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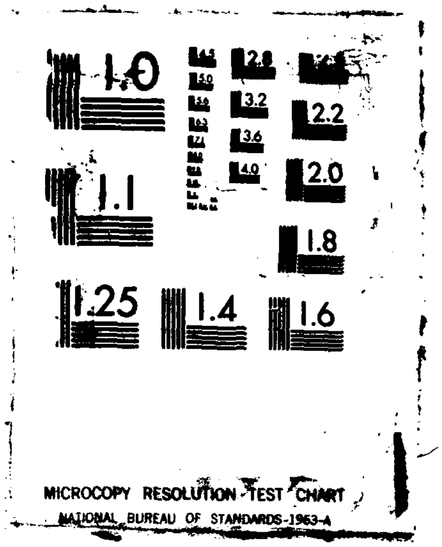
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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

A set of nonlinear and coupled equations governing the thermomechanical deformations of a viscoplastic body undergoing simple shearing deformations is integrated in time by using the Forward-Difference-Galerkin-Finite-Element (FDGFE) method and the Crank-Nicolson-Galerkin-Finite-Element (CNGFE) method. In the latter scheme the number of unknown functions is increased so that the governing equations involve only first order spatial derivatives. It is shown that the solutions obtained by the two methods agree qualitatively but the CNGFE method seems to introduce considerable damping into the system. However, the time increment needed to obtain a stable solution by the CNGFE method is 200 times that required by the FDGFE method.

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## TABLE OF CONTENTS

## I. INTRODUCTION

A thorough study of processes such as metal forming, impact, penetration and adiabatic shear banding requires integration, with respect to time, of a coupled system of nonlinear partial differential equations. For the model representing one of these phenomena to be somewhat realistic, it should incorporate such effects as strain hardening, strain-rate hardening and thermal softening. These effects are exhibited by most metals undergoing large deformations at high strain rates. For homogeneous and simple shearing deformations of such viscoplastic materials, the adiabatic shear stress-shear strain curve is generally concave towards the origin and has a peak in it. At this peak, the effect of thermal softening equals the combined effect of strain and strain-rate hardening. Under further loading, the thermal softening overtakes the strain and strain-rate hardening, and consequently the shear stress required to maintain simple shearing deformations of the body decreases with an increase in shear strain.

In essentially all practical problems enumerated above, one needs to integrate the governing equations well beyond the peak in the stress-strain curve. Whereas it is a trivial matter to carry out this integration when the deformations are homogeneous, it is a rather time consuming endeavor to do so for non-homogeneous deformations even when the deformations are one-dimensional. Herein we discuss our experience with two methods, the forward-difference scheme and the Crank-Nicolson method. In each case, the governing partial differential equations were first reduced to a set of ordinary differential equations by using the Galerkin finite element method. Also in the case of the Crank-Nicolson method the number of unknowns at each point was increased from five to eight so that only first order spatial derivatives of the unknowns appeared in the equations. We should point out that the governing equations are stiff and no artificial viscosity was introduced in either case. Of course, the Crank-Nicolson method has artificial viscosity inherently built into it.

Our numerical experiments reveal that the Crank-Nicolson-Galerkin-Finite-Element (CNGFE) method allows the use of time steps at least two orders of magnitude larger than those permitted by the Forward-Difference-Galerkin-Finite-Element (FDGFE) scheme and still gives an acceptable stable solution. It is conceivable that the efficiency of the forward-difference scheme used herein would improve if auxiliary variables were introduced, as was done for the Crank-Nicolson method, so that only first order spatial derivatives appeared in the governing equations.

We refer the reader to excellent books[1-4] and references given therein for a discussion of various numerical integration techniques. We note that Chandra and Mukherjee[5] recently used the forward-difference method to integrate a stiff set of partial differential equations somewhat akin to ours. They used an Euler type scheme with automatic time-step control. However, selecting parameters that control the time-increment automatically is a hard task.

We add that in an earlier paper[6] the emphasis was on reporting the complete set of solutions, obtained by using the CNGFE method, to equations studied herein. In this paper, we provide details of the two numerical techniques and compare results, for one variable only, obtained by using the two methods.

A. Formulation of the Simple Shearing Problem

We study the simple shearing deformations of a dipolar visco-plastic material and assume that all of the variables have been non-dimensionalized. Thus the body occupies the infinite slab bounded by the planes $y = \pm 1$. Referring the reader to Reference 6 for details, we note that the governing equations are

$$\dot{v} = \frac{1}{\rho}(s - \ell\sigma_{,y})_{,y} , \tag{1}$$

$$\dot{\theta} = k\theta_{,yy} + \Lambda(s^2 + \sigma^2) , \tag{2}$$

$$\dot{s} = \mu(v_{,y} - \Lambda s) , \tag{3}$$

$$\dot{\sigma} = \ell\mu(v_{,yy} - \frac{\Lambda}{\ell}\sigma) , \tag{4}$$

$$\dot{\psi} = \Lambda(s^2 + \sigma^2)/(1 + \frac{\psi}{\psi_0})^n , \tag{5}$$

$$\Lambda = \max\left[0, \left\{\left[\frac{(s^2 + \sigma^2)^{1/2}}{(1 + \frac{\psi}{\psi_0})^n(1 - a\theta)}\right]^{\frac{1}{m}} - 1\right\}/\{b(s^2 + \sigma^2)^{1/2}\}\right] , \tag{6}$$

with boundary conditions

$$v(\pm 1,t) = \pm 1 , \tag{7}$$

$$\theta_{,y}(\pm 1,t) = 0 , \tag{8}$$

$$\sigma(\pm 1,t) = 0 , \tag{9}$$

and a suitable set of initial conditions. Equations (1) and (2) express, respectively, the balance of linear momentum and internal energy. Herein $v$ is the velocity of a material particle, $\rho$ its mass density, $\mu$ its shear modulus, $\ell$ a characteristic material length, $k$ its thermal conductivity, $\theta$ its temperature change from that in the reference configuration, and $s$ and $\sigma$ may be interpreted as the shear stress and the dipolar shear stress. A superimposed dot indicates material time differentiation and a comma followed by $y$ signifies partial differentiation with respect to $y$. The constitutive relations (3) - (6) give one possible model of viscoplastic materials. Equation (6) implies that the plastic parts, $\Lambda s$ and $\Lambda s/\ell$, of the strain rate and the dipolar strain-rate vanish when

$$(s^2 + \sigma^2)^{\frac{1}{2}} \le (1 + \frac{\psi}{\psi_0})^n (1 - a\theta).$$

Because of the non-dimensional variables being used, the initial yield stress equals one in an isothermal and quasistatic reference test. The material parameters $\psi$ and n describe the strain hardening of the material, a the thermal softening, and b and m the strain-rate sensitivity of the material.

We presume that the initial values of $\theta$, s and $\psi$ are symmetric and of v and $\sigma$ antisymmetric in y and seek solutions of equations (1) through (6) with the same symmetry. Thus the problem is to be studied over the spatial domain $[0,1]$ and the boundary conditions become

$$v(1,t) = 1, \qquad v(0,t) = 0, \tag{10}$$

$$\theta_{,y}(1,t) = 0, \qquad \theta_{,y}(0,t) = 0, \tag{11}$$

$$\sigma(1,t) = 0, \qquad \sigma(0,t) = 0. \tag{12}$$

For the initial conditions we take

$$\begin{aligned} &v(y,0) = y, \quad \sigma(y,0) = 0, \quad \psi(y,0) = \tilde{\psi}, \\ &\theta(y,0) = \tilde{\theta}_0 + \tilde{\theta}(y), \\ &s(y,0) \quad \tilde{s}_0 = (1 + \frac{\tilde{\psi}}{\psi_0}) (1 + b\Lambda\tilde{s}_0)^m (1 - a\theta(y,0)). \end{aligned} \tag{13}$$

The values of $\tilde{\theta}_0$, $\tilde{s}_0$ and $\tilde{\psi}$ are such that, during homogeneous deformations of the block, the shear stress $\tilde{s}_0$ and the strain corresponding to $\tilde{\psi}$ lie on the shear stress-shear strain curve for the material. $\Lambda$ in Eq. $(13)_5$ is given by Eq. (6) with $\theta = \tilde{\theta}_0$, $s = \tilde{s}_0$, $\psi = \tilde{\psi}$, $\sigma = 0$. The function $\tilde{\theta}$ describes the aberration in the initial temperature distribution and will result in non-homogeneous deformations of the body.

## B. Numerical Integration of Governing Equations

### 1. Crank-Nicolson-Galerkin-Finite-Element Method.

With the auxiliary variables

$$u = v_{,y} \ , \ \xi = \theta_{,y} \ , \ p = \sigma_{,y} \ , \tag{14}$$

we can rewrite equations (1) - (4) as

$$\dot{v} = \frac{1}{\rho}(s - \ell p)_{,y} \ , \tag{15}$$

$$\dot{\theta} = kg_{,y} + \Lambda(s^2 + \sigma^2) \ , \tag{16}$$

$$\dot{s} = \mu(u - \Lambda s) \ , \tag{17}$$

$$\dot{\sigma} = \ell\mu(u_{,y} - \frac{\Lambda}{\ell}\sigma) \ . \tag{18}$$

Thus only first order spatial derivatives of the unknowns $v,\theta,s,\sigma,u,g$ and $p$ appear in he governing equations. Let $H^1$ denote the space of functions defined on $[0,1]$ the square of whose first order derivative is integrable over $[0,1]$. We approximate the unknown functions $v,\theta,s$ etc. by a linear combination of the finite element basis functions $\{\phi_i(y),\ \overline{i=2,\ldots N}\}$ in an N-dimensional subspace of $H^1$. For example,

$$v(y,t) = v_i(t)\phi_i(y). \tag{19}$$

Throughout this article, a repeated index implies summation over the range of the index. Using Galerkin's[7] method we thus reduce equations (14) through (18) to the following set of equations.

$$M_{ij}u_i = -Q_{ij}v_i \ , \tag{20}$$

$$M_{ij}g_i = -Q_{ij}\theta_i \ , \tag{21}$$

$$M_{ij}p_i = -Q_{ij}\sigma_i \ , \tag{22}$$

$$M_{ij}\dot{v}_i = -\ \tilde{Q}_{ij}s_i + \frac{\ell}{\rho}\tilde{Q}_{ij}p_i \ , \tag{23}$$

$$M_{ij}\dot{\theta}_i = -kQ_{ij}g_i + \Lambda_i P_{ij} \ , \tag{24}$$

$$M_{ij}\dot{s}_i = \mu\, M_{ij}u_i - \mu\, \Lambda_i s_k\, R_{ijk} \ , \tag{25}$$

$$M_{ij}\dot{\sigma}_i = -\mu\ell\, \tilde{Q}_{ij}u_i - \mu\, \Lambda_i\sigma_k\, R_{ijk} \ , \tag{26}$$

where

$$M_{ij} \equiv \int_0^1 \phi_i\phi_j dy = M_{ji} , \quad (27)$$

$$Q_{ij} \equiv \int_0^1 \phi_i\phi_{j,y}\, dy , \quad (28)$$

$$\tilde{Q}_{ij} \equiv Q_{ij} - (\phi_i\phi_j)\Big|_0^1 , \quad (29)$$

$$R_{ijk} \equiv \int_0^1 \phi_i\phi_j\phi_k\, dy = R_{ikj} = R_{kij} , \quad (30)$$

$$P_{ij} \equiv \int_0^1 \phi_i\phi_j(s^2 + \sigma^2)dy = P_{ji} . \quad (31)$$

We note that because of the nonlinear dependence of $P_{ij}$ and $\Lambda$ upon s, $\sigma$, $\psi$ and $\theta$, the coupled set of ordinary differential equations (20)-(26) is not that easy to integrate. The matrices $M_{ij}$, $Q_{ij}$, $\tilde{Q}_{ij}$, $R_{ijk}$ and $P_{ij}$ have been evaluated by using the linear basis functions. Also $v_i(t)$ denotes the velocity of node i at time t.

In the Crank-Nicolson method, equations (20)-(26), assumed to hold at time $(t + \Delta t/2)$, are used to preduct the values of $v,\theta,s,\sigma,g,p,u$ and $\psi$ at time $(t + \Delta t)$ from a knowledge of their values at time t. This is accomplished by approximating $\dot{\theta}_i(t + \Delta t/2)$ by $(\theta_i(t + \Delta t) - \theta_i(t))/\Delta t$, $\theta_i(t + \Delta t/2)$ by $(\theta_i(t + \Delta t) + \theta_i(t))/2$, etc. and by first evaluating the nonlinear terms on the right hand side of (20)-(26) at time t. The resulting system of linear algebraic equations is solved for $v_i(t + \Delta t)$ etc., the right-hand side in equations (20)-(26) is now evaluated at time $(t + \Delta t/2)$ and the system of equations solved again for $v_i(t + \Delta t)$ etc. This iterative process is continued till, at each nodal point,

$$\left|\frac{\Delta v}{v}\right| + \left|\frac{\Delta\theta}{\theta}\right| + \left|\frac{\Delta s}{s}\right| + \left|\frac{\Delta\psi}{\psi}\right| + |\Delta\sigma| + |\Delta g| + |\Delta p| + |\Delta u| \le \varepsilon \quad (32)$$

where subscript i has been dropped from $v_i$ etc., $\Delta v$ denotes the difference between the newly found value of v and that used to compute the right-hand side in (20)-(26), and $\varepsilon$ is a preassigned small number. The initial conditions (13) were used to find $v_i(0)$ etc.

2. Forward-Difference-Galerkin-Finite-Element Method.

In this method the field equations (1) and (2) were first cast into a weak form. Let $\phi$ and $\psi$ be two smooth functions defined on $[0,1]$ such that $\phi(0) = \phi(1)=0$. With equations (1) and (2) multiplied through by $\phi$ and $\xi$ respectively and with use of the boundary conditions (10)-(12), integration by parts over the interval $[0,1]$ gives

$$\int_0^1 \dot{v}\phi dy = -\frac{1}{\rho}\int_0^1 s\phi_{,y}\, dy - \frac{\ell}{\rho}\int_0^1 \sigma\phi_{,yy}dy \ , \tag{33}$$

$$\int_0^1 \dot{\theta}dy = -k\int_0^1 \theta_{,y}\xi_{,y}\, dy + \int_0^1 \Lambda(s^2 + \sigma^2)\xi\, dy \ , \tag{34}$$

Let the interval $[0,1]$ be divided into (N-1) subintervals, not necessarily of equal length. Thus N is the number of nodes in the mesh. Let $\phi_i^0$ , $\phi_i^1$ (i=1,2,...N) be the Hermite basis functions[7], and $\phi_i$ (i=1,2,...N) the finite element basis functions introduced previously (e.g. see Eqn. (19)). We impose the following approximations on v and $\theta$.

$$v(y,t) = v_i(t)\ \phi_i^0(y) + \dot{\gamma}_i(t)\ \phi_i^1(y) \ , \tag{35}$$

$$\theta(y,t) = \theta_i(t)\ \phi_i(y). \tag{36}$$

Here $\dot{\gamma}_i(t)$ is the value of v, at the node i at time t. Hermite basis functions $\phi_i^0$, $\phi_i^1$ can be constructed by matching together element shape functions $\hat{\phi}_1^0$, $\hat{\phi}_2^0$, $\hat{\phi}_1^1$, $\hat{\phi}_2^1$ and similarly $\phi_i(y)$ can be obtained by matching $\tilde{\phi}_1$ and $\tilde{\phi}_2$. In the Galerkin approximation, the same set of basis functions are used to approximate the test functions $\phi$ and $\xi$ as are used for v and $\theta$. Recaling that equations (33) and (34) must hold for arbitrary $\phi$ and $\xi$, we arrive at the following set of ordinary differential equations.

$$\underset{\sim}{M}\ \dot{\underset{\sim}{w}} = -\underset{\sim}{F} \ , \tag{37}$$

$$\underset{\sim}{H}\ \dot{\theta} = -\underset{\sim}{T}\underset{\sim}{\theta} + \underset{\sim}{W}. \tag{38}$$

Here

$$\underset{\sim}{W} = \{v_1, \dot{\gamma}_1, v_2, \dot{\gamma}_2, \ldots, v_N, \gamma_N\}^T ,$$

$$\underset{\sim}{\theta} = \{\theta_1, \theta_2, \ldots, \theta_N\}^T ,$$

$$\underset{\sim}{F} = \{F_1, F_2, \ldots, F_{2N}\}^T = \sum_{J=1}^{N-1} \{f_{JJ}, f_{(J+1)J}, f_{(J+3)J}\}^T ,$$

$$\begin{matrix} f_{JJ} \\ f_{(J+1)J} \\ f_{(J+2)J} \\ f_{(J+3)J} \end{matrix} = \int_{\Omega_J} \left\{ \begin{matrix} s\,\phi^0_{1,y} + \sigma\,\phi^0_{1,yy} \\ s\,\phi^1_{1,y} + \sigma\,\phi^1_{1,yy} \\ s\,\phi^0_{2,y} + \sigma\,\phi^0_{2,yy} \\ s\,\phi^1_{2,y} + \sigma\,\phi^1_{2,yy} \end{matrix} \right\} dy ,$$

$$\underset{\sim}{M} = \sum_{J=1}^{N-1} \int_{\Omega_J} \left\{ \begin{matrix} \phi^0_1\phi^0_1 & \phi^1_1\phi^0_1 & \phi^0_2\phi^0_1 & \phi^1_2\phi^0_1 \\ \phi^0_1\phi^1_1 & \phi^1_1\phi^1_1 & \phi^0_2\phi^1_1 & \phi^1_2\phi^1_1 \\ \phi^0_1\phi^0_2 & \phi^1_1\phi^0_2 & \phi^0_2\phi^0_2 & \phi^1_2\phi^0_2 \\ \phi^0_1\phi^1_2 & \phi^1_1\phi^1_2 & \phi^0_2\phi^1_2 & \phi^1_2\phi^1_2 \end{matrix} \right\} dy ,$$

with similar definitions for $\underset{\sim}{H}$, $\underset{\sim}{T}$ and $\underset{\sim}{W}$. In the above integrations $\Omega_J$ is the region occupied by the Jth element. These integrals are evaluated numerically by using the 4-point Gauss integration rule. Explicit expressions for the matrices in Eq. (38) are not stated above since they are given in many books on the finite element method, e.g. Becker et al.[7]

Equations (37), (38), and (3)-(5) are integrated with respect to time t by using the simple forward-difference method. The solution of equations (37) and (38) gives nodal values of v, $\dot{\gamma}$, and $\theta$ at the next step. From these, values of v, $\dot{\gamma}$, $\theta$, and $v_{,yy}$ at the Gauss points of integration are calculated by using the interpolation relations (35) and (36). For each Gauss point, Eqs. (3)-(5) are integrated to obtain the local values of s, $\sigma$, and $\psi$ at the next time step. Because the integration scheme is only conditionally stable in the linear case, the time step has to be kept very small; its value is dependent on the grid size, the material properties, and the present deformations of the body.

## C. Computation and Discussion of Results

In order to compute numerical results the following values of various non-dimensional parameters that correspond to a typical hard steel were chosen.

$$\rho = 3.928 \times 10^{-5}\ , \quad k = 3.978 \times 10^{-3}\ , \quad a = 0.4973\ , \ \mu = 240.3\ ,$$

$$n = 0.09\ , \qquad \psi_0 = 0.017\ , \qquad b = 5 \times 10^{6},\ m = 0.025\ .$$

For homogeneous deformations of the block, the peak in the shear stress-shear strain curve occurs at a strain of 0.093. The uniform temperature $\theta_0$ = .1033 in the block when $\gamma$ = 0.0692 was perturbed by adding a smooth temperature bump

$$\tilde{\theta}(y) = 0.1\ (1-y^2)^9\ e^{-5y^2}$$

and the resulting initial-boundary value problem was solved by using the aforementioned two methods. In each case no attempt was made to use diagonal matrices equivalent, in some sense, to those computed by using the basis functions. The domain [0,1] was divided into 13 subdomains with nodes at 0, .05, .10, .15, .20, .25, .34375, .43750, .53120, .6250, .71875, .81250, 1.0. For the forward-difference scheme various integrals appearing in the expressions for $\underset{\sim}{F}$, $\underset{\sim}{M}$, $\underset{\sim}{H}$, $\underset{\sim}{T}$ and $\underset{\sim}{W}$ were evaluated by using the 4-point Gauss quadrature rule.

When $\ell$ = 0.0 and 0.01, the forward-difference scheme necessitated taking $\Delta t = .5 \times 10^{-7}$ in order to obtain a stable solution. However, for the Crank-Nicolson method, $\Delta t = .1 \times 10^{-4}$ was found to give a stable and acceptable solution since the results obtained with $\Delta t = .5 \times 10^{-5}$ were found to be indistinguishable from those computed with the larger value of $\Delta t$. As is clear from the two sets of results shown in Figs. 1 and 2, the non-physical damping introduced by the Crank-Nicolson method results in the delayed response as compared to that obtained with the forward-difference method. As is discussed in Reference 6, the development of a late stage plateau is a numerical artifact and does not represent a physical phenomenon. The plateau was also developed in the solution computed by using the forward-difference method even though it is not depicted in the figure.

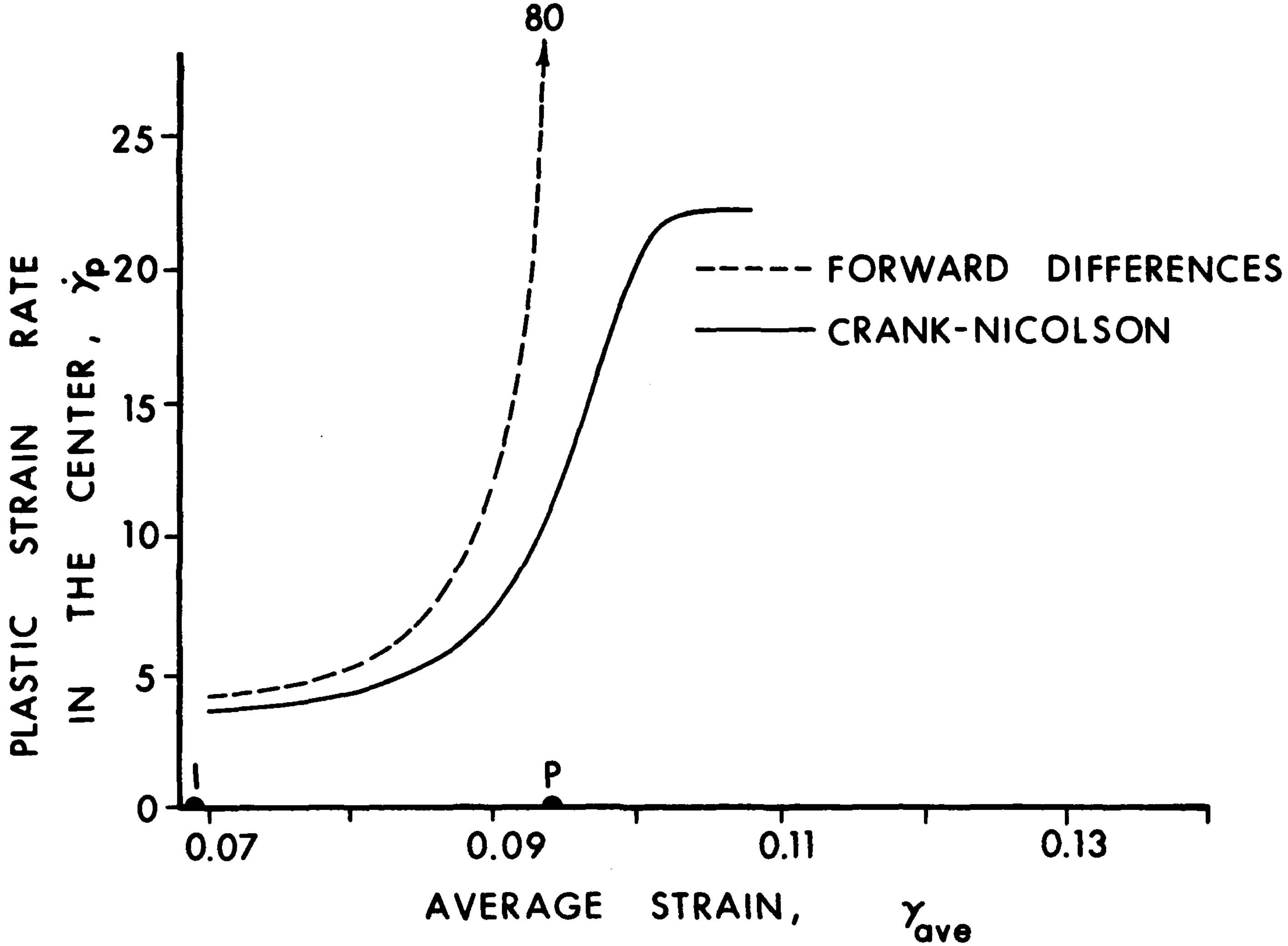


Figure 1: Comparison of Solutions (for $\ell$ = 0.0) by the Two Different Integration Techniques.

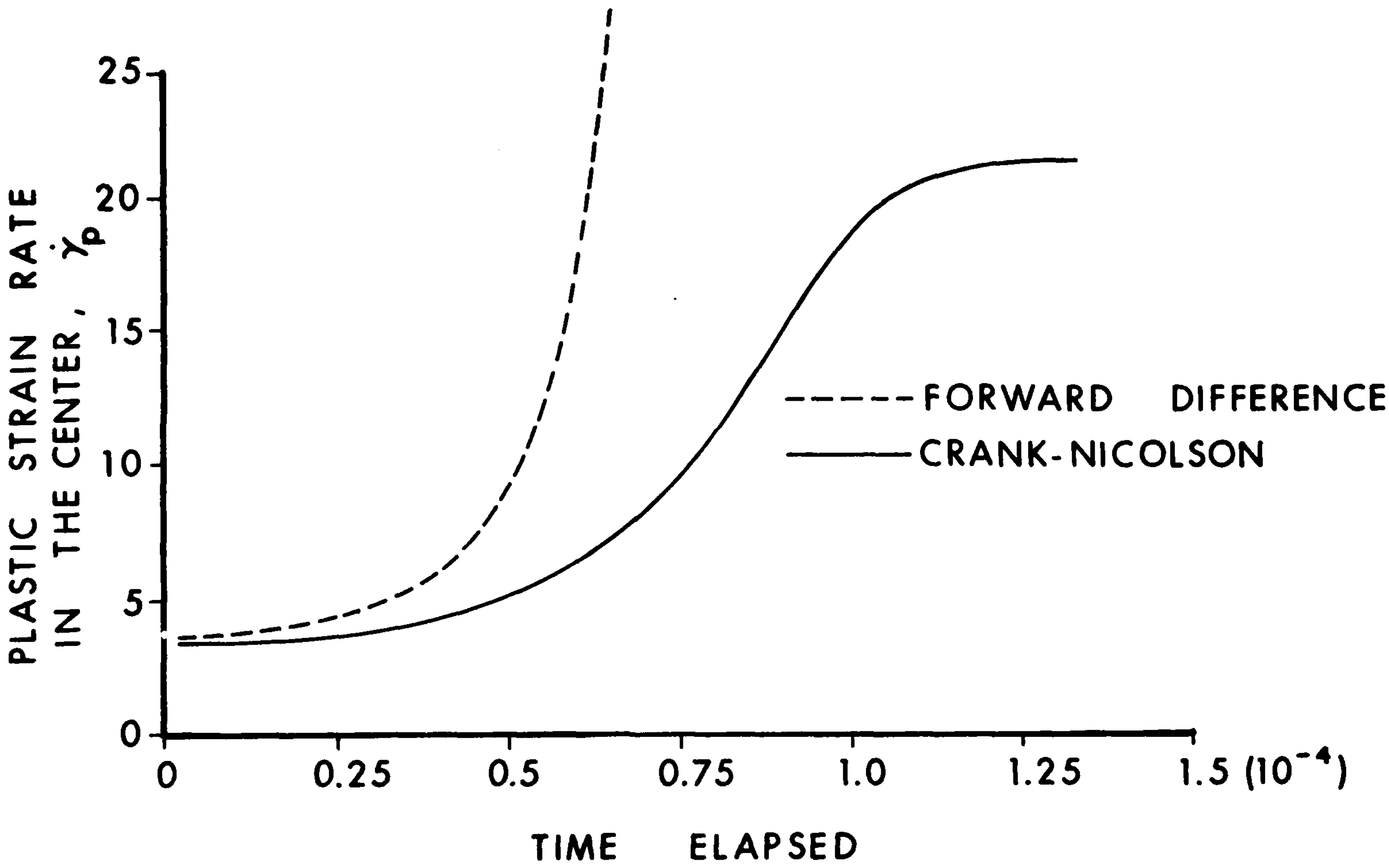


Figure 2: Comparison of Solutions (for $\ell$ = 0.01) by the Two Different Integration Techniques.

The spatial variation of s at late times also indicates some kind of numerical instability. Since the average applied strain rate is unity, the abscissa also represents the time measured from the instant (denoted by I in Fig. 1) the uniform temperature field is perturbed. On an IBM 4381 computer, the CPU time required to compute the solution by the finite-difference method was nearly three times that needed for the other method when $\varepsilon$ in Eqn. (32) was set equal to .01.

Figures 3 and 4 compare the solutions for $\ell = 0.0$ and $\ell = 0.01$ obtained by the Crank-Nicolson method and the forward-difference method. In each case, $\ell = 0.01$ results in a delayed response in the sense that $\dot{\gamma}_p(0,t)$ begins to rise to its maximum value slower and later. However, the two integration techniques depict a similar qualitative difference between the solutions of governing equations for $\ell = 0.0$ and $\ell = 0.01$. We have plotted only $\dot{\gamma}_p(o,t)$ versus t in all of the figures since the $\dot{\gamma}_p(y,t)$ is maximum at y=0 and the rate at which $\dot{\gamma}_p(o,t)$ builds up is important in physical problems. The evolution in time of other variables, the spatial variation of these variables at different times, as well as the effect of choosing different perturbations $\tilde{\theta}(y)$ have been given in [6,8,9].

Whether or not the introduction of auxiliary variables in the FDGFE metehod will permit the use of a larger time step remains to be seen. Also, the use of automatic time-step control as discussed by Chandra and Mukherjee[5] may improve the efficiency of the FDGFE method. Further work in resolving some of the issues raised herein and selecting an optimum value of $\Delta t$ is currently under progress and will be reported on in future.

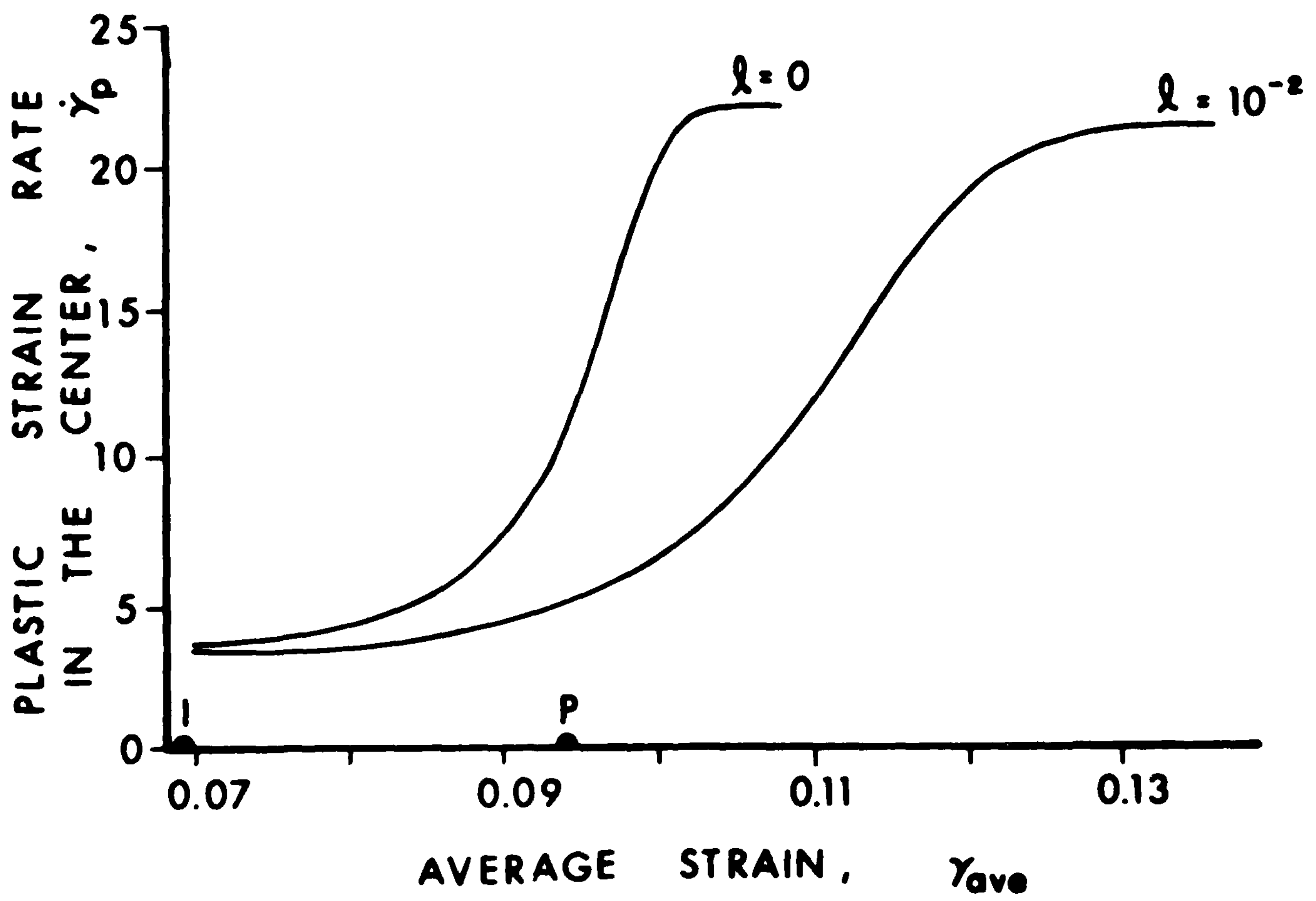


Figure 3: Comparison of Solutions for $\ell = 0.0$ and $\ell = 0.01$ by the CNGFE Method.

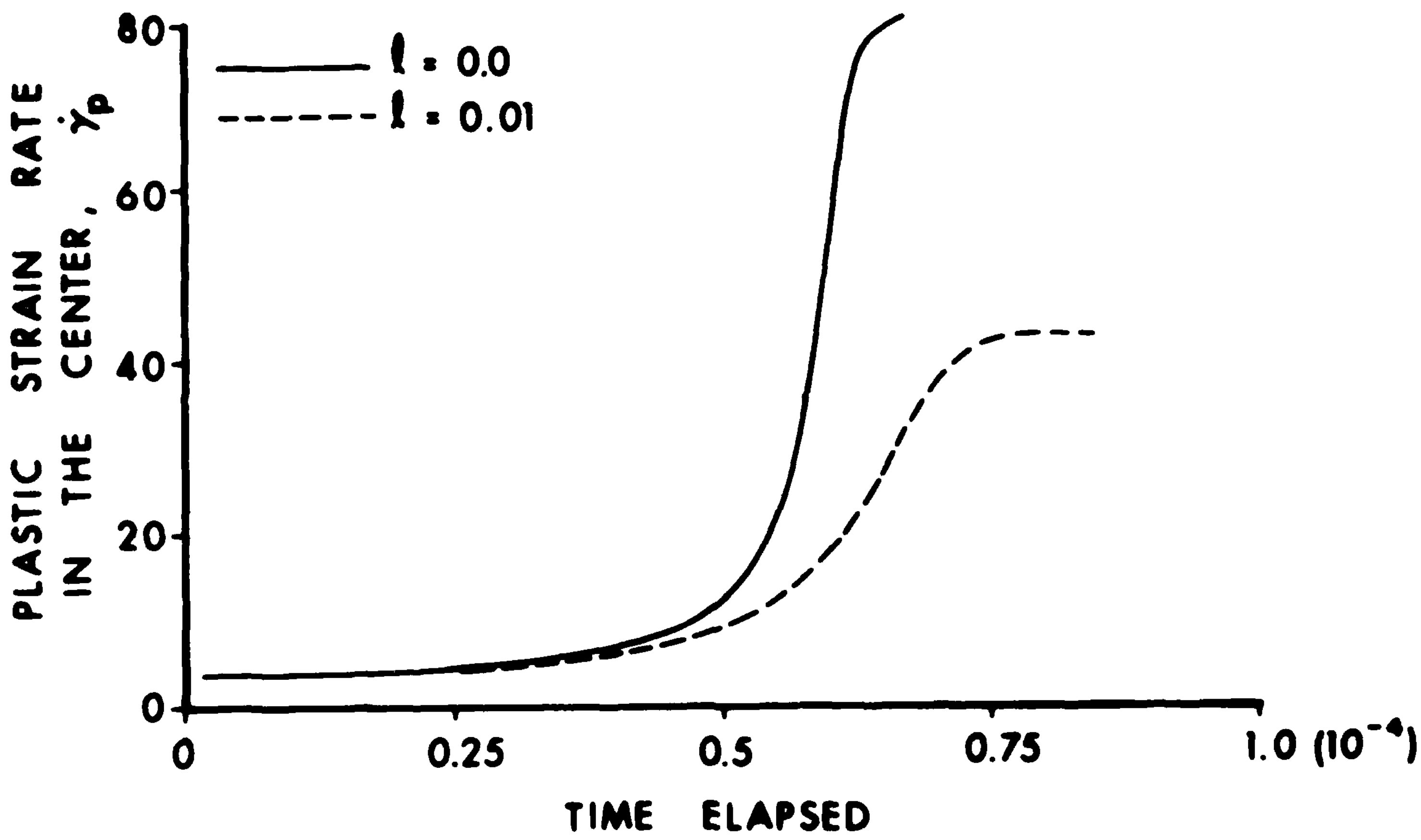


Figure 4: Comparison of Solutions for $\ell$ = 0.0 and $\ell$ = 0.01 by the FDGFE Method.

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# APPENDIX

# CODE LISTING

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      DIMENSION LD(4),ETAU(5),YY(2),ESIG(5), WORK(5,30)
     +  ,VELDOT(31), DDDOT(5,30), SIGMA(5,30)
     + ,PGDOT(5,30),PDDOT(5,30),EGDOT(5,30),TEMP(5,30)
     +   ,PGAMA(5,100),SI(5,100),TMP(31),PD(5,100),TAU(5,30)
C
C
C      READ THE INPUT DATA
C
      READ(5,1000) CM,CN,BETA,A,B,SIO,RHO,CV
      READ(5,1010) NT,NINT,DT,CK,CJ,GDOTO,CMU,CNU
      READ(5,1020) NUMEL,NODES,NPRINT,CL,HT
      READ(5,1025) TIME,RSTART
      CMU=CMU*1000.
C
C
 1000 FORMAT(8F10.4)
 1010 FORMAT(I5,I3,6F10.4)
 1020 FORMAT(3I5,2F10.5)
 1025 FORMAT(E10.4,F10.4)
C
C     PRINT OUT THE INPUT DATA.
C
      WRITE(6,2000) CM,CN,BETA,A,B,SIO,RHO,CV
      WRITE(6,2010) NT,NINT,DT,CK,CJ,GDOTO,CMU,CNU
      WRITE(6,2020) NUMEL,NODES,CL,HT,NPRINT
      WRITE(6,2025) TIME,RSTART
C
C
C       GENERATE NODE NUMBERS AND NON-DIMENSIONAL COORDINATES.
C
      CALL GRID(YCORD,NODE,NODES,NUMEL )
C
C
C     PRINT OUT THE NON-DIMENSIONAL NODAL COORDINATES.
C
      DO 35 J=1,NODES
   35 WRITE(6,2030) J,YCORD(J)
      DO 40 I=1,NUMEL
   40 WRITE(6,2040) I,NODE(1,I),NODE(2,I)
 2000 FORMAT(5X,'M=',E15.5/5X,'N=',E15.5/5X,'BETA=',E15.5/
     + 5X,'A=',E15.5/5X,'B=',E15.5/5X,'SIO=',E15.5/
     + 5X,'RHO=',E15.5/5X,'CV=',E15.5)
 2010 FORMAT(5X,'NO. OF TIME STEPS =',I10/
     + 5X,'NO. OF INTEG. POINTS USED IN NUMERICAL INTEGRATION =',I5/
     + 5X,'TIME INCREMENT =',E15.5/
     + 5X,'THERMAL CONDUCTIVITY =',E15.5/5X
     + ,'FACTOR TO CONVERT FROM JOULES TO KG-M =',E15.5/
     + 5X,'PRESCRIBED STRAIN RATE =',E15.5/5X,'MU = ',E15.5/
     + 5X,'NU = ',E15.5)
 2020 FORMAT(5X,'NUMBER OF ELEMENTS =',I10/5X,
     + 'NUMBER OF NODES =',I10
     + /5X,'MATERIAL LENGTH = ',E15.5/5X,
     + 'HEIGHT OF THE SPECIMEN (H IN METERS ) = ',E15.5
     + /5X, 'PRINT INTERVAL =',I5)
 2025 FORMAT(5X,'TIME AT THE START OF THIS RUN =',E15.5/5X,
     +'RESTART JOB IF RSTART = 0.0;OTHERWISE NOT A',1X,
     +'RESTART JOB.'/5X,'RSTART = ',E15.5)
C
C      COMPUTE NON-DIMENSIONAL NUMBERS
C
      CKAPAO = BETA*(SIO**CN)
```

```
      [illegible]
      A = A*TR
      FACT1 = 4.0*CK/(HT*HT*RHO*CV*GDOTO)
      FACL = 2.*CL/HT
      RHO = RHO*((HT*GDOTO/2.)**2)/CKAPAO
      RHOI=1.0/RHO
      CMU = CMU/CKAPAO
      CNU = CMU*2./(CKAPAO*HT*CL)
      WRITE(6,2055) CKAPAO,FACT1,RHO,CMU,CNU,A,TR
 2055 FORMAT(5X,'KAPAO = ',E15.5/5X,'THERMAL FACTOR = ',E15.5/5X,
     + 'INERTIA FACTOR = ',E15.5/5X,'MUBAR = ',E15.5/5X,
     + 'NU-BAR = ',E15.5/5X,'NON-DIMENSIONAL A = ',E15.5
     + /5X,'REFERENCE TEMPERATURE = ',E15.5)
 2030 FORMAT(5X,I5, F15.5)
 2040 FORMAT(5X,3I10)
C
C     READ THE PARAMETERS CONTROLLING THE DISTURBANCE.
C
      READ(5,1000) ALFA,CNN,EPSILON
      READ(5,1030) HTAU,HGP,HSI,HGPDOT,HTMP,HGT
C     READ (5,1031) TIME,HTAU,HGP,HGT,HTMP,HSI,HGPDOT
C1031 FORMAT(50X,E20.10/10X,E20.10/24X,E20.10/23X,E20.10/
C    + 23X,E20.10/14X,E20.10/30X,E20.10)
C     WRITE(6,2031) HTAU,HGP,HGT,HTMP,HSI,HGPDOT
      WRITE(6,2050) ALFA,CNN,EPSILON
 2050 FORMAT(5X,'VALUES OF VARIABLES CONTROLLING THE DISTURBANCE'/
     + 5X,'ALFA = ',E15.5/5X,'N = ',E15.5/5X,'EPSILON = ',E15.5)
C
C     GENERATE VALUES OF SHAPE FUNCTIONS AND THEIR DERIVATIVES.
C
      CALL SHAPES (NINT)
C
C     CALCUALTE VALUES OF GAMA-DOT AND D-DOT CAUSED BY
C       THE INITIAL DISTURBANCE.
C
      IF(RSTART.NE.0.0)
     +CALL DISTURB(YCORD,NODE,EGDOT,DDDOT, ALFA,CNN, EPSILON
     + ,NUMEL,NINT,TEMP,TMP,NODES,NTMP )
C
C     CALCULATE THE INITIAL YIELD STRESS.
      CK = ((1.0+HSI/SI0)**CN)*(1.0+B*GDOTO*HGPDOT)**CM
C
C    TAUO = TAUO/CKAPAO.
C
      TAUO = HTAU
C
C     SET INITIAL STRESS = YIELD STRESS.  THE INITIAL DISTURBANCE ALTERS
C       THE TOTAL STRAIN RATE AT A NODE POINT.
C
C     SET THE INITIAL STRAIN RATE = STRAIN RATE DUE TO HOMOGENEOUS
C       DEFORMATION + STRAIN RATE CAUSED BY THE DISTURBANCE.
C       NON-DIMENSIONALIZED STRAIN RATE DUE TO HOMOGENEOUS DEFORMATION
C       IS EQUAL TO 1.0
C
      IF(RSTART.EQ.0.0) GO TO 55
      DO 45 NEL = 1, NUMEL
      DO 45 INT = 1, NINT
      EGDOT(INT,NEL) = EGDOT(INT,NEL) + 1.0
   45 CONTINUE
C     SET INITIAL STRESS = STRESS CAUSED BY THE DISTURBANCE + TAUO
C       STRESSES CAUSED BY THE DISTURBANCE ARE TAKEN EQUAL TO 0
C
      DO 46 I = 1,NUMEL
      DO 46 J = 1,NINT
```

```
      TR = CKAPAO/(RHO*CV*CJ)
      A = A*TR
      FACT1 = 4.0*CK/(HT*HT*RHO*CV*GDOTO)
      FACL = 2.*CL/HT
      RHO = RHO*((HT*GDOTO/2.)**2)/CKAPAO
      RHOI=1.0/RHO
      CMU = CMU/CKAPAO
      CNU = CMU*2./(CKAPAO*HT*CL)
      WRITE(6,2055) CKAPAO,FACT1,RHO,CMU,CNU,A,TR
 2055 FORMAT(5X,'KAPAO = ',E15.5/5X,'THERMAL FACTOR = ',E15.5/5X,
     + 'INERTIA FACTOR = ',E15.5/5X,'MUBAR = ',E15.5/5X,
     + 'NU-BAR = ',E15.5/5X,'NON-DIMENSIONAL A = ',E15.5
     + /5X,'REFERENCE TEMPERATURE = ',E15.5)
 2030 FORMAT(5X,I5, F15.5)
 2040 FORMAT(5X,3I10)
C
C     READ THE PARAMETERS CONTROLLING THE DISTURBANCE.
C
      READ(5,1000) ALFA,CNN,EPSILON
      READ(5,1030) HTAU,HGP,HSI,HGPDOT,HTMP,HGT
C     READ (5,1031) TIME,HTAU,HGP,HGT,HTMP,HSI,HGPDOT
C1031 FORMAT(50X,E20.10/10X,E20.10/24X,E20.10/13X,E20.10/
C    + 23X,E20.10/14X,E20.10/30X,E20.10)
C     WRITE(6,2031) HTAU,HGP,HGT,HTMP,HSI,HGPDOT
      WRITE(6,2050) ALFA,CNN,EPSILON
 2050 FORMAT(5X,'VALUES OF VARIABLES CONTROLLING THE DISTURBANCE'/
     + 5X,'ALFA = ',E15.5/5X,'N = ',E15.5/5X,'EPSILON = ',E15.5)
C
C     GENERATE VALUES OF SHAPE FUNCTIONS AND THEIR DERIVATIVES.
C
      CALL SHAPES (NINT)
C
C     CALCUALTE VALUES OF GAMA-DOT AND D-DOT CAUSED BY
C       THE INITIAL DISTURBANCE.
C
      IF(RSTART.NE.0.0)
     +CALL DISTURB(YCORD,NODE,EGDOT,DDDOT, ALFA,CNN, EPSILON
     + ,NUMEL,NINT,TEMP,TMP,NODES,HTMP )
C
C     CALCULATE THE INITIAL YIELD STRESS.
      CK = ((1.0+HSI/SIO)**CN)*(1.0+B*GDOTO*HGPDOT)**CM
C
C    TAUO = TAUO/CKAPAO.
C
      TAUO = HTAU
C
C     SET INITIAL STRESS = YIELD STRESS.  THE INITIAL DISTURBANCE ALTERS
C       THE TOTAL STRAIN RATE AT A NODE POINT.
C
C     SET THE INITIAL STRAIN RATE = STRAIN RATE DUE TO HOMOGENEOUS
C       DEFORMATION + STRAIN RATE CAUSED BY THE DISTURBANCE.
C       NON-DIMENSIONALIZED STRAIN RATE DUE TO HOMOGENEOUS DEFORMATION
C       IS EQUAL TO 1.0
C
      IF(RSTART.EQ.0.0) GO TO 55
      DO 45 NEL = 1, NUMEL
      DO 45 INT = 1, NINT
      EGDOT(INT,NEL) = EGDOT(INT,NEL) + 1.0
   45 CONTINUE
C     SET INITIAL STRESS = STRESS CAUSED BY THE DISTURBANCE + TAUO
C       STRESSES CAUSED BY THE DISTURBANCE ARE TAKEN EQUAL TO 0
C
      DO 46 I = 1,NUMEL
      DO 46 J = 1,NINT
```

```
      SIGMA(J,I) = 0.0
   46 TAU(J,I) = CK*(1.0-A*TEMP(J,I))
  55  CONTINUE
C     FOR THE RESTART JOB, READ VELOCITY,TEMP,SI,TAU,SIGMA.
      IF(RSTART.NE.0.0) GO TO 70
      DO 60 NEL = 1,NUMEL
      DO 60 INT = 1,NINT
      READ(5,1070) TAU(INT,NEL),SIGMA(INT,NEL),PGAMA(INT,NEL)
      READ(5,1071) SI(INT,NEL),PGDOT(INT,NEL),TEMP(INT,NEL),PD(INT,NEL)
      READ(5,1072) EGDOT(INT,NEL),DDDOT(INT,NEL)
 60    CONTINUE
 1070 FORMAT(10X,3E20.12)
 1071 FORMAT(2X,4E19.12)
 1072 FORMAT(2X,2E19.12)
      DO 65 NOD = 1, NODES
 65   READ(5,1075) VELDOT(NOD),TGDOT(NOD),TMP(NOD)
 1075 FORMAT(15X,3E15.7)
      CALL DD(DDDOT,NINT,NUMEL,NODE,VELDOT,TGDOT,YCORD,EGDOT,DT,NZERO,
     +NTT)
  70  CONTINUE
      CMI=1./CM
      GDOTOI = 1.0/GDOTO
C
C
 1030 FORMAT(6F10.4)
      WRITE(6,2031) HTAU,HGP,HSI,HGPDOT,HTMP,HGT
 2031 FORMAT(5X,'HTAU =',E15.5/5X,'HGP =',E15.5/5X,'HSI =',E15.5/
     + 5X,'HGPDOT =',E15.5/5X,'HTMP =',E15.5/5X,'HGT =',E15.5)
C
      NDF=2*NODES
      MB=4
      DO 990 NTT=1,NT
C
C       INITIALIZE THE GLOBAL MASS MATRIX AND THE GLOBAL FORCE VECTOR.
C       THE MASS MATRIX IS GENERALIZED IN THE SENSE THAT IT INCLUDES
C      THE INERTIA TERMS APPROPRIATE FOR D-DOUBLE DOT.  THE FORCE VECTOR
C       EQUALS THE RESULTANT OF FORCES CAUSED BY STRESSES AND
C       DIPOLAR STRESSES DUE TO THE DISTURBANCE.
C
      DO 50 I=1,NDF
      FORCE(I)=0.0
      IF(NTT.NE.1) GO TO 50
      DO 49 J=1,MB
   49 AMASS(J,I) = 0.0
   50 CONTINUE
C
C
      TIME = TIME + DT*GDOTOI
C
      NCOUNT = NTT/NPRINT
      NZERO  = NCOUNT * NPRINT - NTT
C FIND THE HOMOGENEOUS SOLUTION FOR THE PRESENT VALUE OF TIME.
C
      CALL HOMOG(HTMP,HGT,HGP , HTAU,HSI,CMI, CM,A,B,GDOTO
     + , CMU,DT,SIO,TIME, HGPDOT,NZERO)
C
C
C     ASSEMBLE THE GL0BAL MASS MATRIX.
C
      DO 900 NELM=1,NUMEL
      DO 100 I=1,2
      II=NODE(I,NELM)
      YY(I) = YCORD(II)
  100 CONTINUE
```

```
      DO  105 INT = 1, NINT
      ETAU(INT) = TAU(INT,NELM)
  105 ESIG(INT) = SIGMA(INT,NELM)
C
C     CALCULATE THE ELEMENT MASS MATRIX AND THE FORCE VECTOR
C
      CALL ELEMENT(YY,ETAU,ESIG,EFORCE,EMASS,NINT,FACL,NTT)
      DO 110 II=1,2
      JJ=2*II
      I = NODE(II,NELM)*2
      LD(JJ-1)=I-1
  110 LD(JJ)=I
      MC=0
      DO 130 II=1,4
      I=LD(II)
      FORCE(I) = FORCE(I) + EFORCE(II)
      IF(NTT.NE.1) GO TO 130
      DO 129 JJ=1,II
      MC=MC+1
      M=LD(JJ)
      IR=MINO(I,M)
      IC=IABS(I-M)+1
  129 AMASS(IC,IR)=AMASS(IC,IR)+EMASS(MC)
  130 CONTINUE
  900 CONTINUE
      DO 915 J=1,NDF
      FORCE(J) = FORCE(J)*RHOI
  915 CONTINUE
C
C MODIFY THE MASS MATRIX FOR THE PRESCRIBED BOUNDARY CONDITIONS.
C
C    AT THE BOUNDARY NODES (THE FIRST AND THE LAST NODE)
C         GAMA AND VELOCITY IS PRESCRIBED.
C     THEREFORE, WE SET GAMA - DOUBLE-DOT = 0.0 AND ACCELERATION = 0.0
C       AT THESE NODES.
C
      II = 2*NODES-1
      CALL MODIFY(1,0.0,AMASS,FORCE,NDF,MB,MB)
      CALL MODIFY(II,0.0,AMASS,FORCE,NDF,MB,MB)
      IF(NTT.EQ.1)
     +CALL SOLVE(AMASS,FORCE,NDF,MB,MB,1)
      CALL SOLVE(AMASS,FORCE,NDF,MB,MB,2)
C
C
C
C     NOW THE ARRAY FORCE CONTAINS TOTAL NODAL ACCELERATIONS AND NODAL
C   STRAIN DOUBLE-DOTS CAUSED BY THE DISTURBANCE. FROM THESE VALUES FIND
C      VALUES OF DDOT.
C
C     TRANSFER VALUES FROM THE ARRAY FORCE INT ARRAYS VELDOT AND DDOT.
C
 2110 FORMAT(5X,'ACCELERATION DUE TO DISTURBANCE AND TOTAL GAMA-2 DOT'/
     + 5X,'NODE #',5X,'ACCELERATION',5X,'GAMA-DOUBLE DOT')
      DO 200 I = 1, NODES
      II = 2*I
      VELDOT(I) = FORCE(II-1)
      TGDOT(I) = FORCE(II)
 2100 FORMAT(5X,I5,2E15.5)
  200 CONTINUE
C
C
C FIND PLASTIC PARTS OF EGDOT AND DDDOT.  DENOTE THESE
C     BY PGDOT,PDDOT.
C
```

```
      CALL PLASTIC(EGDOT,DDDOT,PGDOT,PDDOT, CMI,GDOTO,SIGMA, CN,
     + A,B,CMU,CNU,DT,TAUO,TEMP,SIO, NTT,NUMEL,NINT,WORK,TAU
     +,HSI,HGP,HGPDOT,PGAMA,SI,RSTART,PD)
C
C      CALCULATE TOTAL DDOT AND TOTAL GDOT.
      CALL DD(DDDOT,NINT,NUMEL,NODE,VELDOT,TGDOT,YCORD,EGDOT,DT,NZERO,
     + NTT)
C
C
C     SOLVE THE THERMAL PROBLEM
C
      CALL THERM(TEMP ,WORK,FACT1, NUMEL,NINT,NTT,DT,TMP
     + ,YCORD,NODES,NODE)
      IF(NZERO. NE. 0) GO TO 990
      DO 985 NEL = 1,NUMEL
      DO 985 INT = 1, NINT
      WRITE(6,2400) NEL,INT,TAU(INT,NEL),SIGMA(INT,NEL),PGAMA(INT,NEL)
      WRITE(6,2401) SI(INT,NEL),PGDOT(INT,NEL),TEMP(INT,NEL),PD(INT,NEL)
      WRITE(6,2401) EGDOT(INT,NEL),DDDOT(INT,NEL)
985   CONTINUE
      DO 980 NOD=1,NODES
980    WRITE(6,2290) NOD,VELDOT(NOD),TGDOT(NOD),TMP(NOD)
 2290  FORMAT(10X,I5,3E15.7)
 2400 FORMAT(2X,2I4,3E20.12)
 2401 FORMAT(2X,4E19.12)
  990 CONTINUE
      STOP
      END
      SUBROUTINE HOMOG(HTMP,HGT,HGP ,HTAU,HSI,CMI, CN,A,B,GDOTO
     + , CMU,DT,SIO,TIME, HGPDOT,NZERO)
      HCKAPA1 = (1.0 + HSI/SIO)**CN
      HCKAPA = HCKAPA1 * (1.0 - A * HTMP)
      QT=HTAU/HCKAPA
      IF(QT.LT.1.0)GO TO 500
      R2=QT**CMI - 1.0
      GAMA = R2/(B*GDOTO *HTAU)
      GO TO 600
  500 WRITE(6,2000) QT,TIME
      GAMA = 0.0
  600 CONTINUE
      HSIDOT = GAMA*(HTAU*HTAU/HCKAPA1)
      HGPDOT = GAMA * HTAU
      HTMPDOT = HGPDOT * HTAU
      HTAUDT=CMU*(1.0-HGPDOT)
      HSI=HSI+HSIDOT*DT
      HTAU=HTAU+HTAUDT*DT
      HGP=HGP+HGPDOT*DT
      HGT=HGT+   DT
      HTMP = HTMP + HTMPDOT*DT
      IF(NZERO.EQ.0)WRITE(6,2005) TIME,HTAU,HGP,HGT,HTMP,HSI,HGPDOT
 2005 FORMAT(5X,'HOMOGENEOUS SOLN. AT PHYSICAL TIME (SECS.) =',E20.10/
     + 5X,'TAU =',E20.10/5X,'GAMA -PLASTIC =',E20.10/
     + 5X,'TOTAL STRAIN =',E20.10/5X,'TEMPERATURE = ',E20.10/
     + 5X,'SI = ',E20.10
     + /5X,'PLASTIC STRAIN-RATE =',E20.10)
      RETURN
 2000 FORMAT(5X,'FOR THE HOMOG. SOLN. THE MATERIAL IS DEFORMING'
     + ,1X,'ELASTICALLY'/
     +5X,'RATIO OF STRESS TO HARDENING FUNCTION =',F15.5/
     +5X,'CURRENT VALUE OF TIME = ',F15.5)
      END
      SUBROUTINE SHAPES(NINT)
      COMMON/SHAPES/SHAPO(2,5),SHAP1(2,5),DSHAPO(2,5),DSHAP1(2,5),
     + DDSHAPO(2,5),DDSHAP1(2,5),SHAPB(2,5),DSHAPB(2,5)
```

```
      DIMENSION SI(4)
      SI(1) = -0.861136311594053
      SI(2) = -0.339981043584856
      SI(3) = - SI(2)
      SI(4) = - SI(1)
C
C EVALUATE THE SHAPE FUNCTIONS AND THEIR DERIVATIVES.
C
      DO 100 I = 1,NINT
      S=SI(I)
      SHAPO(1,I) = (2.0 - 3.0*S + S**3)/4.0
      SHAPO(2,I) = (2.0 + 3.0*S - S**3)/4.0
      SHAP1(1,I) = (1.0 - S - S*S + S**3)/4.0
      SHAP1(2,I) = (-1.0- S + S*S + S**3)/4.0
      DSHAPO(1,I) = (-3.0 +3.0*S*S)/4.0
      DSHAPO(2,I)= (3.0 - 3.0*S*S)/4.0
      DSHAP1(1,I)= (-1.0-2.0*S+3.0*S*S)/4.0
      DSHAP1(2,I)= (-1.0 +2.0*S +3.0*S*S)/4.0
      DDSHAPO(1,I) = (6.0*S)/4.0
      DDSHAPO(2,I) = (-6.0*S)/4.0
      DDSHAP1(1,I) = (-2.0+6.0*S)/4.0
      DDSHAP1(2,I) = ( 2.0 + 6.0*S)/4.0
C
      SHAPB(1,I) = (1.0 - S)/2.0
      SHAPB(2,I) = (1.0 + S)/2.0
      DSHAPB(1,I) = -1./2.
      DSHAPB(2,I) = 1./2.
C
  100 CONTINUE
      RETURN
      END
      SUBROUTINE ELEMENT(YY,ETAU,ESIG,EFORCE,EMASS,NINT,FACL,NTT)
      COMMON/SHAPES/SHAPO(2,5),SHAP1(2,5),DSHAPO(2,5),DSHAP1(2,5),
     + DDSHAPO(2,5),DDSHAP1(2,5),SHAPB(2,5),DSHAPB(2,5)
      DIMENSION YY(2),ETAU(5),WEIGHT(5),EFORCE(4),EMASS(10),ESIG(5)
      WEIGHT(1) = 0.347854845137454
      WEIGHT(2) = 0.652145154862546
      WEIGHT(3) = WEIGHT(2)
      WEIGHT(4) = WEIGHT(1)
      MC=0
C
      DO 100 I = 1,4
      EFORCE(I) = 0.0
      IF(NTT.NE.1) GO TO 100
      DO 50 J= 1, I
      MC=MC+1
 50   EMASS(MC)=0.0
  100 CONTINUE
      DO 200 INT = 1,NINT
      WT=WEIGHT(INT)
      DJAC = (YY(2) - YY(1))*0.50
C     WRITE(6,2010) DJAC
 2010 FORMAT(5X,'DJAC = ',E15.5)
      IF(NTT.NE.1) GO TO 185
      ST2 = WT*DJAC
      EMASS(1) = EMASS(1) + SHAPO(1,INT)*SHAPO(1,INT)*ST2
      EMASS(2) = EMASS(2) + SHAPO(1,INT)*SHAP1(1,INT)*ST2
      EMASS(3) = EMASS(3) + SHAP1(1,INT)*SHAP1(1,INT)*ST2
      EMASS(4) = EMASS(4) + SHAPO(1,INT)*SHAPO(2,INT)*ST2
      EMASS(5) = EMASS(5) + SHAP1(1,INT)*SHAPO(2,INT)*ST2
      EMASS(6) = EMASS(6) + SHAPO(2,INT)*SHAPO(2,INT)*ST2
      EMASS(7) = EMASS(7) + SHAPO(1,INT)*SHAP1(2,INT)*ST2
      EMASS(8) = EMASS(8) + SHAP1(1,INT)*SHAP1(2,INT)*ST2
      EMASS(9) = EMASS(9) + SHAPO(2,INT)*SHAP1(2,INT)*ST2
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      EMASS(10)= EMASS(10)+ SHAP1(2,INT)*SHAP1(2,INT)*ST2
  185 CONTINUE
      IF(DJAC.LE.1.0E-20) GO TO 3000
      WT1=WT*FACL/DJAC
      GO TO 190
 3000 WRITE(6,2020) DJAC
 2020 FORMAT(5X,'DJAC = ',E15.5)
      STOP
  190 CONTINUE
      EFORCE(1) = EFORCE(1) - (ETAU(INT)*DSHAP0(1,INT)*WT +
     + ESIG(INT)*DDSHAP0(1,INT)*WT1)
      EFORCE(2) = EFORCE(2) - (ETAU(INT)*DSHAP1(1,INT)*WT +
     + ESIG(INT)*DDSHAP1(1,INT)*WT1)
      EFORCE(3) = EFORCE(3) - (ETAU(INT)*DSHAP0(2,INT)*WT +
     + ESIG(INT)*DDSHAP0(2,INT)*WT1)
      EFORCE(4) = EFORCE(4) - (ETAU(INT)*DSHAP1(2,INT)*WT +
     + ESIG(INT)*DDSHAP1(2,INT)*WT1)
  200 CONTINUE
C
C     WRITE(6,2030) EFORCE(1),EFORCE(2),EFORCE(3),EFORCE(4)
 2030 FORMAT(5X,4E15.6)
C
      RETURN
      END
      SUBROUTINE SOLVE(A,B,NN,MB,MMAX,KK)
      DIMENSION A(1),B(1)
C
C    SOLUTION OF SYMMETRIC BANDED EQUATIONS IN SINGLE SUBSCRIPT ARITH.
C
      MB1=MB-1
      NNN=NN-1
      IF(KK.EQ.2) GO TO 2000
      II=1
      DO 300 N=1,NNN
      CC=A(II)
      IF(CC.EQ.0.0) GO TO 250
      J1=II+1
      J2=II+MB1
      NE=NN-N
      IF(NE.LT.MB1) J2=II+NE
      M=II-1
      DO 200 J=J1,J2
      M=M+MMAX
      IF(A(J).EQ.0.0) GO TO 200
      C=A(J)/CC
      K=M
      DO 100 I=J,J2
      K=K+1
  100 A(K)=A(K)-C*A(I)
      A(J)=C
  200 CONTINUE
  250 CONTINUE
      II=II+MMAX
  300 CONTINUE
      RETURN
 2000 II=1
      DO 500 N=1,NNN
      CC=A(II)
      IF(CC.EQ.0.0) GO TO 450
      J1=II+1
      J2=II+MB1
      NE=NN-N
      IF(NE.LT.MB1)J2=II+NE
        C=B(N)
```

```
      L=N
      DO 400 J=J1,J2
      L=L+1
  400 B(L)=B(L)-A(J)*C
      B(N)=C/CC
  450 CONTINUE
      II=II+MMAX
  500 CONTINUE
      CC=A(II)
      IF(CC.NE.0.0) B(NN)=B(NN)/CC
      N=NN
      II=MMAX*(NN-2)+1
      DO 700 I=2,NN
      N=N-1
      IF(A(II).EQ.0.0) GO TO 650
      J1=II+1
      J2=II+MB1
      NE=NN-N
      IF(NE.LT.MB1) J2=II+NE
      C=B(N)
      L=N
      DO 600 J=J1,J2
      L=L+1
  600 C=C-A(J)*B(L)
      B(N)=C
  650 CONTINUE
      II=II-MMAX
  700 CONTINUE
      RETURN
      END
      SUBROUTINE MODIFY(N,Q,A,B,NN,MB,MMAX)
      DIMENSION A(MMAX,NN),B(NN)
C
C     MODIFICATION FOR PRESCRIBED ESSENTIAL BOUNDARY CONDITIONS.
C
      DO 100 J=2,MB
      L=N-J+1
      IF(L.LE.0) GO TO 50
      B(L)=B(L) - A(J,L)*Q
      A(J,L)=0.0
   50 L=N+J-1
      IF(L.GT.NN) GO TO 100
      B(L)=B(L) - A(J,N)*Q
  100 A(J,N) = 0.0
      B(N) = Q
      A(1,N) = 0.0
      RETURN
      END
      SUBROUTINE GRID(YCORD,NODE,NODES,NUMEL )
      DIMENSION YCORD(30),NODE(2,30)
C     THIS SUBROUTINE GENERATES NON-DIMENSIONAL CO-ORDINATES OF NODAL
C       POINTS.
C      Y - BAR = Y / (HT/2) .
      DY1 = 0.05
      DY2 = 0.09375000
      DY = DY2
      YI = -1.0
      DO 100 N = 1, NODES
      YCORD(N) = YI
      YI = YI + DY
      DY = DY2
      IF(N.GT.7.AND.N.LT.18) DY = DY1
  100 CONTINUE
C
```

```
C       GENERATE NODE NUMBERS
C
      NO = 1
      DO 200 N = 1,NUMEL
      NODE(1,N) = NO
      NODE(2,N) = NO + 1
      NO = NO + 1
  200 CONTINUE
      RETURN
      END
      SUBROUTINE DISTURB(YCORD,NODE,EGDOT,DDDOT, ALFA,CN,
     + EPSILON,NUMEL,NINT,TEMP,TMP,NODES,HTMP )
      COMMON/SHAPES/SHAPO(2,5),SHAP1(2,5),DSHAPO(2,5),DSHAP1(2,5),
     + DDSHAPO(2,5),DDSHAP1(2,5),SHAPB(2,5),DSHAPB(2,5)
      DIMENSION  YCORD(30),NODE(2,30),EGDOT(5,30),DDDOT(5,30)
     + ,TMP(31),TEMP(5,30)
      WRITE(6,2010)
 2010 FORMAT(5X,'NODAL VALUES OF VARIABLES CAUSED BY THE DISTURBANCE'/
     + 5X,'ELE. # INT. PT. #' ,9X, 'GAMADOT',9X,'D-DOT',15X,'TEMP.')
      DO 100 NEL = 1, NUMEL
      I1= NODE(1,NEL)
      I2 = NODE(2,NEL)
      DO 100 INT = 1, NINT
      S = SHAPB(1,INT)*YCORD(I1) + SHAPB(2,INT)*YCORD(I2)
      FAC = (1.0-S*S)
      FACN = FAC**CN
      EXPV = EXP(-ALFA*S*S)
      EGDOT(INT,NEL) = 0.0
      DDDOT(INT,NEL) = 0.0
      TEMP(INT,NEL) = HTMP + EPSILON*FACN*EXPV
      WRITE(6,2000)NEL,INT,S,EGDOT(INT,NEL),DDDOT(INT,NEL),TEMP(INT,NEL)
 2000 FORMAT(5X,2I5,4E15.5)
  100  CONTINUE
      DO 200 I=1,NODES
      S = YCORD(I)
      TMP(I) = HTMP +  ((1.-S*S)**CN)*EPSILON*(EXP(-ALFA*S*S))
      WRITE(6,2000) I,I,S,TMP(I)
  200 CONTINUE
      RETURN
      END
      SUBROUTINE THERM(TEMP, WORK,FAC1, NUMEL,NINT,NTT,DT,TMP
     +,YCORD,NODES,NODE)
      COMMON/SHAPES/SHAPO(2,5),SHAP1(2,5),DSHAPO(2,5),DSHAP1(2,5),
     + DDSHAPO(2,5),DDSHAP1(2,5),SHAPB(2,5),DSHAPB(2,5)
      DIMENSION TEMP(5,30),WORK(5,30),NODE(2,30),YT(2),YCORD(30)
      DIMENSION    TMP(31),DISSIP(3),HEAT(2,31),
     +   THCOND(31,31),FORCE(31)
     \DIMENSION LD(3),TH(6),EMASS(6),WEIGHT(4)
      DATA WEIGHT/0.347854845137454, 0. 652145154862546,
     + 0.652145154862546,0.347854845137454/
C
C      GENERATE THE MATRICES
C
      MB = 2
      DO 250 I = 1 , NODES
      FORCE(I) = 0.0
      IF(NTT.NE.1) GO TO 250
      DO 230 J = 1, MB
  230 HEAT(J,I) = 0.0
      DO 231 J = 1,NODES
  231 THCOND( J, I ) = 0.0
  250 CONTINUE
      DO 900 NEL = 1,NUMEL
      DO 300 I = 1, 2
```

```
      II = NODE(I,NEL)
      YT(I) =  YCORD(II)
  300 CONTINUE
      IF(NTT.NE.1) GO TO 510
      MC = 0
      DO 400 I = 1, 2
      DO 400 J = 1, I
      MC = MC + 1
      TH(MC) = 0.0
  400 EMASS(MC) = 0.0
      MC = 0
      DO 500 I = 1 , 2
      DO 500 J = 1 , I
      MC = MC + 1
      DJAC = (YT(2) - YT(1))*0.50
      IF(DJAC.LT.1.0E-10) GO TO 999
      DO 450 INT = 1,NINT
      WT = WEIGHT(INT)*DJAC
      WT1 = WEIGHT(INT)/DJAC
      EMASS(MC) = EMASS(MC) + SHAPB(I,INT)*SHAPB(J,INT)*WT
      TH(MC) = TH(MC) + DSHAPB(I,INT)*DSHAPB(J,INT)*WT1
  450 CONTINUE
  500 CONTINUE
  510 CONTINUE
      DO 550 I = 1, 2
      DISSIP(I) = 0.0
      DJAC = (YT(2) - YT(1))*0.50
      DO 550 INT = 1, NINT
      DJAC = DSHAPB(1,INT)*YT(1) + DSHAPB(2,INT)*YT(2)
      WT = WEIGHT(INT) * DJAC
      DISSIP(I) =  DISSIP(I) + SHAPB(I,INT)*WORK(INT,NEL)*WT
  550 CONTINUE
C ASSEMBLE THE GLOBAL MATRICES
C
      DO 600 II = 1,2
      I=NODE(II,NEL)
  600 LD(II) = I
      MC = 0
      DO 700 II = 1,2
      I=LD(II)
      FORCE(I) = FORCE(I) + DISSIP(II)
      IF(NTT.NE.1) GO TO 700
      DO 690 JJ = 1, II
      MC = MC + 1
      M = LD(JJ)
      IR = MINO(I,M)
      IC = IABS(I-M) + 1
      HEAT(IC,IR) = HEAT(IC,IR) + EMASS(MC)
      THCOND(I,M) = THCOND(I,M) + TH(MC)*FAC1
      THCOND(M,I) = THCOND(I,M)
  690 CONTINUE
  700 CONTINUE
  900 CONTINUE
      DO 950 I = 1,NODES
      IM1=I-1
      IF(I.EQ.1) IM1 = 1
      IP1 = I+1
      IF(I.EQ.NODES) IP1 = I
      DO 945 J = IM1,IP1
  945 FORCE(I) = FORCE(I) - THCOND(I,J)*TMP(J)
  950 CONTINUE
      IF(NTT.EQ.1)
     +CALL SOLVE(HEAT,FORCE,NODES,MB,MB,1)
      CALL SOLVE(HEAT,FORCE,NODES,MB,MB,2)
```

```
      DO 990 N=1,NODES
      TMP(N) = TMP(N) + FORCE(N)*DT
  990 CONTINUE
C
      DO 995 NEL = 1,NUMEL
      I1 = NODE(1,NEL)
      I2 = NODE(2,NEL)
      DO 995 INT = 1,NINT
  995 TEMP(INT,NEL) = TMP(I1)*SHAPB(1,INT) + TMP(I2)*SHAPB(2,INT)
      RETURN
  999 WRITE(6,2400) NEL,NINT,DJAC
 2400 FORMAT(5X,'DJAC FOR ELE.',I5,'INT. PT.',I5,'=',E15.5//
     + 5X,'THE EXECUTION OF THE PROGRAM IS BEING STOPPED IN SUB. THERM')
      STOP
      END
      SUBROUTINE DD(DDDOT,NINT,NUMEL,NODE,VELDOT,TGDOT,YCORD,EGDOT
     + , DT,NZERO,NTT)
      COMMON/SHAPES/SHAPO(2,5),SHAP1(2,5),DSHAPO(2,5),DSHAP1(2,5),
     + DDSHAPO(2,5),DDSHAP1(2,5),SHAPB(2,5),DSHAPB(2,5)
      DIMENSION DDDOT(5,30),NODE(2,30),VELDOT(31),TGDOT(31),
     + YCORD(30),EGDOT(5,30)
C
C         THIS SUBROUTINE COMPUTES TOTAL D-DOT AND TOTAL G-DOT.
C
C     IF(NTT.LE.5) WRITE(6,2010)
 2010 FORMAT(5X,'ELE.# INT. PT. #     TOTAL G-DOT',5X,'TOTAL D-DOT'//)
C
      DO 200 NEL = 1, NUMEL
      I1 = NODE(1,NEL)
      I2 = NODE(2,NEL)
      DJACI = 2.0/(YCORD(I2) - YCORD(I1))
      DO 200 INT = 1, NINT
      DDDOT(INT,NEL) = DDDOT(INT,NEL) + (DT*DJACI*DJACI)*
     + (DDSHAPO(1,INT)*VELDOT(I1) + DDSHAPO(2,INT)*VELDOT(I2) +
     +  DDSHAP1(1,INT)*TGDOT(I1)  + DDSHAP1(2,INT)*TGDOT(I2) )
      EGDOT(INT,NEL) = EGDOT(INT,NEL) + DT*DJACI*
     + ( DSHAPO(1,INT)*VELDOT(I1) + DSHAPO(2,INT)*VELDOT(I2) +
     +    DSHAP1(1,INT)*TGDOT(I1)  + DSHAP1(2,INT)*TGDOT(I2))
C     IF(NTT.LE.5)
C    +WRITE(6,2000)NEL,INT,EGDOT(INT,NEL),DDDOT(INT,NEL)
 2000 FORMAT(5X,2I5,2E15.5)
  200 CONTINUE
      RETURN
      END
      SUBROUTINE PLASTIC(EGDOT,DDDOT,PGDOT,PDDOT, CNI,GDOTO,SIGMA,
     +  CN,A,B,CMU,CNU,DT,TAUO,TEMP,SIO, NTT,NUMEL,NINT,WORK,TAU
     + , HSI,HGP,HGPDOT,PGAMA,SI,RSTART,PD)
      DIMENSION EGDOT(5,30),DDDOT(5,30),PGDOT(5,30),PDDOT(5,30)
     + ,PGAMA(5,100),SI(5,100),TEMP(5,30),TAU(5,30),SIGMA(5,30)
     + ,WORK(5,30),PD(5,100)
C
      IF(RSTART.EQ.0.0) GO TO 100
      IF(NTT.NE.1) GO TO 100
C     INITIALIZE SI,PGDOT,PDDOT,TEMP,PGAMA,PD,SIGMA,TAU AT EACH POINT
C
      DO 50 NEL = 1,NUMEL
      DO 50 INT = 1,NINT
      SI(INT,NEL)=HSI
      PGDOT(INT,NEL)=HGPDOT
      PDDOT(INT,NEL)=0.0
      PGAMA(INT,NEL)=HGP
      PD(INT,NEL)=0.0
      TAU(INT,NEL) = TAUO
      SIGMA(INT,NEL) = 0.0
```

```
   50 CONTINUE
C
  100 CONTINUE
C
      DO 500 NEL = 1,NUMEL
      DO 500 INT = 1,NINT
      SII=SI(INT,NEL)
      TMP=TEMP(INT,NEL)
      CKAPA1 =  ((1.0+SII/SIO)**CN)
      CKAPA = CKAPA1 * (1.0 - A*TMP)
      ESTRES = SQRT ( TAU(INT,NEL)*TAU(INT,NEL) +
     + SIGMA(INT,NEL)*SIGMA(INT,NEL))
      QT = ESTRES/CKAPA
      IF(QT.LT.0.0) GO TO 461
      IF(QT.LT.1.0) GO TO 450
      R2 = QT**CNI - 1.0
      GAMA = R2/(B*ESTRES*GDOTO)
      SIDOT = GAMA * ESTRES*ESTRES/CKAPA1
      GO TO 460
  450 CONTINUE
C     WRITE(6,2100) INT,NEL,QT
 2100 FORMAT(5X,'THE INT. PT.',I5,' OF ELE. # ',I3,'IS UNLOADING.'/
     + 5X,'TAU/KAPA = ',E15.5)
      GAMA = 0.0
      SIDOT = 0.0
  460 CONTINUE
      GPDOT = GAMA * TAU(INT,NEL)
      DPDOT = GAMA * SIGMA(INT,NEL)
      PGDOT(INT,NEL)=GPDOT
      PDDOT(INT,NEL) = DPDOT
      SI(INT,NEL) = SI(INT,NEL) + SIDOT*DT
      PGAMA(INT,NEL) = PGAMA(INT,NEL) + GPDOT*DT
      PD(INT,NEL) = PD(INT,NEL) + DPDOT*DT
C     WRITE(6,2000) NEL,INT,SIDOT,GPDOT,DPDOT
 2000 FORMAT(5X,2I5,3E15.5)
      WORK(INT,NEL) = CKAPA1 * SIDOT
      TAUDOT = CMU*(EGDOT(INT,NEL) - GPDOT)
      SIGDOT = CMU*(DDDOT(INT,NEL) - DPDOT)
      TAU(INT,NEL) = TAU(INT,NEL) + TAUDOT*DT
      SIGMA(INT,NEL) = SIGMA(INT,NEL) + SIGDOT*DT
  500 CONTINUE
      GO TO 462
  461 WRITE(6,2010) QT
 2010 FORMAT(////5X,'THE EXECUTION OF THE PROGRAM IS BEING
     + STOPPED. QT = ',E15.5)
      STOP
  462 CONTINUE
      RETURN
      END
```

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