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**TEST METHODS FOR MATERIAL  
CHARACTERIZATION OF COMPOSITE CYLINDERS**

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Y. F. CHENG

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A study has been conducted of test methods for material characterization of composite cylinders. The purpose was to obtain constants in the stiffness matrix $C_{ij}$ which is necessary in the designing process. Test methods for determining engineering constants (Young's moduli, Poisson's ratios, and shear moduli) for composite cylinders with fibers in the axial and circumferential directions have been found. Constants in the compliance matrix $S_{ij}$ can then (CONT'D ON REVERSE)		

## 20. ABSTRACT (CONT'D)

be calculated by means of the well-known equations relating compliance matrix to engineering constants. Finally, the stiffness matrix  $C_{ij}$  is given by the inverse of  $S_{ij}$ . For composite cylinders with other fiber directions, engineering constants may be obtained by the rotation of coordinate axes and rule of mixture. (Keywords: ) →

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## INTRODUCTION

Fiber-reinforced composite materials have been increasingly used in engineering structures as well as in lightweight armament components because of their high strength, stiffness, and significant weight savings. Some armament components have the form of cylinders. The stiffness matrix  $C_{ij}$  is needed in the designing of these cylinders. A study of test methods for material characterization of a filament wound composite cylinder was made with a purpose of obtaining its stiffness matrix.

Stress-strain relations for anisotropic, orthotropic, and transversely isotropic materials are reviewed. Compliance matrix  $S_{ij}$  is given in terms of engineering constants  $E$ ,  $\mu$ , and  $G$  (Young's moduli, Poisson's ratios, and shear moduli, respectively). Test methods for obtaining these constants are shown. Finally, the stiffness matrix is given by the inverse of  $S_{ij}$ .

## STRESS-STRAIN RELATIONS FOR ANISOTROPIC MATERIALS

The generalized Hooke's law relating stresses to strains can be written as

$$\sigma_i = C_{ij}\epsilon_j \quad i, j = 1, 2, \dots, 6$$

where  $\sigma_i$  are the stress components,  $C_{ij}$  the stress matrix, and  $\epsilon_j$  the strain components. The comparison between tensor and contracted notations for stresses and strains is given in Table I, where  $\tau$  and  $\gamma$  are shear stress and shear strain, respectively.

TABLE I. COMPARISON BETWEEN TENSOR AND CONTRACTED NOTATIONS FOR STRESSES AND STRAINS

Stresses		Strains	
Tensor Notation	Contracted Notation	Tensor Notation	Contracted Notation
$\sigma_{11}$	$\sigma_1$	$\epsilon_{11}$	$\epsilon_1$
$\sigma_{22}$	$\sigma_2$	$\epsilon_{22}$	$\epsilon_2$
$\sigma_{33}$	$\sigma_3$	$\epsilon_{33}$	$\epsilon_3$
$\tau_{23} = \sigma_{23}$	$\sigma_4$	$\gamma_{23}$	$\epsilon_4$
$\tau_{31} = \sigma_{31}$	$\sigma_5$	$\gamma_{31}$	$\epsilon_5$
$\tau_{12} = \sigma_{12}$	$\sigma_6$	$\gamma_{12}$	$\epsilon_6$

The stiffness matrix  $C_{ij}$  has 36 components. By virtue of symmetry,  $C_{ij} = C_{ji}$ , and only 21 of the constants are independent. Similarly, we can write

$$\epsilon_i = S_{ij}\sigma_j \quad i, j = 1, 2, \dots, 6$$

where  $S_{ij}$  is the compliance matrix and the inverse of  $C_{ij}$ .

If there is one plane of material property symmetry, the strain-stress relations reduce to

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{pmatrix}$$

where the plane of symmetry is  $z = 0$ . There are 13 independent constants and such a material is termed monoclinic.

If there are three orthogonal planes of material property symmetry, the strain-stress relations reduce to

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

There are nine independent constants and the material is termed orthotropic.

If at every point of the material there is one plane in which the mechanical properties are equal in all directions, then the material is termed transversely isotropic. If, for example, the 1-2 plane is the special plane of isotropy, then the strain-stress relations are

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & S & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

There are only five independent constants.

## ENGINEERING CONSTANTS FOR TRANSVERSELY ISOTROPIC MATERIALS

Engineering constants are Young's moduli  $E$ ; Poisson's ratios  $\mu$ , and shear moduli  $G$ . These are measured in simple tests such as uniaxial tension. The constants obviously have more direct meaning than the components of the compliance and stiffness matrices. Most simple tests are performed with a known load or stress. Then the resulting strain is measured. Thus, the components of the compliance matrix  $S_{ij}$  are determined more directly than those of the stiffness matrix  $C_{ij}$ . For a transversely isotropic material, the components of the compliance matrix in terms of the engineering constants are

$$[S_{ij}] = \begin{bmatrix} 1/E_1 & -\mu_{21}/E_2 & -\mu_{31}/E_3 & 0 & 0 & 0 \\ -\mu_{12}/E_1 & 1/E_2 & -\mu_{32}/E_3 & 0 & 0 & 0 \\ -\mu_{13}/E_1 & -\mu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}$$

and

$E_1 = E_2$ ,  $\mu_{12} = \mu_{21}$ ,  $\mu_{13} = \mu_{31}$ ,  $\mu_{32} = \mu_{23}$ ,  $G_{23} = G_{31}$ ,  $G_{12} = E_1/2(1+\mu_{21})$   
where the 1-2 plane is the special plane of isotropy. Additional relations

$$\mu_{32}/E_3 = \mu_{23}/E_2, \quad \mu_{31}/E_3 = \mu_{13}/E_1$$

must be satisfied from the condition of symmetry in  $S_{ij}$ . There are only five independent engineering constants:  $E_1$ ,  $E_3$ ,  $\mu_{12}$ ,  $\mu_{13}$ , and  $G_{23}$ .

## CYLINDERS WITH FIBERS IN AXIAL DIRECTION

This is a transversely isotropic case where the  $R\theta$  plane is the special plane of isotropy. Letting  $(R,\theta,Z)$  coincide with the  $(1,2,3)$  directions, the compliance matrix in terms of the engineering constants follows:

$$[S_{ij}] = \begin{bmatrix} 1/E_r & -\mu_{\theta r}/E_r & -\mu_{zr}/E_z & 0 & 0 & 0 \\ -\mu_{r\theta}/E_r & 1/E_\theta & -\mu_{z\theta}/E_z & 0 & 0 & 0 \\ -\mu_{rz}/E_r & -\mu_{\theta z}/E_\theta & 1/E_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{\theta z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{zr} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{r\theta} \end{bmatrix}$$

and

$$E_r = E_\theta, \quad \mu_{r\theta} = \mu_{\theta r}, \quad \mu_{rz} = \mu_{\theta z}, \quad \mu_{zr} = \mu_{z\theta}$$

$$G_{\theta z} = G_{zr}, \quad G_{r\theta} = E_r/2(1+\mu_{\theta r})$$

Additional relations

$$\mu_{z\theta}/E_z = \mu_{\theta z}/E_\theta$$

$$\mu_{zr}/E_z = \mu_{rz}/E_r$$

must be satisfied from the condition of symmetry in  $S_{ij}$ . There are only five independent engineering constants:  $E_r$ ,  $E_z$ ,  $\mu_{r\theta}$ ,  $\mu_{rz}$ , and  $G_{\theta z}$ .

A test at uniaxial loading gives the values of  $E_z$  and  $\mu_{z\theta}$ . Data taken at the outside surface of a test under internal pressure with floating ends determine the value of  $E_\theta$ . The values of  $\mu_{\theta z}$  and  $\mu_{rz}$  can then be calculated. Data taken at the inside surface of the same test determine the value of  $\mu_{r\theta}$ . The remaining engineering constant  $G_{\theta z}$  can be determined by means of a torsion test.

#### CYLINDERS WITH FIBERS IN CIRCUMFERENTIAL DIRECTION

This is again a transversely isotropic case where the RZ-plane is the special plane of isotropy. Letting  $(Z,R,\theta)$  coincide with the  $(1,2,3)$  directions, all statements in the previous section hold provided that proper changes of directions are made. Specifically,  $S_{ij}$  has the following form:

$$[S_{ij}] = \begin{bmatrix} 1/E_z & -\mu_{rz}/E_r & -\mu_{\theta z}/E_\theta & 0 & 0 & 0 \\ -\mu_{zr}/E_z & 1/E_r & -\mu_{\theta r}/E_\theta & 0 & 0 & 0 \\ -\mu_{z\theta}/E_z & -\mu_{r\theta}/E_r & 1/E_\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{r\theta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{\theta z} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{zr} \end{bmatrix}$$

The five independent engineering constants are:  $E_z$ ,  $E_\theta$ ,  $\mu_{zr}$ ,  $\mu_{z\theta}$ , and  $G_{r\theta}$ . They can be found by a uniaxial test, an internal pressure test with floating ends, and a torsion test.

#### CONCLUSIONS

Test methods for determining engineering constants, i.e., Young's moduli, Poisson's ratios, and shear moduli, for composite cylinders with fibers in the axial or circumferential directions have been shown. Constants in the compliance matrix  $S_{ij}$  can then be calculated by means of the well-known equations relating compliance matrix to engineering constants. The stiffness matrix is given by the inverse of  $S_{ij}$ .

For composite cylinders with other fiber directions, engineering constants may be obtained by the rotation of coordinate axes and the rule of mixture.

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