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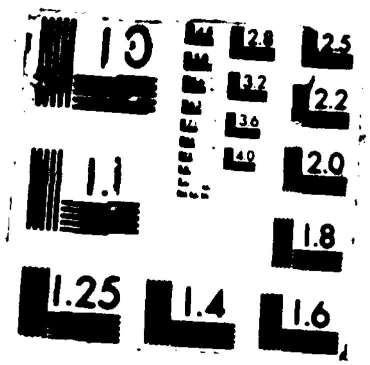
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ANALYSIS OF COMPOSITE SHRINK FITS - TRESCA MATERIAL

PETER C. T CHEN

JULY 1987

US ARMY ARMAMENT RESEARCH, DEVELOPMENT
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A thin composite shrink fit assembly is examined herein using an elastic-plastic analysis. The ring and disk are made of different materials. Interferences large enough to induce plastic deformations in the ring are considered. The ring material is assumed to be a linear strain-hardening material that obeys Tresca's yield condition and the associated flow rule. The explicit expressions for stresses and deformations in the shrink fit assembly have been

(CONT'D ON REVERSE)

20. ABSTRACT (CONT'D)

obtained. Numerical results are presented for shrink fit assemblies with different geometric ratio, hardening parameter, and different combinations of materials. (Keywords:)

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INTRODUCTION

The shrink fit fastening process is widely used in industry to produce tight, precision assemblies where other fastening methods are neither necessary nor practical. By shrinking a thin ring onto a disk of the same thickness, an elastic state of biaxial, hydrostatic stress can be induced in the disk. For sufficiently small values of interference of the fit, the ring and disk remain elastic; for large values of interference, the ring becomes plastic, first at the interference; for yet larger values of interference, it is possible to produce a plastic state in the disk. This problem was analyzed recently by Gamer and Lance (ref 1) considering the same materials for the disk and ring.

In this report we examine a thin composite shrink fit assembly using a plane-stress elastic-plastic analysis. The ring and disk are made of different materials. Interferences large enough to induce plastic deformations in the ring are considered. The ring material is assumed to be a linear strain-hardening material that obeys Tresca's yield condition and the associated flow rule. The stresses and deformations in the shrink fit assembly are obtained as functions of the interference of the fit.

ELASTIC ASSEMBLY

A shrink fit assembly is shown in Figure 1. The assembly may be produced by cooling the disk and/or heating the ring with the manufactured interference I. The common interference radius of the assembly is a. The thickness, h, is small compared to a, and hence, the state of stress may be assumed to be plane. All thermal effects are neglected and the displacement is assumed to be small everywhere.

[1]Gamer, U. and Lance, R. H., "Residual Stresses in Shrink Fits," Int. J. Mech. Sci., Vol. 25, No. 7, 1983, pp. 465-470.

For small values of interference of fit, the stress state in the entire assembly is elastic. The stresses and displacements in the ring are

$$\left.\begin{array}{l}\sigma_r\\ \sigma_\theta\end{array}\right\} = \frac{P}{1 - a^2/b^2}\left[\frac{a^2}{b^2} \mp \frac{a^2}{r^2}\right] \qquad \begin{array}{r}(1a)\\ (1b)\end{array}$$

$$u/r = (P/E)[(1+\nu)(a^2/r^2) + (1-\nu)(a^2/b^2)]/(1-a^2/b^2) \qquad (1c)$$

and in the disk

$$\sigma_r = \sigma_\theta = -P \quad , \quad u/r = -(1-\nu_1)P/E_1 \qquad (2)$$

where E, ν and E_1, ν_1 are the material constants of the ring and disk, respectively. At the interface, u_a (ring) - u_a (disk) = I by the compatibility requirement. The interference pressure (p) is a function of the interference (I) given by

$$p = \frac{EI}{a}\left(1 - \frac{a^2}{b^2}\right)/\left[(1+\nu) + (1-\nu)\frac{a^2}{b^2} + (1-\nu_1)\left(1 - \frac{a^2}{b^2}\right)\frac{E}{E_1}\right] \qquad (3)$$

For sufficiently large values of the interference, the stresses in the ring reach the yield limit. Assuming that Tresca's yield condition governs the behavior of the material, the ring first becomes plastic at the interference when the stresses satisfy

$$\sigma_\theta - \sigma_r = \sigma_o \qquad (4)$$

where σ_o is the initial tensile yield stress. The solution for the critical interference pressure to cause incipient plastic deformation is

$$p^* = \tfrac{1}{2}\,\sigma_o(1 - a^2/b^2) \qquad (5)$$

and it follows from Eq. (3) that the interference for the onset of plastic flow is

$$I^* = \frac{\sigma_o}{E}\,\frac{a}{2}\left[(1+\nu) + (1-\nu)\frac{a^2}{b^2} + (1-\nu_1)\left(1 - \frac{a^2}{b^2}\right)\frac{E}{E_1}\right] \qquad (6)$$

which reduces to $I^* = a\sigma_0/E$ for the special case ($E_1 = E$, $\nu_1 = \nu$) considerea in Reference 1.

PARTIALLY PLASTIC ASSEMBLY

For values of interference larger than that given by Eq. (4), a plastic zone forms in the ring, so that for $a \leqslant r \leqslant \rho$ the ring is plastic, while for $\rho \leqslant r \leqslant b$, the ring material is still in an elastic state. The elastic-plastic interface radius ρ is a function of the interference I.

We assume that the ring is made of a linear work-hardening material which obeys Tresca's yield condition

$$\sigma_\theta - \sigma_r = \sigma \tag{7}$$

where the yield stress σ is a function of the plastic strain ϵ^p. For a linear work-hardening material, we have

$$\sigma = \sigma_0(1+\eta\epsilon^p) \quad \text{and} \quad \eta = (E/\sigma_0)m/(1-m) \tag{8}$$

where η (or m) is the hardening parameter.

Applying the usual flow rule and following the method of analysis reported by Gamer and Lance (ref 1) and Bland (ref 2), the expressions for the stresses and the displacement can be obtained explicitly. The complete solution in $a \leqslant r \leqslant \rho$ is:

$$\sigma_r = \sigma_0(1-m)[\ln\frac{r}{a} - \tfrac{1}{2}(1-\nu)\eta\frac{D}{E}r^{-2}] + C \tag{9}$$

$$\sigma_\theta = \sigma_0(1-m)[1+\ln\frac{r}{a} + \tfrac{1}{2}(1-\nu)\eta\frac{D}{E}r^{-2}] + C \tag{10}$$

$$(1-\nu)^{-1}u = \frac{\sigma_0}{E}(1-m)[r\ln\frac{r}{a} - \tfrac{1}{2}(1-\nu)\eta\frac{D}{E}r^{-1}] + \frac{C}{E}r + \frac{D}{E}r^{-1} \tag{11}$$

[1]Gamer, U. and Lance, R. H., "Residual Stresses in Shrink Fits," Int. J. Mech. Sci., Vol. 25, No. 7, 1983, pp. 465-470.
[2]Bland, D. R., "Elastoplastic Thick-Walled Tubes of Work-Hardening Materials Subject to Internal and External Pressures and Temperature Gradients," J. Mech. Phys. Solids, Vol. 4, 1956, pp. 209-229.

In the elastic zone, $\rho \leq r \leq b$, the stresses and the displacement are:

$$\left.\begin{matrix}\sigma_r \\ \sigma_\theta\end{matrix}\right\} = \frac{E}{1-\nu} A \mp \frac{E}{1+\nu} \frac{B}{r^2} \qquad \begin{matrix}(12) \\ (13)\end{matrix}$$

$$u = Ar + B/r \qquad (14)$$

The constants A, B, C, D, p, and ρ all depend on the interference I, and can be evaluated by considering the following conditions: continuity of stress and displacement at $r = \rho$ requires $\sigma_r(\rho^-) = \sigma_r(\rho^+)$ and $u(\rho^-) = u(\rho^+)$. At the ring-disk interface $\sigma_r(a) = -p$ and at the outer surface of the ring $\sigma_r(b) = 0$. The yield condition in Eq. (7) must be satisfied at $r = \rho$ and finally, compatibility of the displacement field with the interference I requires that $u(a^+) - u(a^-) = I$. These conditions are sufficient to determine all unknown parameters. In this report the constants A, B, C, D are determined as functions of ρ.

$$A = \tfrac{1}{2}(1-\nu)(\sigma_0/E)(\rho/b)^2 \quad , \quad B = \tfrac{1}{2}(1+\nu)(\sigma_0/E)\rho^2$$

$$C = \sigma_0[\tfrac{1}{2}m - (1-m)\ln(b/a) - \tfrac{1}{2}(1-\rho^2/b^2)] \quad , \quad D = \sigma_0\rho^2/(1-\nu) \qquad (15)$$

The dimensionless interference pressure and interference are given, respectively, by

$$\bar{p} = P/\sigma_0 = \tfrac{1}{2}(1-\rho^2/b^2) + (1-m)\ln(\rho/a) + \tfrac{1}{2}m(\rho^2/a^2-1) \qquad (16)$$

$$\bar{I} = (E/\sigma_0)I/a = (\rho/a)^2 - [(1-\nu) - (1-\nu_1)E/E_1](P/\sigma_0) \qquad (17)$$

When the ring and disk are made of the same material, i.e., $E_1 = E$, $\nu_1 = \nu$, Eq. (17) reduces to the simple formula, $(E/\sigma_0)I/a = (\rho/a)^2$. For this special case (ref 1), the constants A, B, C, D, P, and ρ can be expressed explicitly as functions of interference I. In general, the interference pressure (p) is related to the interference (I) implicitly through the elastic-plastic interface (ρ) as shown in Eqs. (16) and (17) for $a \leq \rho \leq b$. The upper limit of the partially

[1]Gamer, U. and Lance, R. H., "Residual Stresses in Shrink Fits," Int. J. Mech. Sci., Vol. 25, No. 7, 1983, pp. 465-470.

plastic assembly is obtained by letting $\rho = b$. The corresponding interference pressure (p^{**}) and interference (I^{**}) are

$$p^{**}/\sigma_0 = (1-m)\ln(b/a) + \tfrac{1}{2}m(b^2/a^2-1)$$

$$(E/\sigma_0)I^{**}/a = (b/a)^2 - [(1-\nu) - (1-\nu_1)E/E_1]p^{**}/\sigma_0 \tag{18}$$

FULLY PLASTIC ASSEMBLY

When the interference I is larger than I^{**}, we have reached the fully plastic state in the ring. In this case, the expressions for the stresses and the displacement in $a \leq r \leq b$ are still the same as those given by Eqs. (9), (10), and (11). The constants C, D, and the interference p are determined with the boundary conditions $\sigma_r(a) = -p$, $\sigma_r(b) = 0$, and the compatibility requirement $u(a^+) - u(a^-) = I$. The results for the constants are

$$C = [pa^2/b^2 - (1-m)\sigma_0 \ln(b/a)]/(1-a^2/b^2)$$

$$D = 2a^2[p - (1-m)\sigma_0\ln(b/a)]/[m(1-\nu)(1-a^2/b^2)] \tag{19}$$

and the interference pressure is given as a function of interference by

$$\frac{p}{\sigma_0} = \frac{m(E\sigma_0/Ia)(1-a^2/b^2) + 2(1-m)\ln(b/a)}{2 - m[(1-\nu) - (1-\nu_1)E/E_1](1-a^2/b^2)} \tag{20}$$

NUMERICAL RESULTS AND DISCUSSIONS

The analysis described above makes it possible to predict the interference pressure in a composite shrink fit assembly, and hence, determine the stress state in the ring and disk as a function of the interference. The numerical results have been obtained for shrink fit assemblies with different geometric ratio ($\alpha = a/b$), hardening parameter (m), and different combinations of materials. For a steel ring with $\alpha = 0.5$, $m = 0.0$, $E = 30\times10^6$ psi, $\nu = 0.3$, $\sigma_0 = 15\times10^4$ psi, we have considered three types of disks: (a) rigid disk with $E_1 = 1000\ E$, $\nu_1 = 0.0$, $\sigma_1 = 1000\ \sigma_0$; (b) steel disk of the same material as the ring;

(c) a disk made of tungsten carbide with $E_1 = 88.5 \times 10^6$ psi, $\nu_1 = 0.258$, $\sigma_1 = 50 \times 10^4$ psi. The numerical results of the interference pressure (p/σ_0) for these three cases are presented graphically in Figure 2 as functions of the interference ($\bar{I}$). The results of the hoop stress at the inside surface of the ring are presented in Figure 3 also for these three cases. As can be seen from these two figures, the results for the composite shrink fit assembly marked (c) fall between the two limits established by cases (a) and (b).

For composite shrink fit assemblies made of tungsten carbide disk and steel ring with $\alpha = 0.5$, $m = 0.0, 0.1, 0.2$, the results are presented in Figures 4 and 5, respectively, for the interference pressure and the hoop stress at the bore as functions of the interference. The effect of hardening parameter (m) on these relations can be seen from these two figures. For the same combination of composite shrink fit assembly with $m = 0.05$, $\alpha = 1/4, 1/3, 1/2, 3/4$, the results showing the effect of geometric ratio (α) are shown in Figures 6 and 7 for the interference pressure and hoop stress at the bore, respectively.

The numerical results of the stresses and displacements in composite shrink fit assemblies have also been obtained, but only some results are presented here. The distributions of hoop stresses in a steel ring with $\alpha = 0.5$ are shown in Figures 8, 9, and 10 for $m = 0.0, 0.1, 0.2$, respectively. In each figure, we have shown the results corresponding to four stages of interference: (a) initial yielding ($\rho/a = 1.0$), $\bar{I}^* = 0.832$; (b) partial yielding ($\rho/a = 1.5$), $\bar{I} = 1.970, 1.960, 1.950$; (c) complete yielding ($\rho/a = 2.0$), $\bar{I}^{**} = 3.689, 3.653, 3.617$; (d) fully plastic state with $\bar{I} = 1.5\ I^{**}$. For an ideally plastic ring ($m = 0.0$), the stress distribution remains unchanged after complete yielding has been reached. For stra...-hardening rings, the stress distributions show large variations, especially for large values of interference. As shown in Figures 8, 9, and 10, the hardening parameter has a significant effect on the stress distributions. Additional stress distributions in the ring with $m = 0.1$ are shown in Figures 11 and 12 for $\alpha = 1/3$ and $1/4$, respectively. The effect of geometric ratio on the distributions can be seen by comparing Figures 9, 11, and 12.

REFERENCES

1. Gamer, U. and Lance, R. H., "Residual Stresses in Shrink Fits," Int. J. Mech. Sci., Vol. 25, No. 7, 1983, pp. 465-470.

2. Bland, D. R., "Elastoplastic Thick-Walled Tubes of Work-Hardening Materials Subject to Internal and External Pressures and Temperature Gradients," J. Mech. Phys. Solids, Vol. 4, 1956, pp. 209-229.

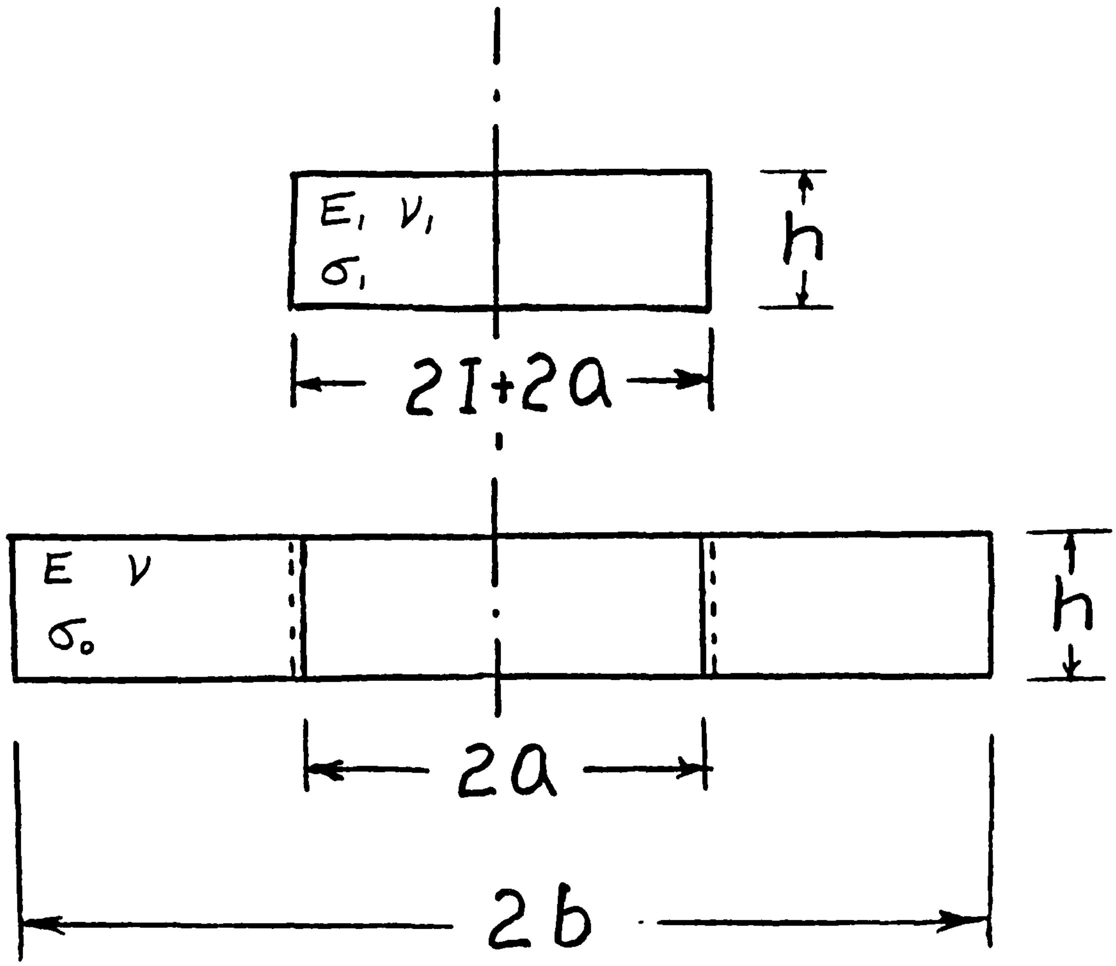

Figure 1. Shrink fit assembly.

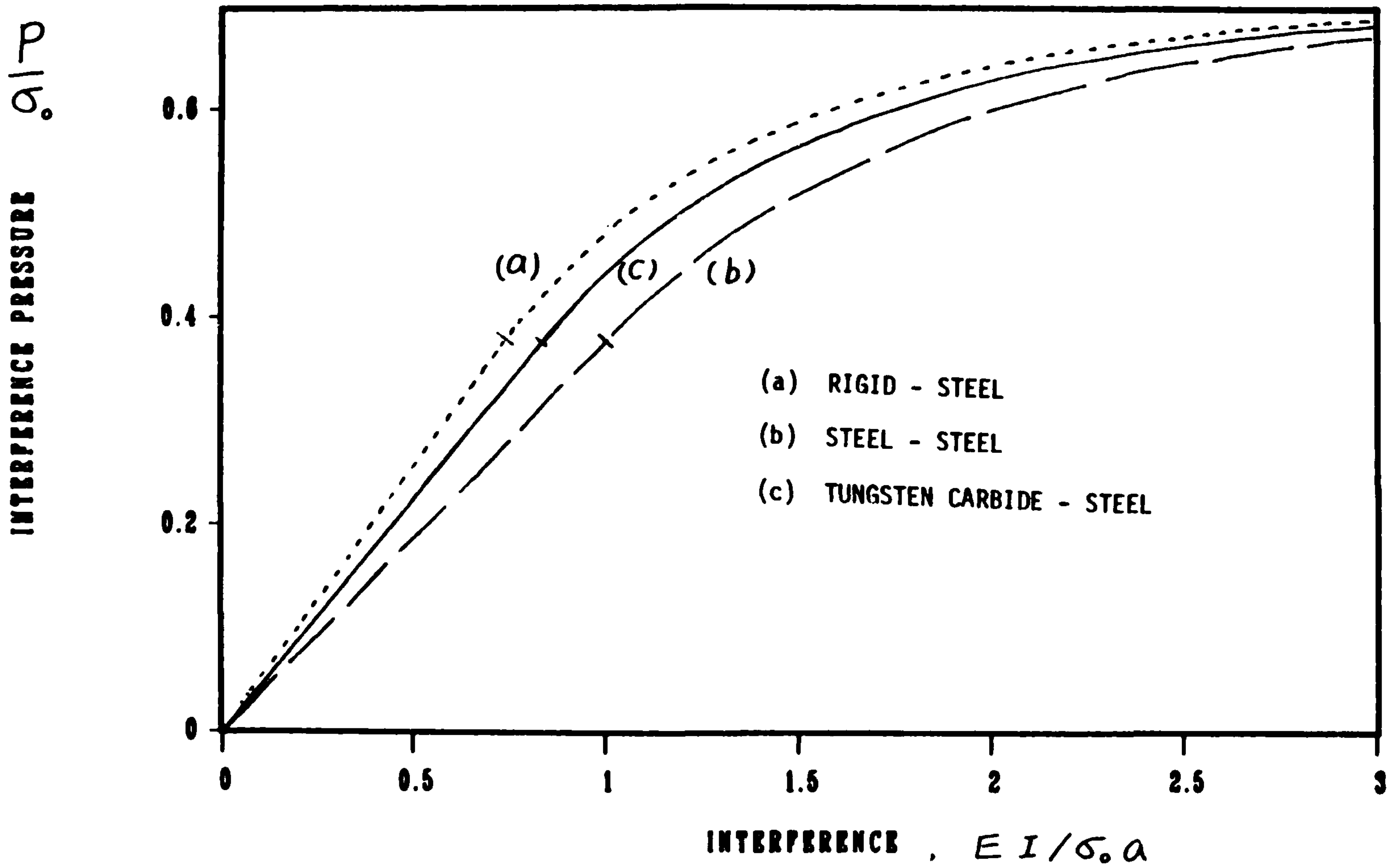

Figure 2. Interference pressure vs. interference for three shrink fit assemblies (α = 0.5, m = 0.0).

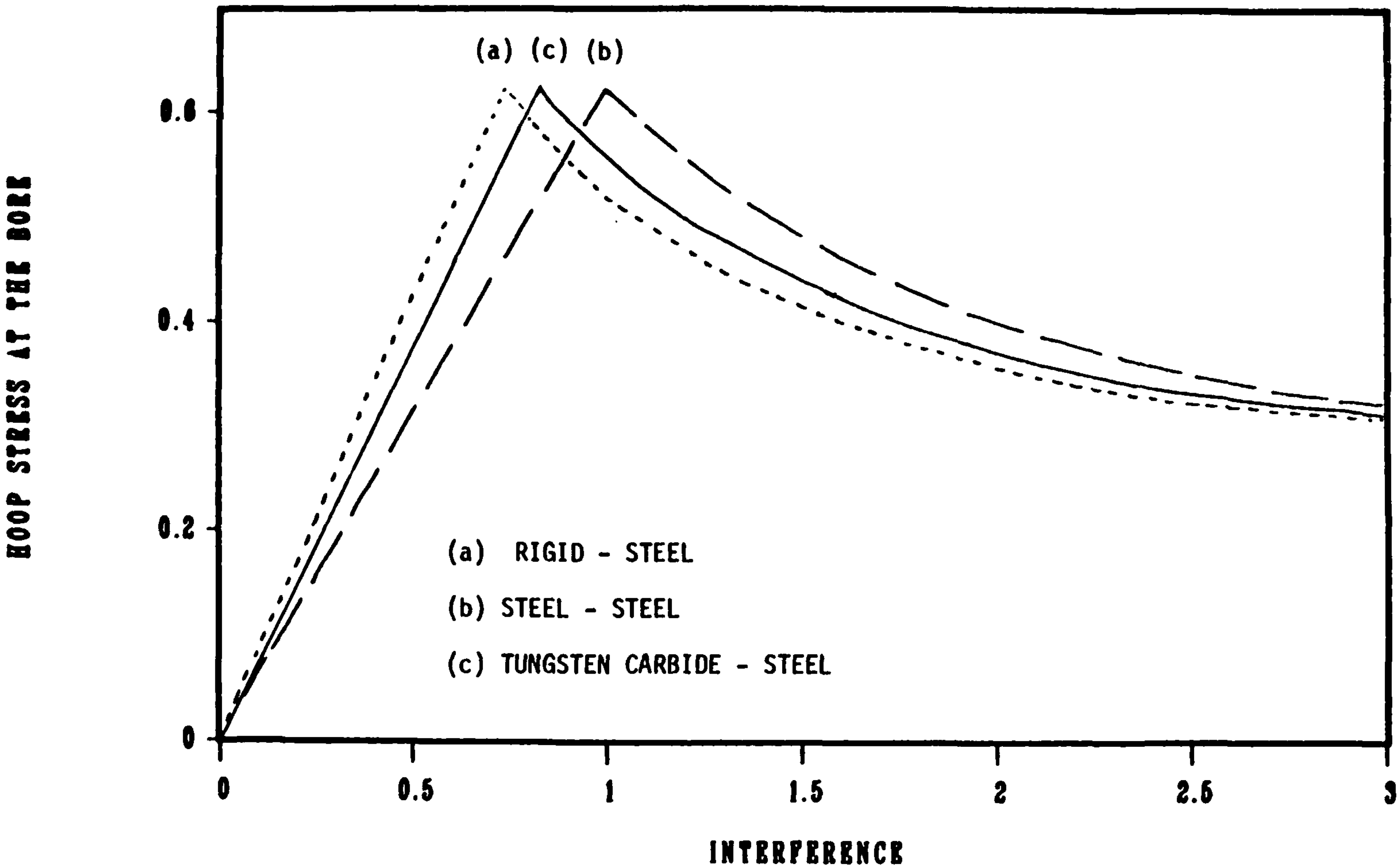

Figure 3. Hoop stress at the bore vs. interference for three shrink fit assemblies (α = 0.5, m = 0.0).

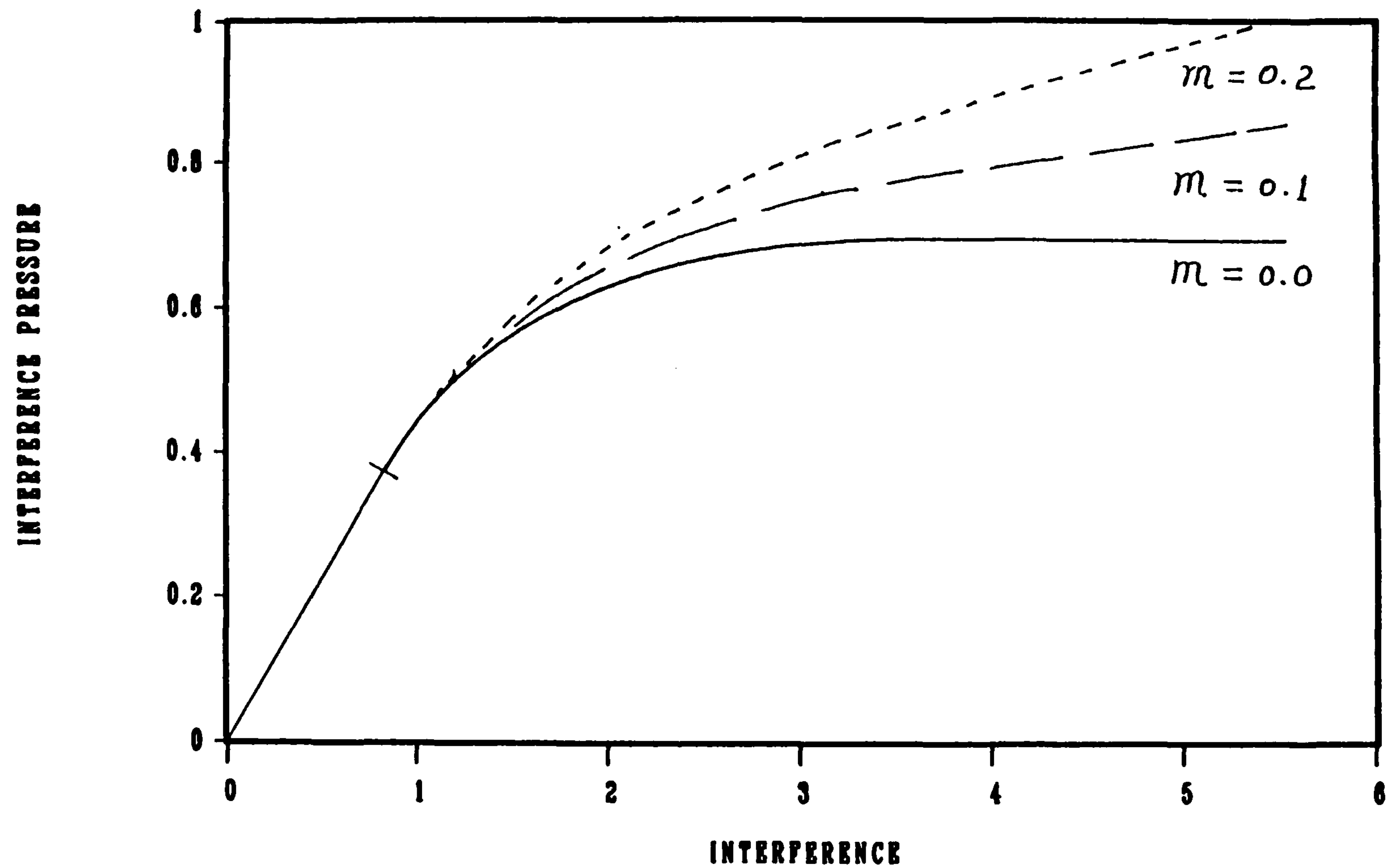

Figure 4. The effect of hardening on interference pressure in a composite assembly (α = 0.5).

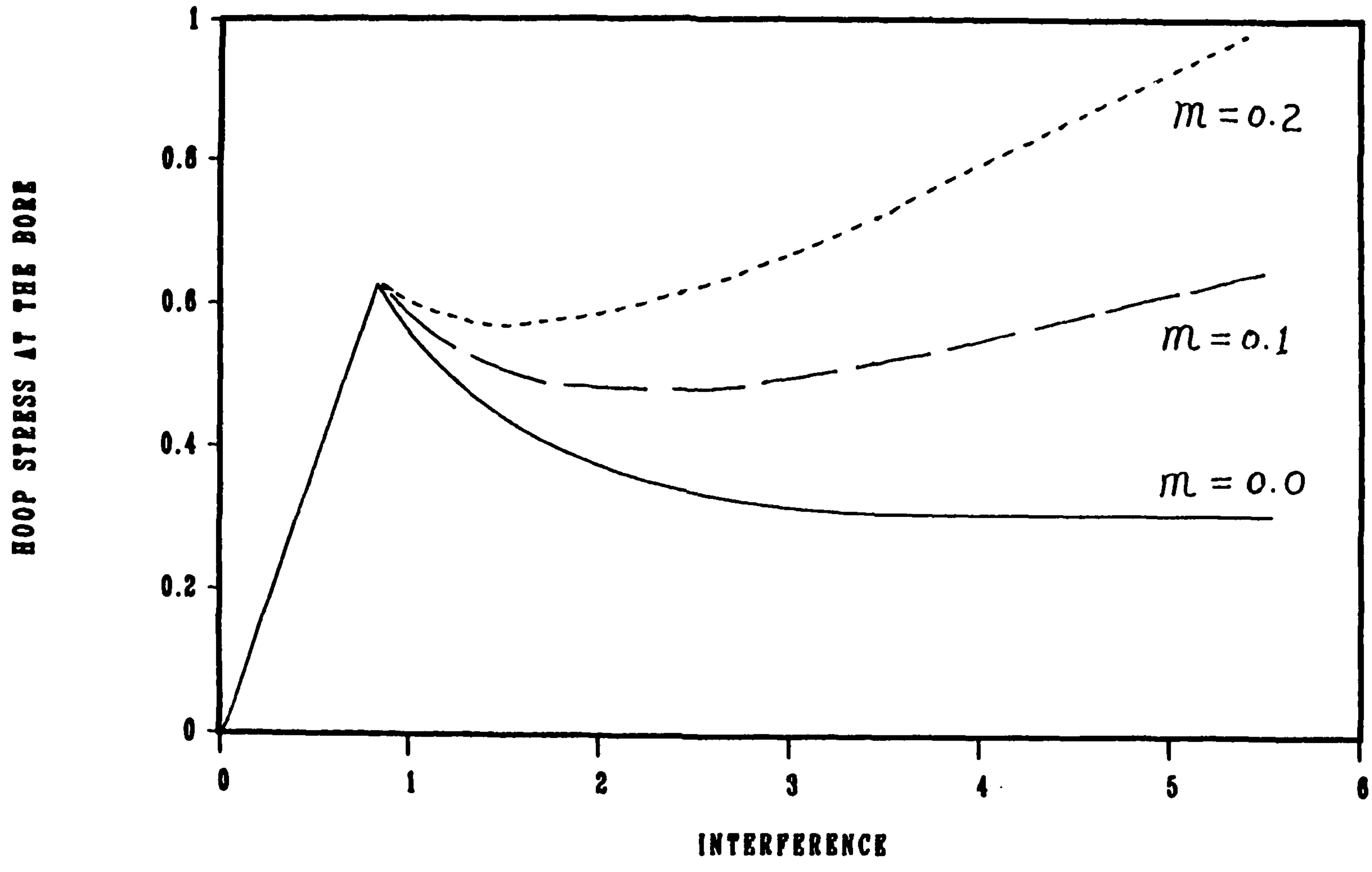

Figure 5. The effect of hardening on the hoop stress at the bore of a steel ring ($\alpha = 0.5$).

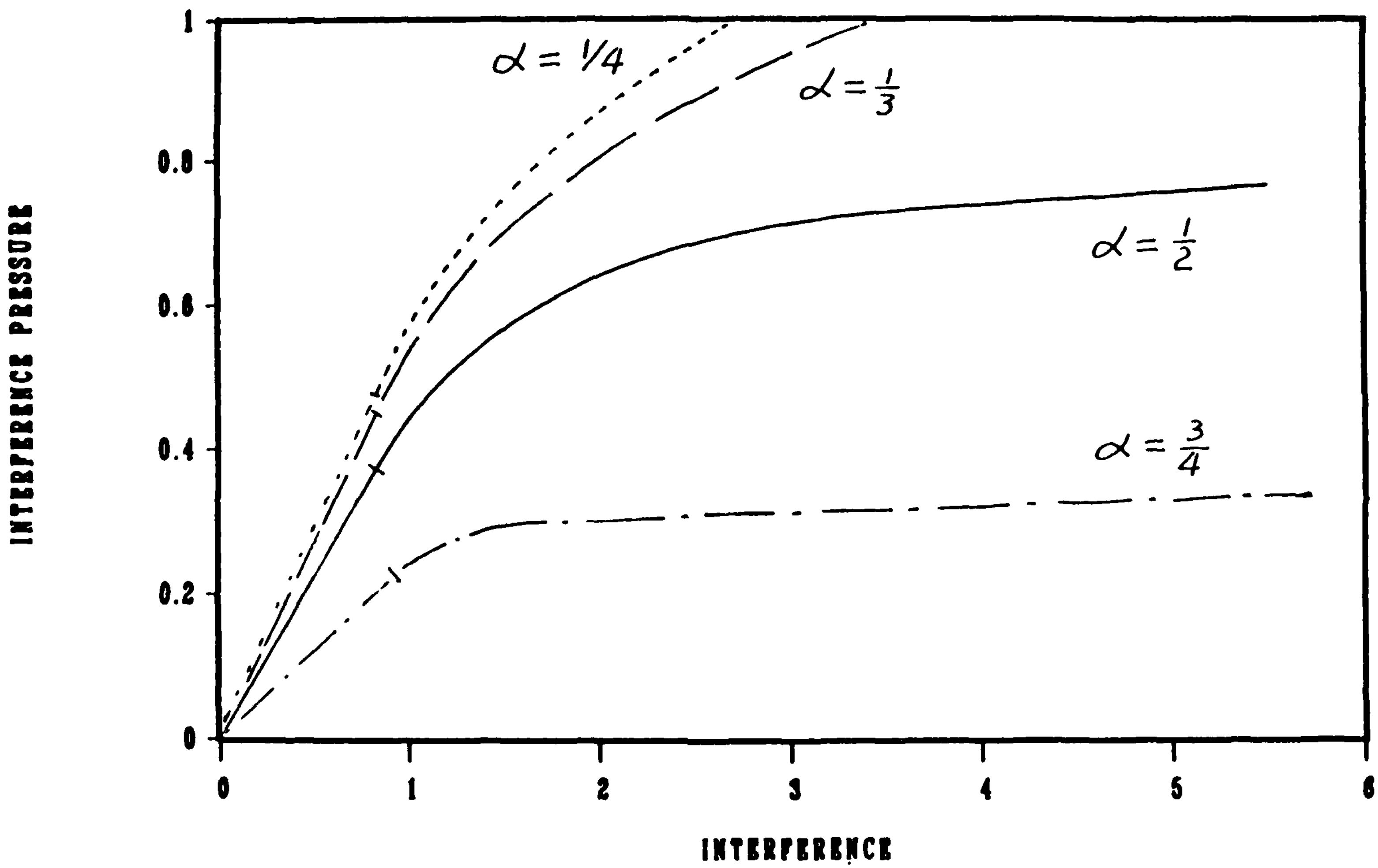

Figure 6. The effect of geometric ratio on interference pressure in a composite assembly (m = 0.05).

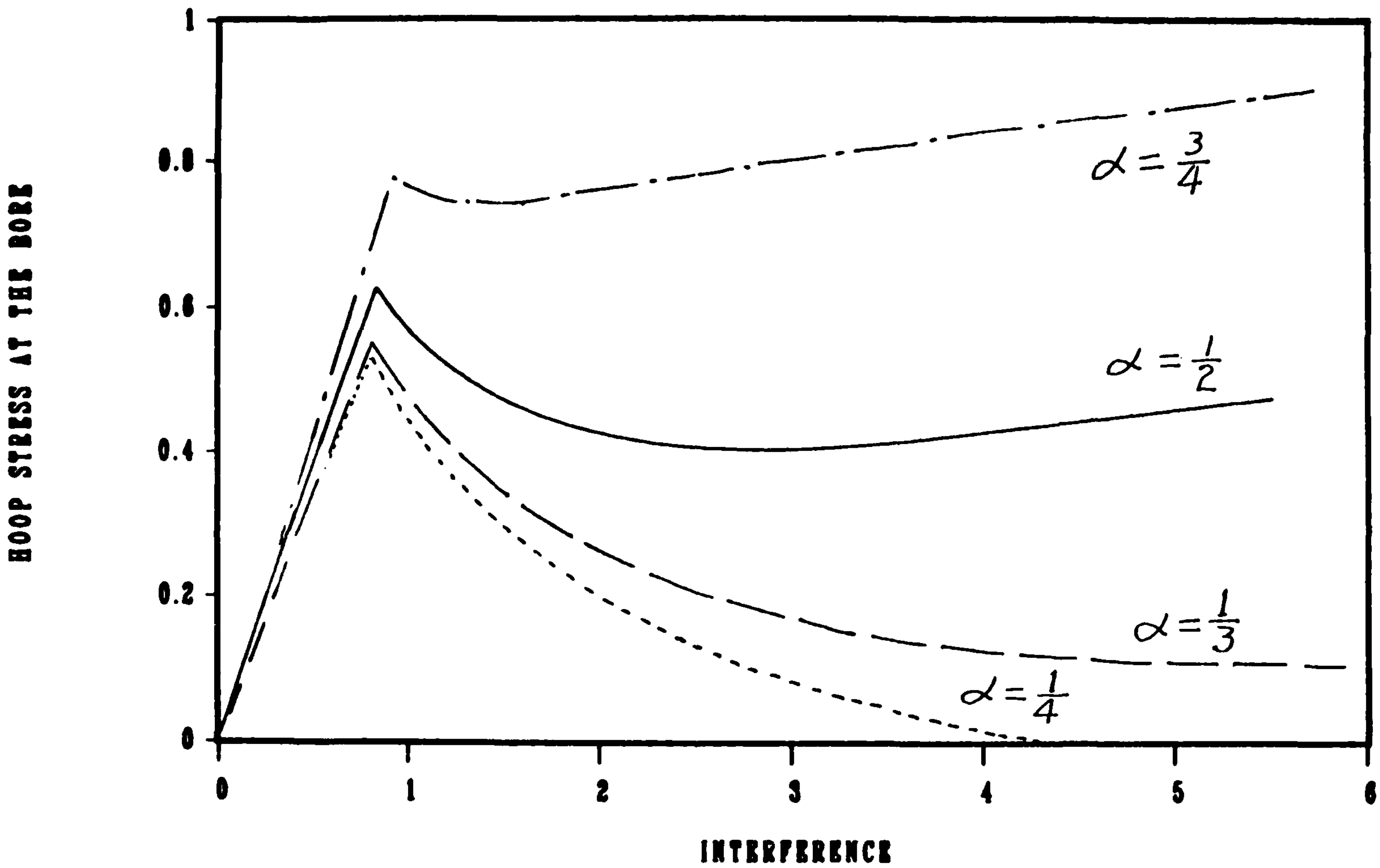

Figure 7. The effect of geometric ratio on the hoop stress at the bore of a steel ring (m = 0.05).

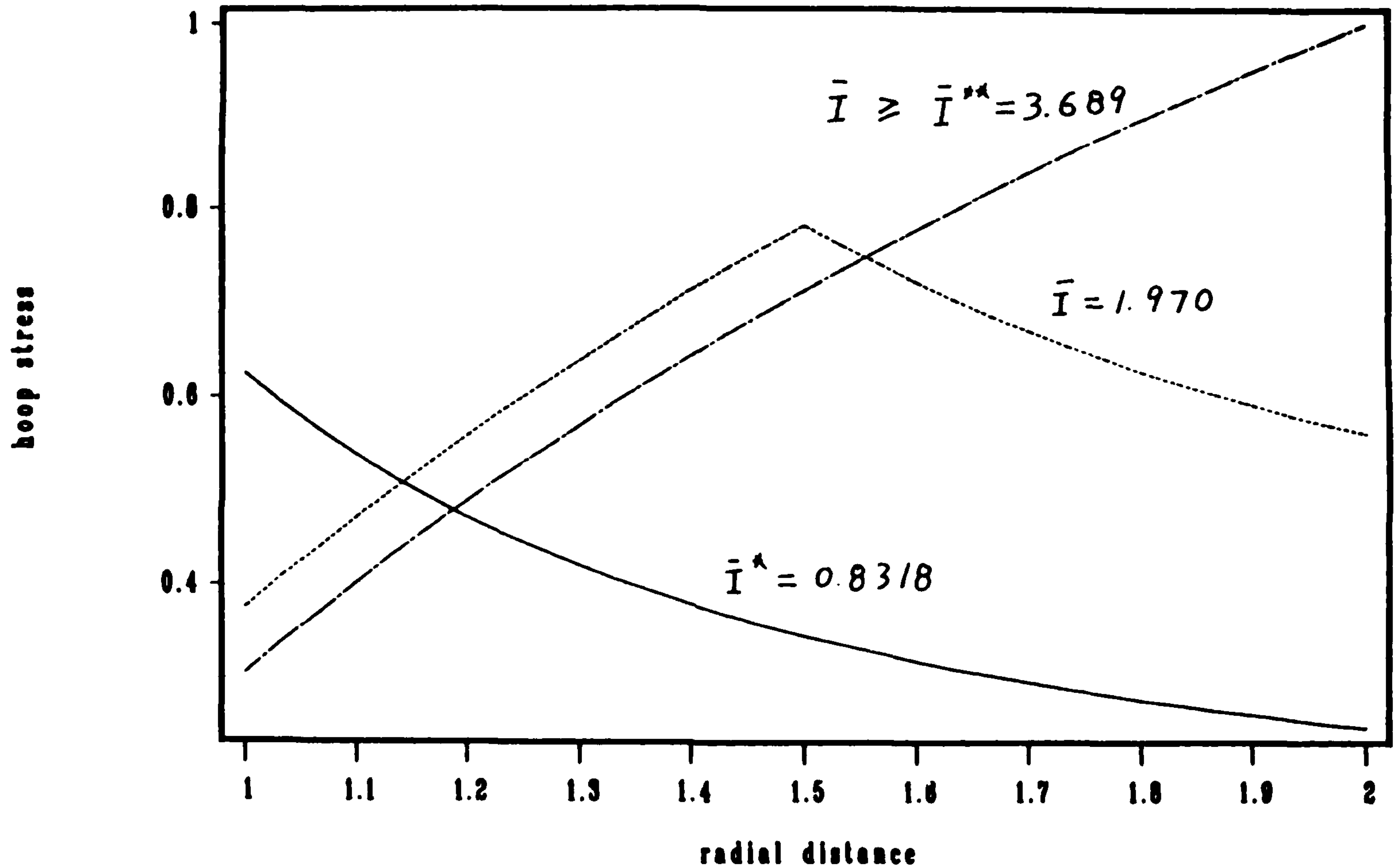

Figure 8. The distribution of hoop stress in a composite assembly ($\alpha = 0.5$, $m = 0.0$).

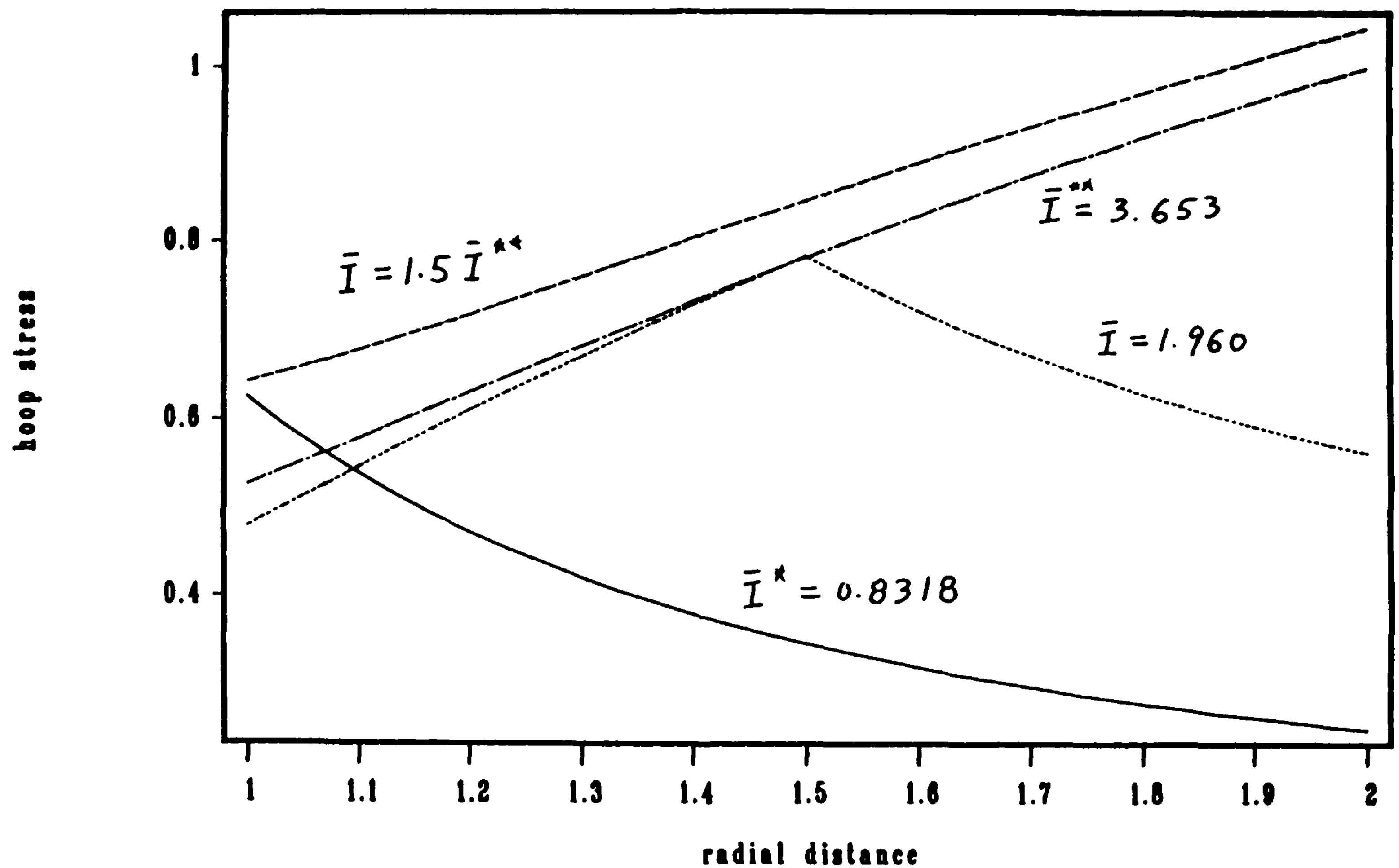

Figure 9. The distribution of hoop stress in a composite assembly (α = 0.5, m = 0.1).

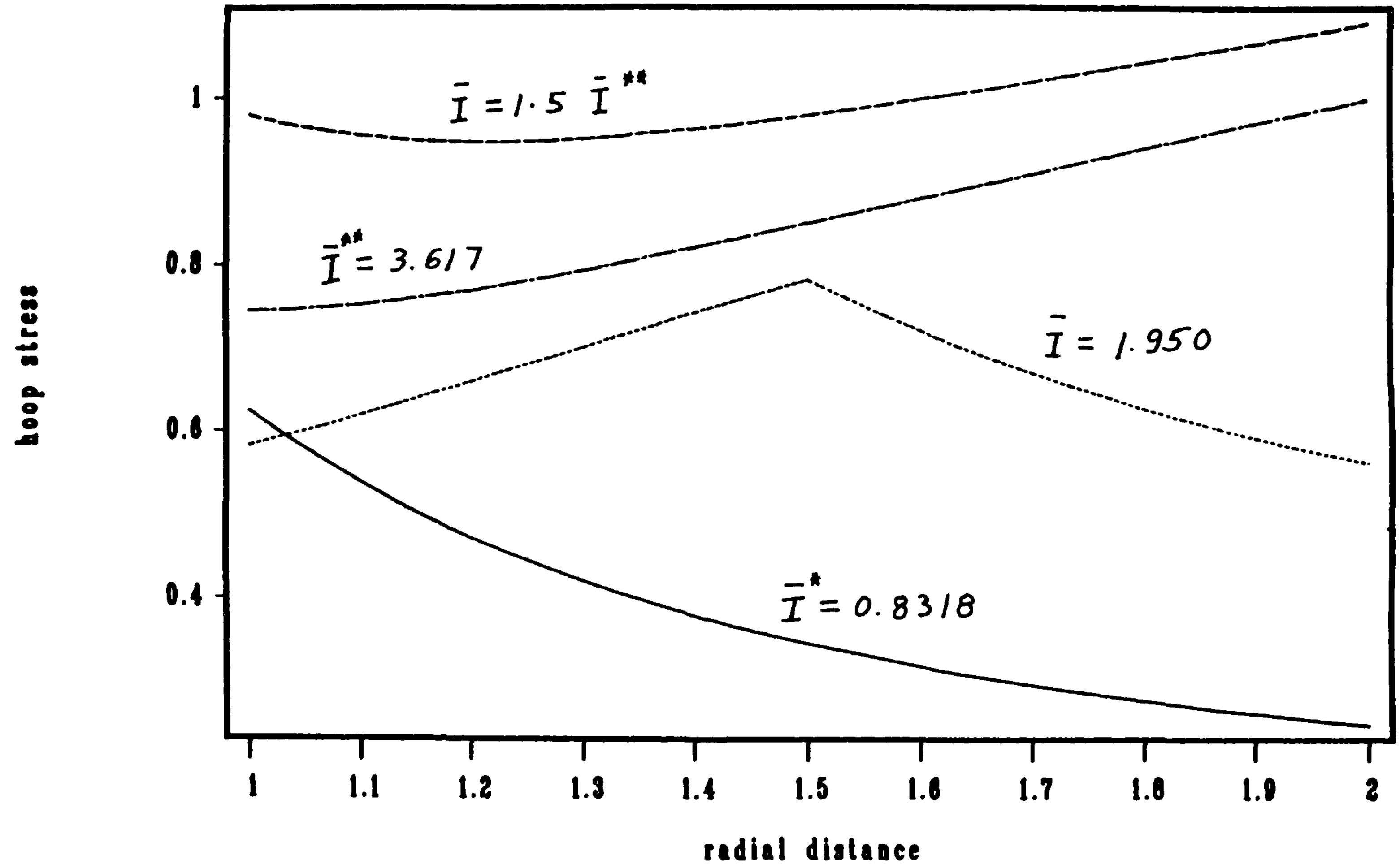

Figure 10. The distribution of hoop stress in a composite assembly (α = 0.5, m = 0.2).

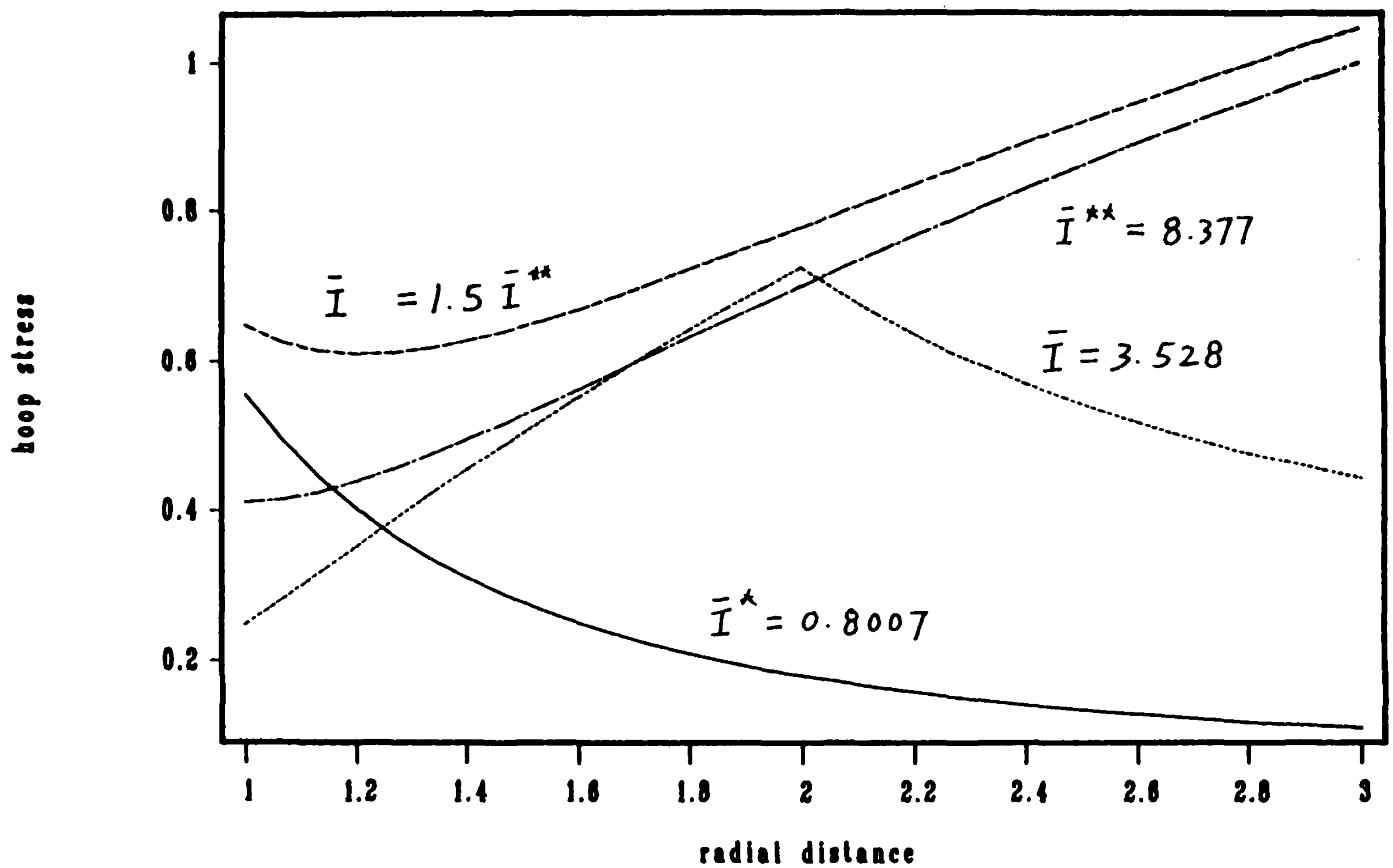

Figure 11. The distribution of hoop stress in a composite assembly (α = 1/3, m = 0.1).

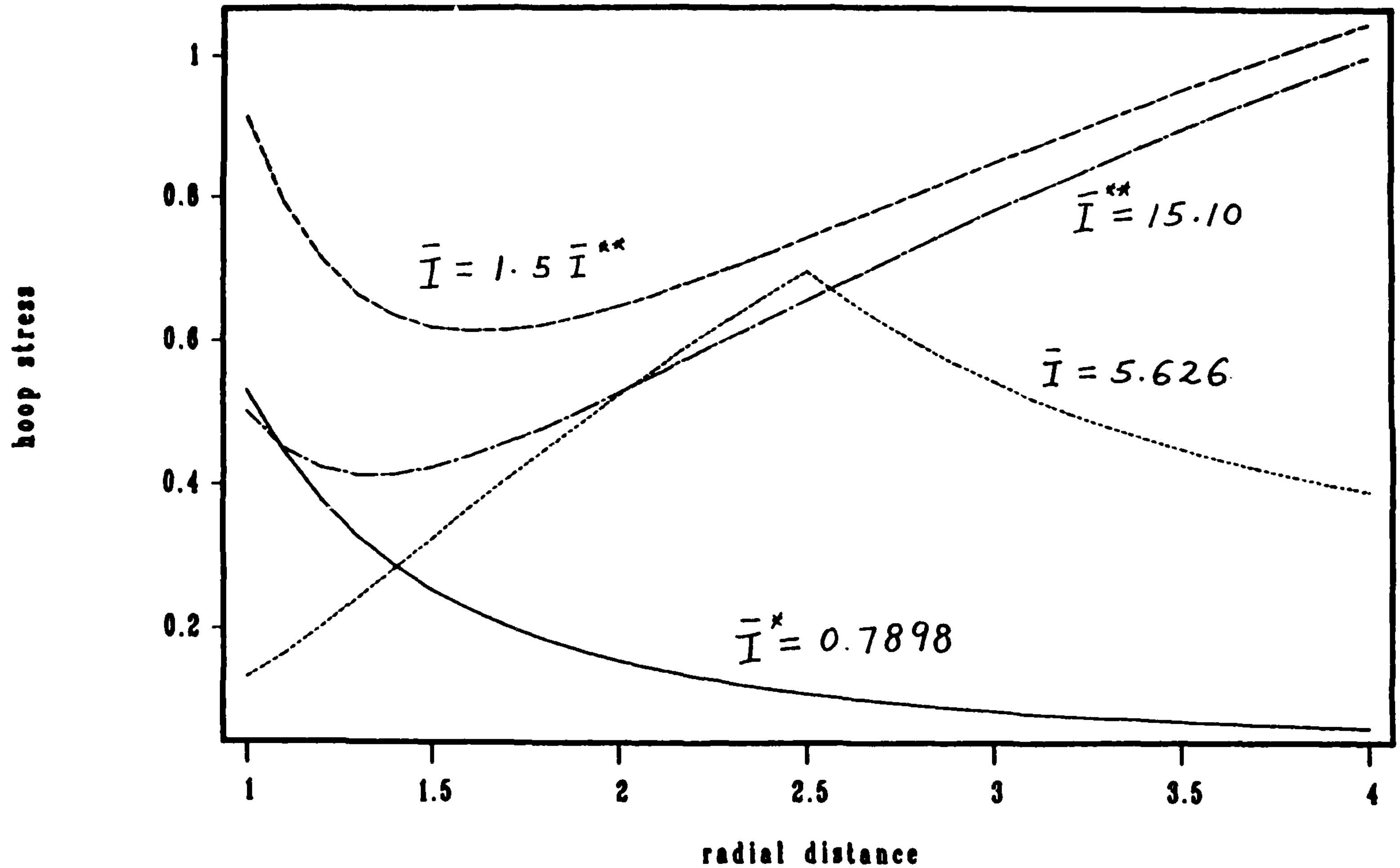

Figure 12. The distribution of hoop stress in a composite assembly (α = 0.25, m = 0.1).

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