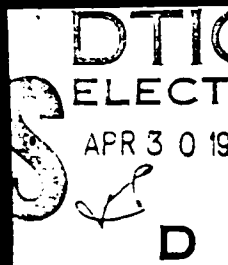


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A Theory of Diagnostic Inference:
Contract Progress Report

Hillel J. Einhorn and Robin M. Hogarth
University of Chicago
Graduate School of Business
Center for Decision Research



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Sponsored by:
Engineering Psychology Programs
Office of Naval Research
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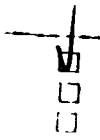
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<p>This report summarizes a program of research on diagnostic inference for the period from October 15, 1983 to February 14, 1987. The central theme underlying the work is that complex judgments and choices result from simple psychological processes that interact with highly variable and complex environments. Thus, attempts have been made to identify such processes, to describe them by parsimonious mathematical models, and to test the implications of the models in experimental tasks. In particular, specific attention has been given to the use of cognitive anchoring-and-adjustment strategies. Four major projects are described. These involve models of judgments of probable cause, belief updating, judgment and choice under ambiguity, and a theory of preference reversals. Discussion of each of the four projects follows the same format: statement of issues motivating the particular topic as well as principles underlying the approach adopted in the research, description of the model, summary of implications and major empirical results obtained to date. The report concludes by listing technical reports and publications which were produced during the contract period and which provide detailed information on all the projects.</p>					
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Introduction

This report summarizes work on psychological processes of diagnostic inference for the period from October 15, 1983 to February 14, 1987. The structure of the report reflects the four major topic areas under which research has been conducted. However, we first emphasize, in general terms, the central theme underlying the work, the research strategy adopted, and certain key notions underlying the research program.

Central theme. The central theme underlying much of the work is that complex judgments and decisions result from simple psychological processes that interact with highly variable and complex environments. In other words, whereas basic psychological processes are quite simple, the complexity of behavior results from the way such processes interact with the complexities of the environment (see Hogarth, 1986). However, it is the complexity of behavior that we seek to elucidate.

Research strategy. Given the above, the research strategy adopted has sought to develop descriptive, quantitative models of judgment and choice by:

1. Identifying basic psychological processes.
2. Developing parsimonious mathematical models of these processes.
3. Constructing tasks and environments in which to test the predictions of the models.
4. Empirically testing the predictions.

Key notions. Three key notions have guided the substantive thrust of the work:

1. Diagnosis depends crucially on judgments of probable cause, uncertainty, and risk. These, in turn, are dependent upon imagination (i.e., mentally simulating "what might be", or "what might have been").

2. Apparently complex mental processes can be modeled by variations of simple anchoring-and-adjustment strategies.

3. Specific tasks affect both the amount and type of imagination used. This makes behavioral responses highly context dependent.

In discussing each of the four major topics of this report, we first outline the issues motivating the research and the principles underlying our approach to the specific problems. After presenting the models developed for each project, we summarize implications and the major empirical results obtained to date.

Judging probable cause

The question of how people form and evaluate causal explanations is of major concern in all theories of diagnostic inference. Furthermore, since information is generally uncertain, causal inference takes place under conditions of uncertainty. In our work we have proposed a model that specifies both the components of probable cause judgments and how those components are combined. In addition, we have examined various inferential errors that can result when intuitions based on causal reasoning conflict with the dictates of probability theory (Einhorn & Hogarth, 1986).

Four general principles underly the probable cause model:

1. Causal inference is triggered by events that are abnormal or deviate from some standard or "causal field". For example, why was one ship much slower than the others? To explain such deviations causal candidates are also perceived to be exceptions to normal or expected states.
2. Causal judgments are affected by probabilistic indicators called "cues-to-causality"; these are temporal order, covariation, contiguity in time and space, and similarity of cause and effect.
3. The assessment of causal strength depends on constructing a causal chain to link the effect with the suspected cause, and then integrating the strength of the link with other cues-to-causality.

4. The strength of a causal explanation is discounted by the strength of specific alternatives. This process follows a sequential anchoring-and adjustment strategy and is thus a special case of the updating of beliefs.

The model:

We posit that the strength of a causal explanation depends on the cues-to-causality. Thus, the suspected cause (X) must precede the effect (Y) (thereby implying consistency with temporal order, Q_T); X and Y must be linked by a causal chain (Q_L); and, X must be a deviation in a background to be considered causally relevant (Q_B) (for example, the car that swerved across the road caused the accident, not the car driving straight ahead). These three factors are treated as necessary for an explanation to be judged as having a probability greater than zero. For example, if one cannot construct a causal chain to link X and Y, X will not be seen as relevant for Y. Moreover, the cues of contiguity in time and space and congruity of cause and effect (long and strong effects are seen to demand long and strong causes), constrain the chain by affecting the number of links. In addition to the three necessary cues of temporal order, deviation in a background, and causal chain strength, both the correlation between X and Y (Q_C) and the similarity of X and Y (Q_S) also affect judged causal strength. The above process can be modeled in the following way: denote $s(X,Y)$ as the strength of a causal explanation before considering specific alternatives and let,

$$s(X,Y) = Q_T Q_B Q_L (\lambda_C Q_C + \lambda_S Q_S) \quad (1)$$

where

Q_T = temporal order of X and Y (0,1)

Q_B = degree to which X is a deviation or difference in the background ($0 \leq Q_B \leq 1$)

Q_L = causal chain strength ($0 \leq Q_L \leq 1$).

Q_C = correlation of X with Y ($0 \leq Q_C \leq 1$)

Q_S = similarity of cause and effect

λ 's = weighting parameters reflecting attention given to the cues of correlation and similarity ($\lambda_C + \lambda_S = 1$).

Note that (1) implies that if either Q_T , Q_B , or Q_L is zero, $s(X,Y)$ will also be zero. Furthermore, because (1) is defined as a multiplicative function of these three cues, even if all are moderately high, their product could still be low. Since Q_L is an important part of (1), we model it as follows: let, c_j be the absolute value of the correlation between variables at the j th link in the causal chain. Then,

$$Q_L = \prod_{j=1} c_j \quad (j = 1, \dots, J) \quad (2)$$

By assuming a multiplicative function, equation (2) implies the following: (a) The strength of a chain is equal to its weakest link. Thus, if c_1 has the lowest link strength in the chain, Q_L is at best equal to c_1 (if all other links are 1). Indeed, note that if any $c_j = 0$, $Q_L = 0$, regardless of the strength of the other links; (b) Because c_j is between 0 and 1, the longer the chain (J), the lower the strength of the whole chain (since one is multiplying fractions). Therefore, longer chains are generally weaker than short ones. However, equation 2 also implies that if all the c_j 's are high, the length of the chain is less important in affecting Q_L . Indeed, if $c_j = 1$ for all j , the length of the chain is irrelevant.

Once the strength of an explanation is assessed via (1), its overall strength can be determined by anchoring on $s(X,Y)$ and then discounting (adjusting) X on the basis of the strengths of specific alternatives. Since this is a special case of updating (which is discussed below), we refrain from discussing the formal model.

Implications:

1. The model integrates (and simplifies) a vast literature on causal reasoning from many literatures.
2. By highlighting the uncertain nature of causal reasoning and contrasting this with

formal probabilistic models, the model identifies the nature and cause of certain types of "errors" people make in probabilistic reasoning. For example, the confusion of diagnostic and prognostic probabilities [i.e., $p(X|Y)$ with $p(Y|X)$] and, the conjunction error, in which conjunctions of events are judged as more probable than their components.

3. The model provides a framework for studying causal reasoning that has not existed before. Some of the important issues for study include: effects of the causal field; differences between causal and correlational reasoning; the direction of inference processes (e.g., prognosis versus diagnosis); assessing the strength of causes within complex scenarios; issues of multiple causation; updating of causal beliefs across time; and, the role of imagination in the construction of causal chains and scenarios. Since the conflict between probabilistic and causal reasoning often leads to serious errors in judgment, the model can be used to develop aids to avoid these errors .

Experimental results:

1. Assessing probable cause against different backgrounds greatly changes judged causal strength (Einhorn & Hogarth, 1986; McGill, 1986).
2. Manipulation of cues-to-causality in scenarios affects probable cause judgments as predicted by the model (Einhorn & Hogarth, 1986).
3. There are differences between correlational and causal reasoning in terms of the weighting of information from 2 x 2 tables (Lipe, 1985). Moreover, causal inference is more affected by whether reasoning is done in a diagnostic mode (from effects to prior causes) versus a prognostic mode (from causes to subsequent effects).

A Model of Belief Updating

A critical aspect of decision making centers on how people update beliefs on the basis of new

information. Consider, for example, updating beliefs of enemy intentions based on reports of troop movements, or evaluating evidence as it is received sequentially in legal proceedings. Whereas Bayes' theorem provides an optimal algorithm for these tasks, much evidence indicates that people do not update opinions in the prescribed manner and that opinions can be influenced by factors such as the order in which information is presented, whether information is presented in aggregate or disaggregated form, and so on. In this work, we develop a descriptive model for updating beliefs.

Three general principles are assumed to underly the updating model:

1. Beliefs (in hypotheses) are updated via an anchoring-and-adjustment process, i.e., anchor on one's present degree of belief and update by adjusting for the impact of new information.
2. New information is encoded as positive, negative, or irrelevant for the hypothesis being judged.
3. Two conflicting forces affect the adjustment process: (a) a force toward adaptation or surprise that captures sensitivity to new evidence; and, (b) an inertial force that dampens the effect of new information.

The model:

It is first assumed that evidence is evaluated as being positive (confirming) or negative (disconfirming) relative to the hypothesis being evaluated. Second, sensitivity to new evidence is modeled by allowing its impact to be directly related to one's current belief state. Specifically, negative evidence is hypothesized to have greater impact the larger one's current belief, whereas positive evidence has greater impact the smaller one's current belief. Thus, as shown on the left of Figure 1 below, if S_k represents one's beliefs after receiving k items of information, and all items are of equal negative strength, belief in the hypothesis would decrease over time at a decreasing rate. For positive evidence, on the other hand, the figure would be reflected in mirror-image as shown on the right.

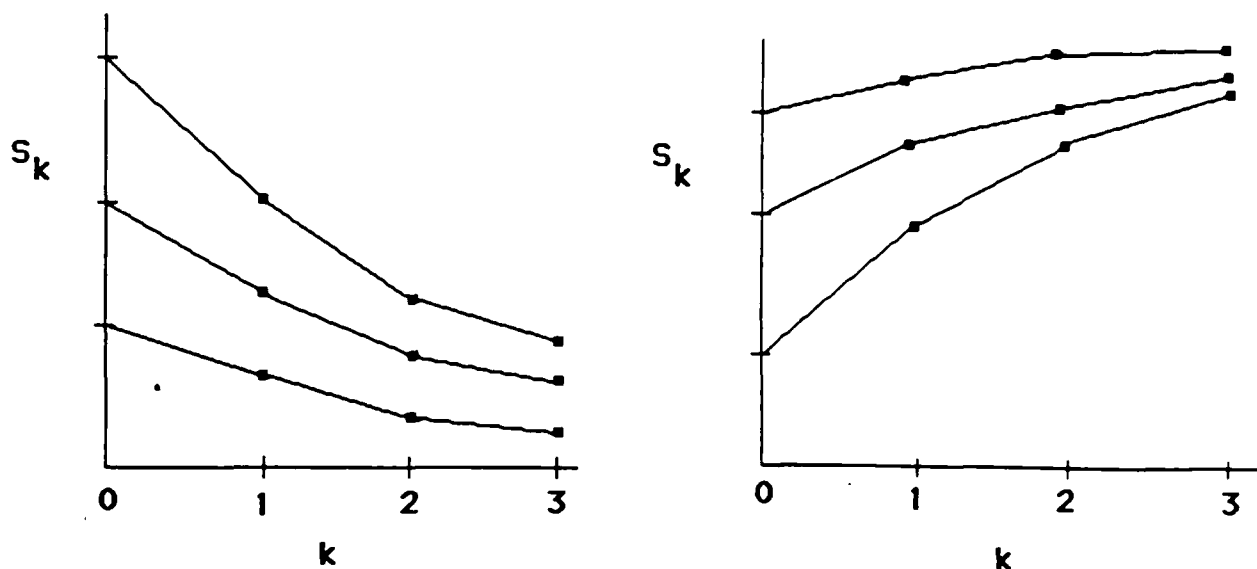


Figure 1: "Opinion curves" resulting from sequential evaluation of evidence

Both of these figures illustrate the case where people are fully sensitive to the information and there is no conflicting force due to inertia. Inertia is modeled by allowing the force toward sensitivity to decline as k increases. However, the rates of decline and increase are moderated by two parameters that represent the individual's sensitivity toward negative and positive evidence, respectively. The effect of inertia on the "opinion curves" shown in the two figures would therefore be to flatten these as k increases. In specific algebraic terms, the model can be represented in the form:

$$S_k = S_{k-1} + S_{k-1} \alpha^k s_k \quad \text{when } s_k \leq 0 \quad (3)$$

$$S_k = S_{k-1} + (1 - S_{k-1}) \beta^k s_k \quad \text{when } s_k \geq 0 \quad (4)$$

where s_k represents the judged evidentiary value of the k th piece of evidence, and α ($0 \leq \alpha \leq 1$) and β ($0 \leq \beta \leq 1$) capture sensitivity toward negative and positive evidence, respectively. Note that the model is simple in form and has only two parameters. However, it is capable of explaining a wide range of phenomena.

Implications:

1. The form of the "belief function" is specified as a function of new information.
2. The model predicts when order effects such as primacy or recency will or will not occur (see below).
3. The model predicts when beliefs will *not* change with new information. Hence, perseverance of beliefs is possible within the model.
4. The model predicts when beliefs will convergence or diverge with the same evidence. Thus, different persons confronted with the same evidence may end up further apart. Note from the above figure that people starting with different beliefs will eventually converge provided their α and β parameters are not too disparate.

Example of model's predictions: Order effects

Model predictions depend on values taken by α and β .

1. When $\alpha = \beta = 1$ (i.e., there is no inertia), the model predicts no order effects for consistent evidence (i.e., all positive or all negative evidence) but recency for mixed evidence – the "fishtail" phenomenon. This is illustrated in the Figure 2 below.

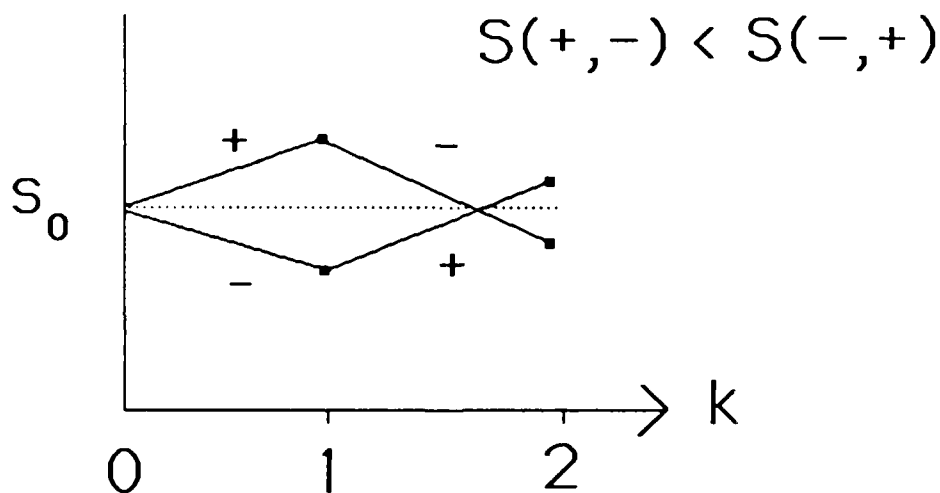


Figure 2: Typical "fishtail" pattern of recency effects for mixed evidence when sensitivity to evidence is high.

2. When $\alpha = 0$ and $\beta = 1$, or $\alpha = 1$ and $\beta = 0$, the model predicts no order effects.
3. When $\alpha = \beta = .5$, the model predicts primacy for consistent evidence, but can predict primacy, recency or no effects with mixed evidence depending on values of initial positions, strength of evidence and so on.

Results:

1. The model allows simple integration of apparently conflicting results in many different literatures.
2. Predictions where $\alpha = \beta = 1$ (no inertia model) have been examined and found to hold (Einhorn & Hogarth, 1985).

Judgment and Choice under Ambiguity

Laboratory studies of decision making typically provide people with precise values of the probabilities that affect outcomes. In more realistic situations, decision makers often have only vague or imprecise knowledge about the size of relevant probabilities. Thus, an important aspect of decision making deals with ambiguity or uncertainty about one's uncertainties. In our work, we have modeled how ambiguity affects both inference (i.e., estimating uncertain probabilities) and choice.

The two general principles underlying the ambiguity model are:

1. Judgments of ambiguous probabilities reflect a compromise between data and imagination ("what is" with "what might be").
2. An anchoring-and-adjustment process provides the mechanism for achieving the compromise.

The model:

The basic idea underlying the model is that people adjust an initial estimate of the probability (which might be based on past information, an initial impression, or a figure provided by experts), by mentally simulating or imagining other values the probability could take. Two factors affect the mental simulation process: (1) perceptions of ambiguity – the greater the ambiguity, the greater the adjustment; and (2) one's attitude toward ambiguity in the circumstances. This factor determines whether more or less weight is given in imagination to values of the probability that are larger or smaller than the initial estimate. The algebraic model used to represent this process has two parameters that link the estimates of the ambiguous probabilities, denoted $S(p)$, to the initial or anchor probabilities, denoted p . One of these parameters, θ , reflects "perceived ambiguity" whereas the other, β , represents "attitude toward ambiguity in the circumstances". The relation between $S(p)$ and p is illustrated graphically in Figure 3 below. In these figures, θ reflects the amount by which ambiguous estimates [i.e., $S(p)$] are distorted from the initial anchor probabilities through the process of imagining alternative values that p could take; the greater the perceived ambiguity (θ), the more the mental simulation, and the greater the "distortion" from p . On the other hand, β reflects the net "direction" of the mental simulation process (i.e., up or down from the anchor). Thus, a cautious person may give more weight to

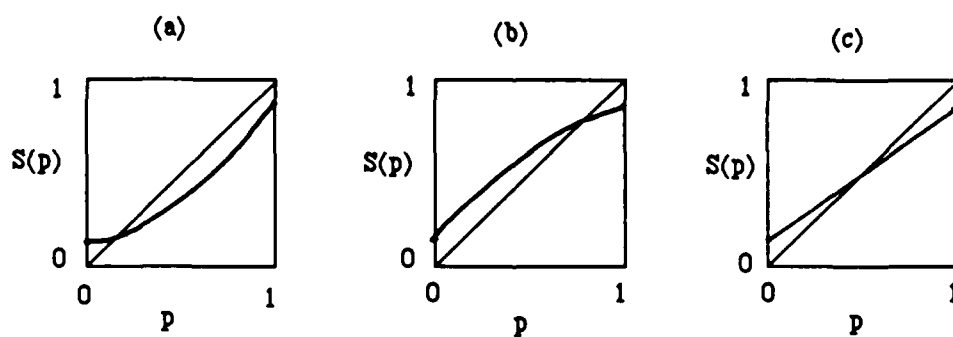


Figure 3: Examples of three "ambiguity" functions

possible values of p below the anchor when faced with estimating the chances of a positive

outcome, but give greater weight to values above p when dealing with potential negative outcomes.

Panel (a) is an example of a typical $S(p)$ function for gains, panel (b) shows the typical case for losses, and panel (c) illustrates the situation where imaginary values of p above and below the anchor are given equal weight in the mental simulation process. To summarize, θ or perceived ambiguity governs the extent to which $S(p)$ deviates from p (i.e., the diagonal in the figures), whereas β or "attitude toward ambiguity" determines the point at which the $S(p)$ function crosses the diagonal.

The algebraic formulation of the above process is provided by the equation,

$$S(p) = p + \theta (1 - p - p^\beta) \quad (4)$$

where $0 \leq \theta \leq 1$ and $\beta \geq 0$. Note that $S(p)$ is bounded between 0 and 1.

Implications:

1. The model specifies when subjective probabilities of mutually exclusive and complementary events do or do not sum to one. This allows for the resolution of certain well-known "paradoxes" in the risk area – e.g., Allais' paradox, Ellsberg's paradox, and other inconsistent choices vis-à-vis expected utility theory.

2. Whereas previous work on ambiguity has focussed on ambiguity avoidance, our model specifies conditions under which ambiguity will be sought, and situations where ambiguity will have little effect. For example, note in panel (a) that $S(p) > p$ for small values of p , whereas $S(p) < p$ for larger values of p . In addition, $S(p) \approx p$ in the region where the $S(p)$ function crosses the diagonal.

3. Changes in θ (perceived ambiguity) and β (attitude toward ambiguity) imply specific changes in $S(p)$ estimates that can explain several issues in risky decision making (see results below).

4. The model can be used to examine individual differences by means of the parameters, θ and β .

Results:

1. In a series of inference tasks, the model provided a good fit to subjects' judgments at both the individual and aggregate levels. These tests included cases where, following the model's predictions, subjects' probability judgments for complementary events did or did not sum to one (Einhorn & Hogarth, 1985).

2. By varying the content of scenarios in inference tasks, we manipulated "perceived ambiguity" and induced shifts in subjects' judgments that were in accord with the model's predictions concerning changes in the parameter θ (Einhorn & Hogarth, 1985).

3. Individual subjects' θ and β parameters were estimated on the basis of an inference task and were then successfully used to predict their responses in a choice task (Einhorn & Hogarth, 1985).

4. A series of studies involving choice from hypothetical urns supported the notion that there is a different ambiguity function for gains (similar to panel a) and losses (similar to panel b). This study, as well as the following, strengthened our interpretation of the β parameter (Einhorn & Hogarth, 1986).

5. An extensive series of tests have involved the pricing of insurance and warranties using students, business executives, and professional actuaries as subjects. Scenarios in these studies have varied (a) perceived ambiguity – ambiguous or non-ambiguous information about probabilities, (b) attitude toward ambiguity – induced by making subjects take the perspective of buyers or sellers, and (c) (initial) probability of loss – varying probabilities from .01 to .90. Results strongly confirmed the model's predictions for all of the different subject populations. For example, sellers of insurance are more sensitive to the implications of ambiguity than buyers. For low probability of loss events, buyers avoid ambiguity (in the sense that they are willing to pay more for insurance in ambiguous as opposed to non-ambiguous circumstances holding p constant). However, ambiguity avoidance decreases as p increases (see panel b of Figure 3 above) and, when p reaches a certain level, buyers are seen to prefer ambiguity. These studies are reported in Einhorn and Hogarth (1986), Hogarth (1986), Hogarth and Kunreuther (1985; 1986).

6. A further study has examined the use of verbal as opposed to numerical expressions of uncertainty and how these interact with ambiguity. Results shows that, consistent with the hypothesized effects of the β parameter in the model, differential weight is attached to the same verbal qualifiers (e.g., "unlikely", "probable") according to whether people are dealing with uncertain gains or losses (study in preparation).

A Theory of Preference Reversals

Consider two gambles of approximately equal expected value. Moreover, let one gamble, denoted A, have a high probability of gaining a small prize whereas the other gamble, B, has a low probability of gaining a larger prize. The preference reversal phenomenon refers to the fact that people who choose gamble A over gamble B often place a smaller minimum selling price on A. This has proven to be one of the most robust and puzzling phenomena in studies of behavioral decision making and is of great theoretical importance for studies of rational choice in both psychology and economics. However this reversal is only one of *six* possible reversals. Moreover, experimental results indicate that all six reversals exist in one-outcome gambles; hence, there are preference reversal *phenomena*.

Expression theory (Goldstein & Einhorn, 1987) has been developed to account for these results. The theory assumes that the basic evaluation of a gamble is expressed on various scales via a subjective interpolation process. A quantitative model is developed that accounts for both the magnitude and direction of the various reversals. The model also predicts that reversals are weakened in two-outcome gambles; that violations of dominance can exist in ratings; and, prices for gambles can be predicted from ratings and vice versa.

There are four general principles underlying the model:

1. Decision making is seen as consisting of three processes: encoding information, evaluating information, and expressing the evaluation on some response scale (a kind of E^3). The

third stage, which has been given little attention, is the focus of this project.

2. In expressing preferences on various response scales (such as ratings and prices), people use a subjective interpolation process to translate preferences from one scale to another.

3. The subjective interpolation process involves the matching of proportional adjustments on the different scales – e.g., if a rating is 20% from the top of the scale, the selling price is judged to be 20% from the maximum price.

4. Although the subjective interpolation process is a reasonable strategy, it leads to large and systematic inconsistencies in the expression of preference.

The model:

It is assumed that the overall utility of a gamble, $u(G)$, is derived from an anchoring-and-adjustment process of the form,

$$u(G) = u(W) - \Delta [u(W) - u(L)] \quad (6)$$

where $u(W)$ is the utility of the amount to be won, $u(L)$ is the utility of the amount to be lost, and Δ is the proportional adjustment in utility for the gamble due to uncertainty and other situational factors ($0 \leq \Delta \leq 1$). Thus, one anchors on the utility of the amount to be won and then adjusts for the difference in the utilities of the payoffs, where Δ is the "adjustment weight." It is further assumed that in choice, gamble 1 will be chosen over gamble 2 when $u(G_1) > u(G_2)$. The key to the model involves the use of Δ as a way of performing the subjective interpolation process. Consider setting a minimum selling price (denoted as MS) for G_1 . Define the proportional adjustment on the monetary scale as Δ' . Let $\Delta' = f(\Delta)$, where f is monotone. The minimum selling price can then be written as,

$$MS = W - \Delta' (W-L) \quad (7)$$

Similarly, in considering ratings of gambles (denoted by R), let Δ'' be the proportional reduction in

the ratings (on a 0 to 100 scale for example) such that,

$$R = 100 - \Delta'' (100 - 0) \quad (8)$$

This model leads to systematic reversals in the following way: imagine a gamble in which there is a very high probability of winning a small amount versus a gamble in which there is a low to moderate probability of winning a large amount (for the sake of simplicity, assume that these gambles have zero losses). For the first gamble, consider that Δ_1 is low since there is little adjustment for uncertainty; for the second gamble, however, Δ_2 is high. Furthermore, let us assume that when people are asked to choose between the gambles, they like the "sure-thing" gamble to the riskier alternative. How is it possible that one could give a higher price to the gamble that one does not choose (i.e., the riskier gamble)? For the sake of simplicity (and without loss of generality), assume that $L = 0$, so that the formula for minimum selling price reduces to,

$$MS = W(1-\Delta') \quad (9)$$

Note that although the riskier gamble has a larger Δ' than that for the sure-thing, the amount to win, W , is also larger. Hence, the minimum selling price for the riskier gamble could be larger than for the sure-thing *even though* the sure-thing is chosen. The same general logic applies to the comparisons of ratings with prices, ratings with choices, and between different types of choices.

Predictions/results:

1. The model correctly predicts the direction and magnitude of the six types of reversals for both one and two-outcome gambles. Moreover, these predictions are quite strong in the sense that certain reversals are predicted to be "impossible."
2. The model correctly predicts that ratings of gambles can *increase* by making losses more severe (and holding all else constant). Thus, a gamble that is dominated by another can still receive a higher rating under specifiable conditions.

3. The model is extended to deal with the prediction of ratings from prices and vice versa. Thus, not only does the model predict the direction and magnitude of reversals, it successfully predicts the exact values of minimum selling prices and ratings.

4. The model is extended to account for various utility elicitation biases found in the literature. For example, it is known that eliciting certainty equivalents for lotteries does not yield the same utility functions as eliciting probability equivalents. Expression theory accounts for these anomalies.

5. Expression theory can be extended to deal with a variety of judgment/choice problems by examining the interactions between the three stages of the decision making process.

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