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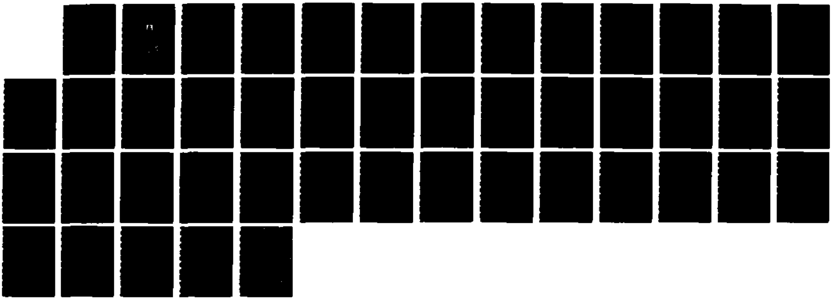

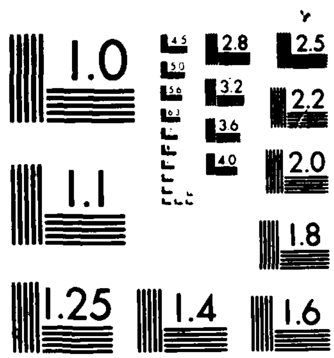

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FOREIGN TECHNOLOGY DIVISION

TRANSMISSION OF DIGITAL INFORMATION ALONG CHANNELS WITH THE STORAGE

Academy of Sciences of the USSR

Institute of Problems of the Transmission of Information

(Selected Articles)

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PARTIALLY EDITED MACHINE TRANSLATION

FTD-ID(RS)T-0534-87 16 July 1987

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TRANSMISSION OF DIGITAL INFORMATION ALONG CHANNELS WITH THE STORAGE (Selected Articles)

Academy of Sciences of the USSR
Institute of Problems of the Transmission of Information

English pages: 37

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In them occurs the state named "night blindness" - hemeralopia, which, according to the current point of view, is a result of damage of the rod-shaped apparatus of the eye.

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However, in recent years it has been shown that with the hereditary pigment degenerations in animals the biochemical changes are observed in all celluar elements of the retina.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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TRANSMISSION OF DIGITAL INFORMATION ALONG CHANNELS WITH THE STORAGE.

Academy of Sciences of the USSR.

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ANALYSIS OF THE CORRECTIVE PROPERTIES OF REPEATED AND CASCADE CODES.

V. V. Zyablov.

Repeated codes have long ago been known [1]. In this work Elayes shows that with the independent errors the probability of erroneous decoding can be done how conveniently small at the constant velocity transmission, if length and dimension of the repeated code unlimitedly grow. The connection of the parameters of the repeated code with the parameters of its subcodes was investigated by Slepian [2]. Although since then left many works, dedicated to the analysis of the corrective properties and advisability of applying the repeated codes, their corrective properties, especially for the channels with the storage, they were investigated insufficiently.

Cascade codes were for the first time proposed by Forney [3]. They are further development of the ideas, placed in the repeated codes.

Fundamental purpose of this work -to show that for real channels, which are characterized by presence of errors, inclined to grouping, and by variability of character of errors in time, of great interest are cascade codes.

1. Cascade codes.

In the book of Forney [3] is given algebraic determination (construction) of cascade codes and are analyzed corrective properties by their way of determining probabilities of their erroneous decoding with independent errors. The given examination of the cascade codes differs in that:

1) are given somewhat different from that being in the book of odds not the construction of the cascade codes;

2) the analysis of the corrective properties of the cascade codes is carried out by the determination of code distance of the binary code, which corresponds to cascade, and quantity of corrected subblocks.

This approach makes it possible to consider probability of erroneous decoding during application of cascade codes both in channels with independent errors and in channels with grouped errors.

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We will call code, which is constructed as follows, cascade:

a) k binary informational symbols are divided/marked off into k_2 subblocks on k_1 of binary symbols in each;

b) subblocks of length k_1 of binary symbols are considered as elements of field $GF(2^{k_1})$ and are coded by appropriate (n_2, k_2)-code of Reed-Solomon with code distance of $d_2=r_2+1$, which we will call code of second step/stage;

c) each symbol of code of second step/stage is considered as k_1

informational symbols of linear binary (n_1, k_1)-code with code distance of d_1, which we will call code of first stage.

Process of this coding is represented in Fig. 1. Let us show that the binary notation of all words of that obtained by such form of the cascade code is the binary linear code (by binary notation it is understood the consecutive recording/record n_2 of the binary words of length n_1). Since the binary notation of all words of the cascade code is the divisible binary code, then it suffices to show that all its words form group during the addition on module 2, i.e., the sum of any two words forms the third word of the cascade code.

During addition of two words of cascade code on module 2 we obtain word of the same code, since:

1) code of second step/stage linear and addition in field $GF(2^{k_1})$ is additions on module 2;

2) the code of first stage is the binary linear code, i.e. the sum of two code words is a code word of the same code.

From diagram of coding we directly obtain following formulas for parameters of cascade code through parameters of its subcodes:

$$n = n_1 n_2,$$
$$k = k_1 k_2, \qquad (1)$$
$$R = \frac{k}{n} = \frac{k_1 k_2}{n_1 n_2} = R_1 R_2.$$

Since code distance of code of second step/stage is equal to d_2, then there is minimum d_2 of nonzero words of code of first stage in any nonzero word of cascade code. Any nonzero word of the code of first stage has a weight d_1. Consequently, the minimum weight of the cascade code (d) is defined as

$$d > d_1 d_2. \quad (2)$$

Generating matrix/die (G) of binary linear code, which corresponds to cascade, can be obtained as

$$G = (B_{k_1} \times G_2) \times G_1 = G_2^{*} \times G_1,$$

where $B_{k_1} \times G_2 = G_2^{*}$ - product of Kronecker of vector of column B_{k_1} from elements $\beta_1 = 10\ldots,0, \beta_2 = 01\ldots0, \ldots, \beta_{k_1} = 00\ldots1$ to generating matrix/die (G_2) of code of second step/stage; $G_2^{*} \times G_1$ - product of Kronecker matrix/die G_2^{*} to generating matrix/die (G_1) of code of first stage.

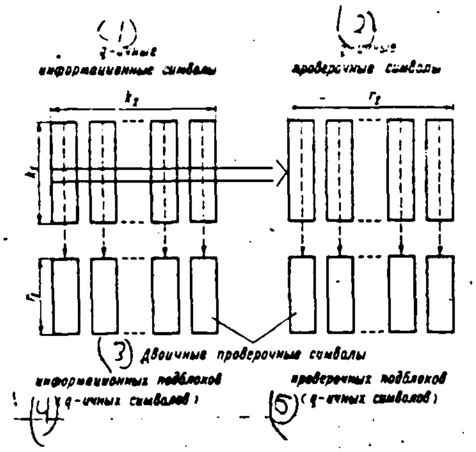

Fig. 1. Diagram of coding two-dimensional cascade code.

Key: (1). q-nary informational symbols. (2). q-nary checkout symbols. (3). Binary checkout symbols. (4). informational subblocks (q-nary symbols). (5). checkout subblocks (q-nary symbols).

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Let us illustrate obtained relationships/ratios by following examples.

Let as code of first stage binary linear code (7.3) with d_1=4 be used, and as code of second step/stage - Reed-Solomon codes above field; GF (2^3); (7.4) with d_2=4, (7.3) with d_2=5 and (7.2) with d_2=6. Parameters of the cascade and corresponding shortened codes of Bose-Chaudhuri-Hocquenhem (B-C-H) with n=49 are given in the table.

Каскадный код (1)		(2) Код Б—Ч—Х		
k	d	k	d	неукороченный код (3)
12	16	15	14	(63,29)
9	20	9	16	(63,23)
6	24	4	21	(63,18)

Key: (1). Cascade code. (2). Code B-C-H. (3). unshortened code.

2. Comparison of the code distances of the repeated and cascade codes.

Code distance of repeated code is always equal to product of code distances of its subcodes [4]. Let us show that for some cascade codes in formula (2) both the equal sign and the sign of absolute inequality can occur.

Let us consider cascade code, in which as code of first stage binary code with one checkout symbol ($d_1=2$), is used, and as code of second step/stage - unshortened code of Reed-Solomon with code distance of

$$d_2 > \frac{1}{2}(k_1+1)k_1(k_2-1). \quad (3)$$

For example, code of first stage (5, 4) with $d_1=2$, and code of second step/stage - Reed-Solomon code (15.2) with $d_2=14$.

Let us show that for such codes in formula (2) inequality sign occurs. The word, which consists of some units, i.e., (α^0, α^0 ...

α^0, α^0), where α - primitive element of field $GF(2^{k_1})$, is a code word of Reed-Solomon code [5]. From this it follows that any nonzero element of field $GF(2^{k_1})$ in Reed-Solomon code is encountered either by n_2 or it is less than k_2 times. Actually, let us assume that in any word element (α^i) is encountered γ times. Then sum on the module for this word with the word of some units, multiplied by element α^i, is a code word and contains γ zero, i.e., has weight $w=n_2-\gamma$. On the other hand, the weight of any word of Reed-Solomon code is determined by relationship/ratio $w \geq d_2=n_2-k_2+1$, whence it follows that

$$\gamma \leqslant k_2 - 1. \quad (4)$$

In field $GF(2^{k_1})$ is only $(k_1+1)k_1/2$ nonzero elements weight of which in binary notation it does not exceed two. Each of these elements in the code word is encountered not more than than k_2-1 times.

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Consequently, if condition (3) is satisfied, then among d_2 nonzero words in the cascade code there will be at least one weights three or more, i.e., in formula (2) inequality sign will occur.

Following example shows that in formula (2) equal sign can occur. Let as the code of first stage be used the binary linear (n_1, n_1-1)-code with $d_1=2$, and as the code of the second step/stage - Reed-Solomon code (n_2, n_2-1) with $d_2=2$. The checkout matrix/die of the code of the second step/stage can be registered in the following form:

$$H_2 = \| \alpha^0 \alpha^1 \alpha^2 \dots \alpha^{q-2} \|.$$

It is not difficult to ascertain that vector ($\alpha^0 0 \dots 0 \alpha^1$) is a code vector of the code of the second step/stage. The weight of the corresponding word of the cascade code is equal to four. Consequently, in formula (2) equal sign will occur.

Let us compare code distance of cascade and repeated codes with $r_2>1$. Let the code of first stage in both cases is one and the same. For any binary code with $n_2>8$ and $r_2>1$ occurs $d_2^*<r_2$, where d_2^* - code distance of the binary code of the second step/stage. Let as the code of the second step/stage of the cascade code Reed- Solomon code be used with $d_2=r_2+1$. Consequently, the code distances of cascade ($d^{(k)}$) and repeated ($d^{(u)}$) of the codes are connected with the following relationship/ratio:

$$d^{(k)} > d_1 (r_2 + 1) > d_1 r_2 > d_1 d_2^* = d^{(u)}. \qquad (5)$$

From relationship/ratio (5) it follows that with independent errors cascade code is always better than repeated, if $r_2>1$. With $r_2=1$ the cascade and repeated codes can be identical.

3. Asymptotic behavior of the corrective ability with the independent errors of the multidimensional cascade and repeated codes.

They call relative corrective capability of code for independent errors

$$D = d/n. \quad (6)$$

From relationship/ratio (2) it follows that relative corrective capability of cascade code for independent errors is determined by relationship/ratio

$$D > d_1 d_2/n_1 n_2 = D_1 D_2. \quad (7)$$

From relationship/ratio (7) it follows that for repeated code relative corrective capability for independent errors at constant velocity of transmission will be not vanishing functions from n, then only then, when relative corrective capabilities for independent errors of all their subcodes are those not vanishing functions from n_i.

Today best nonrandom block codes are codes B-C-H, for which at constant velocity of transmission we have

$$D \approx b/\log n, \quad (8)$$

where b - constant.

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From (7) and (8) we obtain, that relative corrective capability of two-dimensional repeated code for independent errors at constant velocity of transmission will decrease as

$$D^{(u)} = D_1 D_2 = b_1 b_2/\log\ n_1 \log n_2. \quad (9)$$

Since log n=log n_1+log n_2<log n_1 log n_2, then the asymptotic behavior of the relative corrective capability for the independent errors in the two-dimensional repeated code, constructed on the basis of the codes B-C-H, is worse than in the codes B-C-H.

Let as codes of first degree of cascade code code B-C-H be used, and as code of second step/stage - unshortened code of Reed-Solomon, for which

$$D_2 = \frac{r_2+1}{n_2} = b_2 = \text{const}. \tag{10}$$

Then the relative corrective capability for the independent errors in cascade code ($D^{(k)}$), constructed on the basis of the codes B-C-H and the codes of Reed-Solomon, is defined as

$$D^{(k)} > D_1 D_2 = b_1 b_2 / \log n_1.$$

Taking into account that n_1=$k_1$$R_1$ and k_1=log (n_2+1)<log n, we obtain,

$$D^{(k)} > \frac{b_1 b_2}{\log\log n - \log R_1}. \tag{11}$$

Consequently, two-dimensional cascade codes, which are constructed on basis of binary codes B-C-H and codes of Reed-Solomon, have best asymptotic behavior, than codes B-C-H.

Let us consider multidimensional repeated (cascade) codes, for which code distance is defined as

$$d = \prod_{i=1}^{s} d_i = \prod_{i=1}^{s} a_i r_i = \prod_{i=1}^{s} a_i \prod_{i=1}^{s} r_i,$$

where $a_i = d_i/r_i = f(R_i)$ and $1 < a_i \leqslant 2$ - for Reed-Solomon code and $a_i \approx 0{,}2 \div 0{,}5$ for binary linear code. Consequently, the code distance of the multidimensional code is determined in essence by capacity $v = \prod_{i=1}^{s} r_i$. From work [6] it follows that for given ones $k_1, k_2, \ldots, k_s$ and n v will be greatest, if $r_1/k_1 = r_2/k_2 = \ldots = r_s/k_s$ or, that the same,

$$R_1 = R_2 = \ldots = R_s = R_i. \qquad (12)$$

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From condition (12), taking into account that

$$R = 1 - r/n = \prod_{i=1}^{s} R_i = R_e^s,$$

$$R_e = 1 - r_i/n_i = \left(1 - \frac{r}{n}\right)^{\frac{1}{s}} > 1 - r/(n-r)s,$$

we obtain

$$r_i/n_i \leqslant r/(n-r)s. \qquad (13)$$

Consequently, the relative corrective capability for the independent errors in the multidimensional codes can be evaluated on top as follows:

$$D = \prod_{i=1}^{s} D_i < \prod_{i=1}^{s} a_i\,[r/(n-r)s]^s. \qquad (14)$$

From relationship/ratio (14) it follows that relative corrective capability for independent errors in cascade and those in repeated codes rapidly decreases with increase in their dimension. Therefore a good relative corrective capability for the independent errors should be expected only in the cascade codes with small dimensions.

One should note that relative corrective capability for independent errors is although very important, not determining parameter, since S-dimensional cascade and repeated codes with independent errors and $s \to \infty$ can ensure how conveniently small probability of erroneous decoding at constant velocity of transmission.

4. Analysis of the corrective properties of the sub-class of the cascade codes.

Let us consider sub-class of two-dimensional cascade codes, in which as code of first stage is used binary linear (k_1+1, k_1)-code with $d_1=2$, and as code of second step/stage - Reed-Solomon code with foundation $q = 2^{k_1}$ and with $n_2 = 2^{k_1} - 1$, $d_2 = n_2 - k_2 + 1 = r_2 + 1$.

Parameters of such cascade codes are determined from following formulas:

$$\begin{aligned} n &= (2^{k_1} - 1)(k_1 + 1), \\ k &= k_1 k_2, \\ d &> d_1 d_2 = 2r_2 + 2. \end{aligned} \tag{15}$$

Let us compare relative corrective capability for independent errors of this sub-class of cascade codes and binary codes B-C-H. The dependences of the speed of transmission (R) on the relative corrective capability for the independent errors (D) for the cascade codes and the codes B-C-H are given in Fig. 2. As can be seen from graphs, the cascade codes virtually are not inferior to the codes B-C-H in the range of lengths n=75-1016 and speeds of transmission R=0.25-0.6, but they sometimes even exceed them.

Let us consider algorithm of decoding of this sub-class of cascade codes.

With decoding of codes of first stage, which have one checkout symbol, is conducted only detection of errors. Subblocks with the odd number of errors are erased, i.e., with the decoding of the codes of the second step/stage they are considered as erasings.

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With decoding of codes of second step/stage (Reed-Solomon codes) is conducted correction of errors and erasings, for example, on algorithm, proposed in work [7].

With this algorithm of decoding of this sub-class of cascade codes code distance of cascade code [8] completely is realized, and furthermore, is corrected large quantity of errors of multiplicity $t>(d-1)/2$. In the same time the algorithms of the decoding of the codes B-C-H, as a rule, do not make it possible to correct the error of multiplicity $t>(d-1)/2$. Consequently, the cascade codes will ensure with the independent errors the large correctness of transmission, than the binary codes B-C-H.

Let us consider corrective properties of sub-class of cascade codes with respect to errors, which are grouped into one or several error bursts.

Let us determine length of error burst, reliably corrected by data by sub-class of cascade codes.

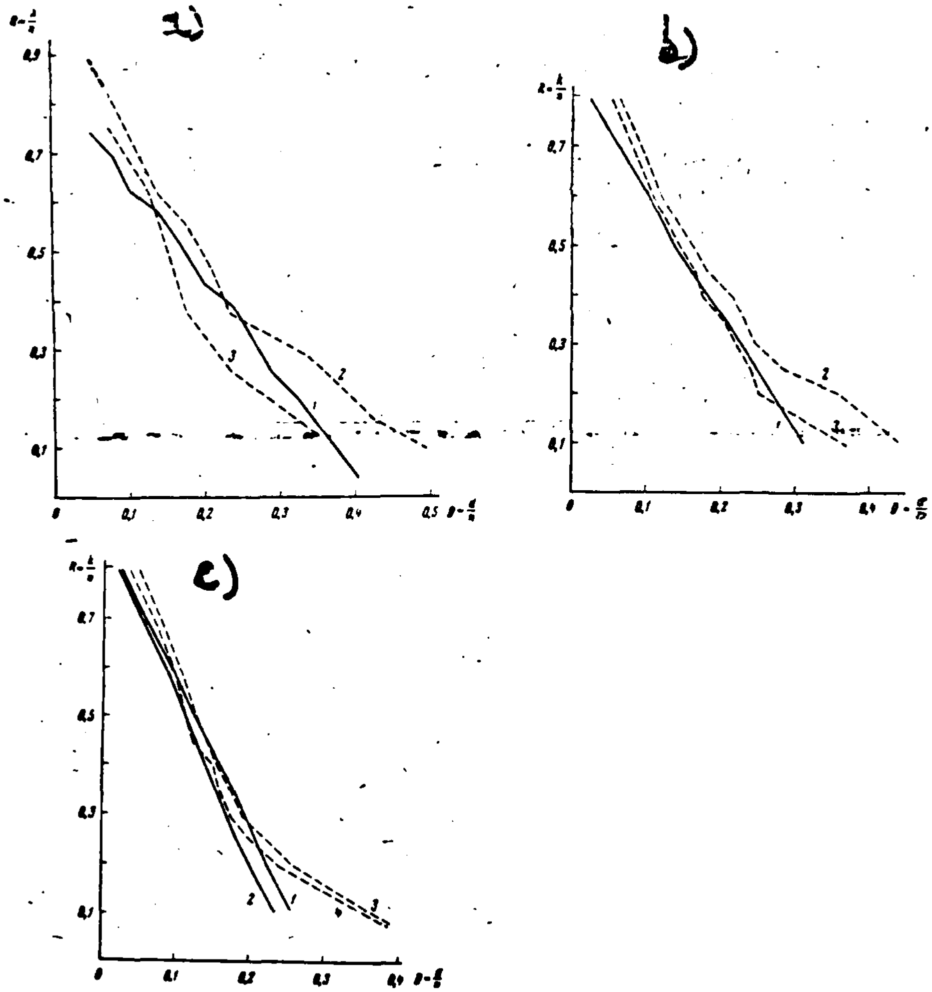

Fig. 2. Dependence of speed of transmission (R) on relative corrective capability for independent errors (D). a) for the cascade codes of length n=75 (1) and codes B-C-H of lengths n=63 (2) and n=73 (3), b) for the cascade codes of length n=186 (1) and codes B-C-H of lengths n=127 (2) and n=255 (3), c) for the cascade codes of lengths n=441 (1) and n=1016 (2) and codes B-C-H of lengths n=551 (3) and 1023 (4).

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Let the subblocks of the code of first stage be consecutively/serially transmitted. Then any burst of errors of the length

$$B \leqslant n_1 [r_2/2] - (k_1 - 1), \quad (16)$$

where [X] - integer part X, is reliably corrected, since it will be placed either in $[r_2/2]$ subblocks and then it is reliably corrected by the code of the second step/stage or in $([r_2/2]+1)$ subblocks, but then in two subblocks it will be on one error, i.e., they will be discovered, and combination from $t_2=[r_2/2] - 1$ errors and $\tau_2=2$ erasings is reliably corrected by the code of the second step/stage.

For number of checkout symbols of linear code, which corrects burst of errors of length b, if b/n is not very small, are used following simple evaluations: $2b \leq r \leq 3b$ or $0.33 \leq b/r \leq 0.5$.

Ratio B/r for sub-class of cascade codes and speeds of transmission R=0.25 and R=0.5 in question is given in Fig. 3. From the graphs it is evident that the sub-class of the cascade codes in question is close to the optimum linear codes, which correct the single error burst.

Let us consider possibility of correction of several error bursts. Assume that in code combination of the cascade code after its transmission along the channel appeared γ the error bursts. Any burst of errors of length B_i will strike not more than l_i subblocks when

$$l_i =]\, B_i/n_1 \,[+ 1,$$

where] X [- the smallest whole number, greater or equal to X. On the other hand, γ the error bursts will be corrected, if

$$l=\sum_{i=1}^{\gamma} l_i \leqslant [r_2/2].$$

Carrying out reasonings, analogous to those, which occurred during determination of single reliable corrected error burst, we obtain following condition for sum of lengths of packets, under which they are reliably corrected:

$$\sum_{i=1}^{\gamma} B_i \leqslant \max\{n_1[r_2/2]-(2\gamma-1)(k_1-1),\ [r_2/2]\}. \quad (17)$$

Dependence of total length of reliably corrected error bursts on number of error bursts for cascade code with n=1016 and R=0.25 is given in Fig. 4.

Let us note that condition (17) assigns lower boundary for sum of lengths of reliably corrected bursts of errors, allocation of lengths of which can be arbitrary. Since allocation of lengths does not have vital importance, but have a value only a number of error bursts and their total length, then the codes with such corrective properties will be good for the channels, where the errors are inclined to the grouping, but allocation of the lengths of the error bursts carries random character.

Algorithm of decoding, which realizes corrective properties of cascade codes examined with respect to errors, which are grouped into one or several packets, the same, as for correction of independent errors. The analysis of the corrective properties of the sub-class of the cascade codes and by the error burst was produced under the condition of their reliable correction.

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Actually with this algorithm of decoding the corrective properties of the sub-class of the cascade codes and to the error bursts will be above due to the correction with a certain probability of error bursts with the larger sum of lengths, than it is guaranteed by condition (17). Actually, assume that the error bursts affected l subblocks. After the decoding of the codes of first stage the subblocks with the odd combination of errors will become erasings. If in t_2 the subblocks are the not detected by the code first steps/stages of error, and in $\tau_2 = l - t_2$ - subblocks - the detected errors, then when $2t_2 + \tau_2 \le r_2$, the decoding will be carried out correctly. Consequently, the probability of incorrect decoding ($P_e(l)$), when errors occur into $l > r_2/2$ subblocks, is equal to the probability of the fact that the errors are discovered less than in $2(l - [r_2/2])$ subblocks. Let p_{11} be probability of the undetected error after the decoding of the codes of first stage (if errors are independent or errors within the packet are independent, then $p_{11} \le 0.5$). Then we obtain the following expression for the probability of the incorrect decoding, when $r_2/2 \le l \le r_2$:

$$P_e(l) = \sum_{i=0}^{2l-r_2-1} C_l^i p_{11}^{l-i} (1 - p_{11})^i.$$

Fig. 5 gives upper bound of probability of incorrect decoding as functions of ratio l/r_2 for cascade code with n=1016 and R=0.25. From the graph it is evident that with small disturbances of condition (17) probability of incorrect decoding is small.

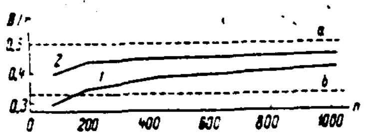

Fig. 3

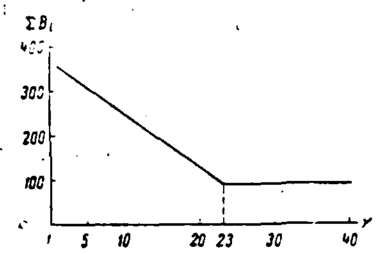

Fig. 4.

Fig. 3. Dependence of ratio of length of corrected error burst to number of checkout symbols (B/r) on length of cascade codes (n) for speeds of transmission R=0.5 (1) and R=0.25 (2) (a and b - border for optimum linear codes).

Fig. 4. Dependence of total length of reliably corrected bursts of errors (ΣB_i) on number of bursts of errors (γ) for cascade code with n=1016 and R=0.25.

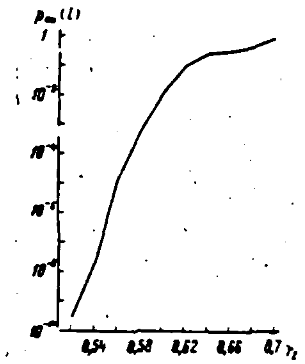

Fig. 5. Upper bound of probability of incorrect decoding $(P_{om}(l))$ as function of ratio of number of subblocks with errors to number of checkout symbols of code of second step/stage (l/r_2) for cascade code with n=1016 and R=0.25.

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Conclusion.

Work examines corrective properties of cascade and repeated codes. The conducted investigations showed that:

1) the cascade codes have the best corrective properties, than the repeated codes;

2) the corrective properties of the cascade and repeated codes deteriorate with an increase in their dimension;

3) the two-dimensional cascade codes, constructed from the binary codes B-C-H and the codes of Reed-Solomon, have asymptotically best parameter D=d/n, than the binary codes B-C-H.

Analysis of corrective properties of sub-class of cascade codes, in which in first stage are used codes with one checkout symbol - and in second step/stage - unshortened codes of Reed-Solomon, it shows that in the range of lengths n=75-1016 and speeds of transmission R=0.25-0.6 they virtually are so good for independent variables, errors as binary codes B-C-H, and with error bursts their corrective properties are close to optimum. The fact that these codes are efficient both for the channels with the independent errors and for the channels with the error bursts, and the algorithm of decoding, which realizes the most fully corrective properties, with any character of errors one and the same, makes with their especially valuable for the real channels, in which the character of errors varies in the time randomly.

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ALGORITHMS OF THE STEP BY STEP DECODING OF THE REPEATED AND CASCADE CODES.

V. V. Zyablov.

Repeated and cascade codes can successfully be used both in channels with independent errors and in channels with errors, inclined to grouping [1, 2]. Its decoding is one of the principal problems during the application of any code. In the general case the decoding of the repeated and cascade codes is simplified due to their step by step decoding, i.e., first are decoded the codes of first stage, and then the second. However, the analysis of that how the corrective properties of the repeated or cascade code fully with this decoding are realized, in the literature is absent. The algorithm of step by step decoding, proposed by Elayson [3], historically was the first algorithm of the decoding of the repeated code. The most general/most common algorithm of the decoding of the repeated and cascade codes is the algorithm of decoding on generalized code distance, proposed by Forni [4].

In work analysis of realizable corrective properties with different simple algorithms of step by step decoding of repeated and cascade codes with independent errors is conducted. The generalized algorithm of step by step decoding, which the completely realizes code distance of the repeated or cascade code is actually the disintegration of the best algorithm of decoding in terms of the generalized code distance on they are simple the algorithms of step by step decoding, is developed/processed.

SIMPLE ALGORITHMS OF STEP BY STEP DECODING.

Algorithm of step by step decoding, with which code of first step is used for correction of all errors to multiplicity i and detection of all remaining errors (combination of code of first stage with discovered errors it is erased), and code of second step/stage is used for correction of errors and erasings in word decoded by code of first stage of code of second step/stage, we will call simple and let us designate A_i.

Let us consider smallest multiplicity of error, which can be not corrected with algorithm of decoding A_i, where i=0, 1, 2, ..., [(d_1-1)/2].

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Let d_1 - code distance of code of first stage, and d_2 - code distance of code of second step/stage, then cascade code has code distance of [5]

$$d_k > d_1 d_2. \quad (1)$$

Subsequently we will speak only about cascade codes, since repeated code is special case of cascade code [5].

In the case, when in relationship/ratio (1) equal sign occurs, highest multiplicity of reliably corrected by cascade code error is determined by relationship/ratio

$$t^k = \left[\frac{d_1 d_2 - 1}{2}\right]. \qquad (2)$$

For determining smallest multiplicity of error, which can be not corrected with algorithm of decoding A_i, let us introduce function

$$f_i(X) = \min_Y \{X(i + I) + Y(d_1 - i)\} \text{ where } X + 2Y + I > d_2, \qquad (3)$$

where X - number of erased subblocks of code of first stage, after its decoding; Y - number of subblocks of code of first stage with errors after coding of code of first stage; $f_i(X)$ - smallest multiplicity of error, which can be not corrected with algorithm of decoding A_i, if erasing X subblocks after decoding of code of first stage occurs.

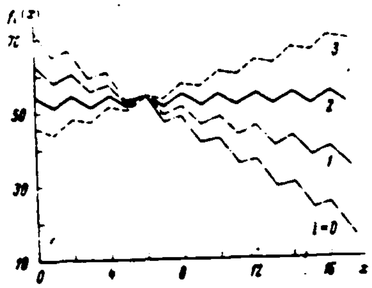
For explanation of aforesaid let us consider following example. Let there be the cascade code, in which as the code of first stage binary cyclic code (23, 11) with $d_1=8$ is used, and as the code of the second step/stage - shortened code of Reed - Solomon (31, 15) with $d_2=17$ above field $GF(2^{11})$. In accordance with (2) this code can reliably correct all errors up to multiplicity $t^k = 67$.

Let t_i be highest multiplicity of reliably corrected error with algorithm of decoding A_i, then we have

$$t_i = \min_X f_i(X) - 1 \quad \text{with} \quad 0 < X < d_2. \qquad (4)$$

From formulas (3) and (4) we obtain t_0=16, t_1=33, t_2=50, t_3=43.

Plotted functions f_i for i=0, 1, 2, 3 are given in figure. Thus, we see that $t_i \le t_k$ with any i. On the other hand, best is algorithm A_2, with which f_2 (X) is virtually parallel to axis X. Let us show that with any values of d_1 and d_2 graphs $f_i(X)$ are similar represented in the figure.



Dependence of the smallest multiplicity of irremediable error on a number of erased with the decoding of the code of first stage subblocks for different simple algorithms of step by step decoding in the cascade of the code with d_1=8 and d_2=17.

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(1) Пусть $2(i+1) \approx (d_1 - i)$, т. е. $i_{опт} = [(d_1 - 2)/3]$, (2) тогда

$$f_{опт}(X) = \begin{cases} [(d_1+1)/3]\, d_2 & \text{(3) при } X = d_2 \bmod 2, \\ [(d_1+1)/3](d_2+1) & \text{(3) при } X \neq d_2 \bmod 2. \end{cases} \qquad (5)$$

Key: (1). Let. (2). then. (3). with.

From relationship/ratio (5) it follows that when $i = i_{опт}$ function $f_i(X)$ oscillates relative to straight/direct, parallel axis X.

(1) Пусть $i = i_{опт} + j$, где $-i_{опт} \leqslant j \leqslant [(d_1 - 1)/2] - i_{опт}$, (2) тогда

$$f_i(X) = \begin{cases} f_{опт}(X) + j[(3X - d_2)/2] & \text{(3) при } X = d_2 \bmod 2, \\ f_{опт}(X) + j(3X - d_2 - 1)/2 & \text{(3) при } X \neq d_2 \bmod 2. \end{cases} \qquad (6)$$

Key: (1). Let. (2). then. (3). with.

From relationship/ratio (6) it follows that when $i \neq i_{опт}$

$$t_i = \min f_i(X) - 1 < t_{опт} = \min f_{опт}(X) - 1. \qquad (7)$$

From obtained results we have, that algorithm of step by step decoding A_i, where $i=(d_1-2)/3$, ensures greatest highest multiplicity of reliably corrected errors in comparison with any simple algorithm of step by step decoding and that $f_{i\,\text{опт}} \approx \frac{2}{3} t^k$.

GENERALIZED ALGORITHM OF STEP BY STEP DECODING.

Generalized algorithm of step by step decoding is complicated algorithm of step by step decoding, i.e., is combination of several simple algorithms of step by step decoding. In the particular case the generalized algorithm of step by step decoding is a combination of all simple algorithms of step by step decoding. The block diagram of the generalized algorithm of step by step decoding is represented below. This algorithm consists of the following:

the combination of the cascade code accepted consecutively/serially is decoded for each simple algorithm A_i, where i=0.1, ..., $[(d_1-1)/2]=\gamma$;

after decoding of combination accepted through algorithm A_i is located distance of combination accepted from that obtained (d_i), if code combination of cascade code was obtained as a result of decoding. Otherwise the word, obtained as a result of decoding on algorithm A_i, is rejected;

obtained distance d_i is equal with $t_k = [(d_k-1)/2]$ and, if $d_i < t_k$, decoding ceases, and word, obtained as a result of decoding on algorithm A_i, is accepted for unknown; otherwise decoding is continued.

if $d_i > t_k$, then obtained distance is equal with d_v, where d_v is minimum of all distances, obtained with previous decodings of this combination accepted on simple algorithms of step by step decoding.

in the case $d_i < d_v$ code combination, obtained with algorithm A_i, is memorized, and d_v is substituted on d_i. Otherwise code combination, obtained with algorithm A_i, is rejected.

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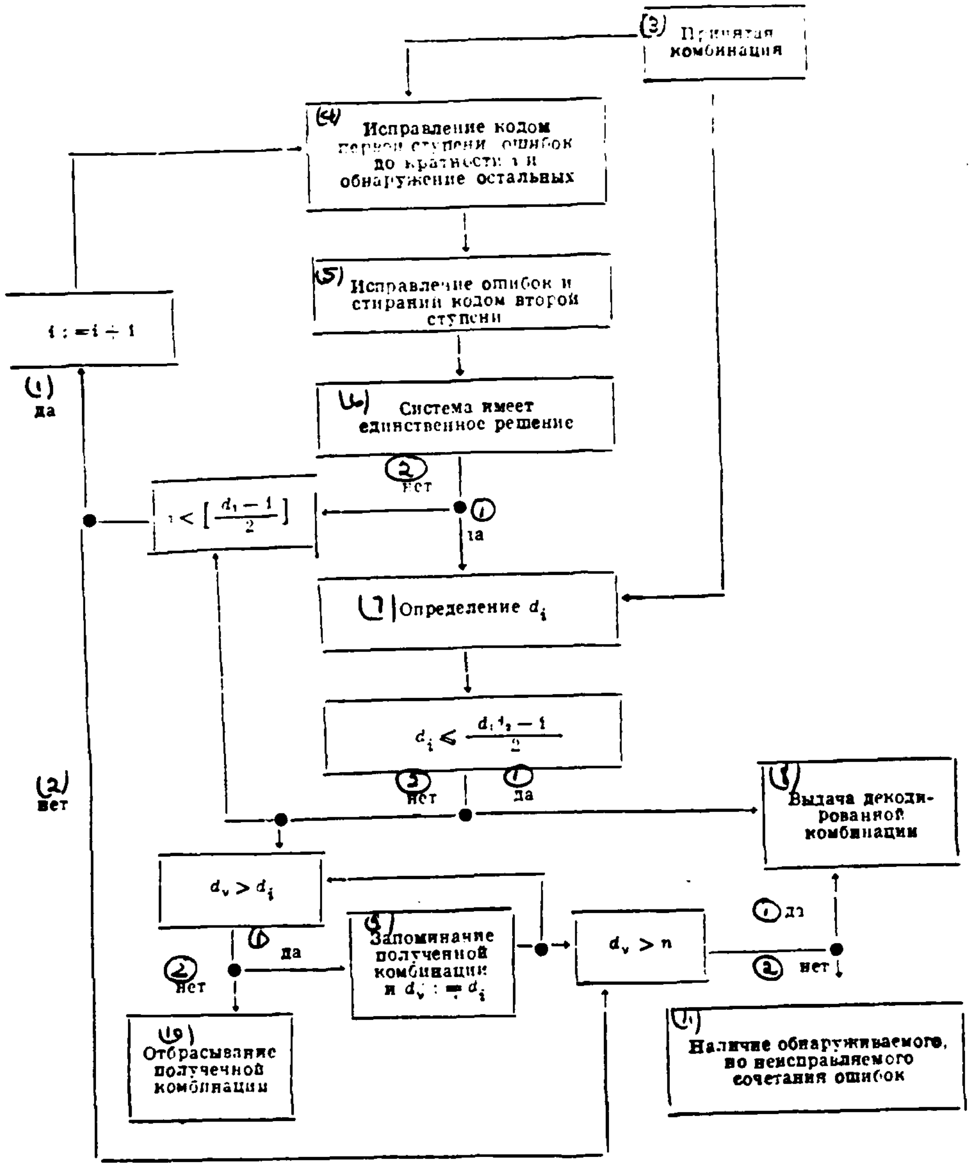
After $i=\gamma$ decoding ceases, and for unknown word, which has distance d_v is accepted.

Generalized algorithm of step by step decoding guarantees selection of best of simple algorithms for step by step decoding of cascade code for this combination of errors. We shall prove that, although for any simple algorithm for step-by-step decoding of a cascode code the highest rate of reliably corrected errors is less than is guaranteed by code distance of cascade code, generalized algorithm of step by step decoding makes it possible to correct all errors up to the rate guaranteed by code distance, if $d=d_1d_2$.



Key: (1). yes. (2). no. (3). Combination accepted. (4). Correction by code of first stage of errors to multiplicity i detection of rest. (5). Correction of errors and erasings by code of second step/stage. (6). System has unique solution. (7). determination. (8). Output of decoded combination. (9). Memorizing of obtained combination and . (10). rejection of obtained combination. (11). Presence of discovered, but not faulty combination of errors.

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Let error of multiplicity t occur, so that is correct

$$d=d_1d_2>2t. \quad (8)$$

On the other hand, the error of multiplicity t can be registered in the following form:

$$t=\sum_{j=1}^{n_1} jX_j, \quad (9)$$

de X_j - number of subblocks of the code of first stage with the errors of multiplicity.

From relationships/ratios (8) and (9) we obtain

$$d_1d_2>2\sum_{j=1}^{n_1} jX_j. \quad (10)$$

If error of multiplicity t, which satisfies (8) and having allocation on subblocks, given by relationship/ratio (9), can be corrected with any simple algorithm A_i, then with generalized algorithm of step by step decoding of cascade code word, accepted with this combination of errors, will be decoded correctly. Consequently, it is necessary to demonstrate that from the validity of relationship/ratio (10) follows validity of at least one of the following $\gamma+1$ inequalities:

$$d_2>\sum_{j=1+i}^{d-1-i} X_j+2\sum_{j=d_1-i}^{n_1} X_j, \quad (i=0, 1, \ldots, \gamma). \quad (11)$$

Let $d_1=2\gamma+1$, then all relationships/ratios (11), except latter, be multiplied by two, we store/add up their left and right sides and, after leading conversions, we obtain

$$(2\gamma+1)d_2=d_1d_2>2\sum_{=1}^{d_1-1}jX_j+2(2\gamma+1)\sum_{j=d_1}^{n_1}X_j. \quad (12)$$

Let $d_1=2\gamma+2$, then all relationships/ratios (11) be multiplied by two, we store/add up their left and right sides and, after leading conversions, we obtain

$$(2\gamma+2)d_2=d_1d_2>2\sum_{j=1}^{d_1-1}jX_j+2(2\gamma+2)\sum_{j=d_1}^{n_1}X_j. \quad (13)$$

From validity of relationship/ratio (10) validity of relationships/ratios (12) and (13) follows, while from validity of latter validity of at least one of relationships/ratios (11) follows.

ALGORITHMS OF THE STEP BY STEP DECODING OF THE CASCADE CODES DURING THEIR APPLICATION IN THE REAL CHANNELS.

As it follows from work [6], error in standard telephone channels is far from independent. The model of the source of errors, proposed in work [6], and well conforming with experimental data, is characterized by the following:

errors in channel are inclined to the grouping, i.e., errors are grouped into the packets, and the error bursts are grouped into the chains/networks.

errors in packets are independent, and packets in chains/networks are independent.

probability of error within error burst is close to 0.5.

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If average/mean length of error burst is commensurate with length of subblock, then are probable following distortions of subblocks:

subblock out of error burst.

subblock wholly within error burst.

error burst begins in this subblock.

error burst is finished in this subblock.

one packet begins, and it is finished with other in this subblock.

error burst wholly lies/rests at this subblock.

The first of distortions examined does not require corrections, but the secondly correction of errors, as a rule is thoughtless. With the decoding of the codes of first stage in the remaining cases it is expedient either only to discover errors or to correct the single error burst and to discover remaining errors. With this relationship/ratio of length the burst of errors and length of subblock is assumed the following algorithm of decoding B_1:

first iteration - codes of first stage are used only for the detection of errors, and the codes of the second step/stage - for the correction of errors and erasings.

second iteration - codes of first stage are used for correction of error bursts, and codes of second step/stage - for correction of errors and erasings.

if code combinations were obtained in both iterations, then for unknown is accepted that, which from that accepted differs in smaller number of subblocks. If only in one of the iterations was obtained code combination, then it is accepted for unknown.

Algorithms of step by step decoding with inclined to grouping errors, just as with independent errors, can be decomposed into simple and complicated algorithms of step by step decoding. In this case by simple algorithm of step by step decoding with the errors inclined to the grouping is understood this algorithm of the step by step decoding, when the code of first stage corrects the i-fold error bursts and it discovers the remaining errors (combination of the code of first stage with the discovered errors it is erased), and the code of the second step/stage is used for the correction of errors and erasings. This algorithm of step by step decoding we will designate C_i (i=0, 1, 2, ...). In particular, algorithm C_0 always coincides with algorithm A_0.

By complicated algorithm of step by step decoding at inclined to grouping errors we will also understand combination from simple algorithms of step by step decoding. In particular, algorithm B_1 is a complicated algorithm of step by step decoding, since it is the combination of algorithms C_0 and C_1.

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Following algorithm of decoding B_2 will be analog of generalized algorithm of step by step decoding with errors, inclined to grouping,:

first iteration - codes of first stage are used for detection of errors, and codes of second step/stage - for correction of errors and erasings.

second iteration - codes of first stage are used for correction of single burst of errors and detection of remaining errors, and codes of second step/stage - for correction of errors and erasings.

third iteration - codes of first stage are used for correction of two bursts of errors and detection of remaining errors, and codes of second step/stage - for correction of errors and erasings and so forth to multiplicity of maximum reliably corrected error;

from code combinations, obtained on different iterations, for unknown is accepted that, which differs from that accepted in smallest number of subblocks.

About advisability of applying these algorithms of decoding it is possible to say following: algorithm C_0 is advisable with length of cascade code on the order of hundreds and thousands of binary symbols and following relationship/ratio between lengths of subcodes $n_1=a \log n_2$; B_1 - with length of cascade code on the order of hundreds and thousands of binary symbols and following relationship/ratio between lengths of subcodes $n_1 = an_2$; B_2 - for three-dimensional cascade codes and cascade codes of higher dimensions or when length of code is so great, that one or several chains/networks wholly are placed in code combination.

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