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THE BOX METHOD FOR LINEAR PROGRAMMING PART 2 TREATMENT
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Part II — Treatment of Problems in Standard Form
with Explicitly Bounded Variables

by
Karel Zikan
Richard W. Cottle

TECHNICAL REPORT SOL 87-9

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THE BOX METHOD FOR LINEAR PROGRAMMING: PART II – TREATMENT OF PROBLEMS IN STANDARD FORM WITH EXPLICITLY BOUNDED VARIABLES

by Karel ZIKAN and Richard W. COTTLE

1. Introduction.

The Box Method is a new interior-point algorithm for linear programming that deals with linear programs whose constraints are all linear inequalities. [See Zikan and Cottle (1987, Algorithm I) .] Hence, from the standpoint of (primal) problems presented in what Dantzig (1963, p. 86) calls “standard form” (i.e., equality constraints, nonnegative variables), the Box Method operates on the *dual problem*; in this sense it is a *dual method*.

Since linear programming problems with explicit upper (and lower) bound constraints are very common, a practical linear programming algorithm must be able to treat them as easily as the Simplex Method does. This report aims to accomplish for the Box Method what Dantzig (1955) did for the Simplex Method: to produce an efficient variant of the algorithm for handling problems with (explicit) upper bounds on the variables. The explicitly bounded variables are assumed to occur in a primal problem having standard form, although the algorithm will solve its dual. Like the Simplex Method, the variant presented here solves the problem almost as efficiently as if the upper bounds were not present. The increases in problem size and computational effort are very small.

Specifically, it will be shown in this paper how a crucial aspect of the Box Method can take advantage of the special structure of the *dual* of a linear program in standard form with explicit upper bounds on the variables. The key step in the Box Method is the determination of a *minimum-weight basis* corresponding to an interior feasible iterate. (This is also called the *combinatorial* or *matroidal problem*.) Minimum-weight bases are used in constructing the “boxes” over which the algorithm’s direction-finding subproblems are solved. Since the other algorithmic effects are insignificant, only matters related to the minimum-weight basis will be discussed. An important consequence of the results presented here is that any efficient scheme pertaining to (primal) problems *without* explicit upper bounds on the variables can be extended to the bounded-variable case. This point will be illustrated in the sequel to this report where the Box Method will be specialized to the important class of (linear) minimum cost flow problems.



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2. Block formulation.

Consider the linear program

$$\text{minimize} \quad b^T y \quad (1.1)$$

$$\text{subject to} \quad A^T y = c \quad (1.2)$$

$$0 \leq y \leq u \quad (1.3)$$

where A^T is an $n \times m$ matrix with linearly independent rows. Accordingly, $\text{rank } A^T = n \leq m$. After slack variables are inserted in (1), the constraint matrix becomes

$$\bar{A}^T = \begin{pmatrix} A^T & 0 \\ I & I \end{pmatrix},$$

the objective function vector and right-hand side vector become, respectively,

$$\bar{b}^T = (b^T, 0^T) \quad \text{and} \quad \bar{c} = \begin{pmatrix} c \\ 0 \end{pmatrix}.$$

It is immediate that $\bar{A}^T$ is an $(n + m) \times 2m$ matrix of full rank. The purpose of an upper-bounding technique is to avoid using such a potentially large matrix and to work instead with A^T itself.

Now assume that the columns of the matrix $\bar{A}^T$ are "weighted" as in the Box Method.¹ This means that to each column of $\bar{A}^T$ is associated a positive real number, its weight. How the weights are assigned is not important at the moment. Given the weight vector, w , the first problem is to find a *minimum-weight basis* consisting of columns of the matrix $\bar{A}^T$ and then an appropriate factorization. A minimum-weight basis is a maximal set of linearly independent columns for which the sum of the corresponding weights is minimum. It will be shown that solving this *matroidal problem* $MP(\bar{A}^T, w)$ can be accomplished with very little effort beyond what is needed to do the analogous job for A^T . Furthermore, the structure of a minimum-weight basis facilitates solving the two equations for which the Box Method requires solutions.

3. Theory.

The propositions and corollaries of this section provide the theoretical underpinnings of the algorithmic developments that follow. The results proceed from the natural (and obvious) pairing between the columns of the $(n + m) \times 2m$ matrix $\bar{A}^T$, namely column i corresponds to column $m + i$. Therefore, let

$$i \longleftrightarrow i' = m + i \quad i = 1, \dots, m. \quad (2)$$

¹The Box Method, as stated, would deal with $\bar{A}$ (rather than $\bar{A}^T$) and hence the rows would be weighted.

The first result is for matrices of more general type than $\bar{A}^T$.

Proposition 1. *Let M be a $k \times \ell$ matrix having no column of zeros. Assume B and D are $k \times k$ invertible and $\ell \times \ell$ invertible diagonal matrices, respectively. Given a fixed ℓ -vector of weights, w , the matroidal problems associated with the matrices M and $\bar{M} = BMD$ are solved by the same set of indices.*

Proof. The matrices M and BMD have exactly the same sets of linearly independent columns. That is, if E denotes a nonempty index set drawn from $\{1, \dots, \ell\}$, then $\bar{M}_{.E}$ has linearly independent columns if and only if $M_{.E}$ has linearly independent columns. Since the two matroidal problems are defined in terms of the same weight vector, w , it follows that the greedy algorithm will make the same column choices in each problem, thereby producing the same index set as solution. ■

Corollary 1. *Assume that the matrix A^T and the vector of weights, w , are fixed. Then the same set of basic indices solves the matroidal problem for every matrix having the block form*

$$\begin{pmatrix} A^T & 0 \\ B & BD \end{pmatrix} \quad (3)$$

where B is any $m \times m$ invertible matrix and D is an invertible diagonal matrix of the same order.

Proof. In view of Proposition 1, it suffices to note that a matrix of the form (3) can be brought to the "canonical form" (2) through multiplication on the left by

$$\begin{pmatrix} I & 0 \\ 0 & B^{-1} \end{pmatrix} \quad (4)$$

and on the right by

$$\begin{pmatrix} I & 0 \\ 0 & D^{-1} \end{pmatrix}. \quad (5)$$

In the case of (4), the identity matrix is of order n whereas in (5) it is of order m . ■

Remark. Assumption (A3) of the Box Method ordinarily calls for some (row) scaling in the $\bar{A}$. Accordingly, one is required to solve matroidal problems in which the matrices have the block form

$$\begin{pmatrix} A^T D & 0 \\ D & I \end{pmatrix}, \quad (6)$$

but such matrices are easily reduced to the form (2).

Corollary 2. Assume the matrix A^T and the vector of weights, w , are fixed. Then the same set of basic indices solves all matroidal problems for matrices of the form

$$\begin{pmatrix} A^T & 0 \\ D_1 & D_2 \end{pmatrix} \quad (7)$$

where D_1 and D_2 are nonsingular diagonal matrices of order m .

Proof. Obvious. ■

Remark. A special case of Corollary 2 is that in which D_1 and D_2 are (possibly distinct) "signed identity matrices" (i.e., their diagonal elements are ± 1 .)

Tie-breaking assumption. In order to avoid clumsiness in the wording of the following results, it will be assumed that ties are broken according to an arbitrary—but fixed—rule. This being settled, no special consideration need be given to ties.

Proposition 2. From each pair of indices (i, i') , at least one index, namely the one with the smaller associated weight, must be selected when the greedy algorithm is applied to the matroidal problem associated with $\bar{A}^T$.

Proof. Given the special structure of the matrix, the column i' can not be linearly dependent on previously selected columns unless its "mate", i , has already been selected. The same holds for column i . Since the greedy algorithm considers the columns according to increasing order of their weights, the assertion readily follows. ■

Proposition 3. All bases, i.e., maximal linearly independent sets of columns, in the matrix $\bar{A}^T$ contain exactly n (i, i') pairs and $m - n$ single elements from the remaining (i, i') pairs.

Proof. The rank of $\bar{A}^T$ is $n + m$, hence this is number of columns in any maximal linearly independent set. Let E denote the set of indices of such a set of columns. There are m pairs of indices (i, i') , and E must contain at least one representative of each pair. (Otherwise, the corresponding matrix would contain a row of zeros and hence would not consist of linearly independent columns.) Assume these m indices have been identified. No matter how the other n indices are chosen, their mates will necessarily belong to E . After this pairing is established, there must remain $m - n$ among the original m elements for which E contains no mate; these are the single elements. ■

The idea is illustrated by the matrix

$$\begin{pmatrix} A_1^T & A_2^T & 0 & 0 \\ (D_1)_1 & 0 & (D_2)_1 & 0 \\ 0 & (D_1)_2 & 0 & (D_2)_2 \end{pmatrix}$$

in which A_1^T denotes a nonsingular $n \times n$ matrix; for $i = 1, 2$, $(D_i)_1$ and $(D_i)_2$ denote nonsingular diagonal matrices of orders n and $m - n$, respectively. In the present case, the pairs of indices would be $(1, 1'), \dots, (n, n')$; this corresponds to the fact using A_1^T to cover the first n rows makes it necessary for $(D_2)_1$ to cover the next n rows. The remaining rows must be covered by columns drawn in a "complementary" way—that is, exactly one of i, i' —from the matrices $(D_1)_2$ and $(D_2)_2$.

Definition. Given $w \in R^{2m}$, let $\hat{w} \in R^{2m}$ be defined by the relations

$$\hat{w}_i = \max \{w_i, w_{m+i}\}, \quad \hat{w}_{m+i} = \min \{w_i, w_{m+i}\}. \quad (8)$$

The following result pertains to the selection of a basis in accordance with the associated matroidal problem.

Proposition 4. For a fixed matrix A^T the solution $E(w)$ of the matroidal problem $MP(\bar{A}^T, w)$ associated with the weight-vector w and the solution $E(\hat{w})$ of the matroidal problem $MP(\bar{A}^T, \hat{w})$ associated with weight-vector $\hat{w}$ contain the same set of pairs of indices.

Proof. Consider the problem $MP(\bar{A}^T, w)$ associated with the matrix $\bar{A}^T$. Assume $w \neq \hat{w}$, for otherwise there is nothing to prove. Define the set

$$U = \{i \in \{1, \dots, m\} : \hat{w}_i > w_i\}.$$

Let $i \in U$ be arbitrary, and perform the following sequence of operations:

1. Multiply $\bar{A}^T$ on the left by the elementary matrix that corresponds to a Gauss-Jordan pivot on the element in row $n + i$ and column i , that is, $\bar{A}_{n+i,i}^T$. (Cf. Proposition 1: left multiplication by nonsingular B .)
2. Change the sign of column $i' = m + i$. (Cf. Proposition 1: right multiplication by a nonsingular diagonal matrix.)
3. Interchange columns i and i' and also interchange the corresponding pair of weights. (Otherwise, the matroidal problems are not the same. Note that a—nondiagonal—permutation matrix is being applied on the right!)
4. Bring the matrix (which has the form (7) with signed identity matrices) to the form (2). (Cf. Corollary 2.)

Now the structure of the matrix is the same as in the beginning except for the roles of the indices i and i' which have been interchanged. Repeat the same procedure for all $i \in U$.

When the process is complete, the original matroidal problem $MP(\bar{A}^T, w)$ will have been transformed into the canonical format of the matroidal problem $MP(\bar{A}^T, \hat{w})$. The difference is that for each $i \in U$, the column in the i position now bears the name i' and vice versa. Therefore, for each $i \in U$, $i \in E(w)$ if and only if $i' \in E(\hat{w})$, and $i' \in E(w)$ if and only if $i \in E(\hat{w})$. Likewise, for each $i \notin U$, $i \in E(w)$ if and only if $i \in E(\hat{w})$, and $i' \in E(w)$ if and only if $i' \in E(\hat{w})$. Thus, $E(w)$ and $E(\hat{w})$ contain the same index pairs. ■

Corollary 3. To solve the matroidal problem $MP(\bar{A}^T, w)$, it suffices to solve $MP(\bar{A}^T, \tilde{w})$ where $\tilde{w}$ denotes the first m components of the vector $\hat{w}$.

Proof. Obvious. ■

Example. Suppose

$$A^T = \begin{pmatrix} 5 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix} \text{ and } w = (5, 8, 6, 9, 4, 7),$$

so that $m = 3$ and $n = 2$. Thus,

$$\hat{w} = (9, 8, 7, 5, 4, 6) \text{ and } U = \{1, 3\}.$$

To solve the matroidal problem $MP(\bar{A}^T, w)$ one must select 5 linearly independent columns from the matrix $\bar{A}^T$ for which the sum of the corresponding weights is as small as possible. The data can be arranged in tabular form as follows:

1	2	3	1'	2'	3'	← Indices
5	8	6	9	4	7	← Weight vector w
5	2	3	0	0	0	← $\bar{A}^T$
1	2	4	0	0	0	
1	0	0	1	0	0	
0	1	0	0	1	0	
0	0	1	0	0	1	

The transformation process described in Proposition 4 ends with the table

1'	2	3'	1	2'	3	← Indices
9	8	7	5	4	6	← Weight vector $\hat{w}$
5	2	3	0	0	0	← $\bar{A}^T$
1	2	4	0	0	0	
1	0	0	1	0	0	
0	1	0	0	1	0	
0	0	1	0	0	1	

The solution to $MP(\bar{A}^T, w)$ is

$$E(w) = \{2, 3', 1, 2', 3\}.$$

By contrast, the matroidal problem $MP(\bar{A}^T, \hat{w})$ is expressed by the table

1	2	3	1'	2'	3'	← Indices
9	8	7	5	4	6	← Weight vector $\hat{w}$
5	2	3	0	0	0	
1	2	4	0	0	0	
1	0	0	1	0	0	← $\bar{A}^T$
0	1	0	0	1	0	
0	0	1	0	0	1	

which only differs from the preceding table in the locations of the primes on the indices (i.e., the columns labels). The solution of $MP(\bar{A}^T, \hat{w})$ is obviously

$$E(\hat{w}) = \{2, 3, 1', 2', 3'\}.$$

Proposition 2 asserts that for every $i = 1, \dots, m$, the index of the smaller weight $w_i, w_{i'}$ belongs to $E(w)$. The definition of $\hat{w}$ implies that $i' \in E(\hat{w})$ for every i , and then focuses attention on solving $MP(A^T, \tilde{w})$ where $\tilde{w}_i = \hat{w}_i$ for $i = 1, \dots, m$.

4. Algorithm.

As seen above, the solution set $E(w)$ contains n pairs of indices (i, i') and $m - n$ other elements, one from each remaining (i, i') pair, namely the one corresponding to the smaller weight. Accordingly, the set $E(w)$ has a natural partitioning into the disjoint union of four subsets: D , D' , S , and S' . The following relationships hold.

- i) D and S are subsets of $\{1, \dots, m\}$;
- ii) D' and S' are subsets of $\{1', \dots, m'\}$;
- iii) $i \in D$ if and only if $i' \in D'$;
- iv) D and D' each contain n elements;
- v) S and S' may each be empty, but their union contains exactly $m - n$ elements.

procedure upper bounds min-weight basis

begin

$S \leftarrow \emptyset;$ (comment: S is the set of single i 's selected)

$S' \leftarrow \emptyset;$ (comment: S' is the set of single i 's selected)

$D \leftarrow \emptyset;$ (comment: D is the set of i 's selected for pairs)

$D' \leftarrow \emptyset;$ (comment: D' is the set of i 's selected for pairs)

for $i = 1, m$ **do**

begin

if ($w_i < w_{i'}$) **then**

$\tilde{w}_i \leftarrow w_{i'}$

$S \leftarrow S \cup \{i\}$

else

$\tilde{w}_i \leftarrow w_i$

$S' \leftarrow S' \cup \{i'\}$

endif

end

(comment: This completes the selection of indices according to Proposition 2.)

begin

solve $MP(A^T, \tilde{w})$ to get D and D' ; (comment: Corollary 3.)

$S \leftarrow S \setminus D$

$S' \leftarrow S' \setminus D'$

return S, S', D , and D'

end

end

Thus, as a result of solving $MP(\bar{A}^T, w)$, one obtains the index set $E(w)$ and its partitioning:

$$E(w) = D \cup D' \cup S \cup S'.$$

Indeed, as will be seen in the next section, it is useful to partition the full sets of indices as follows:

$$\{1, \dots, m\} = D \cup S \cup N \quad \text{and} \quad \{1', \dots, m'\} = D' \cup S' \cup N'.$$

The elements of N and N' are, of course, the indices of nonbasic columns of $\bar{A}^T$. There is a pairing between the elements of S and those of N' . Likewise, there is a pairing between the elements of S' and those of N .

In the example used in Section 3, these six sets are

$$\begin{aligned} D &= \{2, 3\}, & D' &= \{2', 3'\}, \\ S &= \{1\}, & S' &= \emptyset, \\ N &= \emptyset, & N' &= \{1'\}. \end{aligned}$$

5. Solving systems of equations.

Each iteration of the Box Method entails the solution of a box problem, each of which, in turn, calls for the solution of two systems of equations stated in terms of the current minimum-weight basis. [See (12) and (13) in Zikan and Cottle (1987).] This section will discuss how to treat these systems when the original primal problem is of the bounded variable type.

In general, let $\bar{B}^T$ denote the minimum-weight basis $(\bar{A}^T)_{E(w)}$. The first system of equations to be solved is of the form

$$\bar{B}^T \bar{c} = \bar{c}, \quad (9)$$

where

$$\bar{c} = \begin{pmatrix} c \\ u \end{pmatrix}.$$

The second system of equations to be solved is of the form

$$\bar{B}z = \bar{z} \quad (10)$$

where $\bar{z}$ is the solution to a simplified box problem [see (14) in Zikan and Cottle (1987)] that is defined in terms of $\bar{c}$. The solution z of (10) above is the solution to the box problem corresponding to the current minimum-weight basis.

Up to a rearrangement of rows and columns, the matrix $\bar{A}^T$ can be written in the form

$$\begin{pmatrix} A^T_{\cdot D} & A^T_{\cdot S} & A^T_{\cdot N} & 0 & 0 & 0 \\ I_D & 0 & 0 & I_{D'} & 0 & 0 \\ 0 & I_S & 0 & 0 & I_{N'} & 0 \\ 0 & 0 & I_N & 0 & 0 & I_{S'} \end{pmatrix}.$$

After further rearrangement, it follows that

$$\bar{B}^T = \begin{pmatrix} A^T_{\cdot D} & 0 & A^T_{\cdot S} & 0 \\ I_D & I_{D'} & 0 & 0 \\ 0 & 0 & I_S & 0 \\ 0 & 0 & 0 & I_{S'} \end{pmatrix}.$$

Note that the submatrix $A^T_{\cdot D}$ of $\bar{B}^T$ is the minimum-weight basis matrix that would have been obtained if there had not been any explicit upper bounds. From the (essentially block triangular) structure of $\bar{B}^T$, it is evident that solving systems of equations like those indicated in (9) and (10) requires little more work or storage than solving equations (12) and (13) in Zikan and Cottle (1987).

To see this more clearly, consider a system of the form (9). In more detail the system can be written as

$$\begin{pmatrix} A^T_{\cdot D} & 0 & A^T_{\cdot S} & 0 \\ I_D & I_{D'} & 0 & 0 \\ 0 & 0 & I_S & 0 \\ 0 & 0 & 0 & I_{S'} \end{pmatrix} \begin{pmatrix} \tilde{c}_D \\ \tilde{c}_{D'} \\ \tilde{c}_S \\ \tilde{c}_{S'} \end{pmatrix} = \begin{pmatrix} \bar{c}_D \\ \bar{c}_{D'} \\ \bar{c}_S \\ \bar{c}_{S'} \end{pmatrix}. \quad (11)$$

Solving (11) can be done by *sequentially* solving the following set of four smaller systems:

$$I_{S'} \tilde{c}_{S'} = \bar{c}_{S'}; \quad (12)$$

$$I_S \tilde{c}_S = \bar{c}_S; \quad (13)$$

$$A^T_{\cdot D} \tilde{c}_D = \bar{c}_D - A^T_{\cdot S} \tilde{c}_S; \quad (14)$$

$$I_{D'} \tilde{c}_{D'} = \bar{c}_{D'} - I_D \tilde{c}_D. \quad (15)$$

In block form, (10) becomes

$$\begin{pmatrix} A_{D\cdot} & I_D & 0 & 0 \\ 0 & I_{D'} & 0 & 0 \\ A_{S\cdot} & 0 & I_S & 0 \\ 0 & 0 & 0 & I_{S'} \end{pmatrix} \begin{pmatrix} z_D \\ z_{D'} \\ z_S \\ z_{S'} \end{pmatrix} = \begin{pmatrix} \tilde{z}_D \\ \tilde{z}_{D'} \\ \tilde{z}_S \\ \tilde{z}_{S'} \end{pmatrix}; \quad (16)$$

the technique for solving it is analogous to that for (11), and will not be spelled out.

Remark. Due to the possible need for scaling, it may happen that some of the blocks identified here as identity matrices are actually nonsingular diagonal matrices. It is clear that if these conditions obtain, there is very little extra storage or computational effort required. The only significant work stems from equation (14).

The procedure presented here can be modified to handle the case where some, *but not* all of the variables are explicitly upper bounded. The details are easily worked out and hence are omitted.

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SOL 87-9: The Box Method for Linear Programming: Part II — Treatment of Problems in Standard Form with Explicitly Bounded Variables, Karel Zikan and Richard W. Cottle (July 1987, 11 pp.)

A crucial aspect of the Box Method for linear programming is the finding of a "minimum-weight basis" corresponding to a given interior feasible point. This subproblem leads to the formation of the "Box Problem," a special linear program having a closed form solution which provides the search direction at the current iteration. Finding a minimum-weight basis is a matroidal (or, combinatorial) optimization problem that can be handled by a greedy algorithm. This paper suggests a way of efficiently solving the minimum-weight basis problem in cases where the (primal, standard form) linear program contains explicitly bounded variables. It is shown that the main part of the task requires almost no more computational effort or storage space than does a problem of the same size without upper bounded variables. While this result is believed to be valuable in its own right, there is additional benefit to be gained in applications where the finding of a minimum-weight basis (for a linear program without explicit upper bounds on its variables) is done by a special greedy algorithm. Such is the case with minimum-cost network flow problems which will be discussed in Part III of this series.

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