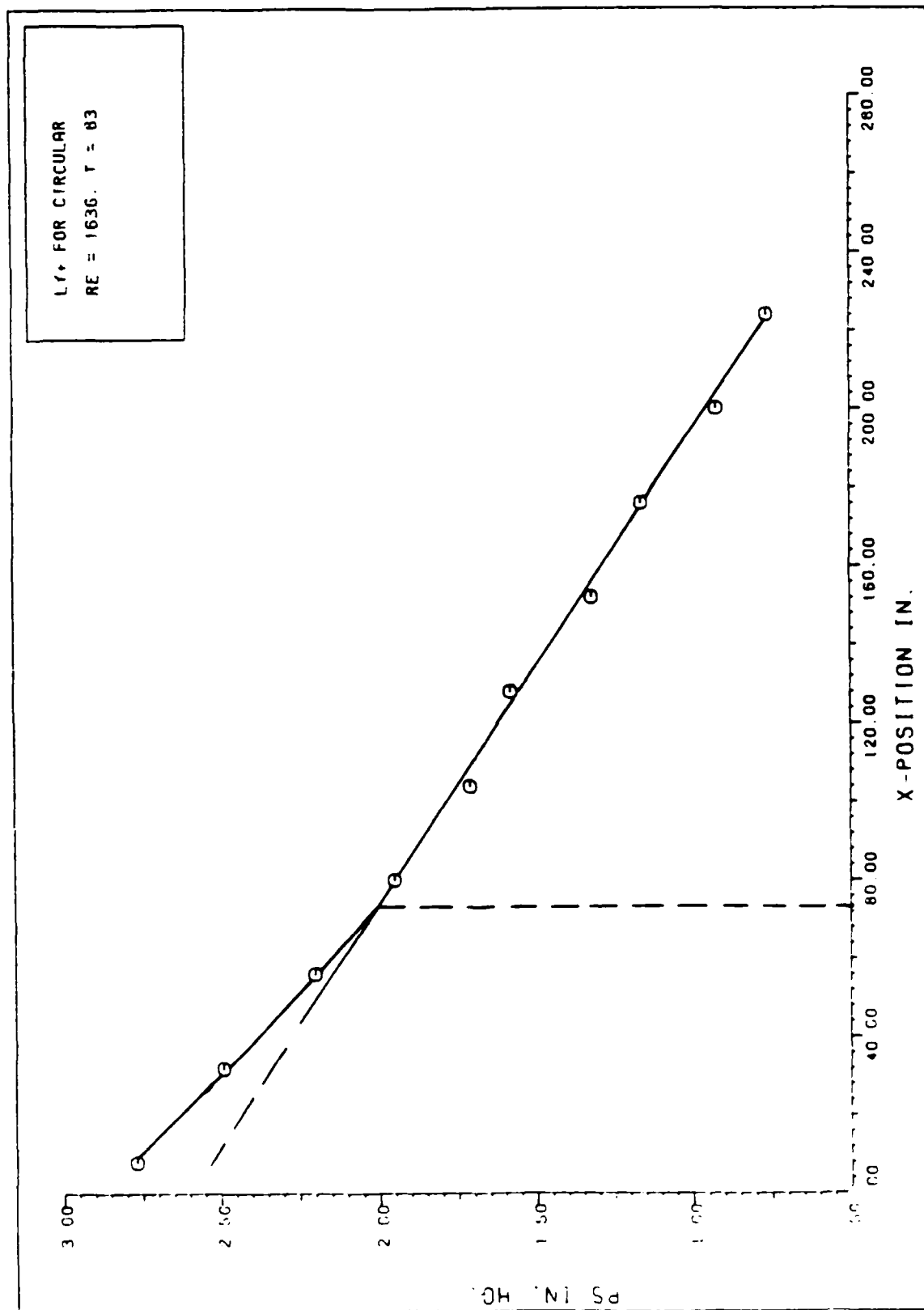


Figure 22. Hydrodynamic Entrance Length for Circular Duct, $Re = 1636$.

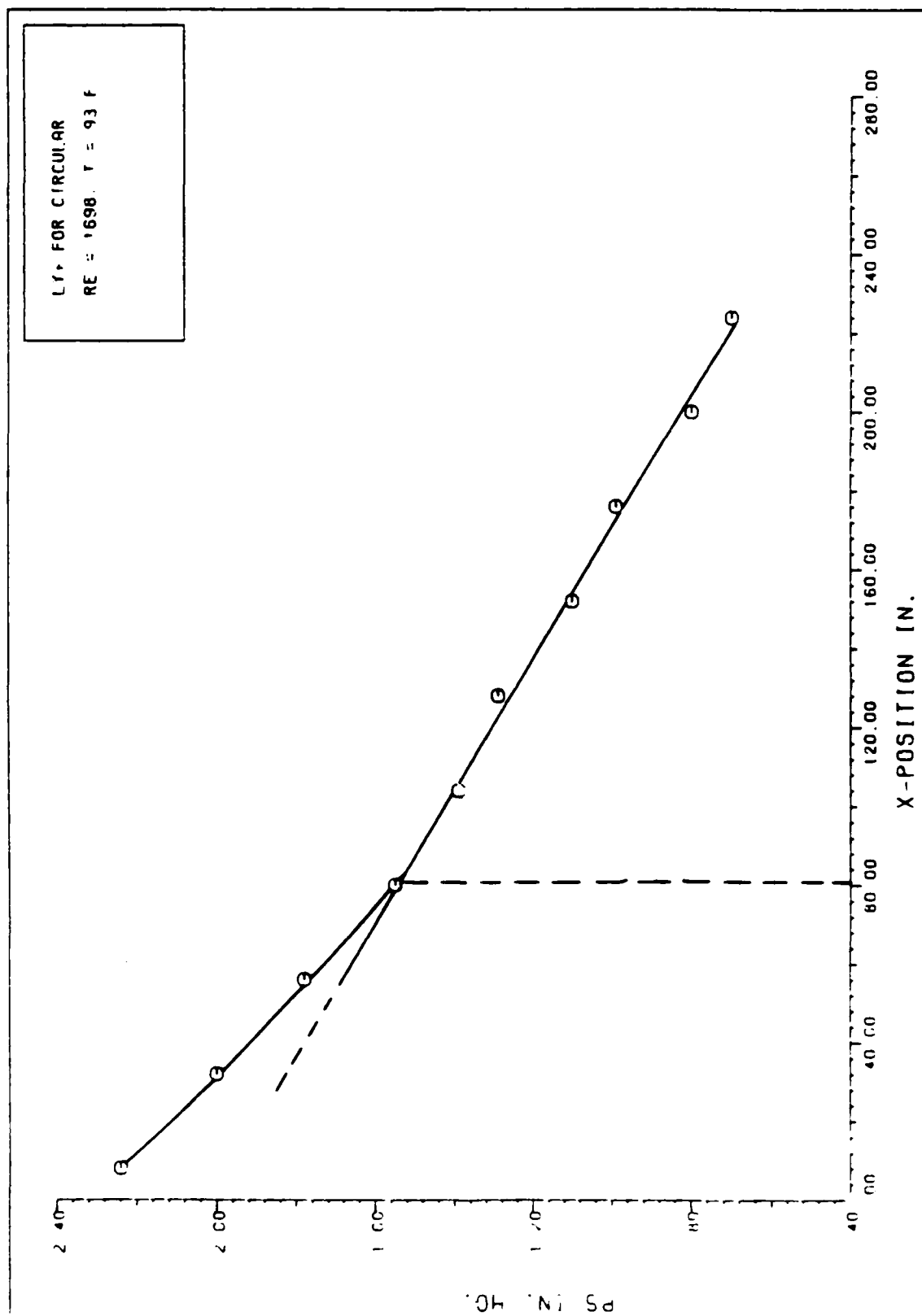


Figure 23. Hydrodynamic Entrance Length for Circular Duct, $Re = 1698$.

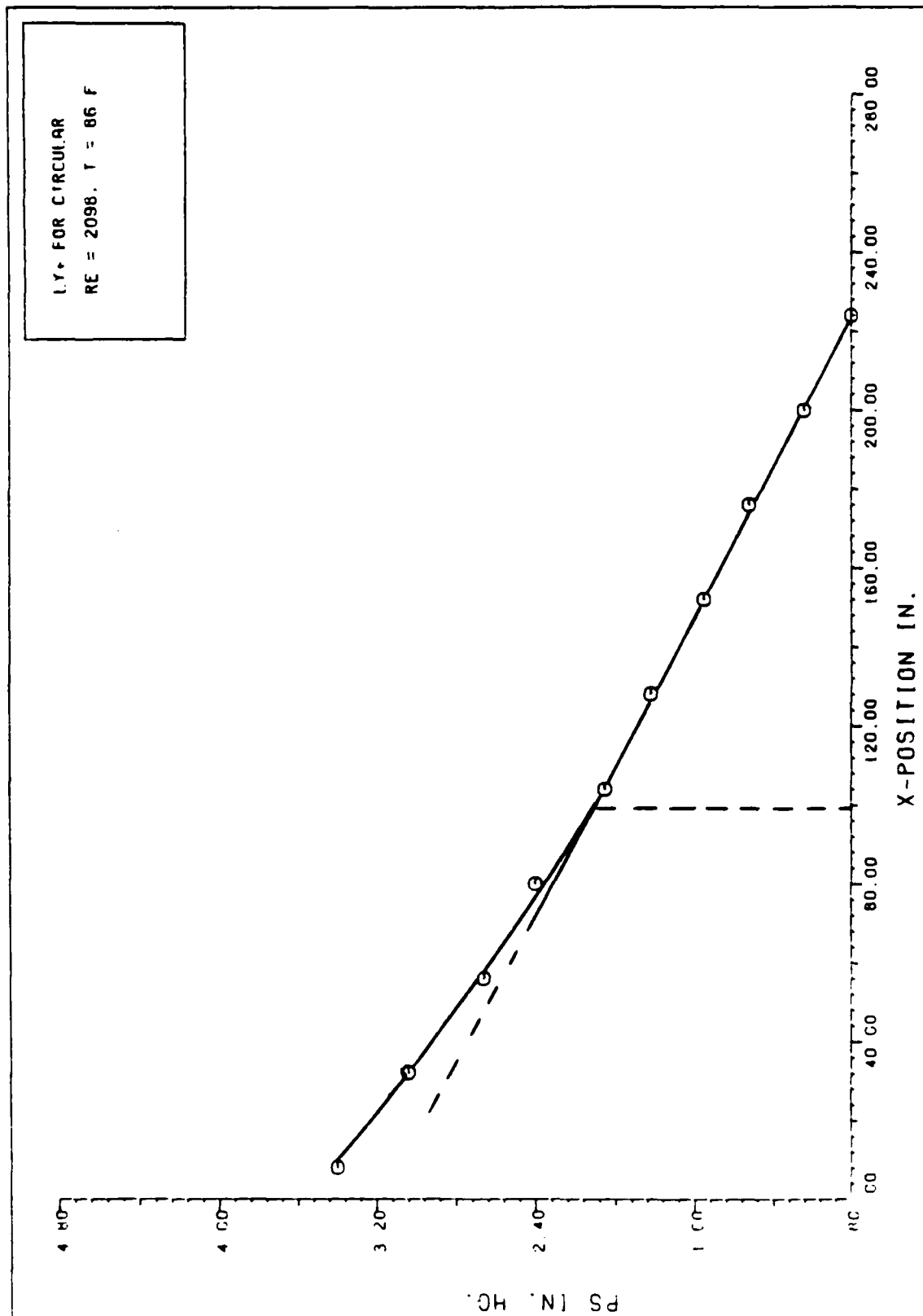


Figure 24. Hydrodynamic Entrance Length for Circular Duct, $Re = 2098$.

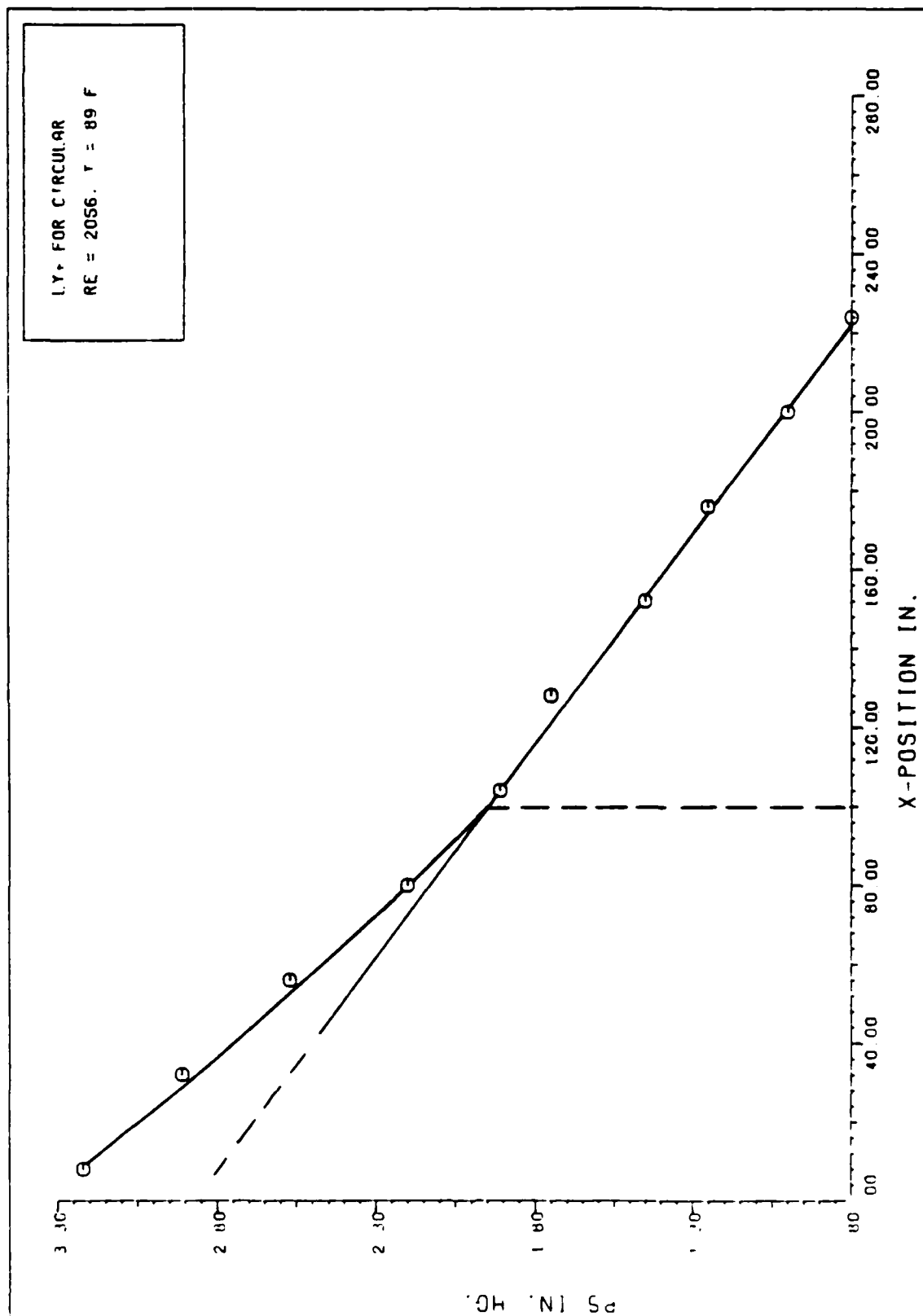


Figure 25. Hydrodynamic Entrance Length for Circular Duct, $Re = 2056$.

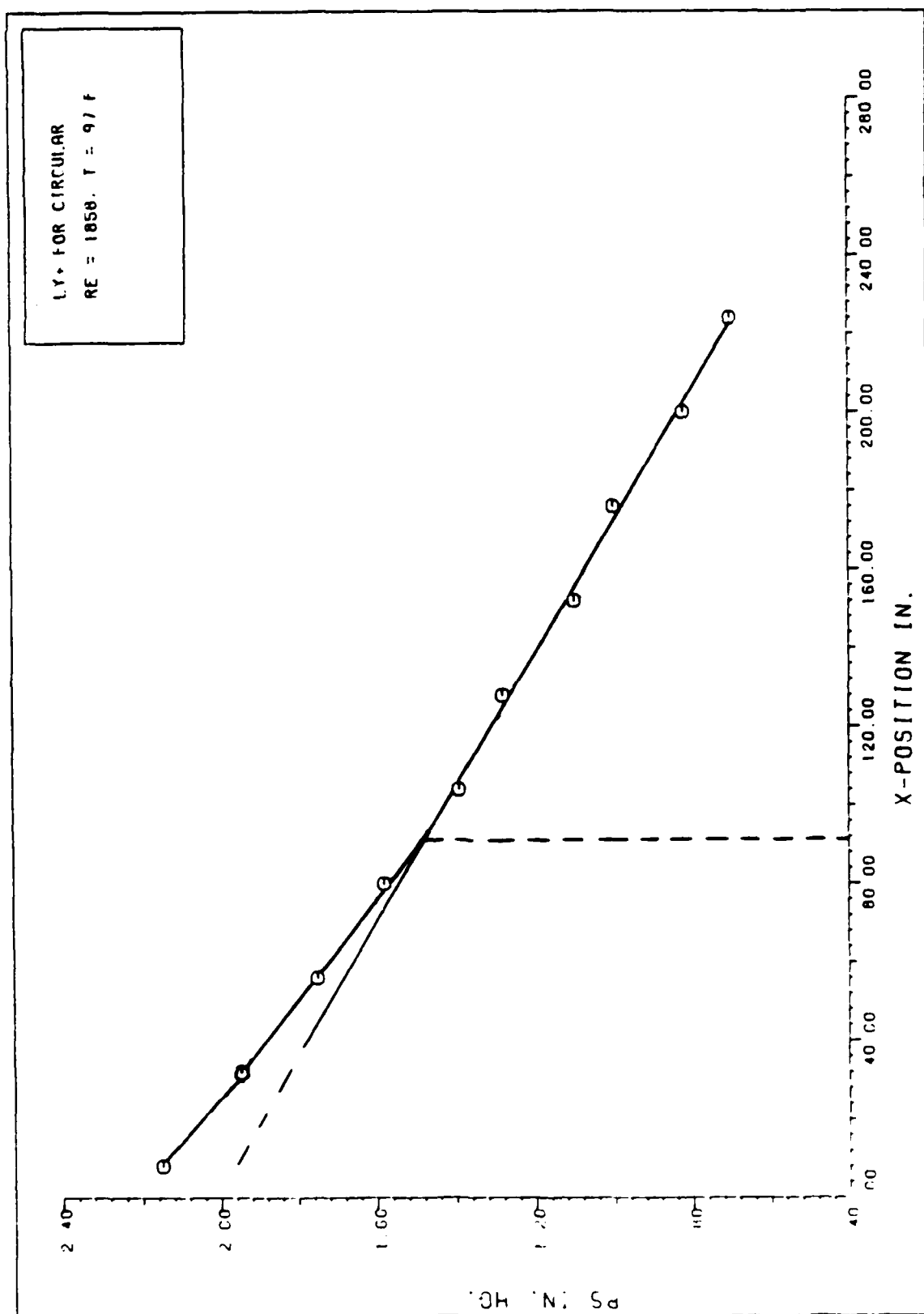


Figure 26. Hydrodynamic Entrance Length for Circular Duct, $Re = 1858$.

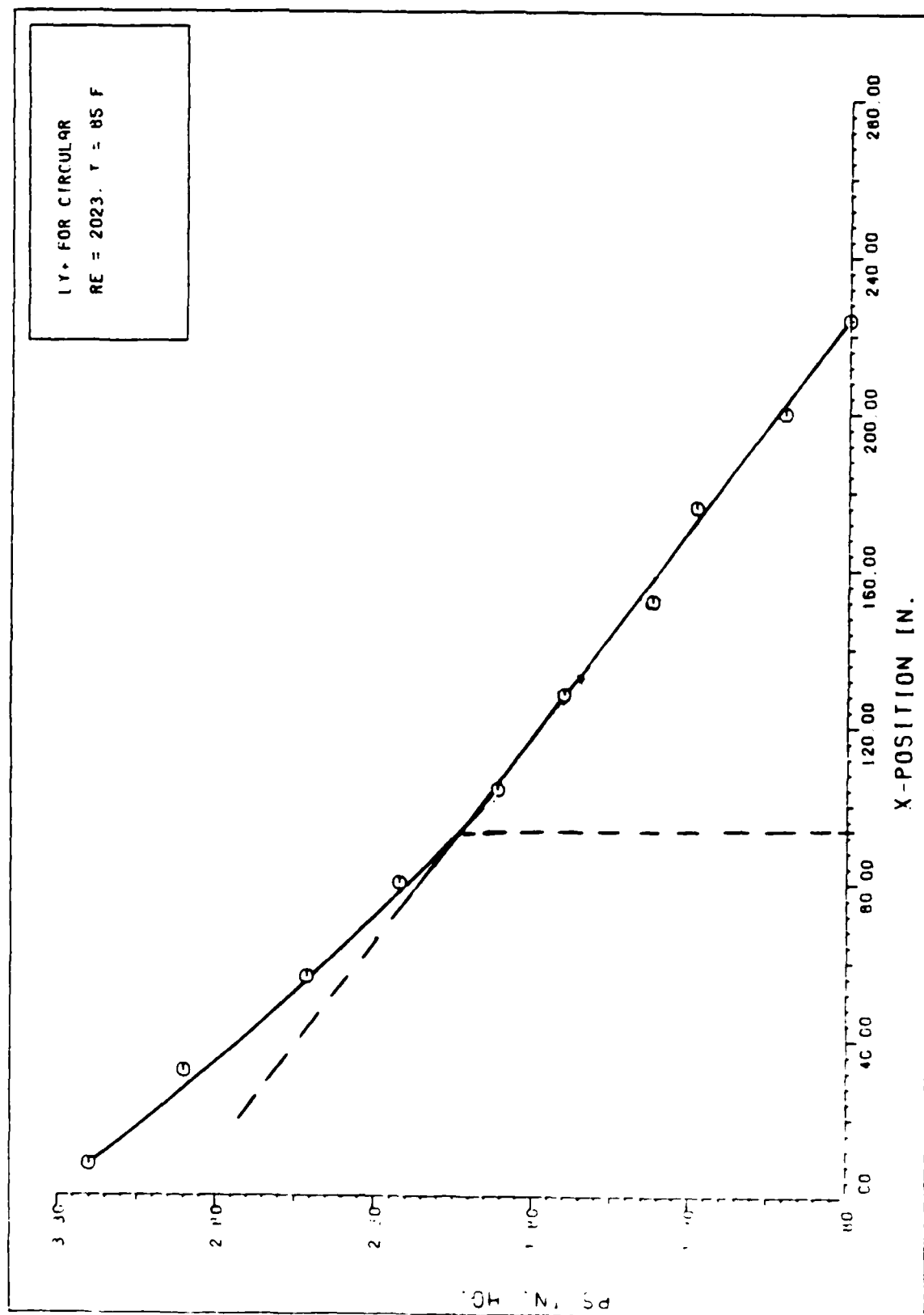


Figure 27. Hydrodynamic Entrance Length for Circular Duct, $Re = 2023$.

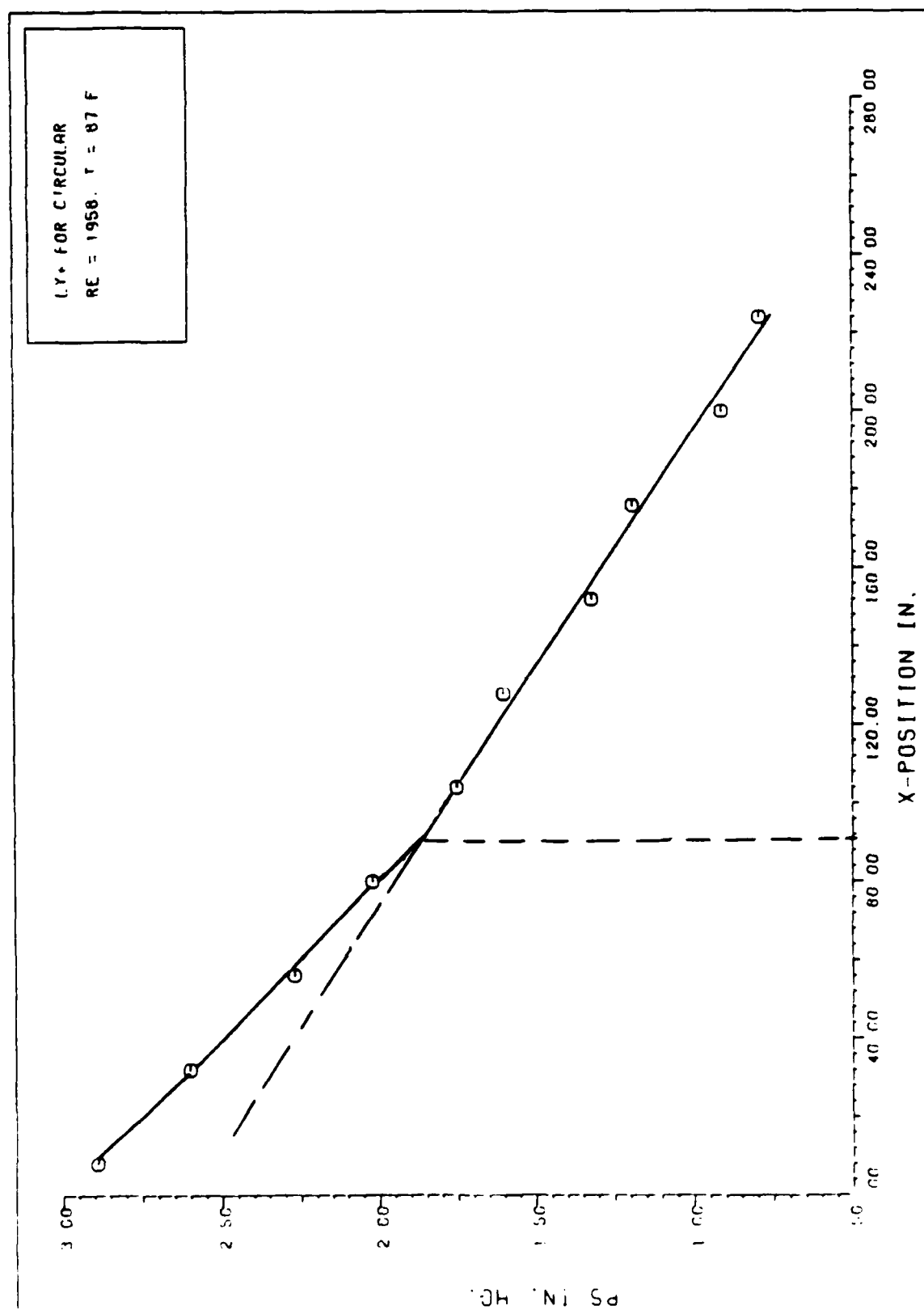


Figure 28. Hydrodynamic Entrance Length for Circular Duct, $Re = 1958$.

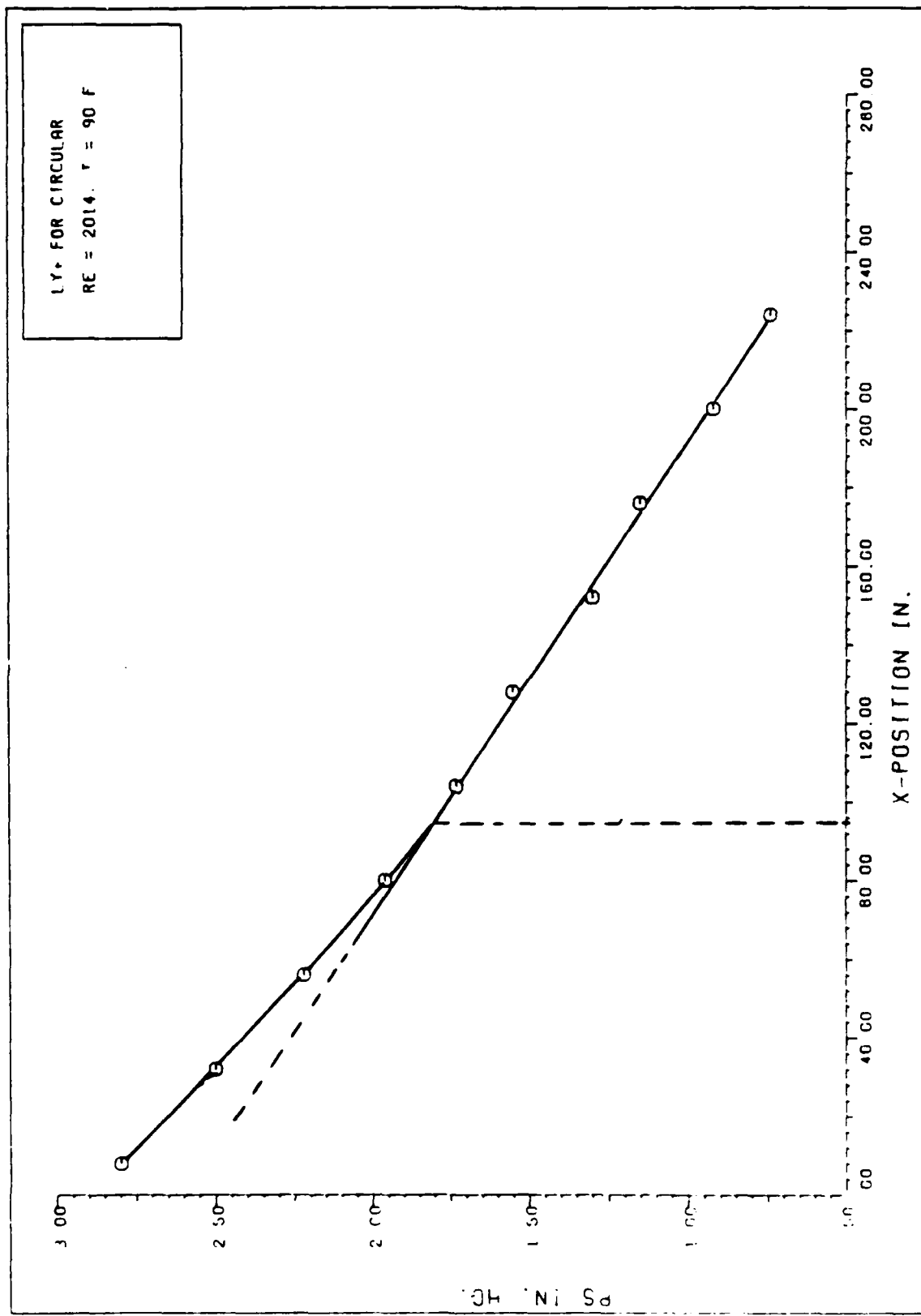


Figure 29. Hydrodynamic Entrance Length for Circular Duct, $Re = 2014$.

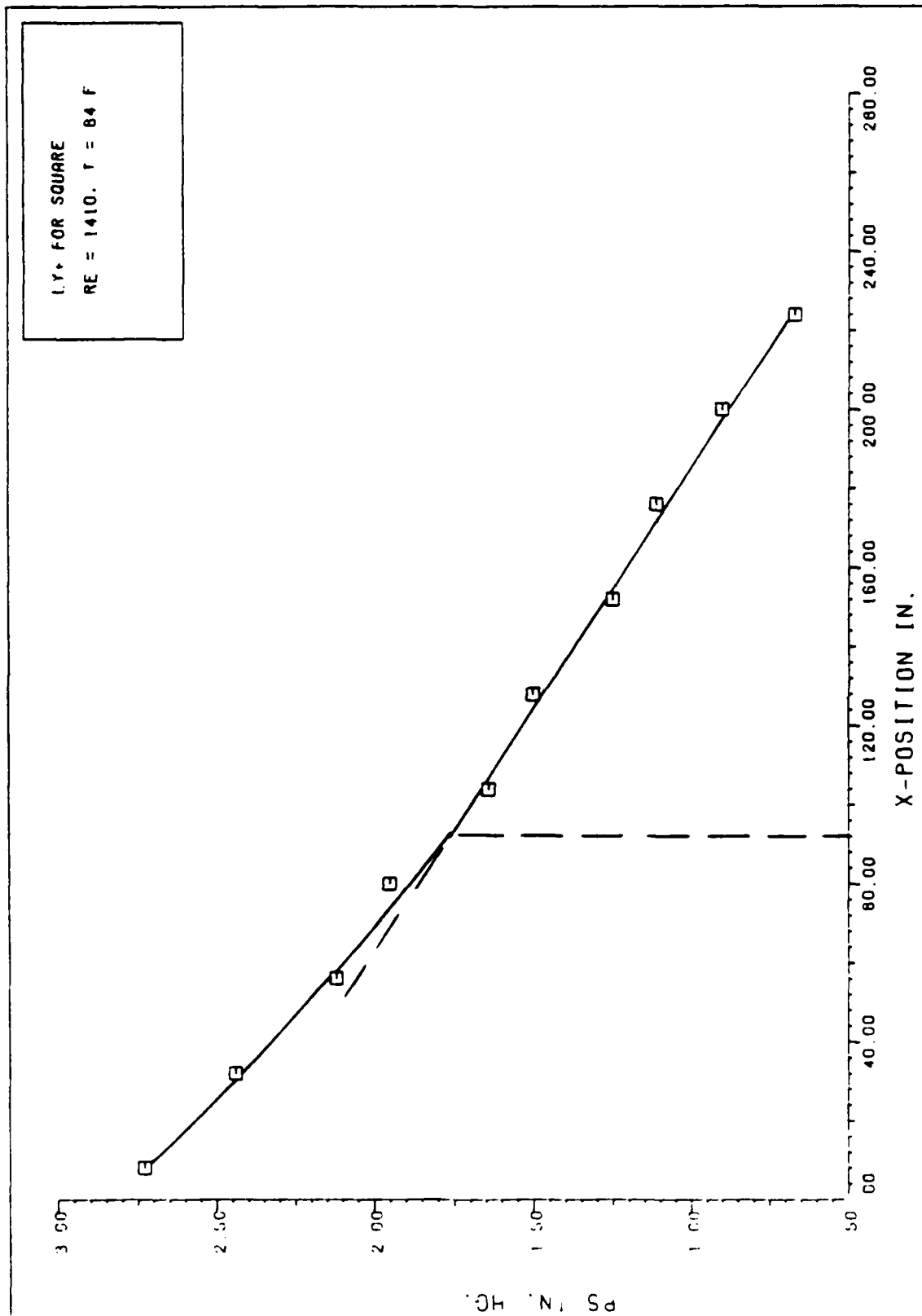


Figure 30. Hydrodynamic Entrance Length for Square Duct, Re = 1410.

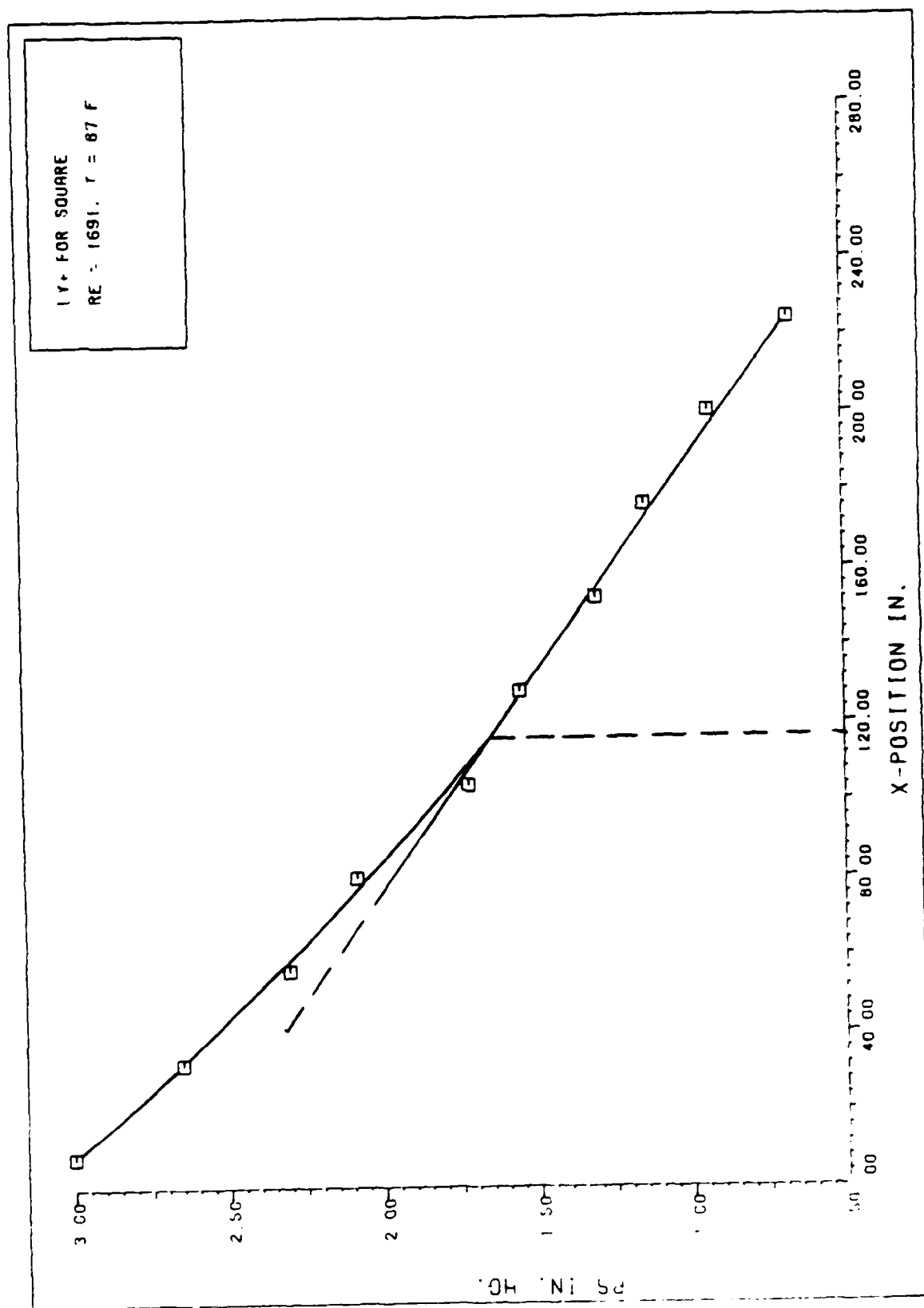


Figure 32. Hydrodynamic Entrance Length for Square Duct, $Re = 1691$.

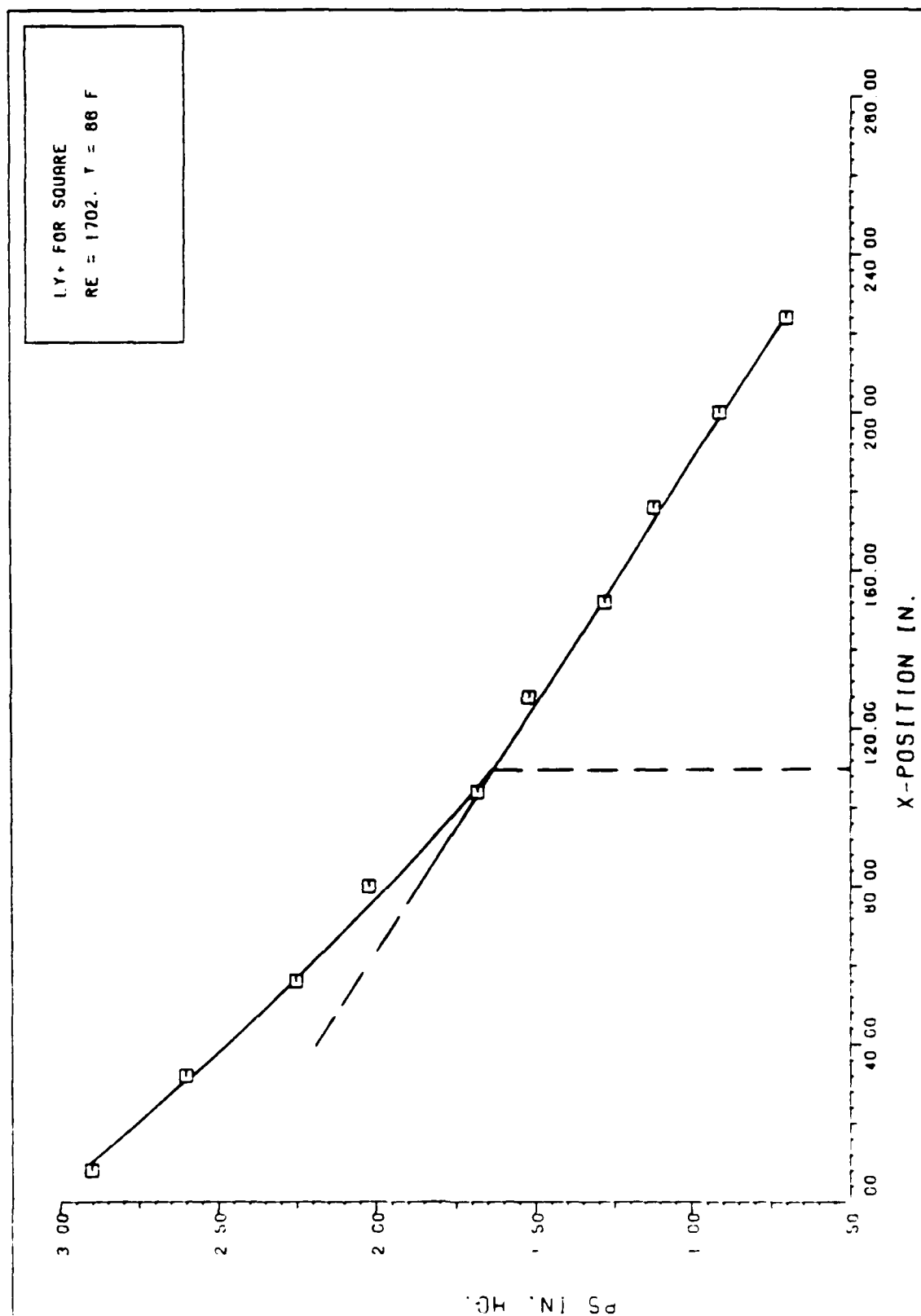


Figure 31. Hydrodynamic Entrance Length for Square Duct, $Re = 1702$.

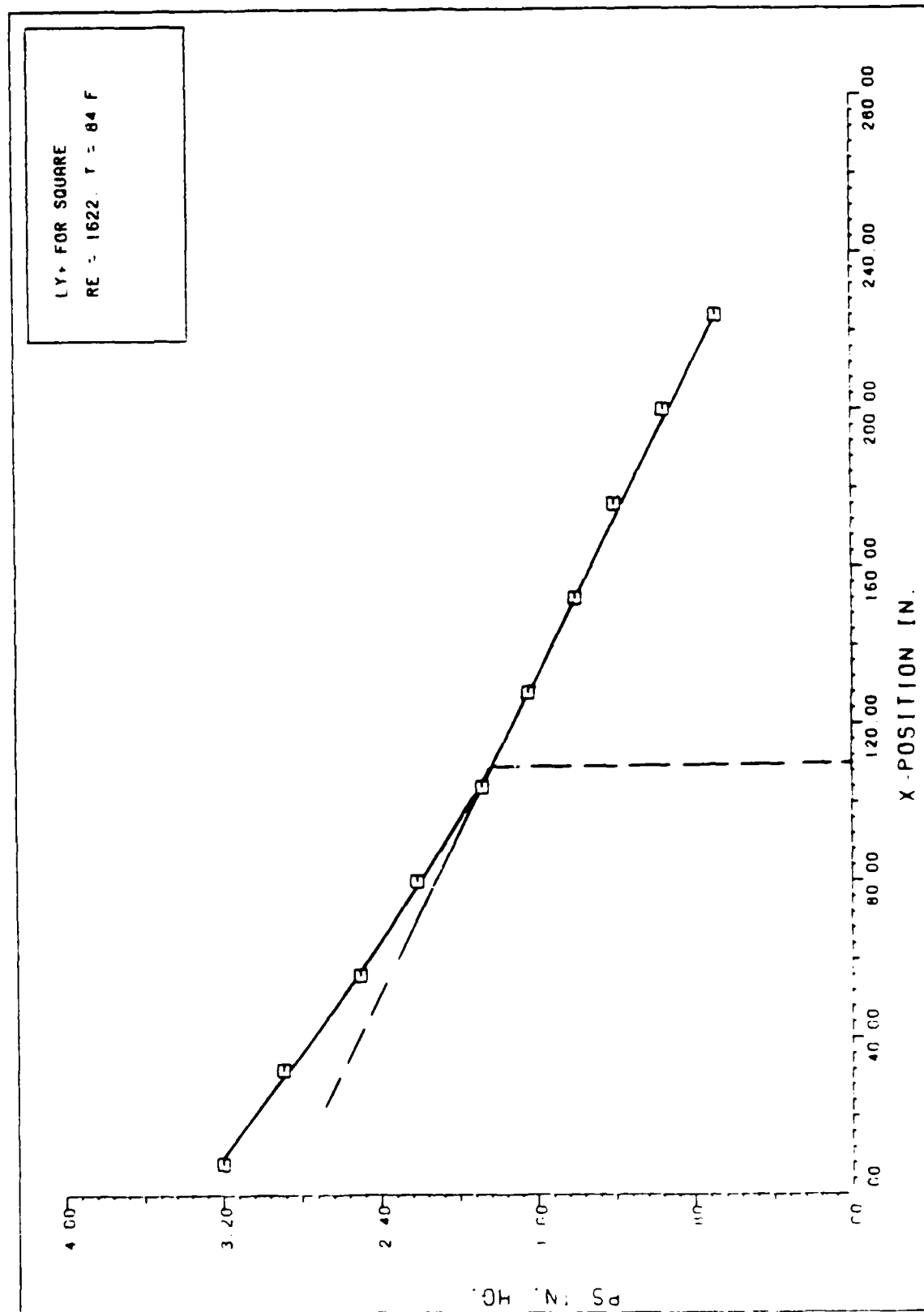
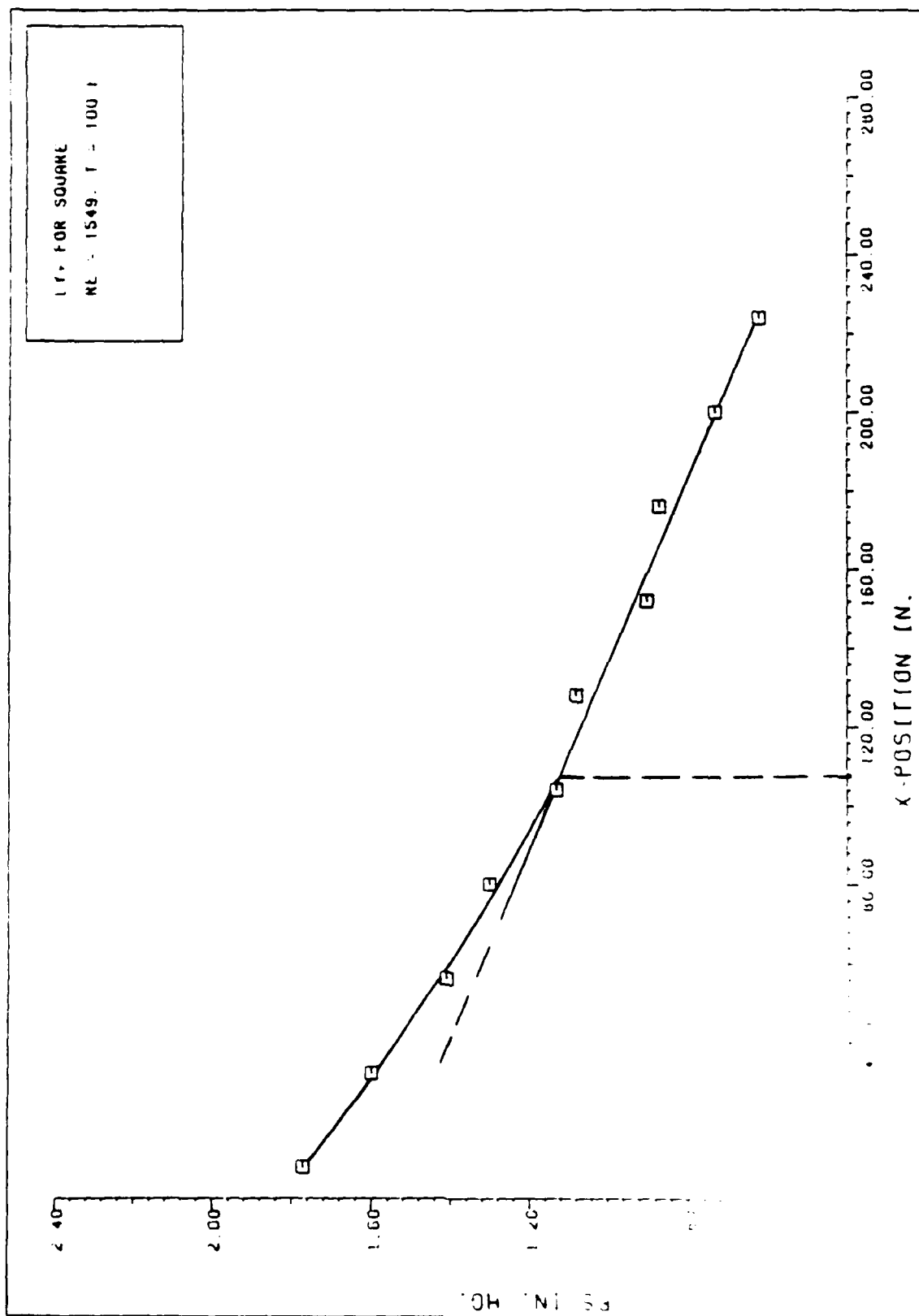


Figure 33. Hydrodynamic Entrance Length for Square Duct, $Re = 1622$.



Distance Length for Square Duct, Re = 1549.

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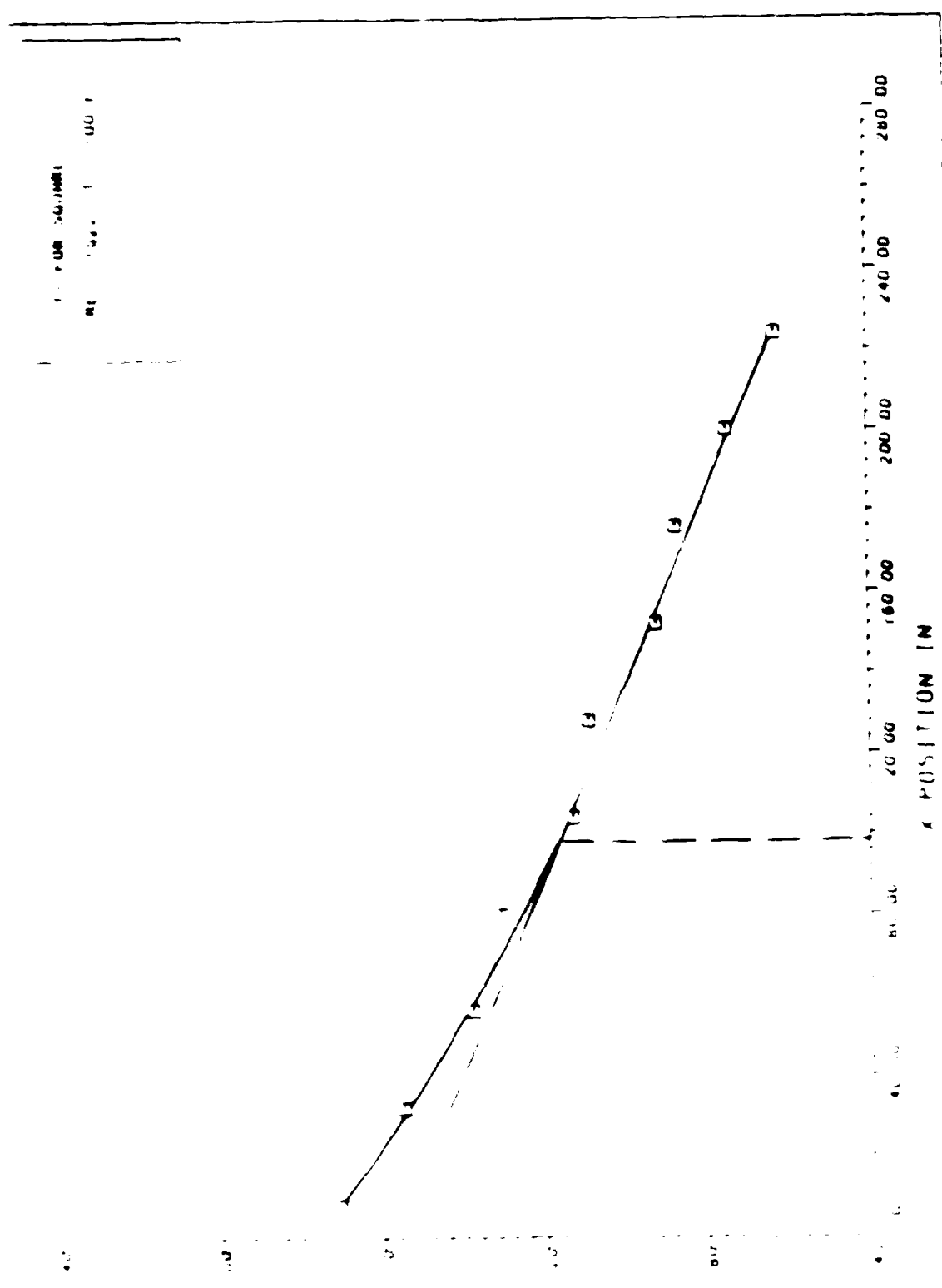


Figure 3. Measured pressure length for square duct, Re = 1500.

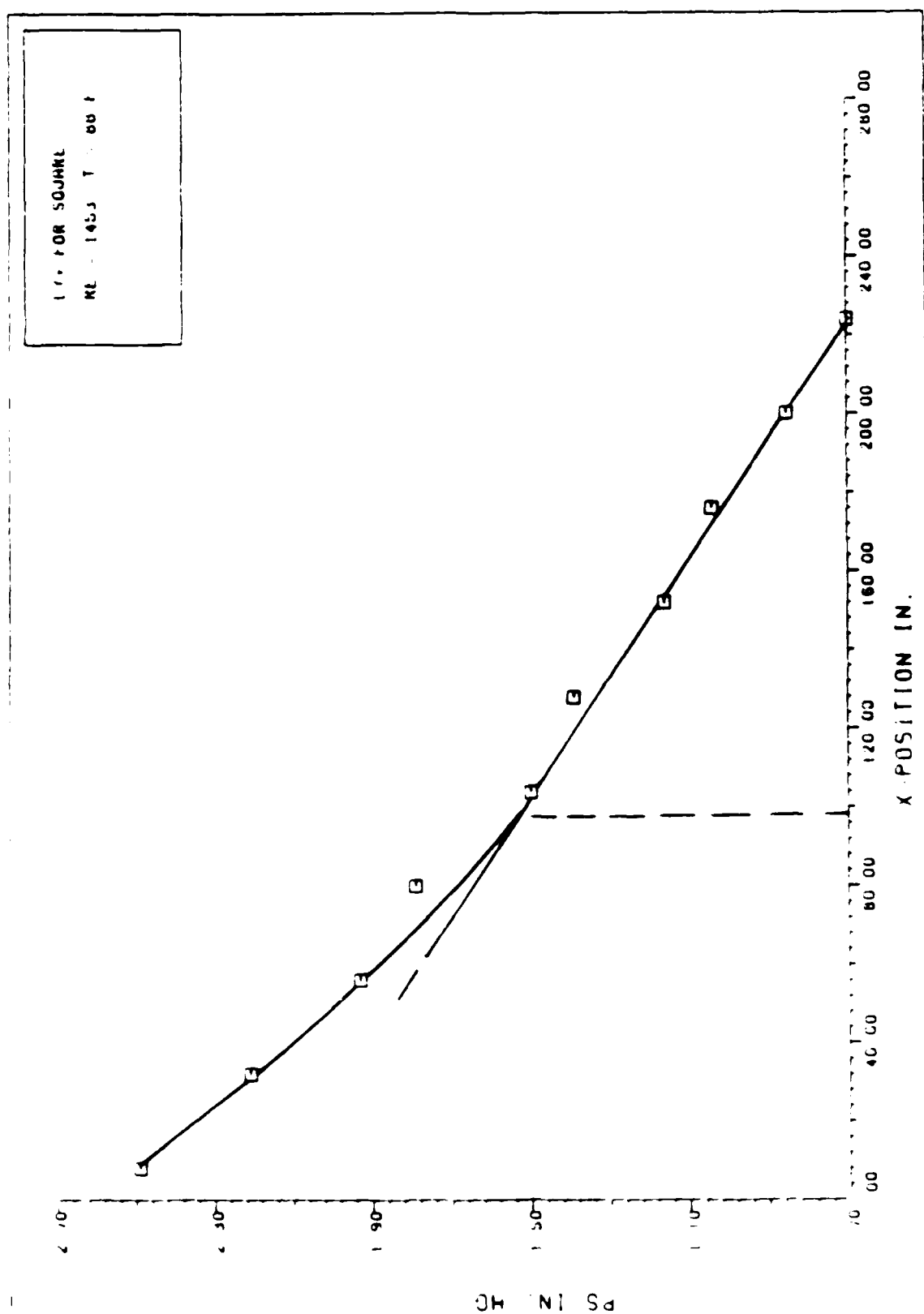


Figure 36. Hydrodynamic Entrance Length for Square Duct, $Re = 1453$.

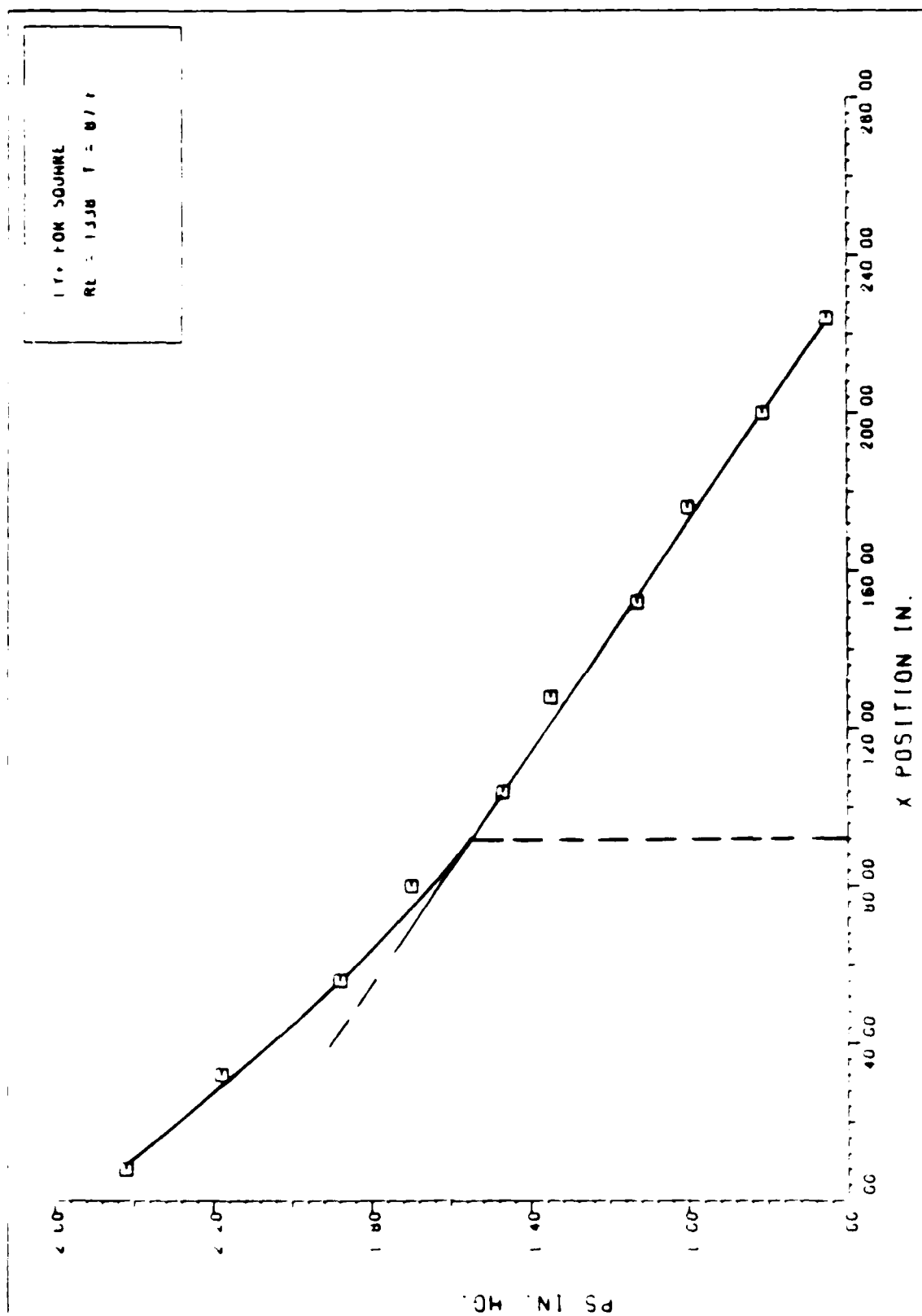


Figure 37. Hydrodynamic Entrance Length for Square Duct, $Re = 1338$.

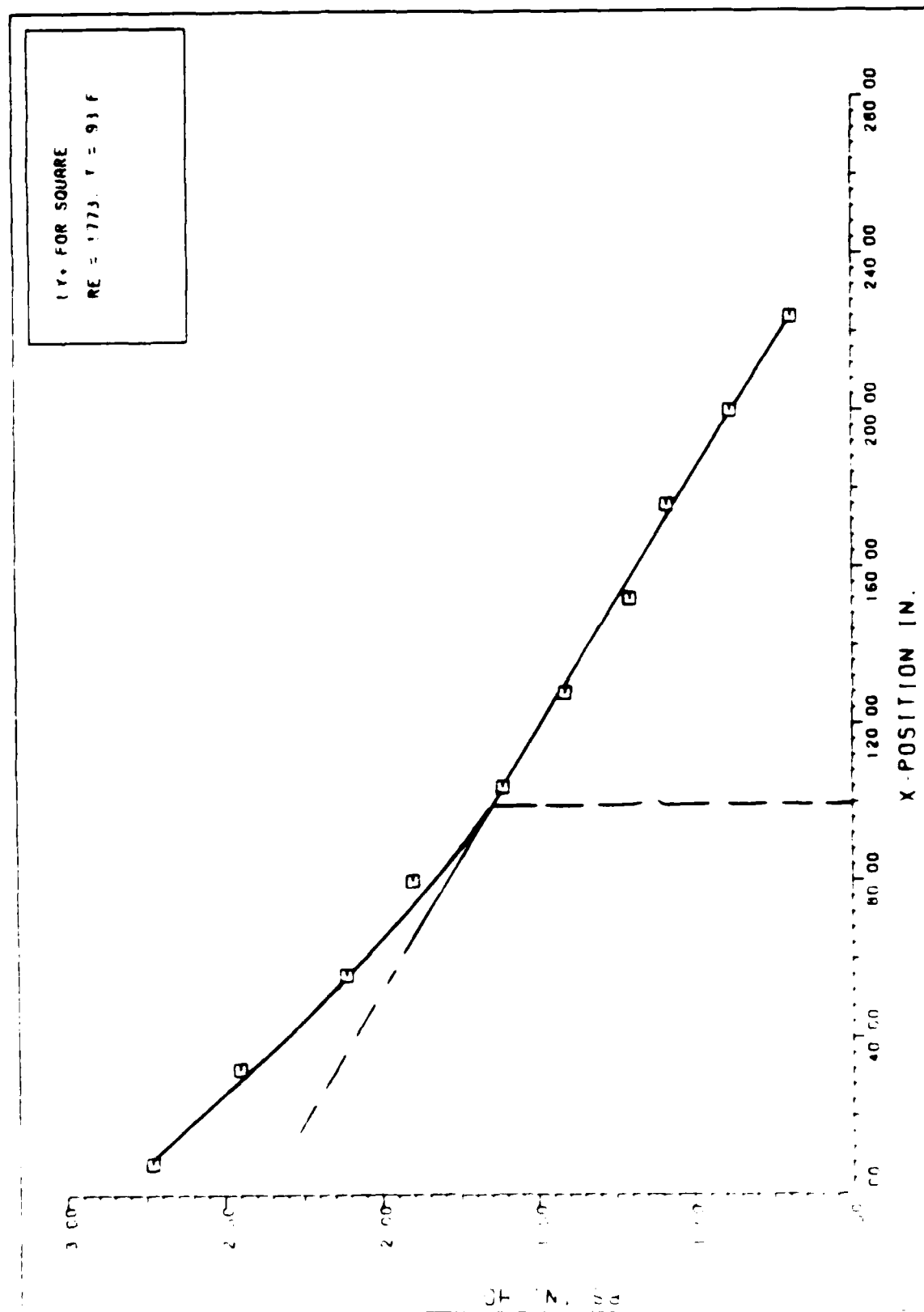


Figure 3b. Hydraulic Entrance length for Square Duct, $Re = 1773$.

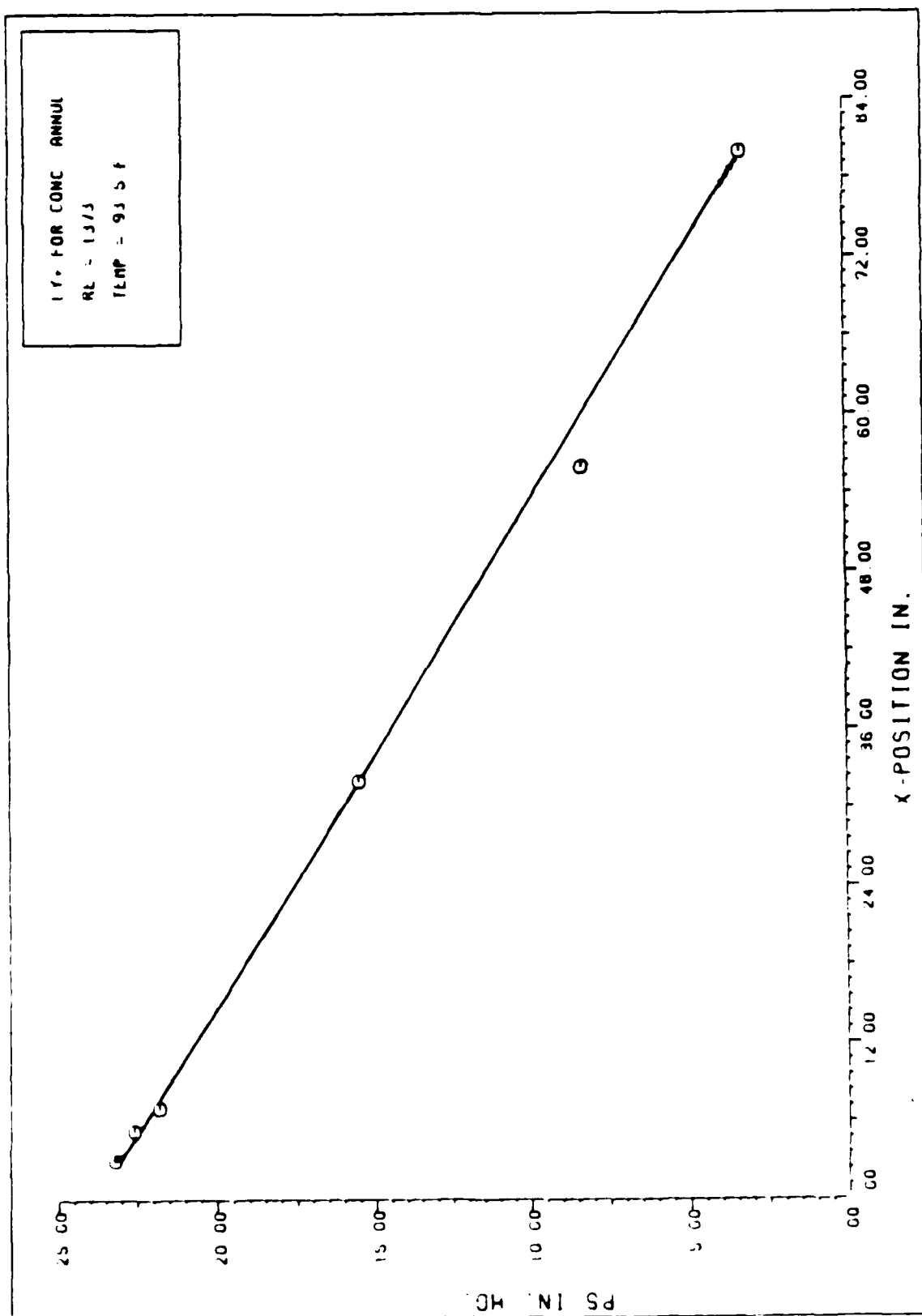


Figure 39. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1373$.

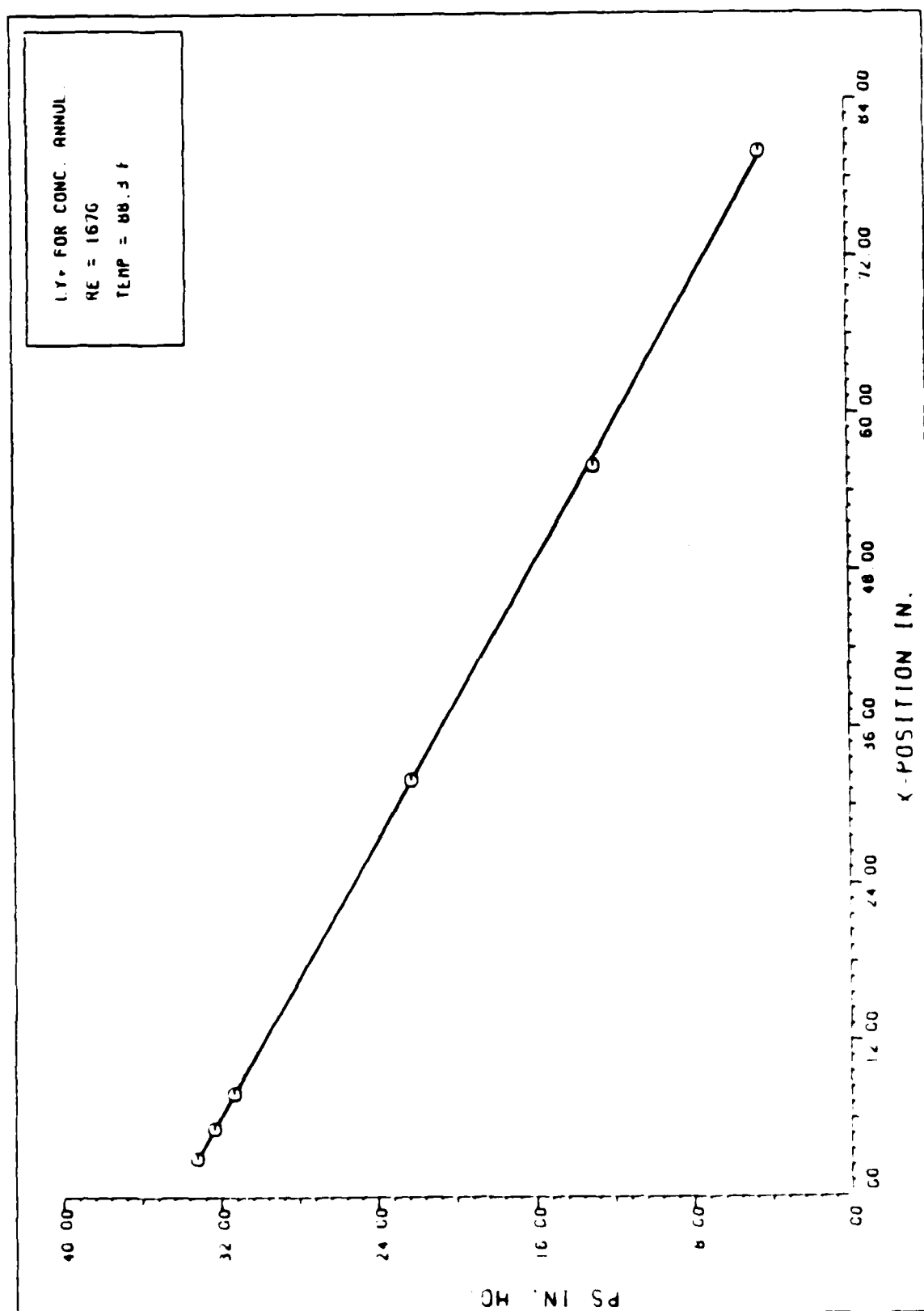


FIGURE 10. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1676$.

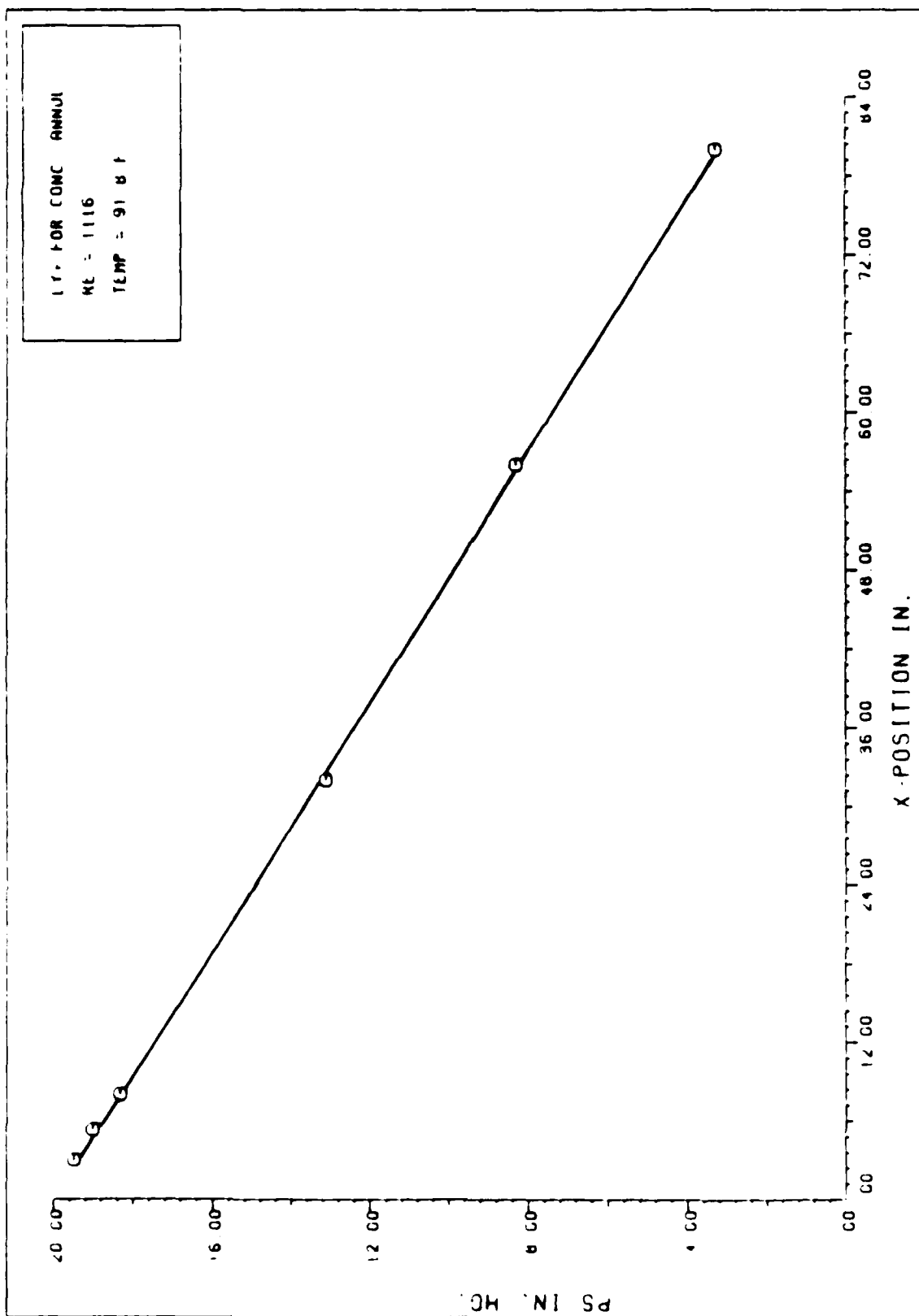


Figure 41. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1116$.

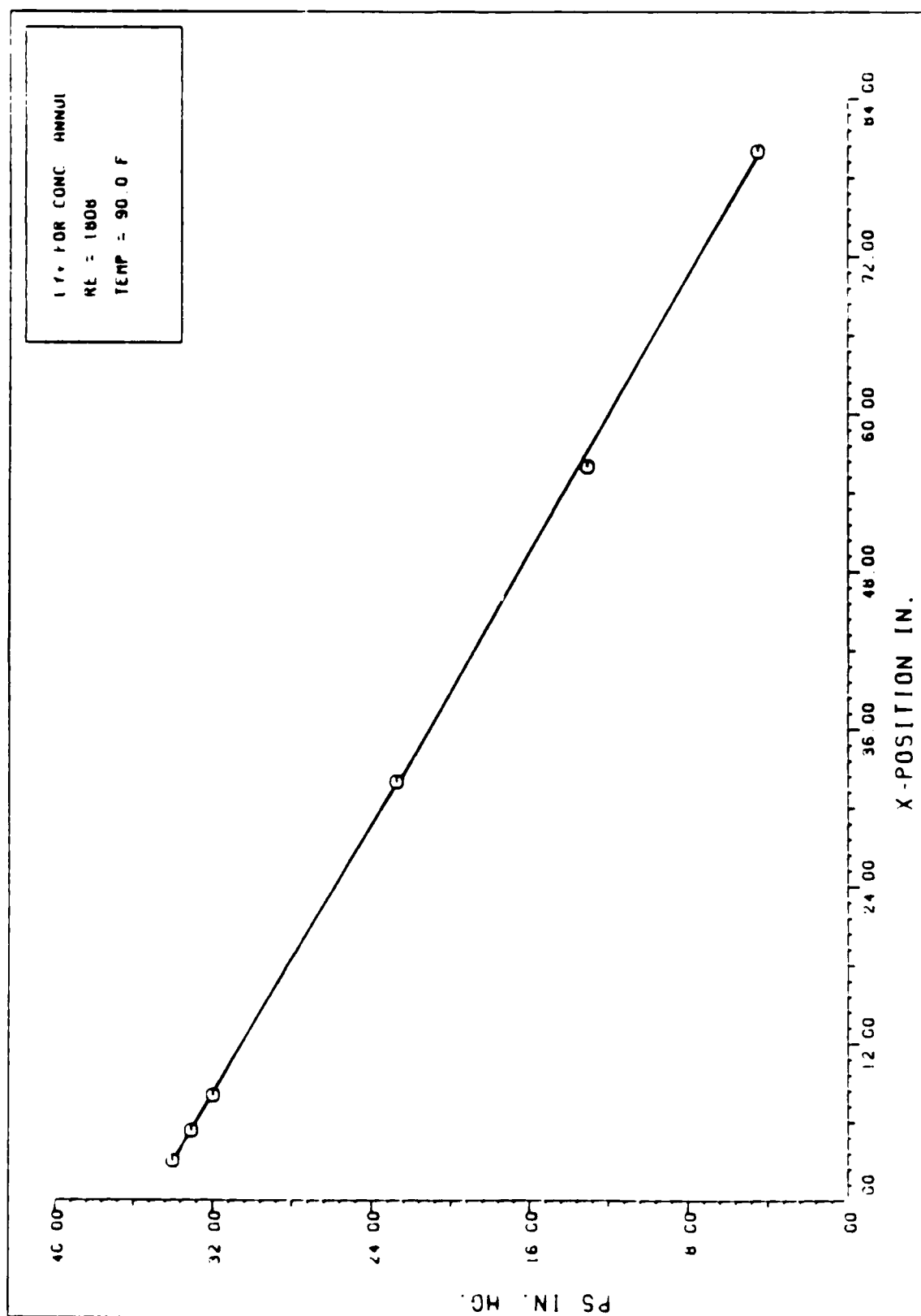


Figure 42. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1808$.

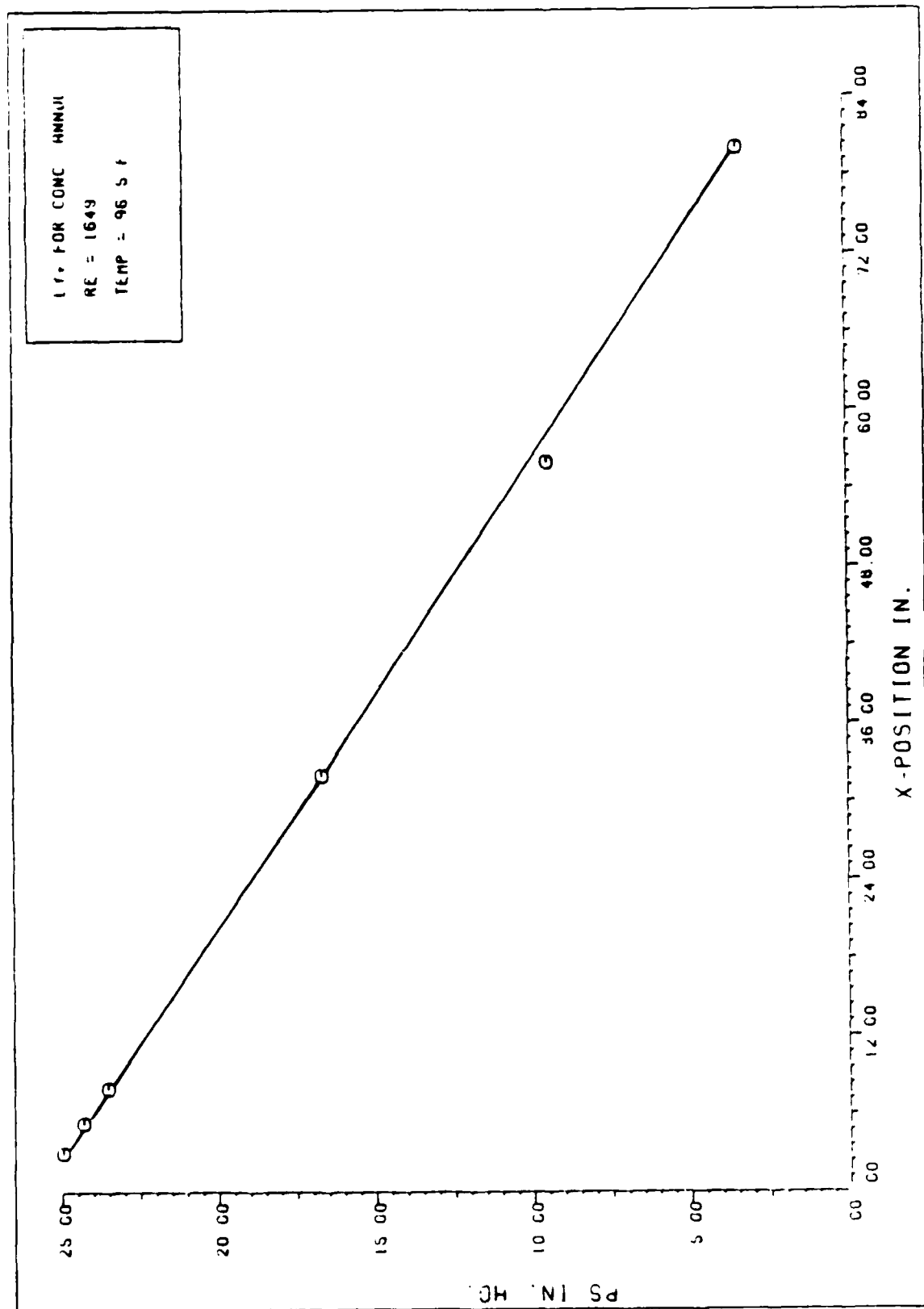


Figure 43. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1649$.

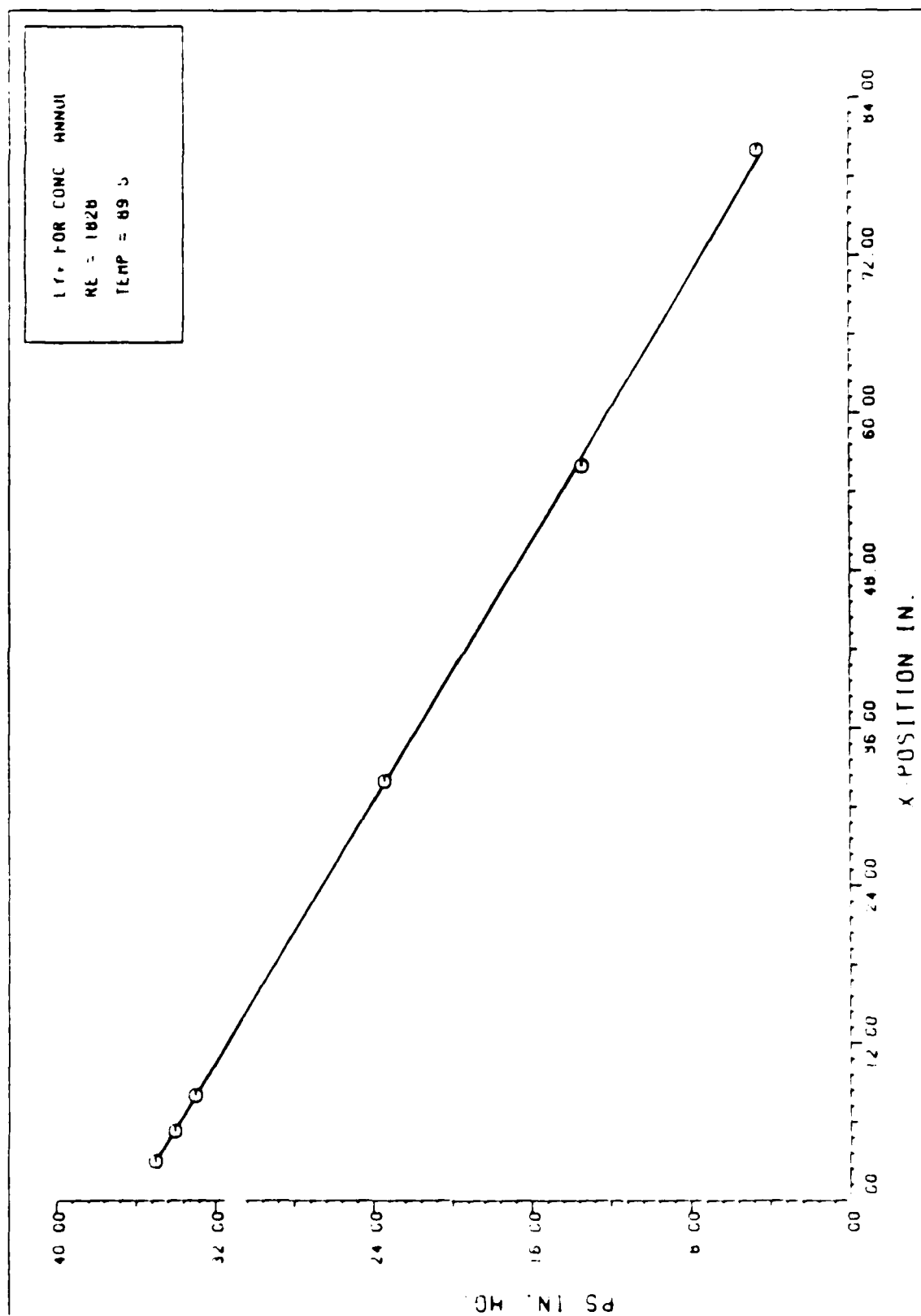


Figure 64. Hydraulic Entrance Length for Concentric Annular Duct, $Re = 1828$.

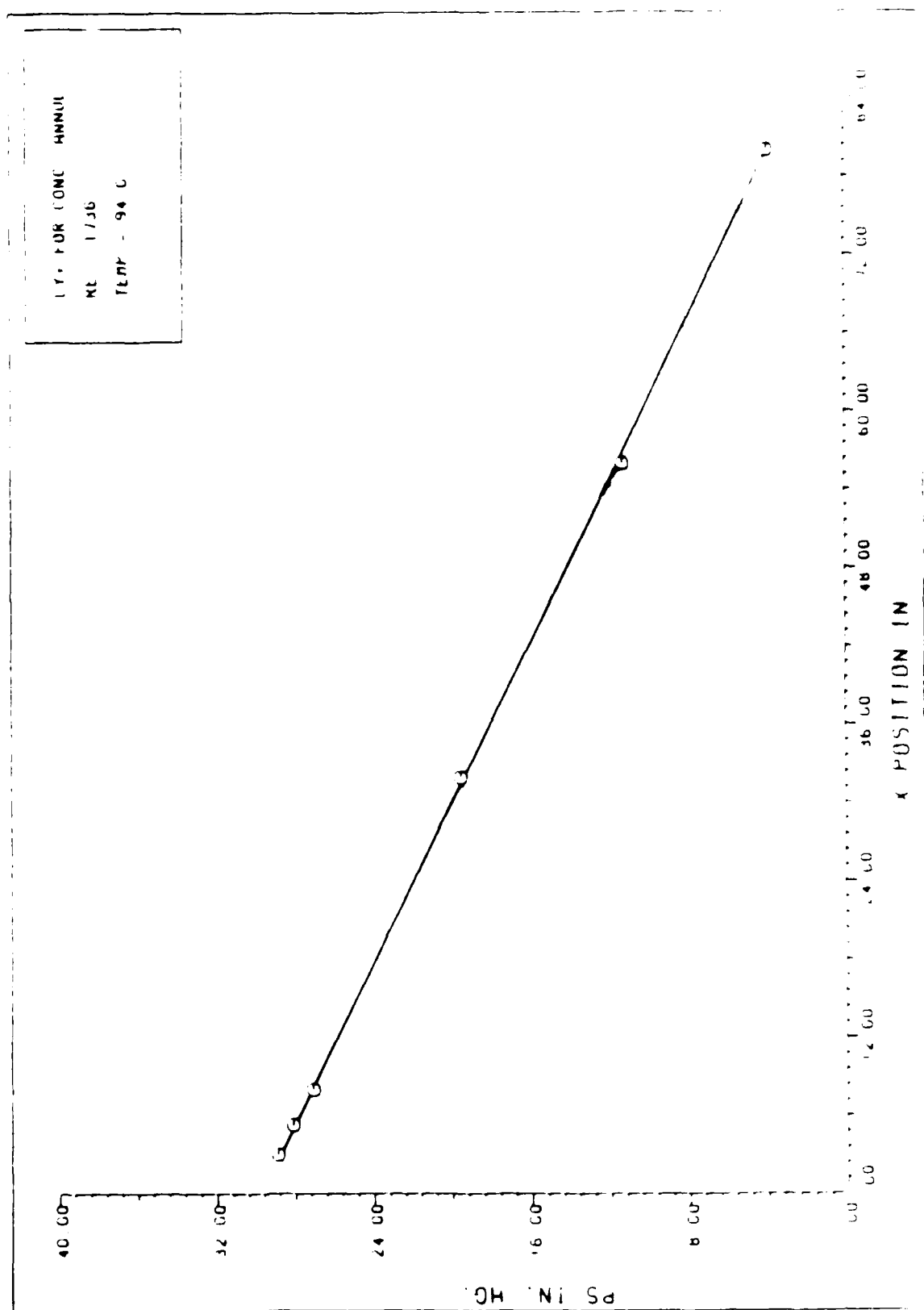


Figure 9. Hydrostatic Entrance Length for Concentric Annular Flow, $Re = 136$.

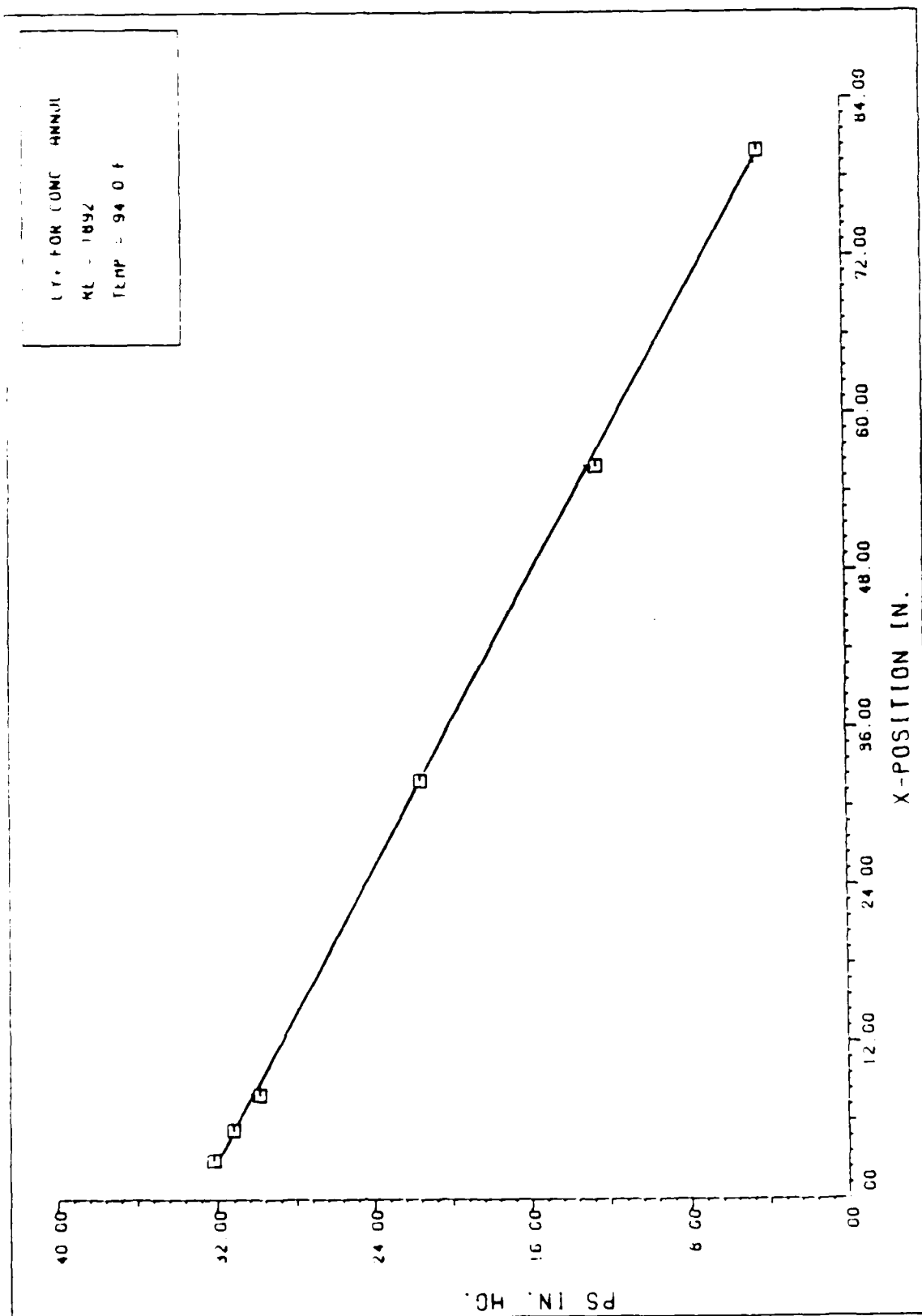


Figure 46. Hydrodynamic Entrance Length for Concentric Annular Duct, $Re = 1892$.

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A179 511

REPORT DOCUMENTATION PAGE

Form Approved
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REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE				
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GAE/AA/86D-17			5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION School of Engineering		6b. OFFICE SYMBOL (If applicable) AFIT/ENY	7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Air Force Institute of Technology Wright-Patterson AFB, OH 45433			7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)			10. SOURCE OF FUNDING NUMBERS	
			PROGRAM ELEMENT NO	PROJECT NO
			TASK NO	WORK UNIT ACCESSION NO
11. TITLE (Include Security Classification) A Redefined Hydraulic Diameter For Laminar Flow				
12. PERSONAL AUTHOR(S) Sutherland, Bruce John, B.S., Captain, USAF				
13a. TYPE OF REPORT Final M.S. Thesis-Draft	13b. TIME COVERED FROM Jul 85 to Dec 86	14. DATE OF REPORT (Year, Month, Day) 1986, Dec., 1	15. PAGE COUNT 111	
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	Laminar Flow, Hydraulic Diameter, Transition Reynolds Number; Hydrodynamic Entrance Length	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) Title: A Redefined Hydraulic Diameter For Laminar Flow Thesis Chairman: Dr. James Hitchcock Cont. on reverse				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. James Hitchcock			22b. TELEPHONE (Include Area Code) (513) 255-3517	22c. OFFICE SYMBOL AFIT/ENY

Approved for public release: LAW AFR 190-11.
13 MAR 87
Air Force Institute of Technology Development
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Abstract

For laminar, steady flow in ducts, the current definition of hydraulic diameter, D_h , does not accurately depict the non-uniform wall shear stress distribution around the perimeter of non-circular duct shapes. In this investigation, a new hydraulic diameter, $D_{h,1}$, was empirically determined. It correlated friction factor data for many non-circular shapes to within approximately 2.4 % of the circular duct value.

An experiment, using the APIT Oil Flow Rig Set-up, was run to determine the effect on transition Reynolds number, Re_{tr} , and hydrodynamic entrance length, L^* , of replacing D_h with $D_{h,1}$. Transition Reynolds number and L^* were determined, based on D_h and $D_{h,1}$, for a circular, square, and concentric annular duct.

Transition Reynolds numbers, based on D_h , for the square and concentric annular ducts were approximately 12.5 % lower than the circular duct Re_{tr} . The Re_{tr} , based on $D_{h,1}$, did not correlate well for the concentric annulus, but did correlate for the square duct.

Hydrodynamic entrance lengths, based on D_h and $D_{h,1}$, were experimentally determined for the circular and square duct only. The square duct L^* , and the analytic concentric annular duct L^* , based on $D_{h,1}$, did not correlate to the circular duct value.

Although the Re_{tr} and L^* for the square and concentric annular ducts did not correlate well when based on $D_{h,1}$, the data obtained is still useful to the engineering community since it provides an additional data base for experimental L^* , and provides experimental data on Re_{tr} for square and concentric annular ducts.

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