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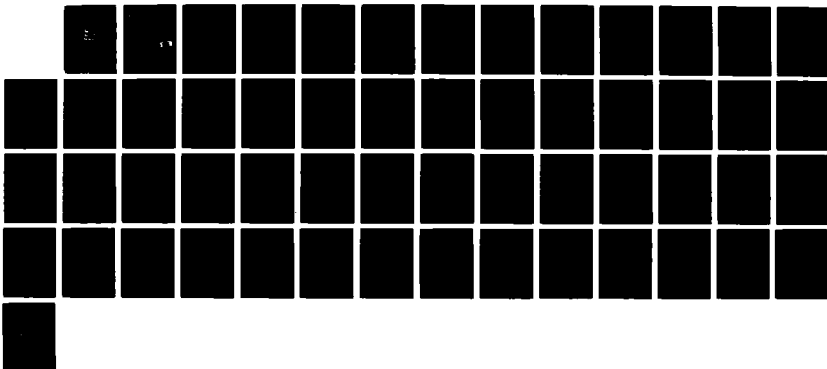
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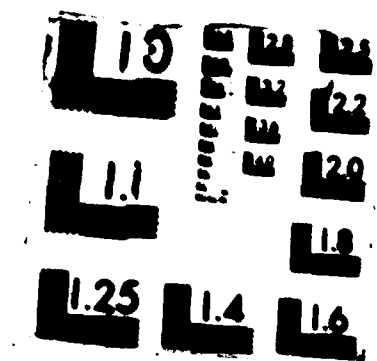
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**A Psychometric Analysis of a Three-Dimensional Spatial Task**

Isaac I. Bejar  
Educational Testing Service  
Princeton, New Jersey

June 1986

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### Abstract

This paper considers the feasibility of incorporating research results from cognitive science into the modeling of performance on psychometric tests and the construction of test items. The paper focuses on the feasibility of modeling performance on a three-dimensional rotation task within the context of Item Response Theory (IRT). Three-dimensional items were chosen because there is a rich literature on the mental models that are used in their solution. To test the feasibility of psychometrically modeling performance on this item type an 80-item, three-dimensional rotation test was constructed. An inexpensive computer system was also developed to administer the test and record performance, including response-time data. Data were collected on high school juniors and seniors. As expected, angular disparity was a potent determinant of item difficulty. The applicability of IRT to these data was investigated by dichotomizing response time at several points and applying standard item parameter estimation procedures. It was concluded that an approach to psychometric modeling that explicitly incorporates information on the mental models examiners use in solving an item is not only workable, but also essential for future developments in psychometrics.

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## A Psychometric Analysis of a Three-dimensional Spatial Task

Isaac I. Bejar

### Introduction

The bulk of psychometric theorizing and testing practice has been unconcerned with very detailed descriptions of the mental processes that underlie performance on test items. Instead, the focus has been on broader constructs, such as aptitude and abilities. As a result, the item, the building block for constructing a test, does not play as large a role as perhaps it should in either test construction or psychometric modeling. Thurstone, I think, deserves some of the credit, or rather some of the blame, for this situation (See Stenner, Smith, & Burdich, 1983). He was opposed to radical behaviorism and in reacting to it urged psychologists not to let the stimulus be the driving force of psychology. Although this reaction was not specifically concerned with testing, it is not hard to imagine that he might carry this perspective to the psychometric arena as well.

Whether Thurstone is responsible or not, it is accurate to say that in much of test construction the items are viewed as replicates and of little intrinsic interest. The alternative perspective is that items, far from being easily replaceable entities, are important in their own right and that accounting for differences among them with respect to their psychometric characteristics will aid our understanding of what a test measures (e.g., Bejar, 1985; Embretson, 1983), just as accounting for differences in total score variation improves our understanding of what a test measures.

Whereas the preferred methodology for modeling test score variation has been factor analysis, the methodologies of cognitive science (see

Miller, Polson, & Kintsch, 1984) appear to be suited to undertake the validation of tests from this perspective. This report focuses on mental rotation research and has the broad objective of exploring the feasibility of incorporating cognitive research with psychometric modeling and item construction. This will be investigated by examining data from a well-studied cognitive task--the three-dimensional mental rotation task--from a psychometric perspective. Specifically, the present research aims to capitalize on the significant amount of research produced in the last fifteen years in the area of spatial cognition. Much of this research has focused on the type of representation used by subjects to solve spatial problems. This progress is significant. Charles Myers, who, at ETS in the '50's, conducted much research on spatial ability for the College Board, said that

In this report we use the term "spatial ability" to represent a complex family of abilities with unknown interrelationships. We do not yet know of a terminology that permits a more precise and efficient language. (Myers, 1958, p. 24)

By contrast, Cooper and Shepard (1984) recently concluded after reviewing their work on mental rotation that

In spite of some unresolved issues, the close match we have found between mental rotation and their counterparts in the physical world leads inevitably to speculations about the functions and origin of human spatial imagination. It may not be premature to propose that spatial imagination has evolved as a reflection of the physics and geometry of the external world. The rules that govern structures and motions in the physical world may, over evolutionary history, have been incorporated into human perceptual machinery, giving rise to demonstrable correspondences between mental imagery and its physical analogues. (p. 114)

In the intervening period significant research and theorizing from the factor-analytic and the experimental perspective have occurred. Much of



that work has been reviewed (Corballis, 1982; Lohman, 1979; McGee, 1979). What emerges from a distillation of the literature is the presence of three mental factors; namely, spatial relations, spatial visualization, and spatial orientation. These factors have been investigated by cognitive psychologists (e.g., Pellegrino & Kail, 1982). But a specific task under the spatial relations factor has received so much attention that Corballis (1982) has raised it to the level of paradigm. That task is the three-dimensional mental rotation task. A typical stimulus used for this kind of research appears in Figure 1.

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Insert Figure 1 About Here  
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The most significant finding from this line of research has been the seemingly universal finding that one feature of these stimuli, namely angular disparity, controls the response time (e.g., Cooper, 1980; Shepard & Metzler, 1971). By contrast, within psychometric settings it is usually difficult to obtain a priori predictions of the psychometric difficulty of the item, let alone of its response time. That is, within psychometrics we are often content to estimate difficulty. But, of course, estimating is not explaining.

It is not unreasonable to suggest that we often focus on estimating rather than explaining difficulties because of the absence of a valid psychological model for solving the item. To the extent that the psychological model is concerned with the effort required to solve the item it may be feasible to accurately predict an item's psychometric characteristics, especially its difficulty. If this can be done with enough

precision it is in principle an alternative procedure, or at least an additional source of information, for estimating the difficulty of items. For example, it may be feasible to obtain valid estimates of difficulty by combining information about the psychological demands of the items and a small sample of subjects, instead of administering the test to a large sample of potential examinees. The implementation of this approach would require procedures for estimating the parameters in a psychometric model that are capable of incorporating "prior" information into the estimation process.

The foundation for incorporating prior information into the estimation of psychometric parameters is being laid (e.g., Bock & Atkin, 1981; Swaminathan & Gifford, 1981; Tsutakawa & Lin, 1984). Moreover, commercially available programs exist capable of handling some forms of prior information (Assessment System Corporation, 1984; Mislevy & Bock, 1982). Since the production of collateral information on the item would be based on an understanding of how examinees solve the item, the possibility also exists that at some point it might be possible to build, on the basis of that knowledge, systems whereby an item writer could receive feedback on the likely psychometric characteristics of a prospective item before it is ever administered to an examinee. (See Bejar, Stabler, & Camp 1986; Bejar & Yocom, 1986).

#### Psychometric Modeling of Spatial Rotation Data

The application of the foregoing to the psychometric modeling of spatial ability suggests as a criterion of success the determination of psychometric difficulty in terms of item attributes, or specifically, in the present case, linking psychometric difficulty to angular disparity. The mildest criterion, perhaps, is that difficulty should increase as

angular disparity increases. A stronger criterion is that difficulty should increase in a linear fashion with angular disparity or some transformation thereof. If such a relationship can be established our interpretation of the data can be considerably more descriptive. In the absence of such a linkage, difficulty in an IRT response model is defined, when guessing is not present, as the point on the ability scale at which there is a 50-50 chance of responding correctly. When difficulty is linked to an item attribute, such as angular disparity, we can reference performance to that attribute. Thus we could speak of ability as the level required to achieve a 50-50 chance of success on a task involving a certain degree of angular disparity. In short, relating psychometric parameters to a mental model of the item solution process is likely to improve the interpretation of psychometric results.

Since the three-dimensional mental rotation item does not require problem solving, the time to obtain a correct response is directly interpretable as the efficiency with which the mental rotation takes place. Therefore, in fitting a psychometric model to these data, both accuracy and response time should be taken into consideration. Considering both responses suggests an expansion of the criterion described above. That is, in addition to expecting an increase in difficulty as a function of angular disparity, we should expect that the relationship between angular disparity and difficulty would remain the same as the time limit to perform the task is increased. Figure 2 illustrates the expected relationship. In general, however, allowance must be made for the possibility that the intercept is not linear in time. That is, in general, the gap between lines may not be a constant.

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Insert Figure 2 About Here  
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### Modeling Response Latency

A strategy for incorporating response latency into psychometric modeling has been proposed recently by Bloxom (1985); therefore, a review of the relevant literature will not be attempted here. Here we will focus on a discussion of modeling response latency as an extension of models for dichotomous data. The approach we follow is to fit a dichotomous item response model to response times to a set of 80 three-dimensional rotation items. The objective is to determine whether a more refined psychometric model should be attempted; not to provide the definite calibration of these data.

A common model for the probability of dichotomous response when time is not a factor is the two parameter logistic model:

$$P(u_i = 1|\theta) = \frac{1}{1 + e^{-1.7a_i(\theta - b_i)}} \quad [1]$$

where  $a_i$  is the discrimination,  $b_i$  is difficulty parameter, and  $\theta$  is ability. Now consider a situation where the interest is on the probability of a correct response after a certain period of time has elapsed. We would expect that, at least with certain item types, the longer an item is considered, the higher the probability of a correct response. Figure 3 conveys this notion. In effect we have an equation such as Equation [1] after increasing amounts of time have elapsed. Figure 3 has a constraint that is essential for interpretability, namely that the curves only differ on their

inflection point. This means that the discrimination parameter is constant across time, but the difficulty parameter varies as a function of time. Put differently, the probability of a correct response after increasing elapsed times is solely a function of time. Micko (1969) has applied this idea by specifying the dichotomous item response model to be the Rasch model.

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Insert Figure 3 About Here  
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Although it is in principle possible to model response time with a dichotomous item response model it is also possible to generalize the dichotomous model as Samejima (1983) has done. In this generalization the continuous response is converted to a 0-1 interval. For response latency this means that the response is expressed as a proportion of the total allowed time for responding to an item. If the time limit is 15 seconds, for example, a response latency of 5 seconds would be .33. Samejima refers to the response expressed in this manner as  $z$ . There is nothing in the model concerning whether the response time is for a correct or an incorrect response. Such a distinction must be made for scoring purposes; however, this will be discussed in a subsequent report. Basically, the idea is to treat responses as correct if, in fact, a correct response is produced after  $s$  seconds, and as not-correct-yet if an incorrect response is given. That is, incorrect responses are treated as incomplete responses indicating that "the last time we looked, namely after  $s$  seconds, the individual had not produced a correct response." In statistical terminology, incorrect responses are treated as censored observations, as in survival analysis (e.g., Miller, 1981).

With this in mind, the probability that someone of ability  $\theta$  takes longer than  $z$  to respond is given by

$$\begin{aligned}
 P_{z_i}^*(\theta) &= \int_{-\infty}^{a_g(\theta - b_{z_i})} \text{Dexp}(-Dt) [1 + \exp(-Dt)]^{-2} dt \quad [2] \\
 &= \left[ 1 + e^{-Da_i(\theta - b_{z_i})} \right]^{-1} \\
 &= \frac{1}{1 + e^{-Da_i(\theta - b_{z_i})}}
 \end{aligned}$$

which is similar to Equation [1] except that the difficulty parameter now is a function of  $z$ , response time. The difficulty function  $b_z$  is not constrained to any particular shape other than that it be monotonic. In this paper we will investigate the fit of a linear function. That is, we are interested in  $b_z$ 's of the form

$$b_{z_i} = \phi_i + \delta_i z \quad [3]$$

$\phi_i$  could be further decomposed into components associated with figure attributes, but our focus here is on the adequacy of a linear difficulty function when angular disparity and time are taken into consideration.

An interesting implication of this model is its possible compatibility with the "slope and intercept" methodology commonly used by cognitive researchers interested in individual differences (e.g., Lansman, Donaldson, Hunt, & Yantis (1982)). The slope and intercept methodology calls for computing the regression of response time on item attributes such as angular disparity, for each subject. The slope and intercept of that regression are taken to be estimates of individual differences parameters. The slope, for example, is taken to be an indicator of speed of processing

while the intercept is interpreted to include a series of "overhead" processes such as encoding. It is not clear from users of this methodology what relationship ought to exist between these two parameters. However, in practice they are often correlated. In the Lansman et al. study, for example, the slope on paper and pencil three-dimensional mental rotation tests correlated .43 and .32 with the slope and intercept, respectively, of a computer version of the same test. That is, the slope and intercept seem to be picking up substantially the same individual difference variable.

The model in Equation [2] predicts that the relationship between response time and item complexity (as reflected by the difficulty parameter) for a given individual is linear and increasing for a fixed level of accuracy; that is, proportion correct. This is consistent with the "slope and intercept" methodology. However, the model also predicts that the slope of that relationship is constant across all levels of ability while the intercept varies with ability. This is not necessarily consistent with the slope and intercept methodology since the slope of the regression of response time on angular disparity has been generally interpreted as the rate at which the subjects mentally rotate the object. Thus, according to the model in Equation [2] the locus of individual difference, when accuracy is constant, is not in the rate of rotation, the slope, but rather on the intercept which is often associated with the encoding and other "overhead" processes of the item. From a substantive point of view the distinction is important since it is precisely the interpretation that subjects mentally rotate the figures that has generated so much interest in this line of research.

### Design

To secure data for the study, we recruited 160 high school students from a local high school. The items used in this study consisted of 80 pairs of three-dimensional Shepard-Metzler figures. The following eight figures were used (A1, A2, A3, C1, C2, C3, E1, and E2).<sup>1</sup> For each figure true and false pairs were constructed by rotating at angular disparities of 20, 60, 100, 140, and 180. The true pairs were constructed by rotating the same figure along the picture axis. The false pairs were constructed by rotating the mirror image instead. Altogether there were 16 items at each angular disparity. The resulting 80 items were videotaped and placed on a videodisc using the 3M mastering process.

The items were presented in two different orders. In one, the examinees saw items at 100, 60, 180, 140, and 20 degrees. At each angular disparity there were 16 possible items, and one of those was chosen at random. With the second ordering, subjects worked items of 20, 140, 180, 60, and 100 degrees. Approximately one-half of the subjects took the item in each order.

The instrumentation for each data collection station consisted of the following components:

- \* A 64K microcomputer with 1-disc drive (Radio Shack 26-3127)
- \* A videodisc player (Pioneer PR 8210)
- \* An Amdek Color I Monitor
- \* A Joystick (Radio Shack, 26-3012)
- \* A computer-to-videodisc interface (especially constructed for the project)

<sup>1</sup> The author is indebted to Professor Roger Shepard of Stanford University for providing these materials.



The microcomputer was programmed to control the videodisc as well as record the responses. Response time was recorded in "ticks" where a tick is a 60th of a second. Subjects responded by means of a joystick connected to the computer. A "yes" response was signaled by moving the joystick forward, while a "no" response was indicated by moving the joystick backward.

Because of the potential unfamiliarity of the equipment, at least as a psychological testing device, careful attention was given to the instructions. Instructions were tested with several students unrelated to the study to insure that they were fully understandable. Students were told that they were to respond by moving the joystick and that they were to respond as quickly as possible without sacrificing accuracy. The instructions appear in Appendix B.

The examinee's first task was to respond to a simple task, namely to indicate whether an arrow was pointing up or down. This was done to familiarize each examinee with the response device as well as to time their reaction time to a task with almost no cognitive load. From these responses it is possible to obtain an estimate of the motor speed of individuals. These data were not analyzed as part of this study, however.

After the arrow task, subjects were given instructions on the rotation test. As part of the instructions they were able to manipulate an animation sequence containing a true and a false item. This allowed the examinee to become familiar with true and false items at all possible angles. Also, seeing the rotation in real time, examinees may have been encouraged to use a rotation strategy to solve the items. The examinees were allowed to do six practice items. The practice items were followed by 80 real items. There was a 15-second time limit for each item. After 15 seconds a "timeout" message was given if the subject had not responded.

However, students could pace themselves in the sense that they controlled when the next item was administered. At the end of each item students were told whether they had responded correctly or incorrectly.

#### Parameter Estimation

With the growing interest in the psychometric modeling of response time (Bloxom, 1985; Scheiblechner, 1985; Thissen, 1983), it is likely that estimation procedures tailored to response time will be forthcoming. In the meantime it is possible to obtain estimates of item parameters through estimation procedures designed for the dichotomous case. That is the approach taken here. In a nutshell, the approach calls for successively dichotomizing response time and fitting a one-parameter logistic model at each dichotomization point. Imposing a one-parameter model across angular disparities implements the constraint that discrimination be constant across time.

Each basic item, i.e., pair of distinct figures, was fitted separately. Figure 4 shows the structure on the data matrix for a given basic item. The notation "0/1," which occurs only on the upper left corner, indicates that the entries in that section of the matrix could be 1, a correct response was given, or 0, a correct response was not given.

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Insert Figure 4 About Here  
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The notation "0/1/2" indicates that responses in that block could be 0, a correct response was not given; 1, a correct response was given, or 2, the "item" was not presented. A response within this type of block was coded 2 if in a preceding block a correct response was not given to that item. For example, if at the end of the three-second interval an examinee

responds "false" to a true item, then that item is coded 2 in subsequent intervals. The notation 2 simply means that all the items in that block were treated as not presented. For each basic figure there were 50 "items" corresponding to the true angular disparities and true/false classification, and 5 time intervals.

Each of the eight data matrices were analyzed separately with BILOG (Mislevy & Bock, 1982) specifying a one-parameter logistic model. The resulting estimates were rescaled with respect to the distribution of ability estimates estimated with the EAP algorithm.

### Results

Unlike the typical mental rotation experiment in which subjects receive a great deal of practice time, the subjects in this study spent altogether no more than forty minutes, including instruction and practice items, on the mental rotation task. Therefore, it is important to verify that the usual finding concerning the linearity of response time on angular disparity is replicated in this case. Figure 5 shows the relationship between angular disparity and response time for correct responses for true and false items across the eight basic figures. Figure 5 suggests that there is, for the most part, a good linear fit to the data. The largest residual is at 100 degrees. Apart from this, there is relatively little scatter around the best-fitted line. In fact, the fit appears better than has been reported in some studies. The impact of angular disparity on response time is less potent for false items, as Figure 5 shows.

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Insert Figure 5 About Here  
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To assess the fit of Equation [2] we will examine two criteria. One

is the relationship of difficulty to angular disparity; the other is the relationship of reaction time and angular disparity for subjects of different ability levels. In the first instance, on the basis of Equation [2] we expect the relationship to be linear with angular disparity and to maintain the same slope as time goes by. Secondly, if Equation 2 is a valid model for these data we expect to find that for single subjects, or groups of subjects, the slope of the relationship of response time on angular disparity is constant across these subjects or groups.

Figure 6 shows the results from the item calibration for each basic item. Figure 7 shows the equivalent plots for the false items. The first expectation is largely fulfilled for the true items. That is, there is a nearly perfect linear relationship between response time and angular disparity. Moreover, the slope of the best-fitting line does not change with time. The major deviation from expectation occurs at 100 degrees. This angular disparity proved to be consistently more difficult than expected. In addition, for some items, the relationship between difficulty and angular disparity appears to be nonlinear beyond 5 seconds. For the false items, however, it is clear that angular disparity is not a determinant of response time in these data (but may be so with highly practiced subjects).

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Insert Figures 6 and 7 About Here  
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Figures 8 and 9 show the mean difficulty estimates across the eight basic figures for true and false items. In Figure 8 we notice again the discrepancy at 100 degrees and a slight deviation from linearity beyond 5 seconds.

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Insert Figures 8 and 9 About Here  
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The second prediction we wish to test is that the individual differences are reflected on the intercept and not the slope. To invalidate this prediction it is sufficient to show that it is not so for individual examinees or groups of examinees that differ in spatial ability. Since we did not have available an independent measure of spatial ability the approach taken here is to compare two groups widely different on SAT-M. Two groups were formed based on the upper- and lower-third scores on SAT-M. The higher group consisted of 38 examinees with a mean SAT-M of 713; the lower group consisted of 31 students with a mean of 444. Figure 10 shows the relationship between proportion correct and mean reaction time (i.e., regardless of whether the response was correct or incorrect)<sup>2</sup> with angular disparity for these two groups. The higher SAT group has a higher slope, suggesting that they are not rotating as fast, but also a smaller intercept. Thus, other things being equal, the higher SAT examinees are faster on the items with smaller angular disparity and about as fast on the higher angular disparities. It is also true, as can be seen in Figure 10, that the accuracy rate is different, being higher for the high SAT-M group on the more discrepant items.

<sup>2</sup>Since the model does not specifically distinguish between correct and incorrect responses and focuses on the modeling of response time, it is more appropriate to plot response time rather than response time of correct responses in the context of model fitting. Nevertheless, the plot with mean correct response time was also done, but similar results were obtained.

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Insert Figure 10 About Here  
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### Discussion

The motivation behind this work has been both substantive and methodological. The substantive interest has been the assessment of the convergence or lack thereof of two approaches to individual differences: a psychometric approach and an approach inspired by cognitive psychology. Hunt and MacLeod (1978) have expressed concerns that the differences between these two approaches may be irreconcilable. They give as an example the tendency for psychometricians to focus on global scores, such as proportion correct, versus the tendency of the cognitive psychologist to focus on more reductionist parameters such as slope and intercepts of performance on task attributes. This paper shows that the dichotomy need not exist, at least not when we adopt IRT as a psychometric framework. For example, person characteristic curve methodology (Carroll, Meade, & Johnson, in preparation; Trabin & Weiss, 1983) is well suited to characterize subject performance in a psychologically meaningful fashion. This paper demonstrates that the IRT framework can be expanded when the response of interest is the response time and in doing so demonstrates the possibility of encompassing both global and reductionist views of individual differences within the same measurement framework.

According to Equation [2] the locus of individual differences is on the intercept and not on the slope. The results presented in Figure 10 show that the relationship of reaction time to angular disparity has a different slope and intercept for groups of presumably different spatial ability. If we believe that the intercept is the correct indicator of

ability to rotate, then we would conclude that the higher SAT group is higher on that ability. That conclusion would be consistent with existing literature (e.g., Fennema & Sherman, 1977). If we believe that the slope is the correct indicator, we would conclude that the high SAT group is of lower ability, a finding which would not be consistent with the literature. A reconciliation of these opposing conclusions lies in an accounting of the differences between the high and low SAT-M group on their accuracy. That is, the slope of the relationship between response time and angular disparity cannot be compared unless accuracy is constant in the two groups, (cf. Kail, 1985).

According to Equation [2], the slope of the relationship between the magnitude of the response,  $z$ , and difficulty can be controlled through the accuracy parameter. (See Appendix A). For a given ability level a smaller slope can be obtained by reducing accuracy as difficulty increases. A larger slope will be obtained, for example, by holding accuracy constant as difficulty increases. Therefore, the differences in slope seen in Figure 10 do not necessarily violate the prediction of the model and could well be eliminated after adjusting for accuracy. Moreover, the adjustment would not have to be done explicitly in practice since, in the estimation of ability, accuracy and speed are taken into account. That is, IRT may provide a solution to the speed-accuracy tradeoff problem (see Thissen, 1983). The estimation of ability will be treated in a subsequent report.

The second motivation for this work has been purely psychometric, and the work focuses on the development of a generative approach to psychometric modeling and test administration (Bejar & Yocom, 1986). By a generative approach I mean a methodology where the generation of the items is controlled by an algorithm encoding sufficient knowledge about the

mental processes underlying performance on the item that it is capable of anticipating the psychometric characteristics of the item before it is administered to examinees. The goals of this methodology are a natural extension of adaptive testing (Weiss, 1983). In an adaptive test a computer retrieves from an existing database (containing previously calibrated items) the item that is most informative for a given individual. However, with a generative approach, an item, instead of being retrieved, is created specifically for an individual in such a way that the anticipated psychometric characteristics of the items are maximally informative for each individual.

One of the goals of adaptive testing has been the achievement of high measurement precision throughout the ability range. This is a special concern with dichotomous items where a balance must be struck between overall level of precision and distribution of precision at different levels of ability. As we move from dichotomous to continuous responses that balance takes care of itself in the sense that for continuous response models information may be high throughout the ability range and in some cases equally high at all ability levels (Samejima, 1973). Therefore, that goal of adaptive testing seems to be automatically satisfied through the use of continuous responses. Nevertheless, there may be advantages to adapting the angular disparity to individuals of different abilities as a means of insuring a common response strategy by all examinees.

The two essential ingredients in implementing a generative approach are (a) a psychometric framework of sufficient flexibility; and (b) a knowledge base about performance on the item. The three-dimensional cube item was chosen for this study because of the potent effect of angular disparity on performance in mental rotation items. It is therefore



possible to establish the relationship between angular disparity and difficulty based on a few points. By manipulating this feature of the item it becomes possible to generate items of any arbitrary difficulty. In a practical implementation of this idea we may have, say, 30 basic items that can be presented at rotations ranging from 20 to 180. Although everyone would be presented all 30 items, the rotation at which they are actually presented would be different for different individuals.

The measurement model that was explored in this paper to explain performance on a three-dimensional rotation item is an extension of existing item response models in current use for the dichotomous response case (Samejima, 1983). Two predictions of this model were explored as a means of testing its fit. One prediction concerned the linearity between difficulty and angular disparity. The results suggested that the prediction was substantially satisfied. The exception was at 100 degrees which was found to be a more difficult angular disparity than expected. Also, beyond 5 seconds a nonlinear relationship appears to emerge.

Implicit in this prediction is the assumption that both true and false versions of an item would have the same slope. This was distinctly not the case. That is, it appears that a two-dimensional model would be required to provide an adequate description of the true and false data. (To assess the bias that may have been introduced into the item parameter estimates as a result of the bi-dimensionality, the model was fitted to true items only, but no noticeable difference could be seen in the resulting estimates.)

The second prediction was concerned with the interpretation of slope and intercept parameters. According to the model, individuals of different ability have the same slope but different intercepts. The results suggested that considering the different accuracy rates of a high- and

low-scoring SAT-M group, the difference in slopes could not necessarily be interpreted as valid indicators of rotation speed. In effect, our results suggest that the bulk of individual differences is on the intercept rather than the slope parameter. This does not necessarily conflict with research suggesting that rotation speed is an important individual-differences variable since examinees were exposed to the items for a relatively short period of time. Indeed, in contradiction of other research, the males and females studied do not differ on the slopes and intercepts (Bejar & Harvey, in progress). Whereas sex differences in tasks involving mental rotation appear to be a well established fact (Linn & Petersen, 1985), and because of the relatively brief exposure that subjects had, our data may not be typical.

While results of the study show that the idea of generating items of arbitrary difficulty is indeed feasible, there are some difficulties that must be borne in mind with even relatively simple stimuli, such as the three-dimensional rotation items. Specifically, performance on false items is not a function of angular disparity; on the surface this finding therefore suggests that performance on the false items is controlled by a different combination of mental processes (see Carter, Pazak, & Kail, 1983). In short, further psychometric examination of the three-dimensional cubes could focus on multidimensional modeling of processing and decision processes and on models that characterize the change in performance as a function of practice.

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Figure 1

Sample True and False Three-Dimensional Rotation Items

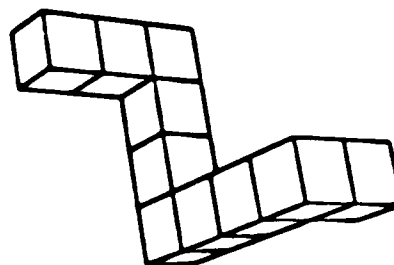
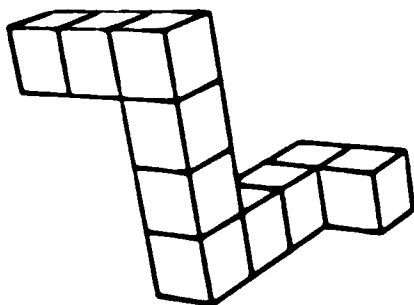
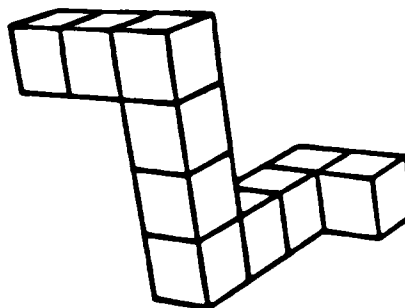
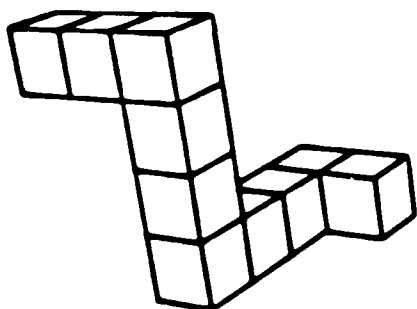


Figure 2

Hypothetical Relationship Between Difficulty  
and Angular Disparity and Function of Time

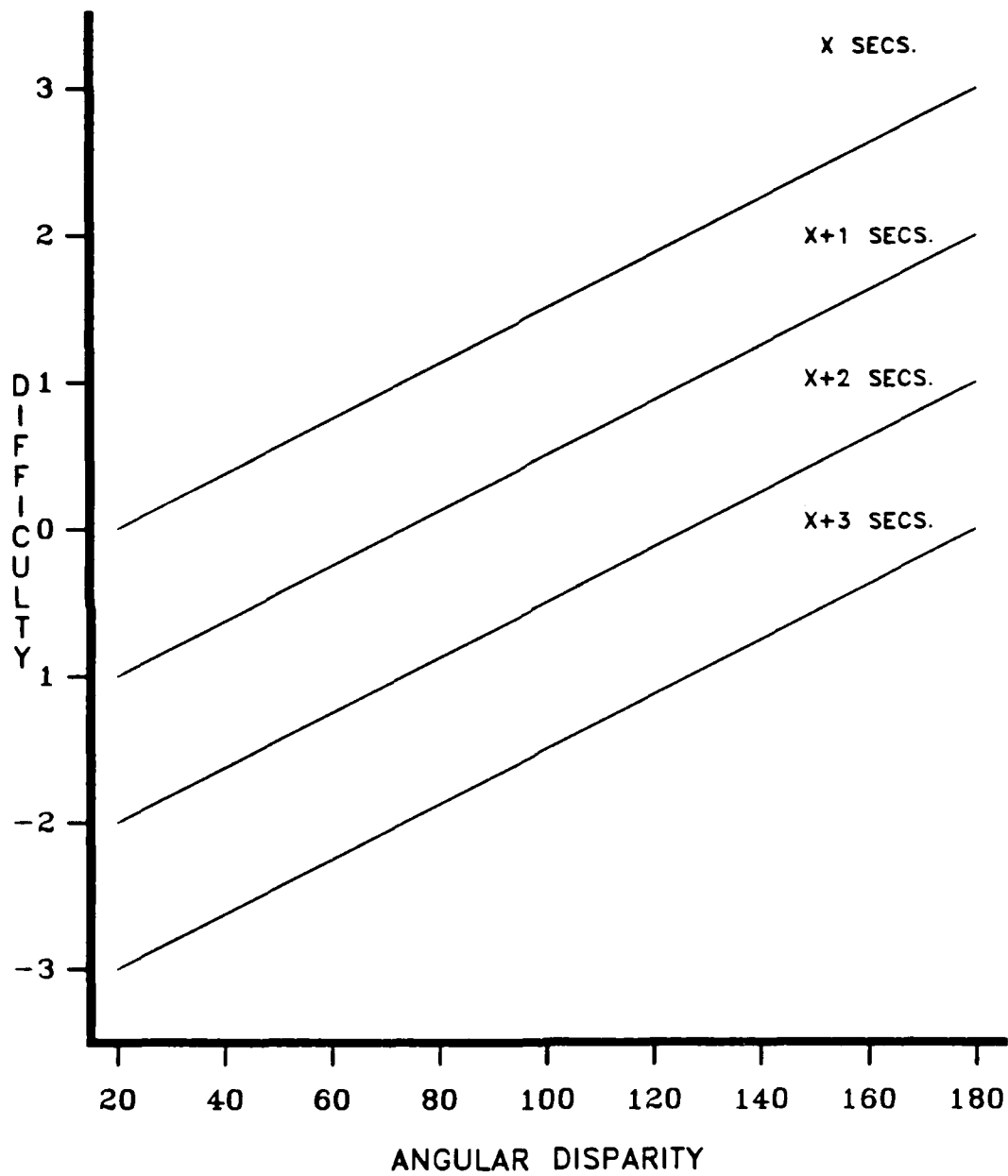




Figure 3

Hypothetical Item Response Functions as a Function of Time

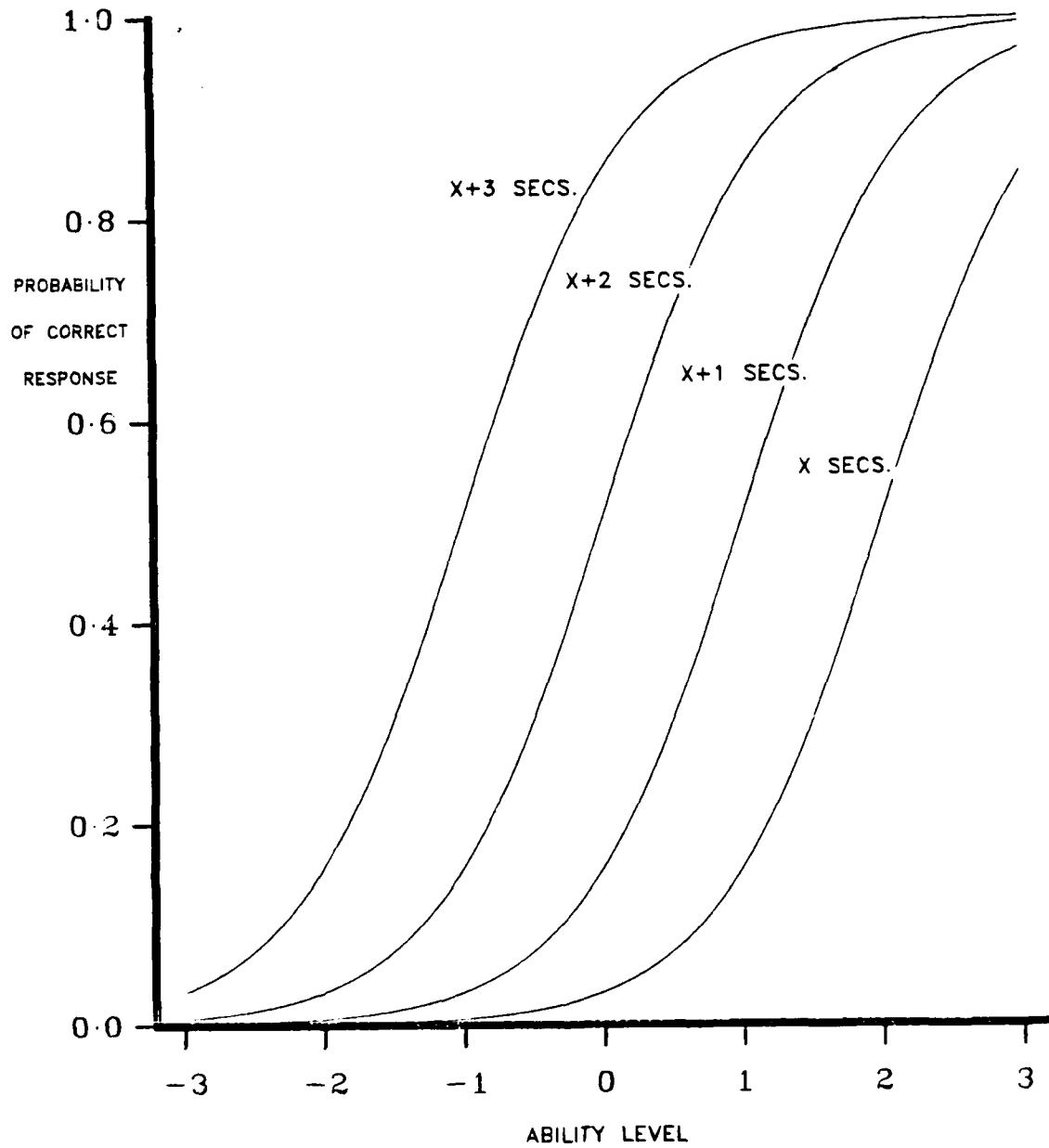


Figure 4  
Data Matrix for True and False Versions of an Item at Successive Dichotomization Points and Angular Disparities

| Seconds    | 3 Seconds |   |     |   |      |   | 4 Seconds |   |      |   |     |   | 5 Seconds |       |      |   |      |       |      |   |
|------------|-----------|---|-----|---|------|---|-----------|---|------|---|-----|---|-----------|-------|------|---|------|-------|------|---|
|            | 20°       |   | 60° |   | 100° |   | 140°      |   | 160° |   | 20° |   | 60°       |       | 100° |   | 140° |       | 180° |   |
|            | T         | F | T   | F | T    | F | T         | F | T    | F | T   | F | T         | F     | T    | F | T    | F     | T    | F |
| Subjects 1 |           |   |     |   |      |   |           |   |      |   |     |   |           |       |      |   |      |       |      |   |
| 3          |           |   |     |   | 0/1  |   |           |   |      |   |     |   |           |       |      | 2 |      |       |      |   |
| 4          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 0/1/2 |      |   |      | 2     |      |   |
| 5          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 2     |      |   |      | 0/1/2 |      |   |
| 6          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 2     |      |   |      | 2     |      |   |
| 7          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 2     |      |   |      | 2     |      |   |
| Subjects 2 |           |   |     |   |      |   |           |   |      |   |     |   |           |       |      |   |      |       |      |   |
| 3          |           |   |     |   | 0/1  |   |           |   |      |   |     |   |           | 2     |      |   |      | 2     |      |   |
| 4          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 0/1/2 |      |   |      | 2     |      |   |
| 5          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 0/1/2 |      |   |      | 0/1/2 |      |   |
| 6          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 2     |      |   |      | 2     |      |   |
| 7          |           |   |     |   | 2    |   |           |   |      |   |     |   |           | 2     |      |   |      | 2     |      |   |

Figure 5

Relationship Between Reaction Time and Angular Disparity for  
True and False Versions Averaged Across the Eight Items

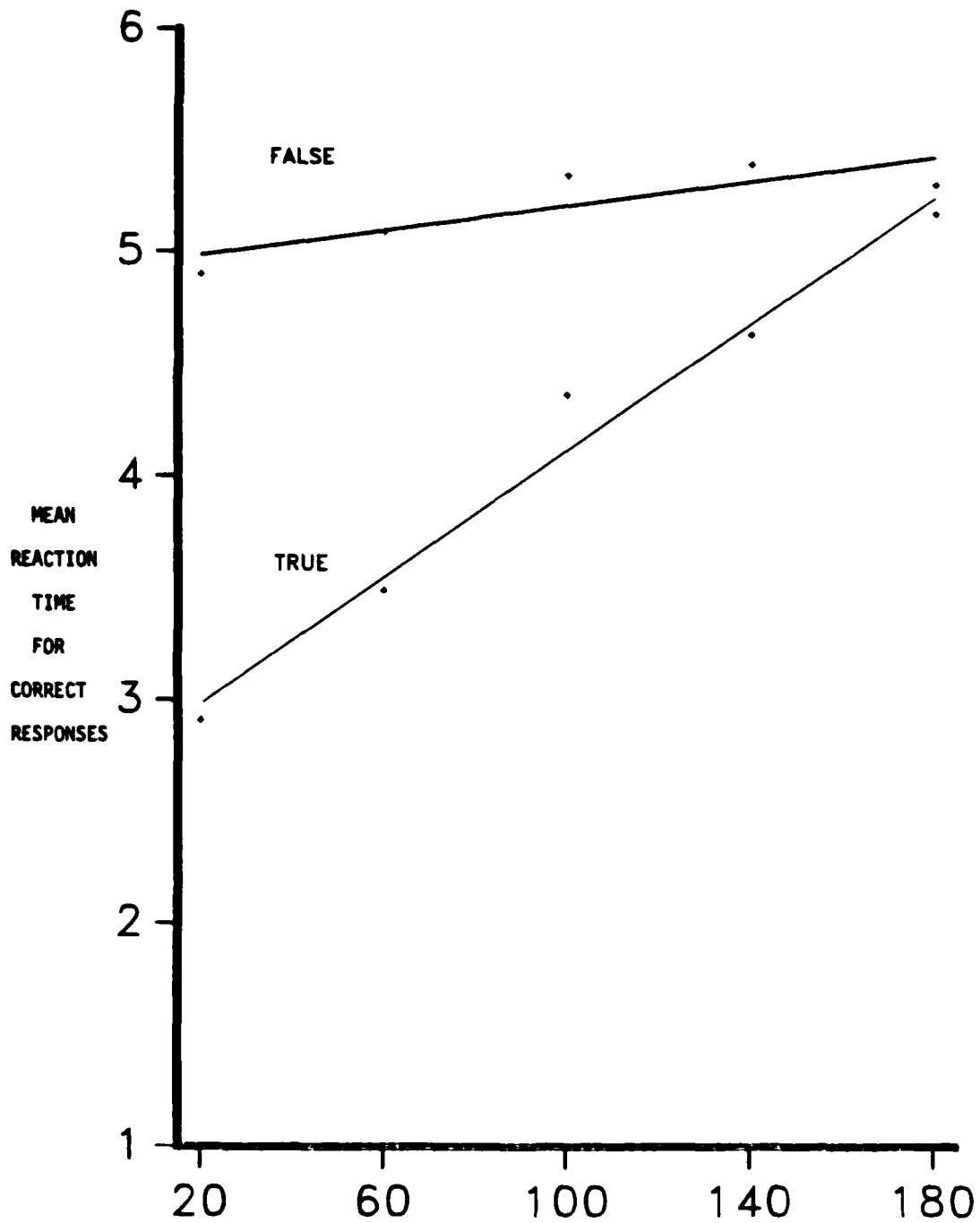
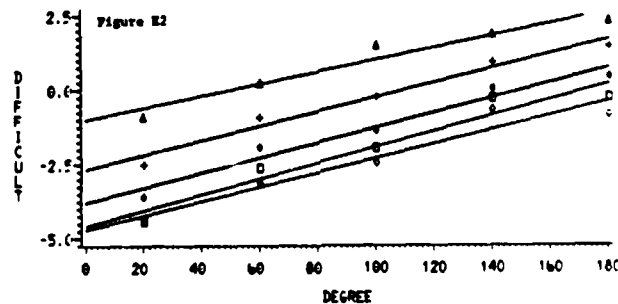
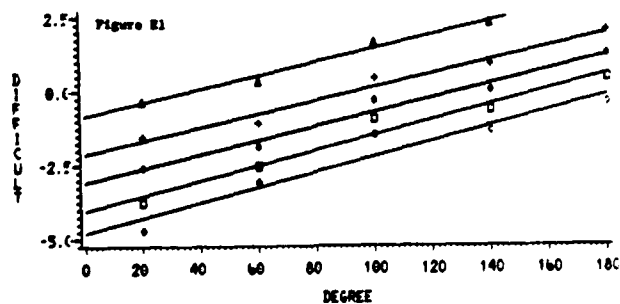
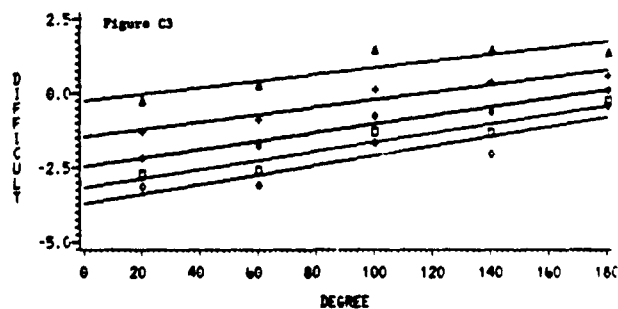
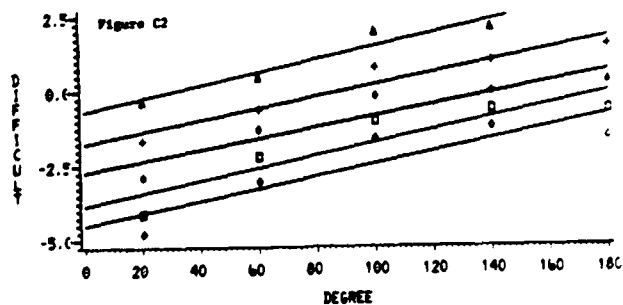
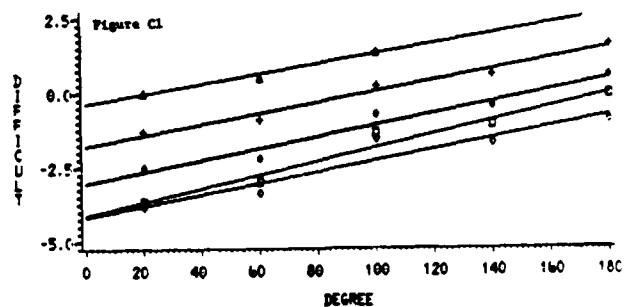
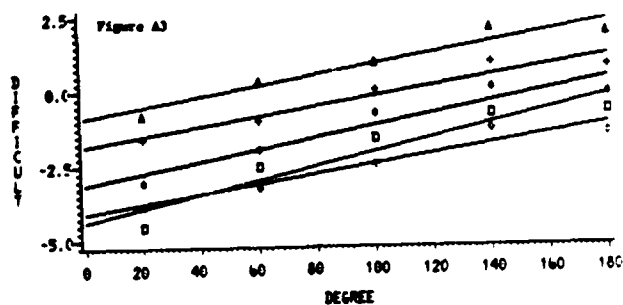
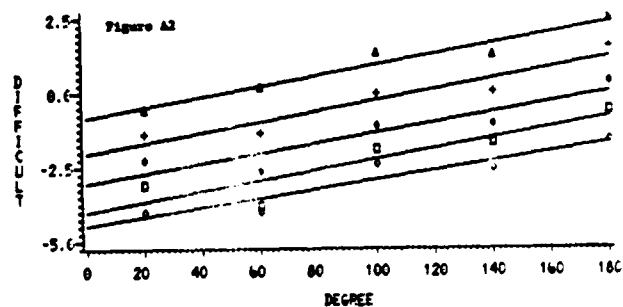
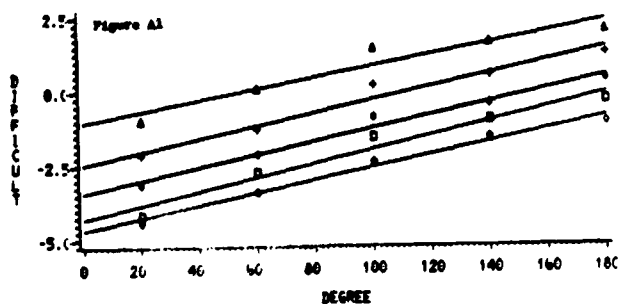
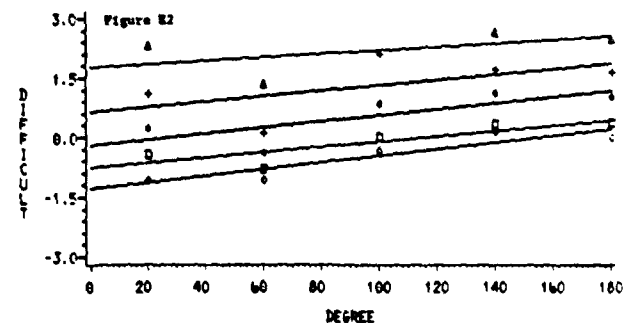
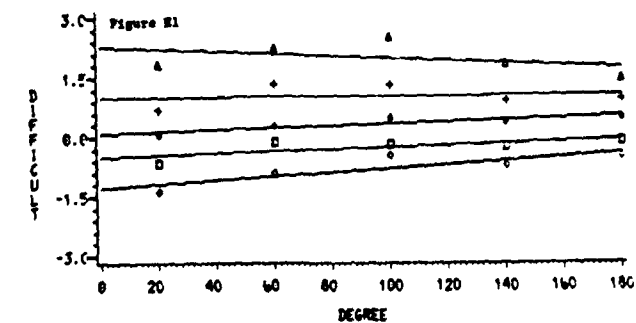
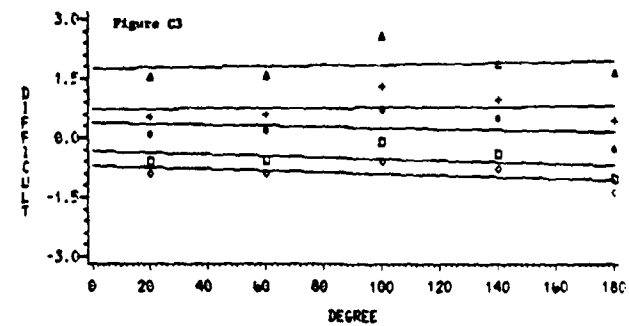
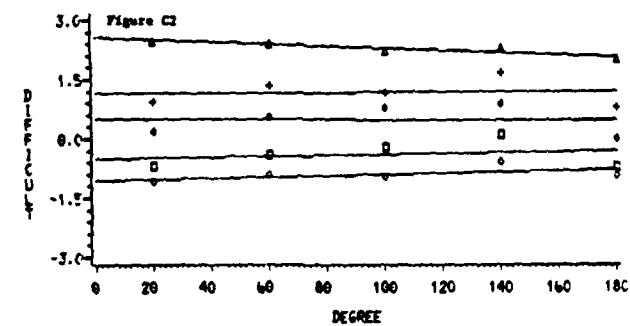
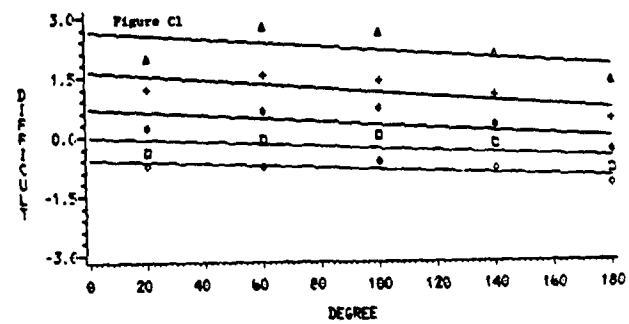
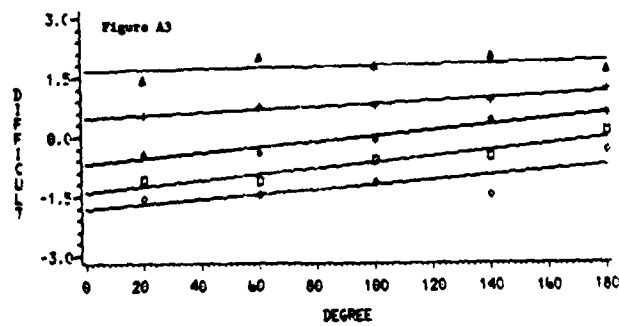
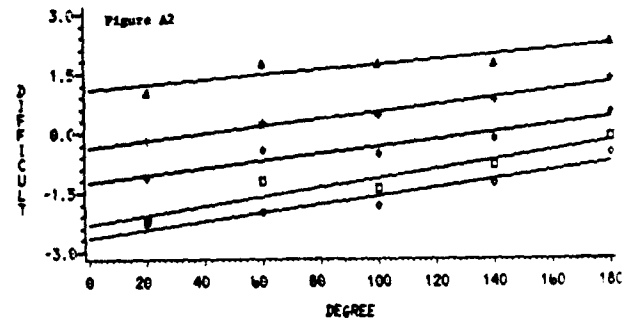
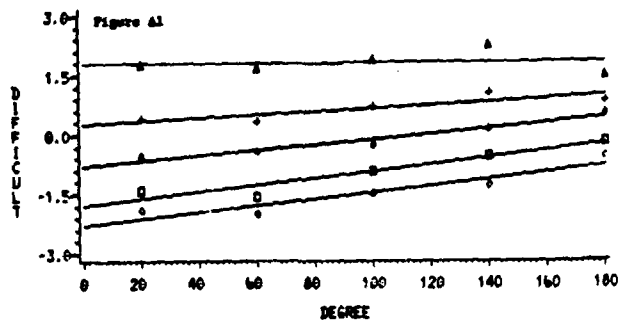


Figure 6  
Relationship Between Difficulty and Angular Disparity for Each  
of the Eight Basic Items as a Function of Time (True Version)



TRIANGLE = 3 SECONDS  
PLUS = 4 SECONDS  
STAR = 5 SECONDS  
SQUARE = 6 SECONDS  
DIAMOND = 7 SECONDS

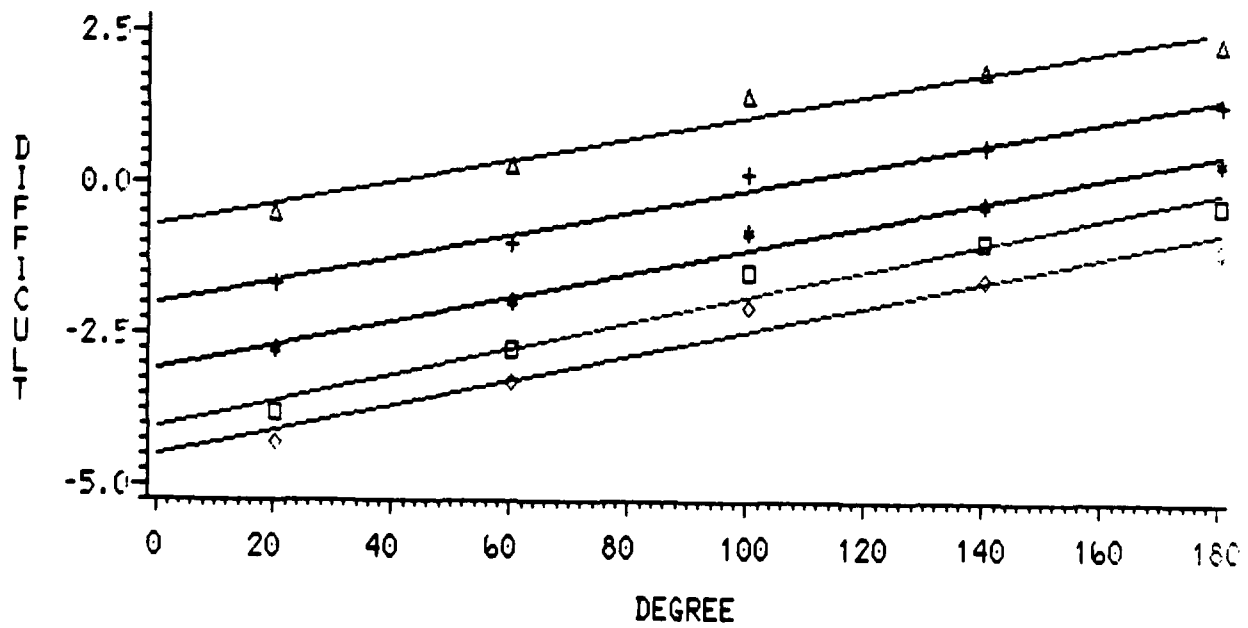
Relationship Between Difficulty and Angular Disparity for Each  
of the Eight Basic Items as a Function of Time (False Version)



TRIANGLE = 3 SECONDS  
PLUS = 4 SECONDS  
STAR = 5 SECONDS  
SQUARE = 6 SECONDS  
DIAMOND = 7 SECONDS

Figure 8

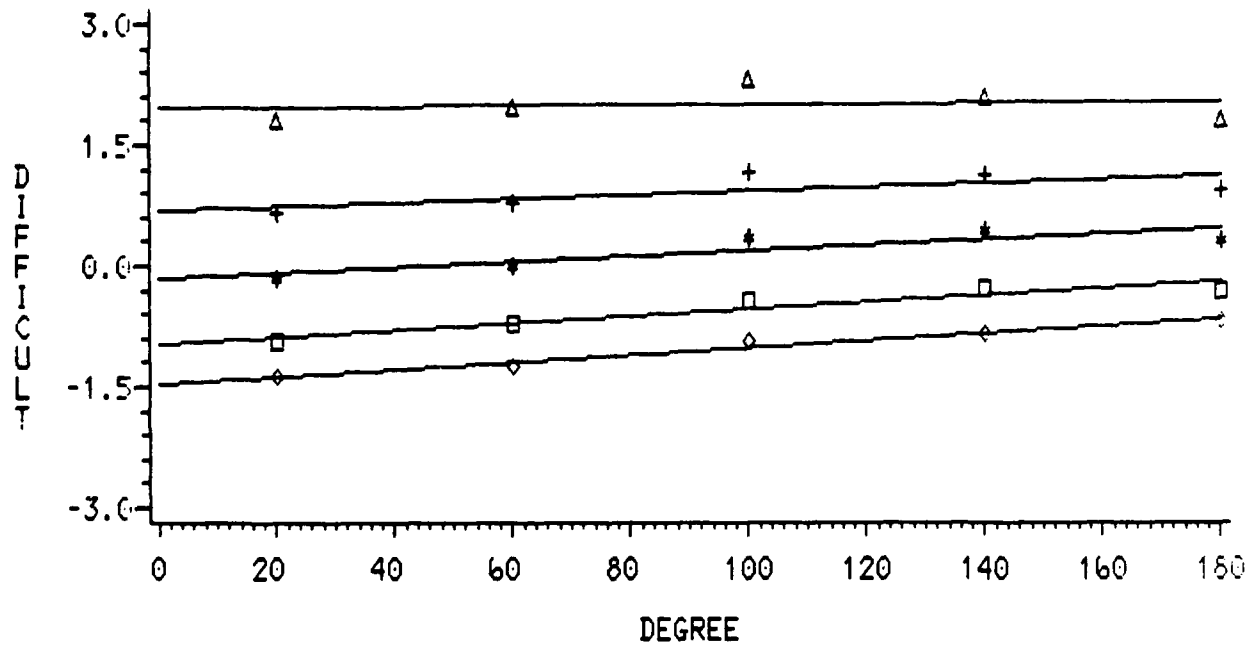
Relationship Between Difficulty and Angular Disparity as a  
Function of Time Averaged Across All Items (True Version)



TRIANGLE = 3 SECONDS  
PLUS = 4 SECONDS  
STAR = 5 SECONDS  
SQUARE = 6 SECONDS  
DIAMOND = 7 SECONDS

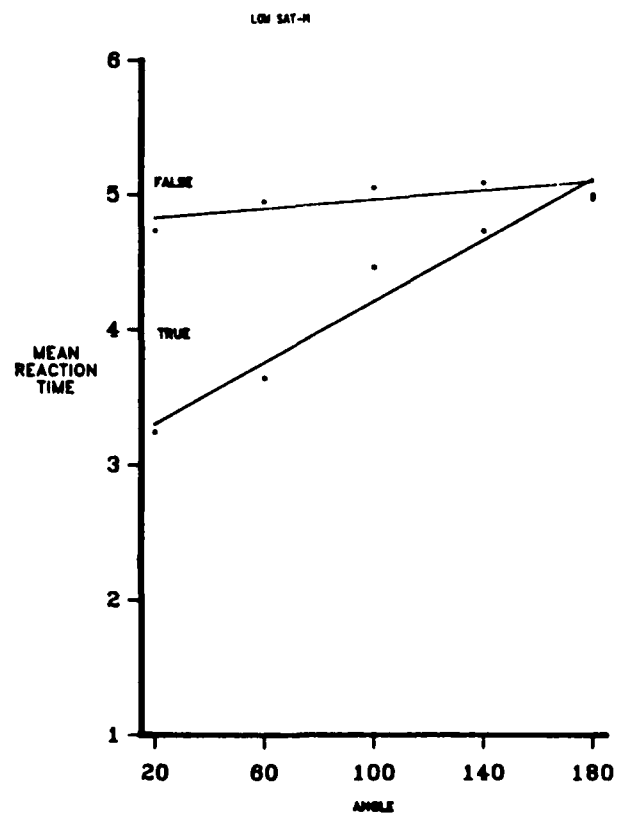
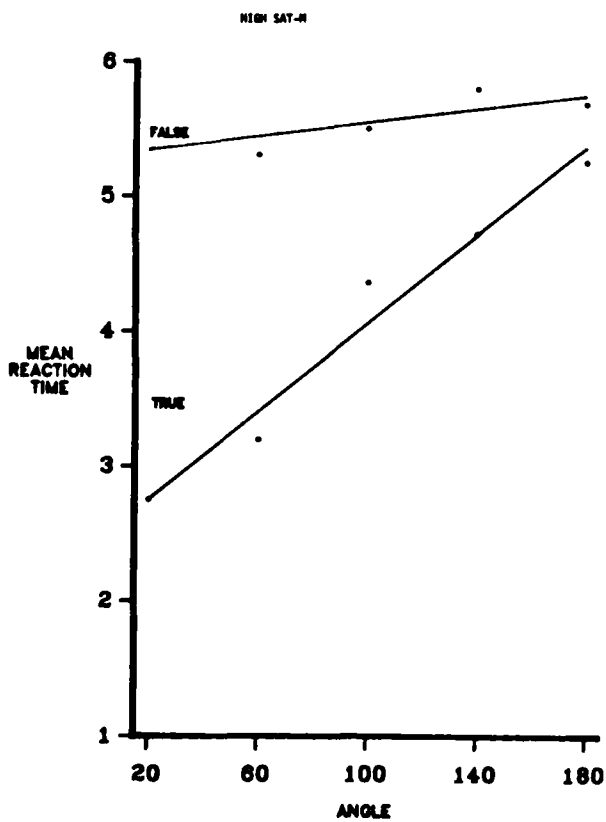
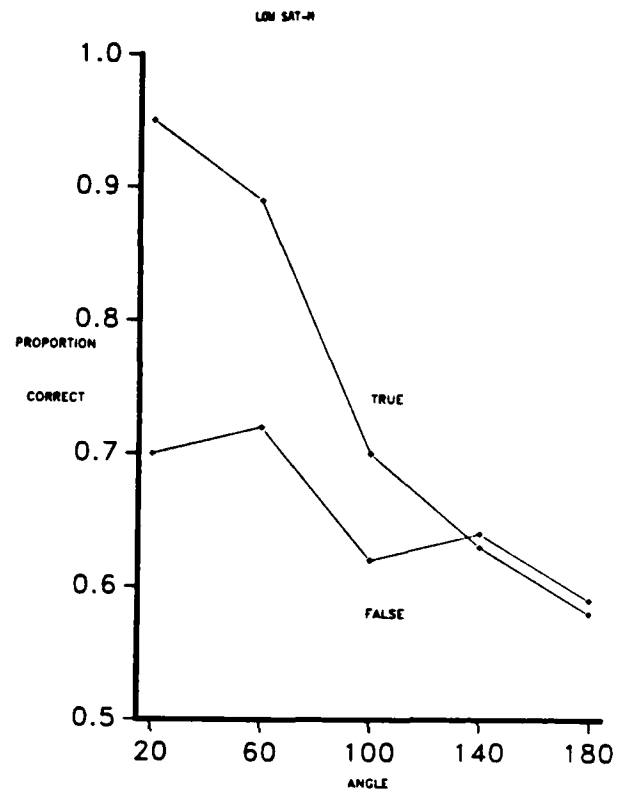
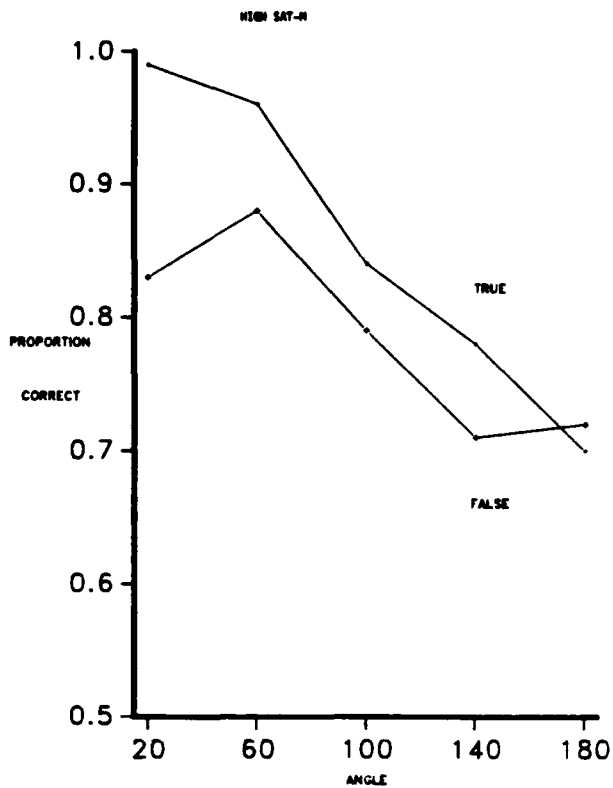
Figure 9

Relationship Between Difficulty and Angular Disparity as a  
Function of Time Averaged Across Eight Items (False Version)



TRIANGLE= 3 SECONDS  
PLUS = 4 SECONDS  
STAR = 5 SECONDS  
SQUARE = 6 SECONDS  
DIAMOND= 7 SECONDS

Relationship Between Proportion Correct and Reaction Time as a Function of Angular Disparity for High and Low Scoring SAT-M Groups





APPENDIX A

To explore the implications of the model it is a matter of substituting values for the parameters. In this appendix we will show the relationship between reaction time and difficulty and show how the slope of the relationship between the response and difficulty can be manipulated by changing accuracy.

The response model is

$$P_z^* = \frac{1}{1 + e^{-Da_i(\theta - b_{z_i})}} \quad [A1]$$

We will interpret  $P_z^*$  as the accuracy and  $z$  as the response time. We wish to observe the response time as a function of accuracy. (Actually, since the model is oriented such that a large response is associated with higher  $\theta$  we will study the relationship with  $i$ - $z$  instead.) For illustration purposes we will assume there are four items of the same discrimination but different difficulty and will examine the results of the model for ability level  $\theta = 1$ . More concretely, assume:

$$a_i = 1.00 \quad \text{for } k=\underline{1}..4$$

$$b_{z_k} = 3z + \phi_k \quad (k=\underline{1}..4)$$

$$\phi_k = .25k - .25 \quad (k=\underline{1}..4)$$

$$\theta = 1.0$$

To simulate different response modes we will vary  $P_z^*$  and observe what happens to  $(1-z)$ . The results for three "response modes" appear in Table A1. Column A has the results for a careless mode. That is the response  $(1-z)$  remains constant as difficulty increases but accuracy

A-2

decreases from .60 to .30. For mode C, however,  $(1-z)$  increases from .75 to .99, and accuracy remains constant. That is, in order to maintain a constant accuracy rate on increasingly difficult items,  $1-z$  must increase.

Table A1

| <u>k</u> | A                         |                         | B                         |                         | C                         |                         |
|----------|---------------------------|-------------------------|---------------------------|-------------------------|---------------------------|-------------------------|
|          | <u><math>(1-z)</math></u> | <u><math>P^*</math></u> | <u><math>(1-z)</math></u> | <u><math>P^*</math></u> | <u><math>(1-z)</math></u> | <u><math>P^*</math></u> |
| 1        | .75                       | .60                     | .75                       | .60                     | .75                       | .60                     |
| 2        | .75                       | .50                     | .79                       | .55                     | .83                       | .60                     |
| 3        | .75                       | .40                     | .83                       | .50                     | .91                       | .60                     |
| 4        | .75                       | .30                     | .88                       | .45                     | .99                       | .60                     |

Figure A1 plots those data. As can be seen the slope is zero for A and highest for C. In short, the slope of reaction time on item attributes is partly a function of the accuracy with which the individual chooses to respond.

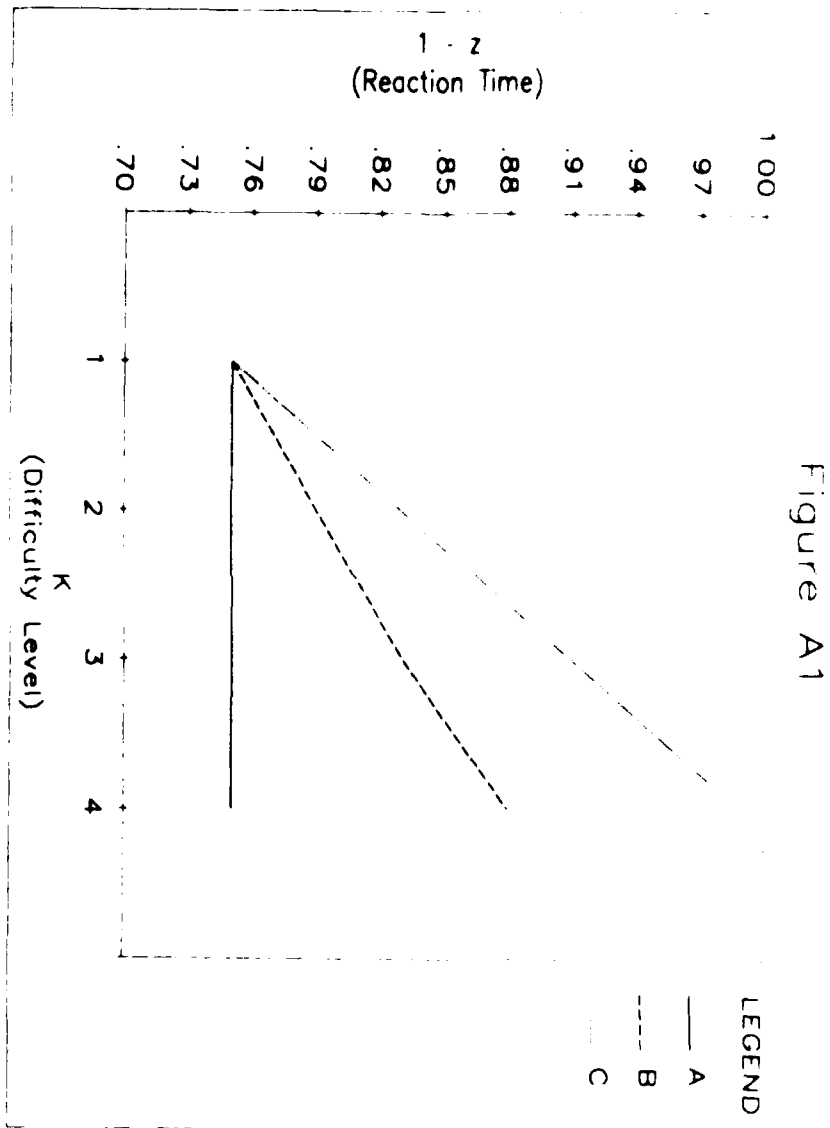
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Insert Figure A1 About Here

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Figure A1

Relationship Between Response Time and Difficulty  
Level for Three Levels of Accuracy



## APPENDIX B

Instruction for Mental Rotation Items

In the following exercises you must respond with the joystick and the red button. To get you used to responding in this way we want you to practice on some samples. In these samples you will see an arrow pointing up or down:

PUSH the stick FORWARD if arrow points UP.

PULL the stick BACK if arrow points down.

RESPOND AS QUICKLY AS YOU CAN.

(Forty arrow items are presented.)

The first kind of exercise you will work on consists of two figures. Your task is to decide whether or not the two figures are the same. It is important to be FAST and CORRECT. Sometimes the two figures will be the same even though the one on the right may be drawn at a different angle. Sometimes the two figures will not be the same.

To show you what we mean, in the following example the two figures are the same but are at a different angle. To see the example press the red button. Each time you press the red button the figure on the right will move closer to the one on the left.

(The subject is able to rotate the figure back and forth.)

If you would like to see this example again push the joystick forward. To see the rest of the instructions press the red button.

You just saw an example of figures that are the same. In the following example the two figures are NOT the same. To see the example press the red button. Each time you press the red button the figure on the right will come closer to the one on the left but they will still not be the same.

If you would like to see this example again push the joystick forward. To see the rest of the instructions press the red button.

First you will take some practice trials. It is important to be FAST and CORRECT. However, you can pace yourself because with the red button you control when to see the next trial. The time you take between trials is not counted.

Respond QUICKLY and CORRECTLY.

PUSH joystick FORWARD

if figures are the SAME.

PULL joystick BACKWARD

if the figures are NOT THE SAME.

Press the red button when you are ready  
for the next trial.

(Six practice items are presented.)



You are now ready for the real trials. Remember!

Respond QUICKLY and CORRECTLY.

PUSH joystick FORWARD

if figures are the SAME.

PULL joystick BACKWARD

if the figures are NOT THE SAME.

Press the red button when you are ready for the  
next trial.

(The 80 items are next)

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