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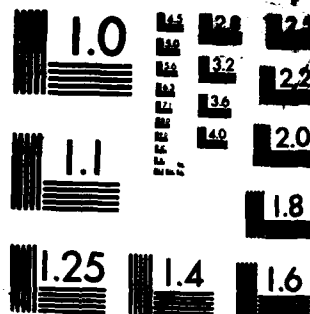EXPERIMENT STATION VICKSBURG MS ENVIR..
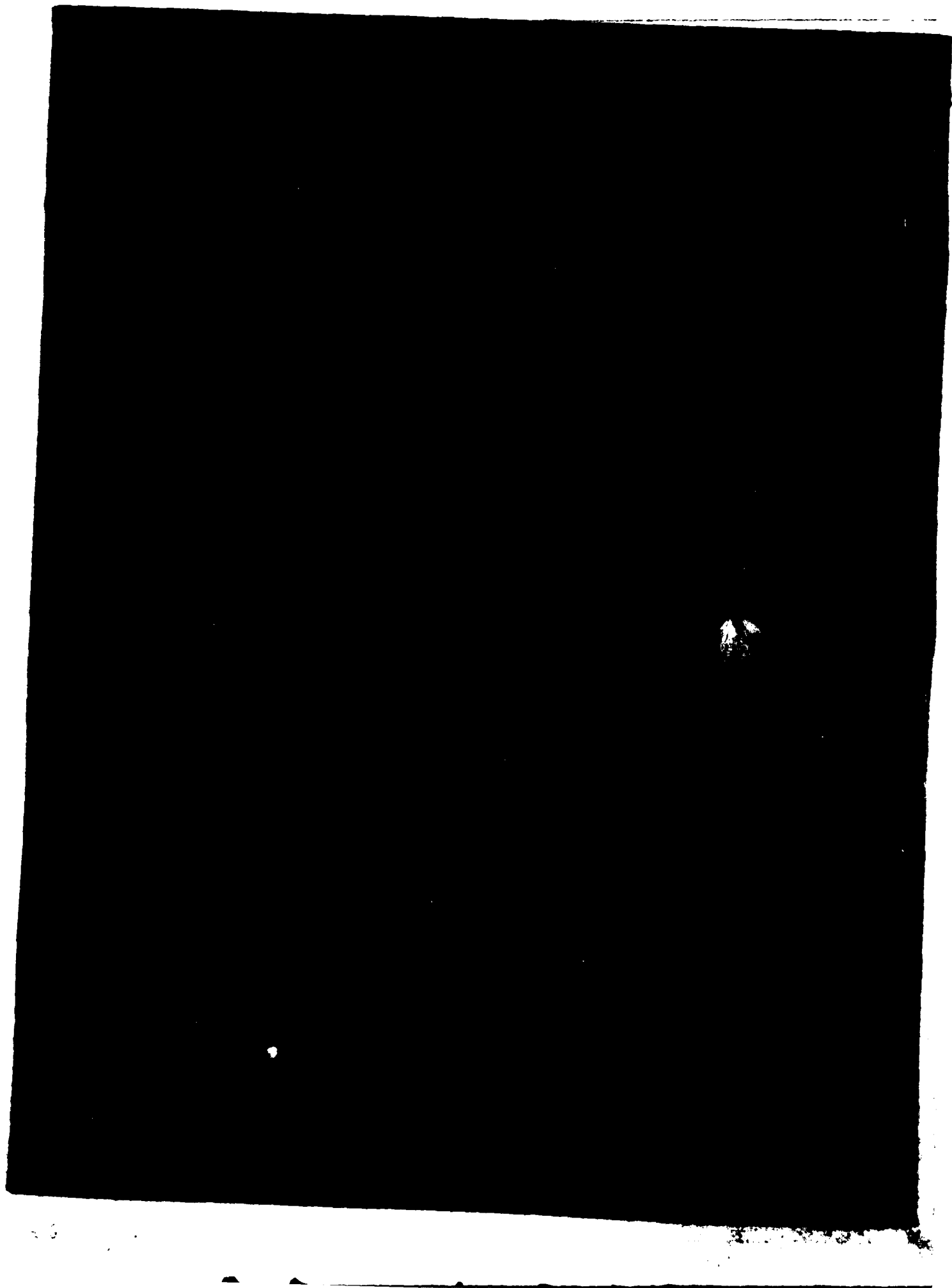UNCLASSIFIED   D R SMITH ET AL. MAR 87 WES/TR/E-87-2   F/G 13/2   NL

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Many significant studies over the past 35 years have combined to produce the present level of understanding of withdrawal processes. Analytical and empirical relationships have been developed and examined for many different density, flow, and geometric configurations. Analytical work has focused on withdrawal from two-layer or linearly stratified reservoirs. In particular, sidewall and bottom point-sink withdrawal are two geometries that have been extensively investigated. The descriptions developed by most investigators were expressions that reduced to the densimetric Froude number. It is troublesome, however, that the critical Froude numbers in these mathematical descriptions at similar flow conditions vary so greatly. In an effort to reduce the variance in these expressions, the authors use the concepts of "withdrawal angle" and symmetry to incorporate the effects of lateral geometry or topography into the critical Froude number description. The authors' development of this relationship greatly simplifies the array of coefficients that is

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Linear stratification	Stratified flow	
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19. ABSTRACT (Continued).

reported in the literature for various outlet configurations and patterns of density stratification. The relationship was developed through analytical evaluation of existing expressions and subsequent laboratory experimental verification. Observed prototype conditions, comparing favorably with predictions made with this technique, further verified its credibility. The authors explicitly included in the development of this more generalized description of withdrawal the effects of arbitrary boundary interference. For linear stratification, the developed relationship is descriptive of intermediate withdrawal and withdrawal with arbitrary interference and reduces to the analytical description of bottom or surface withdrawal. Results of experiments verified the applicability of this relationship for all three potential withdrawal conditions (i.e., intermediate, surface or bottom, and arbitrary interference).

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PREFACE

The research reported herein was conducted under the Environmental and Water Quality Operational Studies (EWQOS) Program, Work Unit IIIA.4, "Describe the Selective Withdrawal Characteristics of Various Outlet Configurations," which was sponsored by the Office, Chief of Engineers (OCE), US Army. The OCE Technical Monitors for EWQOS were Mr. Earl Eiker, Dr. John Bushman, and Mr. James L. Gottesman. Dr. Jerome L. Mahloch was Program Manager of EWQOS.

The research was conducted within the Reservoir Water Quality Branch (RWQB), Hydraulics Laboratory (HL), US Army Engineer Waterways Experiment Station (WES). Messrs. H. B. Simmons, former Chief of the HL; F. A. Herrmann, Jr., Chief of the HL; and J. L. Grace, Jr., Chief of the Hydraulic Structures Division and manager of EWQOS efforts in the HL, directed the study. Dr. Dennis R. Smith, former Chief of the RWQB, provided the basic premise upon which this research was conducted and performed the major portion of analysis. Messrs. Steven C. Wilhelms, Mark S. Dortch, and Jack E. Davis performed the physical modeling for verification of the conclusions drawn herein. Dr. Smith and Messrs. Wilhelms and Jeffery P. Holland, Chief of the RWQB, prepared this text. Mr. Dortch provided suggestions throughout the study and text preparation. The report was edited by Ms. Jessica S. Ruff of the WES Information Products Division.

COL Allen F. Grum, USA, was the previous Director of WES. COL Dwayne G. Lee, CE, is the present Commander and Director. Dr. Robert W. Whalin is Technical Director.

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CONTENTS

IMPROVED DESCRIPTION OF SELECTIVE WITHDRAWAL
THROUGH POINT SINKS

PART I: INTRODUCTION

Background

1. Application of the selective withdrawal concept has become a
very important alternative for controlling reservoir in situ and release
water quality. The significance of selective withdrawal has grown with
the increasing need for high-quality water resources to meet demands for
water supply, recreation, and wildlife. Selective withdrawal capabil-
ities can often provide the operational flexibility to optimally respond
to water quality demands. It is because of these interests and recogni-
tion of the attributes of selective withdrawal that research has con-
tinued on the processes governing withdrawal.

2. As a concept, selective withdrawal is relatively simple. It
is the capability to describe the vertical distribution of withdrawal
from a density-stratified reservoir and then to apply that capability at
appropriate depths to be selective about the quality of water that is
withdrawn. However, the simplicity ends with that statement. Over the
last 30 years, many researchers have examined fluid withdrawal from a
stratified medium. Most have proposed mathematical descriptions of the
processes that affect withdrawal. Results from these researchers form
the basis for the mathematical or numerical descriptions of selective
withdrawal that are used today.

3. As a general rule, analytical development of a withdrawal de-
scription has been idealized with a two-layer or linear stratification
pattern. For more complex stratification, experimentation provided
empirical modifications of the analytical descriptions. Further, most
researchers, analytical and experimental, have evaluated withdrawal
through a "point sink" or a "line sink." These simplifying assumptions
dictate that the outlet dimension be small relative to the depth or

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other critical dimension. The point sink concept would be applied for three-dimensional flow into a "point" (i.e., water quality intake). The line sink concept is applicable for two-dimensional flow into a "line" (i.e., linear intake diffuser).

4. In the literature survey (Part II), expressions describing withdrawal either implicitly or explicitly contain similar dimensionless groupings. However, there is considerable discrepancy among the analytical and experimental coefficients associated with these expressions. For example, eight different values are proposed for the critical Froude number for linear stratification. No systematic investigation of the various equations has been conducted, and thus no attempt has been made to develop equations that can or would be generally accepted for linear or two-layer stratification. In general, there are multiple equations for similar hydrodynamic regimes which fundamentally differ only in certain idealized assumptions or experimental configuration. Thus, a relationship should exist that is common to several of the conditions and expressions. The development of a more general description would greatly enhance and expand the understanding and applicability of selective withdrawal.

Purpose and Scope

5. For many Corps of Engineers (CE) reservoirs, the stratified flow field produced by withdrawal through an outlet portal located on a vertical face can be approximated by point sink equations. The point sink description most frequently used within the CE is that developed by Bohan and Grace (1969). They determined dimensionless groupings that characterized the withdrawal regime and subsequently developed empirical descriptions. The descriptions, developed for stratification and outlet conditions that frequently occur in CE reservoirs, were incorporated into the predictive numerical routine SELECT (Bohan and Grace 1969). Since the original development of SELECT in 1969, numerous improvements to the routine have been made (Bohan and Grace 1973, Davis et al. 1986). The general objectives of this investigation were to make

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additional improvements by extending the range of applicability of existing equations and to develop improved descriptions for certain conditions.

6. Analytical and experimental study was initiated to accomplish these objectives. The purpose of the analytical study was to develop new descriptions applicable to a broader range of stratification and geometric conditions. Numerous equations have been proposed for various idealized conditions. However, only minor attempts have been made to analyze the various equations for consistency and to develop more generalized equations. Thus, withdrawal descriptions were analyzed toward this end. The objective of the experimental program was to demonstrate the validity of newly developed withdrawal descriptions.

PART II: LITERATURE SURVEY AND ANALYSIS

7. With the exception of Bohan and Grace (1969), selective with-drawal studies for point sinks have predominantly addressed idealized stratification. In most studies, reservoir stratification was approximated as linearly stratified or composed of multiple linear segments. Analytical solutions have been limited to multiple fully mixed layers (Figures 1a and 1b) or to linear stratification (Figure 1c). Empirical descriptions have been proposed for arbitrary stratification profiles (Figure 1f), as well as the idealized conditions presented in Figures 1c-1e. The objective of this part of the report is to demonstrate consistency between the various equations that have been proposed for each type of stratification and subsequently to develop equations that are valid for a wider range of flow conditions.
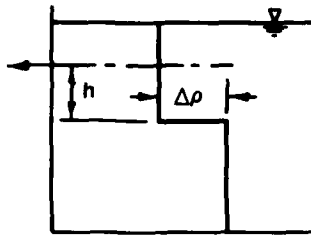
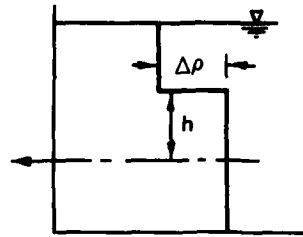### Two-Layer Stratification

Unbounded withdrawal

8. Withdrawal from two-layer stratification in which the with-drawal is from a layer that can be considered infinitely thick has received significant attention. Craya (1949) analytically and Gariel (1949) experimentally investigated withdrawal through a point sink on a vertical face. The sink was located in the upper layer at a distance h above the interface* (Figure 1a). Craya assumed the dimensions of the orifice were infinitesimal with respect to the thickness of the two layers and with respect to the distance between the interface and center line of the orifice. The vertical distance between the center line of the outlet and the water surface elevation was also assumed to be infinite. Craya's objective was to determine the critical discharge (i.e., the maximum discharge) from the upper layer without withdrawing any of the lower layer. The analysis indicated that the critical discharge was given by the following equation:
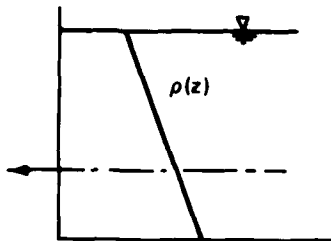
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* For convenience, symbols and abbreviations are listed in the Notation (Appendix B).
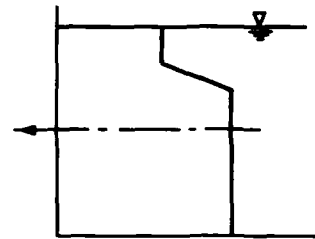
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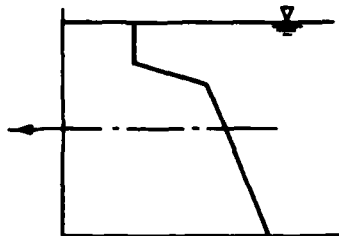a. Two-layer fluid
with sink above the
interface

b. Two-layer fluid
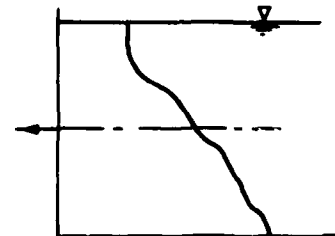with sink below the
interface

c. Linearly stratified
fluid

d. Two fully mixed
layers separated by
linear gradient

e. Piecewise linear ap-
proximation to reservoir
stratification

f. Arbitrary stratifica-
tion typical of reservoir
stratification

Figure 1. Definition diagrams for density profiles

$$\frac{Q_c}{\sqrt{\frac{g}{\rho}\,\Delta\rho h^5}} = 2.54 \qquad\qquad (1)$$

or

$$F_c = 2.54 \qquad\qquad (2)$$

where

Q_c = (critical) maximum discharge possible from one layer without withdrawing from the other layer

g = acceleration due to gravity

ρ = fluid density at the elevation of the orifice center line

$\Delta\rho$ = the density difference of the two layers

h = the vertical distance from the orifice to the interface separating the two layers

F_c = critical Froude number

Gariel approximated these idealized conditions (Figure 2a) and experimentally verified Equation 1. His criterion for incipient flow was that no more than 1 percent of the total release was withdrawn from the lower layer.

9. Craya indicated that the constant would be different if the proximity of the interface relative to the orifice modified the velocity field. Craya also noted that the vertical dimension of the orifice could affect withdrawal. This is consistent with point sink assumptions. By definition, point sink assumptions imply that the sink has infinitesimal dimensions with respect to the far-field withdrawal zone. Craya analytically deduced that the vertical dimension of the orifice has little or no effect upon the validity of Equation 1 as long as h was three to four times greater than the orifice diameter.

10. After reviewing these and other results, Brooks and Koh (1969) made several important observations. Since the density difference between the two layers ($\Delta\rho$) was assumed to be very small compared to ρ (Boussinesq approximation), the geometries may all be inverted, with an interchange of the lighter and heavier fluids. Therefore,

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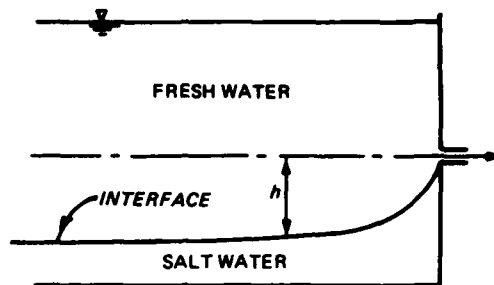
a. Schematic of Gariel (1949) experimental
investigation of point sink withdrawal
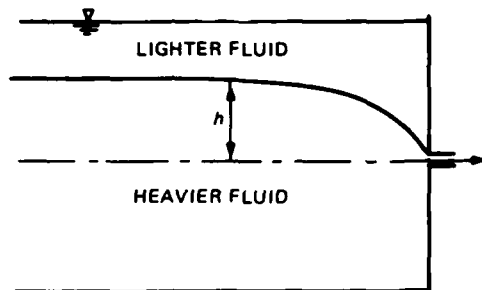
b. Schematic of drawing lighter fluid down

Figure 2. Schematic of conditions for which
Equation 1 is valid

Equation 1 would apply just as well to determining the critical dis-
charge for drawing a lighter fluid down a distance h from above (Fig-
ure 2b) as for drawing a heavier fluid up (Figure 2a) the same distance.
Additionally, the flow patterns in Figures 2a and 2b are similar when
one of them is inverted. From this observation, Brooks and Koh postu-
lated that the critical discharge for the axisymmetric flow condition
shown in Figure 3 should be given by

$$F_c = 5.1 \qquad\qquad (3)$$

11. Equation 3 was obtained by doubling the withdrawal rate to
allow for flow from both sides of the vertical plane shown in Figure 2.
Brooks and Koh also noted that if diameter d was small relative to

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distance h (Figure 3), the geometry of the outlet was unimportant (i.e., it did not matter whether the outlet was in a horizontal or vertical plane).
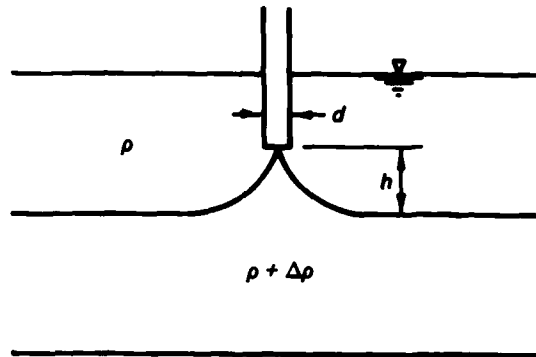

Figure 3.   Schematic of axisymmetric
critical flow

12. Wood (1978) also conducted a series of two-layer tests. Two of those experiments were conducted with an outlet on a vertical wall located below the interface. The stratification, based upon Wood's description, could be approximated as shown in Figure 1d. Initially, the stratification was not truly two-layer. The interface was diffusive but gradually became sharp as the experiments progressed, because of the gradual withdrawal of water at the elevation of the density gradient. Wood reported a critical Froude number of 2.04 as compared to the Craya and Gariel value of 2.54. However, Wood's primary objective was to determine the relative fraction of fluid withdrawn from each layer if the critical discharge was exceeded.

13. Jirka and Katavola (1979) reported the experimental determination of critical Froude numbers that were an order of magnitude less than those suggested by Craya (1949), Gariel (1949), and Wood (1978). However, the fully mixed upper and lower layers in the Jirka and Katavola experiments were separated by a very diffuse interface. The Froude numbers that characterized their interfaces were several orders of magnitude weaker than those investigated by most other researchers. Additionally, significant outlet diameters were used.

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14. Lawrence and Imberger (1979) conducted several axisymmetric withdrawal experiments for stratification conditions similar to those indicated in Figure 1d. Based upon a preliminary analysis, Lawrence and Imberger suggested a critical Froude number of 4.9. Lawrence (1980) noted that the relatively low critical Froude number was the result of diffuse interfaces and that, the sharper the interface, the closer the experimental results agreed with the theoretical value postulated by Brooks and Koh (1969). Lawrence then concluded that $F_c = 5.08$ for axisymmetric withdrawal from an infinite and ideal two-layer system.

15. The concept of "withdrawal angle" must be introduced at this point for inclusion in subsequent analysis and formulation. The withdrawal angle θ is measured in radians on a horizontal plane through the point sink. Figure 4 shows plan view schematics of various withdrawal angles relative to the point sink outlet. Using this concept and generalizing the Brooks and Koh (1969) symmetry arguments, it is readily deduced that all these results for two-layer stratification are described by

$$F_c = 2.54 \left(\frac{\theta}{\pi} \right) \tag{4}$$

16. Equation 4 is in exact agreement with Craya (1949) and Gariel (1949) for withdrawal through a point sink on a vertical face ($\theta = \pi$). For axisymmetric withdrawal ($\theta = 2\pi$), it is consistent with the experimental results of Lawrence and Imberger (1979) and Lawrence (1980). The largest disagreement is with Wood's (1978) results, which imply a critical Froude number approximately 20 percent lower than Equation 4. However, for a given critical discharge, this corresponds to only about a 9-percent difference in the distance from the sink to the interface. This is a remarkable agreement, considering that it is extremely difficult to determine the threshold of incipient flow. Additionally, it is difficult to create and maintain a density structure compatible with two-layer assumptions. Lawrence (1980) indicated that considerable care was required to restrict the interface thickness to less than 1 cm during testing and that diffuse interfaces resulted in an experimentally

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$\theta = \pi/4$

$\theta = \pi/3$

$\theta = \pi/2$

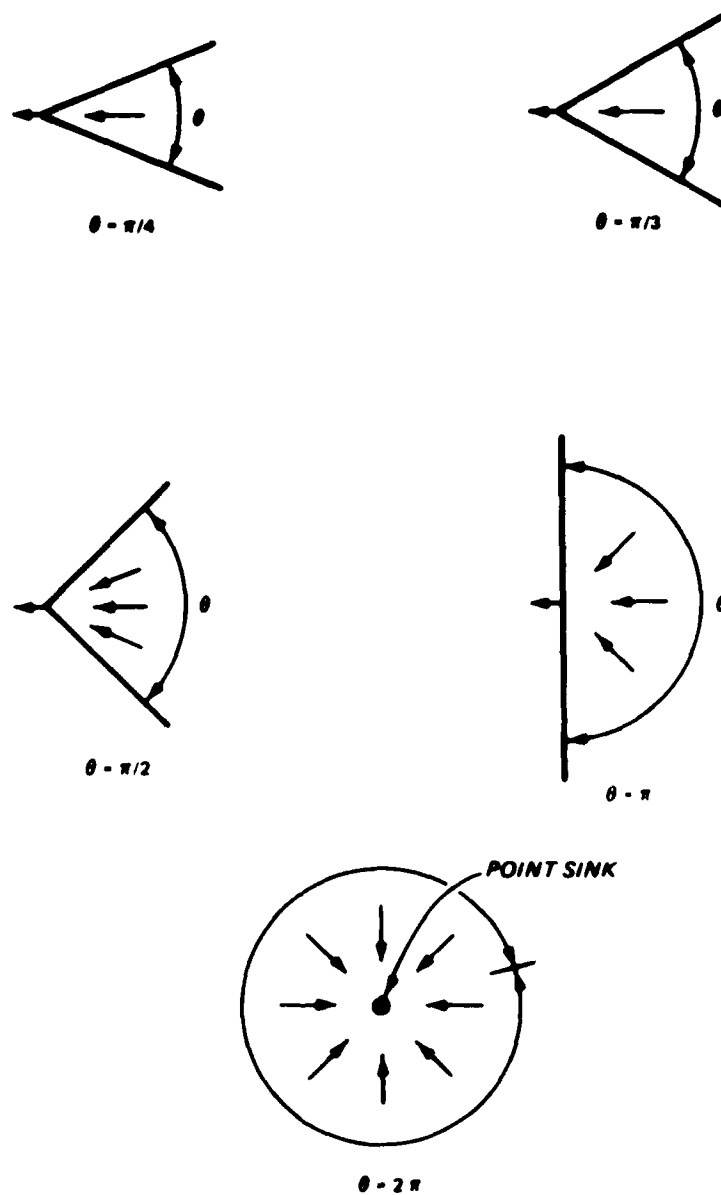$\theta = \pi$

POINT SINK

$\theta = 2\pi$

Figure 4. Schematic plan views of various
withdrawal angles

determined critical Froude number that was less than the theoretical
value.

17. Equation 4 should be valid for point sinks either above
or below the interface. It is rigorously valid, however, only for

idealized two-layer stratification. If used for prediction for more
diffuse interfaces, it will tend to overpredict critical discharge or,
for a given discharge, underpredict the distance h from the sink to
the "limit of withdrawal." For these cases, the actual critical Froude
number will be somewhat less than the analytical value.

Bottom withdrawal

18. As summarized by Yih (1980), Craya (1949) analytically deter-
mined that the critical Froude number for surface withdrawal (through a
vertical face) was 0.75. This result should be equally valid for bottom
withdrawal (based on symmetry arguments), although this aspect of the
Craya analysis has been largely ignored.

19. Harleman, Morgan, and Purple (1959) analytically and exper-
imentally investigated two-layer axisymmetric bottom withdrawal (Fig-
ure 5). The stagnant layer of lighter fluid lay over the flowing fluid

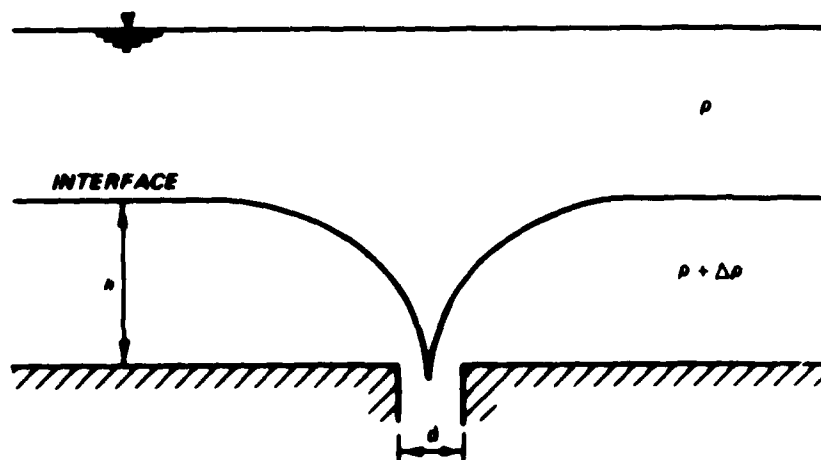Figure 5. Schematic of axisymmetric bottom withdrawal
investigated by Harleman, Morgan, and Purple (1959)

depth h . Yih (1980) noted that the Harleman, Morgan, and Purple
(1959) data were plotted with the initial assumption that Q_c varied
with the diameter d of the sink. Yih noted that Q_c is independent
of d as long as d is not too large. Thus, the critical Froude
number is given by

$$F_c = \frac{Q_o}{h^2 \sqrt{g \frac{\Delta\rho}{\rho} h}} = 1.61 \tag{5}$$

$$F_c = \frac{\pi}{2} \tag{6}$$

20. Wood (1978) presented an elegant analysis of bottom with-drawal from a two-layer stratification. A schematic of the case that he investigated is presented in Figure 6. For this general case, Wood

PLAN

ELEVATION
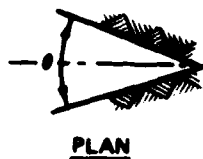
Figure 6. Schematic of the general case investigated by Wood (1978)
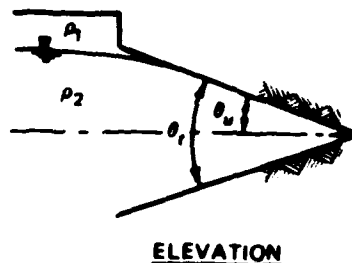
analytically deduced that the critical Froude number is given by

$$F_c = \frac{0.405}{\left(\epsilon_{2c}\right)^{1/2}} \cdot \frac{(1 + \gamma)}{\sin \theta_u} \tag{7}$$

where

ϵ_{2c} = ratio of the velocity in the withdrawal layer at the inter-face to the mean velocity in the withdrawal layer along the arc defined by a radial distance from the sink to the position of virtual control

γ = sin $(\theta_t - \theta_u)$/sin θ_u (see Figure 6)

21. In Wood's (1978) derivation, radial flow was assumed down-stream of the position of virtual control. For cases where θ_u is

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small, this assumption was reasonable; but as θ_u approached $\pi/2$, it was no longer valid. As a result, it was necessary to determine ϵ_{2c} from experimental data for the cases of practical interest, i.e., $\theta_u \approx \pi/2$. Based upon an analysis of two-layer bottom withdrawal, Wood recommended a value of 1.25 for $\epsilon_{2c}^{1/2}$ if $\theta_u \approx \pi/2$. Thus, for bottom withdrawal with $\theta_u \approx \pi/2$, Equation 7 can be written as

$$F_c \approx 1.02 \left(\frac{\theta}{\pi}\right) \tag{8a}$$

22. Binney (1975) analytically and experimentally investigated the outlet geometry indicated in Figure 7. Bryant and Wood (1976)
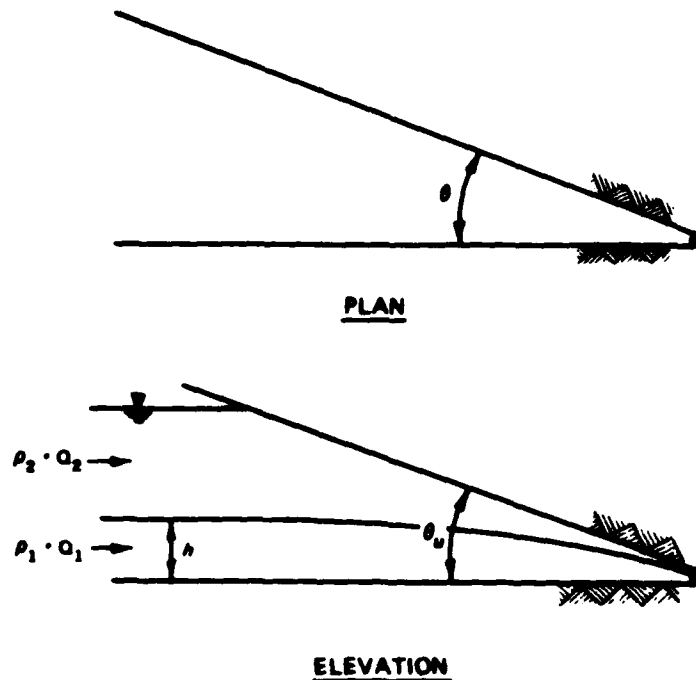

PLAN

ELEVATION

Figure 7. Schematic of rectangular contraction

extended Binney's work by analytically determining the maximum single-layer flow through contractions with square cross sections. For square cross sections, the maximum single-layer flux is given by

$$F_c \approx 0.405 \tag{8b}$$

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23. The results discussed above agree with each other well, espe-
cially considering the idealized assumptions in the derivations and the
experimental difficulty in determining the threshold of critical flow.
This is readily demonstrated by extending the Brooks and Koh (1969) sym-
metry arguments and generalizing Harleman's description (Equation 6) as

$$F_c = 0.81 \left(\frac{\theta}{\pi} \right) \tag{8c}$$

24. The difference in the critical Froude number implied by Equa-
tion 8c and Craya's analytical results (paragraph 18) is 7.4 percent.
Equation 8c can be shown to be in exact agreement with the critical
Froude number analytically determined by Bryant and Wood (1976) for the
maximum single-layer flow and bottom withdrawal ($\theta = \pi/2$) . It does not
agree with Wood's (1978) results as well as desired. This difference in
the critical Froude number results in a difference in the computed crit-
ical distance h of 10 percent for a given discharge and pycnocline.

25. The reason for the disparity may be due to the experimental
procedure employed by Wood, as discussed in paragraph 12. As noted
earlier, the interfaces at initiation of Wood's experiments were often
diffuse. Additionally, it should be noted that it was not Wood's pri-
mary objective to determine the critical Froude number. The coeffi-
cients quantifying critical conditions were determined from experiments
with both layers flowing. Thus, Equation 8c is consistent with experi-
mental and analytical results except for those of Wood (1978), with
Wood's results perhaps having limited applicability in this context.
However, it was the authors' opinion that Equation 8c should be verified
with additional experimental data.

Laboratory verification

26. Laboratory experiments with two-layer stratification and bot-
tom withdrawal were conducted at WES to verify Equation 8c. Two with-
drawal angles were evaluated: π and $\pi/2$. The upper, lighter water
layer was marked with Rhodamine-WT fluorescent dye. Critical discharge
or incipient withdrawal of the upper layer was determined by an increase
in release water fluorescence. When critical discharge was reached, the

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test was stopped (to prevent additional drawdown), and the density
structure was determined. Detailed descriptions of the facility, pro-
cedure, tests, and analysis techniques used for this experimentation are
presented in Appendix A.

27. Based on the experimentally observed stratification data and
the given withdrawal angle, the critical discharge was computed from
Equation 8d and plotted against the observed critical discharge in Fig-
ure 8. The excellent fit of these data clearly verifies the applicabil-
ity of Equation 8d for describing point sink bottom withdrawal from
two-layer stratification. Equally important, this demonstrates the
relevance of the withdrawal angle concept.

$$F_c = \frac{Q_c}{\sqrt{\frac{\Delta\rho}{\rho} gh^5}} = 0.81 \ \frac{\theta}{\pi} \qquad (8d)$$

Linear Stratification

28. Numerous equations have been proposed to predict the limits
produced by point sink withdrawal from a linearly stratified impoundment
(Figure 9). Most investigators have restricted their analysis to inter-
mediate or bottom withdrawal or implicitly treated them as separate en-
tities. The exception is Hino's (1980) analysis. He related intermedi-
ate and bottom withdrawal through symmetry arguments and provided a
basis to compare the various descriptions. Only Hino's results are dis-
cussed here; however, the reader is referred to the original manuscript
for details.

29. Hino used a pertubation method to obtain an approximate
steady-state solution for axisymmetric point sink withdrawal from a lin-
early stratified reservoir. He characterized far-field withdrawal by a
Froude number which used the total thickness of the flow field. For
intermediate withdrawal with no interception of boundaries (bottom
or free surface) by the withdrawal zone (Figure 10a), Hino's results

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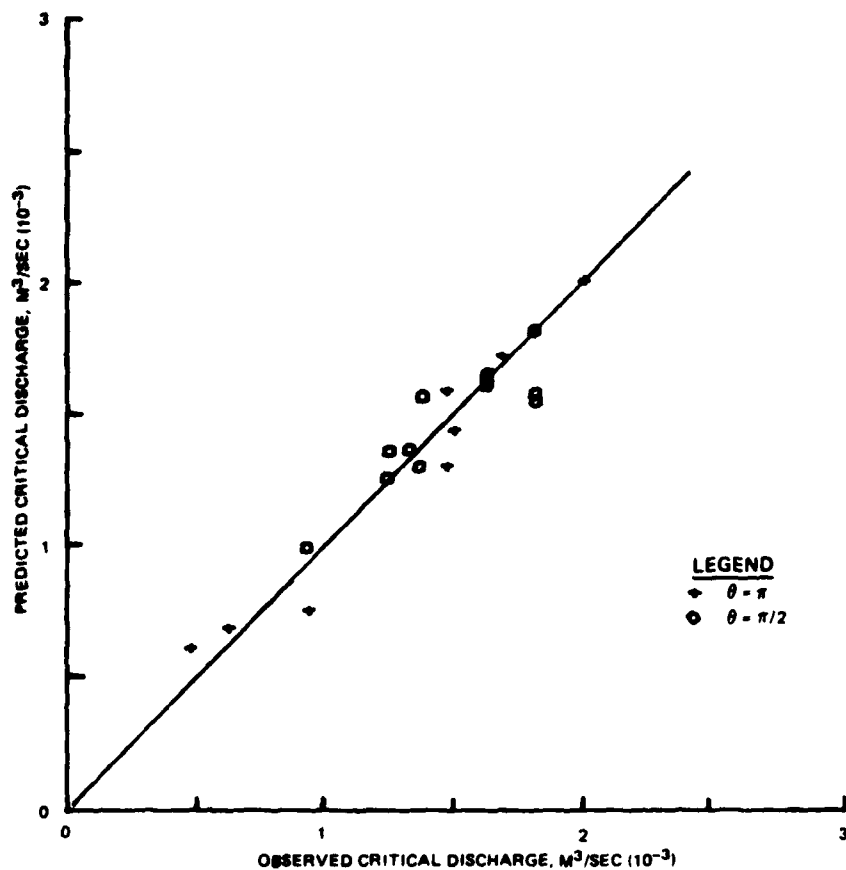
Figure 8. Comparison of critical discharges predicted with
Equation 8d and observed critical discharge
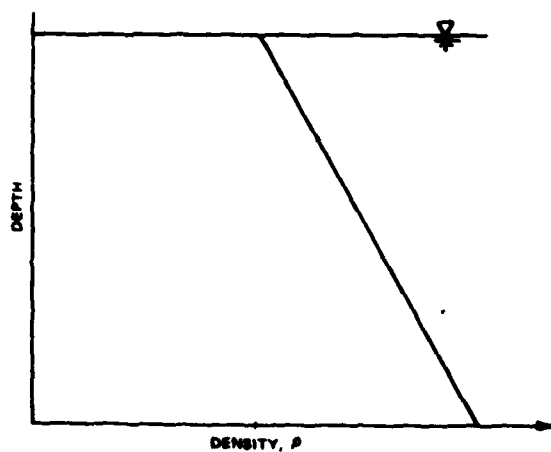
Figure 9. Linear stratification

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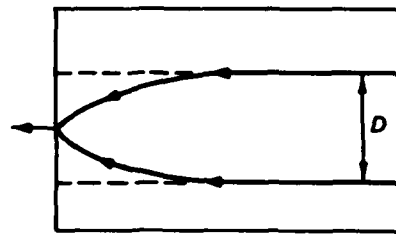
a. Intermediate withdrawal
   with no interference
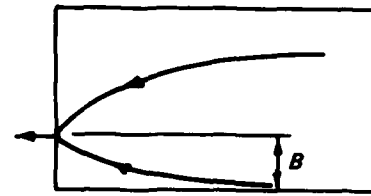
b. Boundary interference
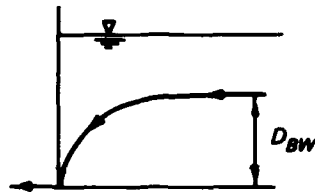
c. Bottom withdrawal
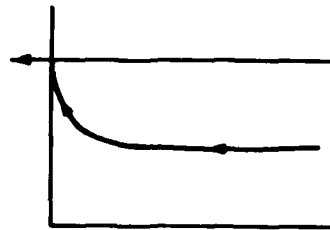
d. Surface withdrawal

Figure 10. Schematic of withdrawal conditions investigated
analytically by Hino (1980)

indicate that the free limits of withdrawal are described by

$$F_T = \frac{Q}{D^3 N} = 0.251 \qquad (9)$$

where

$$N = \sqrt{\frac{g}{\rho} \frac{d\rho}{dz}}$$

and

F_T = densimetric Froude number for total withdrawal zone

Q = withdrawal rate

D = thickness of withdrawal zone

19

N = buoyancy frequency

$d\rho/dz$ = density gradient

Hino indicated that field data obtained by Aki and Shirasuna (1974) and Adachi and Nakamura (1977) and laboratory data obtained by Hino and Furusawa (1969) confirmed Equation 9.

30. When boundary interference occurs, the withdrawal zone is truncated. To compensate for the resulting reduced flow contribution from the truncated portion of the flow field (i.e., that portion theoretically above the water surface), the unaffected limit forms farther from the sink when compared to limits for intermediate or no-interference withdrawal. Hino used Kao's (1965) method for the virtual line sink distribution to develop a theoretical relationship (Figure 11) between the Froude number of the withdrawal zone with boundary interference (such as bottom interference shown in Figure 10b) and the ratio of the distance B between the sink and the boundary of interference and the total thickness D of the withdrawal zone computed as if no interference existed. With this relationship it is possible to determine the location of the unaffected limit of withdrawal for a given set of conditions.

31. Hino noted that symmetry could be used to deduce F_T for axisymmetric bottom (Figure 10c) or surface withdrawal (Figure 10d). For axisymmetric intermediate withdrawal, F_T is given by Equation 11. For bottom withdrawal, symmetry dictates that the geometry and flow conditions correspond to half the depth and half the intake rate such that

$$Q_{BW} = 0.5 \ Q \tag{10}$$

$$D_{BW} = 0.5 \ D \tag{11}$$

where

Q_{BW} = flow through a sink at the bottom of the reservoir

D_{BW} = thickness of the withdrawal zone with bottom or top withdrawal

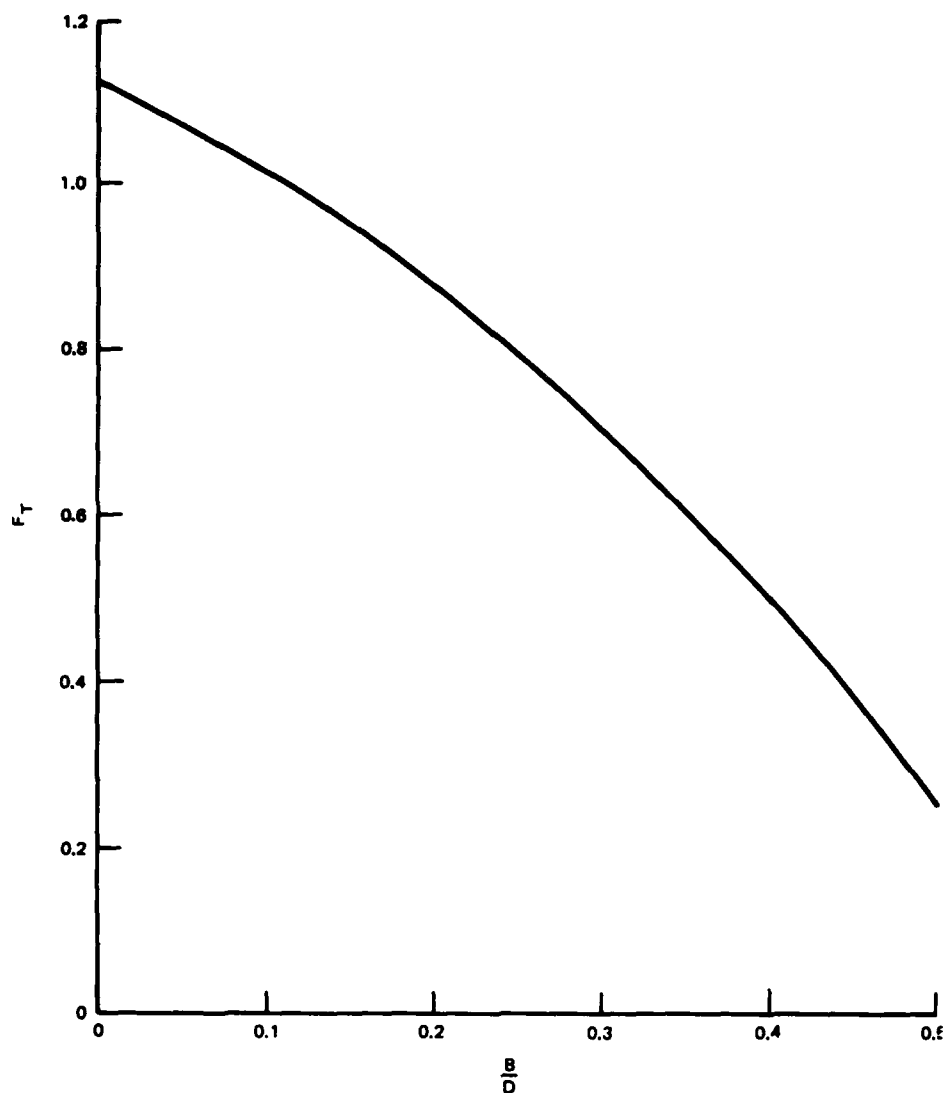and Q and D are as defined for Equation 9. Substituting

Figure 11. Theoretical curve for the Froude number for
boundary interference (Hino 1980)

Equations 10 and 11 into Equation 9, it is easily shown that densimetric
Froude number describing top or bottom withdrawal is

$$F_{BW} = \frac{Q_{BW}}{D_{BW}^3 \ N} = 1.0 \tag{12}$$

21

32. As shown in Figure 11, the Hino (1980) virtual sink analysis implies $F_{BW} = 1.125$ as compared to unity in Equation 12 at $B = 0$ (bottom or top withdrawal). Although this difference corresponds to only a 4-percent difference in the predicted thickness of the withdrawal zone, it creates some doubt about the analytical results presented in Figure 11. Hino (1980) recommended unity (for top and bottom withdrawal) based upon an analysis of Hino and Furusawa's (1969) laboratory experiments. These experiments were conducted with multilayered stratification that approximated linear stratification. Further, for boundary withdrawal, the point sink was located at the bottom boundary.

33. Croach (1971) analytically addressed axisymmetric point sink withdrawal from the bottom of a linearly stratified pool by using a dual stream function approach. He obtained the critical distribution of flow as a function of depth with a variational method based upon an extremal hypothesis that the flow is confined to a minimum layer thickness. For axisymmetric bottom withdrawal, Croach's descriptions of the free limit can be written as

$$\frac{Q_{BW}}{D_{BW}^3 \, N} = 0.789 \tag{13}$$

He made no attempt to generalize the above results for intermediate withdrawal.

34. Farrant (1982) experimentally investigated the intermediate withdrawal zone produced by a release through a point sink on a vertical face. His primary objective was to determine the limits produced by steady-state intermediate withdrawal from a linearly stratified reservoir. Through dimensional arguments, he deduced that the limits could be characterized by

$$\frac{Q}{z_i^3 N} = c_i \tag{14}$$

where

z_i = distance between the center line of the sink and the upper or lower free limit

c_i = experimentally determined constant

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35. To evaluate c_i , Farrant conducted a *series of experiments* in a relatively small tank. Some scatter appears to be inherent in the experimental data because of significant drawdown during testing due to the limited volume of the tank. However, Farrant noted that the experimental limits were affected by two processes. In the early phase of these experiments, there was a tendency for test conditions to asymptotically approach steady-state withdrawal. Later, however, there were gradual changes in the density profile with respect to time. The time required to develop a quasi-steady withdrawal layer was much shorter than the time scale required for significant changes in the background density gradient. As a result, the limits determined for quasi-steady conditions during each experiment can be considered representative of steady-state conditions. However, these quasi-steady conditions did not necessarily correspond to linear conditions throughout the experiment because, as drawdown proceeded, the stratification became nonlinear. An example of this change is illustrated by the density profiles prior to and after withdrawal presented in Figure 12.

36. Because of the above effects, Farrant used two methods of data analysis. Initially, he used the average values of the upper and lower limits and the initial buoyancy frequency and determined that the steady state limits could be described by

$$\frac{Q}{\bar{Z}^3 N} = 0.951 \tag{15a}$$

where $\bar{Z}$ is the average of the distances from the sink to the upper or lower free limits.

37. Subsequently, Farrant used local values in an attempt to account for the asymmetrical stratification produced by drawdown. A regression analysis was used to determine the constant c that best satisfied

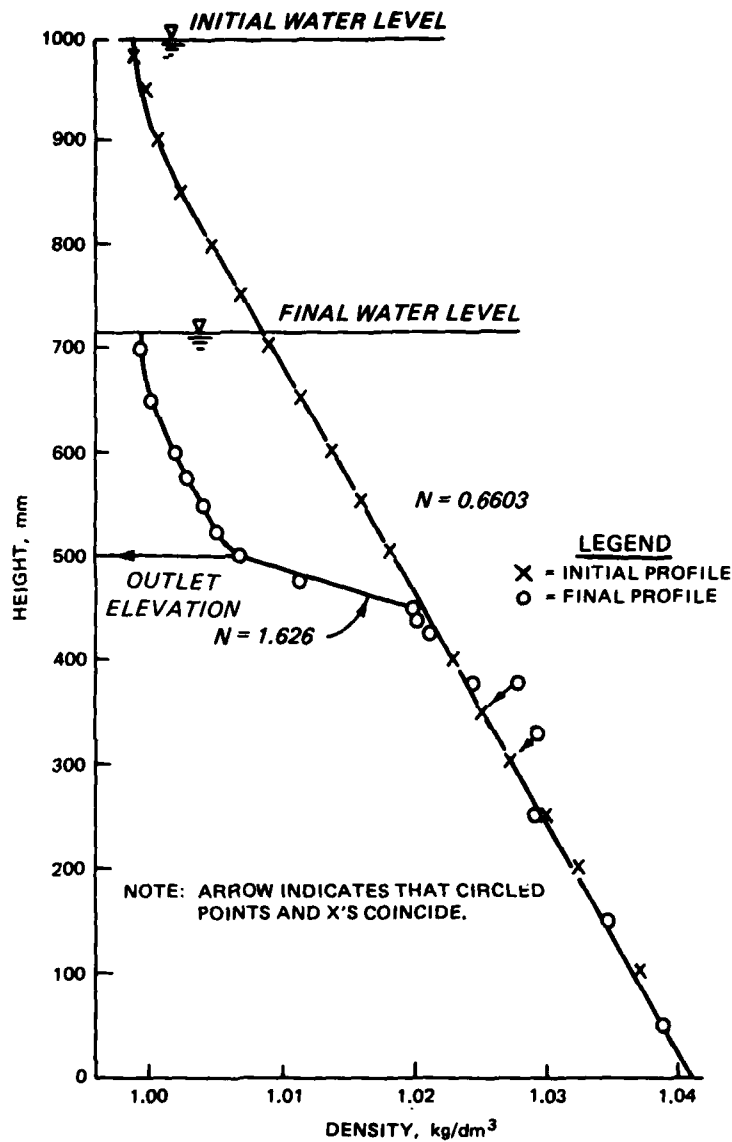$$\frac{Q_i}{z_i^3 N_i} = c \tag{15b}$$

23

Figure 12.   Initial and final density profiles
for Run 6 (Farrant 1982)

where

Q_i = the withdrawal through the respective upper or lower part
of the withdrawal zone

N_i = the average buoyancy frequency between the upper or lower
limit and the center line of the sink

i = upper or lower limit index

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38. With the second approach, Farrant obtained

$$\frac{Q_i}{z_i^3 N_i} = 0.364 \qquad (16)$$

From regression analysis of his experimental data, he found the coefficients of variation of Equations 15a and 16 to be 0.159 and 0.186, respectively. He recommended Equation 16, even though it had a larger coefficient of variation, because he considered it more descriptive of the effects of asymmetrical stratification on withdrawal.

39. Lawrence and Imberger (1979) experimentally investigated axisymmetric withdrawal from a vertical region that was initially linearly stratified. The test procedure was analogous to that of Farrant (1982). The experimental tank was relatively small, and effects similar to those discussed in paragraph 35 occurred, i.e., drawdown resulted in nonlinear stratification. Based upon a preliminary analysis of one of their experiments, Lawrence and Imberger suggested that the axisymmetric withdrawal limits could be approximated by

$$\frac{Q}{z_i^3 N} = 3.48 \qquad (17)$$

Subsequently, Lawrence (1980) reanalyzed these data and suggested that the limits were more accurately described by

$$\frac{Q}{z_i^3 N} = 1.48 \qquad (18)$$

However, Lawrence (1980) states that the constant should be considered preliminary.

40. The stratified flow field produced in an arbitrarily stratified reservoir by a point sink withdrawal through a vertical face was investigated by Bohan and Grace (1969). Part of their objective was to develop descriptions that were applicable to the stratification

conditions normally occurring in reservoirs. Laboratory experiments were conducted with stratification patterns and intensities that encompassed and often exceeded the range typically occurring in reservoirs. By using nonlinear density profiles, unequal upper and lower limits (an asymmetric withdrawal zone about the sink) often occurred. By using a large tank and relatively small releases with respect to the tank capacity, insignificant drawdown (or changes in stratification) occurred.

41. The research of Bohan and Grace (1969) is discussed in this section because their results are applicable to linear stratification as well as arbitrary stratification. Basically, the limits and the corresponding density differences $\Delta\rho$ were experimentally determined and plotted as shown in Figure 13. For prediction of the free limits, they recommended

$$V_o = \frac{Z_i^2}{A_o} \sqrt{\frac{\Delta\rho}{\rho_o} g Z_i}$$ (19)

where

V_o = average velocity through the orifice

A_o = area of the orifice opening

$\Delta\rho$ = density difference between the fluid at elevation of the orifice and the fluid at the upper or lower limit of the zone of withdrawal

ρ_o = fluid density at the elevation of the orifice

Equation 19 can be rewritten as

$$\frac{Q}{Z_i^2 \sqrt{\frac{g\Delta\rho_i Z_i}{\rho_o}}} = 1.0$$ (20)

where $Q = V_o A_o$.

42. Equation 20 was based upon density point values, indicating that the procedure used for data analysis by Bohan and Grace (1969) implicitly assumed density difference to be independent of path. As

26
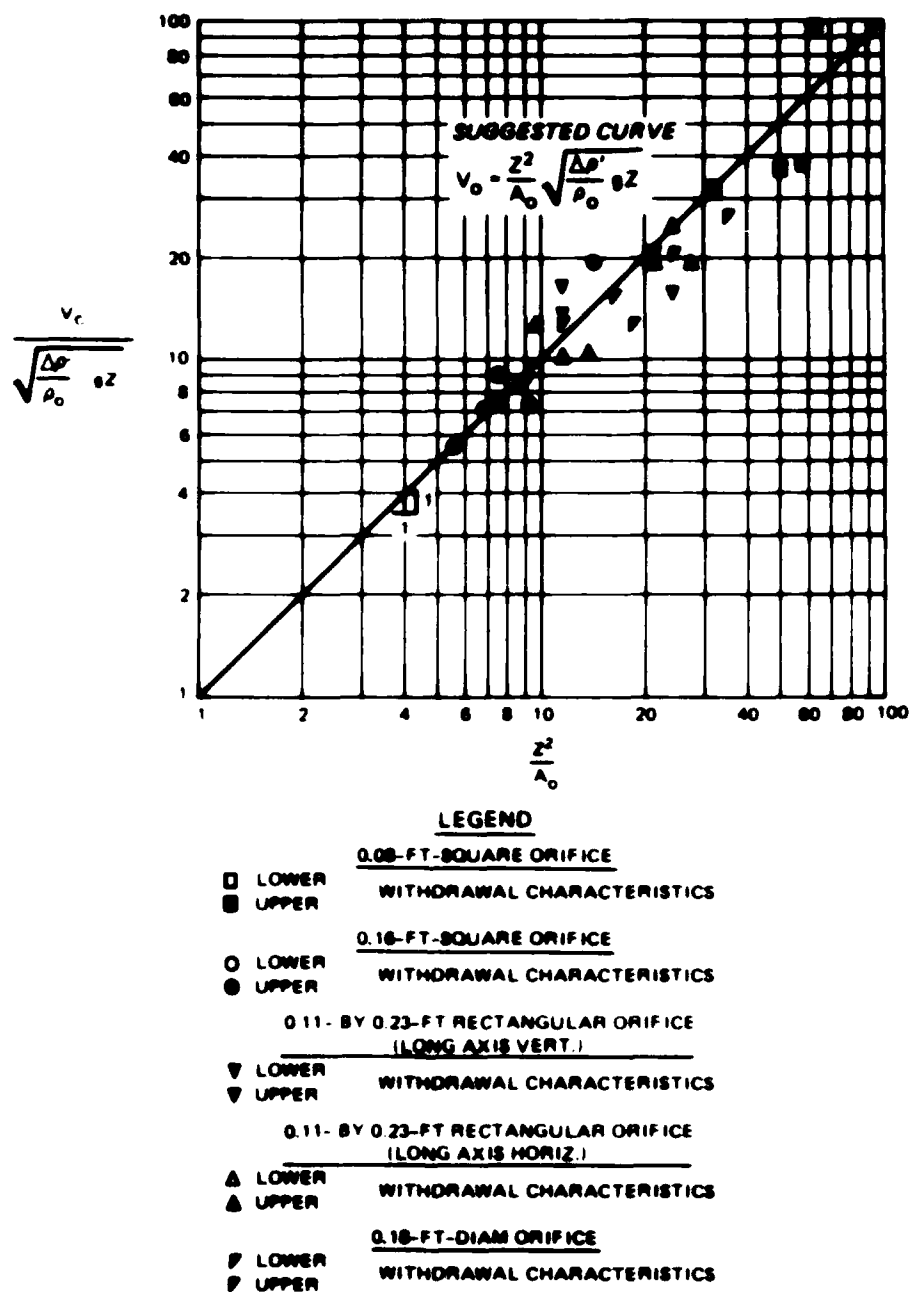
Figure 13. Withdrawal limits determined by Bohan and Grace (1969) (to convert feet to meters, multiply by 0.3048)

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Figure 14 shows, a variety of stratification patterns will satisfy this
assumption. Thus, the Bohan and Grace analysis was equally applicable
to a linear as well as an arbitrary density profile between the center
line of the outlet and the experimentally determined limits. Typical
stratification profiles investigated by Bohan and Grace and the experi-
mentally determined limits are shown in Figure 15.
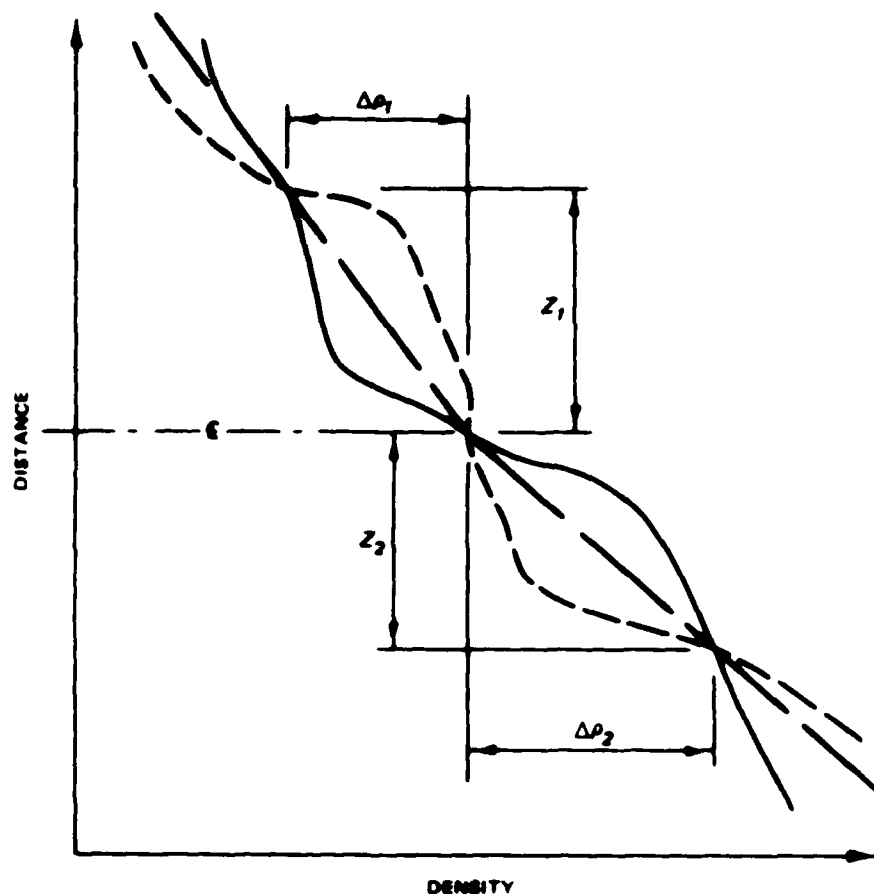
Figure 14. Schematic of various density profiles
that Equation 20 satisfies

Synthesis of Linear Stratification Results

43. Numerous equations have been suggested in the previous para-
graphs for the prediction of the free limits produced by withdrawal from

a linearly stratified reservoir. In general, each equation was devel-
oped for a specific withdrawal condition, as indicated in Table 1. By
using symmetry arguments, the respective equations can be converted to
other types of withdrawal conditions, each of which could be written in
the general form

$$\frac{Q}{z^3 N} = b_1 b_2 \left(\frac{\theta}{\pi}\right) \tag{21}$$

where

   b_1 = 1.0 for intermediate withdrawal; 0.5 for top or bottom
   withdrawal

   b_2 = a constant

The values of b_2 implied by the various equations are shown in
Table 2.

   44. The various results suggest a range for b_2 between 0.73
to 1.74, which seems to indicate that significant scatter exists in its
magnitude. However, this is very misleading. First, the largest value,
1.74, implied by Lawrence and Imberger's (1979) conclusions was consid-
ered preliminary by one of the authors; Lawrence's (1980) later recom-
mendations suggested a value of 0.74. Thus, with the exception of
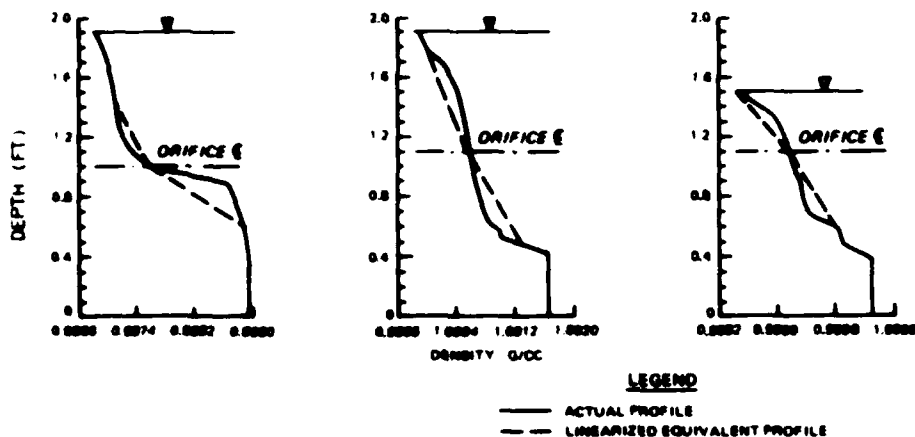
Figure 15. Typical density profiles for the experiments
conducted by Bohan and Grace (1969)

Table 1
Geometric Conditions for Which Point Sink Equations Have Been Developed

Investigator	Equation	Geometry	Withdrawal Angle
Croach (1971)	13	Axisymmetric	2π
Bohan and Grace (1969)	20	Intermediate withdrawal through a vertical face	π
Lawrence and Imberger (1979)	17	Axisymmetric intermediate withdrawal	2π
Lawrence (1980)	18	Axisymmetric intermediate withdrawal	2π
Hino (1980)	9	Axisymmetric intermediate withdrawal	2π
Hino (1980)	12	Axisymmetric top or bottom withdrawal	2π
Farrant (1982)	15, 16	Intermediate withdrawal through a vertical face	π

Table 2
Constants Deduced by Symmetry from Various Selective Withdrawal Descriptions

Investigator	Equation	b_2	Analytical	Experimental
Croach (1971)	13	0.79	X	
Bohan and Grace (1969)	20	1.00		X
Lawrence and Imberger (1979)	17	1.74		X
Lawrence (1980)	18	0.74		X
Hino (1980)	9	1.00	X	X
Farrant (1982)	15	0.95		X
Farrant (1982)	16	0.73		X

Lawrence and Imberger's preliminary recommendation, b_2 varies from 0.73 to 1.00. Second, the limits predicted with the respective equations are not strong functions of b_2. The free limits are inversely proportional to the cube root of b_2. Solving Equation 21 for z gives

$$z = c_1 c_2 \left(\frac{\pi}{\theta} \frac{Q}{N} \right)^{1/3} \tag{22}$$

where

$$c_1 = \left(\frac{1}{b_1} \right)^{1/3}$$

$$c_2 = \left(\frac{1}{b_2} \right)^{1/3}$$

c_1 is a constant depending on intermediate, bottom, or top withdrawal and c_2 ranges between 1.00 and 1.11. With the exception of Lawrence and Imberger's (1979) preliminary result, the limits predicted by the respective equations are within 11 percent of each other. Considering the diversity of conditions from which these general equations were developed, this agreement is remarkable.

45. These results strongly support the opinion that unity should be assumed for b_2 and thus c_2. This is in exact agreement with Bohan and Grace's (1969) and Hino's (1980) results. It is within 2 percent of the equation that best describes the Farrant (1982) data. Additionally, unity is consistent with prototype data obtained by Aki and Shirasuna (1974) and Adachi and Nakamura (1977) as reported by Hino (1980).

46. It was stated in paragraph 43 that b_1 was 0.5 for top or bottom withdrawal and unity for intermediate withdrawal. Figure 16 shows the comparison of experimentally determined (see Appendix A for experimental procedure) intermediate withdrawal limits with predictions made with Equation 22 ($b_1 = 1.0$). Based on these data and the preceding analysis, there can be little question about the value of the
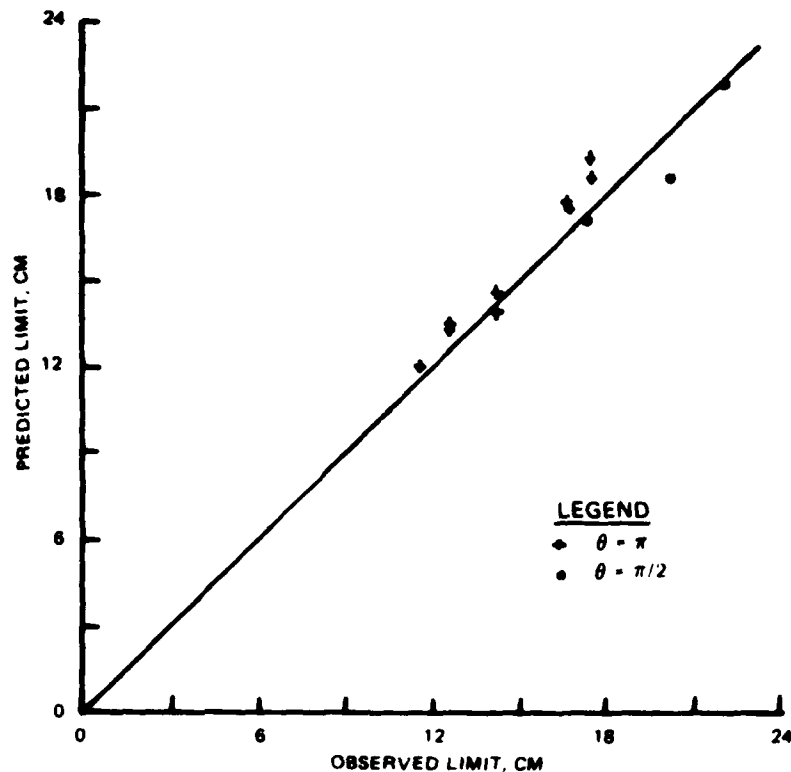
Figure 16. Comparison of limits predicted with
Equation 22 and observed limits, intermediate
withdrawal

coefficient for intermediate withdrawal. For boundary withdrawal, the
value of b_1 was based on symmetry arguments and Hino's (1980) analysis
of experimental data obtained by Hino and Furusawa (1969) for bottom
withdrawal. To further verify the coefficient for surface withdrawal,
several surface withdrawal experiments (described in Appendix A) were
conducted. A comparison of the predicted and observed limits is pre-
sented in Figure 17. Although there is some scatter in the data because
the stratification was not exactly linear, the results clearly support a
value of 0.5 for surface withdrawal as presented. Based on the above
results, Equation 22 can be written as shown below. This equation will
be compared to prototype data in Part III.

$$\frac{Q}{z^3 N} = b_1 \frac{\theta}{\pi} \qquad (23)$$
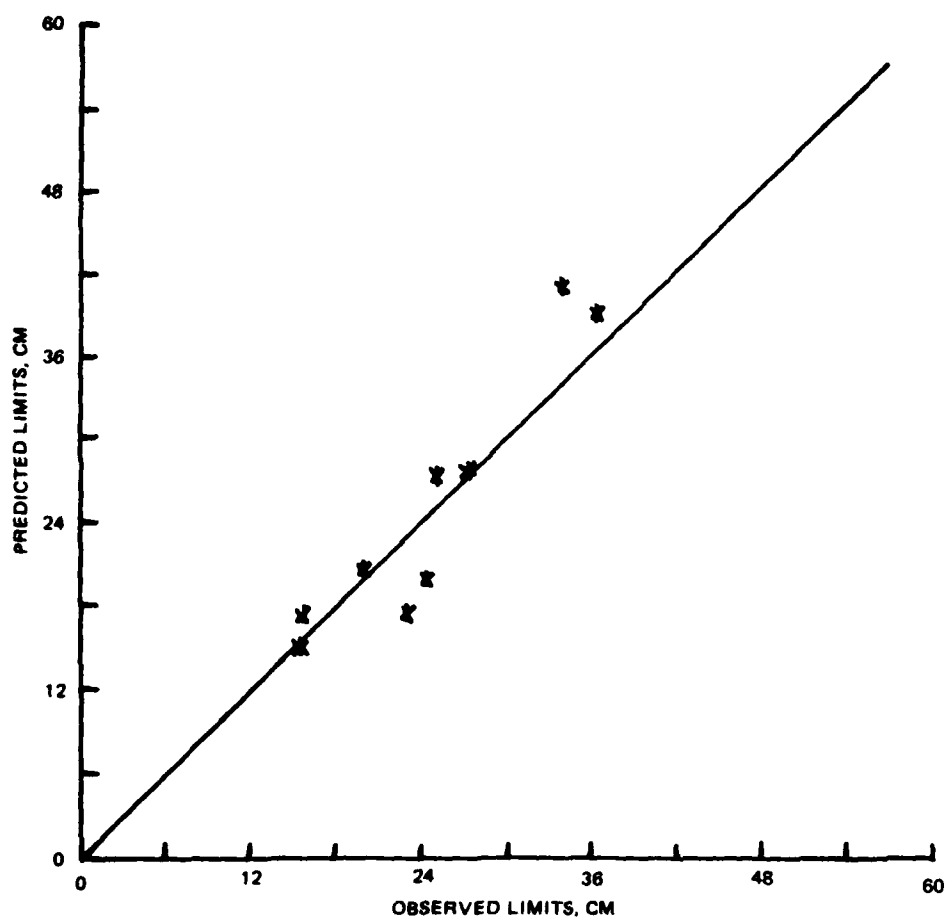
Figure 17. Comparison of withdrawal limits predicted with Equation 23 and observed limits, surface withdrawal $\theta = 2\pi$

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PART III: COMPARISON OF POINT SINK EQUATIONS WITH PROTOTYPE DATA

47. Few attempts have been made to demonstrate the efficacy of point sink equations with prototype data. Hino (1980) indicated his results are supported by prototype data obtained by Aki and Shirasuna (1977) and by Adachi and Nakamura (1977). With this exception, to our knowledge, predictions with point sink equations have not been compared to prototype data. To evaluate the proposed point sink equations, prototype data compatible with point sink assumptions were extracted from the literature, and the observed limits were compared with those predicted by point sink equations.

48. Several field studies were conducted by the Tennessee Valley Authority (TVA), and the results are summarized by Wunderlich and Elder (1973). The basic objective of each field test was to determine the thickness of the withdrawal zone produced under steady-state conditions and subsequently to develop general selective withdrawal descriptions. Quasi-steady flow conditions were established in the reservoir by operating a specified number of turbines at steady state for several hours. Various withdrawal rates were obtained by varying the number of operating turbines during subsequent tests. The most comprehensive field studies appear to be those conducted at Cherokee, Douglass, Fontana, and Ocoee No. 1 Reservoirs. Wunderlich and Elder classified Douglass and Cherokee as bottom withdrawal projects. In both cases the intakes were relatively close to the bottom of the impoundments. Similarly, Fontana and Ocoee No. 1 were classified as intermediate withdrawal projects since the observed withdrawal zones apparently did not interact with the bottom or free-surface boundaries.

49. In some of the field experiments, three or four turbines were operated. For these instances, the point sink equations discussed earlier are invalid. Significant interaction can occur between the withdrawal zones produced by the respective outlets, which is inconsistent with equations as formulated in this report. The field tests with one or two turbines operating are consistent with the point sink

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assumptions. For those tests with releases from two turbines, nearly
equal releases were made through turbines on opposite ends of the power-
house. The lateral spacing of the ports was sufficient to prevent with-
drawal zone interference. Thus, the release through each outlet pro-
duced a withdrawal zone with point sink characteristics.

50. The stratification conditions at these projects for the
reported tests were approximately linear. Withdrawal was made from a
vertical dam face (withdrawal angle $\theta = \pi$). The withdrawal zone
limits for intermediate flow were predicted by rearranging Equation 23
and accounting for the portion of flow through each turbine. This
resulted in

$$D = 2 \left(\frac{Q}{nN}\right)^{1/3} \tag{24a}$$

where

D = total withdrawal zone thickness (2Z for intermediate
    withdrawal)

Q = release rate from the project

n = number of turbines operating

51. For bottom withdrawal, predictions were made with

$$D = 1.26 \left(\frac{Q}{nN}\right)^{1/3} \tag{24b}$$

The buoyancy frequency used in the predictions was calculated from the
density profile data reported by Wunderlich and Elder (1973). In some
instances, the buoyancy frequencies, as reported by Wunderlich and
Elder, appeared to be the average buoyancy frequency of the density
profile rather than the average buoyancy frequency of the withdrawal
zone, as used in this study. The comparison of predicted and observed
withdrawal zones is presented in Figure 18 and in Table 3.

52. Although the quantity of prototype data is much less than
desired, the agreement between the observed and predicted withdrawal
zone thickness is striking. With the exception of Ocoee No. 1, the
difference between the predicted and observed thickness is less than

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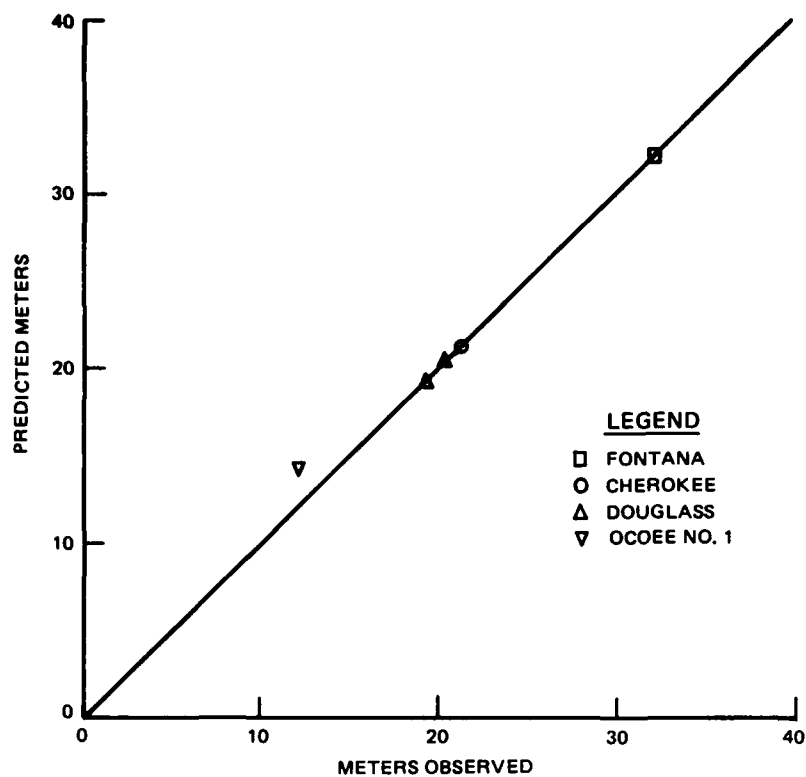
Figure 18. Comparison of measured and predicted with-
drawal zone thickness for several TVA reservoirs

Table 3

Withdrawal Zone Thickness for Four Impoundments

Reservoir	Total Flow $m^3 \cdot sec^{-1}$	N sec^{-1}	Number of Units n	D Predicted m	D Observed m
Douglass	110	0.0230	1	21.2	21.3
Cherokee	227	0.0264	2	20.5	20.5
Cherokee	212	0.0303	2	19.1	19.4
Fontana	161	0.0193	2	32.2	32.3
Ocoee No. 1	34	0.0479	2	14.2	12.3

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2 percent. For Ocoee No. 1, the difference is approximately 15 percent.
The intakes at Ocoee No. 1 are relatively high in the reservoir and it
is quite possible that boundary interference occurred. It should be
noted that the Ocoee No. 1 observed withdrawal zone has a thickness be-
tween that which would be predicted for surface withdrawal and interme-
diate withdrawal. Thus, these results demonstrate that the point sink
equations are consistent with prototype data for intermediate and bottom
withdrawal. The results also suggest that flow portioning is often a
reasonable assumption if the outlets are separated by a large distance.
It is also important to recognize that the withdrawal zone produced by
immediately adjacent ports often can be approximated by point sink equa-
tions by assuming a single point sink and a flow rate equal to the total
flow through both outlets.

PART IV: DEVELOPMENT OF A NEW EQUATION FOR
ARBITRARY BOUNDARY INTERFERENCE

53. As indicated earlier, few attempts have been made to develop
a description of the free limit produced by withdrawal from a linearly
stratified reservoir if boundary interference exists for the other
limit. Improved descriptions are needed, particularly for those cases
in which neither bottom, nor surface, nor intermediate withdrawal is a
realistic assumption (such as for an outlet that is located near, but
not at, the reservoir bottom or surface). Descriptions of the free
withdrawal limit produced under boundary interference can be developed
from elementary symmetry arguments.

54. Hino (1980) noted that symmetry could be used to approximate
the limits when the release is from the top or bottom of the reservoir.
This result is readily deduced from Figure 19. Intermediate point sink
withdrawal from a linearly stratified reservoir produces a symmetrical
velocity distribution with the maximum velocity at the center line of
the outlet. The velocity distribution produced in the top half of the
intermediate withdrawal zone is identical to that produced by bottom
withdrawal, with one-half of the flow residing in each half of the zone.
Further, the upper half of the intermediate withdrawal zone is equiva-
lent to bottom withdrawal if an idealized slip condition exists at the
bottom boundary. Similarly, the lower half of the intermediate with-
drawal zone is equivalent to surface withdrawal for one-half the flow.

55. These arguments can be extended to the case of arbitrary
interference by considering the case of surface interference indicated
in Figure 20. Assume an imaginary plane is passed through the flow
at a distance b above the center line of an outlet, as shown in Fig-
ure 20a. The imaginary plane corresponds to a free surface at a dis-
tance b above the offtake (Figure 20b). The flow in the withdrawal
zone shown in Figure 20b would be equal to that portion of total flow
Q passing through the part of the intermediate withdrawal zone

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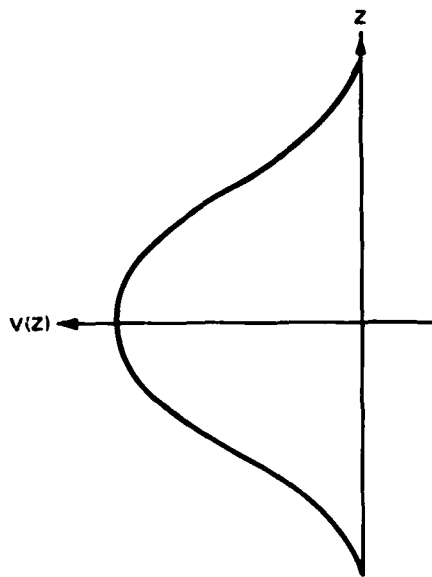
a. Velocity distribution
intermediate withdrawal
discharge = Q
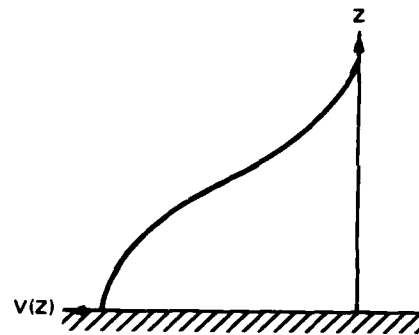
b. Velocity distribution
bottom withdrawal, dis-
charge = Q/Z
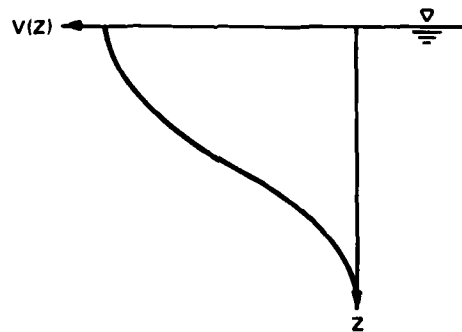
c. Velocity distribution
surface withdrawal, dis-
charge = Q/Z

Figure 19. Idealized velocity distributions for withdrawal
from linearly stratified reservoirs based upon symmetry

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a. Velocity distribution
    intermediate withdrawal,
       linear stratification

b. Velocity distribution
    surface interference,
       linear stratification
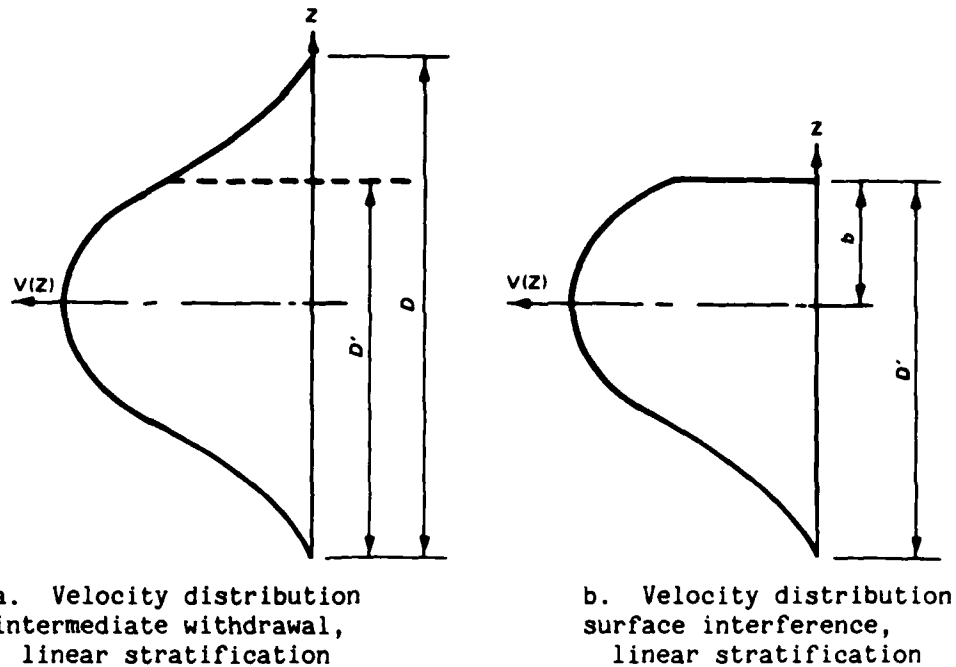
Figure 20. Assumed velocity distribution produced by
       boundary interference based upon symmetry

(Figure 20a) that is below the imaginary plane. Therefore Q' is given
by

$$Q' = \phi Q \qquad (25)$$

where ϕ is the ratio of discharge below the imaginary plane to the
total discharge in the withdrawal zone; i.e., the fraction of Q below
the imaginary plane.

56. The thickness of the withdrawal zone D' below the imaginary
plane is assumed equal to the thickness of the withdrawal zone for sur-
face interference. Thus,

$$D' = \chi D \qquad (26)$$

where χ is the fraction of withdrawal zone thickness D below the
imaginary plane.

40

57. Equation 23 established that the limits for intermediate withdrawal were adequately described by

$$Z = \left(\frac{Q\pi}{N\theta}\right)^{1/3} \qquad (27)$$

Noting that $D = 2Z$ for a linearly stratified system, this can be re-written as

$$\frac{Q}{D^3 N} = 0.125 \left(\frac{\theta}{\pi}\right) \qquad (28)$$

58. Substituting Equations 25 and 26 into Equation 28 results in

$$\frac{Q'}{D'^3 N} = \frac{0.125\phi}{\chi^3} \left(\frac{\theta}{\pi}\right) \qquad (29)$$

or

$$F' = \frac{0.125\phi}{\chi^3} \left(\frac{\theta}{\pi}\right) \qquad (30)$$

where F' is the Froude number of flow through the withdrawal zone with interference.

59. If the velocity distribution for intermediate point sink withdrawal is known, ϕ and χ can be expressed in terms of b and D'. Then, it is relatively simple to solve the resulting equation for D' (the total thickness of the withdrawal zone), since all other quantities will be known for a particular application. Thus, the basic problem is to determine the velocity distribution produced by point sink withdrawal from a linearly stratified reservoir.

60. The normalized velocity distribution suggested by Bohan and Grace (1969) is

$$v = \frac{v}{V} = \left(1 - \frac{y}{Y}\frac{\Delta\rho}{\Delta\rho_M}\right)^2 \qquad (31)$$

where

    v = normalized velocity distribution

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v = local velocity

V = maximum velocity

y = distance between the center line of the offtake and the point of interest

Y = distance between the center line of the offtake and the free limit

$\Delta\rho$ = density difference between the elevation of offtake and elevation y

$\Delta\rho_M$ = density difference between the center line of the offtake and the free limit

Figure 21 reflects the comparison between Equation 31 and the experimental data presented by Bohan and Grace. As discussed in paragraph 41,
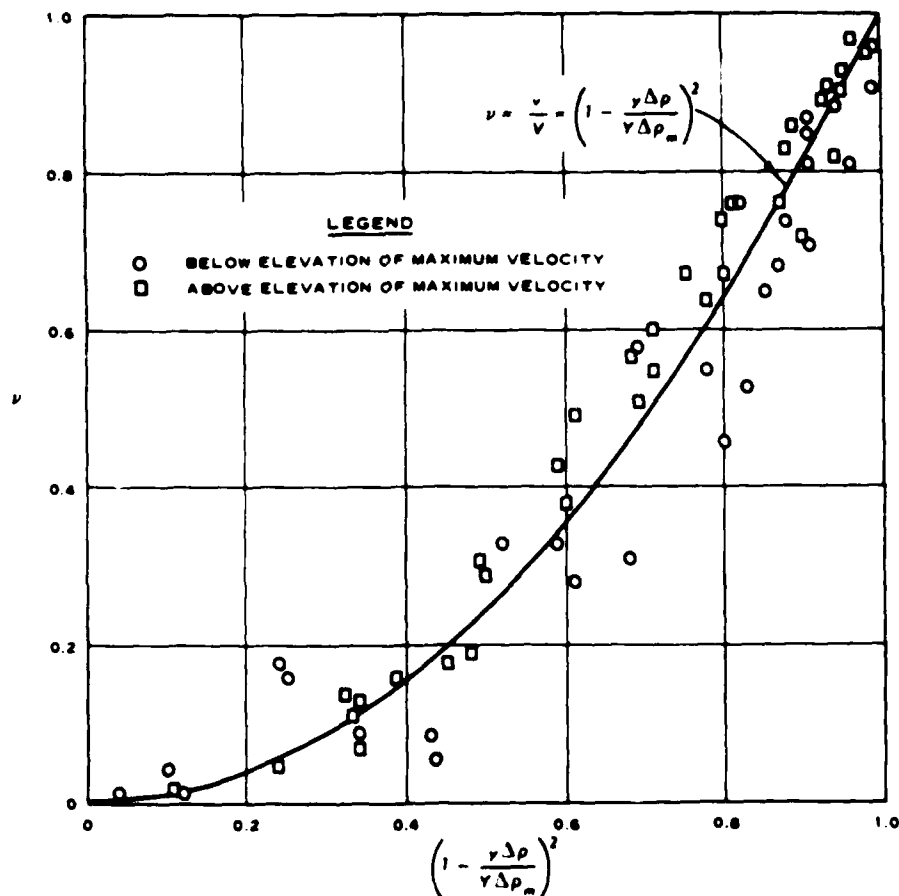

Figure 21. Dimensionless velocity distribution
recommended by Bohan and Grace (1969)

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although Bohan and Grace's data were obtained for arbitrary stratifi-
cation, the data should group about linear conditions because the dens-
ity profiles used in their analysis can be considered as perturbations
about linear stratification.

61. A dimensionless normalized distance can be defined with a
term from Equation 31 as

$$\eta = \frac{y}{Y} \tag{32a}$$

Additionally, for linear stratification,

$$\eta = \frac{\Delta\rho}{\Delta\rho_M} \tag{32b}$$

Therefore, Bohan and Grace's normalized velocity distribution for linear
stratification reduces to

$$v = \left(1 - \eta^2\right)^2 \tag{33a}$$

62. Croach (1971) analytically demonstrated that the normalized
velocity distributions produced by point sink and line withdrawal were
equivalent and were described by

$$v = \cos^2 \frac{\pi}{2} \eta \tag{33b}$$

He demonstrated excellent agreement between Equation 33b and Koh's
(1966) experimental results for line sink withdrawal.

63. For line sink withdrawal from a linearly stratified reser-
voir, Pao and Kao (1974) noted that their experimental results were
accurately described by

$$v = \frac{1}{2} (1 + \cos \eta\pi) \tag{33c}$$

Equation 33c is an identity to the equation proposed by Croach. Fischer
et al. (1979) recommended a similar equation for line sink withdrawal.

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64. A comparison of the normalized velocity distribution from Equations 33a-c is presented in Figure 22. The line sink data of Pao and Kao (1974) and the analytical solution of Croach (1971) are in almost exact agreement with the Bohan and Grace (1969) results. Croach's analytical solution indicated that the normalized velocity distributions for point and line sinks are identical, which is certainly suggested by the results obtained by Bohan and Grace (1969). It should be noted that Bohan and Grace's velocity distribution contains a density difference term which makes the expression applicable to both linear and arbitrary, nonlinear stratification conditions.

65. There is sufficient evidence to assume that the normalized velocity distribution can be approximated by Equations 33a-c. Using Equation 33c for the velocity distribution and integrating it over appropriate portions of the withdrawal zone for Q' and Q results in

$$\phi = \frac{1}{2} \left\{ 1 + \frac{1}{\pi} \sin \left(\frac{b/D'}{1 - b/D'} \pi \right) + \frac{b/D'}{1 - b/D'} \right\} \qquad (34)$$
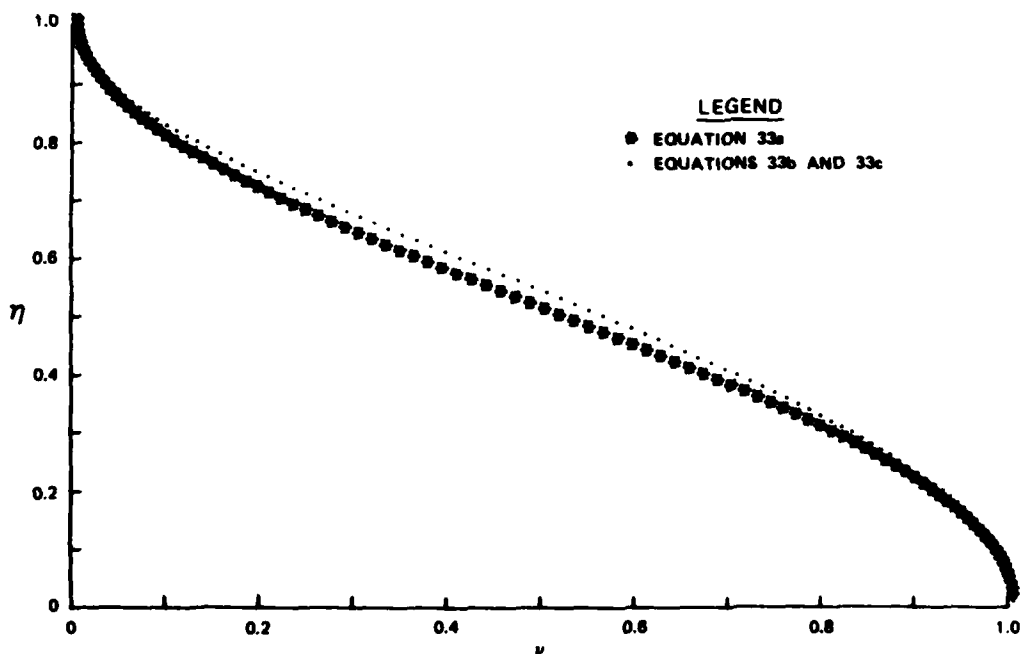

Figure 22. Comparison of normalized velocity distribution recommended by Croach (1971), Bohan and Grace (1969), Pao and Kao (1974)

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66. From Equation 26 and Figure 20, it follows that

$$x = \frac{1}{2}\left(1 + \frac{b/D'}{1 - b/D'}\right) \tag{35}$$

Then, substitution of Equations 34 and 35 into Equation 29 yields

$$\frac{Q'}{D'^3 N} = \frac{\frac{1}{2}\left[1 + \frac{1}{\pi}\sin\left(\frac{b/D'}{1 - b/D'}\,\pi\right) + \frac{b/D'}{1 - b/D'}\right]\frac{\theta}{\pi}}{\left(1 + \frac{b/D'}{1 - b/D'}\right)^3} \tag{36}$$

as a description of the free limit produced from withdrawal experiencing boundary interference.

67. Equation 36 is consistent with all the results presented in Part II. For intermediate withdrawal (b/D = 0.5), Equation 36 reduces to

$$\frac{Q}{D^3 N} = 0.125\,\frac{\theta}{\pi} \tag{37}$$

and, for bottom or top withdrawal (b/D = 0), it reduces to

$$\frac{Q}{D^3 N} = 0.5\,\frac{\theta}{\pi} \tag{38}$$

These results are identical to the recommendations made in Part II for linear stratification based upon the literature review where D = 2Z .

68. Unlike Hino's (1980) virtual sink analysis, this solution for arbitrary interference converges to the result Hino recommended for boundary withdrawal, i.e., surface or bottom withdrawal. Further, the surface withdrawal experiments presented in paragraph 46 (Figure 17) and several bottom withdrawal experiments (described in Appendix A) confirm this result. Figure 23 compares the observed limits for bottom withdrawal and those predicted with Equation 38. Although minor scatter exists because the stratification was not identically linear, Figure 23 clearly indicates that Equation 36 or 38 is valid at least to a first approximation for surface or bottom withdrawal. These results and Figure 17 support Equation 36 at the two limits b/D' = 0.0 and

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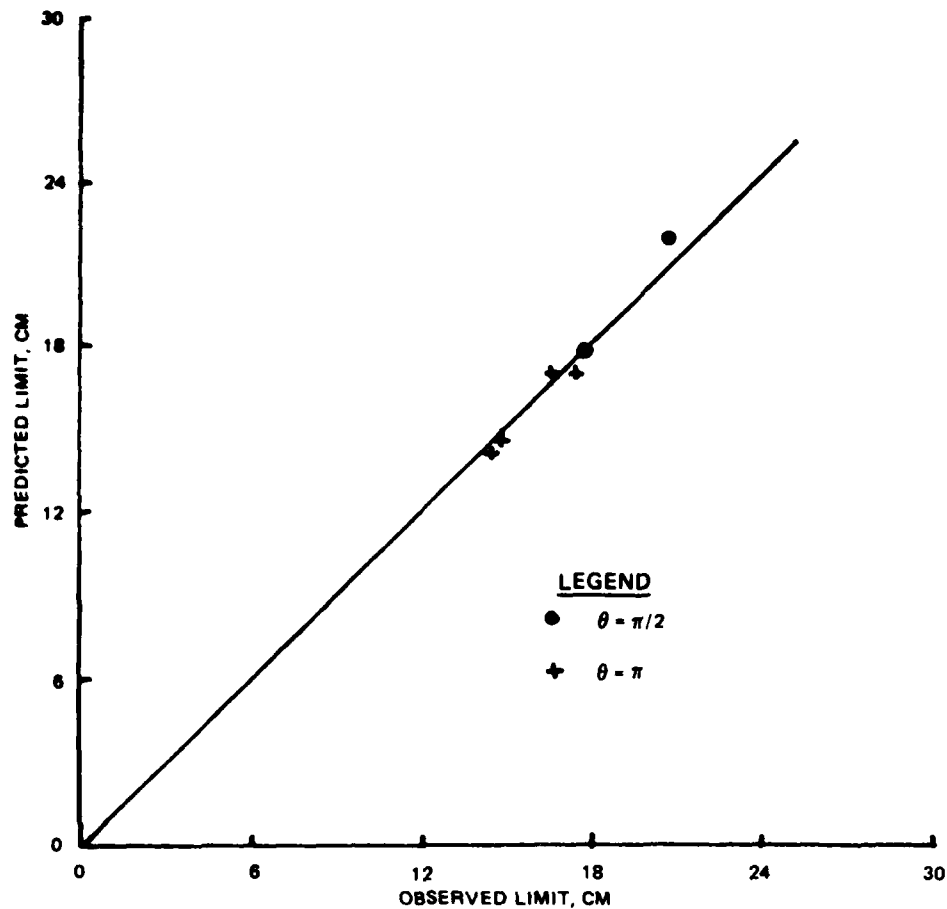
Figure 23. Comparison of observed limits for bottom
withdrawal and those predicted with Equation 38

b/D' = 0.5 . However, they do not demonstrate that it is a reasonable
description of the limits if

$$0.0 \ll \frac{b}{D'} \ll 0.5 \tag{39}$$

69. The assumptions underlying Equation 36 are conceptually inde-
pendent of the type of boundary, i.e., a free surface, a solid boundary,
or a strong pycnocline. The equation simply assumes a slip boundary
condition. Since the Boussinesq assumption is valid, the interference
boundary may be above or below the outlet. To verify Equation 36 for
an arbitrary interference boundary, experiments were conducted with a

significant density gradient below the outlet (see Appendix A for ex-
perimental procedure). The point sink was on a vertical face ($\theta = \pi$).
The density profiles in the experiments were approximated by an ideal-
ized profile, an example of which is indicated in Figure 24. Comparison
of the experimental results with Equation 36 is presented in Figure 25.
The difference between the limits predicted by Equation 36 and the
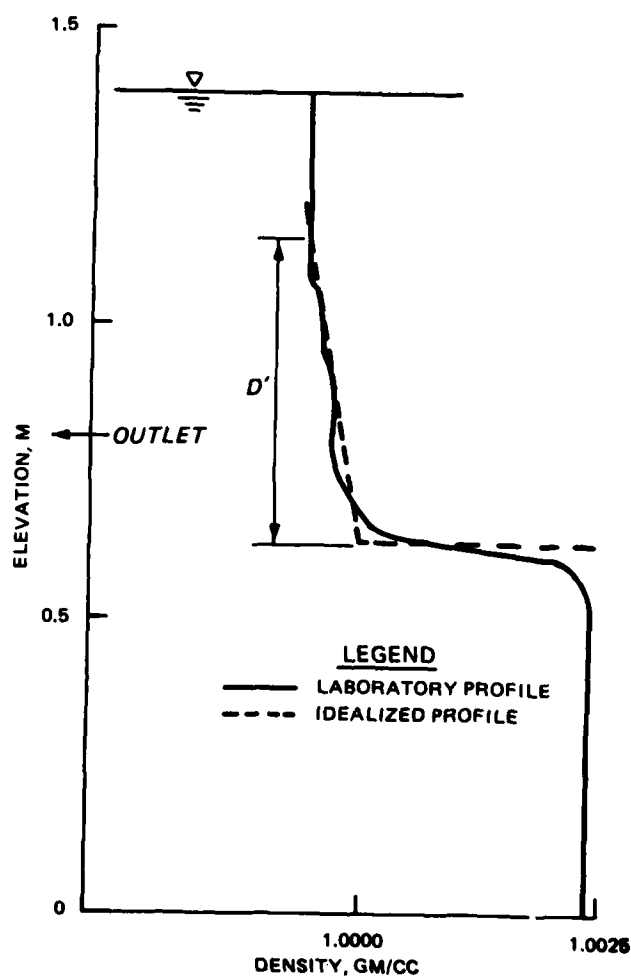
Figure 24. Comparison of laboratory profile
and idealized profile used to evaluate Equa-
tion 37 for boundary interference

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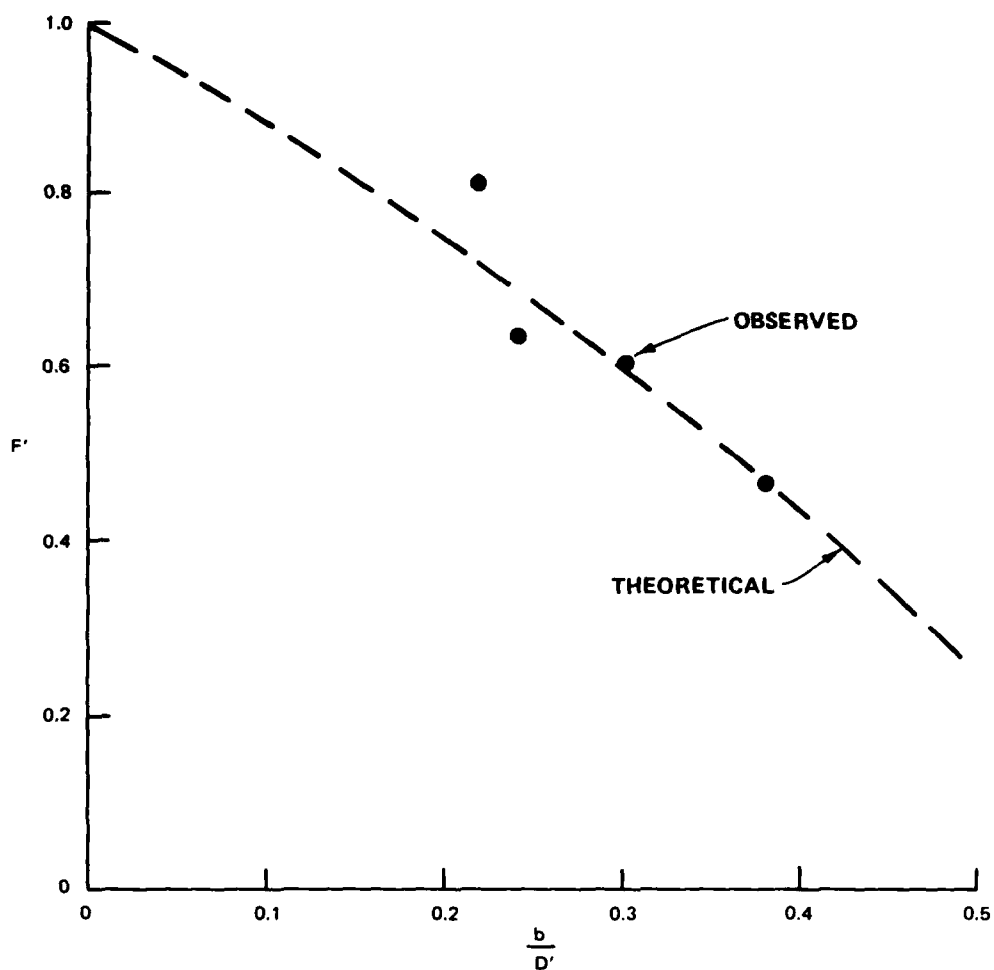
Figure 25. Comparison of experimental and theoretical
(Equation 36) results

experimentally observed limits was less than 6 percent which is on the
order of experimental error. These experimental results are more accu-
rately described by Equation 36 than by Hino's theoretical curve (Fig-
ure 11). Thus, the agreement between experiment and theory is con-
sidered sufficient to recommend Equation 36 for boundary interference
calculation.

PART V: SUMMARY AND CONCLUSIONS

70. The objective of this investigation was to develop improved descriptions of the limits of withdrawal through a point sink. This was achieved by generalizing established equations through symmetry arguments and verifying the more general equations with laboratory and field data previously reported by other investigators. In some instances, acquisition of additional laboratory data was required. The point sink equations were classified into two general categories: linear and two-layer.

71. For linear stratification, it was found that the equations characterizing the limits of withdrawal could be written in the general form (Equation 23)

$$\frac{Q}{Z^3 N} = b_1 \frac{\theta}{\pi}$$

where $b_1 = 0.5$ for bottom or top withdrawal and 1.0 for intermediate withdrawal.

72. The limits of the withdrawal zone, computed from this equation, were found to be consistent with laboratory and prototype data for intermediate, top, and bottom withdrawal. For intermediate withdrawal, Equation 23 is in exact agreement with the analytical equation proposed by Hino (1980) and is consistent with the experimental data of Hino and Furusawa (1969), Bohan and Grace (1969), and Farrant (1982). Additionally, it is consistent with Lawrence's (1980) analysis of data reported by Lawrence and Imberger (1979). The withdrawal thickness computed from the above equation is within 2 percent of that measured at Fontana Reservoir (as reported by Wunderlich and Elder 1973) and within approximately 15 percent of that measured at Ocoee No. 1. The equation is also consistent with prototype data obtained by Aki and Shirasuna (1974) and Adachi and Nakamura (1977), as reported by Hino (1980).

73. The data base for top and bottom withdrawal is much less extensive than for intermediate withdrawal. However, adequate justification exists to recommend Equation 23 for the former cases. It is

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consistent with the symmetry arguments of Hino (1980), experimental data
obtained by Hino and Furusawa (1969) for bottom withdrawal, and experi-
mental data contained in this report for top and bottom withdrawal. The
thickness of the withdrawal zone implied by Equation 23 is within 2 per-
cent of prototype data reported by Wunderlich and Elder (1973) for
Douglas and Cherokee Reservoirs. It should be noted that the limits for
bottom withdrawal, computed from Equation 23, are only 8 percent lower
than those computed from Croach's (1971) analytical equation. Based on
the above consistency, it can only be assumed that this difference is a
result of certain idealizations in Croach's derivation.

74. A theoretical solution for withdrawal limits was derived for
arbitrary interference by a rigid boundary, free surface, or a strong
pycnocline. The solution uses the velocity distribution determined
analytically by Croach (1971) and is supported by experimental data
reported by Bohan and Grace (1969) and Pao and Kao (1974). Using sym-
metry arguments and assuming an idealized slip condition at the plane of
interference, the equation describing the withdrawal (Equation 36) zone
was expressed as

$$\frac{Q'}{D'^3 N} = \frac{\frac{1}{2}\left\{1 + \frac{1}{\pi}\sin\left(\frac{b/D'}{1 - b/D'}\pi\right) + \frac{b/D'}{1 - b/D'}\right\}\frac{\theta}{\pi}}{\left(1 + \frac{b/D'}{1 - b/D'}\right)^3}$$

The solution of Equation 36 converges to the Equation 23 for interme-
diate or boundary withdrawal. Experimental data obtained in the study
verified the solution for flow conditions approximately halfway between
these extremes.

75. The velocity distribution for withdrawal from a linearly
stratified impoundment was a critical element in developing Equation 36.
The description used was analytically derived by Croach (1971). As
noted in paragraph 64, this velocity distribution is nearly identical to
the results given by Bohan and Grace (1969) for linear stratification.
However, this mathematical description of the velocity profile provided
an equation that could be directly integrated and, as such, was chosen

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to develop Equation 36. It must be noted that Croach's description is applicable only for linear stratification and is used only in limit prediction. If used in the computation of the velocity distribution for arbitrary nonlinear stratification, Croach's formulation would produce inaccurate results. The Bohan and Grace velocity distribution description (Equation 31) was developed through experimentation with nonlinear stratification and includes a term that accounts for the changes in velocity distribution resulting from arbitrary stratification. Thus, for prediction of a velocity distribution in a linear or arbitrary stratification, the Bohan and Grace formulation is recommended.

76. Simplified equations were also obtained for withdrawal from two-layer stratification. For point sink withdrawal at the bottom of a two-layer stratified medium or at the surface, the critical flow can be predicted from

$$\frac{Q_c}{h^2 \sqrt{g \frac{\Delta\rho}{\rho} h}} = 0.81 \; \frac{\theta}{\pi}$$

This equation reduces to the results of Harleman, Morgan, and Purple (1959) for $\theta = 2\pi$, is consistent with data obtained in this study for a withdrawal angle of π and 2π , and is in exact agreement with an analytical solution by Bryant and Wood (1976) for bottom withdrawal with a withdrawal angle of $\pi/2$. The critical discharge is only 7.4 percent greater than that implied by Craya's (1949) idealized equation for $\theta = \pi$. The only data with which Equation 8c is inconsistent are those of Wood (1978). However, it was not Wood's objective to evaluate critical conditions but rather to determine the relative flow proportions between various layers after critical conditions were exceeded. Thus, the proposed equation is consistent with all investigations which have explicitly addressed critical conditions.

77. If the sink is some distance h from a density interface and is in a homogeneous region that can be considered infinite in vertical

extent, the critical Froude number for incipient withdrawal of the interface is (from Equation 5)

$$\frac{Q_c}{\sqrt{h^2 \ g \ \frac{\Delta\rho}{\rho} \ h}} = 2.54 \ \frac{\theta}{\pi}$$

This equation is in exact agreement with Craya's (1949) analytical and Gariel's (1949) experimental results for $\theta = \pi$, as well as Lawrence's (1980) conclusions for $\theta = 2\pi$. It is somewhat inconsistent with Wood's (1978) results but, as indicated above, this study did not explicitly address critical conditions. Thus, Equation 5 is consistent with all reported data that focused on quantification of incipient flow.

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APPENDIX A: EXPERIMENTAL FACILITIES, PROCEDURES AND ANALYSIS

Two-Layer Tests

Facility and equipment

  1. The facility used in the bottom withdrawal experiments for two-layer stratification was a 6-m-square by 0.61-m-deep tank (Figure A1). An outlet orifice, 2.5 cm in diameter, was located at the
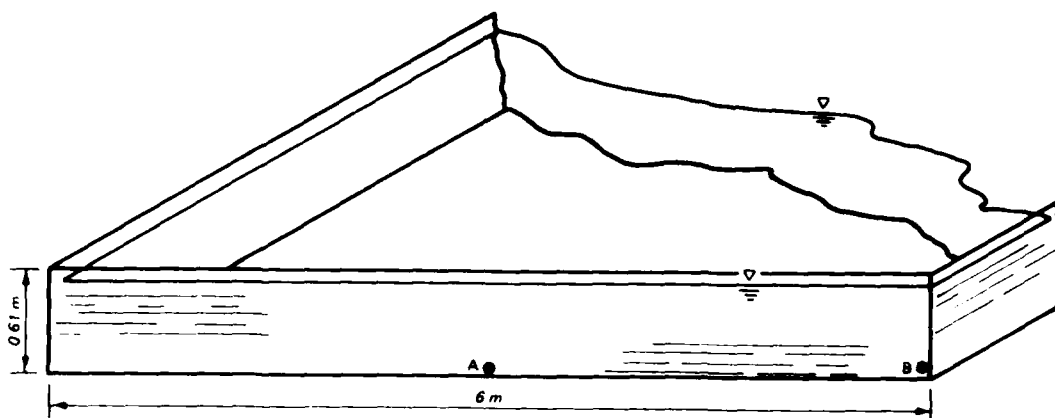

Figure A1. Drawing of the 6-m-square test flume

bottom but centered between the sides of the tank (location A) where $\theta = \pi$. Another vertical outlet orifice was located in the bottom corner of one side of the tank (location B) to simulate bottom withdrawal with a withdrawal angle of $\theta = \pi/2$.

  2. The density stratification for the tests was produced by floating fresh water (dyed with Rhodamine-WT for test purposes) over saline water. The density difference between the two layers ranged from 0.0018 to 0.0025 g/cu cm. Care was taken to minimize mixing between the two layers to keep the interface as distinct as possible.

Instrumentation

  3. The density stratification was measured with a density probe which consisted of conductivity and temperature sensors. The conductivity sensor was calibrated with solutions of known temperature and

specific gravity so that temperature and conductivity values measured in the tests could be converted to density in grams per cubic centimeter. The conductivity sensor, which was constructed with 1-mm-diam platinum-coated contacts spaced 6 mm apart, permitted point measurements. The temperature sensor was a thermistor bead that also allowed point measurements.

4. Fluorescence was used as an end-of-test indicator. Therefore, a fluorometer was used to analyze release water fluorescence. The fluorometer readings were monitored on a strip chart recorder as indicated in the test procedure.

Test procedure

5. Prior to test startup, the density stratification was measured, plotted, and approximated with lines, as shown in Figure A2. The

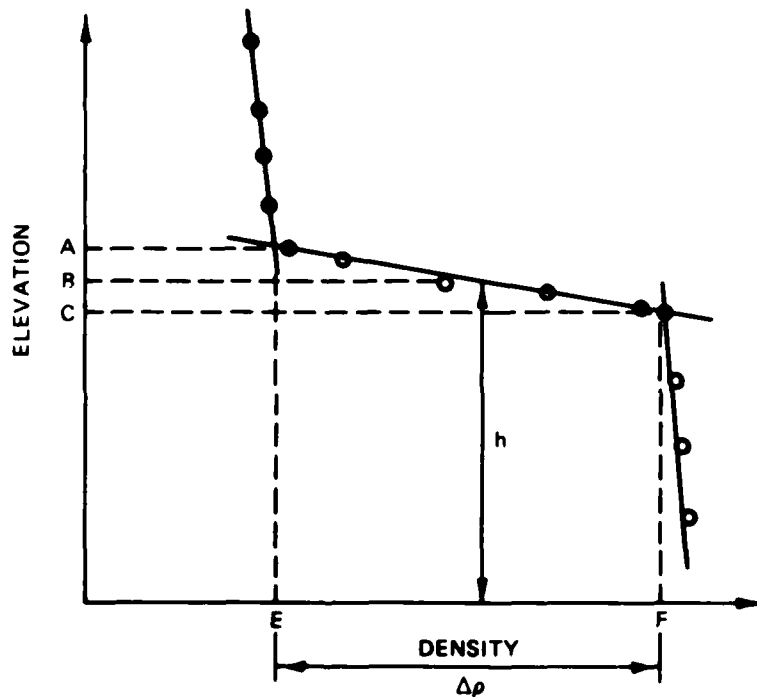
Figure A2. Idealizing an actual two-layer stratification

interface height h was taken as the vertical distance from the bottom to point B, midway between points A and C. The density difference Δρ was computed as the difference between the densities at points E and F.

A2

6. Based on h and $\Delta\rho$ of the initial density profile, a critical discharge was estimated with Equation 8c (see main text). The test was conducted with an actual discharge Q_a, which was less than critical. As water was withdrawn, the depth of water in the tank decreased. When the depth of saline water in the tank dropped to the critical h (for Q_a) where incipient flow of the upper layer was observed (as indicated by increased fluorescence in the release), the test was stopped. The actual discharge was thereby defined to be the critical discharge Q_c for the existing conditions in the tank. Remeasuring the density profile and substituting the newly measured h and $\Delta\rho$ from the density structure into Equation 8c permitted prediction of the critical discharge, for comparison with that observed.

Linear Density Gradient Tests

Facilities and equipment

7. The facilities used in the withdrawal experiments with linear density gradients consisted of two tanks. One tank was the same as that described in paragraph 1; the other tank measured 12.75 m long by 4.85 m wide by 1.2 m deep (Figure A3). In the latter tank, two orifices were located at midheight of the 4.85-m-long tank walls; each was used for intermediate withdrawal tests. The first orifice was centered between the ends of the walls, allowing a withdrawal angle of π (location A). The second orifice was located at the intersection of the walls, placing it in a corner, which allowed a withdrawal angle of $\pi/2$ (location B). Surface withdrawal experiments were conducted with the water surface of this tank at an elevation such that the outlet was partially submerged. Boundary interference tests were also conducted in this tank when the water surface was located well above the outlet and the pycnocline was located below the outlet as shown in Figure 24 of the main text. Bottom withdrawal experiments were conducted in the tank described in paragraph 1 of this appendix. Both tanks employed vertically scaled grids which were used as backdrops to determine withdrawal zone thickness.

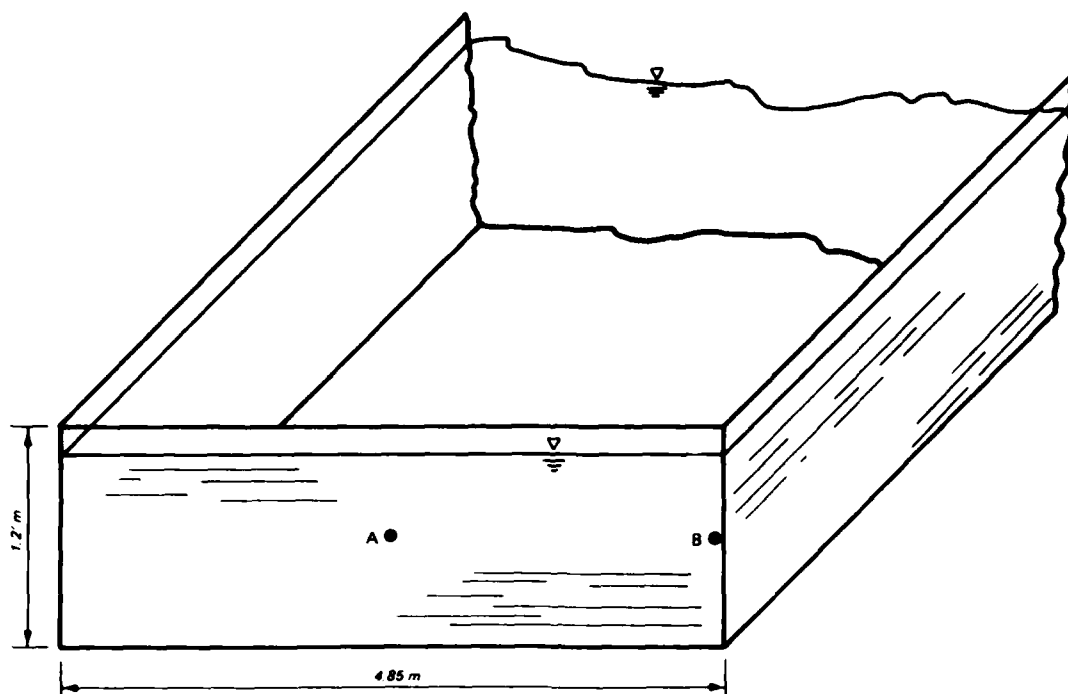8. The linear density gradient used in the tests was generated

A3

Figure A3. Drawing of the 12.75- by 4.85-m test flume

by floating fresh water over saline water and then methodically mixing
the pool until a reasonably linear density gradient was achieved. The
density stratification was measured with the probes described in para-
graph 3.

9. Because of the low magnitude of velocities encountered during
withdrawal, flow visualization techniques were used to determine veloc-
ity distribution and the limits of withdrawal. Video equipment was used
to record each test, thereby allowing a careful review and analysis of
each test.

Test procedure

10. After measuring and verifying the linearity of the density
gradient, a discharge was released from the desired orifice. A dye
particle was dropped into the tank a few feet upstream of the orifice.
This resulted in a vertical dye streak that was displaced horizontally
as fluid was withdrawn. After a period of time, movement of the dye
streak identified the limits of the withdrawal zone. The limits of
the withdrawal zone were determined by analyzing dye streak movement

A4

relative to the scaled-grid backdrop. The observed limits could then be
compared to the limits predicted by Equation 23.

APPENDIX B: NOTATION

A_o Area of the orifice opening

b_1 Assigned value for withdrawal angle coefficient because of geometry; 1.0 for intermediate withdrawal; 0.5 for top or bottom withdrawal

b_2 Withdrawal coefficient shown to be equal to 1.0

B Distance between the sink and the boundary of interference

c Densimetric Froude number constant for assymetrical withdrawal

c_i Experimentally determined constant

C_1 Constant $1/b_1$ depending on intermediate, bottom, or top withdrawal

C_2 Constant $1/b_2$ shown to be equal to 1.0

d Diameter of the sink

$d\rho/dz$ Density gradient

D Thickness of withdrawal zone

D' Thickness of withdrawal zone below the imaginary plane

D_{BW} Thickness of the withdrawal zone with bottom or top withdrawal

F' Froude number of flow through the withdrawal zone with interference

F_{BW} Densimetric Froude number describing top or bottom withdrawal

F_c Critical Froude number

F_T Densimetric Froude number for total withdrawal zone

g Acceleration due to gravity

h Vertical distance from the orifice to the interface separating the two layers

i Upper or lower limit index

n Number of turbines operating

N Buoyancy frequency

N_i Average buoyancy frequency between the upper or lower limit and the center line of the sink

Q Withdrawal rate

Q' Fraction of Q below the imaginary plane

Q_{BW} Flow through a sink at the bottom of the reservoir

Q_c (Critical) maximum discharge possible from one layer without withdrawing from the other layer

B1

Q_i	Withdrawal through the respective upper or lower part of the withdrawal zone
v	Local velocity
V	Maximum velocity
V_o	Average velocity through the orifice
y	Distance between the center line of the offtake and the point of interest
Y	Distance between the center line of the offtake and the free limit
$\bar{Z}$	Average of the distances from the sink to the upper or lower free limits
Z_i	Distance between the center line of the sink and the upper or lower free limit
γ	$\sin(\theta_t - \theta_u)/\sin\theta_u$
$\Delta\rho$	Density difference of the two layers; density difference between elevation of offtake and elevation y
$\Delta\rho_M$	Density difference between the center line of the offtake and the free limit
ε_{2c}	Ratio of the velocity in the withdrawal layer at the interface to the mean velocity in the withdrawal layer along the arc defined by a radial distance from the sink to the position of virtual control
η	Dimensionless normalized distance; y/Y ; for linear stratification, $\Delta\rho/\Delta\rho_M$
θ	Plan view (horizontal) withdrawal angle, radians
θ_t	Vertical withdrawal angle
θ_u	Angle between horizontal and upper boundary
ν	Normalized velocity distribution
ρ	Fluid density at the elevation of the orifice center line
ρ_o	Fluid density at the elevation of the orifice
ϕ	Ratio of discharge below the imaginary plane to the total discharge in the withdrawal zone
χ	Fraction of withdrawal zone thickness D below the imaginary plane

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