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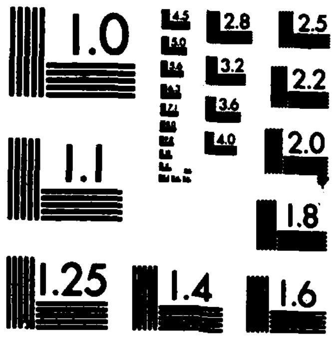

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Efficient Evaluation of Polynomials And Exponentials of Polynomials For Equi-Spaced Arguments

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A k-th order polynomial can be evaluated by means of k additions and no multiplications, when done in a recursive fashion at equi-spaced arguments. The evaluation of an exponential of a k-th order polynomial can be accomplished by k multiplications and no additions or exponentiations. Combinations of rational functions and exponentials can therefore be realized very efficiently by combining these properties.

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LIST OF SYMBOLS

$P_3(x)$	Third-order polynomial of x, (1)
α,β,γ,μ	Polynomial coefficients, (1)
x_0	Starting value of x, (2)
Δ	Increment in x, (2)
$Q_3(n)$	Third-order polynomial of n, (3)
a,b,c,d	Polynomial coefficients, (3),(4)
Q_2,Q_1	Difference polynomials, (5),(6)
$P_3(x)$	Exponential of third-order polynomial, (10)
$Q_3(n)$	Exponential of third-order polynomial, (11)

EFFICIENT EVALUATION OF POLYNOMIALS AND EXPONENTIALS OF POLYNOMIALS FOR EQUI-SPACED ARGUMENTS

INTRODUCTION

The evaluation of polynomials at equi-spaced arguments is a recurring task that arises in many applications. When a k-th order polynomial is written in nested form, its evaluation generally requires k additions and k multiplications at each argument of interest. For a set of equi-spaced arguments, ~~we will~~ it is demonstrated that the multiplications can be entirely circumvented (except during initialization) and that a recursive procedure employing only k additions per stage will suffice to generate the sequence of polynomial values.

For an exponential of a polynomial, an even greater savings is possible; namely, the exponential can be circumvented (except during initialization), and only k multiplications per stage are required in a recursive procedure. Memory storage is also kept at a minimum.

EVALUATION OF POLYNOMIAL

The procedure is best introduced by way of example. Suppose we want to evaluate third-order polynomial

$$P_3(x) = \alpha + \beta x + \gamma x^2 + \mu x^3 \tag{1}$$

at the set of equi-spaced arguments

$$x_n = x_o + n\Delta \quad \text{for } n = 0, 1, 2, \ldots . \tag{2}$$

That is, we are interested in the values

$$Q_3(n) \equiv P_3(x_n) = a + bn + cn^2 + dn^3 \quad \text{for } n = 0, 1, 2, \ldots , \tag{3}$$

where

$$a = \alpha + \beta x_o + \gamma x_o^2 + \mu x_o^3 ,$$

$$b = \Delta(\beta + 2\gamma x_o + 3\mu x_o^2) ,$$

$$c = \Delta^2(\gamma + 3\mu x_o) ,$$

$$d = \Delta^3\mu . \tag{4}$$

To this aim, define difference

$$Q_2(n) = Q_3(n) - Q_3(n-1) = b - c + d + (2c - 3d)n + 3dn^2 . \tag{5}$$

Also define

$$Q_1(n) = Q_2(n) - Q_2(n-1) = 2c - 6d + 6dn , \tag{6}$$

and observe that

$$Q_1(n) - Q_1(n-1) = 6d \ . \tag{7}$$

These last three recursions together read as

$$\left.\begin{aligned} Q_1(n) &= Q_1(n-1) + 6d \\ Q_2(n) &= Q_2(n-1) + Q_1(n) \\ Q_3(n) &= Q_3(n-1) + Q_2(n) \end{aligned}\right\} \text{ for } n = 1,\ 2,\ \dots\ , \tag{8}$$

and require only 3 additions for each n, with no multiplications whatsoever. The starting values for recursion (8) follow immediately from (6), (5), and (3), respectively:

$$\begin{aligned} Q_1(0) &= 2c - 6d\ , \\ Q_2(0) &= b - c + d, \\ Q_3(0) &= a\ . \end{aligned} \tag{9}$$

Extension to a k-th order polynomial is obvious, and requires k additions per stage, with no multiplications.

EVALUATION OF EXPONENTIAL OF POLYNOMIAL

Suppose we want to evaluate the exponential of a third-order polynomial, namely

$$P_3(x) = \exp[\alpha + \beta x + \gamma x^2 + \mu x^3] \tag{10}$$

at the arguments listed in (2). That is, we want the values

$$Q_3(n) \equiv P_3(x_n) = \exp[a + bn + cn^2 + dn^3] \quad \text{for } n = 0,\ 1,\ 2\ \ldots\ , \tag{11}$$

where a,b,c,d are given in (4).

To accomplish this goal, define ratio

$$Q_2(n) = Q_3(n)/Q_3(n-1) = \exp[b - c + d + (2c - 3d)n + 3dn^2]\ . \tag{12}$$

Also define

$$Q_1(n) = Q_2(n)/Q_2(n - 1) = \exp[2c - 6d + 6dn]\ , \tag{13}$$

and observe that

$$Q_1(n)/Q_1(n - 1) = \exp[6d]\ . \tag{14}$$

These last three recursions together read as

$$\left.\begin{aligned} Q_1(n) &= Q_1(n - 1)\ \exp[6d] \\ Q_2(n) &= Q_2(n - 1)\ Q_1(n) \\ Q_3(n) &= Q_3(n - 1)\ Q_2(n) \end{aligned}\right\} \quad \text{for } n = 1,\ 2,\ \ldots, \tag{15}$$

and require only 3 multiplications for each n, with no additions or exponentiations. The starting values for recursion (15) follow immediately from (13), (12), and (11), respectively:

$$
\begin{aligned}
Q_1(0) &= \exp[2c - 6d] , \\
Q_2(0) &= \exp[b - c + d] , \qquad (16) \\
Q_3(0) &= \exp[a] .
\end{aligned}
$$

Initialization requires the evaluation of four exponentials.

Extension to an exponential of a k-th order polynomial is obvious, and requires k multiplications per stage. Initialization requires the evaluation of k + 1 exponentials.

CONCLUSION

All the results above apply to complex coefficients a,b,c,d as well as complex arguments x_0, Δ. Only integer n needs to be real. However, since a complex multiplication involves 4 real multiplications and 2 real additions, the time of execution will naturally be larger.

The applicability of the above results to linear frequency-modulation with Gaussian amplitude-modulation follows readily, by restricting the order of the polynomial in (10) and (11) to $k = 2$; i.e., set $\mu = d = 0$. This particular case has been treated in [1]; in particular, the accuracy of the procedure has been investigated and found adequate for most applications. The evaluation of cosines or sines of real polynomials can be achieved by setting a,b,c,d in (11) to purely imaginary values.

When k equals 2 and a,b,c are complex, the quantities $Q_1(n)$ and $Q_2(n)$ are complex. Since a complex multiplication involves 4 real multiplications and 2 real additions, the number of operations per stage to generate $Q_1(n)$ and $Q_2(n)$ is 8 real multiplications and 4 real additions. As an example, if a,b,c are purely imaginary, $a = ia'$, $b = ib'$, $c = ic'$, then

$$Q_2(n) = \exp[i(a' + b'n + c'n^2)] =$$

$$= \cos[a' + b'n + c'n^2] + i \sin[a' + b'n + c'n^2] , \qquad (17)$$

meaning that the cosine and sine are capable of simultaneous generation at each stage. (Attempts to generate the cosine alone, with a lesser number of operations per stage, have not been successful.)

There is no need to set aside storage arrays for the recursive quantities in (8) or (15), if these numbers are used on the fly as they are generated. Then the computer coding for (8) is simply Q1 = Q1 + D6, Q2 = Q2 + Q1, Q3 = Q3 + Q2, while that for (15) is simply Q1 = Q1*E6, Q2 = Q2*Q1, Q3 = Q3*Q2, in the order listed. Generally, storage of only k temporary variables is required for a k-th order polynomial. On the other hand, if $Q_3(n)$ must be stored for later use, the only change in the coding above is to replace the Q3 lines by Q3(N) = Q3(N - 1) + Q2 and Q3(N) = Q3(N - 1)*Q2, respectively; there is no need to store Q1 or Q2 in arrays.

For general values of k, if the product of a rational function with an exponential of a polynomial must be calculated, it can be broken down into the evaluation of two polynomials and one exponential, as indicated above. Then one additional multiplication and division realizes the desired combination. Extensions to sums and products of such combinations are obvious. The orders, k, of the numerator and denominator polynomials, as well as the polynomial inside the exponential, need not be equal, but are completely arbitrary.

REFERENCES

[1] J. F. Kaiser, "On the Fast Generation of Equally-Spaced Values of the Gaussian Function A $\exp(-at^2)$," submitted to IEEE Trans. Acoust., Speech, Signal Processing, February 1987.

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